

Difference.

Newton's forward Interpolation formula:-  
given

x	$x_0$	$x_1$	...	$x_n$
y	$y_0$	$y_1$		$y_n$

$$x_1 = x_0 + h, \quad x_2 = x_0 + 2h, \quad x_3 = x_0 + 3h \dots$$

$$x_n = x_0 + nh$$

i.e (equal interval)

$$\begin{aligned} y \text{ if } f(x) = & y_0 + p \Delta y_0 + \frac{p(p-1)}{2!} \Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!} \Delta^3 y_0 \\ & + \frac{p(p-1)(p-2)(p-3)}{4!} \Delta^4 y_0 + \dots \\ & \frac{p(p-1)(p-2)(p-3) \dots (p-(n-1))}{(n+1)!} \Delta^{n+1} y_0 \end{aligned}$$

where  $p = \frac{x - x_0}{h}$  i.e  $x = x_0 + ph$ .

Q. forward difference table formation

$x$	$y$	$\Delta y$	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
$x_0$	$y_0$	$\Delta y_0$				
$x_1$	$y_1$	$\Delta y_1 = y_1 - y_0$	$\Delta^2 y_0 = \Delta y_1 - \Delta y_0$	$\Delta^3 y_0 = \Delta^2 y_1 - \Delta^2 y_0$		
$x_2$	$y_2$	$\Delta y_2 = y_2 - y_1$	$\Delta^2 y_1 = \Delta y_2 - \Delta y_1$	$\Delta^3 y_1 = \Delta^2 y_2 - \Delta^2 y_1$	$\Delta^4 y_0 = \Delta^3 y_1 - \Delta^3 y_0$	$\Delta^5 y_0 = \Delta^4 y_1 - \Delta^4 y_0$
$x_3$	$y_3$	$\Delta y_3 = y_3 - y_2$	$\Delta^2 y_2 = \Delta y_3 - \Delta y_2$	$\Delta^3 y_2 = \Delta^2 y_3 - \Delta^2 y_2$	$\Delta^4 y_1 = \Delta^3 y_2 - \Delta^3 y_1$	
$x_4$	$y_4$	$\Delta y_4 = y_4 - y_3$	$\Delta^2 y_3 = \Delta y_4 - \Delta y_3$	$\Delta^3 y_3 = \Delta^2 y_4 - \Delta^2 y_3$		
$x_5$	$y_5$	$\Delta y_5 = y_5 - y_4$				

Q. find the cubic polynomial which takes following values

$f(x)$	$x$	0	1	2	3
	$y$	1	2	1	10

Find  $f(4)$  and  $f'(4) = ?$

(Q) given The forward difference table for given data points.

	$x$	$y$	$\Delta y$	$\Delta^2 y$	$\Delta^3 y$
0	1	1	1		
1	2	2	-1	12	
2	1	1	10		
3	10				

$$\text{here } x_0=0 \quad h=1 \quad p=x$$

$$\Rightarrow p = \frac{x-x_0}{h} = \frac{x-0}{1} = x$$

$$P=x$$

By Newton's forward difference formula

$$\begin{aligned}
 y &= f(x) = y_0 + P \Delta y_0 + \frac{P(P-1)}{2!} \Delta^2 y_0 + \frac{P(P-1)(P-2)}{3!} \Delta^3 y_0 \\
 &= 1 + (x)(1) + \frac{(x)(x-1)(x-2)}{2!} + \frac{(x)(x-1)(x-2)(x-3)}{3!} \\
 &= 1 + x + (f_1)(x-1)(-1) + [(x)(x-1)(x-2)] \\
 &= 1 + x - [(x)(x-1)] + [2x(x-1)(x-2)] \\
 &= 1 + x - [x^2 - x] + [2x[x^2 - 3x + 2]] \\
 &= 1 + x - x^2 + x^3 - 2x^2 + 6x \\
 &= 2x^3 - 7x^2 + 8x + 1
 \end{aligned}$$

Q. forward difference table formation

$x$	$y$	$\Delta y$	$\Delta^2 y$	$\Delta^3 y$
$x_0$	$y_0$	$\Delta y_0$		
$x_1$	$y_1$	$\Delta y_1 = y_1 - y_0$	$\Delta^2 y_0 = \Delta y_1 - \Delta y_0$	
$x_2$	$y_2$	$\Delta y_2 = y_2 - y_1$	$\Delta^2 y_1 = \Delta y_2 - \Delta y_1$	$\Delta^3 y_0 = \Delta^2 y_1 - \Delta^2 y_0$
$x_3$	$y_3$	$\Delta y_3 = y_3 - y_2$	$\Delta^2 y_2 = \Delta y_3 - \Delta y_2$	$\Delta^3 y_1 = \Delta^2 y_3 - \Delta^2 y_2$
$x_4$	$y_4$	$\Delta y_4 = y_4 - y_3$	$\Delta^2 y_3 = \Delta y_4 - \Delta y_3$	$\Delta^3 y_2 = \Delta^2 y_4 - \Delta^2 y_3$
$x_5$	$y_5$	$\Delta y_5 = y_5 - y_4$	$\Delta^2 y_4 = \Delta y_5 - \Delta y_4$	$\Delta^3 y_3 = \Delta^2 y_5 - \Delta^2 y_4$

Q. Find the cubic polynomial which takes following values.

$f(x)$	0	1	2	3
	1	2	1	10

Find  $f(4)$  and  $f'(4) = ?$

(Q) Given The forward difference table for given data points.

$x$	$y$	$\Delta y$	$\Delta^2 y$	$\Delta^3 y$
0	1	1		
1	2	-1	12	
2	1	10		
3	10	g		

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$$\Delta^4 y$$

$$\Delta^5 y$$

$$\text{here } x_0 = 0, y_0 = 1, h = 1$$

$$\Rightarrow p = \frac{x - x_0}{h} = \frac{x - 0}{1} = x$$

$$P = x$$

By Newton's forward difference formula

$$\begin{aligned}
 y &= f(x) = y_0 + p\Delta y_0 + p(p-1)\frac{\Delta^2 y_0}{2!} + p(p-1)(p-2)\frac{\Delta^3 y_0}{3!} \\
 &= 1 + (x)(1) + (x)(x-1)\frac{\Delta^2 y_0}{2!} + (x)(x-1)(x-2)\frac{\Delta^3 y_0}{3!} \\
 &= 1 + x + [(x)(x-1)(-1)] + [(x)(x-1)(x-2)\frac{1}{2}] \\
 &= 1 + x - [(x)(x-1)] + [2x(x-1)(x-2)\frac{1}{2}] \\
 &= 1 + x - [x^2 - x] + [2x[x^2 - 3x + 2]\frac{1}{2}] \\
 &= 1 + x - x^2 + x + [2x[x^2 - 3x + 2]\frac{1}{2}] \\
 &= 2x^3 - 7x^2 + 6x + 1
 \end{aligned}$$

$$\begin{aligned}
 f(4) &= 2(4)^3 - 7(4)^2 + 6(4) + 1 \\
 &= 2(64) - 7(16) + 24 + 1 \\
 &= 128 - 112 + 24 + 1 \\
 &= 16 + 24 + 1 \\
 &= 41
 \end{aligned}$$

$$\begin{aligned}
 f(x) &= 6x^2 - 14x + 6 \\
 f(4) &= 6(16) - 14(4) + 6 \\
 &= 96 - 56 + 6 \\
 &= 40 + 6 \\
 &= 46
 \end{aligned}$$

Q: form the table. estimate the No. of student  
who obtained Marks betw 40 and 45

Marks (X)	30-40	40-50	50-60	60-70	70-80	80
No. of student (Y)	31	42	51	35	31	

Rearranging the table as.

Marks below (X)	No. of student (Y)	$\Delta Y$	$\Delta^2 Y$	$\Delta^3 Y$	$\Delta^4 Y$
40	31		42		
50	73	51	9	-25	
60	124	35	-15	+12	37
70	159	31	-4		
80	190				

$$\text{Let } x = 45 \quad x_0 = 40 \quad h = 10$$

$$P = \frac{x - x_0}{h} = \frac{45 - 40}{10} = 5/10 = 1/2 = 0.5$$

By Newton's forward Difference Interpolation formula.

$$y - f(x) = y_0 + \Delta y_0 (P) + P \frac{\Delta^2 y_0}{2!} + P(P-1) \frac{\Delta^3 y_0}{3!} + P(P-1)(P-2) \frac{\Delta^4 y_0}{4!}$$

$$= 31 + (0.5)(42) + \frac{(0.5)(0.5-1)(9)}{2} + \frac{(0.5)(0.5-1)(0.5-2)}{3!} \times (-25)$$

$$+ \frac{(0.5)(0.5-1)(0.5-2)(0.5-3)}{4 \times 3 \times 2 \times 1} (37)$$

$$= 31 + (0.5)(42) + (0.5) \frac{(-0.5)(9)}{2} + \frac{(0.5)(-0.5)(-1.5) \times (-25)}{6}$$

$$+ \frac{(0.5)(-0.5)(-1.5)(-2.5)}{24} (37)$$

$$= 31 + 21 - 6.125 + 1.4453$$

$$f(45) = 47.87 \approx 48 \quad \text{No. of student having marks.}$$

Q.  $x$   $y$

40	0.8	5.25	10.25	15.25	20.25	(Forward) X
45	1.0	1.6	3.8	8.2	15.4	(Forward) Y

37. No. of student marks betw 40 and 45 = 48 - 31 = 17