$$\forall X > 0. \quad \forall t > 0$$

$$P(X > t) \le \frac{E[X]}{t}.$$

$$E[X] = \int_0^{+\infty} x \cdot p(x)\, dx$$

$$= \underbrace{\int_0^t x\, p(x)\, dx}_{\geq 0} + \int_t^{\infty} x\, p(x)\, dx$$

$$\geq 0 + \underbrace{\int_t^{\infty} x\, p(x)\, dx}_{x \geq t}$$

$$\geq \int_t^{\infty} t \cdot p(x)\, dx$$

$$= t \cdot \underbrace{\int_t^{\infty} p(x)\, dx}_{P(X \geq t)} \qquad \square$$

(2) $\boxed{P(X \geq t' \cdot E[X]) \leq \frac{1}{t'} \quad \forall t' > 0}$ = (2)

(1) $\Leftrightarrow$ (2)

t in (1), $t = t' \cdot E[X]$ ...

(1) $\boxed{P(X \geq t) \leq \frac{E[X]}{t}}$ ... (1)

Simplest dist. X = constant

X	Pr(x)
0	$1 - 1/t'$
1	$1/t'$

$= \frac{1}{t'}$

LHS $= P(X \geq t' \cdot \frac{1}{t'}) = P(X \geq 1) = P(X=1)$

$$P(X=1) = \frac{1}{2}$$

$$P(X=-1) = \frac{1}{2}.$$

Hoeffding's

$$P\left(\sum_{i=1}^{n} X_i \geq t\right) \leq e^{-t^2/2n}$$

Pf. $\underbrace{LHS}$ $\leq e^{\frac{n}{2}\lambda^2 - t\lambda}$ $\quad \lambda > 0.$

$$= P(\lambda \cdot \Sigma X_i \geq \lambda t) \quad \underset{=}{\lambda > 0.}$$

$$= P(\underbrace{e^{\lambda \cdot \Sigma X_i}} \geq e^{\lambda t}) \quad (e^x \uparrow)$$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad \uparrow > 0$

$$\overset{M's}{\leq} \frac{E[e^{\lambda \cdot \Sigma X_i}]}{e^{\lambda t}} \leq \frac{e^{\frac{n\lambda^2}{2}}}{e^{\lambda t}}$$

$$= e^{\frac{n}{2}\lambda^2 - \lambda t}$$

$$\text{num} = E\left[e^{\lambda \cdot \sum X_i}\right]$$

$x_1 \quad x_2$

$$= E\left[\prod_{i=1}^{n} e^{\lambda X_i}\right]$$

$E[X_1 X_2]$

$= E[X_1] \cdot E[X_2]$

$$= \prod_{i=1}^{n} E\left[e^{\lambda X_i}\right]$$

$$\le \prod_{i=1}^{n} e^{\frac{\lambda^2}{2}} = \underline{e^{\frac{n\lambda^2}{2}}}$$

$$E\left[\underline{e^{\lambda X_i}}\right] = e^{\lambda} \cdot \frac{1}{2} + e^{-\lambda} \cdot \frac{1}{2}$$

MGF of X_i

$$= \frac{1}{2}\left(e^{\lambda} + e^{-\lambda}\right)$$

$$\le \underline{e^{\lambda^2/2}}$$

Expert 1 : $\leq \$1000$

2 : $\leq \$500$

3 : $\leq \$100$

$$LHS \leq e^{\frac{n}{2}\lambda^2 - t\lambda} \quad \lambda > 0.$$

λ. s.t. RHS is minimized.

$$n \cdot \lambda - t = 0$$

$$\lambda^* = t/n$$

$$RHS = e^{-t^2/2n}.$$

$\forall$ $X_1, \ldots, X_n$ independent.

$$P(X_i = +1) = 1/2$$

$$P(X_i = -1) = 1/2$$

then:

$$P\left(\sum X_i \geq t\right) \leq e^{-t^2/2n}$$

$\forall\ t > 0.$

$\forall\ a_i \leq X_i \leq b_i$

$$\boxed{P\left(\left|\sum X_i - E[X_i]\right| \geq t\right) \leq 2e^{-\frac{t^2}{\sum (b_i - a_i)^2}}}$$

$X_i = +1$, w.p. 0.6

-1. w.p. 0.4

$S = \sum X_i \geq 100$

$$\boxed{\begin{aligned} E[S] &= 0.2n \\ a_i = -1, &\quad b_i = 1 \end{aligned}}$$

$$P(|S - 0.2n| > t)$$

$$\leq 2 \cdot e^{-\frac{t^2}{n \cdot 4}}$$

w.p. $\underbrace{1 - 2 \cdot e^{-t^2/4n}}_{0.99}$

$$\underbrace{0.2n - t}_{100 = 1} \leq S \leq 0.2n + t.$$

$n > 6500$

$$\begin{cases} 0.2n - t = 100 \\ 2 \cdot e^{-t^2/4n} = 0.01 \end{cases}$$

$n = ?$

$$\begin{cases} 0.2n - t = 100 & \text{---} ① \\ 2 e^{-t^2/4n^2} = 0.01 & \text{---} ② \end{cases}$$

$$② \Rightarrow \frac{1}{2} e^{t^2/4n} = 100$$

$$t^2/4n = \ln 200$$

$$t^2 = (\ln 200) \cdot 4n$$

$$\approx 21 n$$

$$① \Rightarrow t = 0.2n - 100 \qquad \boxed{\approx 1000}$$

~~n≈198~~

$$\Rightarrow (0.2n - 100)^2 = 21n$$

$$0.04n^2 + 10000 - 40n = 21n$$

$$0.04n^2 - 61n + 10000 = 0$$

$$n = \frac{61 + \sqrt{61^2 - 1600}}{0.08} \approx \frac{61 + 45}{0.08}$$

img

label = ?

+1

−1

$X_1 \quad X_2 \quad \cdots \quad X_n \rightarrow 250$

$f(X_1 \cdots X_n)$

$X_i \in \{+1, -1\}$

① make use n labels

② $n = ?$ maj-vote is correct w.p. 0.99.

$+ \; + \; + \rightarrow +$

$+ \; + \; - \rightarrow - / +$

5, $+ \; + \; + \; + \; - \quad +$.

500.

400 $+$

100 $-$

$+$

+

$f(x_1, \ldots, x_n) = \sum x_i \geq 1$

$+ + + + - = 3$

$+ + + - - = 2$

$+ + - - - = -1$

$\vdots$

$n = ?$ w.p. 0.99

$\underline{\sum x_i \geq 1}$ $\underline{n = 250}$

60% good

40%

$$\begin{cases} P(X_i = +1) = 0.6 \\ P(X_i = -1) = 0.4 \end{cases}$$

N images

Goal: all of them are correctly annotated, wp. 0.99

img → 250 → ✓ wp. 0.99

im1 → ~~250~~ 280 → ✓ 0.99 → 0.999

IM2 → ~~250~~ 280 ✓ ~~0.99~~ → 0.999

0.999 × 0.999 = ~~0.98~~ → 0.99

1 $\xrightarrow{320}$ 0.99

2 320 0.99

3 320 0.99

$(0.99)^3 = 0.96$

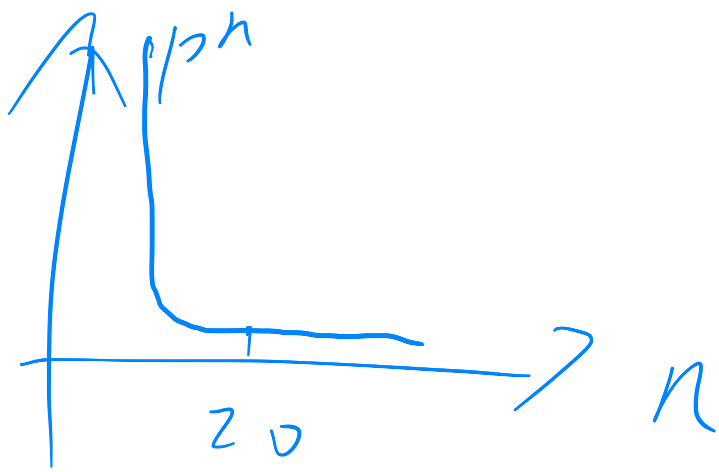
Fix 500

0.99999

$p < 1$.

$1 \to n$.

$p^n \to 0$



500

$\# \text{worker/img} = A \log_a N$

N: size of data set.

Online Learning

Learning from experts.

$1, \ldots, n.$

round 1: $\underbrace{+ \quad + \quad - \quad - \quad \ldots \quad +}_{n}$

	1	2 →	score	
r1,	+	−	1	1/2
	+	+	1	1/2
	+	−	1	1/4
	+	−	1	1/8
	−	+	1/2	1/8

$1, \dots\dots n.$ $\quad \beta \in (0,1)$

$1/n \quad \dots\dots \quad 1/n \dots$

$w_1^1 \quad \dots\dots \quad w_n^1.$

$t = 1, 2, \dots, T.$

$l^t \triangleq \quad l_1^t \quad l_2^t \quad \dots \quad l_n^t \in [0,1]$

$$w_i^{t+1} \leftarrow w_i^t \cdot f(l_i) \qquad \beta^{l_i}$$

Thm: $L_A = \sum_{t=1}^{T} \langle w^t, l^t \rangle$ loss at t-th iter.

$$L_i = \sum_{t=1}^{T} l_i^t$$

$$\frac{1}{T} L_A \le \frac{1}{T} L_{i^*} + \frac{\log n}{T} \rightarrow 0 \quad \forall i$$

$1, \quad \ldots \quad n.$

$w_1 \quad \ldots \quad w_n.$

$1 \quad 2 \quad 3.$

$1/2 \quad 1/4 \quad 1/8$

$\cancel{\langle w^t, \ell^t \rangle}$

$$= \mathbb{E}_{i \sim w^t} [\ell_i^t]$$

$$= \ell_1^t \cdot w_1^t + \ell_2^t \cdot w_2^t + \ldots \ell_n^t \cdot w_n^t$$

$$= \langle w^t, \ell^t \rangle$$

1 2

1/2 1/2

1/2 1/4

2/3 1/3

3/4

1/24