

* Runge kutta Methods -

Consider the ordinary differential equation.

$$\frac{dy}{dx} = f(x, y), \quad y(x_0) = y_0$$

To find $y(x_n) \rightarrow$ where x_n - any value.

~~also~~ By using Runge kutta method.

$$y_{n+1} = y_n + \frac{1}{6} [k_1 + 2k_2 + 2k_3 + k_4]$$

$$\text{where } k_1 = h f(x_n, y_n)$$

$$k_2 = h f(x_n + h/2, y_n + k_1/2)$$

$$k_3 = h f(x_n + h/2, y_n + k_2/2)$$

$$k_4 = h f(x_n + h, y_n + k_3)$$

Q Using Runge kutta method order 4, solve

$$\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2} \quad \text{with } y(0) = 1 \quad x = 0.2$$

Solⁿ given.

$$f(x, y) = \frac{y^2 - x^2}{y^2 + x^2}$$

$$x_0 = 0$$

$$y_0 = y(x_0) = 1$$

taking $h = 0.2$

$$\Rightarrow x_1 = x_0 + h = 0 + 0.2 = 0.2$$

$$x_2 = x_1 + h = 0.2 + 0.2 = 0.4$$

To find $y(0.2) = y_1$ & $y(0.4) = y_2$

By using Range katta 4th order method.

$$y_{n+1} = y_n + \frac{1}{6} [k_1 + 2k_2 + 2k_3 + k_4] \quad \text{--- (1)}$$

Step-1

put $n=0$ into eqⁿ (1)

$$y_1 = y_0 + \frac{1}{6} [k_1 + 2k_2 + 2k_3 + k_4]$$

$$\therefore k_1 = h f(x_0, y_0)$$

$$k_1 = (0.2) \frac{1-0}{1+0} = (1)(0.2) = 0.2$$

$$k_2 = h f(x_0 + h/2, y_0 + k_1/2)$$

$$= (0.2) f(0 + 0.2/2, 1 + 0.2/2)$$

$$= (0.2) f(0 + 0.1, 1 + 0.1)$$

$$= (0.2) f(0.1, 1.1)$$

$$= (0.2) \frac{(1.1)^2 - (0.1)^2}{(1.1)^2 + (0.1)^2} = \frac{1.21 - 0.01}{1.21 + 0.01} = \frac{1.2}{1.22} = 0.9836$$

$$k_3 = h f(x_0 + h/2, y_0 + k_2/2)$$

$$= h f(0 + 0.2/2, 1 + \frac{0.9836}{2})$$

$$= h f(0 + 0.1, 1 + 0.4918)$$

$$= (0.2) f(0.1, 1.4918)$$

$$= (0.2) \frac{(1.4918)^2 - (0.1)^2}{(1.4918)^2 + (0.1)^2} = 0.1967$$

$$\begin{aligned}
 K_4 &= h f(x_0 + h, y_0 + k_3) \\
 &= (0.2) f(0 + 0.2, 1 + 0.1967) \\
 &= (0.2) f(0.2, 1.1967) \\
 K_4 &= 0.1891
 \end{aligned}$$

put K_1, K_2, K_3 and K_4 .

$$y_1 = y(0.2) = 1 + \frac{1}{6} [0.2 + (0.19672)(2) + (2)(0.1967) + 0.1891]$$

$$y(0.2) = y_1 = 1.19599 \approx \underline{\underline{1.196}}$$

Step-II put $n=1$ into eqn (I).

$$y_2 = y(0.4) = y_1 + \frac{1}{6} [K_1 + 2K_2 + 2K_3 + K_4]$$

$$\begin{aligned}
 K_1 &= h f(x_1, y_1) = (0.2) f(0.2, 1.196) \\
 &= (0.2) \left(\frac{x_1^2 - y_1^2}{y_1^2 - x_1^2} \right) = 0.1891
 \end{aligned}$$

$$K_2 = h f(x_1 + h/2, y_1 + K_1/2)$$

$$= (0.2) f(0.2 + 0.2/2, 1.196 + \frac{0.1891}{2})$$

$$= (0.2) f(0.3, 1.196 + 0.09455)$$

$$= (0.2) f(0.3, 1.1966) = 0.1795$$

$$\downarrow \\
 0.1795$$

$$k_3 = h f(x_1 + h/2, y_1 + k_2/2) = (0.2 + 0.1, 1.196 + \frac{0.1795}{2})$$

$$= (0.3, 1.196 + 0.08975)$$

$$= (0.3, 1.28575) (0.2)$$

$$k_3 = 0.1793$$

$$k_4 = h f(x_1 + h, y_1 + k_3)$$

$$= (0.2) (0.2 + 0.2, 1.196 + 0.1793)$$

$$= (0.2) f(0.4, 1.3753)$$

$$= 0.1688$$

putting k_1, k_2, k_3 & k_4 into eqn

$$y_2 = y(0.4) = y_1 + \frac{1}{6} [k_1 + 4k_2 + k_3 + k_4] = 1.196 + \frac{1}{6} [0.1891 + 2(0.1795) + 2(0.1793) + 0.1688]$$

$$y_2 = y(0.4) = 1.3752$$

Q:- Use Runge Kutta 4th order method to find the value of y when $x=1$ given $y(0)=1$ and $\frac{dy}{dx} = \frac{y-x}{y+x}$