The Information Content of Straddles, Risk Reversals and Butterfly Spreads

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Abstract

Options across all strikes at a fixed maturity provide information on the risk-neutral distribution of the underlying security. We use currency options as an example and link the quotes on delta-neutral straddles, risk reversals, and butterfly spreads to the risk-neutral expected values of currency return volatility, skewness, and kurtosis. We propose to compare these risk-neutral expected values from currency options with their corresponding realizations computed from daily currency returns. The average differences define the premiums on the relevant risk exposures.

1. Return moments definitions

Let $r_t \equiv \ln S_t/S_{t-1}$ denote the daily log return on a currency, with S_t denoting the time-t currency price. The population mean (μ), variance $\left(\sigma^2\right)$, skewness (s), and kurtosis (k) of the return distribution

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are defined as

$$
\begin{aligned}
\mu &\equiv \mathbb{E}\left[r_t\right], \\
\sigma^2 &\equiv \mathbb{E}\left[\left(r_t-\mu\right)^2\right], \\
s &\equiv \frac{\mathbb{E}\left[\left(r_t-\mu\right)^3\right]}{\sigma^3}, \\
k &\equiv \frac{\mathbb{E}\left[\left(r_t-\mu\right)^4\right]}{\sigma^4}-3,
\end{aligned}
$$

where $\mathbb{E}[\cdot]$ is the unconditional expectation operator. The square root of the variance, σ, is referred to as the standard deviation, or "volatility." If the currency return is normally distributed, both the skewness s and the kurtosis k become zero. Therefore, skewness and kurtosis can be used to capture the degree of deviation of the distribution from normality. Intuitively, skewness measures the asymmetry of the distribution and kurtosis measures how much "fatter" the tails are as compared to the tails of a normal distribution with the same mean and variance.

2. Inferring risk-neutral return moments from straddles, risk reversals, and butterfly spreads

Breeden and Litzenberger (1978) show that the risk-neutral distribution of the underlying security can be inferred from option prices across all strikes at a certain option maturity n (in years). It is also well-known that when the Black and Scholes (1973) implied volatility is plotted against some measure of moneyness (strikes), the curve often has a smile or smirk pattern, widely referred to as the implied volatility smile. The slope and curvature of the implied volatility smile reveal important information about the skewness and kurtosis of the risk-neutral distribution of the underlying security return. Backus, Foresi, and Wu (1997) formalize this observation and show that the time-t implied volatility smile at time-to-maturity n is approximately related to the time-t expected values of the risk-neutral volatility, skewness, and kurtosis by a quadratic function:

$$
IV_{t,n}(d) \approx \sigma_{t,n}\left(1-\frac{1}{6}s_{t,n}d-\frac{1}{24}k_{t,n}\left(1-d^2\right)\right), \tag{1}
$$

where $IV_{t,n}(d)$ denotes the time-t implied volatility quote at maturity n and moneyness d, with the moneyness defined as

$$d \equiv \frac{\ln F/K + IV^2 n/2}{IV\sqrt{n}}, \tag{2}$$

with F and K referring to the forward and strike prices, respectively. Based on this function, the annualized volatility level $\sigma_{t,n}$ can be approximated by the implied volatility at zero moneyness:

$$IV_{t,n}(0) \approx \sigma_{t,n}\left(1 - \frac{1}{24}k_{t,n}\right) \approx \sigma_{t,n}. \tag{3}$$

Furthermore, the slope and curvature of the implied volatility around zero moneyness are

$$\left.\frac{\partial IV_{t,n}(d)}{\partial d}\right|_{d=0} = -\frac{1}{6}s_{t,n}\sigma_{t,n}, \quad \left.\frac{\partial^2 IV_{t,n}(d)}{\partial d^2}\right|_{d=0} = \frac{1}{12}k_{t,n}\sigma_{t,n}. \tag{4}$$

Under the quadratic assumption, the slope depends on moneyness d but the curvature estimate is constant across all moneyness. Hence, for the curvature definition, the condition of $d = 0$ is not necessary. Nevertheless, it is important to realize that the quadratic function performs well only around $d = 0$. At deep out-of-money, a quadratic function approximation may not work and indeed may present arbitrage opportunities. Hence, we stress that the slope and curvature definition are around at-the-money $d = 0$.

In the OTC currency options market, quotes are in terms of the delta-neutral straddle $(d = 0)$ implied volatility (ATMV), ten-delta and 25-delta risk reversals (RR1) and butterfly spreads (BF). They are defined as

$$\begin{aligned}
ATMV_{t,n} &\equiv IV_{t,n}(0), \\
RR_{t,n}(10) &\equiv IV_{t,n}(d_{10}^c) - IV_{t,n}\left(d_{10}^p\right), \\
RR_{t,n}(25) &\equiv IV_{t,n}(d_{25}^c) - IV_{t,n}\left(d_{25}^p\right), \\
BF_{t,n}(10) &\equiv \left(IV_{t,n}(d_{10}^c) + IV_{t,n}\left(d_{10}^p\right)\right)/2 - ATMV_{t,n}, \\
BF_{t,n}(25) &\equiv \left(IV_{t,n}(d_{25}^c) + IV_{t,n}\left(d_{25}^p\right)\right)/2 - ATMV_{t,n},
\end{aligned}$$

where d_{10}^c, d_{25}^c are the moneyness corresponding to the ten- and 25-delta call options and d_{10}^p, d_{25}^p are the moneyness corresponding to the ten- and 25-delta put options.

If we assume zero interest rates, we can have, approximately

$$\begin{aligned} d_{10}^{p} &\approx -d_{10}^{c} \approx N^{-1}(.9) = 1.2816, \\ d_{25}^{p} &\approx -d_{25}^{c} \approx N^{-1}(.75) = 0.6745, \end{aligned}$$

where $N^{-1}(\cdot)$ denotes the inverse cumulative standard normal function.

Now, if we use the risk reversals as approximate slope measures around zero moneyness and butterfly spreads as approximate curvature measures around zero moneyness. we can link them directly the conditional skewness and kurtosis of the currency return risk-neutral distribution. Using ten-delta risk reversal and butterfly spreads, we have

$$\begin{aligned} -\frac{1}{6} s_{t,n} \sigma_{t,n} &\approx \frac{RR_{t,n}(10)}{d_{10}^{c} - d_{10}^{p}} \approx -\frac{RR_{t,n}(10)}{2 \times 1.2816}, \\ \frac{1}{12} k_{t,n} \sigma_{t,n} &\approx \frac{2BF_{t,n}(10,\tau)}{\left(d_{10}^{p}\right)^{2}} \approx \frac{2BF_{t,n}(10,\tau)}{1.2816^{2}}. \end{aligned}$$

We hence can derive the approximate relation between volatility, skewness, and kurtosis on the one side and straddles, risk reversals, and butterfly spreads on the other:

$$\begin{aligned} \sigma_{t,n} &\approx ATMV_{t,n} \\ s_{t,n} &\approx 2.3409 RR_{t,n}(10) / ATMV_{t,n}, \\ k_{t,n} &\approx 14.6130 BF_{t,n}(10) / ATMV_{t,n}. \end{aligned}$$

Analogously, we can derive the relation between the skewness/kurtosis and the 25-delta relative risk reversals and butterfly spreads:

$$\begin{aligned} s_{t,n} &\approx 4.4478 RR_{t,n}(25) / ATMV_{t,n}, \\ k_{t,n} &\approx 52.7546 BF_{t,n}(25) / ATMV_{t,n}. \end{aligned}$$

3. Time series estimates of daily volatility, skewness, and kurtosis

We can compare the risk-neutral return moments inferred from options quotes to moments computed from time series return data. Given daily currency returns over a certain sample period, and assuming zero mean ($\mu = 0$), we estimate the time-series return moments by

$$
\begin{aligned}
\hat{\sigma} &= \sqrt{\frac{1}{N}\sum_{t=1}^{N}(r_t)^2}, \\
\hat{s} &= \frac{1}{N}\frac{\sum_{t=1}^{N}(r_t)^3}{\hat{\sigma}^3}, \\
\hat{k} &= \frac{1}{N\hat{\sigma}^4}\sum_{t=1}^{N}(r_t)^4 - 3,
\end{aligned}
\tag{5}
$$

where N denotes the actual number of days over the sample period of the calculation.

To match the statistical moments with the option-implied risk-neutral moments, we need to choose the appropriate sample period and do corresponding scaling. Specifically, to match the risk-neutral moments at time t and over an option maturity n, first we collect the N daily returns between time $(t, t+n]$ and do the daily moment estimates according to (5). Second, we do the following appropriate scaling:

$$\hat{\sigma}_{t,n} = \hat{\sigma}\sqrt{365}, \tag{6}$$

$$\hat{s}_{t,n} = \hat{s}/\sqrt{N}, \tag{7}$$

$$\hat{k}_{t,n} = \hat{k}/N, \tag{8}$$

where equation (6) represents annualization based on the actual over 365 day counting convention, given that N denotes the actual number of days between t and $t+n$. Equations (7) and (8) represent time-aggregation scaling adjustments based on the assumption of iid returns. The estimates in (5) generate daily skewness and kurtosis estimates over the sample of N days. However, the option implied skewness and kurtosis is for the return over N days from t to $t+n$. The scaling matches the horizon.

4. Comparing risk-neutral and statistical return moments

The risk-neutral moments estimated from options $(\sigma_{t,n}, s_{t,n}, k_{t,n})$ and the statistical moments computed from daily returns $(\widehat{\sigma}_{t,n}, \widehat{s}_{t,n}, \widehat{k}_{t,n})$ differ in three important aspects:

- The risk-neutral moments $(\sigma_{t,n}, s_{t,n}, k_{t,n})$ are known at time t and are computed from time-t option prices with option maturity n, but the ex-post realized moments $(\widehat{\sigma}_{t,n}, \widehat{s}_{t,n}, \widehat{k}_{t,n})$ are known at time $t+n$, and are computed from daily returns between t and $t+n$.
- The risk-neutral moments are ex ante expectations. The statistical moments are ex post realizations. What is expected ex ante may not happen ex post, even if the ex ante expectation is correct on average.
- The risk-neutral moments are not pure statistically expectations. They contain a risk-adjustment component. Thus, the long-run average of the risk-neutral moments can differ from long-run average of the statistical moments. The average difference between the two reflects the risk premium charged by investors on the relevant risk exposures.

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