Newton's forward Difference Interpolation formula:-

given

x	x_0	x_1	...	x_n
y	y_0	y_1		y_n

$x_1 = x_0 + h$, $x_2 = x_0 + 2h$, $x_3 = x_0 + 3h$ ....

$x_n = x_0 + nh$

i.e (equal interval

$$y = f(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0$$

$$+ \frac{p(p-1)(p-2)(p-3)}{4!}\Delta^4 y_0 + \ldots\ldots$$

$$\frac{p(p-1)(p-2)(p-3)\ldots(p-(n))}{(n+1)!}\Delta^{n+1} y_0$$

where $p = \frac{x - x_0}{h}$ i.e $x = x_0 + ph$.

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Q. forward difference table formation

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
x_0	y_0					
		$\Delta y_0 = y_1 - y_0$				
x_1	y_1		$\Delta^2 y_0 = \Delta y_1 - \Delta y_0$			
		$\Delta y_1 = y_2 - y_1$		$\Delta^3 y_0 = \Delta^2 y_1 - \Delta^2 y_0$		
x_2	y_2		$\Delta^2 y_1 = \Delta y_2 - \Delta y_1$		$\Delta^4 y_0 = \Delta^3 y_1 - \Delta^3 y_0$	
		$\Delta y_2 = y_3 - y_2$		$\Delta^3 y_1 = \Delta^2 y_2 - \Delta^2 y_1$		$\Delta^5 y_0 = \Delta^4 y_1 - \Delta^4 y_0$
x_3	y_3		$\Delta^2 y_2 = \Delta y_3 - \Delta y_2$		$\Delta^4 y_1 = \Delta^3 y_2 - \Delta^3 y_1$	
		$\Delta y_3 = y_4 - y_3$		$\Delta^3 y_2 = \Delta^2 y_3 - \Delta^2 y_2$		
x_4	y_4		$\Delta^2 y_3 = \Delta y_4 - \Delta y_3$			
		$\Delta y_4 = y_5 - y_4$				
x_5	y_5					

Q. Find the cubic polynomial which takes following values

	x	0	1	2	3
$f(x)$	y	1	2	1	10

find $f(4)$ and $f'(4) = ?$

sol^n given The forward difference table for given data points

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
0	$\boxed{1}$			
		$\boxed{1}$		
1	2		$\boxed{-2}$	
		-1		$\boxed{12}$
2	1		10	
		9		
3	10			

here $x_0 = 0 \quad y_0 = 1 \quad h = 1$

$\Rightarrow p = \frac{x - x_0}{h} = \frac{x - 0}{1} = x$

$p = x$

By Newton's forward diffesnce formular

$$y = f(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0$$

$$= 1 + (x)(1) + \frac{(x)(x-1)(-\cancel{2})}{\cancel{2}\times 1} + \frac{(x)(x-1)(x-2)(\cancel{12})}{\cancel{3}\times\cancel{2}\times 1}$$

$$= 1 + x + [(x)(x-1)(-1)] + [x(x-1)(x-2)2]$$

$$= 1 + x - [(x)(x-1)] + [2x(x-1)(x-2)]$$

$$= 1 + x - [x^2 - x] + [2x[x^2 - 3x + 2]]$$

$$= 1 + \cancel{x} - x^2 - \cancel{x} + 2x^3 - 6x^2 + 4x$$

$$= 2x^3 - 7x^2 + 6x + 1$$

Q. forward difference table formation

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
x_0	y_0					
		$\Delta y_0 = y_1 - y_0$				
x_1	y_1		$\Delta^2 y_0 = \Delta y_1 - \Delta y_0$			
		$\Delta y_1 = y_2 - y_1$		$\Delta^3 y_0 = \Delta^2 y_1 - \Delta^2 y_0$		
x_2	y_2		$\Delta^2 y_1 = \Delta y_2 - \Delta y_1$		$\Delta^4 y_0 = \Delta^3 y_1 - \Delta^3 y_0$	
		$\Delta y_2 = y_3 - y_2$		$\Delta^3 y_1 = \Delta^2 y_2 - \Delta^2 y_1$		$\Delta^5 y_0 = \Delta^4 y_1 - \Delta^4 y_0$
x_3	y_3		$\Delta^2 y_2 = \Delta y_3 - \Delta y_2$		$\Delta^4 y_1 = \Delta^3 y_2 - \Delta^3 y_1$	
		$\Delta y_3 = y_4 - y_3$		$\Delta^3 y_2 = \Delta^2 y_3 - \Delta^2 y_2$		
x_4	y_4		$\Delta^2 y_3 = \Delta y_4 - \Delta y_3$			
		$\Delta y_4 = y_5 - y_4$				
x_5	y_5					

Q. Find the cubic polynomial which takes following values

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find $f(4)$ and $f'(4) = ?$

Sol^n given. The forward difference table for given data points

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
0	$\boxed{1}$			
		$\boxed{1}$		
1	2		$\boxed{-2}$	
		-1		$\boxed{12}$
2	1		10	
		9		
3	10			

here $x_0 = 0$ $y_0 = 1$ $h = 1$

$$\Rightarrow p = \frac{x - x_0}{h} = \frac{x-0}{1} = x$$

$$p = x$$

By Newton's forward difference formular

$$y = f(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0$$

$$= 1 + (x)(1) + \frac{(x)(x-1)(-2)}{2\times 1} + \frac{(x)(x-1)(x-2)(12)}{3\times 2\times 1}$$

$$= 1 + x + [(x)(x-1)(-1)] + [x(x-1)(x-2)2]$$

$$= 1 + x - 1[(x)(x-1)] + [2x(x-1)(x-2)]$$

$$= 1 + x - [x^2 - x] + [2x[x^2 - 3x + 2]]$$

$$= 1 + x - x^2 + x + 2x^3 - 6x^2 + 4x$$

$$= 2x^3 - 7x^2 + 6x + 1$$

$f(4) = 2(4)^3 - 7(4)^2 + 6(4) + 1$

$= 2(64) - 7(16) + 24 + 1$

$= 128 - 112 + 24 + 1$

$= 16 + 24 + 1$

$= 41$

$f'(x) = 6x^2 - 14x + 6$

$f'(4) = 6(16) - 14(4) + 6$

$= 96 - 56 + 6$

$= 40 + 6$

$= 46$

Q: form the table. estimate the No. of student who obtained Marks betn 40 and 45

marks (X) :	30-40	40-50	50-60	60-70	70-80
No. of student (y)	31	42	51	35	31

Rearranging the table As.

Marks below (x)	No. of student (y)	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
40	$\boxed{31}$				
		$\boxed{42}$			
50	73		$\boxed{9}$		
		51		$\boxed{-25}$	
60	124		-16		$\boxed{37}$
		35		+12	
70	159		-4		
		31			
$\boxed{80}$	190				

Let $x = 45 \quad x_0 = 40 \quad h = 10$

$$p = \frac{x - x_0}{h} = \frac{45-40}{10} = 5/10 = 1/2 = 0.5$$

By Newton's forward Difference Interpolation Formula.

$$y = f(x) = y_0 + \Delta y_0 (p) + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0$$
$$+ \frac{p(p-1)(p-2)(p-3)}{4!}\Delta^4 y_0$$

$$= 31 + (0.5)(42) + \frac{(0.5)(0.5-1)}{2}(9) + \frac{(0.5)(0.5-1)(0.5-2)}{3 \times 2 \times 1} \times (-25)$$

$$\frac{(0.5)(0.5-1)(0.5-2)(0.5-3)}{4 \times 3 \times 2 \times 1}(37)$$

$$= 31 + (0.5)(42) + \frac{(0.5)(-0.5)}{2}(9) + \frac{(0.5)(-0.5)(-1.5) \times (-25)}{6}$$

$$+ \frac{(0.5)(-0.5)(-1.5)(-2.5)}{24}(37)$$

~~= 31 + 21 - 1.125 + 15.625 - 1.4453~~

$\boxed{f(45) = 47.87 \approx 48}$ (No. of student having marks belowus)

37

$\therefore$ No. of student marks betn 40 and 45 $= 48 - 31$
$= \boxed{17}$

Q.

x	0	5	10	15	20
y	1.0	1.6	3.8	8.2	15.4