# Problem Set #2

UMass Recreational Math Club

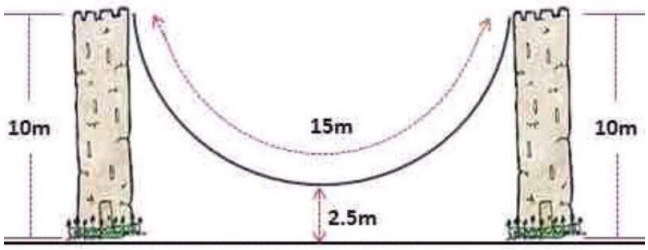
May 2023

**Problem 1.** On an $8 \times 8$ chessboard, a rook is on the top right square. Two players take turns moving the rook either left or down as many squares as they want. The player to move the rook to the bottom left square wins. What is the optimal strategy and who wins?

**Problem 2.** On another chessboard, there is a rook that can only be on (or move through) black squares. Right now it can't move! What is the minimum number of white squares that you have to color black in order for the rook to be able to visit every black square? Assume the rook starts on a black square.

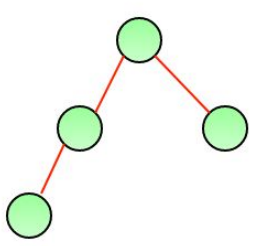
*Bonus.* Generalize your result to a $2n \times 2n$ checkerboard.

**Problem 3.** How far apart are the towers?

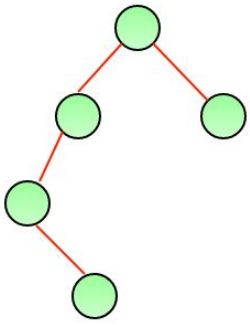



**Problem 4.** The height of a tree is given by the number of levels it has minus one. We call a binary tree balanced if 1) the heights of the left and right subtrees differ by no more than one and 2) every subtree is balanced. Find a tight bound on the number of nodes in a balanced binary tree of height $h$.

*Bonus.* Solve the inverse. That is, find a tight bound on the height of a balanced binary tree with $n$ nodes.



A height balanced tree



Not a height balanced tree