Assignment 7
Probability and Random Variables

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Outline

Question

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Suppose z has an F distribution with (m, n) degrees of freedom.

(a) Show that $\frac{1}{z}$ also has an F distribution with (n, m) degrees of freedom.

(b) Show that mz/(mz + n) has a beta distribution

solution part a

$Z \sim F(m, n)$ Let,

$$Y = \frac{1}{Z}$$

Then ,

$$\begin{aligned} F_Y(y) &= \frac{1}{dy/dz} f_z(1/y) \\ &= \frac{1}{y^2} \frac{(m/n)^{m/2}}{\beta(n/2, m/2)} \frac{1}{y^{m/2-1}} \frac{1}{(1 + m/ny)^{m+n/2}} \\ &= \frac{(n/m)^{n/2}}{\beta(n/2, m/2)} y^{n/2-1} (1 + \frac{n}{my})^{-(m+n)/2} \\ &\sim F(n, m) \end{aligned}$$

solution part b

$$W = \frac{Zm}{Zm + n}$$

$$F_W(w) = P(W \leq w) = P(\frac{Zm}{Zm + n} \leq w)$$

$$= P(Z \leq \frac{nw}{m(1 - w)}) = F_z(\frac{nw}{m(1 - w)})$$

which gives,

$$f_W(w) = \frac{n}{m(1-w)^2} f_z\left(\frac{nw}{m(1-w)}\right)$$

$$= \frac{n}{m(1-w)^2} \frac{(m/n)^{m/2}}{\beta(m/2, n/2)} \left(\frac{nw}{m(1-w)}\right)^{m/2-1} \left(1 + \frac{w}{(1-w)}\right)^{-(m+n)/2}$$

$$= \frac{1}{\beta(m/2, n/2)} w^{m/2-1}, 0 < w < 1$$

Thus W has beta distribution.