Assignment 9 Probability and Random Variables

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Outline

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Question

Question 7.10

We denote by X_m a random variable equal to the number of tosses of a coin until heads shows for the mth time. Show that if

$$P\{h\}=p$$
 , then $E\{x_m\}=rac{m}{p}$.



Solution

As we know,

$$1 + x + x^2 + x^3 + \dots + x^n + \dots = \frac{1}{1 - x}$$

Differentiating, we obtain,

$$1 + 2x + 3x^{2} + 4x^{3} + \dots nx^{n-1} + \dots = \sum_{k=1}^{\infty} kx^{k-1} = \frac{1}{(1-x)^{2}}$$

solution

The Random variable x_1 equals the number of tosses until heads show for the first time. Hence x_1 takes the values 1, 2, 3, ... with $P(x_1 = k) = pq^{k-1}$, Hence,

from above equation,

$$\mathsf{E}(x_1) = \sum_{k=1}^{\infty} k P(x_1 = k) = \sum_{k=1}^{\infty} k p q^{n-1} = \frac{p}{(1-q)^2} = \frac{1}{p}$$



Starting the count after the first head shows,we conclude that the random variable $x_2 - x_1$ has the same statistics as the random variable x_1 . Hence,

we can write

$$\mathsf{E}(x_2-x_1)=E(x_1)$$

$$\mathsf{E}(x_2) = 2E(x_1) = \frac{2}{p}$$

Reasoning similarly, we can conclude that,

from all above results

$$E(x_n - x_{n-1}) = E(x_1)$$
. Hence (induction)

$$E(x_n) = E(x_{n-1}) + E(x_1) = \frac{n-1}{p} + \frac{1}{p} = \frac{n}{p}$$

