$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Working Rule:-

Step I) Given $f(x)=0$ —— ①

find initial root x_0 such that $f(x_0) \sim 0$ (i.e x_0 is near to root of ①).

Step-II

find $f(x_0)$ and $f'(x_0)$

derivative of $f(x)$

Step-III

first approximate root by N-R method

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

find $f(x_1)$ and $f'(x_1)$

Step-IV - 2nd approximater $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

like that 3rd approximate Root

$$x_3 = x_2 - \frac{f(x_2)}{f(x_2)}$$

∴ General formula

$$x_n = x_{n-1} - \frac{f(x_{n-1})}{f'(x_n)}$$

Repeat the Step-IV upto get Correct accurate root.

Q ⇒ find the real root of eqⁿ $x^3 - 3x + 1 = 0$ by N/w Raphson method correct upto 4 decimal places.

Solⁿ given
Let $f(x) = 0$

$x^3 - 3x + 1 = 0$ ——— ①

To find x_0

$f(0) = (0) - 3(0) + 1 = 1 > 0$

$f(1) = 1 - 3(1) + 1 = -1 < 0$

$f(0.5) = (0.5)^3 - 3(0.5) + 1 = -0.375 < 0$

$f(0.4) = (0.4)^3 - 3(0.4) + 1 = -0.136 < 0$

$f(0.3) = (0.3)^3 - 3(0.3) + 1 = 0.127 > 0$

choosing $\boxed{x_0 = 0.3}$

$f(x_0) = 0.127$

$f'(x) = 3x^2 - 3 = 3(0.3)^2 - 3 = -2.73$

1st approximate Root by Newton Rapshon method

$$x_1 = x_0 - \left[\frac{f(x_0)}{f'(x_0)}\right]$$

$$= 0.3 - \left[\frac{0.127}{-2.73}\right] = 0.3 + \left[\frac{127}{273}\right] = 0.346520$$

$f(x_1) = x^3 - 3x + 1 = 0.002048$

$f'(x_1) = 3x^2 - 3 = -2.639771$

2nd approximate Root by N-R method

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 0.34652 - \frac{(0.002048)}{(-2.639771)}$$

$$x_2 = 0.347295$$

$f(x_2) = f(0.347295) = x^3 - 3x + 1 = 0.000035$

$f'(x_2) = f'(0.347295) = 3x^2 - 3 = 3(0.347295)^2 - 3 = -2.638158$

3rd approximate root by N-R method

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 0.347295 - \frac{0.000035}{(-2.638158)}$$

$$= 0.347295 - 0.000013266$$

$$= \underline{0.3472}$$

hence the approximate root correct upto 4 decimal places $\boxed{x = 0.3472}$ - Ans

Q.2 find the real root of eqⁿ cube of 48 by N-R method correct upto 4-decimal places.

Sol^n given

$$x^3 = 48$$
$$x^3 - 48 = 0$$

Let $\therefore f(x) = x^3 - 48 = 0$

Find x_0 $\quad f(0) = -48 \quad f(1) = -47 \quad f(2) = -40 < 0$

$f(3) = 27 - 48 = -21 < 0$

$f(4) = 64 - 48 = 16 > 0$

$\therefore$ Find $f(3.5) = (3.5)^3 - 48 = -5.125 < 0$

$f(3.6) = (3.6)^3 - 48 = -1.344 < 0$

$f(3.7) = (3.7)^3 - 48 = 2.6553 > 0$

by choosing $x_0 = 3.6 \quad f(x_0) = -1.344$

$f'(x) = 3x^2 \quad f'(3.6) = 3(3.6)^2 = 38.88$

1st approximate root by N-R Method.

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 3.6 - \frac{(-1.344)}{(38.88)} = 3.634567$$

$f(x_1) = x^3 - 48 = 0.012910$

$f'(x_1) = 3x^2 = 39.630231$

2nd approximate Root by N-R method.

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 3.634567 - \frac{0.012910}{(39.630231)}$$

$$= 3.634241$$

$f(x_2) = x^3 - 48 = 0.00007$

$f'(x_2) = 3x^2 = 3(3.634241)^2 = 39.623144$

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3rd approximate root by N-R method.

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 3.634241 - \frac{0.000007}{39.623144}$$

$$= \underline{3.634240}$$

hence, the approximate root correct upto 4 decimal places.

$$x = 3.634240.$$