vi) $dy/dx = x+y$ $y(0)=0$ & $h=0.2$ & find $y(0.2)$ $y(0.4)$ $y(0.6)$

Q: Taylor's Series -

We consider the differential equation

$$y' = f(x,y) \quad \text{---} (1)$$

with initial condition $y(x_0) = y_0$

- If $y(x)$ is exact sol$^n$ of $y'$ or eq$^n$ (1) then Taylor's series for $y(x)$ around $x = x_0$ is given by

$$y(x) = y_0 + (x-x_0)y_0' + \frac{(x-x_0)^2}{2!}y_0'' + \frac{(x-x_0)^3}{3!}y_0''' + \frac{(x-x_0)^4}{4!}y_0''''$$

- if the values of $y_0'$, $y_0''$ ... are known then we get the exact sol$^n$ of given ordinary equation.

Q: From Taylor series for $y(x)$, find $y(0.1)$ correct upto four decimal places if $y(x)$ satisfies.

$$y' = x - y^2 \quad \text{and} \quad y(0) = 1$$

Sol$^n$ given

$$y' = x - y^2$$

$$y(x_0) = y_0 \quad \text{i.e.} \quad x_0 = 0 \quad y_0 = 1$$

Page No.:

$\therefore y_0' = x_0 - y_0^2 = 0 - (1)^2 = -1$

$y_0'' = 1 - 2yy_0' = 1 - 2(1)(-1) = 1 + 2 = 3$

$y_0''' = 0 - 2yy_0'' - 2y_0'^2 = -2(1)(3) - 2(-1)^2$

$= -6 - 2 = -8$

$y_0^{III} = -2y_0y_0''' - 2y_0'y_0'' - 2(2y_0'y_0'')$

$= -2y_0y_0''' - 2y_0'y_0'' - 4y_0'y_0''$

$= -2y_0y_0''' - 6y_0'y_0''$

$= -2(1)(-8) - 6(-1)(3)$

$= 16 - 18$

$= 34$

$y_0^{IV} = -2y_0y_0^{IV} - 2y_0''y_0''' - 6y_0'y_0''' - 6y_0''^2$

$= -2y_0y_0^{IV} - 8y_0'y_0''' - 6y_0''^2$

$= -2(1)(34) - 8(-1)(-8) - 6(3)^2$

$= -68 - 64 - 54$

$= -186$

Using values Taylor series is given by

$y(x) = y_0 + (x-x_0)y_0' + \frac{(x-x_0)^2}{2!}y_0'' + \frac{(x-x_0)^3}{3!}y_0''' + \cdots$

Date : / /
Page No.:

$x_0 = 0$

$\therefore$

$$y(x) = y_0 + xy_0' + \frac{x^2}{2!}y_0'' + \frac{x^3}{3!}y_0''' + \frac{x^4}{4!}y_0^{IV} + \frac{x^5}{5!}y_0^{V}$$

putting all values.

$$y(0.1) = 1 + (0.1)(-1) + \frac{(0.1)^2}{2}(3) + \frac{(0.1)^3}{3\times2\times1}(-8) + \frac{(0.1)^4}{4\times3\times2\times1}(34) + \frac{(0.1)^5}{5\times3\times2\times4\times1}(-184)$$

$$y(0.1) = 0.9138$$

Q: Given the differential equation

$$y'' - xy' + y = 0$$

with the conditions $y(0) = 1$ and $y'(0) = 0$ use the Taylors series method to determine the value of $y(0.1)$