

* What is linear eqⁿ?

A linear equation is the eqⁿ written in the form
 $a\pi + b = 0$

where a,b are real numbers and π is variables

This form is sometime called Standard form of
linear equation.

System of linear eqn:-

A system of linear equation is when we have two or more linear equation working together.

$$a_1x + b_1y + c_1z = d_1$$

$$a_2x + b_2y + c_2z = d_2$$

$$a_3x + b_3y + c_3z = d_3$$

This is Standard form of System of linear eqn.

x, y, z = variable

$a_1, b_1, c_1, a_2, b_2, c_2, a_3, b_3, c_3$ and d_1, d_2, d_3 is constant numerical value.

This system of linear eqn we write into form of

$$AX = B$$

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

Working Rule

Let we the following system of linear eqn.

$$\rightarrow a_1x + b_1y + c_1z = d_1$$

$$a_2x + b_2y + c_2z = d_2$$

$$a_3x + b_3y + c_3z = d_3$$

$a_1, a_2, a_3, b_1, b_2, b_3, c_1, c_2, c_3$ are the coefficient of x, y, z simultaneously respectively.

d_1, d_2, d_3 are constant Numerical values.

We write above eqn into matrix form.

$$x = 3.634240.$$

Linear Equation - Cramer's rule:-

- Cramer's rule is an algorithm for solving linear system using determinants.
- Cramer's rule can only be used with linear system that have exactly one soln.
- Cramer's rule is named after Gabriel Cramer's a Swiss mathematician.
- It uses determinants to find the solⁿ of linear eqⁿ/ system
- Given a system of linear equation, Cramer's rule is a handy way to solve just one of the variable without having to solve the whole system of equation i.e Instead of solving the entire system of equation you can use Cramer's rule to solve for just one single variables.

Wronski Rule

Let us the following system of linear eqn.

$$\rightarrow a_1x_1 + b_1y_1 + c_1z_1 = d_1$$

$$a_2x_2 + b_2y_2 + c_2z_2 = d_2$$

$$a_3x_3 + b_3y_3 + c_3z_3 = d_3$$

$a_1, a_2, a_3, b_1, b_2, b_3, c_1, c_2, c_3$ are the coefficient of x_1, y_1, z_1 simultaneously respectively.

d_1, d_2, d_3 are constant numerical values.

We write above eqn into matrix form.

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \rightarrow \text{coefficient matrix}$$

$$= \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} \rightarrow \text{constant value matrix}$$

Take $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

$$D_x = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix} \quad D_y = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}$$

$$D_z = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$$

Standard way to find determinant

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$= a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_2 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$$

$$= a_1 [(b_2 c_3) - (b_3 c_2)] - b_2 [a_2 c_3 - a_3 c_2] + c_1 [a_2 b_3 - a_3 b_2]$$

Now we calculate Dx , Dy , Dz with respect to
according to crammer's Rule.

$$\boxed{x = \frac{Dx}{D} \quad y = \frac{Dy}{D} \quad z = \frac{Dz}{D}}$$

$\therefore \frac{-2}{3+6} - 1$

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Let us use the following system of linear equation.

$$2x + y + z = 3$$

$$x - y - z = 0$$

$$x + 2y + z = 0$$

- we have the left hand side of the system with the variables and right hand side with the numerical values (Answer value).

- Let D be determinant of the coefficient matrix.

(सोंगि) ∵ system of eqⁿ

$$2x + y + z = 3$$

$$x - y - z = 0$$

$$x + 2y + z = 0$$

coefficient Matrix determinant

$$\begin{vmatrix} 2 & 1 & 1 \\ 1 & -1 & -1 \\ 1 & 2 & 1 \end{vmatrix}$$

Answer column value

$$\begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}$$

∴ coefficient Matrix determinant

$$D = \begin{vmatrix} 2 & 1 & 1 \\ 1 & -1 & -1 \\ 1 & 2 & 1 \end{vmatrix}$$

∴ D_x is defined coefficient determinant with answer column values in a column.

$$\therefore \text{we get } D_x = \begin{vmatrix} 3 & 1 & 1 \\ 0 & -1 & -1 \\ 0 & 2 & 1 \end{vmatrix}$$

1) Try D_y on D_2

$$D_y = \begin{vmatrix} 2 & 3 & 1 \\ 1 & 0 & -1 \\ 1 & 0 & 1 \end{vmatrix} \quad \text{and} \quad D_2 = \begin{vmatrix} 2 & 1 & 3 \\ 1 & -1 & 0 \\ 1 & 2 & 0 \end{vmatrix}$$

Evaluating each determinant we get.

$$D = \begin{vmatrix} 2 & 1 & 1 \\ 0 & -1 & -1 \\ 0 & 2 & 1 \end{vmatrix} = 2 \begin{vmatrix} -1 & -1 \\ 2 & 1 \end{vmatrix} - 1 \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} + 1 \begin{vmatrix} 1 & -1 \\ 1 & 2 \end{vmatrix}$$

$$= 2(-1+2) - 1[1-(-1)] + 1[2-(-1)]$$

$$= 2(1) - 1(2) + 1(3)$$

$$= 2 - 2 + 3$$

$$\boxed{D = 3}$$

$$D_x = \begin{vmatrix} 3 & 1 & 1 \\ 0 & -1 & -1 \\ 0 & 2 & 1 \end{vmatrix} = 3 \begin{vmatrix} -1 & -1 \\ 2 & 1 \end{vmatrix} - 1 \begin{vmatrix} 0 & -1 \\ 0 & 1 \end{vmatrix} + 1 \begin{vmatrix} 0 & -1 \\ 0 & 2 \end{vmatrix}$$

$$= 3[-1 - (-2)] - 1[0 - 0] + 1[0 - 0]$$

$$= 3(-1 + 2) - 1(0) + 1(0)$$

$$= 3(1) - 0 + 0$$

$$\boxed{D_x = 3}$$

$$\text{Try } D_y = \begin{vmatrix} 2 & 3 & 1 \\ 1 & 0 & -1 \\ 1 & 0 & 1 \end{vmatrix} = 2 \begin{vmatrix} 0 & -1 \\ 0 & 1 \end{vmatrix} - 3 \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} + 1 \begin{vmatrix} 1 & 0 \\ 1 & 0 \end{vmatrix}$$

$$\begin{aligned}
 &= 2(0-0) - 3[(1-(-1))] + 1[(0-0)] \\
 &= 2(0) - 3(-2) + 1(0) \\
 &= 0 - 6 + 0
 \end{aligned}$$

$$Dy = -6$$

$$\therefore D_2 = \begin{vmatrix} 2 & 1 & 3 \\ 1 & -1 & 0 \\ 1 & 2 & 0 \end{vmatrix} = 2 \begin{vmatrix} -1 & 0 \\ 2 & 0 \end{vmatrix} - 1 \begin{vmatrix} -1 & 0 \\ 1 & 0 \end{vmatrix} + 3 \begin{vmatrix} 1 & 0 \\ 1 & 2 \end{vmatrix}$$

$$\begin{aligned}
 &= 2[(0)-(0)] - 1[(0)-(0)] + 3[(2)-(-1)] \\
 &= 2(0) - 1(0) + 3(3) \\
 &= 0 - 0 + 9
 \end{aligned}$$

$$D_2 = 9$$

according to Crammer's rule.

$$x = Dx \div D$$

$$y = Dy \div D$$

$$z = Dz \div D$$

$$\therefore x = \frac{3}{3} = 1, \quad Dy = -\frac{6}{3} = -2, \quad D_2 = \frac{Dz}{D} = \frac{9}{3} = 3$$