Problem 151: Bring John Glenn Home

Difficulty: Hard

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Problem Background

The movie "Hidden Figures" focused on the work of Katherine Johnson, an African-American woman working at NASA during the Mercury missions. At that time in history, computers were still uncommon, and so Johnson worked as a "human computer," performing complex calculations to ensure that missions were executed successfully. Despite her crucial contributions to the program, Johnson faced a great deal of discrimination from her colleagues. Only recently have Johnson and her fellow African-American colleagues received due recognition for their work in space exploration; she was awarded the Presidential Medal of Freedom in 2015, and the Congressional Gold Medal in 2019. Katherine Johnson died in 2020 at the age of 101.

In one key scene of the movie, Johnson described how to calculate the trajectory of John Glenn's space capsule using "Euler's Method."

Problem Description

Euler's Method is an ancient approximation technique used for differential equations, commonly referred to as "initial value problems." To use it, you solve the equation for the given initial values, then step to the next value based on the slope of the line calculated.

Here, we'll be attempting to approximate the solution for the formula:

$$y = f(x) = \int \frac{\sin(x)}{x} dx$$

If you haven't studied calculus yet, don't worry; we're only going to approximate the solution to this function, not solve it in full. The only bit of calculus we really need is to determine the "derivative" of this function, which is:

$$f'(x) = \frac{\sin(x)}{x}$$

The formula we'll actually be solving for each iteration of Euler's Method is this:

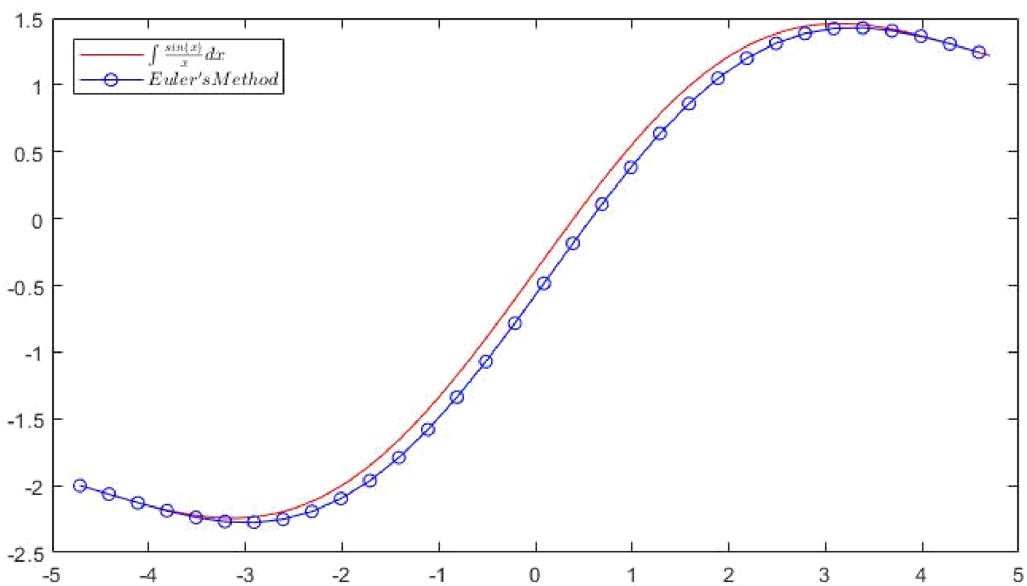
$$y_n = y_{n-1} + h * f'(x_{n-1})$$

In this formula, x and y represent coordinates on a grid, and h is the step size used for the approximation; the amount by which x increases during each step. To use Euler's Method, we have to start with a set of initial values, x_0 and y_0. Whenever the value for x is 0, assume that $f'(0) = 1$.

For example, let's try to approximate this equation using the initial values of:
$x_0 = -1.5\pi, y_0 = -2, h = .3.$

This table and chart show how the values of x and y change following each step of the equation, and how they compare to the actual solution to this equation.

n	$y_n = y_{n-1} + h * \left(\frac{\sin x_{n-1}}{x_{n-1}}\right)$	$x_n = h + x_{n-1}$
0	-2	-1.5π = -4.7124
1	-2.0637	-4.4124
2	-2.1286	-4.1124
3	-2.1888	-3.8124
4	-2.2377	-3.5124
5	-2.2687	-3.2124
6	-2.2753	-2.9124



For this problem, you will need to use Euler's Method to provide several approximated values for the formula given above.

Sample Input

The first line of your program's input, received from the standard input channel, will contain a positive integer representing the number of test cases. Each test case will include a single line, with the following values, separated by spaces:

- x_0, a number representing the initial x value.
- y_0, a number representing the initial y value.
- h, a number representing the constant step value.

- n, a positive integer representing the total number of iterations to perform.

```
2
1 5 0.5 6
-.54 0 0.01 8
```

Sample Output

For each test case, your program must print a single line containing the value of y_n, the value of y obtained after performing n iterations. Values should be rounded to 3 decimal places, not including any trailing zeroes.

```
6.074
0.077
```