

### **Chapter 2: Relational Model**

Database System Concepts, 5th Ed.

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### **Chapter 2: Relational Model**

- Structure of Relational Databases
- Fundamental Relational-Algebra-Operations
- Additional Relational-Algebra-Operations
- Extended Relational-Algebra-Operations
- Null Values
- Modification of the Database





# **Example of a Relation**

| account_number | branch_name | balance |
|----------------|-------------|---------|
| A-101          | Downtown    | 500     |
| A-102          | Perryridge  | 400     |
| A-201          | Brighton    | 900     |
| A-215          | Mianus      | 700     |
| A-217          | Brighton    | 750     |
| A-222          | Redwood     | 700     |
| A-305          | Round Hill  | 350     |





#### **Basic Structure**

- Formally, given sets D₁, D₂, .... Dₙ a relation r is a subset of D₁ x D₂ x ... x Dₙ
   Thus, a relation is a set of n-tuples (a₁, a₂, ..., aₙ) where each aᵢ ∈ Dᵢ
- Example: If
  - customer\_name = {Jones, Smith, Curry, Lindsay, ...}/\* Set of all customer names \*/
  - customer\_street = {Main, North, Park, ...} /\* set of all street names\*/
  - customer\_city = {Harrison, Rye, Pittsfield, ...} /\* set of all city names \*/

```
Then r = { (Jones, Main, Harrison), (Smith, North, Rye), (Curry, North, Rye), (Lindsay, Park, Pittsfield) }
```

is a relation over

customer\_name x customer\_street x customer\_city





### **Attribute Types**

- Each attribute of a relation has a name
- The set of allowed values for each attribute is called the domain of the attribute
- Attribute values are (normally) required to be atomic; that is, indivisible
  - E.g. the value of an attribute can be an account number,
     but cannot be a set of account numbers
- Domain is said to be atomic if all its members are atomic
- The special value null is a member of every domain
- The null value causes complications in the definition of many operations
  - We shall ignore the effect of null values in our main presentation and consider their effect later





#### **Relation Schema**

- $\blacksquare$   $A_1, A_2, ..., A_n$  are attributes
- R =  $(A_1, A_2, ..., A_n)$  is a relation schema Example:

Customer\_schema = (customer\_name, customer\_street, customer\_city)

r(R) denotes a relation r on the relation schema R Example:

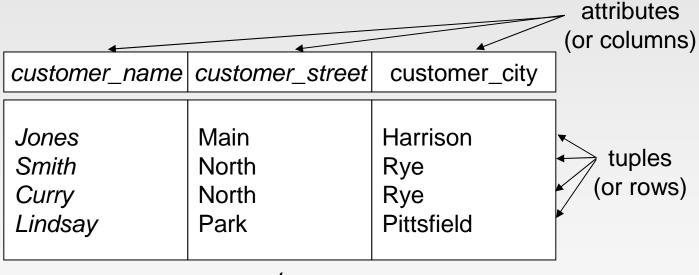
customer (Customer\_schema)





#### **Relation Instance**

- The current values (*relation instance*) of a relation are specified by a table
- An element *t* of *r* is a *tuple*, represented by a *row* in a table



customer



#### **Relations are Unordered**

- Order of tuples is irrelevant (tuples may be stored in an arbitrary order)
- Example: account relation with unordered tuples

| account_number | branch_name | balance |
|----------------|-------------|---------|
| A-101          | Downtown    | 500     |
| A-215          | Mianus      | 700     |
| A-102          | Perryridge  | 400     |
| A-305          | Round Hill  | 350     |
| A-201          | Brighton    | 900     |
| A-222          | Redwood     | 700     |
| A-217          | Brighton    | 750     |





#### **Database**

- A database consists of multiple relations
- Information about an enterprise is broken up into parts, with each relation storing one part of the information

account: stores information about accounts

depositor: stores information about which customer

owns which account

customer: stores information about customers

- Storing all information as a single relation such as bank(account\_number, balance, customer\_name, ..) results in
  - repetition of information
    - e.g.,if two customers own an account (What gets repeated?)
  - the need for null values
    - e.g., to represent a customer without an account
- Normalization theory (Chapter 7) deals with how to design relational schemas





#### The customer Relation

| customer_name | customer_street | customer_city |
|---------------|-----------------|---------------|
| Adams         | Spring          | Pittsfield    |
| Brooks        | Senator         | Brooklyn      |
| Curry         | North           | Rye           |
| Glenn         | Sand Hill       | Woodside      |
| Green         | Walnut          | Stamford      |
| Hayes         | Main            | Harrison      |
| Johnson       | Alma            | Palo Alto     |
| Jones         | Main            | Harrison      |
| Lindsay       | Park            | Pittsfield    |
| Smith         | North           | Rye           |
| Turner        | Putnam          | Stamford      |
| Williams      | Nassau          | Princeton     |





# The depositor Relation

| customer_name | account_number |
|---------------|----------------|
| Hayes         | A-102          |
| Johnson       | A-101          |
| Johnson       | A-201          |
| Jones         | A-217          |
| Lindsay       | A-222          |
| Smith         | A-215          |
| Turner        | A-305          |





### Keys

- Let  $K \subset R$
- $\blacksquare$  K is a **superkey** of R if values for K are sufficient to identify a unique tuple of each possible relation r(R)
  - by "possible r" we mean a relation r that could exist in the enterprise we are modeling.
  - Example: {customer\_name, customer\_street} and {customer\_name}
     are both superkeys of Customer, if no two customers can possibly have
    - In real life, an attribute such as *customer\_id* would be used instead of *customer\_name* to uniquely identify customers, but we omit it to keep our examples small, and instead assume customer names are unique.

the same name



### **Keys (Cont.)**

- K is a candidate key if K is minimal Example: {customer\_name} is a candidate key for Customer, since it is a superkey and no subset of it is a superkey.
- Primary key: a candidate key chosen as the principal means of identifying tuples within a relation
  - Should choose an attribute whose value never, or very rarely, changes.
  - E.g. email address is unique, but may change

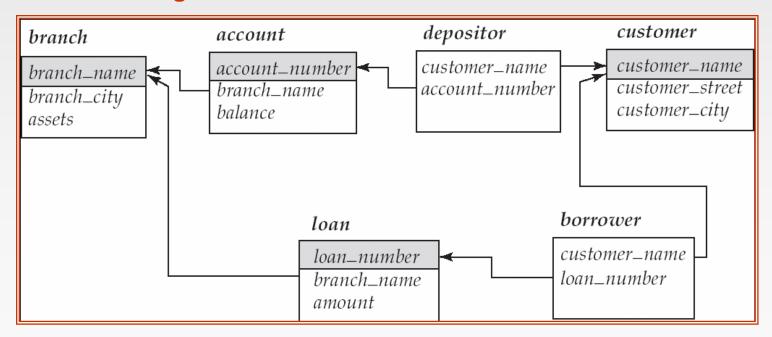




### Foreign Keys

- A relation schema may have an attribute that corresponds to the primary key of another relation. The attribute is called a foreign key.
  - E.g. customer\_name and account\_number attributes of depositor are foreign keys to customer and account respectively.
  - Only values occurring in the primary key attribute of the referenced relation may occur in the foreign key attribute of the referencing relation.

#### Schema diagram







### **Query Languages**

- Language in which user requests information from the database.
- Categories of languages
  - Procedural
  - Non-procedural, or declarative
- "Pure" languages:
  - Relational algebra
  - Tuple relational calculus
  - Domain relational calculus
- Pure languages form underlying basis of query languages that people use.





### **Relational Algebra**

- Procedural language
- Six basic operators
  - select: σ
  - project: ∏
  - union: ∪
  - set difference: –
  - Cartesian product: x
  - rename: ρ
- The operators take one or two relations as inputs and produce a new relation as a result.





# **Select Operation – Example**

Relation r

| Α       | В       | С  | D  |
|---------|---------|----|----|
| α       | α       | 1  | 7  |
| α       | $\beta$ | 5  | 7  |
| β       | $\beta$ | 12 | 3  |
| $\beta$ | β       | 23 | 10 |

$$\bullet$$
  $\sigma_{A=B \land D > 5}(r)$ 

| Α        | В       | С  | D  |
|----------|---------|----|----|
| $\alpha$ | α       | 1  | 7  |
| $\beta$  | $\beta$ | 23 | 10 |





### **Select Operation**

- Notation:  $\sigma_p(r)$
- p is called the selection predicate
- Defined as:

$$\sigma_p(\mathbf{r}) = \{t \mid t \in r \text{ and } p(t)\}$$

Where p is a formula in propositional calculus consisting of **terms** connected by :  $\land$  (**and**),  $\lor$  (**or**),  $\neg$  (**not**) Each **term** is one of:

op or  where op is one of: 
\\$\\$=\\$\\$
,  \\$\neq\\$ , >,  \\$\geq\\$ . <.  \\$\leq\\$

Example of selection:





# **Project Operation – Example**

Relation *r*.

$$\prod_{A,C} (r)$$

$$\begin{array}{|c|c|c|c|c|}
\hline
A & C \\
\hline
\alpha & 1 \\
\alpha & 1 \\
\beta & 1 \\
\hline
\beta & 2 \\
\hline
\end{array}$$



### **Project Operation**

Notation:

$$\prod_{A_1,A_2,\ldots,A_k}(r)$$

where  $A_1$ ,  $A_2$  are attribute names and r is a relation name.

- The result is defined as the relation of *k* columns obtained by erasing the columns that are not listed
- Duplicate rows removed from result, since relations are sets
- Example: To eliminate the branch\_name attribute of account

 $\Pi_{account\_number, balance}$  (account)





# **Union Operation – Example**

Relations *r*, *s*:

| Α        | В |  |
|----------|---|--|
| α        | 1 |  |
| $\alpha$ | 2 |  |
| β        | 1 |  |
| r        |   |  |

| Α | В |  |
|---|---|--|
| α | 2 |  |
| β | 3 |  |
| S |   |  |

 $ightharpoonup r \cup s$ :

$$\begin{array}{c|cc}
A & B \\
\hline
\alpha & 1 \\
\alpha & 2 \\
\beta & 1 \\
\beta & 3 \\
\end{array}$$



### **Union Operation**

- Notation:  $r \cup s$
- Defined as:

$$r \cup s = \{t \mid t \in r \text{ or } t \in s\}$$

- For  $r \cup s$  to be valid.
  - 1. *r*, *s* must have the *same* **arity** (same number of attributes)
  - 2. The attribute domains must be **compatible** (example:  $2^{nd}$  column of r deals with the same type of values as does the  $2^{nd}$  column of s)
- Example: to find all customers with either an account or a loan

$$\Pi_{customer\_name}$$
 (depositor)  $\cup \Pi_{customer\_name}$  (borrower)





# **Set Difference Operation – Example**

Relations *r*, *s*:

| Α | В |  |
|---|---|--|
| α | 1 |  |
| α | 2 |  |
| β | 1 |  |
| r |   |  |

| Α | В |  |
|---|---|--|
| α | 2 |  |
| β | 3 |  |
| S |   |  |

r-s



### **Set Difference Operation**

- Notation r s
- Defined as:

$$r - s = \{t \mid t \in r \text{ and } t \notin s\}$$

- Set differences must be taken between compatible relations.
  - r and s must have the same arity
  - attribute domains of r and s must be compatible





# **Cartesian-Product Operation – Example**

Relations *r*, *s*:

| Α | В |  |
|---|---|--|
| α | 1 |  |
| β | 2 |  |
| r |   |  |

| С   | D                    | E                |
|---|----------------------|------------------|
| $egin{pmatrix} lpha \ eta \ eta \ \gamma \ \end{array}$ | 10<br>10<br>20<br>10 | a<br>a<br>b<br>b |

S

**r** x s:

| Α        | В | С        | D  | Ε |
|----------|---|----------|----|---|
| α        | 1 | α        | 10 | а |
| $\alpha$ | 1 | $\beta$  | 10 | а |
| $\alpha$ | 1 | $\beta$  | 20 | b |
| $\alpha$ | 1 | γ        | 10 | b |
| $\beta$  | 2 | $\alpha$ | 10 | а |
| $\beta$  | 2 | $\beta$  | 10 | а |
| $\beta$  | 2 | $\beta$  | 20 | b |
| $\beta$  | 2 | γ        | 10 | b |



### **Cartesian-Product Operation**

- Notation r x s
- Defined as:

$$r \times s = \{t \mid q \mid t \in r \text{ and } q \in s\}$$

- Assume that attributes of r(R) and s(S) are disjoint. (That is,  $R \cap S = \emptyset$ ).
- If attributes of r(R) and s(S) are not disjoint, then renaming must be used.





### **Composition of Operations**

- Can build expressions using multiple operations
- Example:  $\sigma_{A=C}(r x s)$
- rxs

| Α        | В | С       | D  | E |
|----------|---|---------|----|---|
| α        | 1 | α       | 10 | а |
| $\alpha$ | 1 | $\beta$ | 10 | а |
| $\alpha$ | 1 | $\beta$ | 20 | b |
| $\alpha$ | 1 | γ       | 10 | b |
| $\beta$  | 2 | α       | 10 | а |
| $\beta$  | 2 | $\beta$ | 10 | a |
| $\beta$  | 2 | $\beta$ | 20 | b |
| β        | 2 | γ       | 10 | b |

 $\sigma_{A=C}(r \times s)$ 

| Α                    | В | С       | D  | E |
|----------------------|---|---------|----|---|
| α                    | 1 | α       | 10 | а |
| $oldsymbol{eta}$     | 2 | $\beta$ | 10 | a |
| $\boldsymbol{\beta}$ | 2 | $\beta$ | 20 | b |



### **Rename Operation**

- Allows us to name, and therefore to refer to, the results of relationalalgebra expressions.
- Allows us to refer to a relation by more than one name.
- Example:

$$\rho_X(E)$$

returns the expression *E* under the name *X* 

If a relational-algebra expression E has arity n, then

$$\rho_{_{x(A_{1},A_{2},...,A_{n})}}(E)$$

returns the result of expression E under the name X, and with the attributes renamed to  $A_1$ ,  $A_2$ , ....,  $A_n$ .





### **Banking Example**

branch (branch\_name, branch\_city, assets)

customer (customer\_name, customer\_street, customer\_city)

account (account\_number, branch\_name, balance)

loan (loan\_number, branch\_name, amount)

depositor (customer\_name, account\_number)

borrower (customer\_name, loan\_number)





■ Find all loans of over \$1200

$$\sigma_{amount > 1200}$$
 (loan)

Find the loan number for each loan of an amount greater than \$1200

$$\Pi_{loan\ number} (\sigma_{amount > 1200} (loan))$$

Find the names of all customers who have a loan, an account, or both, from the bank

$$\Pi_{customer\ name}$$
 (borrower)  $\cup \Pi_{customer\ name}$  (depositor)



■ Find the names of all customers who have a loan at the Perryridge branch.

$$\Pi_{customer\_name} (\sigma_{branch\_name="Perryridge"} \\ (\sigma_{borrower.loan\_number=loan.loan\_number} (borrower x \\ loan)))$$

Find the names of all customers who have a loan at the Perryridge branch but do not have an account at any branch of the bank.

$$\Pi_{customer\_name} (\sigma_{branch\_name = "Perryridge"})$$

$$(\sigma_{borrower.loan\_number} = loan.loan\_number (borrower x loan))) - \Pi_{customer\ name} (depositor)$$





- Find the names of all customers who have a loan at the Perryridge branch.
  - Query 1

```
\Pi_{customer\_name} (\sigma_{branch\_name} = "Perryridge" (\sigma_{borrower.loan\_number} = Ioan.loan\_number (borrower x Ioan)))
```

Query 2

```
\Pi_{customer\_name}(\sigma_{loan.loan\_number} = borrower.loan\_number (\sigma_{branch\_name} = "Perryridge" (loan)) x borrower))
```





| customer_name | loan_number |
|---------------|-------------|
| Adams         | L-16        |
| Curry         | L-93        |
| Hayes         | L-15        |
| Jackson       | L-14        |
| Jones         | L-17        |
| Smith         | L-11        |
| Smith         | L-23        |
| Williams      | L-17        |

| loan_number | branch_name | amount |
|-------------|-------------|--------|
| L-11        | Round Hill  | 900    |
| L-14        | Downtown    | 1500   |
| L-15        | Perryridge  | 1500   |
| L-16        | Perryridge  | 1300   |
| L-17        | Downtown    | 1000   |
| L-23        | Redwood     | 2000   |
| L-93        | Mianus      | 500    |

| customer_name | loan_number |
|---------------|-------------|
| Adams         | L-16 \      |
| Curry         | L-93        |
| Hayes         | L-15        |
| Jackson       | L-14        |
| Jones         | L-17        |
| Smith         | L-11        |
| Smith         | L-23        |
| Williams      | L-17        |

| loan_number   | branch_name | amount |
|---------------|-------------|--------|
| <b>√</b> L-11 | Round Hill  | 900    |
| <b>L</b> -14  | Downtown    | 1500   |
| L-15          | Perryridge  | 1500   |
| L-16          | Perryridge  | 1300   |
| L-17          | Downtown    | 1000   |
| L-23          | Redwood     | 2000   |
| L-93          | Mianus      | 500    |





- Find the largest account balance
  - Strategy:
    - Find those balances that are not the largest
      - Rename account relation as d so that we can compare each account balance with all others
    - Use set difference to find those account balances that were not found in the earlier step.
  - The query is:

$$\Pi_{balance}(account) - \Pi_{account.balance}$$
 
$$(\sigma_{account.balance} < \textit{d.balance} (account \times \rho_d (account)))$$





#### **Formal Definition**

- A basic expression in the relational algebra consists of either one of the following:
  - A relation in the database
  - A constant relation
- Let  $E_1$  and  $E_2$  be relational-algebra expressions; the following are all relational-algebra expressions:
  - $E_1 \cup E_2$
  - $E_1 E_2$
  - $E_1 \times E_2$
  - $\sigma_p(E_1)$ , P is a predicate on attributes in  $E_1$
  - $\prod_{S}(E_1)$ , S is a list consisting of some of the attributes in  $E_1$
  - $\rho_X(E_1)$ , x is the new name for the result of  $E_1$





### **Additional Operations**

We define additional operations that do not add any power to the relational algebra, but that simplify common queries.

- Set intersection
- Natural join
- Division
- Assignment





#### **Set-Intersection Operation**

- Notation:  $r \cap s$
- Defined as:
- $r \cap s = \{ t \mid t \in r \text{ and } t \in s \}$
- Assume:
  - r, s have the same arity
  - attributes of *r* and *s* are compatible
- Note:  $r \cap s = r (r s)$





### **Set-Intersection Operation – Example**

Relation *r*, *s*:

| Α | В |
|---|---|
| α | 1 |
| α | 2 |
| β | 1 |

A B α 2 β 3

r

S

 $r \cap s$ 



#### **Natural-Join Operation**

- Notation: r ⋈ s
- Let r and s be relations on schemas R and S respectively. Then,  $r \bowtie s$  is a relation on schema  $R \cup S$  obtained as follows:
  - Consider each pair of tuples t<sub>r</sub> from r and t<sub>s</sub> from s.
  - If  $t_r$  and  $t_s$  have the same value on each of the attributes in  $R \cap S$ , add a tuple t to the result, where
    - t has the same value as  $t_r$  on r
    - t has the same value as t<sub>S</sub> on s
- Example:

$$R = (A, B, C, D)$$

$$S = (E, B, D)$$

- Result schema = (A, B, C, D, E)
- $r \bowtie s$  is defined as:

$$\prod_{r.A, r.B, r.C, r.D, s.E} (\sigma_{r.B=s.B} \land_{r.D=s.D} (r \times s))$$



## **Natural Join Operation – Example**

Relations r, s:

| Α        | В | С       | D |
|----------|---|---------|---|
| α        | 1 | α       | а |
| $\beta$  | 2 | γ       | а |
| γ        | 4 | $\beta$ | b |
| $\alpha$ | 1 | γ       | а |
| $\delta$ | 2 | $\beta$ | b |
| r        |   |         |   |

| В | D | E        |  |
|---|---|----------|--|
| 1 | а | α        |  |
| 3 | a | $\beta$  |  |
| 1 | a | γ        |  |
| 2 | b | $\delta$ |  |
| 3 | b | $\in$    |  |
| S |   |          |  |

■ r⋈s

| A        | В | С        | D | E        |
|----------|---|----------|---|----------|
| α        | 1 | α        | а | α        |
| $\alpha$ | 1 | $\alpha$ | а | γ        |
| $\alpha$ | 1 | γ        | а | $\alpha$ |
| $\alpha$ | 1 | γ        | а | γ        |
| $\delta$ | 2 | $\beta$  | b | $\delta$ |



#### **Division Operation**

- Notation:  $r \div S$
- Suited to queries that include the phrase "for all".
- Let r and s be relations on schemas R and S respectively where

• 
$$R = (A_1, ..., A_m, B_1, ..., B_n)$$

• 
$$S = (B_1, ..., B_n)$$

The result of  $r \div s$  is a relation on schema

$$R - S = (A_1, ..., A_m)$$

$$r \div s = \{ t \mid t \in \prod_{R-S} (r) \land \forall u \in s (tu \in r) \}$$

Where *tu* means the concatenation of tuples *t* and *u* to produce a single tuple





## **Division Operation – Example**

Relations *r*, *s*:

| Α  | В  |
|--|--|
| $\begin{array}{cccccccccccccccccccccccccccccccccccc$ | 1<br>2<br>3<br>1<br>1<br>1<br>3<br>4<br>6<br>1 |
| β  | 2  |
|  | _  |

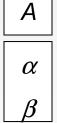
 B

 1

 2

 s

 $r \div s$ :







## **Another Division Example**

Relations *r*, *s*:

| Α  | В | С                | D | E |
|--|---|------------------|---|---|
| α  | а | α                | а | 1 |
| $\begin{array}{ c c c c c c c c c c c c c c c c c c c$ | а | γ                | а | 1 |
| $\alpha$   | а | γ                | b | 1 |
| $\beta$  | а | γ                | а | 1 |
| $\beta$ $\beta$ $\gamma$                               | a | γ<br>γ<br>γ<br>γ | b | 3 |
| γ  | а | γ                | а | 1 |
| γ  | а | γ                | b | 1 |
| γ  | а | $\beta$          | b | 1 |
| r  |   |                  |   |   |

D E
a 1
b 1

 $r \div s$ :

| Α        | В | С        |
|----------|---|----------|
| α        | а | γ        |
| $\gamma$ | a | $\gamma$ |



#### **Division Operation (Cont.)**

- Property
  - Let  $q = r \div s$
  - Then q is the largest relation satisfying  $q \times s \subseteq r$
- Definition in terms of the basic algebra operation Let r(R) and s(S) be relations, and let  $S \subseteq R$

$$r \div s = \prod_{R-S} (r) - \prod_{R-S} ((\prod_{R-S} (r) \times s) - \prod_{R-S,S} (r))$$

#### To see why

- $\prod_{R-S,S}(r)$  simply **reorders** attributes of r
- $\prod_{R-S} (r) \times s$ : all possible pairs between  $_{R-S} (r)$  and s
- $\prod_{R-S}(r) \times s \prod_{R-S,S}(r)$ : those pairs that do not qualify, i.e., do not appear in r(R).
- $\prod_{R-S} (\prod_{R-S} (r) \times s) \prod_{R-S,S} (r)$  gives those tuples t in  $\prod_{R-S} (r)$  such that for some tuple  $u \in s$ ,  $tu \notin r$ .





#### **Assignment Operation**

- The assignment operation (←) provides a convenient way to express complex queries.
  - Write query as a sequential program consisting of
    - a series of assignments
    - followed by an expression whose value is displayed as a result of the query.
  - Assignment must always be made to a temporary relation variable.
- **Example:** Write  $r \div s$  as

$$temp1 \leftarrow \prod_{R-S} (r)$$
  
 $temp2 \leftarrow \prod_{R-S} ((temp1 \times s) - \prod_{R-S,S} (r))$   
 $result = temp1 - temp2$ 

- The result to the right of the ← is assigned to the relation variable on the left of the ←.
- May use variable in subsequent expressions.





#### **Bank Example Queries**

Find the names of all customers who have a loan and an account at bank.

$$\Pi_{customer\_name}$$
 (borrower)  $\cap \Pi_{customer\_name}$  (depositor)

Find the name of all customers who have a loan at the bank and the loan amount

$$\Pi_{customer\_name, loan\_number, amount}$$
 (borrower  $\bowtie$  loan)





#### **Bank Example Queries**

- Find all customers who have an account from at least the "Downtown" and the Uptown" branches.
  - Query 1

$$\Pi_{customer\_name}$$
 ( $\sigma_{branch\_name = "Downtown"}$  (depositor  $\bowtie$  account))  $\cap$   $\Pi_{customer\_name}$  ( $\sigma_{branch\_name = "Uptown"}$  (depositor  $\bowtie$  account))

Query 2

```
\Pi_{customer\_name, \ branch\_name} (depositor \bowtie account) + \rho_{temp(branch\_name)} ({("Downtown"), ("Uptown")})
```

Note that Query 2 uses a constant relation.





#### **Bank Example Queries**

■ Find all customers who have an account at all branches located in Brooklyn city.

$$\prod_{customer\_name, \ branch\_name} (depositor \bowtie \ account)$$

$$\div \prod_{branch\_name} (\sigma_{branch\_city = \text{``Brooklyn''}} (branch))$$





### **Extended Relational-Algebra-Operations**

- Generalized Projection
- Aggregate Functions
- Outer Join





#### **Generalized Projection**

Extends the projection operation by allowing arithmetic functions to be used in the projection list.

$$\prod_{F_1,F_2},...,F_n(E)$$

- E is any relational-algebra expression
- Each of  $F_1$ ,  $F_2$ , ...,  $F_n$  are are arithmetic expressions involving constants and attributes in the schema of E.
- Given relation credit\_info(customer\_name, limit, credit\_balance), find how much more each person can spend:

 $\Pi_{customer\_name, \ limit-credit\_balance}$  (credit\_info)





#### **Aggregate Functions and Operations**

Aggregation function takes a collection of values and returns a single value as a result.

> avg: average value min: minimum value max: maximum value sum: sum of values

count: number of values

Aggregate operation in relational algebra

$$_{G_1,G_2,...,G_n} \vartheta_{F_1(A_1),F_2(A_2,...,F_n(A_n)}(E)$$
  $E$  is any relational-algebra expression

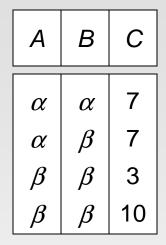
- $G_1, G_2 ..., G_n$  is a list of attributes on which to group (can be empty)
- Each F<sub>i</sub> is an aggregate function
- Each A; is an attribute name





# **Aggregate Operation – Example**

Relation *r*.



 $\mathbf{g}_{\mathbf{sum}(\mathbf{c})}(\mathbf{r})$ 

sum(c)

27



## **Aggregate Operation – Example**

Relation *account* grouped by *branch-name*:

| branch_name | account_number | balance |
|-------------|----------------|---------|
| Perryridge  | A-102          | 400     |
| Perryridge  | A-201          | 900     |
| Brighton    | A-217          | 750     |
| Brighton    | A-215          | 750     |
| Redwood     | A-222          | 700     |

 $branch_name \ \mathcal{G}_{sum(balance)} \ (account)$ 

| branch_name | <b>sum</b> ( <i>balance</i> ) |
|-------------|-------------------------------|
| Perryridge  | 1300                          |
| Brighton    | 1500                          |
| Redwood     | 700                           |





#### **Aggregate Functions (Cont.)**

- Result of aggregation does not have a name
  - Can use rename operation to give it a name
  - For convenience, we permit renaming as part of aggregate operation

branch\_name 9 sum(balance) as sum\_balance (account)





#### **Outer Join**

- An extension of the join operation that avoids loss of information.
- Computes the join and then adds tuples form one relation that does not match tuples in the other relation to the result of the join.
- Uses null values:
  - null signifies that the value is unknown or does not exist
  - All comparisons involving null are (roughly speaking) false by definition.
    - We shall study precise meaning of comparisons with nulls later





## **Outer Join – Example**

#### Relation loan

| loan_number | branch_name | amount |
|-------------|-------------|--------|
| L-170       | Downtown    | 3000   |
| L-230       | Redwood     | 4000   |
| L-260       | Perryridge  | 1700   |

#### Relation borrower

| customer_name | loan_number |
|---------------|-------------|
| Jones         | L-170       |
| Smith         | L-230       |
| Hayes         | L-155       |





### Outer Join – Example

#### Join

*loan* ⋈ *borrower* 

| loan_number | branch_name | amount | customer_name |
|-------------|-------------|--------|---------------|
| L-170       | Downtown    | 3000   | Jones         |
| L-230       | Redwood     | 4000   | Smith         |

#### ■ Left Outer Join

| loan_number | branch_name | amount | customer_name |
|-------------|-------------|--------|---------------|
| L-170       | Downtown    | 3000   | Jones         |
| L-230       | Redwood     | 4000   | Smith         |
| L-260       | Perryridge  | 1700   | null          |



### Outer Join – Example

#### Right Outer Join

*loan* ⋈ *borrower* 

| loan_number | branch_name | amount | customer_name |
|-------------|-------------|--------|---------------|
| L-170       | Downtown    | 3000   | Jones         |
| L-230       | Redwood     | 4000   | Smith         |
| L-155       | null        | null   | Hayes         |

#### ■ Full Outer Join

*loan*⊐⊠ *borrower* 

| loan_number | branch_name | amount | customer_name |
|-------------|-------------|--------|---------------|
| L-170       | Downtown    | 3000   | Jones         |
| L-230       | Redwood     | 4000   | Smith         |
| L-260       | Perryridge  | 1700   | null          |
| L-155       | null        | null   | Hayes         |





#### **Null Values**

- It is possible for tuples to have a null value, denoted by *null*, for some of their attributes
- null signifies an unknown value or that a value does not exist.
- The result of any **arithmetic expression involving** *null* is *null*.
- Aggregate functions simply ignore null values (as in SQL)
- For duplicate elimination and grouping, null is treated like any other value, and two nulls are assumed to be the same (as in SQL)





#### **Null Values**

- Comparisons with null values return the special truth value: unknown
  - If *false* was used instead of *unknown*, then not (A < 5) would not be equivalent to A >= 5
- Three-valued logic using the truth value *unknown*:
  - OR: (unknown or true) = true,
     (unknown or false) = unknown
     (unknown or unknown) = unknown
  - AND: (true and unknown) = unknown, (false and unknown) = false, (unknown and unknown) = unknown
  - NOT: (not unknown) = unknown
  - In SQL "P is unknown" evaluates to true if predicate P evaluates to unknown
- Result of select predicate is treated as false if it evaluates to unknown





#### **Modification of the Database**

- The content of the database may be modified using the following operations:
  - Deletion
  - Insertion
  - Updating
- All these operations are expressed using the assignment operator.





#### **Deletion**

- A delete request is expressed similarly to a query, except instead of displaying tuples to the user, the selected tuples are removed from the database.
- Can delete only whole tuples; cannot delete values on only particular attributes
- A deletion is expressed in relational algebra by:

$$r \leftarrow r - E$$

where r is a relation and E is a relational algebra query.





#### **Deletion Examples**

■ Delete all account records in the Perryridge branch.

$$account - \sigma_{branch\_name = "Perryridge"}(account)$$

Delete all loan records with amount in the range of 0 to 50

loan ← loan − 
$$\sigma$$
 amount ≥ 0 and amount ≤ 50 (loan)

Delete all accounts at branches located in Needham.

$$r_1 \leftarrow \sigma_{branch\_city} = \text{``Needham''} (account \bowtie branch)$$
 $r_2 \leftarrow \Pi_{account\_number, branch\_name, balance} (r_1)$ 
 $r_3 \leftarrow \Pi_{customer\_name, account\_number} (r_2 \bowtie depositor)$ 
 $account \leftarrow account - r_2$ 
 $depositor \leftarrow depositor - r_3$ 





#### Insertion

- To insert data into a relation, we either:
  - specify a tuple to be inserted
  - write a query whose result is a set of tuples to be inserted
- in relational algebra, an insertion is expressed by:

$$r \leftarrow r \cup E$$

where r is a relation and E is a relational algebra expression.

■ The insertion of a single tuple is expressed by letting *E* be a constant relation containing one tuple.



#### **Insertion Examples**

Insert information in the database specifying that Smith has \$1200 in account A-973 at the Perryridge branch.

```
account \leftarrow account \cup \{(\text{``A-973''}, \text{``Perryridge''}, 1200)\}
depositor \leftarrow depositor \cup \{(\text{``Smith''}, \text{``A-973''})\}
```

Provide as a gift for all loan customers in the Perryridge branch, a \$200 savings account. Let the loan number serve as the account number for the new savings account.

```
r_1 \leftarrow (\sigma_{branch\_name = "Perryridge"}(borrower \bowtie loan))
account \leftarrow account \cup \prod_{loan\_number, branch\_name, 200}(r_1)
depositor \leftarrow depositor \cup \prod_{customer\_name, loan\_number}(r_1)
```





#### **Updating**

- A mechanism to change a value in a tuple without charging all values in the tuple
- Use the generalized projection operator to do this task

$$r \leftarrow \prod_{F_1,F_2,\dots,F_L}(r)$$

- Each F<sub>i</sub> is either
  - the I th attribute of r, if the I th attribute is not updated, or,
  - if the attribute is to be updated F<sub>i</sub> is an expression, involving only constants and the attributes of *r*, which gives the new value for the attribute



#### **Update Examples**

Make interest payments by increasing all balances by 5 percent.

$$account \leftarrow \prod_{account\_number, branch\_name, balance * 1.05} (account)$$

Pay all accounts with balances over \$10,000 6 percent interest and pay all others 5 percent

$$\begin{array}{l} \textit{account} \leftarrow \prod_{\textit{account\_number, branch\_name, balance} * 1.06} (\sigma_{\textit{BAL} > 10000}(\textit{account}\,)) \\ \cup \prod_{\textit{account\_number, branch\_name, balance} * 1.05} (\sigma_{\textit{BAL} \leq 10000}(\textit{account}\,)) \end{array}$$





### **End of Chapter 2**

**Database System Concepts**, 5<sup>th</sup> Ed.

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## Figure 2.3. The branch relation

| branch_name | branch_city | assets  |
|-------------|-------------|---------|
| Brighton    | Brooklyn    | 7100000 |
| Downtown    | Brooklyn    | 9000000 |
| Mianus      | Horseneck   | 400000  |
| North Town  | Rye         | 3700000 |
| Perryridge  | Horseneck   | 1700000 |
| Pownal      | Bennington  | 300000  |
| Redwood     | Palo Alto   | 2100000 |
| Round Hill  | Horseneck   | 8000000 |





## Figure 2.6: The *loan* relation

| loan_number   | branch_name | amount |
|---------------|-------------|--------|
| L-11          | Round Hill  | 900    |
| L-14          | Downtown    | 1500   |
| L-15          | Perryridge  | 1500   |
| L-16          | Perryridge  | 1300   |
| L-17          | Downtown    | 1000   |
| L <b>-2</b> 3 | Redwood     | 2000   |
| L-93          | Mianus      | 500    |





## Figure 2.7: The borrower relation

| customer_name | loan_number |
|---------------|-------------|
| Adams         | L-16        |
| Curry         | L-93        |
| Hayes         | L-15        |
| Jackson       | L-14        |
| Jones         | L-17        |
| Smith         | L-11        |
| Smith         | L-23        |
| Williams      | L-17        |





Figure 2.9 Result of  $\sigma_{branch\_name = "Perryridge"}$  (loan)

| loan_number | branch_name | amount |
|-------------|-------------|--------|
| L-15        | Perryridge  | 1500   |
| L-16        | Perryridge  | 1300   |





# Figure 2.10: Loan number and the amount of the loan

| loan_number | amount |
|-------------|--------|
| L-11        | 900    |
| L-14        | 1500   |
| L-15        | 1500   |
| L-16        | 1300   |
| L-17        | 1000   |
| L-23        | 2000   |
| L-93        | 500    |





### Figure 2.11: Names of all customers who have either an account or an loan

customer\_name

Adams

Curry

Hayes

Jackson

Jones

Smith

Williams

Lindsay

Johnson

Turner





### Figure 2.12: Customers with an account but no loan

customer\_name

Johnson Lindsay Turner





#### Figure 2.13: Result of borrower |X| loan

|               | borrower.   | loan.       |             |        |
|---------------|-------------|-------------|-------------|--------|
| customer_name | loan_number | loan_number | branch_name | amount |
| Adams         | L-16        | L-11        | Round Hill  | 900    |
| Adams         | L-16        | L-14        | Downtown    | 1500   |
| Adams         | L-16        | L-15        | Perryridge  | 1500   |
| Adams         | L-16        | L-16        | Perryridge  | 1300   |
| Adams         | L-16        | L-17        | Downtown    | 1000   |
| Adams         | L-16        | L-23        | Redwood     | 2000   |
| Adams         | L-16        | L-93        | Mianus      | 500    |
| Curry         | L-93        | L-11        | Round Hill  | 900    |
| Curry         | L-93        | L-14        | Downtown    | 1500   |
| Curry         | L-93        | L-15        | Perryridge  | 1500   |
| Curry         | L-93        | L-16        | Perryridge  | 1300   |
| Curry         | L-93        | L-17        | Downtown    | 1000   |
| Curry         | L-93        | L-23        | Redwood     | 2000   |
| Curry         | L-93        | L-93        | Mianus      | 500    |
| Hayes         | L-15        | L-11        |             | 900    |
| Hayes         | L-15        | L-14        |             | 1500   |
| Hayes         | L-15        | L-15        |             | 1500   |
| Hayes         | L-15        | L-16        |             | 1300   |
| Hayes         | L-15        | L-17        |             | 1000   |
| Hayes         | L-15        | L-23        |             | 2000   |
| Hayes         | L-15        | L-93        |             | 500    |
|               |             |             |             |        |
|               |             |             | • • • •     |        |
|               |             |             |             | •••    |
| Smith         | L-23        | L-11        | Round Hill  | 900    |
| Smith         | L-23        | L-14        | Downtown    | 1500   |
| Smith         | L-23        | L-15        | Perryridge  | 1500   |
| Smith         | L-23        | L-16        | Perryridge  | 1300   |
| Smith         | L-23        | L-17        | Downtown    | 1000   |
| Smith         | L-23        | L-23        | Redwood     | 2000   |
| Smith         | L-23        | L-93        | Mianus      | 500    |
| Williams      | L-17        | L-11        | Round Hill  | 900    |
| Williams      | L-17        | L-14        | Downtown    | 1500   |
| Williams      | L-17        | L-15        | Perryridge  | 1500   |
| Williams      | L-17        | L-16        | Perryridge  | 1300   |
| Williams      | L-17        | L-17        | Downtown    | 1000   |
| Williams      | L-17        | L-23        | Redwood     | 2000   |
| Williams      | L-17        | L-93        | Mianus      | 500    |





|               | borrower.   | loan.       |             |        |
|---------------|-------------|-------------|-------------|--------|
| customer_name | loan_number | loan_number | branch_name | amount |
| Adams         | L-16        | L-15        | Perryridge  | 1500   |
| Adams         | L-16        | L-16        | Perryridge  | 1300   |
| Curry         | L-93        | L-15        | Perryridge  | 1500   |
| Curry         | L-93        | L-16        | Perryridge  | 1300   |
| Hayes         | L-15        | L-15        | Perryridge  | 1500   |
| Hayes         | L-15        | L-16        | Perryridge  | 1300   |
| Jackson       | L-14        | L-15        | Perryridge  | 1500   |
| Jackson       | L-14        | L-16        | Perryridge  | 1300   |
| Jones         | L-17        | L-15        | Perryridge  | 1500   |
| Jones         | L-17        | L-16        | Perryridge  | 1300   |
| Smith         | L-11        | L-15        | Perryridge  | 1500   |
| Smith         | L-11        | L-16        | Perryridge  | 1300   |
| Smith         | L-23        | L-15        | Perryridge  | 1500   |
| Smith         | L-23        | L-16        | Perryridge  | 1300   |
| Williams      | L-17        | L-15        | Perryridge  | 1500   |
| Williams      | L-17        | L-16        | Perryridge  | 1300   |





customer\_name

Adams Hayes





balance

500

400

700

750

350





## Figure 2.17 Largest account balance in the bank

balance 900





# Figure 2.18: Customers who live on the same street and in the same city as Smith

customer\_name

Curry Smith





### Figure 2.19: Customers with both an account and a loan at the bank

customer\_name

Hayes Jones Smith





| customer_name | loan_number | amount |
|---------------|-------------|--------|
| Adams         | L-16        | 1300   |
| Curry         | L-93        | 500    |
| Hayes         | L-15        | 1500   |
| Jackson       | L-14        | 1500   |
| Jones         | L-17        | 1000   |
| Smith         | L-23        | 2000   |
| Smith         | L-11        | 900    |
| Williams      | L-17        | 1000   |





branch\_name

Brighton Perryridge





branch\_name

Brighton Downtown





| customer_name | branch_name |
|---------------|-------------|
| Hayes         | Perryridge  |
| Johnson       | Downtown    |
| Johnson       | Brighton    |
| Jones         | Brighton    |
| Lindsay       | Redwood     |
| Smith         | Mianus      |
| Turner        | Round Hill  |





#### Figure 2.24: The credit\_info relation

| customer_name | limit | credit_balance |
|---------------|-------|----------------|
| Curry         | 2000  | 1750           |
| Hayes         | 1500  | 1500           |
| Jones         | 6000  | 700            |
| Smith         | 2000  | 400            |





| customer_name | credit_available |
|---------------|------------------|
| Curry         | 250              |
| Jones         | 5300             |
| Smith         | 1600             |
| Hayes         | 0                |





#### Figure 2.26: The pt\_works relation

| employee_name | branch_name | salary |
|---------------|-------------|--------|
| Adams         | Perryridge  | 1500   |
| Brown         | Perryridge  | 1300   |
| Gopal         | Perryridge  | 5300   |
| Johnson       | Downtown    | 1500   |
| Loreena       | Downtown    | 1300   |
| Peterson      | Downtown    | 2500   |
| Rao           | Austin      | 1500   |
| Sato          | Austin      | 1600   |





# Figure 2.27 The pt\_works relation after regrouping

| employee_name | branch_name | salary |
|---------------|-------------|--------|
| Rao           | Austin      | 1500   |
| Sato          | Austin      | 1600   |
| Johnson       | Downtown    | 1500   |
| Loreena       | Downtown    | 1300   |
| Peterson      | Downtown    | 2500   |
| Adams         | Perryridge  | 1500   |
| Brown         | Perryridge  | 1300   |
| Gopal         | Perryridge  | 5300   |





| branch_name | sum of salary |
|-------------|---------------|
| Austin      | 3100          |
| Downtown    | 5300          |
| Perryridge  | 8100          |





| branch_name | sum_salary | max_salary |
|-------------|------------|------------|
| Austin      | 3100       | 1600       |
| Downtown    | 5300       | 2500       |
| Perryridge  | 8100       | 5300       |





# Figure 2.30 The employee and ft\_works relations

| employee_name | street   | city         |
|---------------|----------|--------------|
| Coyote        | Toon     | Hollywood    |
| Rabbit        | Tunnel   | Carrotville  |
| Smith         | Revolver | Death Valley |
| Williams      | Seaview  | Seattle      |
|               |          |              |

| employee_name | branch_name | salary |
|---------------|-------------|--------|
| Coyote        | Mesa        | 1500   |
| Rabbit        | Mesa        | 1300   |
| Gates         | Redmond     | 5300   |
| Williams      | Redmond     | 1500   |





| employee_name | street  | city        | branch_name | salary |
|---------------|---------|-------------|-------------|--------|
| Coyote        | Toon    | Hollywood   | Mesa        | 1500   |
| Rabbit        | Tunnel  | Carrotville | Mesa        | 1300   |
| Williams      | Seaview | Seattle     | Redmond     | 1500   |



| employee_name | street   | city         | branch_name | salary |
|---------------|----------|--------------|-------------|--------|
| Coyote        | Toon     | Hollywood    | Mesa        | 1500   |
| Rabbit        | Tunnel   | Carrotville  | Mesa        | 1300   |
| Williams      | Seaview  | Seattle      | Redmond     | 1500   |
| Smith         | Revolver | Death Valley | null        | null   |



| employee_name | street  | city        | branch_name | salary |
|---------------|---------|-------------|-------------|--------|
| Coyote        | Toon    | Hollywood   | Mesa        | 1500   |
| Rabbit        | Tunnel  | Carrotville | Mesa        | 1300   |
| Williams      | Seaview | Seattle     | Redmond     | 1500   |
| Gates         | null    | null        | Redmond     | 5300   |





| employee_name | street   | city         | branch_name | salary |
|---------------|----------|--------------|-------------|--------|
| Coyote        | Toon     | Hollywood    | Mesa        | 1500   |
| Rabbit        | Tunnel   | Carrotville  | Mesa        | 1300   |
| Williams      | Seaview  | Seattle      | Redmond     | 1500   |
| Smith         | Revolver | Death Valley | null        | null   |
| Gates         | null     | null         | Redmond     | 5300   |