Assignment 9
Probability and Random Variables

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LaTeX

Outline

Question

Question 7.10

We denote by X_m a random variable equal to the number of tosses of a coin until heads shows for the mth time. Show that if

$P\{h\} = p$, then $E\{x_m\} = \dfrac{m}{p}$.

Solution

As we know,

$$1 + x + x^2 + x^3 + ...x^n + ... = \frac{1}{1-x}$$

Differentiating, we obtain,

$$1 + 2x + 3x^2 + 4x^3 + ...nx^{n-1} + ... = \sum_{k=1}^{\infty} kx^{k-1} = \frac{1}{(1-x)^2}$$

solution

The Random variable x_1 equals the number of tosses until heads show for the first time. Hence x_1 takes the values $1, 2, 3, ...$
with $P(x_1 = k) = pq^{k-1}$, Hence,

from above equation,

$$E(x_1) = \sum_{k=1}^{\infty} kP(x_1 = k) = \sum_{k=1}^{\infty} kpq^{k-1} = \frac{p}{(1-q)^2} = \frac{1}{p}$$

Starting the count after the first head shows,we conclude that the random variable $x_2 - x_1$ has the same statistics as the random variable x_1.Hence,

we can write

$$E(x_2 - x_1) = E(x_1)$$

$$E(x_2) = 2E(x_1) = \frac{2}{p}$$

Reasoning similarly, we can conclude that,

from all above results

$$E(x_n - x_{n-1}) = E(x_1). \text{ Hence (induction)}$$

$$E(x_n) = E(x_{n-1}) + E(x_1) = \frac{n-1}{p} + \frac{1}{p} = \frac{n}{p}$$