Robust Least-Squares Optimization for Data-Driven Predictive Control

Submitted in the partial fulfillment of the requirements of the degree of

Master of Technology

by

Shreyas N. B.

(Roll No: 210010061)

Supervisors:

Prof. Ravi Banavar

Prof. Cyrus Mostajeran

Prof. Bamdev Mishra

Center for Systems and Controls

INDIAN INSTITUTE OF TECHNOLOGY BOMBAY 2025

(Draft - October 19, 2025)

Approval Sheet

This thesis entitled *Robust Least-Squares Optimization for Data-Driven Predictive Control* by **Shreyas N. B.** is approved for the degree of *Master of Technology*.

Examiner

____

____

____

Supervisor (s)

____

____

____

Chairperson

____

Date: ____________________

Place: ____________________

Declaration

I declare that this written submission represents my ideas in my own words and where others' ideas or words have been included, I have adequately cited and referenced the original sources.

I also declare that I have adhered to all principles of academic honesty and integrity and have not misrepresented or fabricated or falsified any idea/data/fact/source in my submission.

I understand that any violation of the above will be cause for disciplinary action by the Institute and can also evoke penal action from the sources which have thus not been properly cited or from whom proper permission has not been taken when needed.

I declare that I have not used any of the modern AI tools or any other similar tools for writing the thesis.

____
(Signature)

____
(Name of the student)

____
(Roll No.)

Date: ____________________

Abstract

This thesis introduces a new framework for addressing a geometrically robust least-squares optimization problem, developed in the context of finite-time, data-driven predictive control. Traditional least-squares methods, while foundational in system identification and estimation, often struggle to maintain performance in the presence of model uncertainty or noisy data. To address this, the proposed formulation embeds robustness directly into the optimization process, rather than treating it as an external correction or regularization term. The central idea is to reinterpret the least-squares problem through a geometric lens and formulate it as a minimax problem on a product manifold, allowing for a principled treatment of uncertainty and nonlinearity.

The core formulation considers two sets of variables: one representing the decision variable of interest, and the other representing uncertainty, bounded within a geometric constraint described as a ball. This ball constraint captures possible perturbations or variations in the data, which can arise from measurement noise, modeling errors, or unmodeled system dynamics. By doing so, the method directly encodes robustness against such variations into the optimization problem itself. The resulting minimax structure can be interpreted as the controller or estimator seeking a solution that minimizes the worst-case residual error induced by the uncertainty. This formulation thus bridges ideas from robust optimization, geometric control, and estimation on manifolds.

A key theoretical contribution of this work lies in the explicit solvability of the inner maximization problem. Despite the high-level geometric structure, the maximization over the uncertainty variable admits a closed-form expression, simplifying the overall computation and enabling efficient implementation. This property distinguishes the approach from conventional robust least-squares methods that often rely on iterative or conservative approximations to handle uncertainty. By leveraging the geometry of the manifold and the symmetry of the ball constraint, the inner problem collapses into a tractable form that preserves interpretability while ensuring robustness.

When applied to data-driven predictive control, the proposed method demonstrates strong performance, particularly for linear time-invariant (LTI) systems whose dynamics are not explicitly known but can be inferred from data. Under mild assumptions of controllability and observability, the algorithm is able to generate predictive control inputs that stabilize the system and track desired trajectories effectively. The finite-time formulation ensures that the optimization remains computationally feasible for online implementation, an essential property for real-time control applications. The method's ability to integrate data-driven modeling with geometric robustness makes it particularly suitable for scenarios where accurate models are unavailable or costly to obtain, such as in aerial robotics, autonomous systems, and complex mechanical structures.

Beyond its direct application to predictive control, this thesis contributes conceptually to the intersection of geometry and optimization in control theory. By formulating the problem on a product manifold, it emphasizes the role of intrinsic structure in ensuring stability and convergence properties, while the minimax perspective naturally connects to ideas from game theory and robust estimation. Overall, the work provides both theoretical insight and practical algorithms for robust, geometry-aware control, advancing the broader goal of reliable decision-making from uncertain data.

The thesis is organized as follows: Chapter 1 introduces the problem context and motivation, Chapter 2 performs an extensive literature review of related work, Chapter 3 presents the mathematical preliminaries and necessary background, Chapter 4 details the proposed robust least-squares formulation and its theoretical properties, Chapter 5 discusses the application to data-driven predictive control, Chapter 6 provides numerical simulations and results, and finally, Chapter 7 concludes with a summary of contributions and directions for future research.

Contents

1 Introduction

The objective of this thesis is to study the application of robust least-squares optimization in the context of data-driven predictive control. The fundamental idea lies in the behavioral approach to systems theory, which allows us to represent a dynamical system purely based on its observed input-output data, without requiring an explicit parametric model. This approach is particularly useful in scenarios where the underlying system dynamics are complex or unknown.

Least-squares optimization is a classical problem which has been utilized across many domains in science and engineering over the past few centuries. The earliest documented usage of the problem can be traced back to Legendre (1805) [1], where it was described as an algebraic procedure for fitting linear equations to data. Legendre demonstrated the new method by analyzing the same data as Laplace for the shape of the Earth. On the other hand, Gauss (1809) [2] went beyond Legendre and succeeded in connecting the method of least-squares with the principles of probability. The least-squares technique soon became an indispensable tool in astronomy, geodesy and laid the foundation of many core concepts in modern engineering problems.

1.1 Problem Overview

Robust Least-Squares

The least-squares problem is most commonly seen in the form shown below:

$$\min_{x\in\mathbb{R}^n} \|Ax - b\|_2^2, \tag{1.1}$$

where $A \in \mathbb{R}^{m\times n}$ is a matrix of coefficients, $b \in \mathbb{R}^m$ is a vector of observations, and $x \in \mathbb{R}^n$ is the vector of unknowns to be determined. Thus, by minimizing the residual $\Delta b = Ax - b$, it determines a solution that closely matches b in the Euclidean 2-norm. As the domain of applied statistics progressed, the total least squares problem [3] was studied, which considered perturbations in both dependent and independent variables, i.e., $(\Delta A, \Delta b)$. The optimal solutions may be sensitive to perturbations in the data (A, b). One way to mitigate this is to consider the robust least squares problem:

$$\min_{x\in\mathbb{R}^n} \max_{A\in\mathbb{B}_\rho^{\mathrm{F}}(\hat{A})} \|Ax - b\|_2^2, \tag{1.2}$$

where $\mathbb{B}_\rho^{\mathrm{F}}(\hat{A}) = \{A \in \mathbb{R}^{m\times n} : \|A - \hat{A}\|_2 \leq \rho\}$ is a ball centered around $\hat{A}$ with radius ρ, endowed with the Frobenius norm.

Building on these ideas, alternative perturbation models have been explored, yielding different robust versions of the least-squares problem. For instance, El Ghaoui and Lebret (1997) [4] considered the robust least squares problem

$$\min_{x\in\mathbb{R}^n} \max_{[A\ \ b]\in\mathbb{B}^{\mathrm{F}}_{\rho}([\hat{A}\ \ \hat{b}])} \|Ax-b\|_2^2. \tag{1.3}$$

This approach is motivated by scenarios where the exact data (A, b) are unknown, but belong to a family of matrices $(\hat{A}+\Delta A, \hat{b}+\Delta b)$ and the residual $[\Delta A\ \Delta b]$ lies in a norm-bounded matrix ball.

In this thesis, we introduce a robust optimization framework that accounts for the geometric nature of perturbations found in diverse instances of the problem. Specifically, we consider the optimization problem,

$$\min_{x\in\mathbb{R}^n} \max_{\mathcal{S}\in\mathbb{B}^{d}_{\rho}(\hat{\mathcal{S}})} \|P_{\mathcal{S}}x-b\|_2^2, \tag{1.4}$$

where $P_{\mathcal{S}}$ is the orthogonal projection onto the k-dimensional subspace $\mathcal{S}$ of $\mathbb{R}^n$, and $\mathbb{B}^d_\rho(\hat{\mathcal{S}})$ is a ball centered at $\hat{\mathcal{S}}$ with radius ρ defined by the metric d on the *Grassmannian* $\mathrm{Gr}(k,n)$, which is the set of all k-dimensional subspaces in $\mathbb{R}^n$ endowed with the structure of a smooth Riemannian manifold. This approach is motivated by a diverse range of applications where the linear model A is a matrix representation of a subspace subject to bounded perturbations (due to uncertainty or approximations errors), quantified naturally in terms of distances between subspaces.

Data-Driven Predictive Control

The availability of large datasets coupled with unprecedented storage and computational power has recently reignited interest in direct data-driven control methods, which aim to infer optimal decisions directly from measured data (bypassing system identification). At the heart of this emerging trend lies the behavioral approach to system theory (Willems, 2007 [5]) and a seminal result by Willems and his collaborators [6], commonly referred to as the *fundamental lemma*. The lemma establishes that finite-horizon behaviors of Linear Time-Invariant (LTI) systems can be represented as images of raw data matrices.

The proposed idea behind data-driven predictive control via the geometric approach is posed as:

(1) Collect a single input-output trajectory of the system, denoted by $\{u_k, y_k\}_{k=0}^{N-1}$, where $u_k \in \mathbb{R}^m$ and $y_k \in \mathbb{R}^p$ are the input and output at time step k, respectively. Store trajectories as $w_k = [u_k^\top\ y_k^\top]^\top \in \mathbb{R}^q$ where $q = m + p$.

(2) Construct the Hankel matrix of depth L from the collected data:

$$\mathcal{H}_L(w) = \begin{bmatrix} w_0 & w_1 & w_2 & \cdots & w_{N-L} \\ w_1 & w_2 & w_3 & \cdots & w_{N-L+1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ w_{L-1} & w_L & w_{L+1} & \cdots & w_{N-1} \end{bmatrix} \in \mathbb{R}^{qL\times(N-L+1)}.$$

(3) According to the fundamental lemma, if the input sequence $\{u_k\}$ is persistently exciting of order $L+n$ (where n is the order of the system), then any trajectory of length L can be expressed as a linear combination of the columns of $\mathcal{H}_L(w)$. In other words, the behavior $\mathfrak{B}_L$ of the system over a horizon L is given by:

$$\mathfrak{B}_L = \operatorname{im}(\mathcal{H}_L(w))$$

We identify $\mathfrak{B}_L$ as a k-dimensional subspace of $\mathbb{R}^{qL}$, where $k \leq qL$ is the rank of $\mathcal{H}_L(w)$. Thus, it is an element of the Grassmannian $\operatorname{Gr}(k, qL)$, denoted as $\mathcal{S} \equiv \mathfrak{B}_L$.

(4) At each time step t, solve a constrained least-squares problem to find the optimal trajectory that minimizes the cost function while satisfying the system's behavior. The optimization problem can be formulated as:

$$\min_{x \in \mathbb{R}^{qL}} \max_{\mathcal{S} \in \mathbb{B}_\rho^d(\hat{\mathcal{S}})} \|P_{\mathcal{S}} x - b\|_2^2, \quad \text{s.t.} \quad P_{\mathcal{S}} x \in \mathfrak{B}_L$$

As a result, various data-driven modeling, estimation, filtering, and control problems can be formulated as weighted or constrained least squares problems (Markovsky and Dörfler, 2021 [7]). Moreover, being finite-dimensional subspaces, finite-horizon LTI behaviors can be identified with points on the Grassmannian $\operatorname{Gr}(k, n)$ and uncertainty can be naturally quantified using Grassmannian (subspace) metrics. This approach has demonstrated its effectiveness in data-driven mode recognition and control applications and shows promise to open new avenues in adaptive control (Padoan et al., 2022 [8]).

In summary, the main focus of this thesis is to explore the robust least-squares problem with subspace uncertainty and its application in data-driven predictive control via behavioral systems theory. We investigate the theoretical foundations of this approach, develop efficient algorithms for solving the robust optimization problem, and demonstrate its effectiveness through numerical simulations and real-world applications. The ultimate goal is to provide a geometric approach to robust and reliable framework for data-driven control that can handle uncertainties in the system dynamics and improve the performance of control systems in practice.

2 Literature Survey

Remark 1. This chapter is numbered (or perhaps more precisely "lettered"). This means that is appears in Table of Contents with its letter "**N**", which also prefixes all numbering of environments in this chapter.

On the other hand, Introduction →p.1 and ?? →p.?? are unnumbered (or "unlettered") in this sense.

Example 2.0.1 (Usage of Mathematical Fonts). To make the text more readable and beautiful, we can use different types of mathematical fonts for different types of objects (striving to be at least somewhat consistent):

- $\boldsymbol{Bold}$ often for tensorial object (abstract index).
- Sans for groups, certain spaces, or some operations/maps.
- $\mathfrak{Fraktur}$ for algebras (and densities).
- $\mathcal{Calligraphic}$ (available are only capital letters, and ℓ)
- $\mathscr{Calligraphic}$ (alternative font containing also lowercase letters)
- $\mathbb{Double\text{-}Struck}$ for fields like $\mathbb{R}$, spaces like $\mathbb{S}^n$ and $\mathbb{CP}^n$.
- `Typewriter` for code functions, or other special objects.

Example 2.0.2. You can use ◌ as an argument placeholder.

3 Preliminaries

This chapter introduces mathematical concepts and notations related to optimization theory, behavioral approach to systems theory, and data-driven predictive control that will be used throughout this thesis.

3.1 Geometry

In this section, we briefly review some fundamental concepts from Riemannian geometry and optimization on manifolds. For a more comprehensive treatment, the reader is referred to [9] [10].

Euclidean Spaces

Definition 3.1.1 (Inner Product). An inner product on a real vector space $\mathcal{E}$ is a function $\langle\cdot,\cdot\rangle : \mathcal{E}\times\mathcal{E}\to\mathbb{R}$ that satisfies the following properties for all $u, v, w \in \mathcal{E}$ and $a, b \in \mathbb{R}$:

- Symmetry: $\langle u, v\rangle = \langle v, u\rangle$,
- Linearity: $\langle au + bv, w\rangle = a\langle u, w\rangle + b\langle v, w\rangle$,
- Positive-definiteness: $\langle u, u\rangle \geq 0$ and $\langle u, u\rangle = 0 \iff u = 0$.

Definition 3.1.2 (Euclidean Space). A linear space $\mathcal{E}$ equipped with an inner product $\langle\cdot,\cdot\rangle$ is called a Euclidean space. An inner product induces a norm on $\mathcal{E}$ called the Euclidean norm:

$$\|u\| = \sqrt{\langle u, u\rangle}, \quad \forall u \in \mathcal{E}.$$

The standard inner product on $\mathbb{R}^n$ and the associated norm are given by:

$$\langle u, v\rangle = u^\top v, \quad \|u\|_2 = \sqrt{u^\top u}, \quad \forall u, v \in \mathbb{R}^n. \tag{3.1}$$

Similarly, the standard inner product on the space of real matrices $\mathbb{R}^{n\times k}$ is the Frobenius inner product, with the associated Frobenius norm:

$$\langle A, B\rangle = \mathrm{Tr}(A^\top B), \quad \|A\|_\mathrm{F} = \sqrt{\mathrm{Tr}(A^\top A)}, \quad \forall A, B \in \mathbb{R}^{n\times k}, \tag{3.2}$$

where $\mathrm{Tr}(M) = \sum_i M_{ii}$ denotes the trace of a matrix. We often use the following properties of the above inner product, with matrices U, V, W, A, B of compatible sizes:

$$\begin{aligned} \langle U, V\rangle &= \langle U^\top, V^\top\rangle \\ \langle AB, W\rangle &= \langle A, WB^\top\rangle = \langle B, A^\top W\rangle. \end{aligned} \tag{3.3}$$

Definition 3.1.3 (Differential). Let U, V be two open sets in two Euclidean spaces $\mathcal{E}_1$ and $\mathcal{E}_2$, respectively. A map $F : U \to V$ is said to be *smooth* if it is infinitely differentiable (class $\mathcal{C}^\infty$) on its domain.

Also, F is said to be *differentiable* at a point $x \in U$ if there exists a neighborhood $U'$of x such that $F\mid_{U'}$ is smooth. Thus, if $F : U \to V$ is smooth at x, then its *differential* at x, is the linear map $\mathrm{D}F(x) : \mathcal{E}_1 \to \mathcal{E}_2$ defined as:

$$\mathrm{D}F(x)[v] = \frac{d}{dt} F(x+tv)\Big|_{t=0} = \lim_{t\to 0} \frac{F(x+tv) - F(x)}{t}$$

Definition 3.1.4 (Euclidean Gradient). Consider a smooth function $f : \mathcal{E} \to \mathbb{R}$, where $\mathcal{E}$ is a linear space. The Euclidean gradient with respect to an inner product $\langle \cdot, \cdot \rangle : \mathcal{E} \times \mathcal{E} \to \mathbb{R}$, denoted by $\boldsymbol{\nabla} f(x)$ is a unique element of $\mathcal{E}$ such that, for all $v \in \mathcal{E}$,

$$\langle v, \boldsymbol{\nabla} f(x) \rangle = \mathrm{D}f(x)[v], \quad x, v \in \mathcal{E},$$

where $\mathrm{D}f(x) : \mathcal{E} \to \mathbb{R}$ is the differential of f at x.

Definition 3.1.5 (Euclidean Hessian). The Euclidean Hessian of a smooth function $f : \mathcal{E} \to \mathbb{R}$ is the linear map $\mathrm{Hess} f(x) : \mathcal{E} \to \mathcal{E}$ defined as:

$$\mathrm{Hess} f(x)[v] = \mathrm{D}(\boldsymbol{\nabla} f)(x)[v] = \frac{d}{dt} \boldsymbol{\nabla} f(x+tv)\Big|_{t=0}, \quad \forall x, v \in \mathcal{E}.$$

Definition 3.1.6 (Embedded Submanifold). Let $\mathcal{E}$ be a linear space of dimension k. A non-empty subset $\mathcal{M}$ of $\mathcal{E}$ is a (smooth) *embedded submanifold* of $\mathcal{E}$ of dimension n if either of the following conditions hold:

(1) $n = k$ and $\mathcal{M}$ is open in $\mathcal{E}$, also called an open submanifold; or

(2) $n = k - q$ for some $q \geq 1$ and, for each $x \in \mathcal{M}$, there exists an open neighborhood U of x in $\mathcal{E}$ and a smooth map $h : U \to \mathbb{R}^q$ such that:

(a) If y is in U, then $h(y) = 0$ if and only if $y \in \mathcal{M}$; and

(b) rank $\mathrm{D}h(x) = q$.

Such a function h is called a *local defining function* for $\mathcal{M}$ at x.

Definition 3.1.7 (Tangent Space). Let $\mathcal{M}$ be a subset of $\mathcal{E}$. For all $x \in \mathcal{M}$, the *tangent space* to $\mathcal{M}$ at x, denoted by $T_x\mathcal{M}$, is defined as the set of all vectors that are tangents at x to smooth curves on $\mathcal{M}$, i.e.,

$$T_x\mathcal{M} = \{\gamma'(0) \mid \gamma : (-\varepsilon, \varepsilon) \to \mathcal{M} \text{ is a smooth curve with } \gamma(0) = x\}.$$

Definition 3.1.8 (Tangent Bundle). The *tangent bundle* of a manifold $\mathcal{M}$ is the set of all tangent spaces over all points in $\mathcal{M}$:

$$T\mathcal{M} = \{(x, v) \mid x \in \mathcal{M}, v \in T_x\mathcal{M}\}.$$

Riemannian Geometry

Definition 3.1.9 (Differential). The differential of $F : \mathcal{M} \to \mathcal{M}'$ at the point $x \in \mathcal{M}$ is the linear map $DF(x) : T_x\mathcal{M} \to T_{F(x)}\mathcal{M}'$ defined as:

$$\mathrm{D}F(x)[v] = \frac{d}{dt} F(\gamma(t))\Big|_{t=0} = (F \circ \gamma)'(0),$$

Let $\mathcal{M}$ and $\mathcal{M}'$ be two embedded submanifolds of two Euclidean spaces $\mathcal{E}$ and $\mathcal{E}'$, respectively. Then, the map $F : \mathcal{M} \to \mathcal{M}'$ admits a smooth extension $\bar{F} : U \to \mathcal{E}'$ defined on an open neighborhood U of $\mathcal{M}$ in $\mathcal{E}$. Thus, for all $x \in \mathcal{M}$ and $v \in T_x\mathcal{M}$, the differential of F at x can be computed as:

$$\mathrm{D}F(x) = \mathrm{D}\bar{F}(x) \mid_{T_x\mathcal{M}} .$$

Definition 3.1.10 (Inner Product). An inner product on the tangent space $T_x\mathcal{M}$ is a bilinear, symmetric, positive-definite map $\langle \cdot, \cdot \rangle_x : T_x\mathcal{M} \times T_x\mathcal{M} \to \mathbb{R}$. It induces a norm for tangent vectors: $\|u\|_x = \sqrt{\langle u, u \rangle_x}$.

Definition 3.1.11 (Riemannian Metric). A metric $\langle \cdot, \cdot \rangle$ on $T_x\mathcal{M}$ is a Riemannian metric if it varies smoothly with x, in the sense that for all smooth vector fields V, W on $\mathcal{M}$, the function $x \mapsto \langle V(x), W(x) \rangle_x$ is smooth.

Definition 3.1.12 (Riemannian Submanifold). An embedded submanifold $\mathcal{M}$ of a Euclidean space $\mathcal{E}$ is a Riemannian submanifold if it is endowed with the Riemannian metric induced by the inner product of $\mathcal{E}$, i.e., for all $x \in \mathcal{M}$ and $u, v \in T_x\mathcal{M}$,

$$\langle u, v \rangle_x = \langle u, v \rangle,$$

where the right-hand side is the inner product in $\mathcal{E}$.

Definition 3.1.13 (Riemannian Gradient). Let $f : \mathcal{M} \to \mathbb{R}$ be smooth on a Riemannian manifold $\mathcal{M}$. The Riemannian gradient of f is the vector field $\mathrm{grad} f$ on $\mathcal{M}$ uniquely defined by the following identities:

$$\forall (x, v) \in \mathrm{T}\mathcal{M}, \quad \mathrm{D}f(x)[v] = \langle v, \mathrm{grad} f(x) \rangle_x,$$

where $\mathrm{D}f(x) : T_x\mathcal{M} \to \mathbb{R}$ is the differential of f at x, and $\langle \cdot, \cdot \rangle_x$ is the Riemannian metric on $T_x\mathcal{M}$.

Definition 3.1.14 (Projection). Let $\mathcal{M}$ be an embedded submanifold of a Euclidean space $\mathcal{E}$ equipped with a Euclidean metric $\langle \cdot, \cdot \rangle$. The *orthogonal projection* onto the tangent space $T_x\mathcal{M}$ at a point $x \in \mathcal{M}$ is the linear map $P_x^{\perp} : \mathcal{E} \to T_x\mathcal{M}$ which satisfies the following properties:

(1) *Range*: $\mathrm{im}(P_x^{\perp}) = T_x\mathcal{M}$;

(2) *Projector*: $P_x^{\perp} \circ P_x^{\perp} = P_x^{\perp}$;

(3) *Orthogonal*: $\langle u - P_x^{\perp}(u), v \rangle = 0$, for all $u \in \mathcal{E}$ and $v \in T_x\mathcal{M}$.

Proposition 3.1.1. Let $\mathcal{M}$ be an embedded submanifold of a Euclidean space $\mathcal{E}$ equipped with a Euclidean metric $\langle \cdot, \cdot \rangle$. For a smooth function $f : \mathcal{M} \to \mathbb{R}$, the Riemannian gradient at a point $x \in \mathcal{M}$ is given by:

$$\operatorname{grad} f(x) = P_x^{\perp}(\boldsymbol{\nabla} \bar{f}(x)),$$

where $\bar{f}$ is a smooth extension of f to an open neighborhood of $\mathcal{M}$ in $\mathcal{E}$, and $\boldsymbol{\nabla} \bar{f}(x)$ is the Euclidean gradient of $\bar{f}$ at x.

Grassmannians

The Grassmannian $\mathrm{Gr}(k, n)$ is the set of all k-dimensional subspaces of $\mathbb{R}^n$. It can be endowed with the structure of a smooth Riemannian manifold of dimension $k(n-k)$. Each point $\mathcal{S} \in \mathrm{Gr}(k, n)$ can be represented by an orthogonal matrix $Y \in \mathbb{R}^{n \times k}$ such that $Y^\top Y = I_k$.

The projection operator onto the subspace $\mathcal{S}$ is given by $P_{\mathcal{S}} = YY^\top$ and the orthogonal complement projection is $P_{\mathcal{S}}^{\perp} = I_n - YY^\top$.

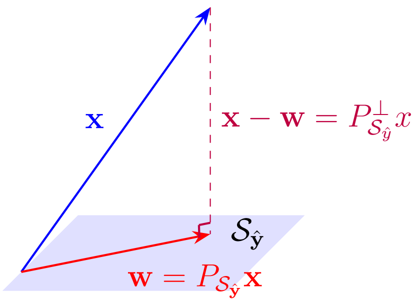

Figure 3.4 / Depiction of the orthogonal projection of a vector onto a subspace $\mathcal{S}_{\hat{\mathbf{y}}}$.

Definition 3.1.15 (Chordal Distance). The chordal distance between two subspaces $\mathcal{S}_1, \mathcal{S}_2 \in \mathrm{Gr}(k, n)$ is defined as:

$$d_2(\mathcal{S}_1, \mathcal{S}_2) = \sqrt{\operatorname{Tr}\left(P_{\mathcal{S}_1}^{\perp} P_{\mathcal{S}_2}\right)} = \frac{1}{\sqrt{2}} \|P_{\mathcal{S}_1} - P_{\mathcal{S}_2}\|_{\mathrm{F}} = \sqrt{\sum_{i=1}^{k} \sin^2(\theta_i)},$$

where $\|\cdot\|_{\mathrm{F}}$ is the Frobenius norm, and $0 \leq \theta_1 \leq \cdots \leq \theta_k \leq \pi/2$ are the principal angles between $\mathcal{S}_1$ and $\mathcal{S}_2$.

Definition 3.1.16 (Gap Distance). The gap distance between two subspaces $\mathcal{S}_1, \mathcal{S}_2 \in \mathrm{Gr}(k, n)$ is defined as:

$$d_\infty(\mathcal{S}_1, \mathcal{S}_2) = \|P_{\mathcal{S}_1} - P_{\mathcal{S}_2}\|_2 = \sin(\theta_k),$$

where $\|\cdot\|_2$ is the spectral norm, and θ_k is the largest principal angle between $\mathcal{S}_1$ and $\mathcal{S}_2$.

3.2 Optimization

Definition 3.2.1 (Convexity). Let $\mathcal{X}$ be a real vector space. Let $f : \mathcal{X} \to \mathbb{R}$ be a function, such that $x \mapsto f(x)$. Then, f is convex over $\mathcal{X}$, for all $x \in \mathcal{X}$, if and only if

$$f(tx_1 + (1-t)x_2) \leq tf(x_1) + (1-t)f(x_2),$$

for all $0 \leq t \leq 1$, and all $x_1, x_2 \in \mathcal{X}$.

Definition 3.2.2 (Constrained Optimization). For the constrained minimization problem,

$$\min_{x\in\mathbb{R}^n} f(x) \quad \text{subject to} \quad \begin{cases} c_i(x) = 0, \ i \in \mathcal{E}, \\ c_i(x) \leq 0, \ i \in \mathcal{I}, \end{cases} \tag{3.5}$$

where f and the functions c_i are all smooth, real-valued functions on a subset of $\mathbb{R}^n$, and $\mathcal{I}$ and $\mathcal{E}$ are two finite sets of indices. We call f the *objective function*, while c_i, $i \in \mathcal{E}$ are the equality constraints and c_i , $i \in \mathcal{I}$ are the inequality constraints. We define the *feasible set* Ω to be the set of points x that satisfy the constraints; that is,

$$\Omega = \{x \mid c_i(x) = 0, i \in \mathcal{E}; c_i(x) \leq 0, i \in \mathcal{I}\},$$

so that we rewrite the problem more compactly as,

$$\min_{x\in\Omega} f(x).$$

Definition 3.2.3 (Lagrangian function). We define the *Lagrangian* function for (3.5) as

$$\mathcal{L}(x, \lambda) = f(x) + \sum_{i\in\mathcal{I}\cap\mathcal{E}} \lambda_i c_i(x)$$

Definition 3.2.4 (First-Order Optimality Conditions). Let x^* be a local solution of (3.5), that the functions f and c_i in (3.5) are continuously differentiable. Then there exists a Lagrange multiplier λ^*, with components $\lambda_i^*, i \in \mathcal{E} \cap \mathcal{I}$, such that the following conditions are satisfied at (x^*, λ^*),

$$\nabla_x \mathcal{L}(x^*, \lambda^*) = 0, \tag{3.6a}$$
$$c_i(x^*) = 0, \quad i \in \mathcal{E}, \tag{3.6b}$$
$$c_i(x^*) \leq 0, \quad i \in \mathcal{I}, \tag{3.6c}$$
$$\lambda^* \geq 0, \quad i \in \mathcal{I}, \tag{3.6d}$$
$$\lambda_i^* c_i(x^*) = 0, \quad i \in \mathcal{E} \cap \mathcal{I}. \tag{3.6e}$$

These conditions (3.6) are often known as the *KKT (Karush-Kuhn-Tucker) conditions*. The conditions (3.6e) are *complementarity slackness conditions*; they imply that either constraint i is active or $\lambda_i^* = 0$, or possibly both.

Definition 3.2.5 (Min-Max Optimization). In general, the min-max problems (see [11], [12]) on Riemannian manifolds that we focus on, are of the kind

$$\min_{x \in \mathcal{M}_x} \max_{y \in \mathcal{M}_y} f(x, y),$$

where f is atleast $\mathcal{C}^2$ and $\mathcal{M}_x, \mathcal{M}_y$ are the Riemannian manifolds containing x, y respectively. Without loss of generality, we assume that minimization takes place first, followed by maximization (cannot be interchanged). These problems are termed *min-max optimization*, whereas the candidate solution points are termed *minimax points*.

Definition 3.2.6 (Global Minimax point). Let $\mathcal{M}_x, \mathcal{M}_y$ be two smooth Riemannian manifolds. Consider two subsets $\mathcal{X}, \mathcal{Y}$ such that $\mathcal{X} \times \mathcal{Y} \subseteq \mathcal{M}_x \times \mathcal{M}_y$. Let $f : \mathcal{X} \times \mathcal{Y} \to \mathbb{R}$, be a function such that $(x, y) \mapsto f(x, y)$. The point (x^*, y^*) is called a global minimax point if for any $(x, y) \in \mathcal{X} \times \mathcal{Y}$, it satisfies:

$$f(x^*, y) \leq f(x^*, y^*) \leq \max_{y' \in \mathcal{Y}} f(x, y').$$

Definition 3.2.7 (Local Minimax point). A point (x^*, y^*) is a local minimax point of $f : \mathcal{X} \times \mathcal{Y} \to \mathbb{R}$ if there exists some $\delta_0 > 0$, and a function h such that $\lim_{\delta \to 0} h(\delta) = 0 \; \forall \; \delta \in (0, \delta_0]$ and for every $(x, y) \in \mathcal{X} \times \mathcal{Y}$ satisfying $d(x, x^*) \leq \delta$ and $d(y, y^*) \leq \delta$ such that

$$f(x^*, y) \leq f(x^*, y^*) \leq \max_{y' : d(y, y') \leq h(\delta)} f(x, y').$$

4 Usage of TEXtured

To quickly familiarize yourself with the TEXtured template, we will go through the basic structure of the template files and explain how to use them. First, take a look at Figure 4.1 →p.11 for a visual representation of the file structure.

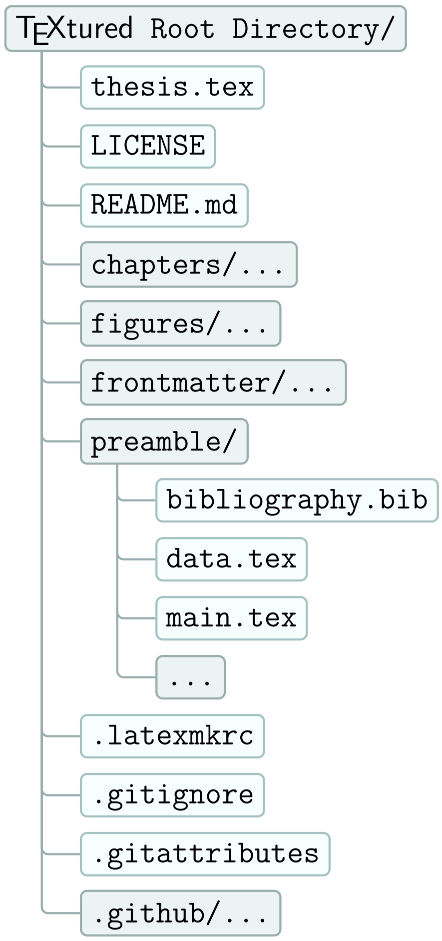

Figure 4.1 / TEXtured template file structure.

The main file is `thesis.tex`. It does not contain the actual content of the document, but instead `\include`s the chapters and *front matter* pages from the corresponding directories.

Make sure to fill all PDF (meta)data — like title, author, etc. — in the `preamble/data.tex` file. The bibliography/reference data is stored in the `preamble/bibliography.bib` file.

All of the TEXtured tweaks and settings located in files under `preamble/...` directories are loaded by the `preamble/main.tex` file, which is itself `\input`ed in the `thesis.tex` file.

The `.latexmkrc` file contains a configuration for the latexmk tool, which provides a convenient way to compile the document.

The usual workflow looks something like this:

- Metadata. Fill in the `preamble/data.tex` file with the necessary information about the document — title, author, and other *metadata*.
- Content. Write the content of the document in the `chapters/` directory. If you need more chapters, just create a new file, and `\include` it in the `thesis.tex` file at appropriate place.
- Figures. To include figures, you can put them in the `figures/` directory. Since this directory is by default included in `\graphicspath`, there is no need to specify full/relative path, and it is enough to use just the filename in the `\includegraphics` command.

- Citations. Using (for example) `\autocite` macro, you can cite in the text any entry added to the `preamble/bibliography.bib` file.

Remark 2 (Toggles). There are a couple of *toggles* in the `thesis.tex` file that can be used to customize style/layout/creation of the document:

- Page Layout — you can choose between *Single-Side* or *Two-Sided* printing by uncommenting the appropriate `\documentclass` line.
- Fancy Style (default: `enabled`) — if the default style is not to your liking, you can disable some of the more "fancy" stylistic elements by using the `\FANCYfalse` line.
- Work-In-Progress Version (default: `disabled`) — if you want to mark the document as a *Draft*, leave the `\WIPtrue` line uncommented (comment out for the final version).
    - Extra Margin (default: `disabled`) — the *Draft* document will include extra right margin (for notes and corrections) when you enable it using `\EXTRAMARGINtrue`.
- Link Boxes (default: `enabled`) — drawing of link/reference boxes can be disabled by `\LINKBOXESfalse` command (improves compilation time).
- Censored Version (default: `disabled`) — if you want to censor chosen parts of the document, include the `\CENSORtrue` line.
- Include Only ... — if you want to compile only a subset of chapters, you can utilize the `\includeonlysmart` command.

Remark 3 (MFF CUNI Template Compatibility). TEXtured can be used out of the box for theses at the Faculty of Mathematics and Physics, Charles University in Prague. Just be sure to include all *front matter* pages and fill out necessary data:

- Title Page with the faculty logo (among other things),
- Declaration,
- Dedication (optional),
- Information Page including the Abstract.

This is done by uncommenting the relevant lines in the main `thesis.tex` file.

Layout of these front matter pages is adapted and modified from the original MFF CUNI template **MaresTemplate**. However, always make sure it is compliant with the faculty guidelines, otherwise please raise an issue on GitHub **TeXtured**.

Remark 4 (License). If you want to make your document publicly available (together with the source code), you should not forget to include an appropriate license of your choice — change the `LICENSE` file, specifying the `CC0 1.0 Universal` license of TEXtured.

5 Features of TEXtured

In the following sections, we will describe the features of TEXtured template, implemented by utilizing various LaTeX packages and custom macros.

Remark 5 (Packages and Macros). We will refer to various LaTeX packages and macros using the following styles:

- `package`→CTAN — a package (together with a link to its CTAN page),
- `\macro` — a command/macro, either built-in or provided by a package,
- `\custommacro` — a custom macro defined in the TEXtured template.

5.1 Code Organization

To avoid large and hard to navigate preamble files, the code is organized into multiple directories/files in the `preamble/` directory, each focusing on a particular function/feature, see Figure 5.1→p.13.

```
preamble/
├── bibliography.bib
├── data.tex
├── toggles.tex
├── main.tex
├── debug/...
├── environments/...
├── general/...
├── hacks/...
├── layout/...
├── math/...
├── misc/...
├── pdfA-compliance/...
├── references/...
└── user/...
```

Figure 5.1 / Structure of the `preamble/` directory.

It is critical that the `preamble/pdfA-compliance/glyphtounicode.tex` file ensuring the PDF/A compliance is sourced before `\documentclass`.

The `preamble/toggles.tex` files defines various toggles, which should be appropriately set right after. Finally, the rest of (preamble) files are then loaded through the `preamble/main.tex` file.

When possible, add your own tweaks and macros to the `preamble/user/` directory reserved for this purpose. This way, you can easily update to newer versions of TEXtured (hopefully) without conflicts.

Remark 6 (Pointers to Directories/Files). If you want to tweak some aspect of the template — or learn how a given feature is implemented — pointers to the relevant directories/files are provided next to the subsequent section/subsection titles to help you navigate the code.

Remark 7 (Custom User Macros). Store your own macros in the `preamble/user/` directory, which is reserved precisely for this purpose. Then, if you would like to update to a newer version of TEXtured, you will be having easier time — less mixing of your code with the template code will result in fewer conflicts you must resolve manually.

Remark 8 (Auxiliary Files). To avoid cluttering the directories with *auxiliary files* generated during the compilation, it is recommended to use the `aux_dir` setting in the `.latexmkrc` file (enabled by default, the `aux_dir` being `.aux/`). All auxiliary files are then stored in a separate directory, leaving the rest tidy.

Remark 9 (Suggestion: One Sentence Per Line). It is a good practice to follow "one sentence per line" rule (or something similar), since it improves diffs for versioning systems like `git`. Tools like `latexindent` can help.

> **Note.** My config for `latexindent` mostly works, but some corner cases can surface. Will share someday.

If multiple sentences are on the same line, changing just one word results in the whole line being marked as changed, making it harder to see how much the text was actually changed in a given commit.

5.2 Page Layout and Style

We will first describe the page layout and style, which includes page dimensions, headers and footers, page numbering, and heading style.

Page Dimensions, Printing Layout

Using `geometry`→CTAN package — set up the page layout (supported single/double-sided printing). Apply `\flushbottom` — try to make text body on all pages have the same height.

Page Headers and Footers

Using `fancyhdr`→CTAN package — page headers and footers — consistent style also for initial page of a chapter (not totally different style with numbering in the bottom center . . .).

Page Numbering

Placing custom `\frontmatter`, `\mainmatter`, and `\backmatter` macros at appropriate places in `thesis.tex`, *Roman numbering* is set up for *front matter*, that is until the start of first numbered chapter, and then *Arabic numbering* for the rest of the document.

Heading Style `preamble/layout/titles.tex`

Pretty chapter heading style — big calligraphic number/letter behind the title.

5.3 Sane Typographical Defaults `preamble/general/`

Now we will concern ourselves with more intricate and detailed typography, more at level of paragraphs, sentences, words, and even letters.

Paragraphs `preamble/general/typesetting.tex`

No paragraph indentation, proper space between paragraphs — `parskip`→CTAN.

Floats, Captions `preamble/general/floats.tex`

Caption styling includes a slight hang, `\footnotesize` font, and a bold sans label. See Appendix A→p.27 for a showcase of the different caption types.

Font and Related Stuff `preamble/general/typesetting.tex`

The default choice are *Latin Modern* fonts — a classic really. Various families and shapes are typically used for different purposes:

- *Serif* family for the main text
- *Slanted* shape for emphasis using `\emph` macro (instead of the default *Italic* shape, which is reserved mainly for math formulas)

  Remark 10 (Nested Emphasis). Nested emphasis is displayed in *Italic* shape. It is rather rare to nest an *additional emphasis inside an emphasis.*
- *(Bold) Sans* family for headings and other structural elements
- *Typewriter* family for computer code and similar stuff

Example 5.3.1. Quick showcase of some font families and shapes:

This is Latin Modern Serif $\alpha = 2^2$
This is Latin Modern Serif Oblique $\alpha = 2^2$
This is Latin Modern Serif Bold $\boldsymbol{\alpha = 2^2}$
This is Latin Modern Serif Bold Oblique $\boldsymbol{\alpha = 2^2}$

This is Latin Modern Sans $\alpha = 2^2$
This is Latin Modern Sans Oblique $\alpha = 2^2$
This is Latin Modern Sans Bold $\boldsymbol{\alpha = 2}^2$
This is Latin Modern Sans Bold Oblique $\boldsymbol{\alpha = 2}^2$

Note. Sans math font has problems with showing properly all bold symbols (sub/superscripts don't work automatically).

For consistent quotation use `\enquote` macro provided by `csquotes`→CTAN.

Micro-Typography

Enable micro-typographic extensions with package `microtype`→CTAN, most prominently character protrusion and font expansion.

Following quote from `microtype`→CTAN documentation nicely explains what it is about:

> Micro-typography is the art of enhancing the appearance and readability of a document while exhibiting a minimum degree of visual obtrusion. It is concerned with what happens between or at the margins of characters, words or lines. Whereas the macro-typographical aspects of a document (i.e., its layout) are clearly visible even to the untrained eye, micro-typographical refinements should ideally not even be recognizable. That is, you may think that a document looks beautiful, but you might not be able to tell exactly why: good micro-typographic practice tries to reduce all potential irritations that might disturb a reader.

5.4 Document Structure

It is important to have a clear and consistent structure of the document. This can be achieved by using various environments for different types of content, and by providing clear and informative titles for each part of the document, thus making it easier to navigate and understand.

Structure Environments

Inspired by the structured mathematical texts, enclosing various parts of the document in the corresponding environments can help to make the document more structured and easier to read. Implemented mostly `tcolorbox`→CTAN package and `keytheorems`→CTAN (modern key–value interface for `amsthm`→CTAN).

Remark 11 (Default Environments). There are predefined boxed "theorem-like" environments for Definition, Theorem, Lemma, Corollary, Proposition, and non-boxed "remark-like" environments for Remark, Proof, Example, Derivation, Calculation, Idea, and Tip (these have at least a mark indicating the end of the environment).

Names of the corresponding environments are lowercase, for example `definition`, `remark`, and so on. They also accept an optional argument for a short description.

Some additional points about the *structure environments*:

- provide clear structure, enables high level of interlinking

- they make the text easy to skim through, quickly get an idea, and know roughly what to expect
- have shared numbering, together with tables, figures, equations — leads to a linear increase of the reference number, making them easier to locate
- not only for physics/math texts, can be generally used to highlight key ideas

  Tip 5.4.1 (Custom Structure Environments). You can easily create additional "structure" environments, see Section 6.1 →p.22.

- avoid using emphasis for the whole body of "theorem-like" environments, since we already have a whole box around it to make them stand out

Remark 12. There are also helper environments for Todo-like notes. By default, there are Todo, Note, Suggestion, and Question environments, but you can easily create your own.

To avoid conflicts with possible existing macros/environments, names of these environments are capitalized, for example `Todo`, `Note`, and so on.

> **Note.** No "code listing" setup yet. PRs welcome.

References and Links `preamble/hacks/custom-reference-boxes.tex`

Custom reference/link/citation styles using `tcolorbox` →CTAN package.

> **Note (Slight Inconvenience — Line Breaks).** There is a slight inconvenience due to small flexibility around line breaks. It would be nice to have a proper workaround.

Remark 13 (Rationale). I like to have clearly distinguished references, links, and citations. By default, `hyperref` →CTAN provides frames around links, but they are not that pretty, and the PDF viewer must support them. Using just colors can sometimes look better, but I still wasn't satisfied.

Sometimes it is nice to know the precise location of the reference, especially when the document is printed and you cannot simply click on them. Therefore, the page number is (by default) included with `\Cref`, see Remark 14 →p.17. Use the starred variant `\Cref*` to omit it.

Remark 14 (Automatic Reference Type Detection). Package `zref-clever` →CTAN provides `\zcref` command — similarly to the older, no longer maintained, `cleveref` →CTAN package — which automatically detects the type of reference, and formats it accordingly. This behavior is adapted in TEXtured with the macro `\Cref`, which wraps the link in nice box, and also shows a corresponding page number of the target.

If you want the link to show the reference title, use `\Nref` — or the starred variant `\Nref*` to omit the page number — which utilizes `zref-titleref` →CTAN.

Table of Contents and Outline/Index

Clear and elegant Table of Contents, which includes all the important parts — also (unnumbered) subsections, but in a more compact style.

Similarly, automatically populate the PDF Outline/Index (digital Table of Contents in PDF viewer). It is very handy for navigating longer documents, and includes also other important pages other than just initial pages of main chapters: Title Page, Contents, Introduction, References, and so on.

Remark 15. I use `Zathura` as my PDF viewer, with the Outline/Index just one `Tab` away, allowing me to quickly jump to the desired part of the document.

Remark 16 (List of Figures, Tables, . . .). If you want/need to include a List of Figures, List of Tables, and so on, you can easily do so by uncommenting the relevant lines in the `\contentsandlists` macro.

5.5 Bibliography/References

Pretty and functional Bibliography/References, via `biblatex`→CTAN package.

Bibliography Style

Entries in References→p.30 have a clean consistent style, which builds on the `ext-numeric-verb` style from `biblatex-ext`→CTAN package.

Tip 5.5.1 (Bibliography Data). Make sure to gather all the relevant data you need for every reference. If you later decide you want to reduce the amount of presented information, `biblatex`→CTAN can help you with that. For example, it is possible to automatically

- remove `url` field if `doi` field is present,
- ignore unwanted fields (`pages`, `number`, `volume`, `series`, `location`, . . .).

Extra Fields

Support extra `github` field.

Custom External Links

Have the external DOI/arXiv/URL/GitHub links displayed in custom boxes, and place them on the new line.

Backreferences

Include *backreferences*, which point from the bibliography to the pages where the reference was cited.

Citation Style `preamble/references/cite.tex`

Include `[` and `]` characters around citation number inside the link (and wrap in `tcolorbox`→CTAN . . .), for example **TeXtured**.

5.6 PDF/A Compliance `preamble/pdfA-compliance/`

Proper metadata setup (via `hyperref`→CTAN and `\DocumentMetadata`).

Remark 17 (Document Data). Various data about the work should be entered in `preamble/data.tex` file. When the relevant entries contain LaTeX commands (for example to obtain specific formatting of the title), it is necessary to provide "plaintext" variations, so that `hyperref`→CTAN can properly set up PDF metadata.

Next we will describe various common violations of PDF/A standard, and how to fix them.

Glyph to Unicode Map `.../pdfA-compliance/glyphtounicode.tex`

To obtain PDF/A compliant PDF, we need to have Unicode mapping for all glyphs used in the document. It can happen — mainly when using fonts providing extra mathematical symbols — that certain glyphs are not covered by mappings loaded in `preamble/pdfA-compliance/glyphtounicode.tex`.

In the `preamble/pdfA-compliance/glyphtounicode.tex` file you can also find an example veraPDF output for a PDF with a problematic glyph. It also points to a guide located in `preamble/pdfA-compliance/LaTeX-find-glyph-name/` directory, which explains how to find out the glyph name, and how to provide the *glyph to Unicode* mapping with `\pdfglyphtounicode` command.

PDF `/Interpolation` Key

Some PDFs can have enabled the `/Interpolation` key, for example `Inkscape` generated PDFs with blur parts. However, PDF/A requires it to be disabled.

This is automatically fixed by `figures/Inkscape/inkscape-export-to-latex` shell script.

5.7 Miscellaneous

Math-Related Tweaks — $e^{\mathbb{I}\pi}$ `preamble/math/`

Some of the math-related tweaks:

- Use `\boldmath` automatically for `\textbf` text (useful mainly in headings).
- Possible to use sans italic font for math via `\mathsfit`.

- Better extendable arrows with TikZ→CTAN.
- *(Optional, disabled by default)* Automatically change usage of `\textcolor` to `\mathcolor` in math mode, so that we get proper math spacing, for example

$$a \times b \text{ (right spacing)} \qquad \text{versus} \qquad a{\times}b \text{ (wrong spacing)} .$$

  However, it is recommended to explicitly use `\mathcolor` when appropriate, since it leads to easier maintenance of the code (copy-pasting to other projects will work without problems).

Following practice is highly recommended.

Tip 5.7.1 (Define Your Own Math Macros). Frequently define macros for notation used more than once. Advantages are for example:

- Code is easier to read/write, since it is more “semantic”.
- To tweak notation, you only need to change it in one place.
- Easier to find all occurrences of a certain notion.

GitHub Actions

Describe implemented GitHub Actions:

- Automatic `latexmk` build of the latest PDF version.
- PDF/A verification via veraPDF.
- Deploy to `gh-pages` branch. One can furthermore enable (in repo settings) GitHub Pages for `gh-pages` branch, which will automatically upload latest PDF to `https://username.github.io/reponame/thesis.pdf`. This enables convenient sharing of your (even continuously evolving) work without needing to commit the PDF (resulting in large repository size) or compiling the PDF on the receiving side.

  Remark 18 (Private Repositories). Even for private repositories such link is publicly accessible. This is why GitHub Pages setup is not done automatically for you. If you want to share the work more “privately”, there are other solutions, for example GitHub Action which uploads PDF to Google Drive, and sharing via a private link. Also look at Section 5.7→p.20.

Censoring

Censoring/redaction using censor→CTAN package. Use `\censor`, `\blackout`, or `\censorbox`. For example, [redacted].

x

Inkscape Integration

Put your Inkscape figures into figures/Inkscape/ directory, and include them using `\includeInkscapeSVG` macro (in place of `\includegraphics`), which has the following features:

- Automatic export after changing the svg (need to enable `--shell-escape` for pdfTEX or LuaTEX, done via .latexmkrc).
- Watermark via a PostScript injection.

  Remark 19 (Watermark String). By default, the watermark string is composed as "©⟨*year*⟩ ⟨*author's name*⟩", where the author's name is extracted from `\ThesisAuthorPlaintext`. You can customize it in the shell script figures/Inkscape/inkscape-export-to-latex to your liking.
- Automatic fix of `/Interpolation` key problem.
- All text is processed by LATEX, ensuring consistent typesetting experience. In particular, you can enter math as usual through `$...$`.

5.8 Non-Features

These features were deemed unnecessary, or even counterproductive, and thus were not implemented/not customized. This does not mean that it is hard or not compatible to use them with TEXtured.

Footnotes

- they break the flow of reading, can be distracting
- either it is important and you want it there — no need to use footnotes — or it is not so important (maybe just a reminder/remark), but then there are in my opinion better ways to handle such situation
  - grayed out/smaller text, sidenotes are better alternative, if the page layout enables them
  - it is not bad to remind reader of something in the main text...

Index, Glossary

- since the text is primarily intended for electronic use, finding usage of certain terms is easy
- text should be ideally structured in such a way, that finding definitions of important terms is straightforward — interlinking/referencing in proper places to indicate where the notion to be used was defined/discussed

6 Tips & Tricks

In this chapter we will see how to utilize and even extend capabilities of TEXtured. Additionally, there will be sprinkled miscellaneous tips on how to improve the quality of your document.

6.1 Structure

Headings

- numbered and "lettered" chapters

  Todo. Describe `\chapternotnumbered`, and "lettered" chapters in front matter.

- Use nicely named subsections — much easier to navigate, since it leads to better ToC and Index

  Todo. Describe `\texorpdfstring`.

Structure Environments

- Utilize structure (remark, definition, ...) environments to make the document more structured and easier to read. Including a brief description as an optional argument can help to foreshadow the content of the environment. Important concepts will then stick out more and will be remembered better.

  Remark 20 (Spacing at the End of Structure Environments). Structure environments ending with displayed math or a list may need a bit of tweaking to ensure proper spacing at their end.

  This is most easily achieved using the `\qedhere` macro on the line, which should be the last one in the environment. This uses the mechanism of the `\qedhere` macro from `amsthm`→CTAN package, but now has also a starred variant for extra vertical space (for equations containing big operators), or even an optional argument for a completely custom vertical shift.

  Todo. Describe creation of new "structure" environments.

- Try to motivate every definition/theorem with "normal" text, do not let the document degenerate just into a listing of definitions/theorems/proofs/...

- Use references to other remarks/definitions/sections to make the document more interconnected, which can help the reader to look at a bigger picture, recollect necessary information to proceed further, or to understand the context better.

Todo. Show using `\autocite{TODO}` in the text **TODO**. Helps to not forget to add the citation later.

6.2 Typography

- use `~` to enter a non-breakable space, or also after a dot in the initials or after academic titles (otherwise one gets bigger space than is proper), for example `M.Sc.~Name Surname`
- proper usage of hyphens/dashes — learn when to use hyphen - (`-`), when en-dash – (`--`), and when em-dash — (`---`)
- use *emphasis* with `\emph` for the names of new and important concepts
- for quotation marks use `\enquote` from `csquotes`→CTAN package
- sometimes using gray text instead of parentheses may result in a cleaner look, for example instead of “(pseudo-)Riemannian” just gray out “pseudo-” like “pseudo-Riemannian”
- choose capitalization style of titles, and stick with it — I chose “titlecase”

6.3 Mathematics & Physics

Math Typesetting

Learn stuff in `amsmath`→CTAN/`eqnlines`→CTAN and `mathtools`→CTAN packages. Then it is possible to write pretty multi-line equations like the following inclusion map

$$\begin{aligned} \iota\colon (\mathbb{S}^1, \mathbb{R}_{\geq 0}, \mathbb{S}^{d-1}) &\longrightarrow \mathsf{AdS}_{d+1}/\mathbb{Z} \\ (\ t\ ,\ \ r\ \ ,\ \omega^{\bullet}\ ) &\longmapsto X = \iota(t, r, \omega^{\bullet}) \equiv \begin{cases} X^{-1} = \sqrt{\ell^2 + r^2}\cos(t/\ell), \\ X^0 = \sqrt{\ell^2 + r^2}\sin(t/\ell), \\ X^i = r\omega^i \quad \text{for } i \in \{1, \dots, d\}\,. \end{cases} \end{aligned}$$

Remark 21 (Math Ending Punctuation). Make sure to use `\eqend` or `\eqcomma` macro (when appropriate) to properly end a math environment with a period or a comma, respectively. They add a small space before the punctuation to make the formula look better.

Todo. Maybe show diagrams with `TikZ`→CTAN package.

Numbers and Units

Use `siunitx`→CTAN package for convenient typesetting numbers and units. Examples are shown in Table 6.1→p.24.

Note. The `siunitx`→CTAN package is very powerful and flexible. It can be even used to nicely align numbers in tables. As of now, this feature is not customized in any way in TeXtured. Suggestions for improvements are welcome.

Command	Output	Usage
`\num{123.45 e-8}`	$123.45 \cdot 10^{-8}$	numbers
`\si{\meter\per\second\squared}`	$\mathrm{m/s^2}$	units
`\SI{123.45}{m/s^2}`	$123.45\,\mathrm{m/s^2}$	numbers with units
`\SIrange{1}{10}{\kilo\meter}`	1–10 km	ranges
`\SIlist{1;3;5}{A}`	1 A, 3 A and 5 A	lists
`\SI{1.23 +- 0.45}{\celsius}`	$(1.23 \pm 0.45)\,°\mathrm{C}$	uncertainties

Table 6.1 / Examples of `siunitx`→CTAN package usage.

6.4 LaTeX Coding

Todo. Describe how to create custom macros with `\NewDocumentCommand`, `\RenewDocumentCommand`, `\NewCommandCopy`, ...

Question. Difference between “macro” and “function” in LaTeX? Which nomenclature is appropriate?

Remark 22 (Macro Space Handling). Using macro inside text in the form `\foo` can swallow the following whitespace. When this is not the desired behavior, call the macro like `\foo{}`. In this way an empty argument is passed to the macro, leaving the following whitespace intact.

Todo. Describe `\makeatletter` and `\makeatother`.

Todo. Describe `\ensuremath`. When math macro is used often outside math mode (alone as `...\(\foo\)...`), defining it wrapped in `\ensuremath` can lead to perhaps easier use (as just `...\foo{}...`).

Todo. Describe `\includeonlysmart`.

Note. Be careful about implicit end of line spaces in function definitions, sometimes necessary to use `%` after last command on the line. TODO: Describe this in more detail.

Todo. Describe WIP mode (particularly with LuaLaTeX).

Note. Some comments in source code refer to files from TeXLive installation on Arch Linux. On other distributions or operating systems the paths might be different.

26

Summary and Outlook

Summary and Outlook.

Example of Appendix Chapter

Example A.0.1 (Figure Caption Tweaking). Now we will show off some figures with tweaked position/extent of the captions. Figure A.1 →p.27 has a side-caption, while Figure A.2 →p.27 has a caption that spans just the width of the figure. This utilizes the `floatrow` →CTAN package and is inspired by the ITT LaTeX template **ITTtemplate**.

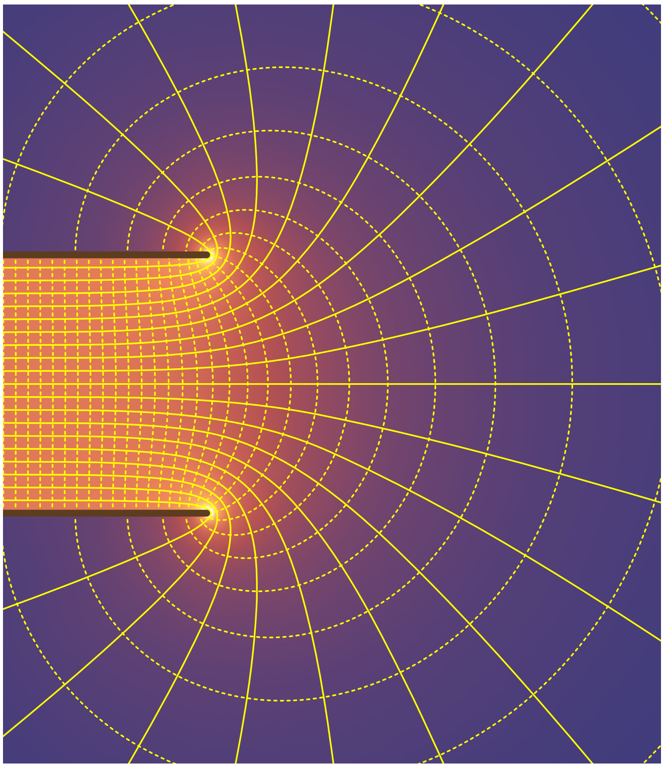

Figure A.1 / Example of a figure with a side-caption.

It displays the two-dimensional electric field near one end of a parallel plate capacitor.

Legend:

- equipotentials
- field lines
- capacitor plate

You can also optionally use a footnote for the figure caption.

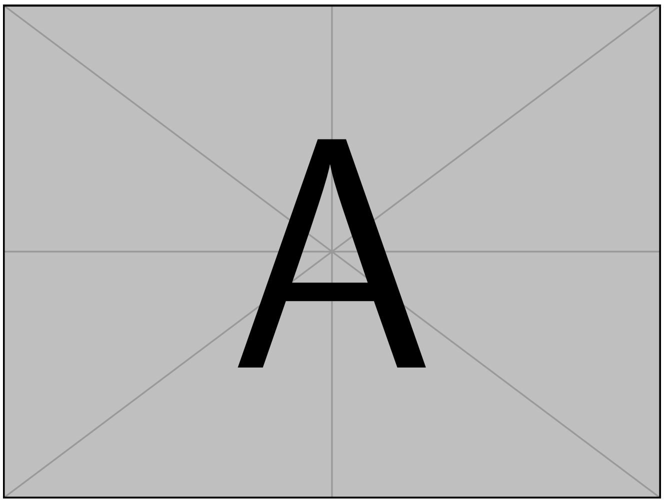

Figure A.2 / Example of a figure with a caption spanning just the width of the figure.

Example A.0.2 (Multi-Paragraph Figure Caption with Verbatim Text). It is possible to have multi-paragraph captions for figures. One must remember to provide a short description as `\caption[This is a Short Description]{...}`, or else LaTeX will complain.

See Figure A.3 →p.28 for an example, demonstrating also a workaround for typesetting verbatim text in contexts where "fragile" commands are not allowed.

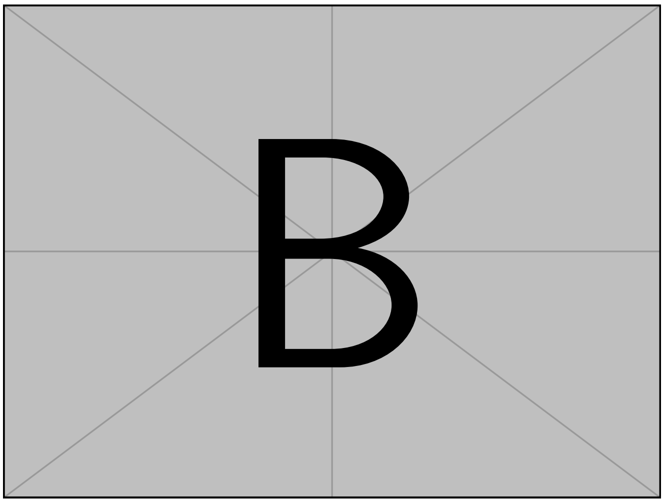

Figure A.3 / Example of a figure with a multi-paragraph caption.

Notice the spacing between the paragraphs. It was customized using the `parskip` key in `\captionsetup` provided by the `caption` →CTAN package.

To typeset verbatim text in the caption, use the `\fakeverb{...}` command instead of the usual `\verb|...|`, which is not allowed in captions.

A.1 Appendix Section

Note the numbering of various environments in the appendix.

Definition A.1.1 (Math in the Description — $\sin(\alpha) \approx \alpha$). This is an example definition in an Appendix. Note the automatic switch to the alternative sans math font in the Definition description.

Remark 23. The page header reflects that this is an appendix page.

Example A.1.1 (Equation Numbering and Referencing). As was mentioned already in Section 5.4 →p.16, equations share numbering with *structure environments*. For example, the equation

$$\phi^* \boldsymbol{g}' \overset{!}{=} \Omega^2 \boldsymbol{g} \equiv \mathrm{e}^{2\omega} \boldsymbol{g} \tag{A.4}$$

is numbered as (A.4) in the appendix.

We can reference this equation using `\Cref` as Equation (A.4) →p.28. Starred variant `\Cref*` results in Equation (A.4). If you desire less verbose output, you can use `\eqref`, which gives (A.4).

Theorem 1 (Example with Math at the End). Theorem ending with math, with proper spacing by utilizing `\qedhere` (can even use an optional argument to finetune the spacing)

$$a^2 + b^2 = c^2 .$$

References

Back-references to the pages where the publication was cited are given by •.

[1] Adrien-Marie Legendre. Nouvelles méthodes pour la détermination des orbites des comètes. F. Didot, **1805**.
URL: https://catalog.hathitrust.org/Record/008630090 1

[2] Carl Friedrich Gauss. Theoria motus corporum coelestium in sectionibus conicis solem ambientum. F. Perthes and I.H. Besser, **1809**.
URL: https://archive.org/details/bub%5C_gb%5C_ORUOAAAAQAAJ 1

[3] Gene H. Golub and Charles F. van Loan. An Analysis of the Total Least Squares Problem. *SIAM Journal on Numerical Analysis*, **1980**.
DOI: 10.1137/0717073 eprint: https://doi.org/10.1137/0717073 1

[4] Laurent El Ghaoui and Hervé Lebret. Robust Solutions to Least-Squares Problems with Uncertain Data. *SIAM Journal on Matrix Analysis and Applications*, **1997**.
DOI: 10.1137/S0895479896298130
eprint: https://doi.org/10.1137/S0895479896298130 2

[5] Jan C. Willems. The Behavioral Approach to Open and Interconnected Systems. *IEEE Control Systems Magazine*, **2007**.
DOI: 10.1109/MCS.2007.906923 2

[6] JC Willems, P Rapisarda, Markovsky, and BLM De Moor. A note on persistency of excitation. English. *Systems & Control Letters*, **2005**.
DOI: 10.1016/j.sysconle.2004.09.003 2

[7] I. Markovsky and F. Dörfler. Behavioral systems theory in data-driven analysis, signal processing, and control. *Annual Reviews in Control*, **2021**.
DOI: 10.1016/j.arcontrol.2021.09.005 3

[8] Alberto Padoan, Jeremy Coulson, Henk J. Van Waarde, John Lygeros, and Florian Dorfler. Behavioral uncertainty quantification for data-driven control. English. In: *2022 IEEE 61st Conference on Decision and Control, CDC 2022*. IEEE, **2022**.
DOI: 10.1109/CDC51059.2022.9993002 3

[9] P.-A. Absil, R. Mahony, and R. Sepulchre. Optimization Algorithms on Matrix Manifolds. Princeton University Press, **2007** 5

[10] Nicolas Boumal. An introduction to optimization on smooth manifolds. Cambridge University Press, **2023**.
DOI: 10.1017/9781009166164 5

[11] Andi Han, Bamdev Mishra, Pratik Jawanpuria, and Junbin Gao. Nonconvex-nonconcave min-max optimization on Riemannian manifolds. *Transactions on Machine Learning Research*, **2023**.
URL: https://openreview.net/forum?id=EDVIHPZhFo 10

[12] Andi Han, Bamdev Mishra, Pratik Jawanpuria, and Akiko Takeda. A Framework for Bilevel Optimization on Riemannian Manifolds. **2024**.
arXiv: 2402.03883 [math.OC] URL: https://arxiv.org/abs/2402.03883 10