

Note:- Row operation - There are 3 kind of elementary Row operation.

- ① Switching of Rows:- A row can be Interchange with another row $R_i \leftrightarrow R_j$

Date: / /

Page No.:

② Row multiplication:- A non zero number can be multiplied or to every eliminate in a row.

$$aR_j \rightarrow R_i \text{ where } a \neq 0$$

② Row addition (subtraction):- we may replace a row by the sum of element of row and a multiple of corresponding element of another row.

$$R_i + aR_j \rightarrow R_i \text{ when } a \neq 0, i \neq j$$

$$RRPP \rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$+PE+PE, 8 = \underline{\underline{0+0}} + PE - 1 \quad E = S + P + PE$$

15

Gauss Elimination Method:-

Also called forward elimination method.

- used to solve the linear system of linear equation by using $Ax=b$ augmented matrix (consisting of equation in n unknown)

$$[A|b] = \left[\begin{array}{cccc|c} a_{11} & a_{12} & \dots & a_{1n} & b_1 \\ a_{21} & a_{22} & \dots & a_{2n} & b_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} & b_m \end{array} \right]$$

To an upper triangular matrix form

$$\left[\begin{array}{cccc|c} c_{11} & c_{12} & \dots & c_{1n} & d_1 \\ 0 & c_{22} & \dots & c_{2n} & d_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & c_{nn} & d_n \end{array} \right]$$

Working Rule:-

Step-1 Consider the System of linear equation.

$$\begin{aligned} (1) \quad & a_1x + b_1y + c_1z = d_1 \\ (2) \quad & a_2x + b_2y + c_2z = d_2 \\ (3) \quad & a_3x + b_3y + c_3z = d_3 \end{aligned}$$

i.e $AX=B$

$$(E \rightarrow \left[\begin{array}{ccc|c} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \end{array} \right] \left[\begin{array}{c} x \\ y \\ z \end{array} \right] = \left[\begin{array}{c} d_1 \\ d_2 \\ d_3 \end{array} \right] \right)$$

Step-1 find augmented matrix for given system.
 $C = [A : B]$

Step-III Transform augmented matrix into upper triangular form i.e. row echelon form using some Row operation.

Step-IV find equations corresponding to upper triangular matrix.

Step-V using back substitution find (x, y, z) sol'n of given system of eqn.

Q. solve the system of linear equation by Gauss elimination method.

equations

$$2x - y + 3z = 9$$

$$x + y + z = 6$$

$$x - y + z = 2$$

Given system of equation can be written as

$$Ax = B$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & -1 & 1 & 2 \\ 2 & -1 & 3 & 9 \end{array} \right]$$

Step-II Now find augmented matrix given system.

$$C = [A:B]$$

$$C = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & -1 & 1 & 2 \\ 2 & -1 & 3 & 9 \end{array} \right]$$

Step-III transform augmented matrix into upper triangular matrix by

Date: / /

Page No.:

$$R_2 \rightarrow R_2 - R_1 \quad \text{and} \quad R_3 \rightarrow R_3 - 2R_1$$

$$C = \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 6 \\ 0 & -2 & 0 & ; & -4 \\ 0 & -3 & 1 & | & -3 \end{array} \right] \xrightarrow{\begin{matrix} \\ \\ + \end{matrix}} \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 6 \\ 0 & -2 & 0 & ; & -4 \\ 0 & -6 & 2 & | & 8 \end{array} \right]$$

$$\text{By } R_3 \rightarrow 2R_3 - 3R_2 \quad \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 6 \\ 0 & -2 & 0 & ; & -4 \\ 0 & -6 & 0 & | & 12 \end{array} \right] \xrightarrow{\begin{matrix} \\ + \\ \hline \end{matrix}} \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 6 \\ 0 & -2 & 0 & ; & -4 \\ 0 & 0 & 2 & | & 4 \end{array} \right]$$

$$x+y+z=6$$

$$-y = -4$$

$$z=6$$

$$P = 5x + y - z$$

$$P = 5 + 2 - 6$$

$$P = -1$$

$$z = 6/2 = 3$$

$$y = 2$$

$$x+2+3=6$$

$$x = +1$$

$$\boxed{x=1 \\ \therefore x=1, y=2, z=3}$$