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PS 6
Due 10/23/20
I pledge my honor that I have abided by the Stevens Honor System.

1) G is a CFG in CNF
Terminal symbols length: $n \geq 1$

Say we have a start symbol, "S". We must substitute this using the form $A \to BC$ which adds another non-terminal symbol. If we want to end up with a string of length n, we must subsitute n-1 times using $A \to BC$ form. Once we have all of these symbols we subsitute all (n) of those non-terminal symbols with terminal symbols n times, using $A \to a$ form. Therefore, it takes $n-1+n = \boxed{2n-1}$ steps.

Scanned with CamScanner

2) $G = (V, \Sigma, R, S)$

$V = \{S, T, U\}$

$\Sigma = \{0, \#\}$

R: $S \to TT \mid U$

$T \to 0T \mid T0 \mid \#$

$U \to 0U00 \mid \#$

This language produces strings along 2 different grammars depending on if you choose TT or U from the start symbol. If you choose TT, the language will be $\{0^i \# 0^j \# 0^k : i, j, k \geq 0\}$, and if you choose U, the language will be $\{0^i \# 0^{2i}\ i \geq 0\}$.

Proof G is not regular:

Assume $U = \{0^i \# 0^{2i} : i \geq 0\}$ is regular

Pumping length p

$s = xyz \in A$ $\quad \underbrace{\overbrace{0 \ldots 0}^{k}}_{p} \# \underbrace{10 \ldots\ldots 01}_{2p}$

$|xy| \leq p$ ✓

$|y| \geq 0$ ✓

We let $y = 0^k$ where $1 \leq k \leq p$, and we can say $x = \epsilon$. If we pump up to $xyyz$, the string $\notin U$ because the amount of 0's on the left of the #, would no longer = the length of half the 0's on the right.

3) $\{a^i b^i c^k d^k : i, k \geq 0\} \cup \{a^i b^k c^k d^i : i, k \geq 0\}$

For $\{a^i b^i c^k d^k ; i, k \geq 0\}$

$S \to \epsilon \mid aXbY \mid Y$

$X \to \epsilon \mid aXb$

$Y \to \epsilon \mid cYd$

For $\{a^i b^k c^k d^i : i, k \geq 0\}$

$S \to X$

$X \to aXd \mid bYc \mid \epsilon$

$Y \to bYc \mid \epsilon$

Rules

$S \rightarrow S_1 \mid S_2$

$S_1 \rightarrow \epsilon \mid a X_1 b Y_1 \mid Y_1$

$X_1 \rightarrow \epsilon \mid a X_1 b$

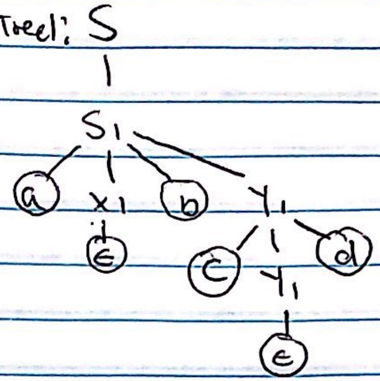
$Y_1 \rightarrow \epsilon \mid c Y_1 d$

$S_2 \rightarrow X_2$

$X_2 \rightarrow a X_2 d \mid b Y_2 c \mid \epsilon$

$Y_2 \rightarrow b Y_2 c \mid \epsilon$

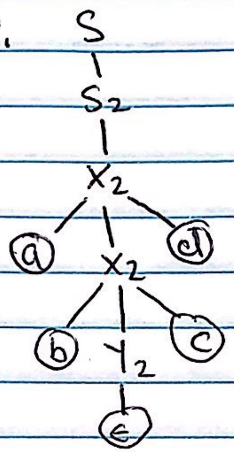
ParseTreeI:



ParseTreeI result

$a \epsilon b c \epsilon d = \boxed{abcd}$

ParseTreeII:



Parse tree II result

$a b \epsilon c d = \boxed{abcd}$

parse tree result 1 = parse tree 2

$\boxed{\text{Ambiguous}}$

4) $L_{add} = \{a^i b^{i+j} c^j : i,j \geq 0\}$

$= \{a^i b^i b^j c^j : i,j \geq 0\}$

CFG:

$S \to XY \mid \epsilon$

$X \to aXb \mid \epsilon$

$Y \to bYc \mid \epsilon$

We see that Ladd is a CFG

$L_{mult} = \{a^i b^{ij} c^j : i,j \geq 0\}$

Pumping Length p.

String S, $|S| \geq p$ $S = uvxyz$, $|uxy| \leq p \wedge |vy| > 0$

then $\exists i : uv^ixy^iz \notin L_{mult}$

Consider string $a^p b^{p^2} c^p$:

$\underbrace{\boxed{a \ldots}}_{p}\ \boxed{a \mid\mid b}\ \underbrace{\boxed{\ldots\ldots b}}_{p^2}\ \boxed{b \mid\mid c}\ \underbrace{\boxed{\ldots c}}_{p}$

5 pieces such that $|uxy| \leq p$

- If we pump only a's, violates PL because the length of a's x length of c's will not = length of b's
- If we pump a's and b's, length of a's x length of c's will not = length of b's
- If we pump only b's, the length of a's x length of c's will not = length of b's
- If we pump b's and c's, length of a's x length of c's will not = length of b's.

- If we pump only c's, then length of a's x length of c's will not = length of b's.

All cases violate PL

If we pump any windows, then $S \notin L_{mult}$.