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PS-5
I pledge my honor that I have abided by the Stevens Honor System.

1) a) $\{0^i 1^j : i < j\} = A$

Lets call this A and assume it is regular.
By P.L, it has pumping length, say p.

Choose $s \in A$ st. $|s| \geq p$

$\underbrace{0 \ldots 0}_{i}\underbrace{1 \ldots 1}_{j}$ $s = xyz$
$|xy| \leq p$
$|y| > 0$

lets say $i = p$
$j > i$ by some amount, say 1, so let's say $j = p+1$

$\overbrace{\underbrace{0 \ldots 0}_{p}}^{x}\underbrace{1 \ldots 1}_{p+1}$ or $0^p 1^{p+1}$

xy must only be 0's if $|xy| \leq p$
y consists of some # ≥ 1 of 0's
let $y = 0^k$ $1 \leq k \leq p$
$xyyz \notin A$ because $p + k \geq p + 1$
and there must be less 0's than 1's
so this violates pumping lemma because it is stated that $i < j$.

b) $\{0^i 1^j : i > j\} = B$

Let's assume B is regular.
By P.L. it has pumping length, say p.

Choose $s \in B$ s.t. $|s| \geq p$

$\underbrace{0 \ldots 0}_{i}\underbrace{1 \ldots 11}_{j}$

$s = xyz$
$|xy| \leq p$
$|y| > 0$

Let's say $j = p$ and $i = p+1$ as $i > j$

$\underbrace{0 \ldots 0}_{p+1}\underbrace{1 \ldots 1}_{p}$ or $0^{p+1} 1^p$

Say x can be some length of 0s and $y = 0^k$.
$|xy| \leq p$.
Since $|y| \geq 1$ that means $1 \leq k \leq p$
If we pump down to $s = xy^0z$
we get $p+1-k$ 0's and p 1's.
Since we said we have i 0's and j 1's,
and $i > j$ we must say $p+1-k > p$.
Since $k \geq 1$, we see that actually
$p+1-k \leq p$ which contradicts $i > j$.

2) $B = \{0^i 1^j : i \neq j\}$.
lets say $C = \{0^i 1^i : i \geq 0\}$ and we Know C is nonregular.

We can find the complement of B or $\overline{B}$ which is any string not in B. If we intersect $\overline{B}$ with $0^* 1^*$ i.e. $\overline{B} \cap 0^* 1^*$, we get $\{0^i 1^i : i \geq 0\}$ which $= C$.
Through regular operations on B, we got the nonregular language, C, proving that B is non regular.

3) 1) 0001^*

The pumping length is 4 because $|s| \geq p$ and $|xy| \leq p$. Our x would be 000 and we would pump y, or 1^*.

2) $0^* 1^*$

Pumping length of 1 since we can pump y as 0 or 1. Since $|s| \geq p$ and in this case, x and z can be ε the length is 1.

3) $0^* 1^* 0^* 1^* \cup 10^*1$

Pumping length for $0^* 1^* 0^* 1^* = 1$ because we can single out either 0^* or 1^* and make the other parts ε.

pumping length for $10^*1 = 3$ because we must pump 0^* where $s = xyz$, $|s| \geq p$ and $x = 1$; $y = 0$; and $z = 1$.

Because of the union we choose the max pumping length of both individual languages, giving us 3.

4) $(01)^*$

Pumping length of 2. We can say $s = xyz$ where $x = \varepsilon$, $y = 01$, and $z = \varepsilon$. Since $|xy| \leq p$ and $x = \varepsilon$, $y = 2$

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3 continued

5) $1^*01^*01^*$

3 ways to do this by pumping the first, second, or third 1^*. while

$S = xyz$

First 1^*: reducing other 1^*'s to ε

$x = \varepsilon$, $y = 1$, $z = 00$

Second 1^*: reducing other 1^*'s to ε

$x = 0$, $y = 1$, $z = 0$

Third 1^*: reducing other 1^*'s to ε

$x = 00$, $y = 1$, $z = \varepsilon$

In each of these cases $|S| = 3$, because we reduce all 1^*s to ε, except the one we are focusing on. Since $|s| \geq p$ and s is at its minimum length for each scenario, we get a pumping length of 3.

$|xy| \le p$ $L = \{a^i, b^j, c^k : i, j, k \ge 0$ and $i = 1 \Rightarrow$
$|y| > 0$ $j = k\}$
$\forall n \ge 0: xy^n z \in L$ (using n because i is used to express amount of a's)

4) a) $p = 2$

Case 1: $i = 0$
We get a string of any number of b's and any number of c's. $\{b^j c^k : j, k \ge 0\}$
We can say $x = \varepsilon$, $y = b$, and $z = b^{j-1} c^k$
Since $|xy| \le 2$, for $k \ge 0$, $xy^n z = b^n b^{j-1} c^k$

This satisfies $|xy| \le 2$; $|y| > 0$, and we can write this case in $xy^n z$ form as $b^{n+j-1} c^k$ which $\in L$.

Case 2: $i = 1$
We get a string with 1 a and an equal # of b's and c's. $\{a b^j c^j : j \ge 0\}$
We can say $x = \varepsilon$, $y = a$, and $z = b^j c^j$

This satisfies $|xy| \le 2$; $|y| > 0$, and we can write this in $xy^n z$ form as
$a^n b^j c^j \in L$

Case 3: $i > 1$
We get a string of any # of a's, b's, and c's. $\{a^i b^j c^k : i, j, k \ge 0\}$
We can say $x = \varepsilon$, $y = aa$, and $z = b^j c^k$

This satisfies $|xy| \le 2$; $|y| > 0$, and in $xy^n z$ form we get $(aa)^n b^j c^k \in L$

All of these cases satisfy the conditions of the pumping lemma.

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b) $L = \underbrace{b^* c^*}_{A} \cup \underbrace{aaa^* b^* c^*}_{B} \cup \{\underbrace{a b^i c^i : i \geq 0}_{C}\}$

A, B, and C are all disjoint in $L = A \cup B \cup C$ because the amount of a's in each expression varies.

We can say $(A \cup B) \cap C = \{\}$
So $L - (A \cup B) = C$

A and B are both regular, so if L was to be regular, $L - (A \cup B)$ should also be regular. Since $L - (A \cup B) = C$, L is not regular because C is not regular.

c) The pumping lemma does not state that non-regular language can't satisfy the 3 conditions of the lemma. Because the P.L. is used to prove that an assumed regular language is non-regular, this shows that passing a non-regular language through the lemma could still satisfy the conditions.