Assignment 13
Probability and Random Variables

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Outline

Question

Question 11.2

Find the innovations filter of process $x(t)$ if

$$s(\omega) = \frac{\omega^4 + 64}{\omega^4 + 10\omega^2 + 9}$$

Solution

The power spectrum S(s) of a regular process can be written as product

$$L(s)L(-s) \quad S(\omega) = |L(j\omega)|^2$$

where $L(s)$ is an innovations filter of $x(t)$
A rational spectrum is the ratio of two polynomials in ω^2 because

$$S(\omega) = S(-\omega)$$

$$S(\omega) = \frac{A(\omega^2)}{B(\omega^2)}$$

solution

This shows that if s(i) is a root of S(s) , then -S(i) is also a root of S(s).Further more all roots are either real or complex conjugates.From this it follows that the roots of S(s) are symmetrical with respect to $j\omega$ axis.Hence they can be separated in two groups,one with $Re(s) < 0$ and right group with $RE(s) > 0$.The minimum phase factor or innovation factor is the ratio of two polynomials formed with left roots.

Now,

$$s_x(\omega) = \frac{\omega^4 + 64}{\omega^4 + 10\omega^2 + 9}$$
$$= \frac{\omega^2 + 4\omega + 8}{\omega^2 + 4\omega + 3} \quad \frac{\omega^2 - 4\omega + 8}{\omega^2 - 4\omega + 3}$$

Therefore,

$$L(s) = \frac{\omega^2 + 4\omega + 8}{\omega^2 + 4\omega + 3}$$