1 The Diffusive Limit of a Run and Tumble Particle Motion

The diffusive limit is

$$\lim_{v,\gamma\to\infty} \frac{v^2}{\gamma} \to 2D \tag{1}$$

The expression known to us for the exit time $\mathbf{E}T_{\text{exit}}$ is

$$\lim_{v,\gamma\to\infty} \mathbf{E}T_{\text{exit}} = \lim_{v,\gamma\to\infty} \left[\frac{-2\gamma + r}{r(2\gamma + r)} + \frac{2\gamma + r}{r(2\gamma + r)} \cosh\left(\frac{L}{2v} \sqrt{r(2\gamma + r)} \right) + \frac{v\sqrt{r(2\gamma + r)}}{r(2\gamma + r)} \sinh\left(\frac{L}{2v} \sqrt{r(2\gamma + r)} \right) \right] \tag{2}$$

$$\lim_{v,\gamma\to\infty} \mathbf{E}T_{\text{exit}} = \lim_{v,\gamma\to\infty} \left[\cancelto{-\frac{1}{r}}{\frac{(-2\gamma - r)}{r(\gamma + r)}} + \cancelto{\frac{1}{r}}{\frac{(2\gamma + r)}{r(2\gamma + r)}} \cosh\left(\frac{L}{2v} \sqrt{r(2\gamma + r)} \right) + \frac{v}{\sqrt{r(2\gamma + r)}} \sinh\left(\frac{L}{2v} \sqrt{r(2\gamma + r)} \right) \right] \tag{3}$$

$$\lim_{v,\gamma\to\infty} \mathbf{E}T_{\text{exit}} = \lim_{v,\gamma\to\infty} \left[-\frac{1}{r} + \ + \frac{1}{r} \cosh\left(\frac{L}{2v} \sqrt{r(2\gamma + r)} \right) + \frac{v}{\sqrt{r(2\gamma + r)}} \sinh\left(\frac{L}{2v} \sqrt{r(2\gamma + r)} \right) \right] \tag{4}$$

Next, see that

$$\begin{aligned} \lim_{v,\gamma\to\infty} \frac{\sqrt{r(2\gamma + r)}}{v} &= \lim_{v,\gamma\to\infty} \sqrt{\frac{2r\gamma}{v^2} + \frac{r^2}{v^2}} && (5) \\ &= \sqrt{\frac{2r}{2D} + 0} && (6) \\ &= \sqrt{\frac{r}{D}} && (7) \end{aligned}$$

Thus, we arrive at the expression

$$\lim_{v,\gamma\to\infty} \mathbf{E}T_{\text{exit}} = \left[-\frac{1}{r} \ + \frac{1}{r} \cosh\left(\frac{L}{2} \sqrt{\frac{r}{D}} \right) + \sqrt{\frac{D}{r}} \sinh\left(\frac{L}{2} \sqrt{\frac{r}{D}} \right) \right] \tag{8}$$