2019101058 Shrey Gupta

Q1) Both people start at origin.

a) To meet after N steps each, they must take equal no. of steps to the right and to the left, but the order in which the steps are taken can be different.

Now,

Let steps taken to the right be i.

$\therefore$ Steps taken to the left are equal to $N-i$

$\therefore$ Probability to be at $x = i-(N-i)$

$x = 2i-N$ positions

$$= \underbrace{\left(\frac{1}{2}\right)^{i}}_{\frac{1}{2}\text{ prob. to step right for } i \text{ steps}} \times \underbrace{\left(\frac{1}{2}\right)^{N-i}}_{\frac{1}{2}\text{ prob to go left}} \times \underbrace{{}^{N}C_{i}}_{\text{ways to arrange left and right steps.}}$$

$\therefore$ Prob. of meeting at $x=2i-N$ is $\boxed{\underbrace{\left(\frac{1}{2}\right)^{N} {}^{N}C_{i}}_{\text{Each person must be at } x=2i-N \text{ after } N \text{ moves.}}}^{2}$

Now, $0 \le i \le N$.

Vandermonde's Identity $\sum_{k=0} \binom{m}{k}\binom{n}{r-k} = \binom{m+n}{r}$

$\therefore$ Prob. of meeting $= \sum_{i=0}^{N} \frac{{}^{N}C_{i} \cdot {}^{N}C_{i}}{2^{2N}}$

Prob. of meeting $= \frac{1}{2^{2N}} \sum_{i=0}^{N} {}^{N}C_{i}\, {}^{N}C_{N-i} = \boxed{\frac{{}^{2N}C_{N}}{2^{2N}}}$

Ans.

b) Now,

For a person to be at the origin after N steps, the no. of right steps should be equal to the no. of left steps. i.e., if the person starts at origin and wants to end at origin after N steps, N must be even with $\frac{N}{2}$ steps in either direction.

$$\therefore \text{Prob. of ending at origin} = \underbrace{\left(\frac{1}{2}\right)^{N/2}}_{\text{Prob. of stepping left}} \underbrace{\left(\frac{1}{2}\right)^{N/2}}_{\text{Prob. of stepping right.}} \underbrace{{}^{N}C_{N/2}}_{\text{Ways to chose } \frac{N}{2} \text{ left and right steps}}$$

$$\therefore P(\text{end at origin if start at origin}) = \begin{cases} \dfrac{{}^{N}C_{N/2}}{2^N}, & N \text{ is even and positive} \\ 0, & \text{otherwise} \end{cases}$$

c) Let d_i represent the Random Variable representing the ith step to be taken.

$$d_i = \begin{cases} +1, & \text{right step} \\ -1, & \text{left step.} \end{cases}$$

Now,

$$\text{Prob.}(d_i = +1) = P(d_i = -1) = \underbrace{\frac{1}{2}}_{\text{Both are equally probable}}$$

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$\therefore E[d_i] = \frac{1}{2}(+1) + \frac{1}{2}(-1)$

$= 0$

Now,

Let d represent the net displacement of the drunk after N steps.

$\therefore d = d_1 + d_2 + d_3 + \_\_ + d_N$

$\therefore E[d] = E[d_1] + E[d_2] + \_\_ + E[d_N]$
$= \sum_{i=1}^{N} E[d_i]$

$E[d] = N \cdot 0$

$\Rightarrow \boxed{E[d] = 0} \Rightarrow$ Mean displacement is 0 - Ans

d) Now,

Again using same definition of d_i as in ~~Pr~~ previous part.

$\therefore E[d_i^2] = \frac{1}{2}(+1)^2 + \frac{1}{2}(-1)^2 = \frac{1}{2} + \frac{1}{2}$

$= 1$

Also,

$E[d_i d_j] = E[d_i] E[d_j]$

This is true because each step is independent of the previous steps. Each steps is an independent event.

$$\Rightarrow E[d_i d_j] = \cancel{E} E[d_i]\, E[d_j]$$

$$= 0 \cdot 0$$

$$\therefore E[d_i d_j] = 0$$

Now, square

Let d^2 represent net square displacement of drunk after N steps.

$$\therefore d^2 = (d_1 + d_2 + d_3 + \dots + d_N)^2$$

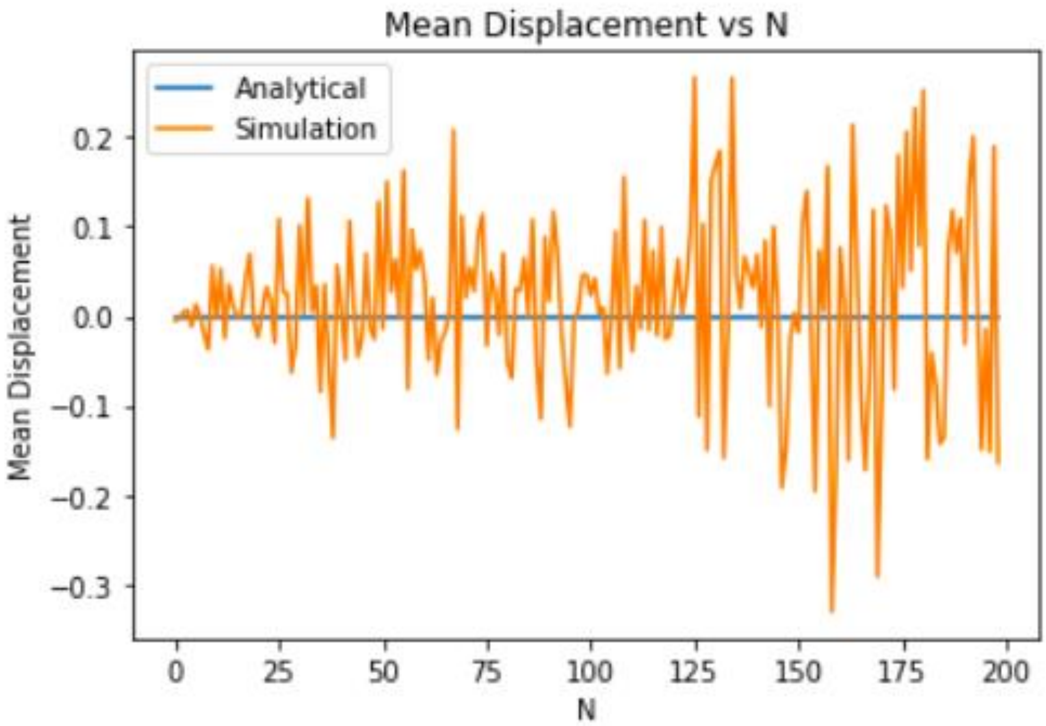
$$\Rightarrow d^2 = d_1^2 + d_2^2 + \dots + d_N^2 + 2(d_1 d_2 + d_1 d_3 + \dots)$$

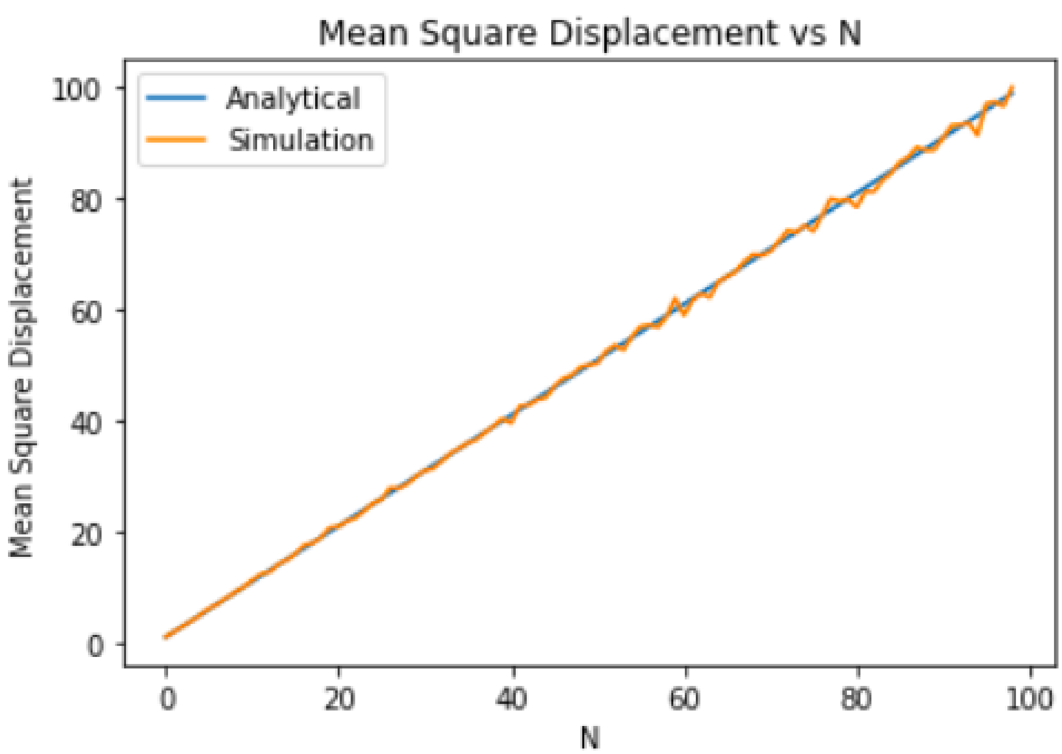
$$\therefore E[d^2] = E[d_1^2 + d_2^2 + \dots + d_N^2] + 2\, E[d_1 d_2 + d_1 d_3 + \dots]$$

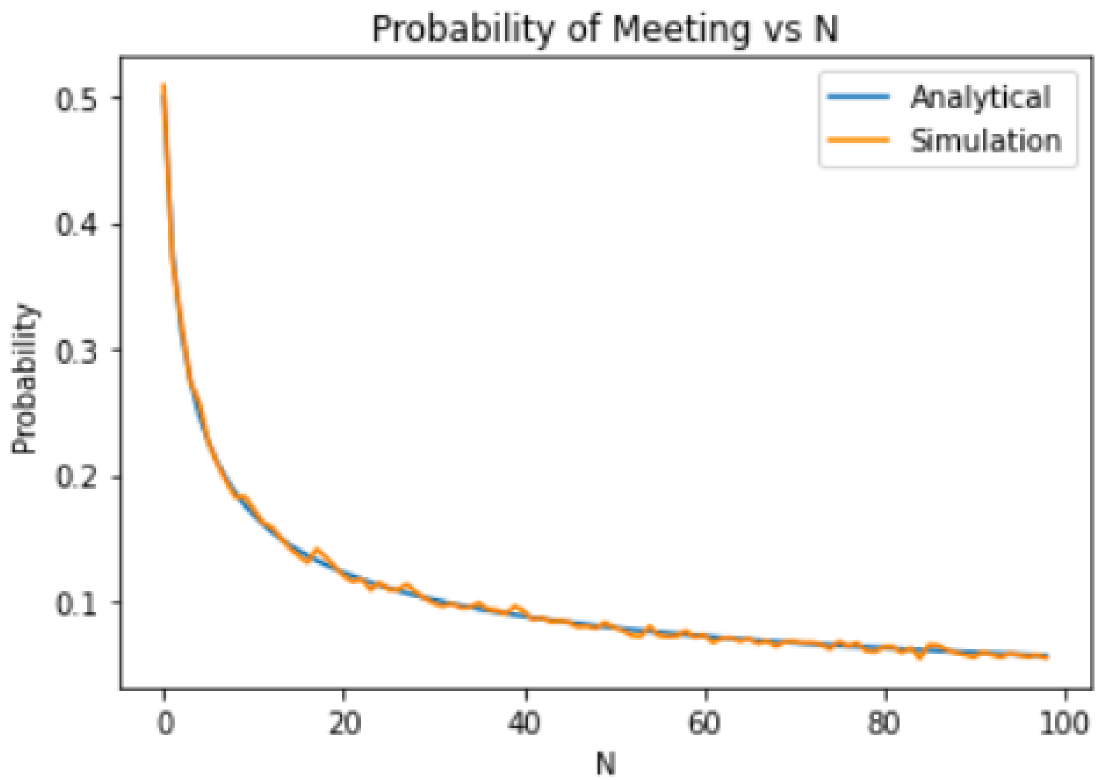
$$\Rightarrow E[d^2] = \sum_{i=1}^{N} E[d_i^2] + 2 \sum_{i<j} E[d_i d_j]$$

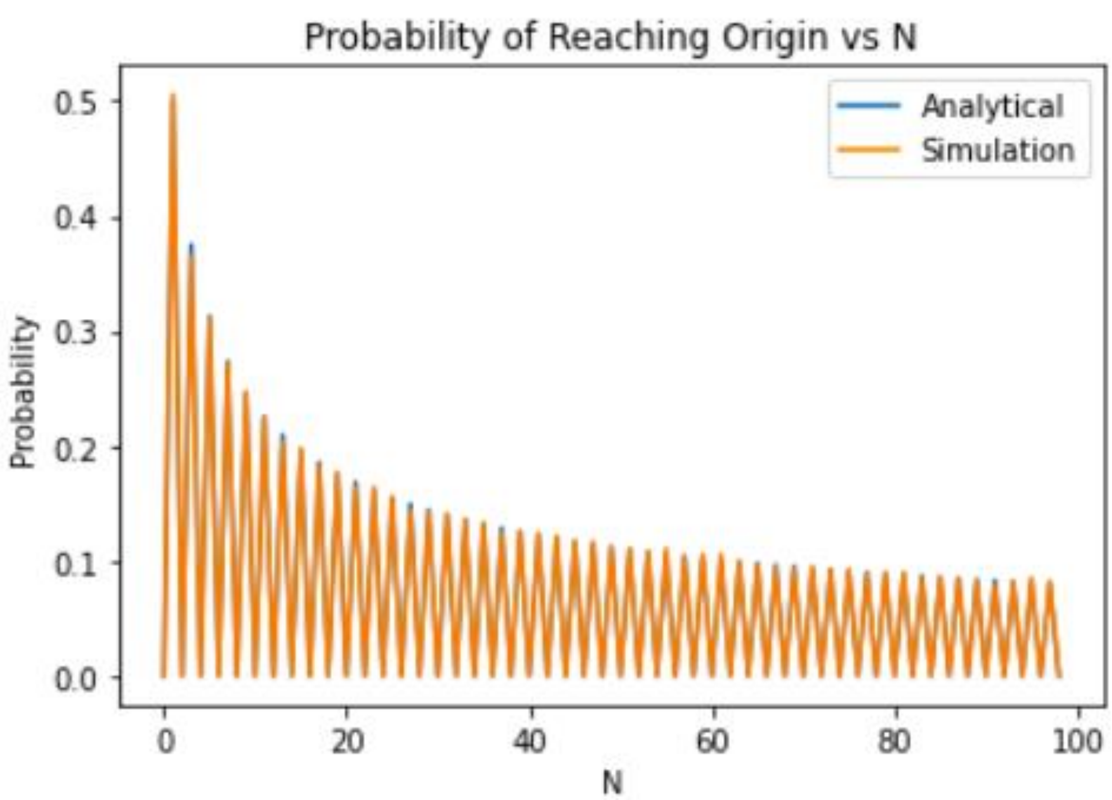
$$= N \cdot 1 + 2 \cdot 0$$

$$\Rightarrow \boxed{E[d^2] = N}$$ $\Rightarrow$ Mean square displacement is N-ans.


Mean Displacement vs N
Analytical
Simulation
Mean Displacement
0.2
0.1
0.0
−0.1
−0.2
−0.3
0
25
50
75
100
125
150
175
200
N


Mean Square Displacement vs N
Analytical
Simulation
Mean Square Displacement
100
80
60
40
20
0
0
20
40
60
80
100
N


Probability of Meeting vs N
Analytical
Simulation
Probability
0.1
0.2
0.3
0.4
0.5
0
20
40
60
80
100
N


Probability of Reaching Origin vs N
Analytical
Simulation
Probability
0.0
0.1
0.2
0.3
0.4
0.5
0
20
40
60
80
100
N