Unit-I

# Solution of transcendal Polynomial eqⁿ and Lineer eqⁿ.



## (1) Bisection Method:-

- In this method ~~it~~ states that if function $f(x)$ is continous betⁿ $a$ and $b$ and $f(a)$ and $f(b)$ are of opposite signs. ∴ There exists atleast one root betⁿ $a, b$.

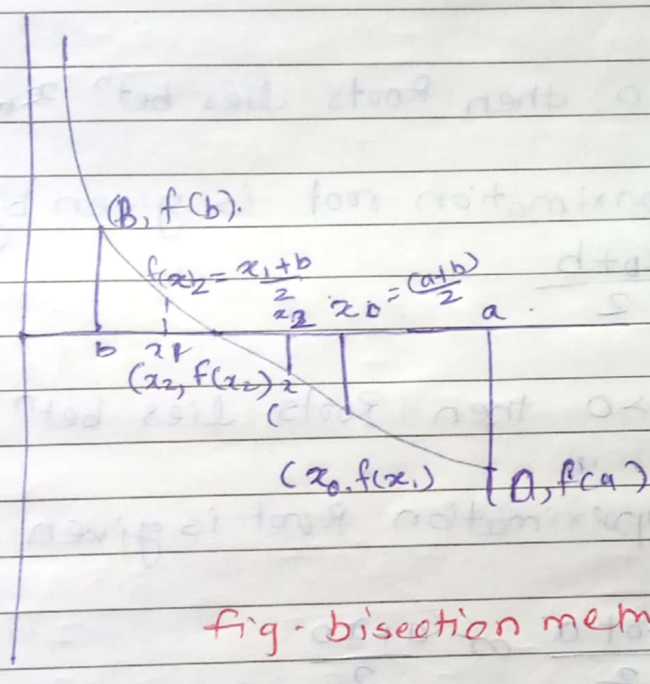


fig. bisection method.

## (1) working Rule:-

Let $f(x)$ is given
∴ $f(x) = 0$. beginer equation.

1 Step-I
find $a$ & $b$ such that
$f(a) < 0$ and $f(b) > 0$
or
$f(a) > 0$ and $f(b) < 0$

step-II
find first approximate root using bisection m.

$$x_0 = \frac{a+b}{2}$$

calculate $f(x_0)$ and examine.

sub step- II.I)
If $f(x_0) < 0$ then Roots lies betn $x_0$ and b.

∴ 2nd approximation root is given by

$$x_1 = \frac{x_0+b}{2}$$

otherwise

step-II II if $f(x_0) > 0$ then Roots lies betn $x_0$ and a.

∴ 2nd approximation Root is given

$$x_1 = \frac{x_0+a}{2} \text{ or } \frac{a+x_0}{2}$$

Calculate $f(x_1)$ and Repeate step-sub step II.I and II.II. for $x_2$

Until the required accuracy of root.

Q. find the real Root of equation $x^3 - x - 4 = 0$ using Bisection method correct upto 3 decimal places.

Soln given
$f(x) = x^3 - x - 4$
by method.

$f(x) = 0$

i.e. $x^3 - x - 4 = 0$

step - I find a, and b.

$f(0) = (0)^3 - 0 - 4 = -4 < 0$

$f(1) = 1 - 1 - 4 = -4 < 0$

$f(2) = 8 - 2 - 4 = 2 > 0$

$f(1.5) = (1.5)^3 - (1.5) - 4 = -2.125 < 0$

$f(1.6) = (1.6)^3 - (1.6) - 4 = -1.501 < 0$

$f(1.7) = (1.7)^3 - (1.7) - 4 = -0.787 < 0$

$f(1.8) = (1.8)^3 - 1.8 - 4 = 0.032 > 0$

$\therefore$ Root lies betn 1.7 and 1.8.

1st approximate root

$\therefore x_0 = \frac{a+b}{2} = \frac{1.7+1.8}{2} = \frac{3.5}{2} = 1.75$

$\therefore f(1.75) = (1.75)^3 - 1.75 - 4 = -0.39 < 0$

$\therefore f(x_0) < 0$ $\therefore$ Root's lies betn $(x_0)$ and b

i.e. 1.75 and 1.8

2nd approximate root.

$x_1 = \frac{x_0 + b}{2}$ $x_1 = \frac{1.75+1.8}{2} = \frac{3.55}{2} = 1.775$

$\therefore f(1.775) = (1.775)^3 - 1.775 - 4 = -0.182 < 0$

$\therefore f(x_1) < 0$

$\therefore$ Root lies betn $x_1$ and b

i.e. 1.775 and 1.8.

3rd approximate Root

$x_2 = \frac{x_1 + b}{2} = \frac{1.775+1.8}{2} = \frac{3.575}{2} = 1.78\ldots$

$f(x)=0$

i.e. $x^3-x-4=0$

step-I find a, and b.

$f(0)=(0)^3-0-4=-4<0$

$f(1)=1-1-4=-4<0$

$f(2)=8-2-4=2>0$.

$f(1.5)=(1.5)^3-(1.5)-4=-2.125<0$

$f(1.6)=(1.6)^3-(1.6)-4=-1.501<0$

$f(1.7)=(1.7)^3-(1.7)-4=-0.787<0$

$f(1.8)=(1.8)^3-1.8-4=0.032>0$.

∴ Root lies betn 1.7 and 1.8.

1st approximate root

$\therefore x_0=\frac{a+b}{2}=\frac{1.7+1.8}{2}=\frac{3.5}{2}=1.75$

$\therefore f(1.75)=(1.75)^3-1.75-4=-0.39<0$

$\because f(x_0)<0$ ∴ Root's lies betn $(x_0)$ and b

i.e 1.75 and 1.8.

2nd approximate root

$x_1=\frac{x_0+b}{2}$ $\quad x_1=\frac{1.75+1.8}{2}=\frac{3.55}{2}=1.775$

$\therefore f(1.775)=(1.775)^3-1.775-4=-0.182<0$

$\therefore f(x_1)<0$

∴ Root lies betn $x_1$ and b

i.e. 1.775 and 1.8.

3rd approximate Root

$x_2=\frac{x_1+b}{2}=\frac{1.775+1.8}{2}=\frac{3.575}{2}=1.78\ldots$

$\therefore f(x_2) = (1.7875)^3 - 1.7875) - 4$
$= -0.076 < 0$

$\because f(x_2) < 0 \quad \therefore$ Root lies betn $x_2$ and $b$
i.e. $1.7875$ and $1.8$

4th approximate root

$$x_3 = \frac{x_2 + b}{2} = \frac{1.7875 + 1.8}{2} = \frac{3.5875}{2} = 1.79375$$

$\therefore f(1.79375) = (1.79375)^3 - 1.79375) - 4$
$= -0.22 < 0$

$\therefore f(x_3) < 0$
$\therefore$ Root lies betn $x_3$ and $b$.
i.e $1.79375$ and $1.8$

$\therefore$ 5th approximate Root

$$x_4 = \frac{x_3 + b}{2} = \frac{1.79375 + 1.8}{2} = \frac{3.59375}{2} = 1.796875$$

$\therefore f(x_4) = f(1.796875) = (1.796875)^3 - 1.796875 - 4$
$= +0.0048 > 0$

$\therefore f(x_4) > 0$
$\therefore$ root lies betn $x_4$ and $x_3$
i.e root lies $1.796875$ and $1.79375$

6th approximate value

$$x_5 = \frac{x_4 + x_3}{2} = \frac{1.79375 + 1.796875}{2} = \frac{3.590625}{2}$$
$$= 1.795312$$

$f(x_5) = f(1.795312) = (1.795312)^3 - 1.79532 - 4$
$= -0.0087 < 0$

$\therefore f(x_5) < 0$

$\therefore$ root lies betn. $x_4$ and $x_5$

i.e 1.796875 and 1.795312

7th approximate value.

$$x_6 = \frac{x_4 + x_5}{2} = \frac{1.796875 + 1.795312}{2}$$

$$= 1.796093.$$

$\therefore$ so hence approximate root correct upto 3 decimal place

$$\boxed{x = 1.796}$$

Q Find the real root of equation $x^3 - 18 = 0$ ~~upto~~ correct

Q.1.

| n | a | b | x | f(x) |
|---|---|---|---|---|
| 1 | 1.7 | 1.8 | 1.75 | -0.39 |
| 2 | 1.75 | 1.8 | 1.775 | -0.182 |
| 3 | 1.775 | 1.8 | 1.7875 | -0.076 |
| 4 | 1.7875 | 1.8 | 1.79375 | -0.022 |
| 5 | 1.79375 | 1.8 | 1.796875 | +0.6048 |
| 6 | 1.79375 | 1.796875 | 1.795312 | -0.0087 |
| 7 | 1.795312 | 1.796875 | 1.7960 | - |