

* What is Linear eqⁿ?

A linear equation is the eqⁿ written in the form
 $ax + b = 0$

where a, b are real numbers and x is variables

∴ This form is sometime called Standard form of linear equation.

System of linear eqⁿ:-

A system of linear equation is when we have two or more linear equation working together.

$$a_1x + b_1y + c_1z = d_1$$

$$a_2x + b_2y + c_2z = d_2$$

$$a_3x + b_3y + c_3z = d_3$$

This is standard form of system of linear eqⁿ.

x, y, z = variable

$a_1, b_1, c_1, a_2, b_2, c_2, a_3, b_3, c_3$ and d_1, d_2, d_3 is constant numerical value.

This system of linear eqⁿ we write into form of

$$AX = B$$

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

Working Rule

Let we the following system of linear eqⁿ.

$$\rightarrow a_1x + b_1y + c_1z = d_1$$

$$a_2x + b_2y + c_2z = d_2$$

$$a_3x + b_3y + c_3z = d_3$$

$a_1, a_2, a_3, b_1, b_2, b_3, c_1, c_2, c_3$ are the coefficient of x, y, z simultaneously respectively.

d_1, d_2, d_3 are constant numerical values.

We write above eqⁿ into matrix form.

$$\lambda = 3.634240.$$

Lineare Equation - Cramer's rule:-

- Cramer's rule is an algorithm for solving linear system using determinants.
- Cramer's rule can only be used with linear system that have exactly one solⁿ.
- Cramer's rule is named after Gabriel Cramer's a Swiss mathematician.
- It uses determinants to find the solⁿ of linear eqⁿ/system
- Given a system of linear equation, Cramer's rule is a handy way to solve just one of the variable without having to solve the whole system of equation
i.e. Instead of solving the entire system of equation you can use Cramer's rule to solve for just one single variables.

Working Rule

Let us take the following system of linear eqⁿ.

$$\rightarrow a_1x + b_1y + c_1z = d_1$$

$$a_2x + b_2y + c_2z = d_2$$

$$a_3x + b_3y + c_3z = d_3$$

$a_1, a_2, a_3, b_1, b_2, b_3, c_1, c_2, c_3$ are the coefficient of x, y, z simultaneously respectively.

d_1, d_2, d_3 are constant Numerical values.

We write above eqⁿ into matrix form.

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$$

→ coefficient of matrix.

$$= \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} \rightarrow \text{constant value matrix.}$$

Take

$$D = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$$

$$D_x = \begin{bmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{bmatrix}$$

$$D_y = \begin{bmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{bmatrix}$$

$$D_z = \begin{bmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{bmatrix}$$

Standard way to find determinant:

$$D = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$$

$$= a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$$

$$= a_1 [b_2 c_3 - b_3 c_2] - b_1 [a_2 c_3 - a_3 c_2] + c_1 [a_2 b_3 - a_3 b_2]$$

||| try we calculate Dx, Dy, Dz

according to crinners Rule.

$x = \frac{Dx}{D}$	$y = \frac{Dy}{D}$	$z = \frac{Dz}{D}$
--------------------	--------------------	--------------------

$$\begin{matrix} -2 & 1 \\ 3 & 6 \end{matrix}$$

$$\begin{matrix} -2 & 1 \\ 3 & 6 \end{matrix}$$

Let use the following system of linear equation.

$$2x + y + z = 3$$

$$x - y - z = 0$$

$$x + 2y + z = 0$$

- we have the left hand side of the system with the variables and right hand side with the numerical values (Answer value).

∴ Let D be determinant of the coefficient matrix.

Solⁿ: ∴ system of eqⁿ

$$2x + y + z = 3$$

$$1x + 1y - 1z = 0$$

$$1x + 2y + 1z = 0$$

coefficient Matrix determinant

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & -1 & -1 \\ 1 & 2 & 1 \end{bmatrix}$$

Answer column value

$$\begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}$$

coefficient Matrix determinant

$$D = \begin{vmatrix} 2 & 1 & 1 \\ 1 & -1 & -1 \\ 1 & 2 & 1 \end{vmatrix}$$

∴ D_x is ^{defined} coefficient determinant with answer column values in a column.

we get $D_x = \begin{vmatrix} 3 & 1 & 1 \\ 0 & -1 & -1 \\ 0 & 2 & 1 \end{vmatrix}$

11)xy Dy on Dz

$$Dy = \begin{vmatrix} 2 & 3 & 1 \\ 1 & 0 & -1 \\ 1 & 0 & 1 \end{vmatrix} \quad \text{and} \quad Dz = \begin{vmatrix} 2 & 1 & 3 \\ 1 & -1 & 0 \\ 1 & 2 & 0 \end{vmatrix}$$

Evaluating each determinant we get.

$$\begin{aligned} D &= \begin{vmatrix} 3 & 1 & 1 \\ 0 & -1 & -1 \\ 0 & 2 & 1 \end{vmatrix} = 3 \begin{vmatrix} -1 & -1 \\ 2 & 1 \end{vmatrix} - 1 \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} + 1 \begin{vmatrix} 1 & -1 \\ 1 & 2 \end{vmatrix} \\ &= 3[-1+2] - 1[1-(-1)] + 1[2-(-1)] \\ &= 3(-1+2) - 1[(1+1)] + 1[2+1] \\ &= 3(1) - 1(2) + 1(3) \\ &= 3 - 2 + 3 \end{aligned}$$

$$\boxed{D = 3}$$

$$Dx = \begin{vmatrix} 3 & 1 & 1 \\ 0 & -1 & -1 \\ 0 & 2 & 1 \end{vmatrix} = 3 \begin{vmatrix} -1 & -1 \\ 2 & 1 \end{vmatrix} - 1 \begin{vmatrix} 0 & -1 \\ 0 & 1 \end{vmatrix} + 1 \begin{vmatrix} 0 & -1 \\ 0 & 2 \end{vmatrix}$$

$$\begin{aligned} &= 3[-1-(-2)] - 1[0-0] + 1[0-0] \\ &= 3(-1+2) - 1(0) + 1(0) \\ &= 3(1) - 0 + 0 \end{aligned}$$

$$\boxed{Dx = 3}$$

$$\text{11)xy } Dy = \begin{vmatrix} 2 & 3 & 1 \\ 1 & 0 & -1 \\ 1 & 0 & 1 \end{vmatrix} = 2 \begin{vmatrix} 0 & -1 \\ 0 & 1 \end{vmatrix} - 3 \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} + 1 \begin{vmatrix} 1 & 0 \\ 1 & 0 \end{vmatrix}$$

$$\begin{aligned}
 &= 2(0-0) - 3[(1-(-1))] + 1[(0-0)] \\
 &= 2(0) - 3(-2) + 1(0) \\
 &= 0 - 6 + 0
 \end{aligned}$$

$$D_y = -6$$

$$\therefore D_z = \begin{vmatrix} 2 & 1 & 3 \\ 1 & -1 & 0 \\ 1 & 2 & 0 \end{vmatrix} = 2 \begin{vmatrix} -1 & 0 \\ 2 & 0 \end{vmatrix} - 1 \begin{vmatrix} 1 & 0 \\ 1 & 0 \end{vmatrix} + 3 \begin{vmatrix} 1 & -1 \\ 1 & 2 \end{vmatrix}$$

$$\begin{aligned}
 &= 2[(0)-(0)] - 1[(0)-(0)] + 3[(2)-(-1)] \\
 &= 2(0) - 1(0) + 3(3) \\
 &= 0 - 0 + 9
 \end{aligned}$$

$$D_z = 9$$

According to Cramer's rule.

$$x = D_x \div D$$

$$y = D_y \div D$$

$$z = D_z \div D$$

$$\therefore x = \frac{3}{3} = 1 \quad D_y = -\frac{6}{3} = -2 \quad D_z = \frac{D_z}{D} = \frac{9}{3} = 3$$