SC 625 - Systems Theory

Assignment 4

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Problem 1.

Solution. □

We can prove that (A, B) is controllable and observable using standard PBH tests where we get rank of observablity matrix and controllability matrix is 4.

$$\mathcal{C} = \begin{bmatrix} 0 & 1 & 0 & g \\ 1 & 0 & g & 0 \\ 0 & 1 & 0 & 2g \\ 1 & 0 & 2g & 0 \end{bmatrix}$$

$$\mathcal{O} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & g & 0 \\ 0 & 0 & 0 & g \end{bmatrix}$$

Thus, we find a K such that $A + BK$ is exponentially stable. Let us take $A_c = -\mu I - A$ for some $\mu > 0$. Thus, we can find a W_c such that

$$A_c W_c + W_c A_c^T = -BB^T$$

where $W = \int_0^\infty e^{A_c t} BB^T e^{A_c^T t} dt$

Using these we can conclude that $\mu > \sqrt{2g}$. Choosing $\mu = 5$, we get the following:

$$W_c = \begin{bmatrix} 0.0044 & -0.0219 & 0.0062 & -0.0325 \\ -0.0219 & 0.1578 & -0.0295 & 0.2034 \\ 0.0062 & -0.0295 & 0.0093 & -0.0465 \\ -0.0325 & 0.2034 & -0.0465 & 0.2823 \end{bmatrix}$$

Hence, we choose $K = -\frac{1}{2} B^T W_c^{-1}$, to get:

$$K = \begin{bmatrix} 409.684 & 183.8736 & -709.684 & -203.8736 \end{bmatrix}$$

We follow the similar process for observabililty using the duality that (A^T, C^T) is controllable if (C, A) is observable. Hence, we can find an L such that $A^T + L^T C^T$ is exponentially stable. Thus, we get the Lyapunov equation as $A_c^T W_o + W_o A_c = -C^T C$ where $W_o = \int_0^\infty e^{A_c^T t} C^T C e^{A_c t} dt$ Thus,

$$W_o = \begin{bmatrix} 0.1 & -0.01 & 0.0122 & -0.0012 \\ -0.01 & 0.002 & -0.0043 & 0.0005 \\ 0.0122 & -0.0043 & 0.0334 & -0.0064 \\ -0.0012 & 0.0005 & -0.0064 & 0.0013 \end{bmatrix}$$

Thus, we choose $L = -\frac{1}{2}CW^{-1}$ to get:

$$L = \begin{bmatrix} -20 \\ -300 \\ -223.8736 \\ -1009.684 \end{bmatrix}$$

Finally, our output feedback controller required an observer. Putting it all together we get the following standard equations:

$$\dot{\hat{x}} = A\hat{x} + Bu + L(C\hat{x} - Cx)$$

$$u = K\hat{x}$$

Plotting the results:

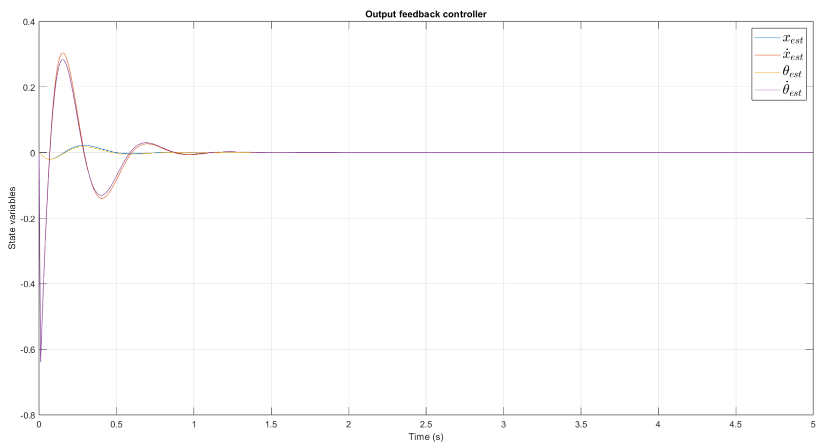

Figure 1: Initial conditions $x(0) = 0, \dot{x}(0) = 0, \theta(0) = 0.1, \dot{\theta}(0) = 0$

This is very similar to the results of the state-feedback controller we have seen earlier.

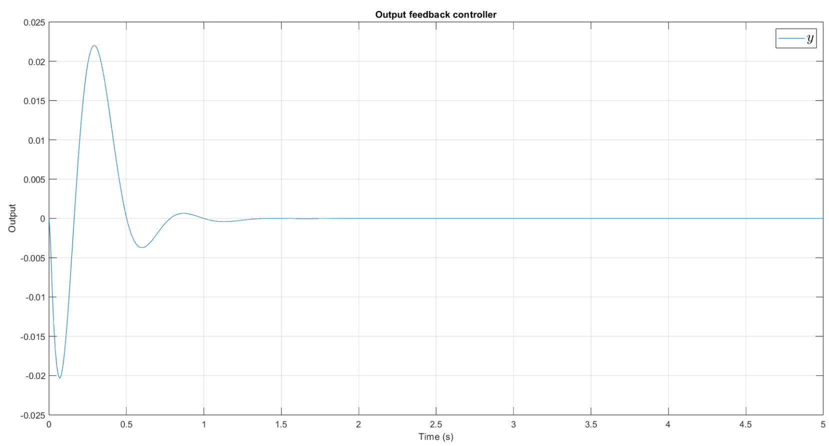

Figure 2: Output $y(t) = Cz(t)$

Problem 2.

Solution. $\square$

The nonlinear ODEs did not give implicit or explicit solutions in MATLAB. The code for this has been attached.