

Note:- Row operation — There are 3 kind of elementary Row operation

- ① Switching of Rows:- A row can be Interchange.  
with another row  $R_i \leftrightarrow R_j$

- ② Row multiplication:- A non zero number can be multiplied ~~or~~ to every element in a row.
- $$aR_j \rightarrow R_j \quad \text{where } a \neq 0$$

- ② Row addition (subtraction):- we may replace a row by the sum of element of row and a multiple of corresponding element of another row.
- $$R_i + aR_j \rightarrow R_i \quad \text{when } a \neq 0, i \neq j$$

$$RRP \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

## Gauss Elimination Method:-

also called forward elimination method.

- used to solve the linear / system of linear equation by using  $Ax=b$  augmented matrix (consisting of equation in  $n$  unknown)

$$[A | b] = \left[ \begin{array}{cccc|c} a_{11} & a_{12} & \dots & a_{1n} & b_1 \\ a_{21} & a_{22} & \dots & a_{2n} & b_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} & b_m \end{array} \right]$$

to an upper triangular matrix form

$$\left[ \begin{array}{cccc|c} c_{11} & c_{12} & \dots & c_{1n} & d_1 \\ 0 & c_{22} & \dots & c_{2n} & d_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & c_{nm} & d_m \end{array} \right]$$

Working Rule:-

Step-I Consider the system of linear equation.

$$\begin{aligned} a_1x + b_1y + c_1z &= d_1 \\ a_2x + b_2y + c_2z &= d_2 \\ a_3x + b_3y + c_3z &= d_3 \end{aligned}$$

i.e.  $Ax=B$

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

Step-II find augmented matrix for given system.  
 $C = [A : B]$



Step-I Transform augmented matrix into upper triangular form / row echelon form using some Row operation.

Step-II find equations corresponding to upper triangular matrix.

Step-III using back substitution find  $(x, y, z)$  soln of given system of eqn.

Q. solve the system of linear equation by Gauss elimination method.

equations

$$2x - y + 3z = 9$$

$$x + y + z = 6$$

$$x - y + z = 2$$

Soln given system of equation can be written as  $AX=B$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 2 \\ 9 \end{bmatrix}$$

Step-I Now find augmented matrix given system.

$$C = [A:B]$$

$$C = \left[ \begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & -1 & 1 & 2 \\ 2 & -1 & 3 & 9 \end{array} \right]$$

Step-II Transform augmented matrix into upper triangular matrix by

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$$R_2 \rightarrow R_2 - R_1 \quad \text{and} \quad R_3 \rightarrow R_3 - 2R_1$$

$$C = \left[ \begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -2 & 0 & -4 \\ 0 & -3 & 1 & -3 \end{array} \right]$$

$$\text{By } R_3 \rightarrow 2R_3 - 3R_2$$

$$\left[ \begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -2 & 0 & -4 \\ 0 & 0 & 2 & 6 \end{array} \right]$$

$$x + y + z = 6$$

$$-2y = -4$$

$$2z = 6$$

$$z = 6/2 = 3$$

$$y = 2$$

$$x + 2 + 3 = 6 - 5$$

$$x = 1$$

$$\boxed{\begin{array}{l} x = 1 \\ \therefore x = 1, y = 2, z = 3 \end{array}}$$