Intrepolation :-

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Let us consider the statement

$$y = f(x) \qquad x_0 \le x \le x_n.$$

This means that corresponding to every value of x in the rang $x_0 \le x \le x_n$, there exists one or more values of y.

- Assuming that $f(x)$ is single-valued and continuous then the values of $f(x)$ corresponding to certain given values of x, say $x_0, x_1, x_2 \ldots x_n$ can easily be computed and tabulated.
- Given the set of tabular values $(x_0, y_0), (x_1, y_1) \ldots (x_n, y_n)$ satisfying the relation $y = f(x)$ where explicit nature of $f(x)$ is not known.
- It is required to find a simpler function say $\phi(x)$ such that $f(x)$ and $\phi(x)$ agree at the set of tabulated points. Such a process is called interpolation.

If $\phi(x)$ is polynomial, then the process is called polynominal interpolation and $\phi(x)$ is called the interpolating polynomial.

eg. given $y = f(x)$

eg.

x	x_0	x_1	x_2	x_3
y	y_0	y_1	y_2	y_3

* value of x is called argument (Independent variable) here we have ~~to~~ for every value of x there exisit a particularly values of y (dependent variable) here we called values of y is entry.

and we read x_0 corresponds to y_0

x_1 —— " —— y_1

⋮

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eg.

x	1	3	5	7
y	10	15	21	52

In this case, there are 4 data is given.
Now we want to find values of y when $x=4$.
We must now from given table values of y at $x=4$
must be lies betn 15 and 21.
∴ The estimate the values. of y we use INTERPOLATION.

There are four methods of Interpolation.

1) Langranges Interpolation.
2) Newton's forward Interpolatio } equal interval.
3) Newtons backward Interpolation
4) ~~Inverse Interpolation~~. (Divided differnce /.
Newton Divided diffence inter.

unequal interval

* Langranges Interpolation :-

This methods is used when unequal interval. ie given values of x and y in table and $x_0 - x_1 x_2 \ldots$ value is ie $(x_1 - x_0)$ $(x_2 - x_1) \neq$ same values.
Consider the. data points with unequal interval. then the by Langrange's formula for any x, $y(x)$ is given by.

x	x_0	x_1	x_2	x_3	x_4
y	y_0	y_1	y_2	y_3	y_4

~~$y(x) = (x-x_0)(x-x_2)$~~

$$y(x) = \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)} \times (y_0) +$$

$$\frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)} \times (y_1) +$$

$$\frac{(x-x_0)(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)} \times y_2 +$$

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$$\frac{(x-x_0)(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)}(y_3) +$$

$$\frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_0)(x_4-x_1)(x_4-x_2)(x_4-x_3)}(y_4)$$

Q) Using langrange's Interpolation method find the form of the fun y(x) from following table

x	5_{x_0}	7_{x_1}	11_{x_2}	13_{x_3}	17
y	150_{y_0}	392_{y_1}	1452_{y_2}	~~5202~~ 2366_{y_3}	5202

at $x = 9$ ie $y(9)$.

Sol^n^ give

X	$5(x_0)$	$7(x_1)$	$11(x_2)$	$13(x_3)$	$17(x_4)$
Y	$450(y_0)$	$392(y_1)$	$1452(y_2)$	~~5202~~ $2366(y_3)$	5202

$x = 9 \Rightarrow y = ?$

Using Langranges Interpolation formula

$$y(x) = \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)} y_0 +$$

$$\frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x-x_4)} y_1 +$$

$$\frac{(x-x_0)(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)} y_2 +$$

$$\frac{(x-x_0)(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)} y_3 +$$

$$\frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_1)(x_4-x_2)(x_4-x_3)(x_4-x_0)} y_4$$

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$$y(9) = \frac{(9-7)(9-11)(9-13)(9-17)}{(5-7)(5-11)(5-13)(5-17)}(150) +$$

$$\frac{(9-5)(9-11)(9-13)(9-17)}{(7-5)(7-11)(7-13)(7-17)}(392) +$$

$$\frac{(9-5)(9-7)(9-13)(9-17)}{(11-5)(11-7)(11-13)(11-17)}(1452) +$$

$$\frac{(9-5)(9-7)(9-11)(9-17)}{(13-5)(13-7)(13-11)(13-17)}(2366) +$$

$$\frac{(9-5)(9-7)(9-11)(9-13)}{(17-5)(17-7)(17-11)(17-13)}(5202)$$

$$= \left(\frac{-50}{3}\right) + \frac{3136}{15} + \frac{3872}{3} - \frac{2366}{3} + \frac{578}{5} =$$

$$= \frac{-250 + 3136 + (5)(3872) - (5)(2366) + (578)(3)}{15}$$

$$= \frac{-250 + 3136 + 19360 - 11830 + 2890}{15} = \underline{\underline{810}}.$$

$25386 - 12080$

Using Langranges interpolation formula find fu^n $y(x)$.

x	0_{x_0}	1_{x_1}	3_{x_2}	4_{x_3}
y	-12_{y_0}	0_{y_1}	12_{y_2}	24_{y_3}

By Langranges Interpolation formula.