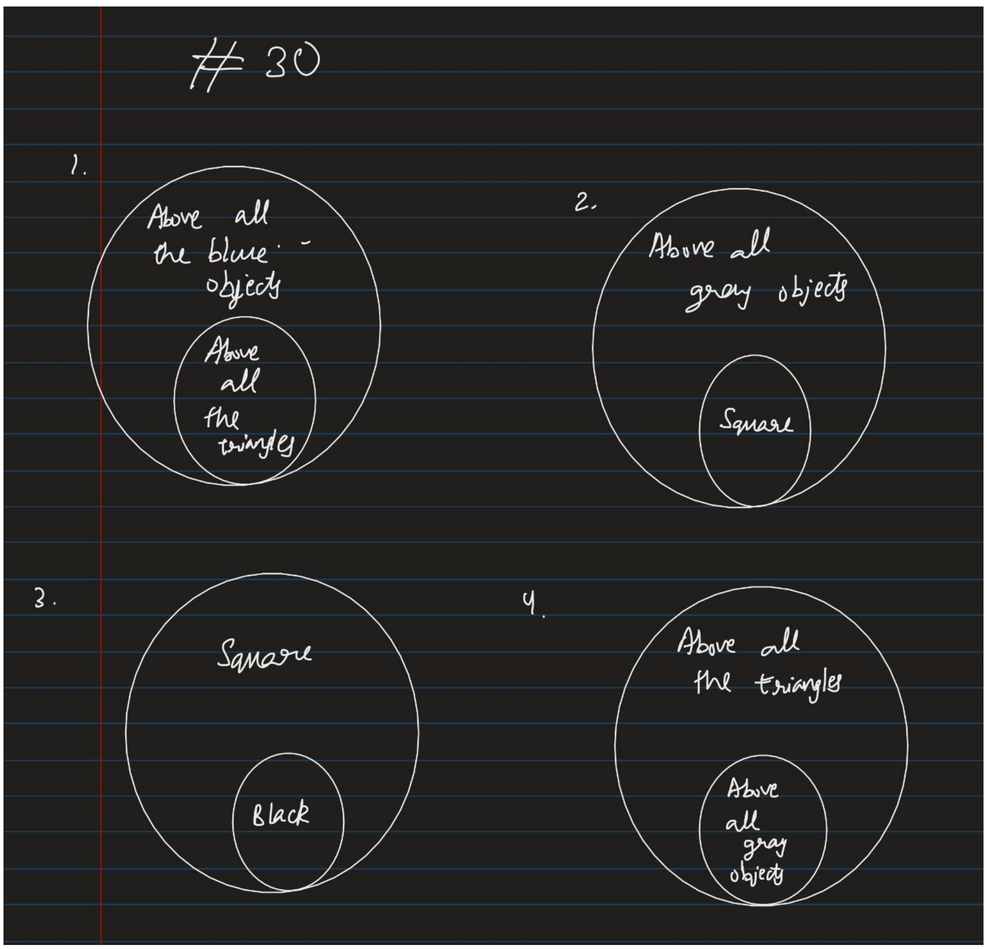
**1)**
**#30**

**a)**
1. ∀ objects x, if x is above all the triangles, then x is above all the blue objects
2. ∀ objects x, if x is a square, then x is above all the gray objects
3. ∀ objects x, if x is black, then x is a square
4. ∀ objects x, if x is above all the gray objects, then x is above all the triangles

∴ ∀ objects x, if x is black, then x is above all the blue objects.

**b)**



**c)**
3. ∀ objects x, if x is black, then x is a square
2. ∀ objects x, if x is a square, then x is above all the gray objects
4. ∀ objects x, if x is above all the gray objects, then x is above all the triangles
1. ∀ objects x, if x is above all the triangles, then x is above all the blue objects

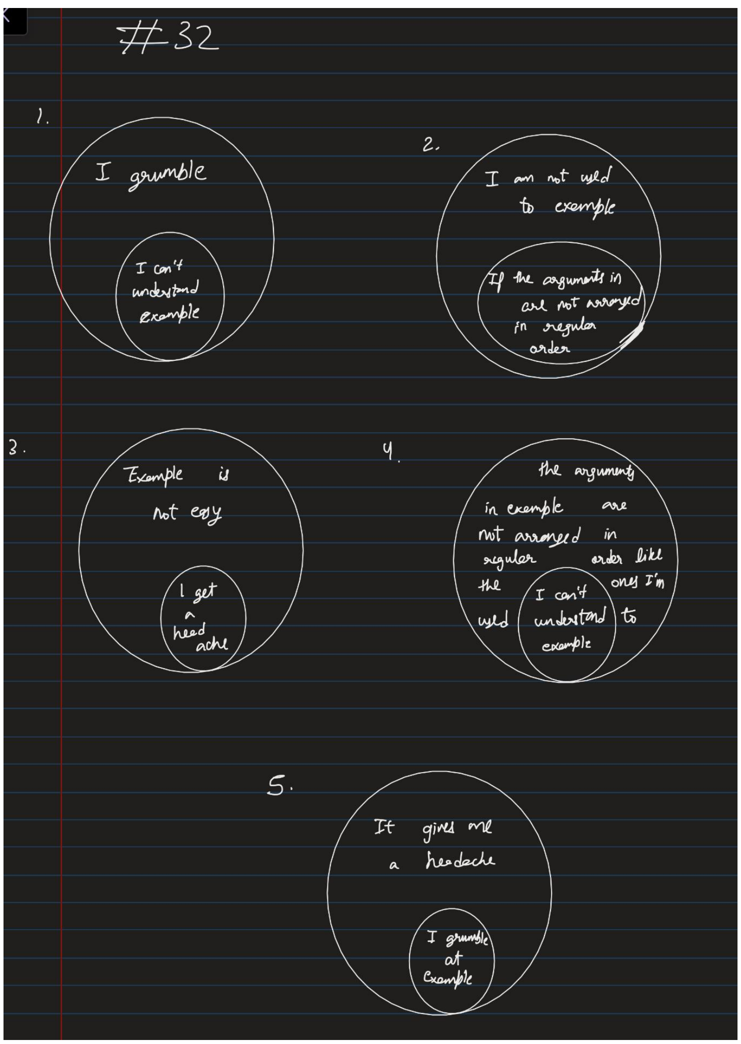
∴ ∀ objects x, if x is black, then x is above all the blue objects.

**2)**
**#32**

**a)**
1. ∀ example x, if I can't understand x I work, then I grumble.
2. ∀ example x, if the arguments in x are not arranged in regular order, then I am not used to x.
3. ∀ example x, if I get a head ache, then x is not easy.
4. ∀ example x, if the arguments in x are not arranged in regular order like the ones I am used to, then I can't understand x.
5. ∀ example x, if I grumble at x, then it gives me a headache.

∴ ∀ example x, x is not easy.

**b)**



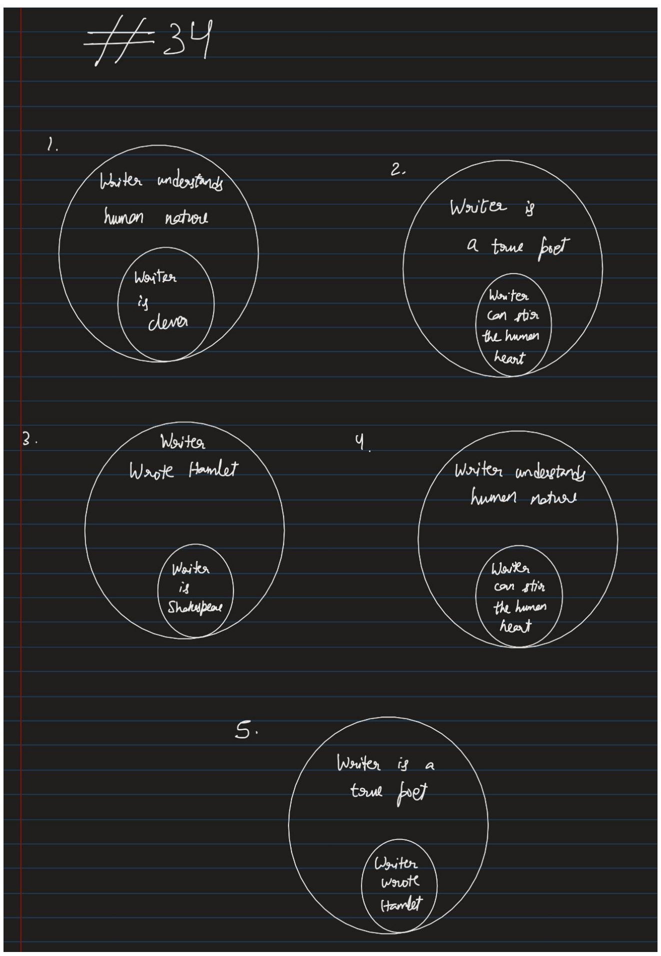

**c)**
2. ∀ example x, if the arguments in x are not arranged in regular order, then I am not used to x.

4. ∀ example x, if the arguments in x are not arranged in regular order like the ones I am used to, then I can’t understand x.
1. ∀ example x, if I can’t understand x I work, then I grumble.
5. ∀ example x, if I grumble at x, then it gives me a headache.
3. ∀ example x, if I get a head ache, then x is not easy.
∴ ∀ example x, x is not easy.

**#34**

**a)**
1. ∀ writer x, if x understands human nature, then x is clever.
2. ∀ writer x, if x is a true poet, then x can stir the human heart.
3. ∀ writer x, if x is Shakespeare, then x wrote Hamlet.
4. ∀ writer x, if x can stir the human heart, then x understands human nature.
5. ∀ writer x, if x has written Hamlet, then x is a true poet.
∴ Shakespeare was clever and a true poet.

**b)**



**c)**

3. ∀ writer x, if x is Shakespeare, then x wrote Hamlet.
5. ∀ writer x, if x has written Hamlet, then x is a true poet.
2. ∀ writer x, if x is a true poet, then x can stir the human heart.
4. ∀ writer x, if x can stir the human heart, then x understands human nature.
1. ∀ writer x, if x understands human nature, then x is clever.

∴ Shakespeare was clever and a true poet.

**3)**
**#7**

There are real numbers a and b such that $\sqrt{a + b} = \sqrt{a} + \sqrt{b}$
Proof: Let a = 0 and b = 1, a and b are real numbers such that
$= \sqrt{0 + 1} = \sqrt{0} + \sqrt{1}$
$= \sqrt{0 + 1} = 0 + 1$
$= \sqrt{1} = 1$
$= 1 = 1$

**#11**

There is an integer n such that $2n^2 - 5n + 2$ is prime

Proof: Let n = 0, n is an integer such that $2(0)^2 - 5(0) + 2 = 2$, which is prime, as 1 and 2 are the only divisors of 2.

**#16**

For every integer n, if n is even then $n^2 + 1$ is prime.

Negation for the statement: For some integer n, n is even and $n^2 + 1$ is not prime.

Counterexample: Let n=8, where n is an integer and is even, such that $8^2 + 1 = 64 + 1 = 65$, which is composite as its divisors include 1,5,13,65.

**4)**

**#22**

For each integer n, with $1 \leq n \leq 10$, $n^2 - n + 11$ is a prime number.

$11 = 1^2 - 1 + 11$ ; $13 = 2^2 - 2 + 11$ ; $17 = 3^2 - 3 + 11$ ; $23 = 4^2 - 4 + 11$ ; $31 = 5^2 - 5 + 11$; $41 = 6^2 - 6 + 11$ ; $53 = 7^2 - 7 + 11$ ; $67 = 8^2 - 8 + 11$ ; $83 = 9^2 - 9 + 11$ ; $101 = 10^2 - 10 + 11$, where 11, 13, 17, 23, 31, 41, 53, 67,83 & 101 are prime numbers where their only divisors are 1 and the number itself.

**#29**

a)
∀ integers n, if n is even, then -n is even.
∀ even integers n, -n is even.
If n is any even integer, then -n is even.

b)
a. definition of even number
b. negating both sides
c. product of two integers is an integer
d. definition of even number

**5)**

**#8**

For any integers m and n, if m is even and n is odd then 5m+3n is odd

Proof: Let m be any particular but arbitrarily chosen even integer and n be any particular but arbitrarily chosen odd integer.

By definition of even number, m = 2p, for some integer p

By definition of even number, n = 2q+1, for some integer q

Then

$$5m + 3n = 5(2p) + 3(2q+1) \quad \text{,by substitution}$$
$$5m + 3n = 10p + 6q + 3$$
$$5m + 3n = 2(5p + 3q + 1) + 1 \quad \text{,by algebra}$$

5p + 3q + 1 is an integer , by sum and product of integers is an integer

Let k = 5p + 3q + 1, where k is an integer

5m + 3n = 2(5p + 3q + 1) + 1 = 2k + 1 is odd by definition of odd, that is 5m + 3n equals twice some integer plus 1.

**#9**

If an integer greater than 4 is a perfect square, then the immediately preceding integer is not prime.

Proof: Suppose x be any particular but arbitrarily chosen integer greater than 4 such that x is a perfect square.

By definition of perfect square, x = n*n, where n is some integer

x > 4

$n^2 > 4,\ by\ substitution$

$n > \sqrt{4},\ by\ squaring\ both\ sides$

$n > |2|$

$n > 2\ or\ n < (-2)$

x > 4

$n^2 > 4,\ by\ substitution$

$n^2 - 1 > 4 - 1,\ by\ subtracting\ 1\ on\ both\ sides$

$n^2 - 1 > 3$

$n^2 - 1 = (n - 1)(n + 1)$, by Algebra

$(n - 1)(n + 1)$ > 3

Since (n-1) > 1 or (n-1) < - 3 and (n+1) > 3 or (n+1) < -1

Let p = n-1 and q = n+1, where p and q are integers

such that p,q $\neq$ 1

Hence, $n^2 - 1$ is not a prime number.

Therefore, if an integer greater than 4 is a perfect square, then the immediately preceding integer is not prime.

**#14**

There exists an integer $k \geq 4$ such that $2k^2 - 5k + 2$ is prime

Proof: To prove that the statement is false, let's assume its negation is true.

Statement: $\exists\, k \in \mathbb{Z},\ k \geq 4$ such that $2k^2 - 5k + 2$ is prime

Negation: $\forall\, k \in \mathbb{Z},\ k \geq 4$ such that $2k^2 - 5k + 2$ is not prime

$2k^2 - 5k + 2 = (2k - 1)(k - 2),\ by\ algebra$

$k \geq 4$

$2k - 1 \geq 2(4) - 1,\ multiplying\ 2\ and\ subtracting\ 1\ on\ both\ sides$

$2k - 1 \geq 7$

$k - 2 \geq 4 - 2$

$k - 2 \geq 2$

Let p = $2k - 1$, q = $k - 2$, where p and q are integers

such that p,q ≠ 1

Hence, $2k^2 - 5k + 2$ is not a prime number.

∴ $\exists\, k \in \mathbb{Z},\ k \geq 4$ such that $2k^2 - 5k + 2$ is prime is a false statement.

**6)**

**#18**

There's $mn = (2p)(2q+1) = 2(2pq+p)$ missing. There is no reasoning to prove how $2pq + p$ is an integer, which should have "product and sum of integers is an integer." they also didn't assign r to a value that is $2pq + p$.

**#19**

We cannot use k in $m = 2k$, as it's used already in the statement and similarly in $n = 2k$.
Instead, $m = 2p$ where p is some integer and $n = 2q$ where q is some integer.
$m+n = 2p + 2q = 2(p+q) = 2r$, where $r = p+q$ is an integer by addition of integers is an integer
$m + n = 2(2r) = 4k$, where $k = r$, an integer.

**7)**

**#26**

For all integers a, b, and c, if a, b, and c are consecutive, then a+b+c is even.

False.

Proof: To prove that the statement is false, let's assume its negation is true.

Statement: ∀ integers a, b, and c, if a, b, and c are consecutive, then a+b+c is even.

Negation: ∃ integers a, b, and c, if a, b, and c are consecutive, then a+b+c is not even.

[Case - I]

Assume a is odd, then by being consecutive, b is even, and c is odd.

By definition of being odd, a = 2p+1, c = 2q+1 and by definition of even, b = 2s, where p, q, s are some integers.

a+b+c = 2p+1 + 2s + 2q + 1 = 2(p+s+q+1), where p+s+q+1 is an integer by sum of integers

Let r = p + s + q + 1 be an integer

Hence, a+b+c = 2r, which is even, by definition of even.

This supports the statement and not the contradiction assumed before.

[Case - II]

Assume a is even, then by being consecutive, b is odd, and c is even.

By definition of being even, a = 2e, c = 2f and by definition of odd, b = 2u+1, where e, f, u are some integers.

a+b+c = 2e + 2u + 1 + 2f = 2(e+u+f)+1, where e+u+f is an integer by sum of integers

Let t = e+u+f be an integer

Hence, a+b+c = 2t +1, which is odd, by definition of odd.

∴ ∃ integers a, b, and c, if a, b, and c are consecutive, then a+b+c is not even.

∴ ∀ integers a, b, and c, if a, b, and c are consecutive, then a+b+c is even is a False statement.

**#30**

For every integer m, if m > 2, then $m^2 - 4$ is composite.

False.

Proof:

To prove that the statement is false, let's assume its negation is true.

Statement: ∀ integer m, if m > 2 $\rightarrow$ $m^2 - 4$ is composite

Negation: ∃ integer m, such that m > 2 $and$ $m^2 - 4$ is not composite

Let us assume m to be any particular but arbitrarily chosen integer such that m> 2.

$m^2 - 4 = (m + 2)(m - 2),\ by\ factorisation\ and\ algebra$

If m>2, then m+2 > 4 and m-2>0,

Let p = m+2, q = m-2, where p and q are some integers, q can be equal to 1.

For $m^2 - 4$ to be prime, m+2 = $m^2 - 4$, by definition of prime number

$m + 2 = (m + 2)(m - 2),\ by\ substitution$

$1 \times \frac{m+2}{m+2} = m - 2$, by multiplication identity and by algebra

$1 = m - 2$

$m - 2 = 1$
$m = 1 + 2 = 3,\ by\ algebra$
Hence, p=m+2=5, by algebra, q=m-2=1, by algebra
Hence $3^2 - 4 = 5 = pq = 5 \times 1$, where p = 5 and q = 1
Since a prime number only has two divisors, 1 and the number itself, by the definition of prime
$m^2 - 4\ is\ prime\ for\ m = 3$
∴ ∃ integer m, such that m > 2 $and\ m^2 - 4$ is not composite
∴ ∀ integer m, if m > 2 $\rightarrow m^2 - 4$ is composite is a False statement

**#31**

For every integer n, $n^2 - n + 11$ is a prime number.
True.
Proof:
Statement: ∀ integer n, $n^2 - n + 11$ is a prime number.
Let $n^2 - n + 11 = pq,\ where\ p\ and\ q\ are\ some\ integers\ and\ factors\ of\ n^2 - n + 11$
$n^2 - n + 11$ is not factorable besides into 1 and $n^2 - n + 11$ itself
Hence, $n^2 - n + 11$ is a prime number