

Romberg's Method:-

- ~~Trapezoidal Rule.~~

- This Method is often used to improve the approximate results obtain by the finite-difference method.

- It's applⁿ to the numerical evaluation of definite integrals. e.g. in the use of trapezoidal rule,

- we consider the definite integrals

$$I = \int_a^b y dx.$$

$$\left[2f + \left(\frac{h}{a} \right) f_1 + (2f + pf + sf + lf) f_2 + lf_3 \right] \frac{h}{3}$$

We evaluate it by the trapezoidal rule with two or more different subintervals of width, h_1, h_2, h_3 obtain approximate value I_1, I_2, I_3 respectively.

$$\text{we take } h_1 = b, h_2 = b/2, h_3 = b/4$$

find,
 $\therefore I(h), I(h/2), I(h/4)$.

working rule

Step-I given definite integral

$$I = \int_a^b f dx.$$

Step-II find val evaluate it by trapezoidal Rule with interval $h, h/2, h/4$ and obtain $I_1 = I(h), I_2 = I(h/2), I_3 = I(h/4) \dots$ respectively.

Step-III form following table.

$$I_1 = I(h)$$

$$I_4 = I(h, h/2)$$

$$I_2 = I(h/2)$$

$$I_6 = I(h, h/2, h/4)$$

$$I_5 = I(h/2, h/4)$$

$$I_3 = I(h/4)$$

$$\therefore I(h, h/2) = I_2 + \frac{1}{3}(I_2 - I_1)$$

$$I(h/2, h/4) = I_3 + \frac{1}{3}(I_3 - I_2)$$

$$I(h, h/2, h/4) = I_5 + \frac{1}{3}(I_5 - I_4).$$

Q. Use Romberg's method to compute:-

$$I = \int_0^1 \frac{1}{1+x} dx.$$

(Correct upto four decimal place)