Machine Learning HW1

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Question 1:

Find the transpose of the following matrices and vectors (10 pts):

1.

Matrix:

$$\begin{bmatrix} 7 & 4 & 5 & 6 \\ 2 & 8 & 9 & 7 \\ 1 & 0 & 2 & 3 \end{bmatrix}$$

Transpose:

$$\begin{bmatrix} 7 & 2 & 1 \\ 4 & 8 & 0 \\ 5 & 9 & 2 \\ 6 & 7 & 3 \end{bmatrix}$$

2.

Matrix:

$$\begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}$$

Transpose:

$$\begin{bmatrix} a & c & e \\ b & d & f \end{bmatrix}$$

3.

Matrix:

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 2 & 0 & 0 & 8 \end{bmatrix}$$

Transpose:

$$\begin{bmatrix} 1 & 5 & 2 \\ 2 & 6 & 0 \\ 3 & 7 & 0 \\ 4 & 8 & 8 \end{bmatrix}$$

4.

Matrix:

$$\begin{bmatrix} 2 & 2 & 8 & 10 \end{bmatrix}$$

Transpose:

$$\begin{bmatrix} 2 \\ 2 \\ 8 \\ 10 \end{bmatrix}$$

5.

Matrix:

$$\begin{bmatrix} 1 \\ 2 \\ 2 \\ 8 \end{bmatrix}$$

Transpose:

$$\begin{bmatrix} 1 & 2 & 2 & 8 \end{bmatrix}$$

Question 2:

Perform the following operations. If the dimension doesn't match, just indicate "dimension mismatch". (10 pts):

1.

$$\begin{bmatrix} -4 \\ -7 \end{bmatrix} - \begin{bmatrix} 7 \\ 3 \end{bmatrix} = \begin{bmatrix} -4-7 \\ -7-3 \end{bmatrix} = \begin{bmatrix} -11 \\ -10 \end{bmatrix}$$

2.

$$\begin{bmatrix} -2 \\ 0 \\ 3 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -2+2 \\ 0+1 \\ 3+1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 4 \end{bmatrix}$$

3.

$$\begin{bmatrix} 1 & 2 & 5 \end{bmatrix} + \begin{bmatrix} -7 \\ 8 \\ 3 \end{bmatrix} = \text{"dimension mismatch"}$$

4.

$$\begin{bmatrix}1 & 2 & 5\end{bmatrix} + \begin{bmatrix}-7 & 8 & 3\end{bmatrix} = \begin{bmatrix}1-7 & 2+8 & 5+3\end{bmatrix} = \begin{bmatrix}-6 & 10 & 8\end{bmatrix}$$

5.

$$\begin{bmatrix}1\\2\\5\\0\end{bmatrix} + \begin{bmatrix}-1\\1\\3\\3\end{bmatrix} = \begin{bmatrix}1-1\\2+1\\5+3\\0+3\end{bmatrix} = \begin{bmatrix}0\\3\\8\\3\end{bmatrix}$$

Question 3:

Given

$$x = \begin{bmatrix}2\\2\\6\end{bmatrix}, \quad X = \begin{bmatrix}-4 & 7 & 5\\1 & -9 & 2\\8 & 3 & 6\end{bmatrix}, \quad Y = \begin{bmatrix}2 & 4 & 6\\1 & 0 & 13\\-4 & 5 & 5\end{bmatrix}, \quad Z = \begin{bmatrix}4 & -1 & 0\\1 & 1 & 1\end{bmatrix}$$

Calculate the following (10 pts):

1. $x^\top XY$

$$x^\top = \begin{bmatrix}2 & 2 & 6\end{bmatrix}$$

$$x^\top X = \begin{bmatrix}2 & 2 & 6\end{bmatrix}\begin{bmatrix}-4 & 7 & 5\\1 & -9 & 2\\8 & 3 & 6\end{bmatrix} = \begin{bmatrix}44 & 4 & 52\end{bmatrix}$$

$$\begin{bmatrix}44 & 4 & 52\end{bmatrix}\begin{bmatrix}2 & 4 & 6\\1 & 0 & 13\\-4 & 5 & 5\end{bmatrix} = \begin{bmatrix}(44\cdot 2 + 4\cdot 1 + 52\cdot -4) & (44\cdot 4 + 4\cdot 0 + 52\cdot 5) & (44\cdot 6 + 4\cdot 13 + 52\cdot 5)\end{bmatrix}$$

$$= \begin{bmatrix}88 + 4 - 208 & 176 + 0 + 260 & 264 + 52 + 260\end{bmatrix}$$

$$= \begin{bmatrix}-116 & 436 & 576\end{bmatrix}$$

2. YX

$$YX = \begin{bmatrix}2 & 4 & 6\\1 & 0 & 13\\-4 & 5 & 5\end{bmatrix}\begin{bmatrix}-4 & 7 & 5\\1 & -9 & 2\\8 & 3 & 6\end{bmatrix}$$

$$= \begin{bmatrix}(-8+4+48) & (14-36+18) & (10+8+36)\\(-4+104) & (7+39) & (5+78)\\(16+5+40) & (-28-45+15) & (-20+10+30)\end{bmatrix}$$

$$= \begin{bmatrix}44 & -4 & 54\\100 & 46 & 83\\61 & -58 & 20\end{bmatrix}$$

3. $2X + 2Y + X^\top Y$

$$2X = \begin{bmatrix} -8 & 14 & 10 \\ 2 & -18 & 4 \\ 16 & 6 & 12 \end{bmatrix}, \quad 2Y = \begin{bmatrix} 4 & 8 & 12 \\ 2 & 0 & 26 \\ -8 & 10 & 10 \end{bmatrix}$$

$$2X + 2Y = \begin{bmatrix} -8+4 & 14+8 & 10+12 \\ 2+2 & -18+0 & 4+26 \\ 16+-8 & 6+10 & 12+10 \end{bmatrix} = \begin{bmatrix} -4 & 22 & 22 \\ 4 & -18 & 30 \\ 8 & 16 & 22 \end{bmatrix}$$

$$X^\top = \begin{bmatrix} -4 & 1 & 8 \\ 7 & -9 & 3 \\ 5 & 2 & 6 \end{bmatrix}$$

$$X^\top Y = \begin{bmatrix} -4 & 1 & 8 \\ 7 & -9 & 3 \\ 5 & 2 & 6 \end{bmatrix} \begin{bmatrix} 2 & 4 & 6 \\ 1 & 0 & 13 \\ -4 & 5 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} (-8+1-32) & (-16+0+40) & (-24+13+40) \\ (14-9-12) & (28+0+15) & (42-117+15) \\ (10+2-24) & (20+0+30) & (30+26+30) \end{bmatrix}$$

$$= \begin{bmatrix} -39 & 24 & 29 \\ -7 & 43 & -60 \\ -12 & 50 & 86 \end{bmatrix}$$

$$2X + 2Y + X^\top Y = \begin{bmatrix} -4 & 22 & 22 \\ 4 & -18 & 30 \\ 8 & 16 & 22 \end{bmatrix} + \begin{bmatrix} -39 & 24 & 29 \\ -7 & 43 & -60 \\ -12 & 50 & 86 \end{bmatrix} = \begin{bmatrix} -43 & 46 & 51 \\ -3 & 25 & -30 \\ -4 & 66 & 108 \end{bmatrix}$$

4. $YZ^\top$

$$Z^\top = \begin{bmatrix} 4 & 1 \\ -1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$YZ^\top = \begin{bmatrix} 2 & 4 & 6 \\ 1 & 0 & 13 \\ -4 & 5 & 5 \end{bmatrix} \begin{bmatrix} 4 & 1 \\ -1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} (2 \cdot 4 + 4 \cdot -1 + 6 \cdot 0) & (2 \cdot 1 + 4 \cdot 1 + 6 \cdot 1) \\ (1 \cdot 4 + 0 \cdot -1 + 13 \cdot 0) & (1 \cdot 1 + 0 \cdot 1 + 13 \cdot 1) \\ (-4 \cdot 4 + 5 \cdot -1 + 5 \cdot 0) & (-4 \cdot 1 + 5 \cdot 1 + 5 \cdot 1) \end{bmatrix}$$

$$= \begin{bmatrix} (8-4+0) & (2+4+6) \\ (4+0+0) & (1+0+13) \\ (-16-5+0) & (-4+5+5) \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 12 \\ 4 & 14 \\ -21 & 6 \end{bmatrix}$$

5. $Z^\top Z x$

$$Z^\top Z = \begin{bmatrix} 4 & 1 \\ -1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & -1 & 0 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} (4\cdot 4+1\cdot 1) & (4\cdot -1+1\cdot 1) & (4\cdot 0+1\cdot 1) \\ (-1\cdot 4+1\cdot 1) & (-1\cdot -1+1\cdot 1) & (-1\cdot 0+1\cdot 1) \\ (0\cdot 4+1\cdot 1) & (0\cdot -1+1\cdot 1) & (0\cdot 0+1\cdot 1) \end{bmatrix}$$

$$= \begin{bmatrix} 17 & -3 & 1 \\ -3 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 17 & -3 & 1 \\ -3 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 6 \end{bmatrix} = \begin{bmatrix} 17\cdot 2 + -3\cdot 2 + 1\cdot 6 \\ -3\cdot 2 + 2\cdot 2 + 1\cdot 6 \\ 1\cdot 2 + 1\cdot 2 + 1\cdot 6 \end{bmatrix} = \begin{bmatrix} 34-6+6 \\ -6+4+6 \\ 2+2+6 \end{bmatrix} = \begin{bmatrix} 34 \\ 4 \\ 10 \end{bmatrix}$$

Question 4:

Given

$$X = \begin{bmatrix} 1 & 7 \\ -8 & 6 \\ 3 & 1 \end{bmatrix}, \quad Y = \begin{bmatrix} 0 & 4 & 6 & -8 \\ 1 & -3 & 5 & 7 \\ 11 & 13 & 9 & 9 \\ -1 & 0 & 8 & 7 \end{bmatrix}, \quad Z = \begin{bmatrix} 0 & 4 & 8 & 1 \\ -3 & 5 & 17 & 11 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 17 \\ 2 & 14 \end{bmatrix}, \quad b = \begin{bmatrix} 19 & -11 & 13 \end{bmatrix}, \quad c = \begin{bmatrix} 12 \\ -11 \\ 10 \end{bmatrix}, \quad x = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad y = \begin{bmatrix} 3 & 3 & 1 \end{bmatrix}$$

Calculate the following both by hand and by numpy (20 pts):

1. $xyy^\top x^\top$

$$xyy^\top x^\top = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 3 & 3 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 3 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 19 \end{bmatrix} = 19 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 19 \\ 38 \\ 57 \end{bmatrix}$$

2. $3c - 9$

$$3c - 9 = 3 \begin{bmatrix} 12 \\ -11 \\ 10 \end{bmatrix} - 9 = \begin{bmatrix} 36 \\ -33 \\ 30 \end{bmatrix} - \begin{bmatrix} 9 \\ 9 \\ 9 \end{bmatrix} = \begin{bmatrix} 27 \\ -42 \\ 21 \end{bmatrix}$$

3. $\langle A, A \rangle \cdot A$

$$\langle A, A \rangle = \text{trace}(A^\top A) = \text{trace}\left(\begin{bmatrix} 1 & 2 \\ 17 & 14 \end{bmatrix} \begin{bmatrix} 1 & 17 \\ 2 & 14 \end{bmatrix} \right) = \text{trace}\left(\begin{bmatrix} 5 & 45 \\ 45 & 485 \end{bmatrix} \right) = 5{+}485 = 490$$

$$\langle A, A \rangle \cdot A = 490 \cdot \begin{bmatrix} 1 & 17 \\ 2 & 14 \end{bmatrix} = \begin{bmatrix} 490 & 8330 \\ 980 & 6860 \end{bmatrix}$$

4. $2XZ$

$$2XZ = 2\begin{bmatrix}1 & 7\\ -8 & 6\\ 3 & 1\end{bmatrix}\begin{bmatrix}0 & 4 & 8 & 1\\ -3 & 5 & 17 & 11\end{bmatrix} = 2\begin{bmatrix}-21 & 39 & 127 & 78\\ 24 & -2 & -88 & -58\\ -3 & 17 & 41 & 14\end{bmatrix} = \begin{bmatrix}-42 & 78 & 254 & 156\\ 48 & -4 & -176 & -116\\ -6 & 34 & 82 & 28\end{bmatrix}$$

5. $(bc)A$

$$bc = \begin{bmatrix}19 & -11 & 13\end{bmatrix}\begin{bmatrix}12\\ -11\\ 10\end{bmatrix} = 19\cdot 12+(-11)\cdot(-11)+13\cdot 10 = 228+121+130 = 479$$

$$(bc)A = 479\begin{bmatrix}1 & 17\\ 2 & 14\end{bmatrix} = \begin{bmatrix}479 & 8143\\ 958 & 6706\end{bmatrix}$$

6. bXA

$$bXA = \begin{bmatrix}19 & -11 & 13\end{bmatrix}\begin{bmatrix}1 & 7\\ -8 & 6\\ 3 & 1\end{bmatrix}\begin{bmatrix}1 & 17\\ 2 & 14\end{bmatrix}$$

$$= \begin{bmatrix}19 & -11 & 13\end{bmatrix}\begin{bmatrix}15 & 115\\ -10 & -94\\ 5 & 65\end{bmatrix} = \begin{bmatrix}460 & 4064\end{bmatrix}$$

7. $c \odot x$

$$c \odot x = \begin{bmatrix}12\\ -11\\ 10\end{bmatrix} \odot \begin{bmatrix}1\\ 2\\ 3\end{bmatrix} = \begin{bmatrix}12\\ -22\\ 30\end{bmatrix}$$

8. $b \odot y$

$$b \odot y = \begin{bmatrix}19 & -11 & 13\end{bmatrix} \odot \begin{bmatrix}3 & 3 & 1\end{bmatrix} = \begin{bmatrix}57 & -33 & 13\end{bmatrix}$$

9. $c \otimes x$

$$c \otimes x = \begin{bmatrix}12\\ -11\\ 10\end{bmatrix} \otimes \begin{bmatrix}1\\ 2\\ 3\end{bmatrix} = \begin{bmatrix}12 & 24 & 36\\ -11 & -22 & -33\\ 10 & 20 & 30\end{bmatrix}$$

10. $YZ^\top$

$$YZ^\top = \begin{bmatrix}0 & 4 & 6 & -8\\ 1 & -3 & 5 & 7\\ 11 & 13 & 9 & 9\\ -1 & 0 & 8 & 7\end{bmatrix}\begin{bmatrix}0 & -3\\ 4 & 5\\ 8 & 17\\ 1 & 11\end{bmatrix}$$

$$= \begin{bmatrix}39 & 127\\ -10 & -94\\ 73 & 241\\ 24 & 115\end{bmatrix}$$

Question 5:

Write the code that performs the following operations using numpy. Make sure you print the results. (20 pts):

- Create two matrices and two vectors:

$$X = \begin{bmatrix} 2 & 3 \\ 9 & 1 \end{bmatrix}, Y = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}, x = \begin{bmatrix} 1 \\ 3 \end{bmatrix}, y = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

- Write the code that extracts the 1st column of X.
- Write the code that extracts the 2nd row of Y.
- Write the code that extracts the bottom right value of X.
- Write the code that adds $X + Y$.
- Write the code that performs $X + YX$.
- Write the code that performs the Hadamard product $x \odot y$.
- Write the code that performs the outer product $x \otimes y$.
- Write the code that performs the dot product XY.
- Write the code that performs the inner product $\langle X, Y \rangle$.

Question 6:

Given vectors

$$v = \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}, w = \begin{bmatrix} 8 \\ 1 \\ 1 \end{bmatrix}, x = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}, y = \begin{bmatrix} 2 \\ 1 \\ 0 \\ 3 \end{bmatrix}, z = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$$

1. L1-norm:

$$\text{L1-norm}(v) = |5| + |0| + |0| = 5$$

$$\text{L1-norm}(w) = |8| + |1| + |1| = 10$$

$$\text{L1-norm}(x) = |3| + |1| + |2| = 6$$

$$\text{L1-norm}(y) = |2| + |1| + |0| + |3| = 6$$

$$\text{L1-norm}(z) = |2| + |5| = 7$$

2. L2-norm:

$$\text{L2-norm}(v) = \sqrt{5^2 + 0^2 + 0^2} = \sqrt{25} = 5$$

$$\text{L2-norm}(w) = \sqrt{8^2 + 1^2 + 1^2} = \sqrt{64 + 1 + 1} = \sqrt{66} \approx 8.12$$

$$\text{L2-norm}(x) = \sqrt{3^2 + 1^2 + 2^2} = \sqrt{9 + 1 + 4} = \sqrt{14} \approx 3.74$$

$$\text{L2-norm}(y) = \sqrt{2^2 + 1^2 + 0^2 + 3^2} = \sqrt{4 + 1 + 0 + 9} = \sqrt{14} \approx 3.74$$

$$\text{L2-norm}(z) = \sqrt{2^2 + 5^2} = \sqrt{4 + 25} = \sqrt{29} \approx 5.39$$

3. L-infinity norm:

$$\text{L-infinity norm}(v) = \max(|5|, |0|, |0|) = 5$$

$$\text{L-infinity norm}(w) = \max(|8|, |1|, |1|) = 8$$

$$\text{L-infinity norm}(x) = \max(|3|, |1|, |2|) = 3$$

$$\text{L-infinity norm}(y) = \max(|2|, |1|, |0|, |3|) = 3$$

$$\text{L-infinity norm}(z) = \max(|2|, |5|) = 5$$

Question 7:

Use Numpy to write the code that loads the hw1.csv file and prints out the data matrix. Please show the code as well as the output. (10 pts)

All coding questions are in .ipynb file