Eric Rudzin + Shreyas Keerthi
Problem Set 7
CS 334
I pledge my honor that I have abided by the Stevens Honor System.

1) High Level Description:
2PDA of $L_{mult} = \{a^i b^k c^i d^k : i, k \geq 0\}$

Since this is a 2PDA, lets say that we have 2 stacks, say S_1 and S_2

- Start in the start state, read nothing, push a \$ onto S_1 and S_2, move to State 2.
- Loop: read "a" and push "a" onto S_1.
- Once you read a "b", push "b" onto S_2, and move to State 3.
- Loop: For every "b" read, push "b" onto S_2.
- Once you read a "c", pop "a" from S_1, and move to State 4.
- Loop: For every "c" read, pop "a" from S_1.
- Once you read a "d", pop "b" from S_2, and move to State 5.
- Loop: For every "d" read, pop "b" from S_2.
- Enter accept state when S_1 and S_2 contain only "\$" in each.

Problem 2:

2PDA for $L_{mult} = \{a^i b^{ij} c^j : i,j \geq 0\}$

High Level Description: 2 Stacks (S_1 and S_2)

1) Push a \$ onto S_1 and S_2.
2) Begin reading "a"s. For each "a" read, push "a" onto S_1.
3) Now, once we can no longer read "a"s, begin reading "b"s.
4) For every "b" read, pop an "a" from S_1, and push an "a" onto S_2, until there are no more "a"s in S_1.
5) If there are no a's in S_1 (only \$), read nothing and for each "a" in S_2, pop that "a" and push an "a" onto S_1. Once there is only a "c", or a "\$" in S_2, push a "c" on to S_2.
6) Now, repeat steps 4 and 5 until we can no longer read any "b"s. If we can no longer read "b"s, and there are "a"s in both stacks, reject.
7) Once we begin reading "c"s, there should be no "a"s in S_2. Now, for each "c" read, pop a "c" from S_2.
8) Accept if S_2 is empty (has a \$) after all "c"s are read.

3) Prove that the intersection of a CFL and a regular language is always context free.

Let's say L_1 is a DFA that represents a regular language, and L_2 is a PDA that represents a CFL

$L_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$
$L_2 = (Q_2, \Sigma, \Gamma, \delta_2, q_2, F_2)$
$L = L_1 \cap L_2$

We must now make a PDA that runs through L_1 and L_2 in parallel. Call this L.
$L = (Q, \Sigma, \Gamma, \delta, q_0, F)$ where...
$Q = Q_1 \times Q_2$, q_0 occurs at $[q_1, q_2]$ - accept at the same time, and $F = F_1 \times F_2$
(p, q) represents the states of L ("p" being a state of L_1 and "q" being a state of L_2)
We can say the combined transition function is:
$\delta((p,q), x, a) = ((r, \delta_1(q, a)), b)$ if $(r, b) \in \delta_2(q, x, a)$

Since we can successfully represent $L_1 \cap L_2$ as a PDA (L), we see that L is a CFL.

Scanned with CamScanner