A Spatial, Agent-Based Analog to the Lotka-Volterra Model

Group 3

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1 Background

While the classical Lotka-Volterra Model refers to dual-species dynamics, our project aims to implement a model using spatial dynamics and answer several questions, such as the effect of the addition of a third species (grass in our model), the effect of energy levels, as well as an inference on spatial restraints of the species and how spatial dynamics affect the model.

2 The Lotka-Volterra Model

The Lotka Volterra Model describes a set of equations for modelling prey-predator dynamics where the populations change through time according the pair of equations:

$$\frac{dx}{dt} = \alpha x - \beta xy \tag{1}$$

$$\frac{dy}{dt} = \delta xy - \gamma y \tag{2}$$

where the variable x is the population density of the prey, y is the population density of some predator, and t represents time. α, β are the prey's parameters describing the maximum prey per capita growth rate and the affect of the presence of predators on the prey death rate respectively. γ, δ respectively describe the predators per capital death rate and the effect of the presence of prey on the predator's growth rate.

In the classical approach, the prey are assumed to have an unlimited supply of food and reproduce exponentially, except when subjected to predation. The rate of predation on the prey is assumed to be proportional to the rate at which the predators and the prey meet (obviously if either x or y are zero, there can be no predation). The interpretation of the equations is simply : "the rate of change of the prey's population is given by its own growth rate minus the rate at which it is preyed upon". Similarly, the second equation represents: "the rate of change of the predator's population depends upon the rate at which it consumes prey, minus its intrinsic death rate".

Thus, the assumptions of the initial model are the presence of unlimited food for the prey, the rate of change of population is proportional to its size, the environment cannot change in favour of one species and genetic adaptation is impossible, predators unlimited appetite, and both populations can be described by a single variable. This amounts to assuming that the populations do not have a spatial age or age distribution that contributes to the dynamics.

3 Model Extension to Three Species

Our own graphical model is agent based, so our course of action was to observe the similarities between our agent based model and a hypothetical Lotka-Volterra Model with three species, which we describe below. Consider 3 species x, y, and z with their behaviour being modeled by the differential equations below:

$$\frac{dx}{dt} = ax - bxy \tag{1}$$

$$\frac{dy}{dt} = -cy + dxy - eyz \tag{2}$$

$$\frac{dz}{dt} = -fz + gyz \tag{3}$$

This model has similar parameters as the classic model: x has a growth rate denoted by ax while y and z have a natural death rate denoted by $-cy$ and $-fz$, respectively. dxy and gyz denote the growth rate of y and z from hunting their prey and $-bxy$ and $-eyz$ denote the death rate of the prey as a result of being hunted. The species y has a death rate (the negative sign in front of $-cy$) instead of a growth rate in the absence of a predator z. This is to avoid inaccuracy in the case that species x goes extinct at some point. If it had a positive growth rate (instead of death rate), then y's population would still increase, despite having no food source. This model is a generalization of the two species model: it adds a third species, but it only allows for a chain of predation (i.e. this model does not allow us to consider the case where both y and z prey on x or where x and y are at the bottom of the food chain and z preys on both).

4 Method

We first created an agent based model using tunable parameters. We tune these parameters in an attempt to demonstrate how the Lotka-Volterra population dynamics change with the introduction of a third species and spatial dynamics. We thus added an additional species (grass), as well as limitations on space and the addition of energy values to both the pre-existing species.

4.1 Energy

For our agent based model, with the addition of grass to the model, we naturally added certain energy parameters to every individual in the model, while also adding conditions (for instance, when the energy level reaches 0 after a certain period of time or movement, the individual dies immediately). If a predator (wolves) attack a prey (sheep), they also inherit that prey's energy value, essentially prolonging their life. This is in stark contrast to the classical Lotka-Volterra system where energy plays no role (instead, predators die at a constant rate and prey only die when predated upon).

4.2 Spatial Dynamics

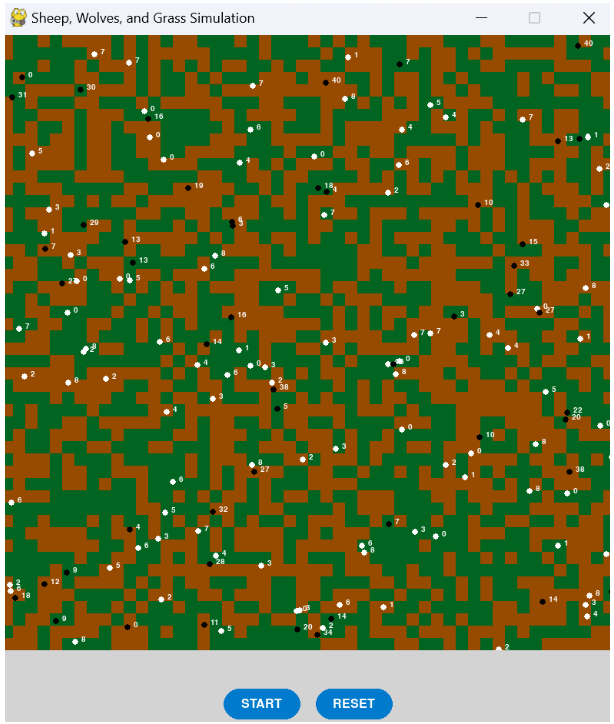
In our model, the spatial dynamics are evident in the simulation environment, where the entities (sheep, wolves, and grass) interact within a grid-based landscape. In the Lotka-Volterra model, spatial dynamics are markedly absent. Spatial dynamics introduce complexities such as habitat heterogeneity, movement constraints, and resource distribution, all of which influence population dynamics.

Firstly, the spatial distribution of resources, represented by grass patches in the grid, affects the dynamics. Grass serves as the primary food source for sheep, and its availability varies across the landscape. In areas with abundant grass, sheep populations thrive, leading to higher reproduction rates. Conversely, scarcity of grass restricts the growth of sheep populations, impacting the carrying capacity of the system.

Secondly, the movement of animals within the spatial domain influences predator-prey interactions. Both sheep and wolves move in a random direction at each time step and predate on any available prey. Moreover, movement causes energy expenditure, influencing the survival and reproduction of individuals.

Additionally, spatial heterogeneity introduces localized dynamics within the population. Clustering of resources or individuals may lead to localized outbreaks or extinctions, influencing the stability of the entire ecosystem. Furthermore, spatial connectivity between patches affects population dispersal and colonization, shaping the metapopulation dynamics of the system.

Clearly, spatial dynamics play a crucial role in our agent based model by modulating resource availability, movement patterns, and localized interactions. Incorporating spatial complexity enhances the realism of our ecological model, providing insights into the dynamics of predator-prey interactions and ecosystem stability. The representation of the spacial effect in our system is shown below:



Further, our simulation also creates a continually updating graph based on the populations of grass, wolves, and sheep in the system at any given point, this is also shown below:

Figure 1: Agent Based Model Graph

5 Final Inferences

Our simulation integrates energy dynamics and spatial limitations, enriching the traditional Lotka-Volterra model and yielding various insights into predator-prey dynamics. By introducing energy constraints, the model captures the finite nature of resources, affecting the survival, reproduction, and interactions of organisms. Energy availability, influenced by grass patches and predation, regulates population growth rates and alters the predator-prey equilibrium. We also demonstrate spatial heterogeneity, affecting movement patterns and encounter rates between species. Animals must navigate the grid-based landscape, facing differing resource gradients, which shape their distribution and behavior. This spatial complexity leads to localized dynamics, such as habitat patches with varying population densities and extinction risks. Consequently, the model's inference reflects a more realistic portrayal of ecological systems, emphasizing the interplay between energy dynamics, spatial structure, and population interactions.

5.1 Lotka-Volterra Model Inference

The output graph from the Lotka-Volterra model displays a population where instead of reaching statistical equilibrium, the populations all die out due to overfeeding and starvation. This is likely due to the model lacking the randomised spatial dynamics, and energy loss, in our spatial dynamics and energy model. Since our model introduces randomness in the chances of meetings b/w predator and prey, the growth rates of the populations get affected and modified in each time interval. Also, if a specific animal is alive too long without gaining the required resources for survival, it will die out. This also helps act as a variable carrying capacity for the various species, allowing for self correction in population sizes

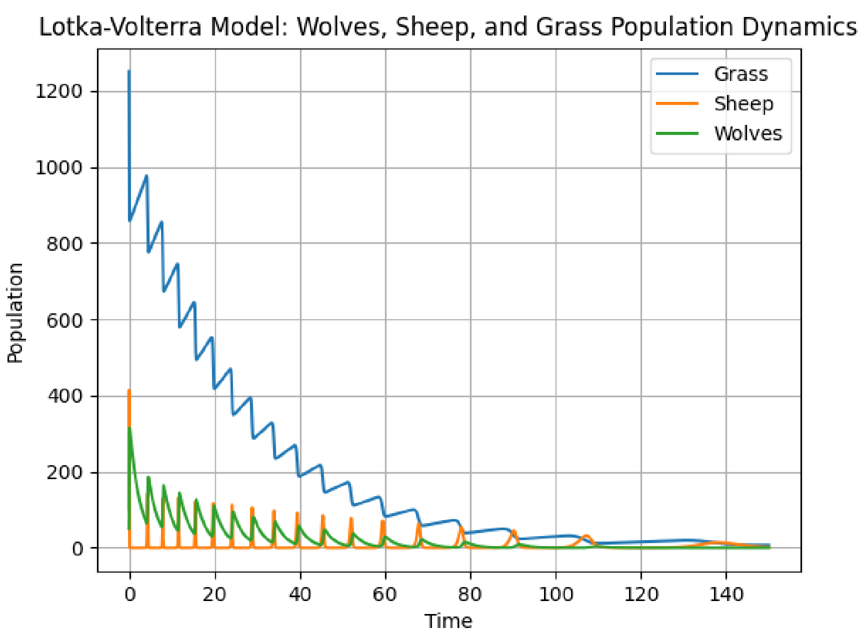


Figure 2: ODE Graph with the same parameters

Compared to the classic Lotka-Volterra model, which primarily focuses on population interactions without considering energy constraints or spatial dynamics, the agent based model yields several differences. Firstly, energy limitations introduce non-linear effects on population dynamics, influencing growth rates and stability thresholds. Organisms must balance energy intake and expenditure, leading to fluctuations in population sizes and altering the equilibrium points. Secondly, spatial constraints introduce spatial autocorrelation and dispersal limitations, impacting population dispersal and colonization patterns. The spatial arrangement of resources and individuals modulates encounter rates and population connectivity, influencing the persistence of species within the landscape. Our inclusion of energy and space limitations enhances the realism and predictive power of the model, providing a more comprehensive understanding of predator-prey dynamics in ecological systems.