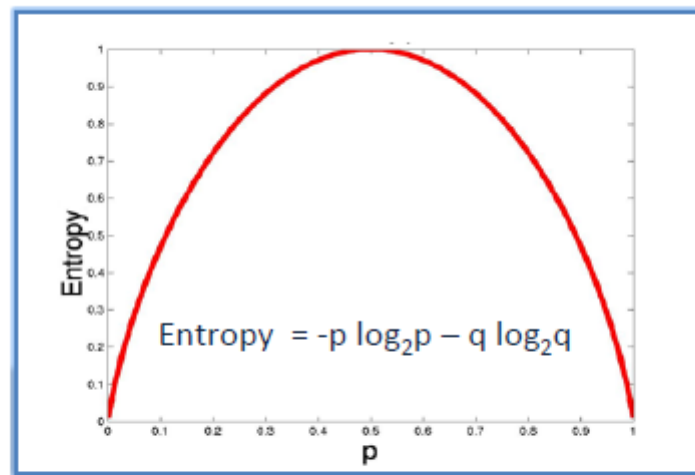




Entropy curve



$$\text{Entropy} = -0.5 \log_2 0.5 - 0.5 \log_2 0.5 = 1$$

To build a decision tree, we need to calculate two types of entropy using frequency tables as follows:

$$E(S) = \sum_{i=1}^c -p_i \log_2 p_i$$

Play Golf	
Yes	No
9	5



$$\begin{aligned} \text{Entropy}(\text{PlayGolf}) &= \text{Entropy}(5,9) \\ &= \text{Entropy}(0.36, 0.64) \\ &= -(0.36 \log_2 0.36) - (0.64 \log_2 0.64) \\ &= 0.94 \end{aligned}$$

b) Entropy using the frequency table of two attributes:

$$E(T, X) = \sum_{c \in X} P(c) E(c)$$

		Play Golf		
		Yes	No	
Outlook	Sunny	3	2	5
	Overcast	4	0	4
	Rainy	2	3	5
				14



$$\begin{aligned} E(\text{PlayGolf}, \text{Outlook}) &= P(\text{Sunny}) * E(3,2) + P(\text{Overcast}) * E(4,0) + P(\text{Rainy}) * E(2,3) \\ &= (5/14) * 0.971 + (4/14) * 0.0 + (5/14) * 0.971 \\ &= 0.693 \end{aligned}$$

### Information Gain

The information gain is based on the decrease in entropy after a dataset is split on an attribute. Constructing

finding attribute that returns the highest information gain.

*Step 1:* Calculate entropy of the target.

$$\begin{aligned}
 \text{Entropy}(\text{PlayGolf}) &= \text{Entropy}(5,9) \\
 &= \text{Entropy}(0.36, 0.64) \\
 &= -(0.36 \log_2 0.36) - (0.64 \log_2 0.64) \\
 &= 0.94
 \end{aligned}$$

*Step 2:* The dataset is then split on the different attributes. The entropy for each branch is calculated. Then it is added proportionally, to get total entropy for the split. The resulting entropy is subtracted from the entropy before the split is the Information Gain, or decrease in entropy.

		Play Golf	
		Yes	No
Outlook	Sunny	3	2
	Overcast	4	0
	Rainy	2	3
Gain = 0.247			

		Play Golf	
		Yes	No
Temp.	Hot	2	2
	Mild	4	2
	Cool	3	1
Gain = 0.029			

		Play Golf	
		Yes	No
Humidity	High	3	4
	Normal	6	1
Gain = 0.152			

		Play Golf	
		Yes	No
Windy	False	6	2
	True	3	3
Gain = 0.048			

$$Gain(T, X) = Entropy(T) - Entropy(T, X)$$

$$\begin{aligned}
 G(\text{PlayGolf}, \text{Outlook}) &= E(\text{PlayGolf}) - E(\text{PlayGolf}, \text{Outlook}) \\
 &= 0.940 - 0.693 = 0.247
 \end{aligned}$$

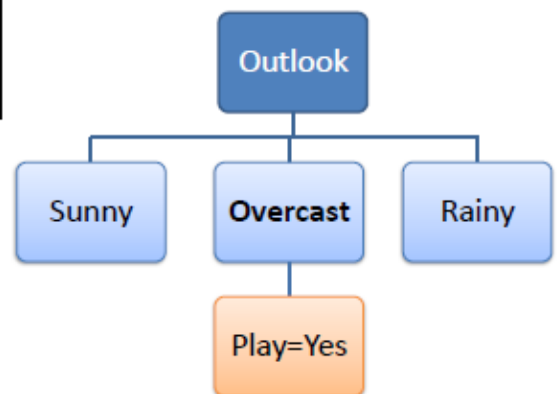
*Step 3:* Choose attribute with the largest information gain as the decision node, divide the dataset by its branches and repeat the same process on every branch.

		Play Golf	
		Yes	No
★	Sunny	3	2
	Overcast	4	0
	Rainy	2	3
Gain = 0.247			

Outlook	Temp	Humidity	Windy	Play Golf
Sunny	Mild	High	FALSE	Yes
Sunny	Cool	Normal	FALSE	Yes
Sunny	Cool	Normal	TRUE	No
Sunny	Mild	Normal	FALSE	Yes
Sunny	Mild	High	TRUE	No
Overcast	Hot	High	FALSE	Yes
Overcast	Cool	Normal	TRUE	Yes
Overcast	Mild	High	TRUE	Yes
Overcast	Hot	Normal	FALSE	Yes
Rainy	Hot	High	FALSE	No
Rainy	Hot	High	TRUE	No
Rainy	Mild	High	FALSE	No
Rainy	Cool	Normal	FALSE	Yes
Rainy	Mild	Normal	TRUE	Yes

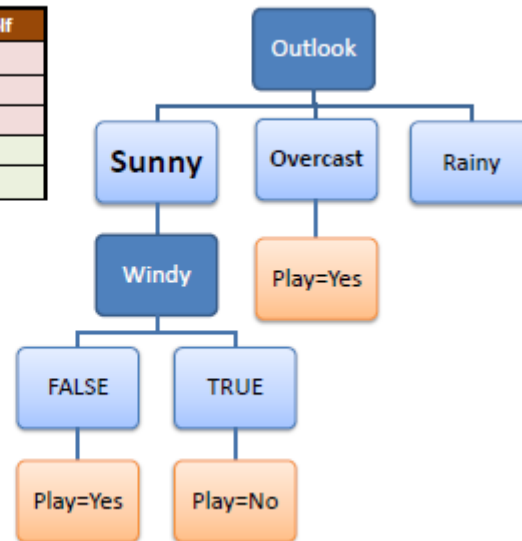
Step 4a: A branch with entropy of 0 is a leaf node.

Temp	Humidity	Windy	Play Golf
Hot	High	FALSE	Yes
Cool	Normal	TRUE	Yes
Mild	High	TRUE	Yes
Hot	Normal	FALSE	Yes



Step 4b: A branch with entropy more than 0 needs further splitting

Temp	Humidity	Windy	Play Golf
Mild	High	FALSE	Yes
Cool	Normal	FALSE	Yes
Mild	Normal	FALSE	Yes
Cool	Normal	TRUE	No
Mild	High	TRUE	No

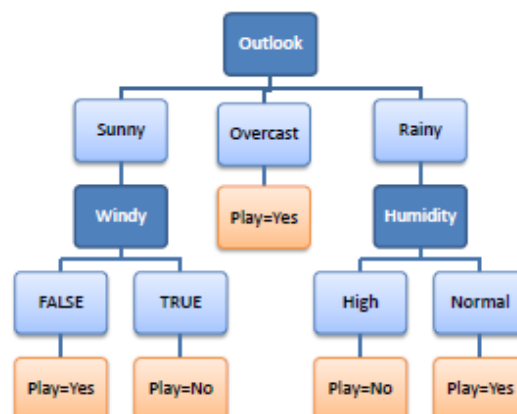


Step 5: The ID3 algorithm is run recursively on the non-leaf branches, until all data is classified.

## Decision Tree to Decision Rules

A decision tree can easily be transformed to a set of rules by mapping from the root node to the leaf nodes one.

$R_1$ : IF (Outlook=Sunny) AND (Windy=FALSE) THEN Play=Yes  
 $R_2$ : IF (Outlook=Sunny) AND (Windy=TRUE) THEN Play=No  
 $R_3$ : IF (Outlook=Overcast) THEN Play=Yes  
 $R_4$ : IF (Outlook=Rainy) AND (Humidity=High) THEN Play=No  
 $R_5$ : IF (Outlook=Rain) AND (Humidity=Normal) THEN Play=Yes



Reference: <http://hunch.net/~coms-4771/quinlan.pdf>