Response to Referee 2

We thank the referee for reviewing our work and providing their valuable feedback. We are glad that we were able to engage the referee's attention with our presentation. Here we respond in detail to his/her comments.

1. The referee says: *In this paper the authors consider a simple model for a vegetated channel flow and analyse the resulting velocity profile on the basis of a modified Orr-Sommerfeld equation containing both a viscous (Reynolds number) term and a term representing the drag of the vegetated layer at the bottom of the channel. They compare some experimental data from an earlier paper from Nepfs group with the implications of their theory and, via a detailed analysis of the instability, conclude (perhaps not surprisingly) that it differs from traditional KH instability because of the presence of this vegetated drag force.*

   We would like to take this opportunity to clarify our position, as well as our understanding of the literature. Existing literature on synchronous waving of terrestrial and aquatic vegetation repeatedly implicated KH instability as the underlying mechanism, even in the presence of the vegetation. Therefore, we would invoke the flexibility in the referee's comment to argue that, while it is not surprising *a posteriori*, this possibility has not been recognized before in the existing literature.

2. The referee says: *p.1 Fig.1 requires attention. The legend needs to be repositioned for greater clarity (perhaps to the left of the velocity profile).*

   We have accepted the referee's suggestion. The legend is repositioned. We have also made other changes to the figure to improve its clarity.

3. The referee says: *p.2, +6 fitted using the experimental observations.*

   We have corrected the error.

4. The referee says: *p.3, +4 I think x (hat) needs to be defined here (the unit vector?).*

   We now introduce $\hat{\mathbf{x}}$.

5. The referee says: *p.3, §3 Its not quite clear to me how (3.1) leads to U(y)=const. below $y = h_g$. I note (from what follows on p.4) that the shear stress (i.e. U(y) presumably) is matched from upper and lower layers, but this seems insufficient. A little more explanation here would be helpful.*

   This follows from a matched asymptotic analysis of (3.1). For the referee's perusal, we have included a version of it at the end of this response. However, while the method and the solution is not unique (because different solutions may differ in accounting for higher orders), it is a standard procedure in perturbation methods with standard textbooks on the topic. We now cite one such textbook (Hinch, 1991) for readers who may be unfamiliar with the topic.

6. The referee says: *p.4, +1,2 On a related point, why does continuity of shear stresses result in a boundary layer of thickness δ? No natural scale (δ) arises from the solution of (3.1) does it? I cannot see qualitatively what distinguishes this 'boundary layer' from the entire 'shear layer' (indeed, the authors suggest that they are in fact analogous). The layer is (looking at fig. 1) much less well defined than, say, the strong shear layer just above an urban canopy, where the velocity gradient is clearly very much greater than in the boundary layer above or the canopy below.*

   We refer the referee to the solution of (3.1) we present at the end of this response. Many features of this solution, we assume, will be familiar to readers of the *Journal of Fluid Mechanics*. There is a natural boundary layer thickness that arises from the a competition between turbulent viscous stress and vegetation drag. It is this thickness we call δ. The boundary layer on this thickness scale exists inside the vegetation, but not outside (the velocity profile outside is simply parabolic). The shear established in the boundary layer is, indeed, equal to the shear in the unvegetated region, as a consequence of matching the shear stress. However, the shear gradient ($U_{yy}(y)$) in the vegetated region is $O(1/\delta)$ larger than that in the unvegetated region. It is this shear gradient that underlies the traditional picture of KH instability in a shear flow. Furthermore, the presence of the inflection point in the velocity profile, which is often invoked to suggest the presence of an instability, is absent in our profile. Instead the shear gradient switches sign at $y = h_g$.

To clarify the statement in the manuscript about the matching of shear stress determining the boundary layer thickness, we are sure the referee realizes that there are two conditions imposed at $y = h_g$, the continuity of velocity and the shear stress. However, only the continuity of shear stress influences the boundary layer thickness, as can be seen from the solution we present at the end of this response.

7. The referee says: *p.4, +9: It would be better to say the shear layer discussed by Ghisalberti and Nepf (2002, 2004).*

   We thank the referee for pointing this out. We prefer the term "invoked by Ghisalberti and Nepf (2002, 2004)", and assume that this revision is in-line with the referee's suggestion.

8. The referee says: *p.5 +10,11, Although experimentally observed wavelengths are not available, they do arise from the analysis of presumably, so some comment about the values and whether they seem physically realistic (and/or how they compare with the scales of typical motions in the turbulent flow above the canopy which are believed to be linked with monami) would be instructive.*

   We thank the referee for pointing out for the usefulness mentioning the wavelengths of the dominant mode. For parameters corresponding to the lab scale experiments ($4 < H/\delta < 10$ in figure 2 of the manuscript), the dimensionless wavenumber of dominant mode is very close to unity, and therefore the dimensional wavelength is approximately half channel width ($2\pi H$). We have revied the manuscript to say:

   Figure 2 also shows that the predicted dimensionless wavenumber of the dominant mode for parameters corresponding to the lab scale experiments ($4 < H/\delta < 10$) is very close to unity, and therefore the dimensional wavelength is approximately $2\pi H$.

9. The referee says: *The final sentence, if I understand this correctly, implies that all the available data suggests that only mode 1 (i.e. only the left hand panels in fig. 4) is actually relevant to real situations. On the other hand, the statement at the bottom on p.7 (mode 2 is distinct from KH), and the conclusions, seem together to imply that the observed monami are mode 2. I became confused at this point and would welcome greater clarity. And if mode 2 is not observed in experiments, the extended analysis of its character is perhaps not particularly relevant and should be shortened.*

   We understand that our presentation was not perfect, and have now clarified the confusion. The fact of the matter is that it is taken for granted based on superficial similarities with the KH instability, that *monami*is based on KH instability. However, a careful analysis is needed to determine if it is really the case. We have presented such an analysis, and find that the picture is not simple. We find that there are two kinds of unstable modes for large vegetation density, and they merge together for smaller vegetation density. Both mechanisms of instability are presumably active when the two modes appear merged. All experiments lie in this regime where the vegetation density is small. Therefore, we are unable to distinguish based on the experimental observations and our analysis, if it is Mode 1 or Mode 2 that dominates in region where the two modes merge. Future experiments may be cleverly designed to resolve this ambiguity.

   To clarify this picture, we now say in the manuscript:

   All experimental data we have found corresponds to a vegetation density for which the unstable region in the $R - k$ space has not split into two, so we are unable to determine if flow instability in the lab scale experiments (Ghisalberti & Nepf, 2002) are due to Mode 1 or Mode 2.

   and in the conclusion:

   We are unable to determine based on observations, and therefore have refrained from identifying, which mode is observed in experiments and it still remains a subtle question and subject of future investigation. Since the two modes merge for the experimental parameters, KH may not be assumed to underlie *monami*.

10. The referee says: *P.4 +8: It's not clear how a boundary layer thickness can be equated to a velocity ratio! This comment also appears in the caption of fig.1. Furthermore, the caption states that the profile is from Ghislaberti and Nepf - case I. This case has H=12.3 cm and hg =9 cm and the caption says that δ = 5.02 cm. This implies an H/δ rather lower than any of the experimental data shown in fig.2 (left), which seems inconsistent.*

    Since the shear stress U_y is continuous across the grass tip we can equate the scaling estimate of shear stress just above the grass (h_g^+) with that of just below the grass (h_g^-). Above the grass tip the base flow is a parabolic velocity profile and the estimate of U_y at the grass tip is U_0/H, whereas below the grass tip the shear stress can be estimated to be U_{bl}/δ, equating the shear stress above the grass and below the grass gives us $\delta/H = U_{bl}/U_0$.

    We think referee has understood the case-I mentioned for Figure 1 as the one from 2002 paper (Mixing layer and coherent structures in vegetated aquatic flows, *J. Geophys. Res. 107*), whereas we are referring to the case-I from the 2004 paper (The limited growth of vegetated shear layers, *Water Resource Research 40(7)*). In this case the channel width is $41cm$, implying $H = 20.5cm$ hence $H/\delta = 4.02$. We would also like to point out that we have used a logarithmic scale on y-axis in Figure 2, and indeed $H/\delta \approx 5$ for all the experimental observation shown in Figure 2.

11. The referee says: *p.5, -9: 'boundary layer, the' (i.e. singular).*

    We have accepted the referee's suggestion.

12. The referee says:

    Fig 2 &3: In both captions a value for the eddy viscosity is given. Was there any physical basis for this value? If not, how was it chosen? And how critical are the comparisons between theory and experiment to its value?

    The constant eddy viscosity of 0.1 Pa s is chosen based on eddy viscosity profile shown in Figure 5 for lab scale experiments of Ghisalberti and Nepf (2004). As the referee realized, the exact value of the eddy viscosity varies with depth in the water column. But overall, the eddy viscosity arises from the eddies the vegetation generates. The scale of velocity for these eddies is the free stream speed ($U \sim 0.1$ m/s), and the size of the eddies is the width of the grass blade ($d \sim 0.01$ m). According to Ghisalberti and Nepf (2004), the eddy viscosity is approximately $\mu \approx 0.1\rho U d = 0.1$ Pa s). The Reynolds number R is the only parameter that depends on the eddy viscosity. Any uncertainty in the eddy viscosity translates into an uncertainty in the critical Reynolds number for the onset of the instability. **Any changes to the manuscript.**

    We have now revised the manuscript to include this discussion. The manuscript now reads:

    The measured values of eddy viscosity vary with depth in the water column (Ghisalberti & Nepf, 2004); we use $\mu = 0.1$ Pa s as a representative value from this range for comparison with experiments.

    and we have removed the statement of the value of eddy viscosity from the captions of figures 2 and 3.

Matched asymptotic solution

Here we present an asymptotic solution of equation (3.1) for $N_g \gg 1$, which should be helpful in clarifying many questions the referee has related to the boundary layer near the grass top. For clarity of presentation in this particular calculation we assume the grass to extend from $y = 0$ to $y = H$, i. e. the submergence ratio is 0.5. Using H as length scale, and $U_0 = (-dP/dx)H^2/\mu$ as velocity scale, we rescale $U = U_0\bar{U}$ and $y - H = H\bar{y}$. Equation (3.1) is then transformed to

$$\begin{aligned} \bar{U}_{\bar{y}\bar{y}} + 1 - R\tilde{N}_g\bar{U}^2 = 0, &\quad \text{for } -1 < \bar{y} < 0 \\ \bar{U}_{\bar{y}\bar{y}} + 1 = 0, &\quad \text{for } 1 > \bar{y} > 0. \end{aligned} \tag{1}$$

For notational convenience, we will drop the overbar and define a drag parameter as $D_g = R\tilde{N}_g$ for $-1 < y \le 0$, $D_g = 0$ for $0 < y \le 1$, grass top is at $y = 0$ and the bottom surface is at $y = -1$. Equation (3.1) now reads

$$U_{yy} + 1 - D_g U^2 = 0. \tag{2}$$

We use the zero shear boundary condition on both the boundaries ($y = -1, 1$) as mentioned in the manuscript.

Asymptotic Solution of 3.1 below the grass

We observe that $U = \sqrt{1/D_g}$ is a stationary point of above equation, where both $U_{yy} = 0$ and $U_y = 0$. By multiplying with U_y and integrating once, we get

$$\frac{1}{2}\left(\frac{dU}{dy}\right)^2 + U - \frac{1}{3}D_g U^3 + C = 0, \tag{3}$$

where C is a constant of integration, which can be determined by observing that as U approaches $1/\sqrt{D_g}$, U_y approaches zero. Applying this condition gives $C = -\dfrac{2}{3\sqrt{D_g}}$. Using the value of C, (3) becomes

$$\frac{dU}{dy} = \sqrt{\frac{2}{3}D_g U^3 + \frac{4}{3\sqrt{D_g}} - 2U} \tag{4}$$

We further use the substitution $U = \dfrac{1}{\sqrt{D_g}}u$ in the above equation, which simplifies it to

$$\frac{du}{(u-1)\sqrt{u+2}} = D_g^{1/4}\sqrt{\frac{2}{3}}dy \tag{5}$$

Above equation can be integrated to obtain the solution in the vegetated region as

$$\left.\begin{aligned} u &= 3\coth\left(\frac{C_1 - yD_g^{1/4}}{\sqrt{2}}\right)^2 - 2 \\ U &= \frac{1}{\sqrt{D_g}}\left(3\coth^2\left(\frac{C_1 - yD_g^{1/4}}{\sqrt{2}}\right) - 2\right) \end{aligned}\right\} \quad \text{for } -1 \le y \le 0, \tag{6}$$

where C_1 is constant of integration, which can be found by matching the shear stress applied by the flow above the grass at the tip of the grass. Deeper inside grass ($h_g - y \gg \delta$) the solution saturates to $U(y) = \frac{1}{\sqrt{D_g}}$ independent of the value of C_1 due to the dominant balance between drag and pressure gradient.

Solution above the grass

Above the grass, the dimensionless form of equation (3.1) translates to

$$U_{yy} + 1 = 0 \tag{7}$$

which can be integrated together with the zero shear boundary condition at top to provided

$$U(y) = y - y^2/2 + C_2 \qquad \text{for } 0 < y < 1 \tag{8}$$

Where C_2 is constant of integration.

Matching the solution obtained by (6) **and** (8)

The value of C_1 and C_2 can be determined by matching the solution (6) and (8) at the grass tip. Matching the shear stress at the grass top results in

$$\frac{3\sqrt{2}}{D_g^{1/4}} \coth\left(\frac{C_1}{\sqrt{2}}\right) \operatorname{csch}^2\left(\frac{C_1}{\sqrt{2}}\right) = 1; \tag{9}$$

Above equation can be solved numerically, but here we will solve it asymptotically for $D_g \gg 1$. We expect $C_1 \ll 1$ in this limit, and the terms in (9) may be expanded in this limit to get

$$\begin{gathered} \frac{3\sqrt{2}}{D_g^{1/4}} \left(\frac{\sqrt{2}}{C_1}\right)\left(\frac{2}{C_1^2}\right) = 1 \\ C_1 = \frac{(12)^{1/3}}{D_g^{1/12}}, \end{gathered} \tag{10}$$

consistent with our expectation. The boundary layer length scale may be derived from examining the scale of y for which the argument of (6) changes asymptotic order. This examination implies $C_1 \sim O(yD_g^{1/4})$ or $y = O(D_g^{-1/3})$. The scale for velocity in the boundary layer is the value of (6) at $y = 0$.

$$U_{\text{top}} = \frac{1}{\sqrt{D_g}}\left(3\coth^2\left(\frac{C_1}{\sqrt{2}}\right) - 2\right) \tag{11}$$

$$\approx \frac{1}{\sqrt{D_g}}\left(\frac{6}{C_1^2} - 2\right) \tag{12}$$

$$\approx \frac{1}{\sqrt{D_g}} \frac{6D_g^{1/6}}{12^{2/3}} \tag{13}$$

$$\approx \left(\frac{3}{2}\right)^{1/3} D_g^{-1/3}. \tag{14}$$

We have employed one of the many possible methods to understand the structure of the solution of (3.1); other methods can also be used to reach the same conclusions. In the manuscript we have presented the outline of an essentially identical method. For a general treatment of the topic, we refer the referee to the book by Hinch (1991).

References

GHISALBERTI, M. & NEPF, H. M. 2002 Mixing layer and cohrent structures in vegetated aquatic flows. *J. Geophys. Res.* **107**.

GHISALBERTI, M. & NEPF, H. M. 2004 The limited growth of vegetated shear layers. *Water Resources Research* **40** (7).

HINCH, E. J. 1991 *Perturbation methods*. Cambridge university press.