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Ordinary Differential Equation

Introduction:-

Differential eqⁿ introduction

Generally we have to know

$$y = f(x)$$

$$\downarrow$$

$$dy/dx,\ d^2y/dx^2,\ d^3y/dx^3 \ldots$$

we write

$$F(x, y, dy/dx, d^2y/dx^2 \ldots) = 0$$

we say that or means that

Differential equation is relation betⁿ dependent variable, Independent variable and it's various derivatives.

There are two type of diffe ① ordinary ② partial.

Ordinary differential equation:-

- Ordinary differential equation is that have one independent variable and one or more dependent variable.

eg. $dy/dx + x^2 y = \sin x$

$dy/dt + \frac{dx}{dt} = \sin t$

order and degree DE.

higest order derivation → degree of highest order derivative

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eg. $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 + y^2 = 0$

degree-2 - order-1

$\sqrt{\left(\frac{dy}{dx}\right)^2 + 1} = \left(\frac{d^2y}{dx^2}\right)$

$\left(\frac{dy}{dx}\right)^2 + 1 = \left(\frac{d^2y}{dx^2}\right)^2$

order-2 degree-2

$\left(\frac{dy}{dx}\right)^{1/3} = \left(\frac{d^2y}{dx^2}\right)$

$\left(\frac{dy}{dx}\right) = \left(\frac{d^2y}{dx^2}\right)^3$

or-2 degree-3

formation of ordinary DE.

Q: $y = C_1 \cos 2x + C_2 \sin 2x$

Diff. w.r. to x

$\frac{dy}{dx} = -2C_1 \sin 2x + 2C_2 \cos 2x$

Another Diff. w.r. to x

$\frac{d^2y}{dx^2} = -4C_1 \cos 2x - 4C_2 \sin 2x$

$= -4\left[C_1 \cos 2x + C_2 \sin 2x\right]$

$\left(\frac{d^2y}{dx^2}\right) = -4y$

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Q: $y = A\cos^2 x + B\sin^2 x$

sol^n diff. w. r. to x

$dy/dx = -A\sin^2 x \cdot 2x + B\cos^2 x \cdot 2x$

$= 2x \cdot [-A\sin^2 x + B\cos x^2]$

$= 2x \cdot [\cancel{A\sin^2 x} - A\cos^2 x \cdot 2x - B\sin^2 x \cdot 2x]$

$+ [-A\sin^2 x + B\cos^2 x] \cdot 2$

$= -4x^2 (A\cos x^2 + B\sin^2 x) + 2 [A\sin^2 x + B\cos^2]$

$$\left[dy/dx = -4x^2 y + \frac{1}{x} dy/dx \right]$$

sol^n of diff. eq^n - general sol^n - complete sol^n.

↓ Assign value assing - particular sol^n.

① Euler ~~eq^n~~ method.

consider the differential eq^n

$dy/dx = f(x, y) \quad y(x_0) = y_0$

$y(x_n) = y_n = ?$

~~Euler~~ Euler method say that

$y_n = y(x_n) = y_{n-1} + h\,(f(x_{n-1}, y_{n-1}))$

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where $h = x_n - x_{n-1}$

$$\boxed{x_n = x_0 + nh}$$

when we put $x_n = 1$.

$y_1 = y(x_1) = y_0 + hf(x_0, y_0)$

$y_2 = y(x_2) = y_1 + hf(x_1, y_1)$

$y_3 = y(x_3) = y_2 + hf(x_2, y_2)$

Q. find $y(2.2)$ using Euler's method from the equation. $dy/dx = -xy^2$ with $y(2) = 1$.

sol^n

given

$dy/dx = -xy^2$

$\therefore f(x, y) = -xy^2$

$x_0 = 2 \quad y_0 = 1$

Let $n = 4$

$h = \frac{2.2 - 2}{4} = \frac{0.2}{4} = 0.05$

$x_1 = x_0 + h = 2 + 0.05 = 2.05$

$x_2 = x_1 + h = 2.05 + 0.05 = 2.1$

$x_3 = x_2 + h = 2.1 + 0.05 = 2.15$

$x_4 = x_3 + h = 2.15 + 0.05 = 2.2$.

$x_1 = x_0 + h = 2 + 0.05 = 2.05$
$x_2 = x_0 + 2h = 2 + 0.1 = 2.1$
$x_3 = x_0 + 3h = 2 + 0.15 = 2.15$
$x_4 = x_0 + 4h = 2 \cancel{+} 0.2 = \underline{2.2}$

$n = 1$

$y_1 = y_0 + h f(x_0 y_0)$
$y_1 = y_0 + h(-x_0 y_0^2)$
$y_1 = y_0 + (0.05) . (-2 \times 1^2)$
$y_1 = 1 + (0.05)(-2)$

$y_1 = 1 - 0.1$
$y_1 = 0.9$

$n = 2$

$y_2 = y_1 + h f(x_1 y_1)$
$= 0.9 + (0.05)(-x_1 y_1^2)$

$= 0.9 + (0.05)(-2.05 \times (0.9)^2)$

$= 0.9 + (0.05)[-2.05 \times (0.9)^2]$
$= 0.9 - 0.08302$
$= 0.81697$

$n = 3$

$y_3 = y_2 + hf(x_2 y_2)$

$= y_2 + h \times (-x_2 y_2^2)$
$= 0.81697 + (0.05)(-2.1 \times 0.81697)$
$= 0.81697 -$
$y_3 = 74689$

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$n=4$

$$y_4 = y_3 + h\, f(x_3, y_3)$$
$$= 0.74689 + (0.05)\left[(2.15)(0.74689)^2\right]$$

$$y_4 = y(2.2) = 0.68692 \,/\, \underline{0.68692}$$

Q: find $y(0.04)$ using Euler's method from the equation $y' = -y$ with $\underline{y(0) = 1}$

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