Is the following function a proper distance function? Why? Explain your answer. Measure the distance between (0, 0, 0), (0, 1, 0), (0, 1, 1), and (1, 1, 1)

$$\boldsymbol{d(x, y) = \Sigma\ (|x_{i-\ Y_i}|\ ^2)}$$

For a function to be a proper distance function, it should follow the following three properties:

1. Non-negativity or separation axiom
2. Identity of indiscernible
3. Symmetry
4. Subadditivity or triangle inequality

Let us assume that, X= (0,0), Y= (0,1) and Z= (1,1)

Then, the function follows the first three properties, but it fails to follow the fourth property. Let us see how,

Distance from X to Y = d (x, y) = Σ ($|0\text{-}0|^2 + |0\text{-}1|^2$) = 1

Distance from Y to Z = d (y, x) = Σ ($|0\text{-}1|^2 + |1\text{-}1|^2$) = 1

Distance from X to Z = d (x, z) = Σ ($|0\text{-}1|^2 + |0\text{-}1|^2$) = 2

Hence, the third property is not satisfied.

Therefore, the given function is not a proper distance function.

Measure the distance between (0, 0, 0), (0, 1, 0), (0, 1, 1), and (1, 1, 1)

Let us take A= (0,0,0), B = (0,1,0), C = (0,1,1), D = (1,1,1)

So, AB = $\sqrt{}(x_2\text{-}x_1)^2 + (y_2\text{-}y_1)^2 + (z_2\text{-}z_1)^2 = \sqrt{}(0\text{-}0)^2 + (1\text{-}0)^2 + (0\text{-}0)^2 = 1$

BC = $\sqrt{}(x_2\text{-}x_1)^2 + (y_2\text{-}y_1)^2 + (z_2\text{-}z_1)^2 = \sqrt{}(0\text{-}0)^2 + (1\text{-}1)^2 + (1\text{-}0)^2 = 1$

CD = $\sqrt{}(x_2\text{-}x_1)^2 + (y_2\text{-}y_1)^2 + (z_2\text{-}z_1)^2 = \sqrt{}(1\text{-}0)^2 + (1\text{-}1)^2 + (1\text{-}1)^2 = 1$

AC = $\sqrt{}(x_2\text{-}x_1)^2 + (y_2\text{-}y_1)^2 + (z_2\text{-}z_1)^2 = \sqrt{}(0\text{-}0)^2 + (1\text{-}0)^2 + (1\text{-}0)^2 = \sqrt{2}$

AD = $\sqrt{}(x_2\text{-}x_1)^2 + (y_2\text{-}y_1)^2 + (z_2\text{-}z_1)^2 = \sqrt{}(1\text{-}0)^2 + (1\text{-}0)^2 + (1\text{-}0)^2 = \sqrt{3}$

BD = $\sqrt{}(x_2\text{-}x_1)^2 + (y_2\text{-}y_1)^2 + (z_2\text{-}z_1)^2 = \sqrt{}(1\text{-}0)^2 + (1\text{-}1)^2 + (1\text{-}0)^2 = \sqrt{2}$