

## \* Runge kutta Method:-

Consider the ordinary differential equation:

$$\frac{dy}{dx} = f(x, y), \quad y(x_0) = y_0$$

To find  $y(x_n)$  → where  $x_n$  - any value.

After By using Runge kutta method.

$$y_{n+1} = y_n + \frac{1}{6} [k_1 + 2k_2 + 2k_3 + k_4]$$

$$\text{where } k_1 = h f(x_n, y_n)$$

$$k_2 = h f\left(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$$

$$k_3 = h f\left(x_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right)$$

$$k_4 = h f(x_n + h, y_n + k_3)$$

Q Using Runge kutta method order 4, solve

$$\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2} \quad \text{with } y(0) = 1 \quad x=0, 2$$

Sol " given .

$$f(x, y) = \frac{y^2 - x^2}{y^2 + x^2} \quad x_0 = 0 \quad y_0 = y(x_0) = 1$$

taking  $h = 0.2$

$$\Rightarrow x_1 = x_0 + h = 0 + 0.2 = 0.2$$

$$x_2 = x_1 + h = 0.2 + 0.2 = 0.4$$

To find  $y(0.2) = y_1$  &  $y(0.4) = y_2$

By using Range-Kutta 4th order method.

$$y_{n+1} = y_n + \frac{1}{6} [k_1 + 2k_2 + 2k_3 + k_4] \quad \text{--- (1)}$$

Step-1 put  $n=0$  into eqn (1)

$$y_1 = y_0 + \frac{1}{6} [k_1 + 2k_2 + 2k_3 + k_4]$$

$$\therefore k_1 = h f(x_0, y_0)$$

$$k_1 = (0.2) \frac{1-0}{1+0} = (1)(0.2) = 0.2$$

$$k_2 = h f(x_0 + \frac{h}{2}, y_0 + k_1 \frac{h}{2})$$

$$= (0.2) f(0 + 0.1, y_0 + 0.1 \cdot 0.1)$$

$$= (0.2) f(0.1, 1.1)$$

$$= (0.2) \frac{(0.1)^2 - (0.1)^2}{(0.1)^2 + (0.1)^2} = \frac{1.21 - 0.01}{1.21 + 0.01} = 0.1967$$

$$0.19672$$

$$k_3 = h f(x_0 + \frac{h}{2}, y_0 + k_2 \frac{h}{2})$$

$$= h f(0 + 0.1, y_0 + 0.19672)$$

$$= h f(0.1, 1.9836)$$

$$= (0.2) f(0.1, 1.9836)$$

$$= 0.2 \frac{(1.9836)^2 - (0.1)^2}{(1.9836)^2 + (0.1)^2} = 0.1967$$

$$K_4 = h f(x_0 + h, y_0 + k_3) \\ = (0.2) f(0 + 0.2, 1 + 0.1967) \\ = (0.2) f(0.2, 1.1967) \\ K_4 = 0.1891$$

put  $K_1, K_2, K_3$  and  $K_4$ .

$$y_1 = y(0.2) = 1 + \frac{1}{6} [0.2 + (0.1967)(2) + (2)(0.1967) + 0.1891]$$

$$y(0.2) = y_1 = 1.19599 \approx \underline{\underline{1.196}}$$

Step-II put  $n=1$  into eqn ①

$$y_2 = y(0.4) = y_1 + \frac{1}{6} [k_1 + 2k_2 + 2k_3 + k_4]$$

$$k_1 = h f(x_1, y_1) = (0.2) f(0.2, 1.196) \\ = (0.2) \left[ \frac{(x_2^2 - x_1^2)}{y_2^2 - x_2^2} \right] = 0.1891$$

$$k_2 = h f(x_1 + h/2, y_1 + k_1/2)$$

$$= (0.2) f\left(0.2 + 0.2/2, 1.196 + \frac{0.1891}{2}\right)$$

$$= (0.2) f(0.3, 1.196 + 0.09455)$$

$$(0.2) f(0.3, 1.1966) = 0.1795$$

0.1795

$$k_3 = h f(x_1 + h/2, y_1 + k_2/2) = (0.2 + 0.1, 1.196 + \frac{0.1795}{2})$$

$$= (0.3, 1.196 + 0.08975)$$

$$= (0.3, 1.28575)(0.2)$$

$$k_3 = 0.1793$$

$$k_4 = h f(x_1 + h, y_1 + k_3)$$

$$= (0.2) (0.2 + 0.2), 1.196 + 0.1793$$

$$= (0.2) f(0.4, 1.3753)$$

$$= 0.1688$$

Putting  $k_1, k_2, k_3$  &  $k_4$  into eqn

$$y_2 = y(0.4) = y_1 + 1.196 + \frac{1}{6} [0.1891 + 2(0.1795) + 2(0.1793) + 0.1688]$$

$$y_2 = y(0.4) = 1.3752$$

Q:- Use Runge Kutta 4<sup>th</sup> order method to find the value of  $y$  when  $x = 1$  given  $y(0) = 1$ .

$$\text{and } \frac{dy}{dx} = \frac{y-x}{y+x}$$