# Homework 2 Solutions

## Problems

1. Problem 1:
   Hello

2. Problem 2:
   Since, we want 6 digit accuracy for the $cos(x)$ function, we say that our maximum error should be $10^{-6}$. The error is expressed by the following formula:

$$\mathbb{E} \leq \frac{1}{(n+1)!} \max_x f^{n+1}(x) \prod_{i=0}^{n} (x - x_i), for\ x \in [0, \pi] \tag{1}$$

   where $n$ is the degree of polynomial used to interpolate the given function.

   (a) Linear Interpolation: For linear interpolation, $n = 1$. Hence, using (1):

$$10^{-6} \leq \frac{1}{2!} \max_x |\frac{d^3 cos(x)}{dx^3}| \max_x |\prod_{i=0}^{1} (x - x_i)| \tag{2}$$

$$\max_x |\frac{d^3 cos(x)}{dx^3}| = \max_x |sin(x)| = 1, for\ x \in [0, \pi]$$

   To get the max of product, we differentiate it with respect to $x$ as follows:

$$\frac{d}{dx}(\prod_{i=0}^{1} (x - x_i)) = 0,$$

$$\implies 2x - (x_0 + x_1) = 0$$

$$\implies x = \frac{(x_0 + x_1)}{2}$$

   Substituting $x$ in (2):

$$10^{-6} \leq \frac{1}{2!} \max |\frac{x_0 - x_1}{2} \frac{x_1 - x_0}{2}|$$

   Let $h = x_1 - x_0$ be the table spacing,

$$10^{-6} \leq \frac{1}{2!} |\frac{-h^2}{4}|$$

$$\implies h \geq \sqrt{8 \times 10^{-6}} = 0.0028$$

   Hence, the required table spacing is $h = 0.0028$.

   (b) Quadratic Interpolation: For quadratic interpolation, $n = 2$. Hence, using (1):

$$10^{-6} \leq \frac{1}{3!} \max_x |\frac{d^4 cos(x)}{dx^4}| \max |\prod_{i=0}^{2} (x - x_i)| \tag{3}$$

$$\max_x |\frac{d^4 cos(x)}{dx^4}| = \max_x |cos(x)| = 1, for\ x \in [0, \pi]$$

To get the max of product, we differentiate it with respect to $x$ as follows:

$$\frac{d}{dx}(\prod_{i=0}^{2}(x - x_i)) = 0,$$

$$\implies x = \frac{(x_0 + x_1 + x_2)}{3}$$

Substituting $x$ in (2):

$$10^{-6} \leq \frac{1}{2!} \max |\frac{(x_1 - x_0) + (x_2 - x_0)}{3} \frac{(x_0 - x_1) + (x_2 - x_1)}{3} \frac{(x_0 - x_2) + (x_1 - x_2)}{3}|$$

Let $h = x_1 - x_0 = x_2 - x_1$ be the table spacing,

$$10^{-6} \leq \frac{1}{3!} |\frac{2h^3}{3}|$$

$$\implies h \geq \sqrt[3]{9 \times 10^{-6}} = 0.0208$$

Hence, the required table spacing is $h = 0.0208$.

(c) To get the number of entries:

i. For Linear Interpolation:

$$N = \frac{\pi - 0}{h} = \frac{\pi}{0.0028} \approx 1111$$

ii. For Quadratic Interpolation:

$$N = \frac{\pi - 0}{h} = \frac{\pi}{0.0208} \approx 151$$