# Productivity Growth, Exports, and Consumption Dynamics in Europe

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**Abstract**

This report presents the findings of the data analysis and computation tasks for the LSE Centre for Macroeconomics pre-doctoral application. **Task A** provides an empirical analysis of Total Factor Productivity (TFP) growth across five European economies over 2000–2023, documenting procyclical productivity patterns and a positive cross-sectional relationship between TFP and export intensity. Using Eurostat national accounts data, I compute sector-level TFP and estimate fixed-effects regressions linking productivity to trade exposure. **Task B** discusses Parle (2022) on ECB communication tone, suggesting improvements through multi-channel extensions and modern transformer-based sentiment models. **Task C** solves a heterogeneous-agent consumption-savings model with income uncertainty using Value Function Iteration, deriving the Euler equation, computing stationary distributions, and calibrating the discount factor to match empirical estimates of the Marginal Propensity to Consume (MPC). The results highlight how borrowing constraints and precautionary motives shape household consumption dynamics.

**Keywords:** Total Factor Productivity, Solow Residual, Export Intensity, Dynamic Programming, Marginal Propensity to Consume, Value Function Iteration.

**JEL Classification:** E21, E23, F14, D15.

*AI Disclosure: Generative AI tools were used for code suggestions, debugging, LaTeX assistance, and understanding the theory behind Task C. All economic interpretations and final outputs were carefully verified by the author.*

*Technical assessment completed within the 24-hour window. Code and replication files are provided in the accompanying submission.

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# 1 Task A: Empirical Analysis

This section investigates the relationship between productivity growth and exports at the sectoral level in Europe. I construct TFP measures using Eurostat national accounts data, analyze cyclical patterns, and estimate the productivity-export relationship econometrically.

## 1.1 Data and Methodology

### Data Sources

All data are sourced from Eurostat's annual national accounts:

- **Output ($Y$):** Gross value added at current prices (million euro) from `nama_10_a10`
- **Labor ($L$):** Total hours worked (thousand hours) from `nama_10_a10_e`
- **Capital ($K$):** Net capital stock at current replacement costs (million euro) from `nama_10_nfa_st`
- **Exports:** Exports by NACE Rev. 2 activity (million euro) from `ext_tec09`

### Sample

The analysis covers five European economies: Germany (DE), Spain (ES), Portugal (PT), Austria (AT), and the Netherlands (NL); over the period 2000–2023. Sectors analyzed include the total economy (TOTAL) and six NACE Rev. 2 sectors: Agriculture (A), Manufacturing (C), Construction (F), Information & Communication (J), Financial & Insurance (K), and Real Estate (L).

## 1.2 TFP Calculation and Total Economy Analysis (Q1a)

Total Factor Productivity (TFP) is computed as the Solow residual from a Cobb-Douglas production function:

$$Y_{cst} = A_{cst} K_{cst}^{\alpha} L_{cst}^{1-\alpha} \tag{1}$$

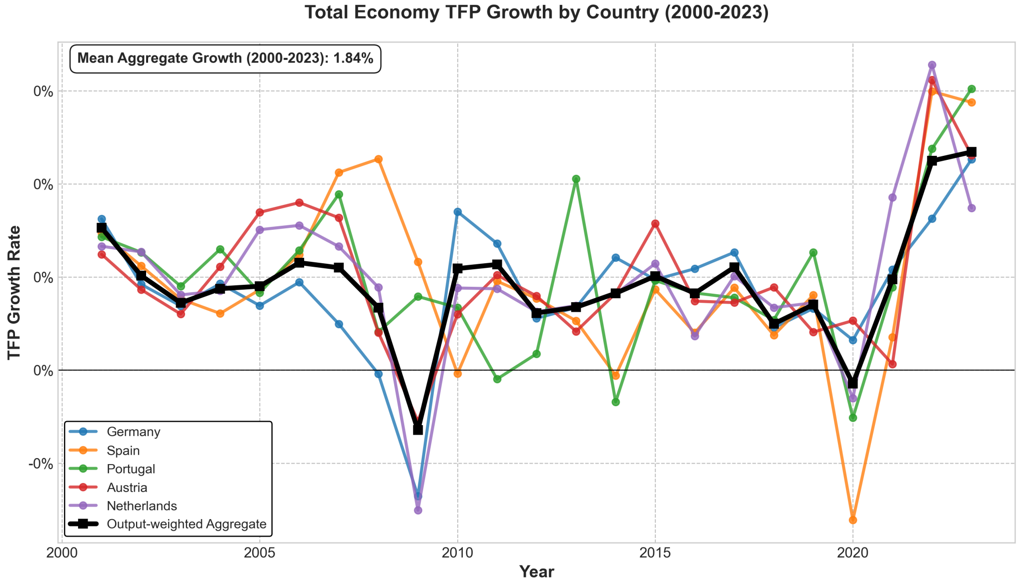
Solving for TFP:

$$A_{cst} = \frac{Y_{cst}}{K_{cst}^{\alpha} L_{cst}^{1-\alpha}} \tag{2}$$

where $c$ indexes countries, $s$ sectors, $t$ years, and $\alpha = 0.33$ represents the capital share of income. Annual TFP growth is calculated as the percentage change: $g_{cst} = (A_{cst} - A_{cs,t-1})/A_{cs,t-1}$.

Figure 1 presents TFP growth for the total economy. The output-weighted aggregate across the five countries reveals substantial cyclical variation, with sharp declines during the 2008–09 financial crisis and the 2020 COVID-19 recession.

**Figure 1:** Total Economy TFP Growth (2000–2023)




*Notes:* This figure plots annual TFP growth rates for Germany (blue), Spain (orange), Portugal (green), Austria (red), and the Netherlands (purple). The bold black line represents the output-weighted aggregate across all five countries. Mean aggregate TFP growth over 2001–2023 is reported in the annotation. **Data source:** Eurostat (`nama_10_a10`, `nama_10_a10_e`, `nama_10_nfa_st`).

**Key Finding:** The mean aggregate TFP growth over 2001–2023 is approximately **1.84%**, with considerable heterogeneity across countries. The Netherlands consistently exhibits higher productivity growth, while Portugal and Spain show more volatile patterns.

## 1.3 Economic Interpretation of $\alpha$ (Q1b)

In the Cobb-Douglas framework, $\alpha$ represents the **output elasticity with respect to capital**. Under the assumptions of perfect competition, constant returns to scale, and profit maximization, factors are paid their marginal products, implying that $\alpha$ equals capital's share of total income:

$$\alpha = \frac{rK}{Y} = \frac{\text{Capital Income}}{\text{Total Income}} \tag{3}$$

**Measurement in Practice:** The capital share can be estimated from national accounts as the ratio of gross operating surplus to gross value added. Empirically, $\alpha \approx 0.33$ is a standard approximation for developed economies, though recent literature documents a rising capital share over time (Karabarbounis and Neiman, 2014).

Moreover, imposing a uniform $\alpha = 0.33$ across sectors is a simplification that may introduce measurement bias. Capital intensity varies substantially across industries:

- **Capital-intensive sectors:** Real Estate (L), Manufacturing (C) — likely $\alpha > 0.4$

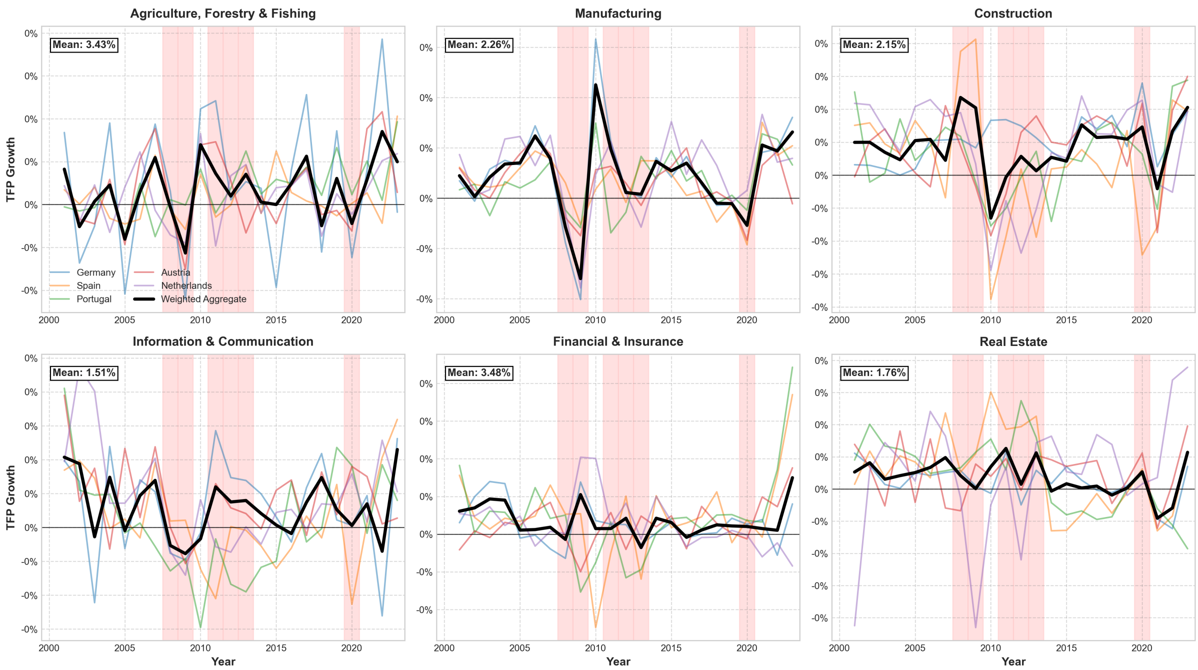
- **Labor-intensive sectors:** Information & Communication (J), Financial Services (K) — likely $\alpha < 0.3$

Sector-specific $\alpha$ values would improve TFP measurement accuracy but require detailed data on factor income shares by industry.

## 1.4 Sectoral TFP Growth with Recession Indicators (Q1c)

Figure 2 presents sectoral TFP growth with recession periods indicated. Each panel displays country-level series alongside an output-weighted sectoral aggregate.

**Figure 2:** Sectoral TFP Growth with Recession Indicators (2000–2023)




*Notes:* Each panel shows TFP growth for a specific sector. Colored lines represent individual countries; the bold black line is the output-weighted aggregate. Red shaded regions indicate recession years (identified using the provided recession indicator). Mean aggregate TFP growth is annotated in each panel.

Table 1 summarizes mean aggregate TFP growth by sector.

## 1.5 Interpretation of Sectoral Patterns (Q1d)

Several economically meaningful patterns emerge from Figure 2:

1. **Procyclicality of TFP:** All sectors exhibit procyclical productivity, with sharp declines during the 2008–09 and 2020 recessions. This reflects both genuine productivity effects and measurement artifacts (labor hoarding, variable capacity utilization).
2. **Manufacturing (C):** Displays the most pronounced cyclicality, consistent with high trade exposure and demand sensitivity. The sector experienced a >10% TFP decline during 2009.

**Table 1:** Mean Aggregate TFP Growth by Sector (2001–2023)

| Code | Sector | Mean TFP Growth (%) |
|---|---|---|
| A | Agriculture, Forestry & Fishing | 3.43 |
| C | Manufacturing | 2.26 |
| F | Construction | 2.15 |
| J | Information & Communication | 1.51 |
| K | Financial & Insurance | 3.48 |
| L | Real Estate | 1.76 |

*Notes:* Mean TFP growth computed as the average of the output-weighted aggregate series for each sector over 2001–2023. Source: Author's calculations using Eurostat data.

3. **Construction (F):** Shows persistent weakness following the 2008 crisis, particularly in Spain, reflecting the housing market collapse and prolonged deleveraging.
4. **Financial Services (K):** Exhibits extreme volatility around 2008–09, capturing the banking sector's central role in the financial crisis. High mean growth partly reflects measurement challenges in financial output.
5. **Agriculture (A):** High volatility driven by weather-dependent production and commodity price fluctuations, less synchronized with business cycles.
6. **Real Estate (L):** Relatively stable TFP growth, as imputed rents smooth measured output fluctuations.

## 1.6 Export Data and Descriptive Statistics (Q2a)

Export data from `ext_tec09` are merged with output data to construct export intensity:

$$\text{Export Share}_{cst} = \frac{\text{Exports}_{cst}}{\text{GVA}_{cst}} \times 100 \tag{4}$$

Table 2 presents descriptive statistics for export share by sector.

**Key Observations:**

- Manufacturing exhibits the highest export intensity (mean 158%), reflecting Europe's role as a manufacturing export hub. Values exceeding 100% occur because exports include re-exported intermediates.
- Construction and Real Estate show minimal export shares (<3%), confirming their non-tradeable nature.
- Substantial cross-country variation exists, with the Netherlands showing consistently higher export orientation.

**Table 2:** Descriptive Statistics: Export Share (% of Output), 2012–2023

| Sector | N | Mean | Std Dev | Min | Median | Max |
|---|---|---|---|---|---|---|
| Agriculture (A) | 51 | 14.84 | 12.55 | 3.25 | 8.77 | 40.67 |
| Manufacturing (C) | 51 | 157.82 | 25.14 | 126.62 | 159.28 | 241.23 |
| Construction (F) | 51 | 2.67 | 1.45 | 0.88 | 2.00 | 7.50 |
| Info & Comm (J) | 51 | 4.58 | 4.23 | 1.69 | 2.75 | 19.31 |
| Financial (K) | 45 | 2.41 | 4.12 | 0.03 | 0.97 | 21.10 |
| Real Estate (L) | 45 | 0.28 | 0.20 | 0.03 | 0.24 | 0.95 |
| Total Economy | 51 | 39.27 | 13.55 | 23.12 | 37.83 | 85.71 |

*Notes:* Export share calculated as exports divided by gross value added. Manufacturing export shares exceed 100% because exports include re-exported intermediate goods. Sample restricted to 2012–2023, as mentioned in the task description.

## 1.7 Productivity-Export Relationship (Q2b)

Figures 3 and 4 examine the relationship between log TFP and export intensity.

**Interpretation:**

- **Total Economy:** Strong positive relationship ($\beta \approx 36$, $R^2 \approx 0.49$). More productive economies exhibit higher export intensity, consistent with comparative advantage theories.
- **Agriculture (A):** Positive relationship ($\beta \approx 14$), reflecting productivity-driven comparative advantage in agricultural exports.
- **Manufacturing (C):** Weak within-sector relationship ($R^2 \approx 0.01$) despite high export levels, suggesting that country-level factors dominate sector-level productivity variation.
- **Construction (F) & Real Estate (L):** Near-zero or negative slopes confirm these sectors are fundamentally non-tradeable.
- **Services (J, K):** Modest positive relationships, reflecting the growing tradability of digital and financial services.

## 1.8 Econometric Analysis (Q3a)

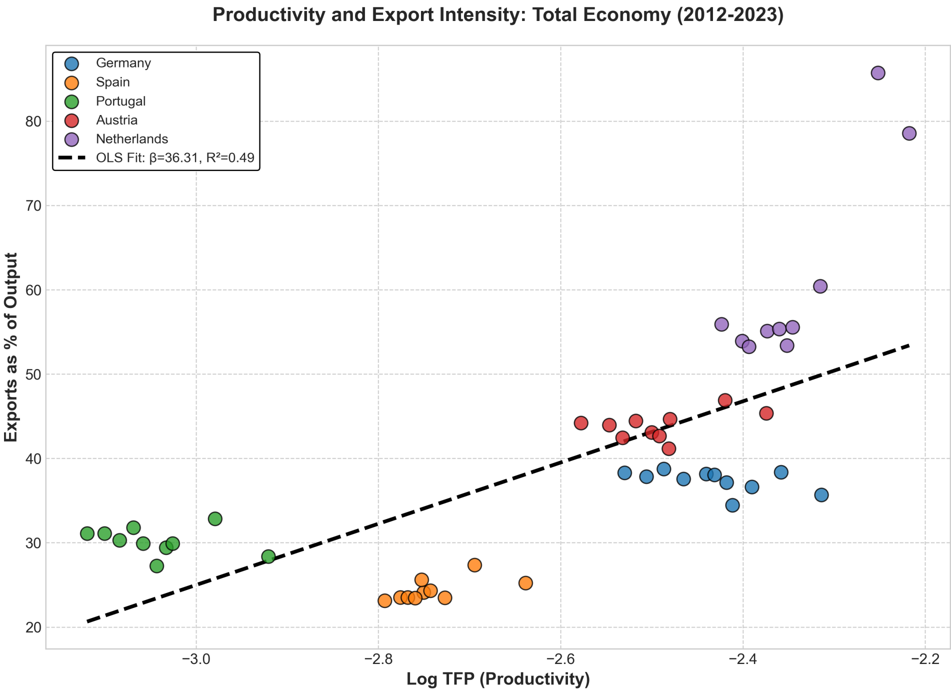
### Model Specification

To formally assess the productivity-export relationship while controlling for unobserved heterogeneity, I estimate:

$$\text{ExportShare}_{cst} = \alpha + \beta \ln(\text{TFP})_{cst} + \gamma_c + \delta_s + \tau_t + \varepsilon_{cst} \tag{5}$$

where $\gamma_c$, $\delta_s$, and $\tau_t$ denote country, sector, and year fixed effects, respectively.

**Figure 3:** Productivity and Export Intensity: Total Economy (2012–2023)




*Notes:* Scatter plot of log TFP against export share for the total economy. Each point represents a country-year observation. The dashed line shows the OLS regression fit. Slope coefficient ($\beta$) and $R^2$ reported in legend.

**Figure 4:** Productivity and Export Intensity by Sector (2012–2023)


*Notes:* Each panel shows the relationship between log TFP and export share for a specific sector. Colors indicate countries. Regression slope ($\beta$) and $R^2$ reported in each panel.

**Rationale:** Country fixed effects absorb time-invariant national characteristics (geography, institutions, trade agreements). Sector fixed effects control for inherent differences in tradability across industries. Year fixed effects capture common macroeconomic shocks (e.g., global recessions) affecting all units simultaneously. This isolates the within-country, within-sector relationship.

### Results

**Table 3:** Regression Results: Export Share on Log TFP

| | Export Share (% of GVA) | | | |
|---|---|---|---|---|
| **Variable** | (1) | (2) | (3) | (4) |
| ln(TFP) | -4.638 | -6.137* | 6.432*** | 5.081*** |
| | (4.084) | (3.650) | (1.992) | (1.894) |
| Observations | 294 | 294 | 294 | 294 |
| $R^2$ | 0.002 | 0.016 | 0.973 | 0.974 |
| Adj. $R^2$ | -0.001 | -0.001 | 0.972 | 0.972 |
| Country FE | No | Yes | Yes | Yes |
| Sector FE | No | No | Yes | Yes |
| Year FE | No | No | No | Yes |

*Notes:* Dependent variable is export share (% of GVA). Robust (HC1) standard errors in parentheses. Model (1) is pooled OLS. Model (2) adds country fixed effects. Model (3) adds sector fixed effects. Model (4) adds year fixed effects. Sample: 5 countries × 6 sectors × 2012–2023. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$.

**Interpretation:** The estimated coefficient on ln(TFP) is **6.432**, which is statistically significant at the 1% level. Since the dependent variable is measured in percentage points (0–100) and the independent variable is in natural logs, this coefficient implies that a **1% increase in TFP is associated with approximately a 0.064 percentage point increase in the export share** (calculated as $\beta/100$). Alternatively, a doubling of TFP (a 100% increase) is associated with a roughly **6.4 percentage point** rise in the export-to-output ratio. The positive sign confirms that, within the same country and sector, periods of higher productivity coincide with greater export orientation.

### Identification Concerns

1. **Reverse Causality:** Exporting may *cause* productivity gains through learning-by-exporting, technology transfer from foreign buyers, and scale economies. This would bias $\hat{\beta}$ upward.

2. **Omitted Variable Bias:** Unobserved factors such ass R&D investment, management quality, access to finance may drive both productivity and export decisions simultaneously.
3. **Measurement Error:** TFP is computed as a residual and contains measurement error from capacity utilization, markups, and input mismeasurement. Classical measurement error attenuates $\hat{\beta}$ toward zero.

**Proposed Solutions**

1. **Instrumental Variables:** Use lagged TFP (exploiting persistence) or sector-level R&D intensity as instruments for current productivity.
2. **Trade Policy Shocks:** Exploit exogenous variation from EU trade agreement implementations or tariff changes as instruments for export market access.
3. **Granger Causality Tests:** Examine whether lagged productivity predicts current exports (and vice versa) to assess temporal precedence.

## 2 Task B: Literature Review

**Favorite Paper: "The Financial Market Impact of ECB Monetary Policy Press Conferences: A Text Based Approach"**

**Author:** Conor Parle (2022)

**Journal:** *European Journal of Political Economy*, 74, 102230

**What I Like About It:**

I encountered this paper during my Master's thesis on ECB communication, and it fundamentally shaped my understanding of how central bank language moves markets. Parle (2022) demonstrates that the narrative tone of ECB press conferences, quantified through dictionary-based sentiment scoring—significantly affects equity returns even after controlling for policy rate surprises. The paper's elegance lies in adapting Tadle (2022) FOMC methodology to the ECB context, showing that markets extract information from how policy is communicated, not merely what is decided. This distinction between "hard" policy signals and "soft" rhetorical cues has profound implications for monetary transmission.

**How I Would Improve It:**

I would extend the analysis across multiple ECB communication channels—speeches, Monetary Policy Accounts, and press releases—to test whether tone effects are channel-specific. Additionally, incorporating modern transformer-based NLP models (e.g., FinBERT or ECB-tuned language models) could capture context, negation, and hedging that dictionary methods miss (Gambacorta et al., 2024; Deng et al., 2024), potentially revealing stronger or more nuanced tone effects obscured by measurement error.

# 3 Task C: Computational Exercise

This section solves a canonical consumption-savings problem with income uncertainty and a borrowing constraint. I derive the theoretical framework, describe the numerical solution, compute stationary distributions, and analyze the marginal propensity to consume.

## 3.1 Dynamic Programming Setup (Q1)

**Environment**

A representative household maximizes expected lifetime utility:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t), \quad u(c) = \frac{c^{1-\sigma}}{1-\sigma} \tag{6}$$

where $\beta = 0.95$ is the discount factor and $\sigma = 2$ is the coefficient of relative risk aversion (CRRA). Each period is interpreted as a quarter.

The household holds risk-free assets $a_t$ yielding gross return $R = 1 + r$ with $r = 0.03$, and receives stochastic labor income $y_t$. Borrowing is prohibited: $a_{t+1} \geq 0$.

### 3.1.1 State and Choice Variables (Q1a)

**State variables:** $(a_t, y_t)$ — current asset holdings and income realization.

**Choice variable:** $a_{t+1}$ — next-period asset holdings (equivalently, consumption $c_t$).

**Budget constraint:**

$$c_t + a_{t+1} = y_t + (1 + r)a_t \tag{7}$$

**Bellman equation:**

$$V(a, y) = \max_{a' \geq 0} \left\{ u(c) + \beta \sum_{y'} \pi(y'|y) V(a', y') \right\} \tag{8}$$

where $c = y+(1+r)a-a'$ from the budget constraint, and $\pi(y'|y)$ is the Markov transition probability from income state $y$ to $y'$.

**Income Process**

As described in the Task description, labor income follows a 5-state Markov chain with $y \in \{0.35, 0.70, 1.00, 1.30, 1.65\}$ and transition matrix:

$$\Pi = \begin{pmatrix} 0.50 & 0.30 & 0.10 & 0.05 & 0.05 \\ 0.20 & 0.50 & 0.20 & 0.05 & 0.05 \\ 0.10 & 0.20 & 0.40 & 0.20 & 0.10 \\ 0.05 & 0.10 & 0.20 & 0.50 & 0.15 \\ 0.05 & 0.05 & 0.10 & 0.30 & 0.50 \end{pmatrix} \tag{9}$$

### 3.1.2 Euler Equation Derivation (Q1b)

**Step 1:** Form the Lagrangian with multiplier $\lambda \geq 0$ on the borrowing constraint:

$$\mathcal{L} = u(y + (1+r)a - a') + \beta \mathbb{E}[V(a', y')|y] + \lambda a' \tag{10}$$

**Step 2:** First-order condition with respect to $a'$:

$$-u'(c) + \beta \mathbb{E}\left[\frac{\partial V(a', y')}{\partial a'}\bigg| y\right] + \lambda = 0 \tag{11}$$

**Step 3:** Envelope condition (differentiating value function with respect to $a$):

$$\frac{\partial V(a, y)}{\partial a} = u'(c) \cdot (1+r) \tag{12}$$

**Step 4:** Apply envelope condition to next period and substitute into FOC:

$$u'(c) = \beta(1+r)\mathbb{E}[u'(c')|y] + \lambda \tag{13}$$

**Step 5:** Euler equation:

$$u'(c_t) \geq \beta(1+r)\mathbb{E}_t[u'(c_{t+1})] \tag{14}$$

with equality when $a_{t+1} > 0$ (borrowing constraint slack).

### 3.1.3 Economic Interpretation (Q1c)

The **Euler equation** (14) characterizes optimal intertemporal allocation. At an interior solution, the marginal utility of consuming one unit today equals the expected discounted marginal utility of saving that unit, earning return $(1 + r)$, and consuming tomorrow.

When the borrowing constraint binds ($a_{t+1} = 0$), the household would prefer higher current consumption but cannot borrow, generating the inequality.

The **borrowing constraint** ($a_{t+1} \geq 0$) prevents households from pledging future income as collateral. This creates a **precautionary saving motive**: households accumulate buffer-stock assets to self-insure against adverse income realizations, as they cannot smooth consumption through borrowing during low-income periods.

**Income risk** amplifies precautionary saving. With concave utility ($u'' < 0$), the marginal utility of consumption is convex, so by Jensen's inequality, income uncertainty raises expected marginal utility of future consumption, inducing greater current saving.

#### 3.1.4 Income Persistence (Q1d)

An income process is **persistent** if current income is a strong predictor of future income—high income today implies high income is likely tomorrow. In the transition matrix $\Pi$, persistence is reflected in large diagonal elements (0.40–0.50), indicating high probability of remaining in the same income state.

## 3.2 Numerical Solution (Q2)

### 3.2.1 Code Description (Q2a)

The provided code solves the household problem using **Value Function Iteration (VFI)**:

1. **Discretization:** Assets discretized on a grid of 500 points over $[0, 20]$; income has 5 discrete states, yielding $500 \times 5 = 2500$ total states.
2. **Pre-computation:** For all state-choice pairs $(a, y, a')$, consumption $c = y + (1 + r)a - a'$ and utility $u(c)$ are computed and stored in matrices.
3. **Iteration:** At each iteration:
   - Compute expected continuation value $\mathbb{E}[V(a', y')|y]$ for each $(a', y)$
   - Evaluate $u(c) + \beta\mathbb{E}[V(a', y')|y]$ for all $(a, y, a')$
   - Optimal policy: $a'^*(a, y) = \arg\max_{a'}\{u(c) + \beta\mathbb{E}[V(a', y')|y]\}$
   - Update value function: $V^{new}(a, y) = \max_{a'}\{u(c) + \beta\mathbb{E}[V(a', y')|y]\}$
4. **Convergence:** Iterate until $\|V^{new} - V^{old}\|_\infty < 0.001$.

The algorithm converged in **137 iterations**.

### 3.2.2 Efficiency Improvements (Q2b)

Without rewriting the code, several modifications could improve performance:

1. **Exploit Monotonicity:** The asset policy function $a'^*(a, y)$ is increasing in $a$. When searching for the optimal $a'$ at asset level $a$, start from the optimal choice at $a - 1$ rather than zero.
2. **Exploit Concavity:** The objective is concave in $a'$. Once values begin decreasing, stop the search—no need to evaluate all 500 grid points.
3. **Howard's Policy Iteration:** After finding a new policy, iterate on the value function holding policy fixed for several iterations before re-optimizing. Reduces total iterations.
4. **Vectorization:** Replace Python loops with NumPy matrix operations to leverage compiled linear algebra routines.
5. **Multigrid Methods:** Solve first on a coarse grid, then interpolate to initialize the fine grid solution.
6. **Continuous Choice with Interpolation:** Allow $a'$ to take any value in $[0, a_{max}]$ and interpolate $V(a', y')$, improving accuracy without increasing grid points.

## 3.3 Stationary Distribution (Q3)

### 3.3.1 Computing the Distribution (Q3a)

Given the policy function $a' = g(a, y)$, the stationary distribution $\mu(a, y)$ satisfies:

$$\mu(a', y') = \sum_{a,y} \mu(a, y) \cdot \mathbb{1}[g(a, y) = a'] \cdot \pi(y'|y) \tag{15}$$

I compute $\mu$ by iterating on the distribution:

1. Initialize with uniform distribution: $\mu^0(a, y) = 1/(N_a \times N_y)$
2. Apply transition operator: $\mu^{n+1} = T(\mu^n)$
3. Iterate until $\|\mu^{n+1} - \mu^n\|_\infty < 10^{-8}$

Convergence occurred after **227 iterations**.

### 3.3.2 Distribution Plots (Q3b)

Figure 5 presents the stationary marginal distributions.

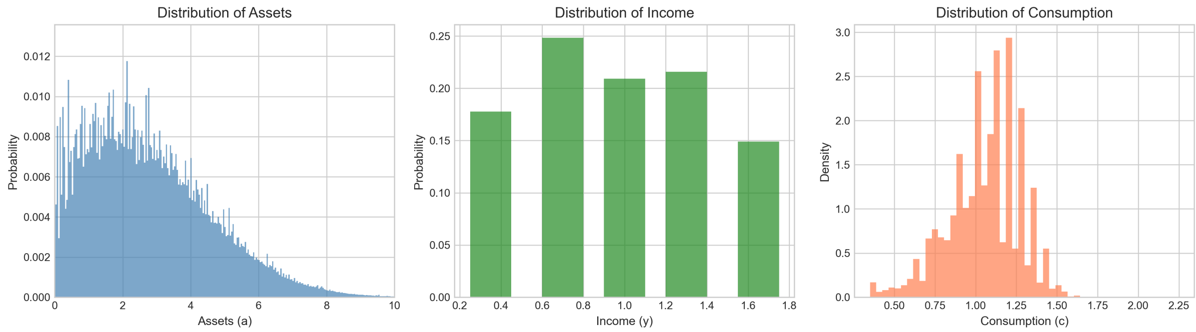
**Key Statistics:**

- Mean assets: $\bar{a} = 2.746$
- Mean income: $\bar{y} = 0.972$
- Mean consumption: $\bar{c} = 1.054$

**Verification:** The budget constraint is satisfied in expectation: $\bar{c} = \bar{y} + r\bar{a} = 0.972 + 0.03 \times 2.746 = 1.054$. ✓

**Economic Interpretation:**

**Figure 5:** Stationary Distributions of Assets, Income, and Consumption




*Notes:* Left panel shows the marginal distribution of assets, middle panel shows income distribution (ergodic distribution of Markov chain), right panel shows consumption distribution. Distributions computed by iterating on the stationary distribution equation.

- The asset distribution is right-skewed with substantial mass near zero, indicating many households are close to the borrowing constraint.
- The consumption distribution is smoother than income, reflecting consumption smoothing through saving.
- Mean assets of 2.75 represent approximately 2.8 quarters of mean income—a modest buffer stock.

### 3.3.3 Alternative Approach (Q3c)

**Monte Carlo Simulation:**

1. Initialize $N$ households (e.g., 10,000) at random $(a_0, y_0)$ states
2. For $T$ periods: apply policy function to get $a_{t+1} = g(a_t, y_t)$; draw $y_{t+1}$ from $\pi(\cdot|y_t)$
3. Discard burn-in periods (e.g., first 500)
4. Construct empirical distribution from simulated panel

**Intuition:** By the ergodic theorem, the time-average of a single long simulation (or cross-sectional average of many households) converges to the stationary distribution. This approach is computationally simpler and easily parallelizable.

## 3.4 Marginal Propensity to Consume (Q4)

### 3.4.1 Computing the MPC (Q4a)

The MPC measures the consumption response to an unexpected one-unit asset windfall:

$$\text{MPC}(a, y) = \frac{c(a + \Delta a, y) - c(a, y)}{\Delta a}, \quad \Delta a = 1 \tag{16}$$

For states where $a + 1$ falls between grid points, I use linear interpolation of the consumption policy function.

**Results:**

- **Average MPC** (distribution-weighted): **0.096** (9.6%)
- MPC range: $[0.00, 0.39]$

Households near the borrowing constraint exhibit high MPCs ($\approx 0.4$), while wealthy households with large asset buffers have MPCs near zero.

### 3.4.2 Calibration Target (Q4b)

**Target:** MPC = 0.25 (25%)

**Justification:** Johnson et al. (2006) estimate MPCs of 20–40% from the 2001 U.S. tax rebates using consumer expenditure data. Fagereng et al. (2021) find average MPCs around 0.5 in Norwegian administrative data. I target 0.25 as a conservative benchmark.

**Parameter Adjusted:** Discount factor $\beta$

**Rationale:** Lower $\beta$ implies greater impatience. Impatient households save less, hold smaller asset buffers, and are more likely to be near the borrowing constraint—where MPCs are highest.

**Calibration Results:**

**Table 4:** Calibration: Discount Factor and Average MPC

| $\beta$ | Average MPC |
|---|---|
| 0.90 | 0.186 |
| 0.92 | 0.154 |
| 0.94 | 0.117 |
| 0.95 | 0.096 |
| 0.96 | 0.070 |

To match the target MPC of 0.25, $\beta \approx 0.88$ would be required—substantially below the standard calibration of 0.95–0.99.

### 3.4.3 Parameter Effects on MPC (Q4c)

- **$\beta$ (discount factor):** $\downarrow \beta \Rightarrow$ more impatient $\Rightarrow$ lower savings $\Rightarrow$ closer to constraint $\Rightarrow$ **higher MPC**
- **$\sigma$ (risk aversion):** $\uparrow \sigma \Rightarrow$ stronger precautionary motive $\Rightarrow$ larger buffer stock $\Rightarrow$ **lower MPC**
- **Income persistence:** $\uparrow$ persistence $\Rightarrow$ shocks last longer $\Rightarrow$ need larger buffer $\Rightarrow$ **lower MPC**
- **Interest rate ($r$):** $\uparrow r \Rightarrow$ higher return to saving $\Rightarrow$ more assets accumulated $\Rightarrow$ **lower MPC**

## 3.5 Behavioral Extensions: Incomplete Optimization (Q5)

### 3.5.1 Approaches to Bounded Rationality (Q5a)

Standard dynamic programming assumes perfect optimization, which may be unrealistic. Several modeling approaches relax this assumption:

1. **Rule-of-Thumb Consumers:** A fraction $\lambda$ of households simply consume current income ($c = y$) rather than optimizing. Aggregate consumption becomes a weighted average of optimizing and hand-to-mouth behavior.

2. **Sparse Decision-Making** (Gabaix, 2014): Households optimize over a simplified, "sparse" representation of the state space, paying attention to only the most important variables. Less salient states are shrunk toward default values.

3. **$\varepsilon$-Greedy Exploration:** With probability $\varepsilon$, households choose a random action instead of the optimal one. Captures experimentation, mistakes, or bounded computational capacity.

4. **Calvo-Style Infrequent Adjustment:** Each period, households re-optimize with probability $\theta$; otherwise, they follow their previous consumption rule or a simple heuristic.

5. **Rational Inattention:** Households face a cost of processing information. They optimally choose how much attention to allocate, leading to state-dependent responsiveness.

### 3.5.2 Implementation: Infrequent Adjustment (Q5b)

**Chosen Approach:** Calvo-style infrequent optimization

**Model Modification:**

- With probability $\theta$: household re-optimizes, setting $c_t = c^*(a_t, y_t)$
- With probability $1-\theta$: household follows a simple rule, e.g., $c_t = \bar{c}$ (fixed consumption) or $c_t = \alpha(y_t + (1+r)a_t)$ (fixed fraction of cash-on-hand)

**Solution Steps:**

1. Solve the standard Bellman equation to obtain optimal policy $g^*(a, y)$
2. Define the rule-of-thumb policy $g^{rule}(a, y)$
3. Effective policy: $g^{eff}(a, y) = \theta \cdot g^*(a, y) + (1-\theta) \cdot g^{rule}(a, y)$
4. Compute stationary distribution using $g^{eff}$
5. Analyze how $\theta$ affects consumption volatility, wealth inequality, and MPC

**Expected Effects:**

- **Higher consumption volatility:** Inattentive households fail to smooth consumption optimally
- **Higher average MPC:** Sluggish adjustment means households over-consume relative to permanent income
- **Excess smoothness:** Aggregate consumption may respond too slowly to aggregate income changes

This extension helps reconcile the low MPCs in standard models with higher empirical estimates, as behavioral frictions push more households toward constrained, high-MPC behavior.

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