Q1)

#15

b) True. All negative numbers in D are even.
d) True. All numbers with 2 as their one's digit have their tens digit as 3 or 4.
e) False. Counterexample = 36, where 6 is in the one's place and 3 is in the tens place

Q2)

#12
False.
Correct Negation: The product of some irrational number and some rational number is not irrational. Or. There exists at least one rational number and one irrational number whose product is not irrational.

#40

If a number is divisible by 8, then it is divisible by 4.

#46

Statement: Having a large income is not a necessary condition for a person to be happy.

If Statement: Having a large income is a necessary condition for a person to be happy.

∀x persons, such that if all x people do not have a large income, then all x people are not happy.

Negation: ∃x person, such that person x does not have a large income and x is happy.

Final Answer: ∃x person, such that person x does not have a large income and x is happy.

Q3)

#43

Statement: For $\lim_{x \to a} f(x) = L$: For every real number ε > 0, there exists a real number > 0 such that for every real number x, if a - < x < a + and x ≠ a, then L - ε < $f(x)$ < L + ε.

Statement with symbols: For $\lim_{x \to a} f(x) = L$: ∀ ε ∈ $\mathbb{R}^+$, ∃ ∈ $\mathbb{R}^+$ such that ∀ x ∈ $\mathbb{R}$, if a - < x < a + and x ≠ a, then L - ε < $f(x)$ < L + ε.

Negation with symbols: For $\lim_{x \to a} f(x) \neq L$: ∃ ε ∈ $\mathbb{R}^+$, ∀ ∈ $\mathbb{R}^+$ such that ∃ x ∈ $\mathbb{R}$ and a - < x < a + and x ≠ a and either L - ε ≥ $f(x)$ or $f(x)$ ≥ L + ε.
Negation: For $\lim_{x \to a} f(x) \neq L$: There is at least a real number ε > 0, for every real number > 0 such that there is at least one real number x and a - < x < a + and x ≠ a and either L - ε ≥ $f(x)$ or $f(x)$ ≥ L + ε.

#44

a) The statement is True. The only real number value x can take is 1, which is unique.
b) The statement is False. There is more than 1 integer value that x can take (such as 1, -1) for 1/x to be an integer, which makes it not unique.
c) The statement is True. The only real number value x can take is the negative of that number and 0 for 0, which is unique.

#45

Re-write without symbols: There exists a unique x value that belongs to the domain D, such that the Predicate of x is True.
Re-write with symbols: There exists a unique x ∈ D such that $\mathscr{P}(x)$.

Q4)

#56

If $\exists x \in D$ and $(P(x) \wedge Q(x))$ is True, $(\exists x \in D, P(x))$ will have to have one x value for which it is True, and at the same time $(\exists x \in D, Q(x))$ will have to have one x value for which it is True. As a result, $(\exists x \in D, P(x)) \wedge (\exists x \in D, Q(x))$ is also True and **has the same values.**

#57

If $\forall x \in D$ and $(P(x) \vee Q(x))$ is True, $(\forall x \in D, P(x))$ can have a False value for $\forall x \in D, Q(x)$ such that Q(x) is also False. As a result, $(\forall x \in D, P(x)) \vee (\forall x \in D, Q(x))$ is False and **it has opposite values.**