# Assignment 2

## EE3900: Linear Systems and Signal Processing

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## Discrete-time Signal Processing

Oppenheim and Schafer

**Problem 2.16 (c):** For the given difference equation

$$y[n] - \frac{y[n-1]}{4} - \frac{y[n-2]}{8} = 3x[n] \tag{1}$$

show that causal LTI system is stable and anticausal LTI system is unstable.

**Solution:** Let $y_h[n]$ be the homogenous solution that solves the difference equation.
It is in the form of $y_h[n] = \sum A(c)^n$, where c's solve the quadratic equation

$$c^2 - \frac{c}{4} - \frac{1}{8} = 0 \tag{2}$$

So, for $c = \frac{1}{2}$ and $c = \frac{-1}{4}$, the general form of homogenous solution is

$$y_h[n] = A_1(\frac{1}{2})^n + A_2(\frac{-1}{4})^n \tag{3}$$

Taking inverse Z- transform on both sides we get,

$$H(z) = \frac{3}{1 - \frac{z^{-1}}{4} - \frac{z^{-2}}{8}} \tag{4}$$

$$= \frac{1}{1 + \frac{z^{-1}}{4}} + \frac{2}{1 - \frac{z^{-1}}{2}} \tag{5}$$

The causal impulse response corresponds to assuming that the region of convergence extends outside the outermost pole,making

$$h_c(n) = [(\frac{-1}{4})^n + 2(\frac{1}{2})^n]u[n] \tag{6}$$

The anticausal impulse response corresponds to assuming that the region of convergence is inside the innermost pole making

$$h_{ac}(n) = -[(\frac{-1}{4})^n + 2(\frac{1}{2})^n]u[-n-1] \tag{7}$$

From the above two equations it is very clear that $h_c[n]$ is absolutely summable while $h_{ac}[n]$ grows without bounds.
Thus causal LTI system is stable while anticausal LTI system is unstable.