An Empirical Analysis of the Capital Asset Pricing Model in the Bond Market

Group 26

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Contents

1 Introduction

Asset pricing models, i.e. the CAPM, are critical for obtaining insightful risk estimations to understand their relation with expected returns (Fama and French, 2004). With the dataset, we examine whether the CAPM and multi-factor models can accurately capture the observed risk-return dynamics and aim to identify their strengths and limitations while exploring the role of an additional momentum factor. We assess model effectiveness in pricing portfolios, consider whether results align with theoretical expectations, and explore potential enhancements or new approaches to improve practical utility when discrepancies or gaps are found.

2 Methodology

We performed two-stage regression with further GRS tests to evaluate the CAPM model and compared the results as motivated by the CAPM model that asset risk premia should be proportional to its beta.

2.1 The "Two-Stage" Regression

Firstly, with MKTB, the first-stage regression estimates the time-series model

$$Portfolio_{k,t} = \alpha_k + \beta_k \cdot MKTB_t + \varepsilon_{k,t}$$

for each $portfolio_k$ obtain estimated $\hat{\alpha_k}$ and $\hat{\beta_k}$ with significance levels and R^2 values also presented. Moreover, GLS estimates are also obtained as the model might suffer from heteroskedasticity problems. The second-stage regression incorporates average monthly excess returns for each $portfolio_k$ ($mean\_return_k$) and regress cross-sectionally on the first-stage estimated $\hat{\beta_k}$ as follow

$$mean\_return_k = \gamma_0 + \gamma_1 \cdot \hat{\beta_k} + \eta_k$$

where $\hat{\gamma_1}$ and its significance level with R^2 value are obtained. The GRS test is performed to test if the model is generally accepted.

Secondly, with the duration-adjusted MKTDB, similar regressions were performed as follows

$$Portfolio_{k,t} = \alpha_k + \beta_k^{'} \cdot MKTDB_t + \varepsilon_{k,t}$$

$$mean\_return_k = \gamma_0 + \gamma_1 \cdot \hat{\beta_k^{'}} + \eta_k$$

The first stage regresses excess returns on market risks to get α and β with different risk factors. The second stage evaluates β in explaining risk premiums that are time averages of $portfolio_k$'s excess returns.

2.2 The Momentum Factor

The 12-month lag was chosen as it captures the intermediate-term momentum effect, where past performance has been shown to persist while avoiding short-term noise and long-term mean reversion, as supported by Chan, Jegadeesh, and Lakonishok (1996), see Appendix A for detailed explanation. Thus the following second-stage regression models are estimated

$$mean\_return_k = \gamma_0 + \gamma_1 \cdot \hat{\beta}_k + \gamma_2 \cdot MOM_k + \eta_k$$

$$mean\_return_k = \gamma_0 + \gamma_1 \cdot \hat{\beta_k^{'}} + \gamma_2 \cdot MOM_k + \eta_k$$

$$mean\_return_k = \gamma_0 + \gamma_1 \cdot \hat{\beta}_k + \gamma_2 \cdot \hat{\beta_k^{'}} + \gamma_3 \cdot MOM_k + \eta_k$$

and without the momentum factor but both MKTB and MKTDB for comparison

$$mean\_return_k = \gamma_0 + \gamma_1 \cdot \hat{\beta}_k + \gamma_2 \cdot \hat{\beta_k^{'}} + \eta_k$$

with similar first-stage regressions.

The inclusion of the momentum factor is motivated by verifying whether past price performance can be used to extract future gains. This analysis of long the best-performers and short the worst ones from the previous year is based on the premise that momentum, might also apply to corporate bond portfolios. If this factor exists and is significant, it could show potential for an empirically relavant test for systematic mispricing.

3 Data and Empirical Results

3.1 The Dataset

Descriptive and summary statistics are presented in the Appendix B.

All variables reveal constant-mean reversion without a trend that could be considered stationary as shown in the following figures.

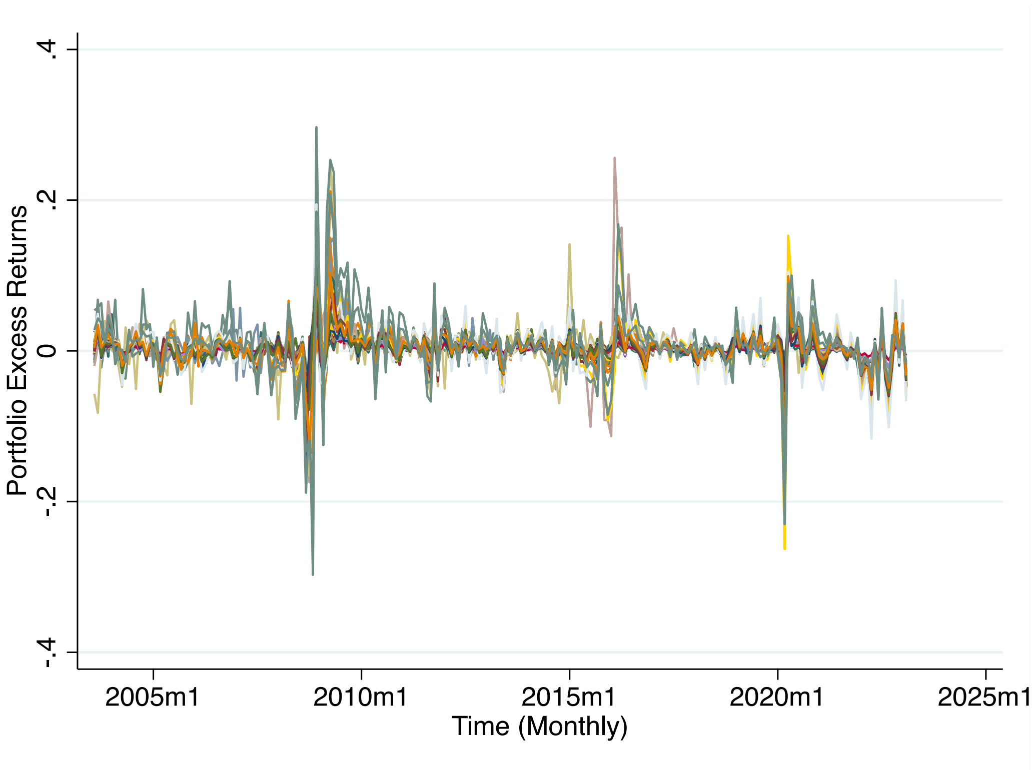

Figure 1: Time Series Plots of Portfolios Excess Returns

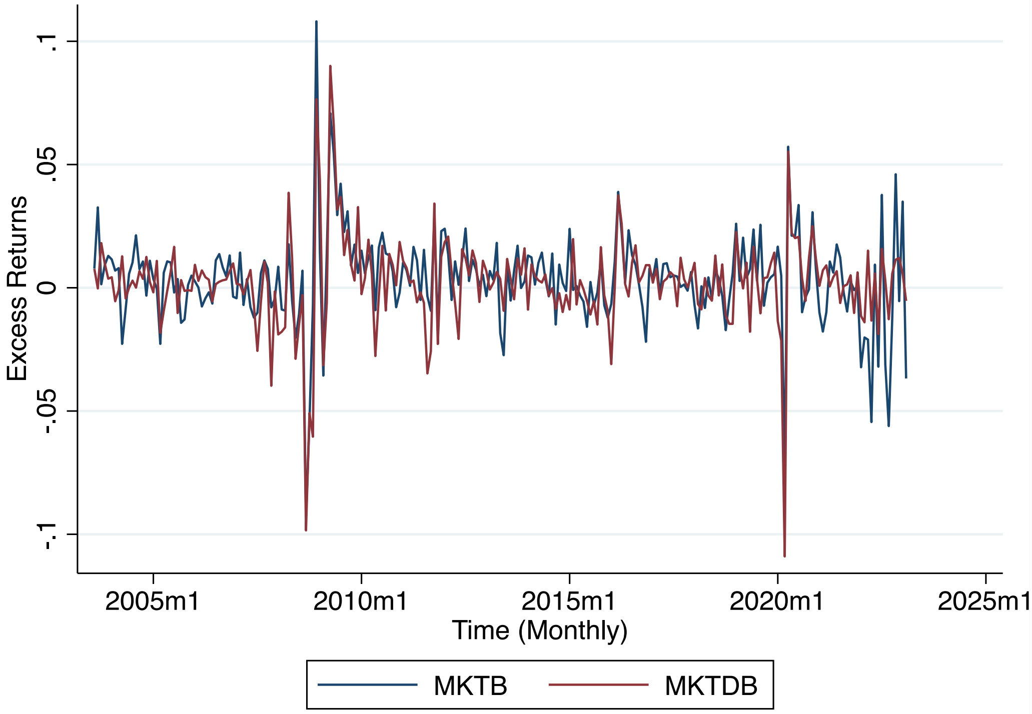

Figure 2: Time Series Plots of Market Risk Factors

3.2 The Single-Factor CAPM

3.2.1 With MKTB

```
R 4.4.1 · C:/Users/Asus/Desktop/2024-25 (WBS)/Course/Asset Pricing/Group Project/AssetP/
> print("Cross-Sectional Regression Summary:")
[1] "Cross-Sectional Regression Summary:"
> print(cross_sec_summary)

Call:
lm(formula = average_returns ~ betas)

Residuals:
       Min         1Q     Median         3Q        Max 
-1.685e-03 -5.661e-04  7.534e-05  4.413e-04  3.135e-03 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept) -0.0004289  0.0003862  -1.111    0.272    
betas        0.0042322  0.0003711  11.404 2.89e-15 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.000836 on 48 degrees of freedom
Multiple R-squared:  0.7304,	Adjusted R-squared:  0.7248 
F-statistic:   130 on 1 and 48 DF,  p-value: 2.892e-15
```

Figure 3: MKTB Cross-sectional Regression

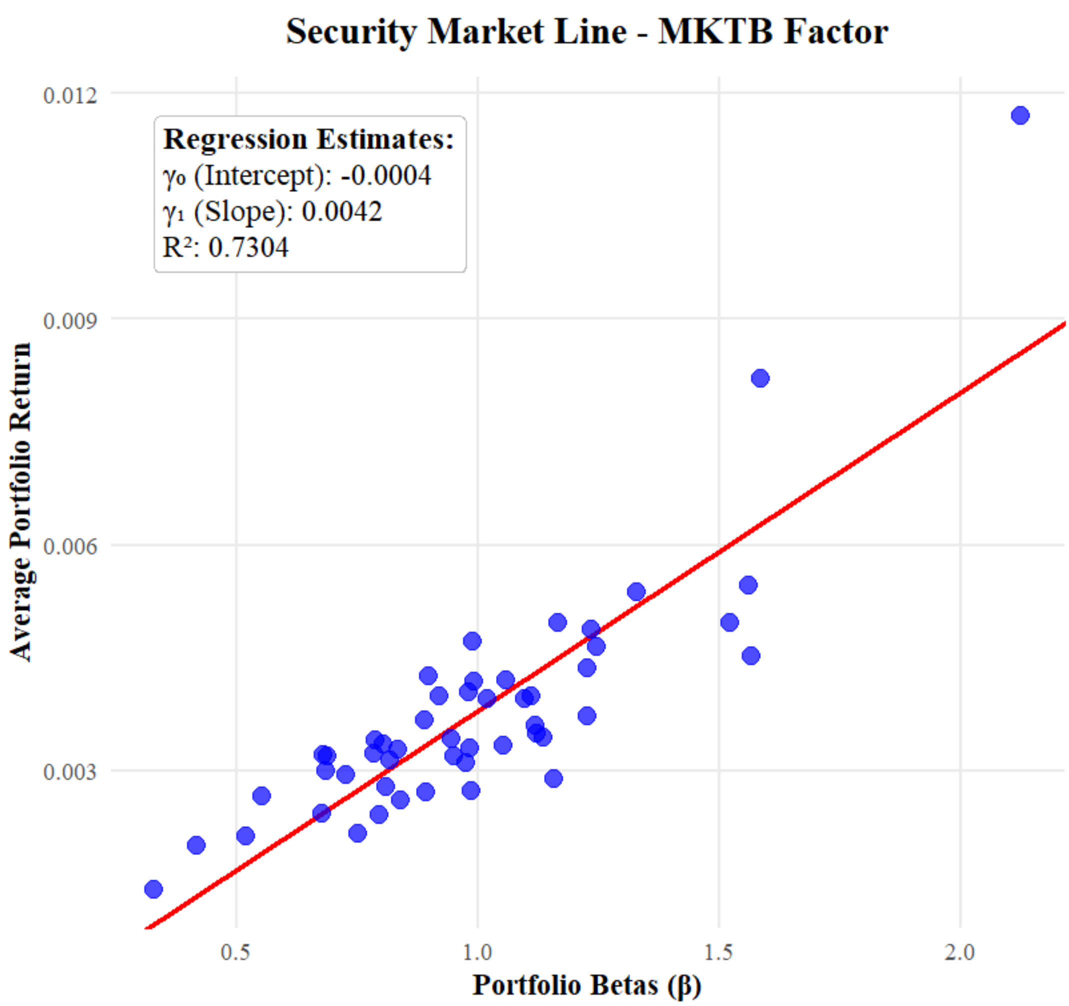

Figure 4: The Security Market Line for MKTB-CAPM Model

The single factor CAPM demonstrates moderate success in explaining the cross-section of portfolio returns, as indicated by its R^2 value of 73.04%. However, discrepancies in alphas between OLS and GLS (see Figures 5 and 6) suggest model misspecification or heteroskedasticity in the model. The Breusch–Pagan test found that 29 portfolios (see Figure 7) exhibited heteroskedasticity, further supporting the need for GLS adjustments.

	Portfolio	Alpha	Alpha_P_Value	Beta	Beta_P_Value	R_Squared
31	Portfolio 35	-0.00121715326882659	0.0218484805293307	1.15817494312502	4.46073924608566e-114	0.890899638409528
38	Portfolio 42	0.000752800760719005	0.0223912701537314	0.68648190741399	6.87691606736717e-110	0.881490006732608

Figure 5: Portfolios with Significant Alphas with MKTB

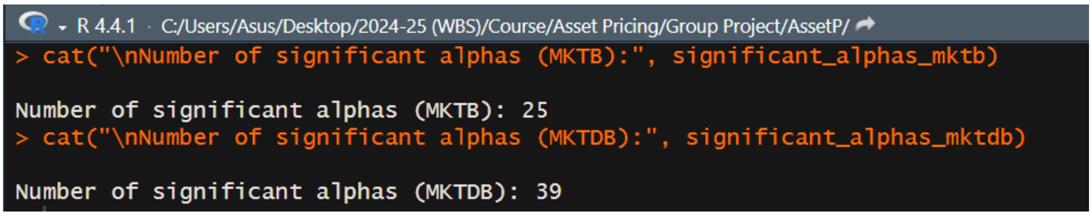

Figure 6: Number of Significant Alphas with MKTB and MKTDB

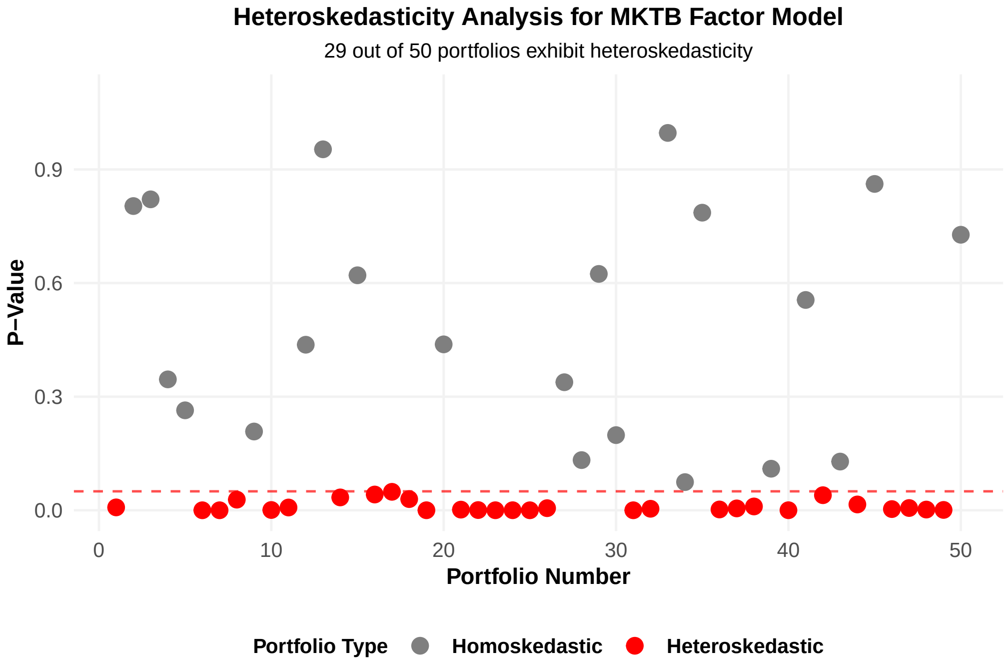

Figure 7: Heteroskedasticity Analysis for MKTB Factor Model

While CAPM captures systematic risk, its limitations are evident as significant alphas indicate some portfolios' excess returns remain unexplained. This implies the potential omission of critical risk factors in the model. In contrast, the low magnitudes of these alphas in both the OLS and GLS regressions indicate a relatively minor impact. This suggests that, despite its limitations, the CAPM provides a reasonably good approximation of expected returns for corporate bonds. Nonetheless, the inclusion of additional risk factors could significantly enhance the model's explanatory power.

The significant gamma estimate (see Figure 3) suggests that MKTB betas explain some risk premia. However, due to its low magnitude, its economic significance should be interpreted with caution. This caution extends to applying the CAPM to corporate bonds. While the high R^2 and significant beta values suggest that the MKTB factor captures some systematic risks, the presence of heteroskedasticity and significant alphas point to potential model misspecification and omitted variables. Likely, this necessitates supplementation by additional risk factors to address omitted variable bias and non-linearities in the model. For example, incorporating bond market specific factors such as term structure variables and credit spreads could further enhance the model's ability to capture the complexities of corporate bond pricing.

3.2.2 With MKTDB

```
> print("Cross-Sectional Regression Summary:")
[1] "Cross-Sectional Regression Summary:"
> print(crosssec_summary_mktdb)

Call:
lm(formula = average_returns ~ Beta)

Residuals:
       Min         1Q     Median         3Q        Max 
-1.472e-03 -3.493e-04 -3.290e-06  3.996e-04  1.508e-03 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept) 0.0011768  0.0001758   6.694 2.17e-08 ***
Beta        0.0035495  0.0002101  16.896  < 2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.0006108 on 48 degrees of freedom
Multiple R-squared:  0.8561,	Adjusted R-squared:  0.8531 
F-statistic: 285.5 on 1 and 48 DF,  p-value: < 2.2e-16
```

Figure 8: MKTDB Cross-sectional Regression

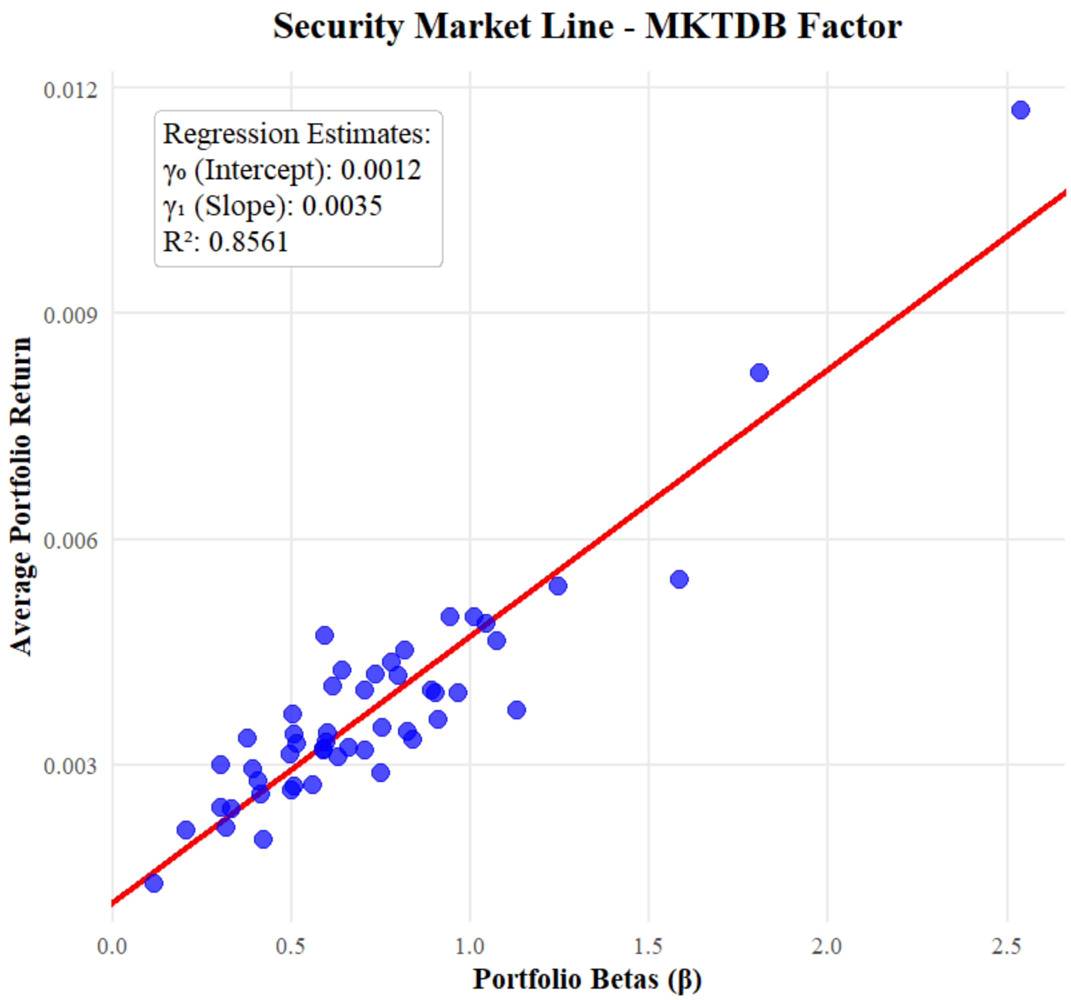

Figure 9: The Security Market Line for MKTDB-CAPM Model

The Cross-sectional R^2 of 85.61% (see Figure 8) indicates a notable improvement over the MKTB model's R^2 of 73.04%. This suggests that adjusting the market factor for bond durations improves its explanatory power for excess returns in corporate bond portfolios. This result aligns with ex-ante expectations because duration adjustments should make the factor more relevant for corporate bonds, which are sensitive to interest rate changes.

However, the number of significant alphas under OLS (27) and GLS (39) is considerably higher than in the MKTB model, implying persistent pricing errors despite the improved overall fit. Therefore, bonds are still exposed to other risk channels not captured by the model. The comparatively higher significant GLS alphas along with the BP test point to the presence of heteroskedasticity in the model.

Furthermore, the significant gamma estimates indicate that betas are meaningfully priced in the cross-sectional regression, consistent with the idea that systematic risk (as captured by MKTDB) influences bond excess returns. However, the disproportionately high number of significant alphas, which on average exhibit greater magnitude than those in the MKTB model, underscores persistent mispricing by the duration-adjusted model.

Metric	MKTB	MKTDB	Key Insights
Cross-sectional R^2	73.04%	85.61%	MKTDB performs better in explaining bond excess returns, likely due to its consideration of interest rate sensitivity via duration adjustments.
OLS Significant Alphas	2	27	MKTDB increases pricing errors under OLS, suggesting that duration adjustments introduce additional sources of mispricing (e.g., imperfect adjustments).
GLS Significant Alphas	25	39	GLS alphas rise significantly with MKTDB, reflecting cross-sectional dependencies and limitations of a single-factor approach.
Significant Gamma Estimates	Yes	Yes	Both models successfully price systematic risk, but MKTDB does so more effectively due to its bond-specific adjustments.

Figure 10: Comparison between Single-Factor Models

While MKTDB enhances the model's explanatory power by 12% compared to MKTB (see Figure 10), both models fall short of capturing the full complexity of risk exposures in corporate bond markets. This limitation suggests the presence of additional systematic or idiosyncratic risk factors that are not adequately addressed. Consequently, a more nuanced, multifactorial approach is necessary to more accurately account for the risk premia in corporate bond returns.

3.3 Multivariate CAPM Models

```
R 4.4.1 · C:/Users/Asus/Desktop/2024-25 (WBS)/Course/Asset Pricing/Group Project/AssetP/
> # Print the adjusted R-squared values
> cat("Adjusted R-squared values:\n")
Adjusted R-squared values:
> cat("MKTB Model:", round(adj_r2_mktb, 4), "\n")
MKTB Model: 0.7248
> cat("MKTDB Model:", round(adj_r2_mktdb, 4), "\n")
MKTDB Model: 0.8531
> cat("Multi-factor Model:", round(adj_r2_multi, 4), "\n")
Multi-factor Model: 0.8561
>
```

Figure 11: Summary of Adjusted R^2

One-factor models such as MKTB and MKTDB achieve an R^2 of 0.7248 and 0.8531 as in Figure 11, i.e., they explain 72.48% and 85.31% variation in the portfolio returns, respectively. The multi-factor model with momentum explains 85.61% of the variation in portfolio returns, indicating its superiority in capturing portfolio returns. Adjusted R^2 is used as it accounts for the number of independent variables and penalizes those that don't add value to the model.

The number of significant alphas obtained from the GLS regression supports this conclusion, where MKTB and MKTBD have 25 and 39 significant alphas, respectively, while the multi-factor model has only 14, highlighting its superior ability to capture factor returns

```
Cross Sectional Regression Summary:
> print(cross_sec_summary_multi)

Call:
lm(formula = average_returns ~ Beta_MKTB + Beta_MKTDB + Beta_MOM,
    data = multi_betas)

Residuals:
       Min         1Q     Median         3Q        Max
-1.582e-03 -3.393e-04 -1.079e-05  2.744e-04  1.491e-03

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.0008512  0.0003430   2.482  0.01679 *
Beta_MKTB    0.0028829  0.0003376   8.540 4.80e-11 ***
Beta_MKTDB   0.0033186  0.0002662  12.466 2.36e-16 ***
Beta_MOM    -0.0044773  0.0014103  -3.175  0.00267 **
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.0006046 on 46 degrees of freedom
Multiple R-squared:  0.8649,    Adjusted R-squared:  0.8561
F-statistic: 98.14 on 3 and 46 DF,  p-value: < 2.2e-16
```

Figure 12: Multivariate Cross-sectional Regression

In Figure 12, the coefficients of factors MKTB (0.0028829) and MKTDB (0.0033186) are positive and statistically significant (p-value < 0.001), indicating that these factors contribute positively to returns and earn a positive risk premium.

However, the coefficient of the momentum factor is negative (-0.0044773) and statistically significant (p-value < 0.01). Jegadeesh and Titman (1993) demonstrate that momentum strategies, indicating a positive relationship with returns. Contrarily, the results show a negative relationship, implying that greater exposure to momentum results in reduced returns.

Daniel and Moskowitz (2014) highlight that momentum strategies tend to underperform, resulting in negative coefficients during market rebounds following bear markets, as seen during the 2008 financial crisis and the COVID-19 pandemic, driven by a substantial recovery in past losers.

```
> cat("Risk Premia Estimates:\n")
Risk Premia Estimates:
> print(cross_sec_summary_multi$coefficients)
                 Estimate   Std. Error   t value     Pr(>|t|)
(Intercept)  0.0008512171 0.0003430091  2.481616 1.679362e-02
Beta_MKTB    0.0028828951 0.0003375926  8.539569 4.798140e-11
Beta_MKTDB   0.0033185986 0.0002662166 12.465784 2.355800e-16
Beta_MOM    -0.0044773007 0.0014102597 -3.174806 2.674863e-03
>
```

Figure 13: Summary of Risk Premia Estimates

The p-values for MKTB and MKTBD are extremely low at 4.798110-11 and 2.355810-16 respectively. These values are well below the 0.1% significance level, indicating that both are highly significant in explaining portfolio returns (aligns with CAPM proposed by Sharpe in 1964). The momentum factor's p-value of 0.00267 is below the 1% significance level, indicating that it also has a statistically significant impact on returns in Figure 13.

The pricing performance of combinations of different models is explored in the following Figure 14‘

Model Comparison: Corporate Bond Excess Returns

A detailed evaluation of single-factor and multi-factor models

Model Type	Cross Sectional R^2 (%)	Adjusted R^2 (%)	Significant Alphas (OLS)	Statistical Significance of Risk Premia	Signs of Risk Premia	Inference
One Factor (MKTB)	73.04	72.48	2	Significant	Positive (MKTB)	Basic single-factor model, the lowest goodness of fit.
One Factor (MKTDB)	85.61	85.31	27	Significant	Positive (MKTDB)	Improved explanatory power over MKTB model.
Multivariate Model 1: MKTB + MKTDB + MOM	86.49	85.61	14	All Significant	Positive (MKTB, MKTDB), Negative (MOM)	Best fit but MOM introduces residual errors.
Multivariate Model 2: MKTB + MKTDB	85.90	85.30	9	All Significant	Positive (MKTB, MKTDB)	Balanced model with lowest pricing errors.
Multivariate Model 3: MKTB + MOM	85.00	84.36	17	All Significant	Positive (MKTB), Negative (MOM)	MOM improves explanatory power over single factor MKTB model.
Multivariate Model 4: MKTDB + MOM	86.23	85.65	43	All Significant	Positive (MKTDB), Negative (MOM)	High R^2 but MOM complicates pricing.

Figure 14: Model Comparison Table

Model 2 emerges as the most efficient and effective model with a high cross-sectional R-squared of 85.9% and Adjusted R-squared of 85.3% compared to more complex models. It minimizes residual mispricing with the least significant alphas (9). The model's positive and significant risk premia further emphasise its robustness.

Gebhardt et al.(2005) identify that investment-grade bonds lack intrinsic momentum and highlight the spillover effect from equity returns to bond returns due to an under-reaction to equity-related information. Thus, the MKTB+MKTBD model, which prioritises market and credit factors, provides a more accurate representation of bond returns and performs better.

```
Console   Terminal ×   Background Jobs ×
R 4.4.1 · C:/Users/Asus/Desktop/2024-25 (WBS)/Course/Asset Pricing/Group Project/AssetP/
> print(results_table_mktb, row.names = FALSE)
 Factor GRS_Statistic     P_Value DF1 DF2            Conclusion
   MKTB        2.5809 2.3244e-06  50 184 Reject Null Hypothesis
> print(results_table_mktdb, row.names = FALSE)
 Factor GRS_Statistic     P_Value DF1 DF2            Conclusion
  MKTDB         2.407 1.1892e-05  50 184 Reject Null Hypothesis
> print(grs_multivariate_table, row.names = FALSE)
            Factor GRS_Statistic     P_Value DF1 DF2            Conclusion
 MKTB + MKTDB + MOM        2.8472 1.9634e-07  50 182 Reject Null Hypothesis
```

```
> print(grs_multivariate_table, row.names = FALSE)
              Factor GRS_Statistic     P_Value DF1 DF2           Conclusion
 MKTB + MKTDB + MOM        2.8472 1.9634e-07  50 182 Reject Null Hypothesis
> print(grs_combinations_table, row.names = FALSE)
 Factor_Combination GRS_Statistic     P_Value DF1 DF2           Conclusion
       MKTB + MKTDB        2.5805 2.3816e-06  50 183 Reject Null Hypothesis
         MKTB + MOM        2.9150 1.0049e-07  50 183 Reject Null Hypothesis
        MKTDB + MOM        3.0900 1.9111e-08  50 183 Reject Null Hypothesis
```

Figure 15: GRS Test Results

The GRS test shows MKTB and MKTDB significantly impact portfolio returns, rejecting the null hypothesis in Figure 15. Additionally, the rejection of the CAPM model, supported by the GRS results and significant alphas, highlights its limitations in explaining asset return variations.

Based on the above results, an effective pricing model should include additional factors like in the Fama-French five-factor model, along with bond-specific factors such as credit spread, liquidity and term structures. Analysing data over different time periods would ensure adaptability to evolving market conditions, enhancing the model's accuracy and reliability for decision making.

4 Conclusion

This study evaluated asset pricing models in the corporate bond market. While CAPM shows reasonable explanatory power, it fails to fully capture cross-sectional bond return. The significant risk premia for MKTB and MKTDB highlight their relevance, however low alphas and OLS-GLS discrepancies indicate heteroskedasticity and model misspecifications.

Multi-factor models outperform one-factor models, though the inclusion MOM yields only marginal improvements and exposes risks of overfitting. The negatively signed MOM risk premium raises questions about its validity as a systematic risk factor, possibly reflecting unexplored behavioural biases.

These findings emphasize trade-offs in model construction: while complexity enhances fit, it cannot fully resolve structural misspecifications. CAPM remains to be the preferred model for its simplicity, but incorporating factors from frameworks like Fama-French offers a more comprehensive approach. Future research should explore bond-specific risks, dynamic models involving nonlinearities, and time-varying risk premia to better reflect the complexities of corporate bond returns.

5 References

Chan, L.K., Jegadeesh, N. and Lakonishok, J., 1996. Momentum strategies. *The journal of Finance*, 51(5), pp.1681-1713.

Daniel, K. and Moskowitz, T.J., 2014. *Momentum crashes* (No. w20439). National Bureau of Economic Research.

Fama, E.F. and French, K.R., 2004. The capital asset pricing model: Theory and evidence. *Journal of economic perspectives*, 18(3), pp.25-46.

Fama, E.F. and French, K.R., 2015. A five-factor asset pricing model. *Journal of financial economics*, 116(1), pp.1-22.

Gebhardt, W.R., Hvidkjaer, S. and Swaminathan, B., 2005. Stock and bond market interaction: Does momentum spill over?. *Journal of Financial Economics*, 75(3), pp.651-690.

Sharpe, W.F., 1964. Capital asset prices: A theory of market equilibrium under conditions of risk. *The journal of finance*, 19(3), pp.425-442.

6 Appendices

6.1 Appendix A

Steps for Constructing the Momentum Factor

Step 1: Generate lagged portfolio returns to ensure the lagged returns account for one year

$$l\_R_{k,t} = R_{ik,t} - R_{k,t-12}$$

Step 2: Calculate the 12-month lagged mean excess returns for each portfolio k

$$l\_\bar{R}_k = \frac{1}{T}\sum_{t=1}^{T} l\_R_{k,t}$$

Step 3: Sort by generating ranks based on the excess return and group them into deciles, each including five companies. Consequently, generate the momentum factor by going long in the five best-performing portfolios (top decile) and shorting the bottom decile

$$MOM_t = R_{top5,t} - R_{bottom5,t}$$

The decision to employ a 12-month lag in returns to construct the MOM factor is motivated by prior empirical evidence. Chan, Jegadeesh, and Lakonishok (1996) documented that momentum strategies based on intermediate horizons of three to twelve months are particularly effective in capturing the continuation of returns. Their findings highlight the gradual adjustment of stock prices to past information, which supports using a 12-month lag to ensure the strategy captures these persistent return drifts effectively.

6.2 Appendix B

Contains data from PRRF.dta
Observations: 235
Variables: 54 1 Dec 2024 13:21

Variable name	Storage type	Display format	Value label	Variable label
Date	int	%td..		Date
Portfolio1	double	%10.0g		Portfolio 1
Portfolio2	double	%10.0g		Portfolio 2
Portfolio3	double	%10.0g		Portfolio 3
Portfolio4	double	%10.0g		Portfolio 4
Portfolio6	double	%10.0g		Portfolio 6
Portfolio8	double	%10.0g		Portfolio 8
Portfolio9	double	%10.0g		Portfolio 9
Portfolio10	double	%10.0g		Portfolio 10
Portfolio12	double	%10.0g		Portfolio 12
Portfolio13	double	%10.0g		Portfolio 13
Portfolio14	double	%10.0g		Portfolio 14
Portfolio16	double	%10.0g		Portfolio 16
Portfolio17	double	%10.0g		Portfolio 17
Portfolio18	double	%10.0g		Portfolio 18
Portfolio19	double	%10.0g		Portfolio 19
Portfolio20	double	%10.0g		Portfolio 20
Portfolio21	double	%10.0g		Portfolio 21
Portfolio22	double	%10.0g		Portfolio 22
Portfolio23	double	%10.0g		Portfolio 23
Portfolio24	double	%10.0g		Portfolio 24
Portfolio25	double	%10.0g		Portfolio 25
Portfolio26	double	%10.0g		Portfolio 26
Portfolio27	double	%10.0g		Portfolio 27
Portfolio28	double	%10.0g		Portfolio 28
Portfolio29	double	%10.0g		Portfolio 29
Portfolio30	double	%10.0g		Portfolio 30
Portfolio31	double	%10.0g		Portfolio 31
Portfolio32	double	%10.0g		Portfolio 32
Portfolio33	double	%10.0g		Portfolio 33
Portfolio34	double	%10.0g		Portfolio 34
Portfolio35	double	%10.0g		Portfolio 35
Portfolio36	double	%10.0g		Portfolio 36
Portfolio37	double	%10.0g		Portfolio 37
Portfolio38	double	%10.0g		Portfolio 38
Portfolio39	double	%10.0g		Portfolio 39
Portfolio40	double	%10.0g		Portfolio 40
Portfolio41	double	%10.0g		Portfolio 41
Portfolio42	double	%10.0g		Portfolio 42
Portfolio43	double	%10.0g		Portfolio 43
Portfolio44	double	%10.0g		Portfolio 44
Portfolio45	double	%10.0g		Portfolio 45
Portfolio46	double	%10.0g		Portfolio 46
Portfolio47	double	%10.0g		Portfolio 47
Portfolio48	double	%10.0g		Portfolio 48
Portfolio49	double	%10.0g		Portfolio 49
Portfolio50	double	%10.0g		Portfolio 50
Portfolio51	double	%10.0g		Portfolio 51
Portfolio52	double	%10.0g		Portfolio 52
Portfolio53	double	%10.0g		Portfolio 53
Portfolio54	double	%10.0g		Portfolio 54
MKTB	double	%10.0g		MKTB
MKTDB	double	%10.0g		MKTDB
Mon	float	%tm		

Sorted by: Mon

Figure 16: Descriptive Statistics

Descriptive Statistics

Variable	Obs	Mean	Std. Dev.	Min	Max
Date	235	19509.132	2069.218	15948	23069
Portfolio1	235	.003	.016	-.076	.06
Portfolio2	235	.002	.018	-.07	.091
Portfolio3	235	.004	.019	-.06	.096
Portfolio4	235	.004	.024	-.127	.15
Portfolio6	235	.003	.017	-.074	.081
Portfolio8	235	.003	.018	-.079	.099
Portfolio9	235	.004	.02	-.086	.108
Portfolio10	235	.003	.024	-.091	.113
Portfolio12	235	.004	.022	-.104	.088
Portfolio13	235	.003	.018	-.086	.065
Portfolio14	235	.003	.02	-.077	.069
Portfolio16	235	.003	.018	-.086	.058
Portfolio17	235	.005	.036	-.174	.256
Portfolio18	235	.004	.033	-.263	.166
Portfolio19	235	.004	.021	-.103	.105
Portfolio20	235	.004	.022	-.089	.125
Portfolio21	235	.003	.022	-.112	.124
Portfolio22	235	.003	.018	-.102	.101
Portfolio23	235	.004	.022	-.12	.141
Portfolio24	235	.005	.024	-.109	.093
Portfolio25	235	.003	.017	-.095	.067
Portfolio26	235	.003	.02	-.107	.103
Portfolio27	235	.005	.036	-.165	.248
Portfolio28	235	.003	.02	-.144	.137
Portfolio29	235	.005	.029	-.122	.226
Portfolio30	235	.001	.008	-.025	.07
Portfolio31	235	.002	.012	-.046	.085
Portfolio32	235	.002	.015	-.056	.098
Portfolio33	235	.003	.018	-.076	.099
Portfolio34	235	.003	.02	-.103	.091
Portfolio35	235	.003	.024	-.161	.103
Portfolio36	235	.003	.023	-.088	.08
Portfolio37	235	.005	.026	-.115	.104
Portfolio38	235	.005	.035	-.142	.212
Portfolio39	235	.012	.057	-.297	.297
Portfolio40	235	.002	.01	-.081	.067
Portfolio41	235	.003	.012	-.073	.098
Portfolio42	235	.003	.014	-.087	.082
Portfolio43	235	.003	.016	-.092	.066
Portfolio44	235	.004	.021	-.1	.118
Portfolio45	235	.004	.022	-.094	.104
Portfolio46	235	.003	.022	-.1	.112
Portfolio47	235	.004	.025	-.098	.123
Portfolio48	235	.005	.031	-.114	.169
Portfolio49	235	.005	.034	-.116	.195
Portfolio50	235	.002	.017	-.073	.09
Portfolio51	235	.003	.02	-.125	.115
Portfolio52	235	.003	.02	-.104	.105
Portfolio53	235	.004	.023	-.134	.104
Portfolio54	235	.008	.039	-.187	.208
MKTB	235	.004	.02	-.093	.108
MKTDB	235	.002	.019	-.109	.09
Mon	235	640	67.983	523	757

Figure 17: Summary Statistics

6.3 Appendix C

Significant Alphas (MKTDB)

Portfolio	Alpha	Alpha_P_Value	Beta	Beta_P_Value	R_Squared
Portfolio 1	0.00233304601874001	0.0161870208739298	0.304165123896228	4.22582868838907E-09	0.13796134673016
Portfolio 3	0.00259283051677888	0.0206178414504527	0.502243278872937	4.98468578709823E-16	0.24648487396273
Portfolio 6	0.00209925401222361	0.0388978739904279	0.393128442262601	1.16968882602746E-12	0.195353586231037
Portfolio 8	0.00253812720142266	0.0183244244594077	0.37741833310518	7.27483401216698E-11	0.166854453982295
Portfolio 9	0.00287430535836709	0.00499073215268025	0.641973067447359	6.16325029483418E-27	0.391442063313698
Portfolio 12	0.00246266616043109	0.0140043302807333	0.798002438202058	9.29312755103959E-38	0.508085845674785
Portfolio 13	0.00217144873754741	0.0316263745204323	0.513382208064722	1.82246018384364E-19	0.295440646014802
Portfolio 19	0.00272456063949747	0.0186516162349973	0.615288555173948	6.94339774292436E-21	0.314746238593257
Portfolio 20	0.00261511594591252	0.0173760134265166	0.734965645606957	1.75104293722258E-29	0.421135439818717
Portfolio 22	0.00207285522191593	0.0388968389816356	0.496012472408884	1.23933902774727E-18	0.283870763213037
Portfolio 23	0.0024624080329784	0.0324932554611226	0.706883876429916	5.86693022703655E-26	0.379636722514973
Portfolio 24	0.00344366649568438	0.0122359534876726	0.591984666097301	4.6087773681424E-15	0.232151682850988
Portfolio 25	0.00231058140841954	0.0126122812866549	0.506545794819527	7.356549170225E-22	0.327710189653896
Portfolio 26	0.00212728929796273	0.0437341334640335	0.599451965259236	4.28339208038589E-23	0.343788633607334
Portfolio 30	0.00116013782493899	0.0317749420698326	0.117489850594152	3.35585375966594E-05	0.0713264820600767
Portfolio 31	0.0016716018479096	0.0281709901114713	0.20759257010159	2.63291509647377E-07	0.107727483661843
Portfolio 37	0.0023300298898082	0.0256108882994988	1.07304067728595	1.71391472881982E-52	0.632026479348395
Portfolio 39	0.00624068559431081	0.00144400236466809	2.53875112301396	5.80968700714519E-69	0.734098872950056
Portfolio 40	0.00110243985815948	0.00895597284398686	0.420564404898164	1.09373591362534E-50	0.618696953923656
Portfolio 41	0.00157748201569062	0.00148847109816059	0.498708847921286	2.28763872893005E-51	0.623770834379985
Portfolio 42	0.00192380799346712	0.000960212759289392	0.588600120172192	8.57169189686554E-52	0.626919704441861
Portfolio 43	0.00181332127018452	0.0083209470502704	0.658714413458183	2.93028774721676E-48	0.600004538646343
Portfolio 44	0.00201549075304557	0.0116843370248324	0.903947551957022	6.32254423360519E-59	0.675869239274111
Portfolio 45	0.00207588691239828	0.0294656700830505	0.891002355173666	1.39066295912491E-46	0.586569615580272
Portfolio 47	0.00267744446502548	0.0386588523547916	0.779630308233384	3.29477758179382E-25	0.370444323090089
Portfolio 53	0.00187481480951025	0.0462845045978668	0.96442987708368	3.01155725441452E-52	0.630246167423981
Portfolio 54	0.00432889119995924	0.000222911276600058	1.80833483720685	9.16012723827245E-83	0.797512301465752

Figure 18: Regression Results with MKTDB

6.4 Appendix D

BP Test for MKTB

Portfolio	BP_Statistic	P_Value	Conclusion
Portfolio 1	7.11572875736338	0.00764105360871801	Heteroskedasticity detected
Portfolio 2	0.0619902609544108	0.803377462834492	No heteroskedasticity
Portfolio 3	0.0511142720747237	0.821135577703725	No heteroskedasticity
Portfolio 4	0.889159035825139	0.345705302975526	No heteroskedasticity
Portfolio 6	1.24800897048036	0.263933094096553	No heteroskedasticity
Portfolio 8	20.869929564056	4.9155100143186E-06	Heteroskedasticity detected
Portfolio 9	16.808933213589	4.133824554081E-05	Heteroskedasticity detected
Portfolio 10	4.82383615459078	0.0280688092704249	Heteroskedasticity detected
Portfolio 12	1.585213339279	0.208011349359234	No heteroskedasticity
Portfolio 13	11.3478811310614	0.000755340973475534	Heteroskedasticity detected
Portfolio 14	7.19569192310925	0.00730788074848045	Heteroskedasticity detected
Portfolio 16	0.603700696513851	0.437169516138006	No heteroskedasticity
Portfolio 17	0.00343711160196287	0.953249296665646	No heteroskedasticity
Portfolio 18	4.49914141316423	0.0339118766537502	Heteroskedasticity detected
Portfolio 19	0.245017661532601	0.620605352991445	No heteroskedasticity
Portfolio 20	4.15167768450939	0.0415932942657475	Heteroskedasticity detected
Portfolio 21	3.88380558838202	0.0487539293354372	Heteroskedasticity detected
Portfolio 22	4.73747692664774	0.0295123231006901	Heteroskedasticity detected
Portfolio 23	15.6487386341927	7.62634307183072E-05	Heteroskedasticity detected
Portfolio 24	0.601131658964399	0.438146572393593	No heteroskedasticity
Portfolio 25	9.87302641655426	0.00167719402897762	Heteroskedasticity detected
Portfolio 26	11.8201009829454	0.0005859468202923	Heteroskedasticity detected
Portfolio 27	15.7408633260961	7.26379196845112E-05	Heteroskedasticity detected
Portfolio 28	14.5125444101165	0.000139229321429669	Heteroskedasticity detected
Portfolio 29	27.275184665209	1.76461632121532E-07	Heteroskedasticity detected
Portfolio 30	7.70126331576941	0.00551821898448193	Heteroskedasticity detected
Portfolio 31	0.917499385145567	0.338132409805951	No heteroskedasticity
Portfolio 32	2.26268154406061	0.132524410104544	No heteroskedasticity
Portfolio 33	0.240039856859149	0.624177329528581	No heteroskedasticity
Portfolio 34	1.65363472753193	0.198464946358533	No heteroskedasticity
Portfolio 35	21.1077661106283	4.34165002044706E-06	Heteroskedasticity detected
Portfolio 36	8.28654937816818	0.00399398114645478	Heteroskedasticity detected

Portfolio 37	1.90191177488607E-05	0.996520363574821	No heteroskedasticity
Portfolio 38	3.18337173260375	0.0743910770678966	No heteroskedasticity
Portfolio 39	0.073785394100922	0.785902938404817	No heteroskedasticity
Portfolio 40	9.45548118916473	0.00210519058500814	Heteroskedasticity detected
Portfolio 41	7.91335741223366	0.00490711144187732	Heteroskedasticity detected
Portfolio 42	6.61866453357308	0.0100915476342752	Heteroskedasticity detected
Portfolio 43	2.55827200869716	0.109718449296841	No heteroskedasticity
Portfolio 44	14.6286890517628	0.000130906485812662	Heteroskedasticity detected
Portfolio 45	0.347747982219455	0.55539071940961	No heteroskedasticity
Portfolio 46	4.22502566631986	0.0398320168103284	Heteroskedasticity detected
Portfolio 47	2.30917840565887	0.128612014063028	No heteroskedasticity
Portfolio 48	5.87699233801441	0.0153400039538921	Heteroskedasticity detected
Portfolio 49	0.0302996198707132	0.861812131597358	No heteroskedasticity
Portfolio 50	8.72977924890418	0.0031305439692339	Heteroskedasticity detected
Portfolio 51	7.56089418596813	0.00596483891002128	Heteroskedasticity detected
Portfolio 52	9.67237138960068	0.00187059710107417	Heteroskedasticity detected
Portfolio 53	11.0291387043476	0.000896908099960707	Heteroskedasticity detected
Portfolio 54	0.121332535132001	0.727593717593713	No heteroskedasticity

Figure 19: BP Test for MKTB Regressions Results

BP Test for MKTDB

Portfolio	BP_Statistic	P_Value	Conclusion
Portfolio 1	4.94159041993824	0.0262179018466033	Heteroskedasticity detected
Portfolio 2	0.0159906731086297	0.899372374721078	No heteroskedasticity
Portfolio 3	0.001103027419081	0.973505643136641	No heteroskedasticity
Portfolio 4	0.0940244346828083	0.759121940574223	No heteroskedasticity
Portfolio 6	0.539542720172509	0.46262229982592	No heteroskedasticity
Portfolio 8	0.940251674380132	0.332213125853012	No heteroskedasticity
Portfolio 9	2.1370934018733	0.143774071426118	No heteroskedasticity
Portfolio 10	0.00115562191897187	0.972881584752126	No heteroskedasticity
Portfolio 12	0.150698235542163	0.697868990502226	No heteroskedasticity
Portfolio 13	9.92831430440627	0.00162755519664333	Heteroskedasticity detected
Portfolio 14	2.68947218632293	0.101013269528899	No heteroskedasticity
Portfolio 16	6.23760478423671	0.0125065506532009	Heteroskedasticity detected
Portfolio 17	0.143874277129274	0.704459435374635	No heteroskedasticity
Portfolio 18	13.1649628986255	0.000285232434858637	Heteroskedasticity detected
Portfolio 19	1.91990324676595	0.165867326751497	No heteroskedasticity
Portfolio 20	0.0545725829482617	0.815289507112337	No heteroskedasticity
Portfolio 21	8.15310340072776	0.00429875334249253	Heteroskedasticity detected
Portfolio 22	0.00676630460496303	0.934441909815484	No heteroskedasticity
Portfolio 23	1.79105864125383	0.180797231239996	No heteroskedasticity
Portfolio 24	3.74067657174748	0.0531029389614593	No heteroskedasticity
Portfolio 25	11.6148009519013	0.000654290400187855	Heteroskedasticity detected
Portfolio 26	0.466321839545016	0.494684178798093	No heteroskedasticity
Portfolio 27	1.32507063776348	0.249684053277126	No heteroskedasticity
Portfolio 28	4.81326773746556	0.0282414436384088	Heteroskedasticity detected
Portfolio 29	14.9496593324361	0.000110417997761051	Heteroskedasticity detected
Portfolio 30	3.16361866770972	0.075296107764913	No heteroskedasticity
Portfolio 31	0.00495284243050837	0.943894047856911	No heteroskedasticity
Portfolio 32	0.596787123850501	0.439806516227173	No heteroskedasticity
Portfolio 33	0.27423594408502	0.600504315536079	No heteroskedasticity
Portfolio 34	2.77984082663094	0.0954576551532424	No heteroskedasticity
Portfolio 35	18.6736904825601	1.5510798942709E-05	Heteroskedasticity detected
Portfolio 36	3.07144777807494	0.0796784825346563	No heteroskedasticity

Portfolio 37	2.10501397456488	0.146817001939139	No heteroskedasticity
Portfolio 38	0.685023435371841	0.407862270666184	No heteroskedasticity
Portfolio 39	0.405485679872855	0.524269735140237	No heteroskedasticity
Portfolio 40	7.98234181779617	0.00472357997977952	Heteroskedasticity detected
Portfolio 41	5.27918804042535	0.0215818101939059	Heteroskedasticity detected
Portfolio 42	1.35456950881589	0.244480854934509	No heteroskedasticity
Portfolio 43	7.96931065876391	0.00475770536825338	Heteroskedasticity detected
Portfolio 44	0.000306971157016474	0.98602130503107	No heteroskedasticity
Portfolio 45	0.554956987756996	0.456299315458047	No heteroskedasticity
Portfolio 46	1.18998753003743	0.275332071640646	No heteroskedasticity
Portfolio 47	0.195773446583922	0.658154247832856	No heteroskedasticity
Portfolio 48	0.235602597974223	0.627400348522552	No heteroskedasticity
Portfolio 49	0.79261815785265	0.373309624371355	No heteroskedasticity
Portfolio 50	1.31545373054976	0.25140959665487	No heteroskedasticity
Portfolio 51	4.94218914429218	0.0262088221208019	Heteroskedasticity detected
Portfolio 52	8.19614207406208	0.00419795599238449	Heteroskedasticity detected
Portfolio 53	11.9688289606869	0.00054097928931818	Heteroskedasticity detected
Portfolio 54	3.09660974576486	0.0784555211487887	No heteroskedasticity

Figure 20: BP Test for MKTDB Regressions Results

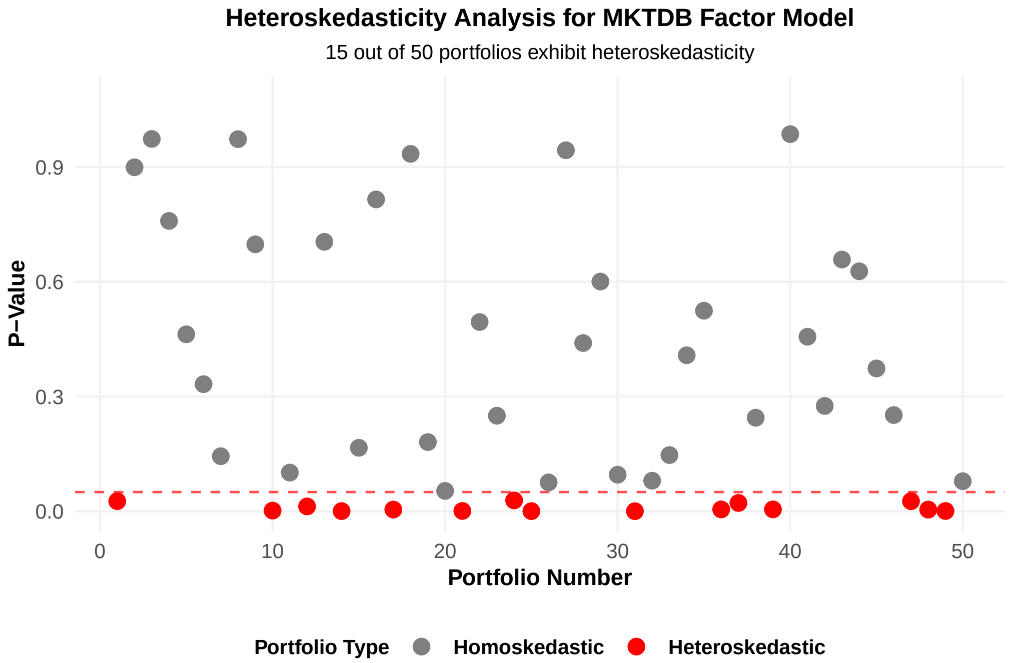

Figure 21: Graph Illustration of Heteroskedasticity of the MKTDB-CAPM Model

BP Test for Multivariate Model

Portfolio	BP_Statistic	P_Value	Conclusion
Portfolio 1	9.15280555975894	0.0273267514678469	Heteroskedasticity detected
Portfolio 2	17.1966956887144	0.000643867556451899	Heteroskedasticity detected
Portfolio 3	7.25397894977928	0.064228416530379	No heteroskedasticity
Portfolio 4	0.8811854486139	0.829965654880577	No heteroskedasticity
Portfolio 6	1.6299197651641	0.652624902060371	No heteroskedasticity
Portfolio 8	16.1841766959122	0.00103952772576266	Heteroskedasticity detected
Portfolio 9	19.5397437221743	0.000211409907911974	Heteroskedasticity detected
Portfolio 10	6.96599003371424	0.0729897222597471	No heteroskedasticity
Portfolio 12	3.11856674510555	0.37370331570729	No heteroskedasticity
Portfolio 13	26.2995437218137	8.25469943354273E-06	Heteroskedasticity detected
Portfolio 14	7.04806151461986	0.0703815634467599	No heteroskedasticity
Portfolio 16	23.5971792227215	3.03154188990715E-05	Heteroskedasticity detected
Portfolio 17	2.17196783311029	0.537490918398369	No heteroskedasticity
Portfolio 18	22.3576136821229	5.49568362813174E-05	Heteroskedasticity detected
Portfolio 19	12.9925823883001	0.00465267393708592	Heteroskedasticity detected
Portfolio 20	19.0611629965119	0.000265551923103691	Heteroskedasticity detected
Portfolio 21	11.1193982592029	0.0110974572164128	Heteroskedasticity detected
Portfolio 22	3.01908670632109	0.388691732998341	No heteroskedasticity
Portfolio 23	14.5308758147063	0.00226477767941838	Heteroskedasticity detected
Portfolio 24	11.1768533284059	0.0108070103303576	Heteroskedasticity detected
Portfolio 25	37.4077758095428	3.77226415149427E-08	Heteroskedasticity detected
Portfolio 26	8.37573191550093	0.0388523490319737	Heteroskedasticity detected
Portfolio 27	12.0496255932353	0.00721508314131268	Heteroskedasticity detected
Portfolio 28	23.5020785538918	3.1732471912446E-05	Heteroskedasticity detected
Portfolio 29	32.2116393895129	4.72261094370936E-07	Heteroskedasticity detected
Portfolio 30	36.5908725633615	5.61611668676029E-08	Heteroskedasticity detected
Portfolio 31	41.7065113054073	4.63073794225849E-09	Heteroskedasticity detected
Portfolio 32	29.2618559798686	1.97290423153258E-06	Heteroskedasticity detected
Portfolio 33	18.5288623422267	0.000342101782183717	Heteroskedasticity detected
Portfolio 34	7.10143318310398	0.0687340710010052	No heteroskedasticity
Portfolio 35	32.5582686245305	3.99121451899755E-07	Heteroskedasticity detected
Portfolio 36	9.12547767877381	0.0276682838902265	Heteroskedasticity detected

Portfolio 37	2.93738011494962	0.401381038998323	No heteroskedasticity
Portfolio 38	31.6408064731778	6.22984623827618E-07	Heteroskedasticity detected
Portfolio 39	15.5470442475842	0.00140410619327357	Heteroskedasticity detected
Portfolio 40	17.5165033389023	0.000553294025904986	Heteroskedasticity detected
Portfolio 41	15.9499239756339	0.00116110810544144	Heteroskedasticity detected
Portfolio 42	20.8223617924973	0.000114607849281598	Heteroskedasticity detected
Portfolio 43	12.4431250144742	0.00600956825074845	Heteroskedasticity detected
Portfolio 44	21.1140234543173	9.96891167430105E-05	Heteroskedasticity detected
Portfolio 45	18.7492131207948	0.000308059692071451	Heteroskedasticity detected
Portfolio 46	6.65474739978413	0.0837554081591566	No heteroskedasticity
Portfolio 47	12.4044486124727	0.00611863634678482	Heteroskedasticity detected
Portfolio 48	6.42622639894108	0.0926178010602787	No heteroskedasticity
Portfolio 49	7.9657652191541	0.0467245594520151	Heteroskedasticity detected
Portfolio 50	19.9292251953706	0.000175572575501869	Heteroskedasticity detected
Portfolio 51	26.942177156036	6.05399405172322E-06	Heteroskedasticity detected
Portfolio 52	73.0361278345716	9.5486947132494E-16	Heteroskedasticity detected
Portfolio 53	20.8496203616783	0.000113124152143313	Heteroskedasticity detected
Portfolio 54	22.7898495656326	4.4667736191367E-05	Heteroskedasticity detected

Figure 22: BP Test for Multivariate Regressions Results

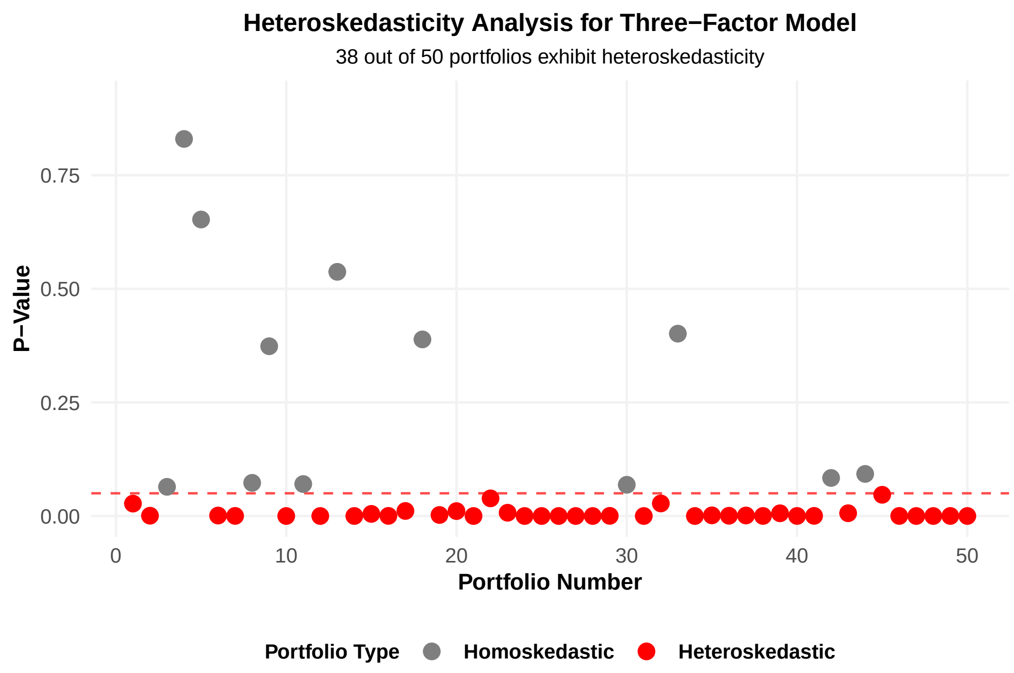

Figure 23: Graph Illustration of Heteroskedasticity of the Multivariate CAPM Model