

## Q: Newton backward difference Formula

Consider the following Data points

$x$	$x_0$	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$\dots$	$x_n$
$y=f(x)$	$y_0$	$y_1$	$y_2$	$y_3$	$y_4$	$y_5$	$\dots$	$y_n$

Backward difference

with  $x_1 = x_0 + h$ ,  $x_2 = x_0 + 2h$ ,  $x_3 = x_0 + 3h$  ...  
means with equal interval.

$\therefore$  Newton's backward difference Formula

$$y=f(x) = y_n + p \nabla y_n + \frac{p(p+1)}{2!} \nabla^2 y_n + \frac{p(p+1)(p+2)}{3!} \nabla^3 y_n + \frac{p(p+1)(p+2)(p+3)}{4!} \nabla^4 y_n + \dots + \frac{p(p+1)(p+2)(p+3)\dots(p+(n-1))}{(n!)} \nabla^n y_n$$

where  $p = \frac{x - x_n}{h}$  i.e.  $x = x_n + ph$ .

formation of backward difference Tables-

$x$	$y$	$\nabla y$	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
$x_0$	$y_0$				
$x_1$	$y_1$	$\nabla y_1 = y_1 - y_0$			
$x_2$	$y_2$	$\nabla y_2 = y_2 - y_1$	$\nabla^2 y_2 = \nabla y_2 - \nabla y_1$		
$x_3$	$y_3$	$\nabla y_3 = y_3 - y_2$	$\nabla^2 y_3 = \nabla y_3 - \nabla y_2$	$\nabla^3 y_3 = \nabla^2 y_3 - \nabla^2 y_2$	
$x_4$	$y_4$	$\nabla y_4 = y_4 - y_3$	$\nabla^2 y_4 = \nabla y_4 - \nabla y_3$	$\nabla^3 y_4 = \nabla^2 y_4 - \nabla^2 y_3$	$\nabla^4 y_4 = \nabla^3 y_4 - \nabla^3 y_3$

Q: Using Newtons Backward difference interpolation formula find  $f(42)$  for following data

X	20	25	30	35	40	45
Y	354	332	291	260	231	204

Sol<sup>n</sup> given

X	Y	$\nabla y$	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$	$\nabla^5 y$
20	354					
		-22				
25	332		-19			
		-41		29		
30	291		10		-37	
		-31		-8		45
35	260		2		8	
		-39		0		
40	231		2			
		-27				
45	204					

Let  $x = 42$   $x_n = 45$   $h = 5$

$p = \frac{x - x_n}{h} = \frac{42 - 45}{5} = -3/5 = -0.6$

Now Newton Backward difference

$$\begin{aligned}
 y = f(x) &= y_n + p \nabla y_n + \frac{p(p+1)}{2!} \nabla^2 y_n + \frac{p(p+1)(p+2)}{3!} \nabla^3 y_n + \\
 &\quad \frac{p(p+1)(p+2)(p+3)}{4!} \nabla^4 y_n + \frac{p(p+1)(p+2)(p+3)(p+4)}{5!} \nabla^5 y_n \\
 &= 204 + (-0.6)(-27) + \frac{(-0.6)(-0.6+1)(2)}{2!} + \\
 &\quad \frac{(-0.6)(-0.6+1)(-0.6+2)(0)}{3!} + \frac{(-0.6)(-0.6+1)(-0.6+2)(-0.6+3)}{4!} (8) \\
 &\quad + \frac{(-0.6)(-0.6+1)(-0.6+2)(-0.6+3)(-0.6+4)}{5!} \times 45
 \end{aligned}$$

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$$= 204 + 16.2 - 0.24 + 0 - 0.8064$$

$$= 219.2 - 1.0464 = 218.1536$$

$$f(x_2) = \underline{\underline{219}}$$

Q from the following table find  $f(7.5)$

$x$	1	2	3	4	5	6	7	8
$y=f(x)$	27	64	125	216	343	512		

$$x_n = 8$$

$$y_n = 512$$

$$h = 1$$

$$x = 7.5$$