

Q: Newton's backward difference formula

Consider the following Data points

x	x_0	x_1	x_2	x_3	x_4	x_5	\dots	x_n
$y = f(x)$	y_0	y_1	y_2	y_3	y_4	y_5	\dots	y_n

Backward difference

with $x_1 = x_0 + h$, $x_2 = x_0 + 2h$, $x_3 = x_0 + 3h \dots$
means with equal interval.

∴ Newton's backward difference formula

$$y = f(x) = y_n + \frac{p\Delta y_n}{1!} + \frac{p(p+1)\Delta^2 y_n}{2!} + \frac{p(p+1)(p+2)\Delta^3 y_n}{3!} + \dots + \frac{p(p+1)(p+2)(p+3)\Delta^4 y_n}{4!} + \dots + \frac{p(p+1)(p+2)(p+3)\dots(p+(n-1))\Delta^n y_n}{(n-1)!}$$

where $p = \frac{x - x_n}{h}$ i.e. $x = x_n + ph$.

formation of backward difference tables-

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
x_0	y_0				
x_1	y_1	$\Delta y_1 = y_1 - y_0$	$\Delta^2 y_2 = \Delta y_2 - \Delta y_1$	$\Delta^3 y_3 = \Delta^2 y_3 - \Delta^2 y_2$	$\Delta^4 y_4 = \Delta^3 y_4 - \Delta^3 y_3$
x_2	y_2	$\Delta y_2 = y_2 - y_1$			
x_3	y_3	$\Delta y_3 = y_3 - y_2$	$\Delta^2 y_4 = \Delta y_4 - \Delta y_3$		
x_4	y_4	$\Delta y_4 = y_4 - y_3$			

Q: Using Newton's Backward difference interpolation formula find $f(42)$ for following data.

x	20	25	30	35	40	45
y	354	332	291	260	231	204

Sol given -

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$	$\nabla^5 y$
20	354		-22			
25	332		-19			
30	291		10		-37	
35	260	2		8		45
40	231		2			
45	204	-27				

$$\text{Let } x = 42 \quad x-n=45 \quad h=5$$

$$P = \frac{x-x_0}{h} = \frac{42-45}{5} = -\frac{3}{5} = -0.6$$

Now Newton Backward difference

$$\begin{aligned}
 y = f(45) &= y_0 + P \nabla y_{05} + \frac{P(P+1)}{2!} \nabla^2 y_{05} + \frac{P(P+1)(P+2)}{3!} \nabla^3 y_{05} + \\
 &\quad \frac{P(P+1)(P+2)(P+3)}{4!} \nabla^4 y_{05} + \frac{P(P+1)(P+2)(P+3)(P+4)}{5!} \\
 &= 204 + (-0.6)(-27) + \frac{(-0.6)(-0.6+1)(2)}{2!} + \\
 &\quad \frac{(-0.6)(-0.6+1)(-0.6+2)(0)}{3!} + \frac{(-0.6)(-0.6+1)(-0.6+2)}{4!} \\
 &\quad \frac{(-0.6)(-0.6+1)(-0.6+2)(-0.6+3)}{5!} \times 45 \\
 &= \frac{(-0.6)(-0.6+1)(-0.6+2)(-0.6+3)(-0.6+4)}{120} \times 45
 \end{aligned}$$

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Method to convert into standard form : 0

$$\begin{aligned} &= 204 + 16.2 - 8.24 + 0 - 0.8064 \\ &= 280.2 - 1.0464 \quad 0.8064 \\ &= 279.1536 \end{aligned}$$

$$f(2.2) = \underline{\underline{219}} \quad 250 \text{ not } 200 \text{ cd} \quad 510 \text{ not } 0 \text{ pmf}$$

Q from the following table find $f(7.5)$ (P)

x	1	2	3	4	5	6	7	8
$y = f(x)$	87.0	89.0	91.0	93.0	95.0	97.0	99.0	101.0

$$x_n = 8 \quad y_n = 5(2)(h) + (x = 7.5)(85.0) + (i)$$

0.80