

Simson's 1/3rd rule:- Let the interval $[a, b]$ be divided into n equal subintervals such that $a = x_0 < x_1 < x_2 \dots x_n = b$ where $x_n = x_{n+1}$.
 \therefore Simson's 1/3rd rule is given by

$$I = \int_{x_0}^{x_n} y \, dx = \frac{h}{3} [y_0 + 4(y_1 + y_3 + \dots + y_{n-1}) + 2(y_2 + y_4 + \dots + y_{n-2}) + y_n]$$

Q: A solid of revolution is formed by rotating about x-axis the area between the x-axis, $x=0$ and $x=1$ and curve through the points with the following coordinates.

x	0.00	0.25	0.50	0.75	1.0000
y	1.0000	0.9896	0.9589	0.9089	0.8415

Estimate the volume of the solid form giving the Answer to 3 decimal places.

If V is the volume of solid formed, then we know that

$$V = \pi \int y^2 \, dx$$

hence, we need the values of y^2 and these tabulated below. (Correct to four decimal pl.)

x	y
0.00	1.0000
0.25	0.9793
0.50	0.9195
0.75	0.8201
1.00	0.7081

With $h=0.25$ Simson's Rule.

$$V = \frac{\pi (0.25)^3}{3} [1.000 + 4(0.9793 + 0.8261) + 2(0.9195) + 0.7081] - 2.6$$

$$= 2.8192$$

★ Simson's 3/8th Rule (n=3)

$$I = \int_{a=x_0}^{b=x_n} y dx = \frac{3h}{8} [y_0 + 3(y_1 + y_2 + y_4 + y_5 + \dots + y_{n-1}) + 2(y_3 + y_6 + y_9 + \dots + y_{n-3}) + y_n]$$

(n should be multiple of 3)

Q. Evaluate $\int_0^6 \frac{1}{1+x^2} dx$ by using Simson's 3/8th Rule.
h=0

$$h = \frac{6-0}{6} = \frac{6}{6} = 1$$

X	0	1	2	3	4	5	6
Y	1	0.5	0.2	0.1	0.0588	0.0385	0.027

$$I = \int_0^6 y dx = \frac{3h}{8} [y_0 + 3(y_1 + y_2 + y_4 + y_5) + 2(y_3) + y_6]$$

$$= \frac{3(1)}{8} [1 + 3(0.5 + 0.2 + 0.1 + 0.0588 + 0.0385) + 2(0.1) + 0.027]$$

$$= 1.3571$$