Assignment 8
Probability and Random Variables

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Outline

Question

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Show that for any x,y real or complex

(a)$\mid E\{xy\} \mid^2 \leq E\{(\mid x \mid)^2\}E\{(\mid y \mid)^2\}$

(b)(triangular inequality)$\sqrt{E\{\mid x+y \mid^2} \leq \sqrt{E(\mid x \mid)^2} + \sqrt{E(\mid y \mid)^2}$

solution part a

Since $\mid E(xy) \mid \leq E(\mid x \mid)E(\mid y \mid)$we can assume that the Random variables x,y are real.
part a))$\mid E\{xy\} \mid^2 \leq E\{(\mid x \mid)^2\}E\{(\mid y \mid)^2\}$
Lets consider the equation,

$$\begin{aligned} E\{[zx - y]^2\} &= E\{z^2x^2 + y^2 - 2zxy\} \\ &= z^2E(x^2) - 2zE(xy) + E(y^2) \end{aligned} \tag{1}$$

The above Equation (1) is quadratic in Z and we also know that its non negative.thus, Discriminant is negative.

$$[E(xy)]^2 \leq [E(x)^2][E(y)^2]$$

The above condition Leads to the required solution.

solution part b

part b)(triangular inequality) $\sqrt{E\{\mid x+y \mid^2} \leq \sqrt{E(\mid x \mid)^2} + \sqrt{E(\mid y \mid)^2}$
Using (a) we obtain,

$$
\begin{aligned}
&E(|x|^2) + E(|y|^2) + 2\sqrt{E(x)^2 E(y)^2} \\
&\geq E(x^2) + E(y^2) + 2E(xy) \\
&= E(|x+y|^2)
\end{aligned}
$$

Thus the above triangular inequality holds.