Assignment 4
Probability and random Variables

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Outline

figure

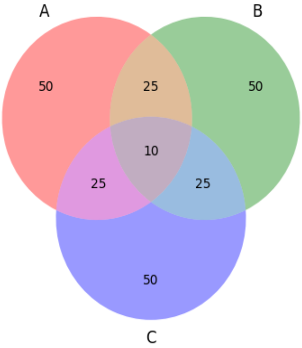

Figure: fig-1

Example

Papoulis book example 2.19

The below example is an illustration that emphasizes that three events can be independent in pairs but not independent

Suppose that the events A,B,C of Fig-1 have the same probability.

$$P(A) = P(B) = P(C) = \frac{1}{5}$$

and the intersections AB,AC,BC,ABC also have the same probability

$$p = P(AB) = P(BC) = P(AC)$$

Independence of events

Independence of three events:
The events $A_1, A_2 and A_3$ are called (mutually) independent if they are independent in pairs:

$$P(A_iA_j) = P(A_i) \times P(A_j) \tag{1}$$

and

$$P(A_1A_2A_3) = P(A_1) \times P(A_2) \times P(A_3) \tag{2}$$

statements

- if p= $\dfrac{1}{25}$

  then these events are independent in pairs but they are not independent because

$$P(ABC) \neq P(A) \times P(B) \times P(C)$$

- if p= $\dfrac{1}{125}$

  then

$$P(ABC) = P(A) \times P(B) \times P(C)$$

  but the events are not independent because

$$P(AB) \neq P(A) \times P(B)$$

Explanation

From the independence of the events A, B, and C it follows that:

1. Anyone of them is independent of the intersection of the other two. Indeed, from (1) and (2) it follows that

$$P(A_1A_2A_3) = P(A_1) \times P(A_2) \times P(A_3) = P(A_1) \times P(A_2A_3) \quad (3)$$

   Hence the events A_1 and A_2A_3 are independent

2. If we replace one or more of these events with their complements, the resulting events are also independent.Indeed since,

$$A_1A_2 = A_1A_2A_3 \cup A_1A_2\overline{A}_3; \quad P(\overline{A}_3) = 1 - P(A_3)$$

   we conclude with (3) that

$$P(A_1A_2\overline{A}_3) = P(A_1A_2) - P(A_1A_2) \times P(A_3) = P(A_1) \times P(A_2) \times P(\overline{A}_3)$$

3. Anyone of them is independent of the union of the other two.

To show that the events A_1 and $A_2 \cup A_3$ are independent, it suffices to show that the events A_1 and $\overline{A_2 \cup A_3} = \overline{A}_2\overline{A}_3$ are independent. This follows from 1 and 2

Generalization

Generalization: The independence of n events can be defined inductively: Suppose that we have defined independence of k events for every $k < n$. We then say that the events $A_1, ..., A_n$ are independent if any $k < n$ of them are independent and

$$P(A_1....A_n) = P(A_1)....P(A_n) \tag{4}$$

This completes the definition for any n because we have already defined independence for $n = 2$