

Romberg's Method:-

- ~~Trapezoidal Rule~~.
- This Method is often be used to improved the approximate results obtain by the finite-difference method.
- It's applⁿ to the numerical evaluation of definite integrals. eg. in the use of trapezoidal rule,
- we consider the definite integral

$$I = \int_a^b y \, dx.$$

we evaluate it by the trapezoidal rule with two or more different subintervals of width, h_1, h_2, h_3 obtain approximate value I_1, I_2, I_3 respectively.

we take $h_1 = b$, $h_2 = b/2$, $h_3 = b/4$

find,

$\therefore I(h), I(h/2), I(h/4)$.

working rule

Step-I

given definite integral

$$I = \int_a^b y dx.$$

Step-II

find val evaluate it by trapezoidal Rule with interval $h, h/2, h/4$ and obtain $I_1 = I(h), I_2 = I(h/2), I_3 = I(h/4)$ respectively.

Step-III

form following table.

$$I_1 = I(h)$$

$$I_4 = I(h, h/2)$$

$$I_2 = I(h/2)$$

$$I_6 = I(h, h/2, h/4)$$

$$I_5 = I(h/2, h/4)$$

$$I_3 = I(h/4)$$

$$\therefore I(h, h/2) = I_2 + \frac{1}{3}(I_2 - I_1)$$

$$I(h/2, h/4) = I_3 + \frac{1}{3}(I_3 - I_2)$$

$$I(h, h/2, h/4) = I_5 + \frac{1}{3}(I_5 - I_4)$$

Q. Use Romberg's method to compute:-

$$I = \int_0^1 \frac{1}{1+x} dx.$$

Correct upto four decimal place