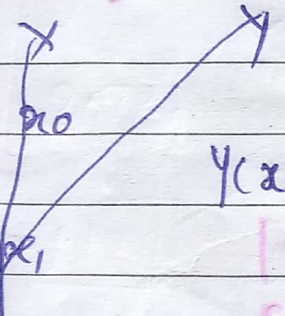
Stirling (central) Difference Interpolation Formula

Consider the following Data points with equal interval.

X	x_0	x_1	x_2	x_3	x_4	x_5
Y	y_0	y_1	y_2	y_3	y_4	y_5

$x = x_i = f(x_i) = y(x_i) = ?$



$$y(x) = y_0 + \left(\frac{\Delta y_0 + \Delta y_{-1}}{2}\right)p + \frac{p^2}{2!}(\Delta^2 y_{-1}) + \frac{p(p^2-1)}{3!}\left(\frac{\Delta^3 y_{-1} + \Delta^3 y_{-2}}{2}\right) + \frac{p^2(p^2-1^2)}{4!}\Delta^4 y_{-2} + \frac{p(p^2-1^2)(p^2-2^2)}{5!}\left(\frac{\Delta^5 y_{-2} + \Delta^5 y_{-3}}{2}\right) + \frac{(p^2)(p^2-1^2)(p^2-2^2)}{6!}\Delta^6 y_{-3} + \cdots$$

where $p = \frac{(x - x_0)}{h}$

Formation of Central Difference Interpolation table

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
x_{-3}	y_{-3}			
		$\Delta y_{-3} = y_{-2} - y_{-3}$		
x_{-2}	y_{-2}		$\Delta^2 y_{-3} = \Delta y_{-2} - \Delta y_{-3}$	
		$\Delta y_{-2} = y_{-1} - y_{-2}$		$\Delta^3 y_{-3}$
x_{-1}	y_{-1}		$\Delta^2 y_{-2} = \Delta$	
		$\Delta y_{-1} = y_0 - y_{-1}$		$\Delta^3 y_{-2}$
x_0	y_0		$\Delta^2 y_{-1} =$	
		$\Delta y_0 = y_1 - y_0$		$\Delta^3 y_{-1}$
x_1	y_1		$\Delta^2 y_0 =$	
		$\Delta y_1 = y_2 - y_1$		
x_2	y_2			

$\Delta^4 y$ $\Delta^5 y$

$\Delta^4 y_{-3}$ — $\Delta^5 y_{-3}$

$\Delta^4 y_{-2}$

Q: Solve the following data point using central difference interpolation to find $x = 35$.

x	20	30	40	50
y	512	439	346	243
		Δy	$\Delta^2 y$	$\Delta^3 y$

Let $x_0 = 40$ then central table for given data.

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
20 y_{-2}	512 y_{-2}			
		$\Delta y_{-2} = -73$		
30 y_{-1}	439 y_{-1}		$\Delta^2 y_{-2} = -20$	
		$\Delta y_{-1} = -93$		$\Delta^3 y_{-2} = +10$
40 x_0	346 y_0		$\Delta^2 y_{-1} = -10$	
		$\Delta y_0 = -103$		
50 x_1	243 y_1			

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Given $x=35$ $x_0=40$ $h=10$

$$p=\frac{35-40}{10}=-\frac{5}{10}=-0.1/2=-0.5$$

Now Centeral Difference stirling formula

$$P_n(x)=y(35)=y_0+p\left(\frac{\Delta y_0+\Delta y_{-1}}{2}\right)+\frac{p^2}{2!}(\Delta^2 y_{-1})$$

$$=346+(-0.5)\left(\frac{-93-103}{2}\right)+\frac{(0.5)^2}{2}(-10)$$

$$=346+(0.5)(98)+\frac{(0.25)(-10)}{2}$$

$$=346-1.25+49.0$$

$$=395-1.25$$

$$=393.75\cong 394$$

Q: Use centeral diff. Int. formula to evaluate. $f(1.22)$

x	1.0	1.1	1.2	1.3
$f(x)$	8.403	8.781	9.129	9.451

$p=\frac{1.22-1.2}{0.1}$