1 Equity

CAPM

$$\mathbb{E}[r_{it}] - r_f = \beta_{iM}\left(\mathbb{E}[r_{Mt}] - r_f\right)$$

Assumptions: mean-variance agents, one period investment horizon, no information asymmetry, risk-free rate r_f, only financial wealth, no taxes and TC
CAPM doesn't hold empirically

Cross Section [1]

Portfolio sorting: in each period, sort stocks into K portfolios; track realized returns at $t = 1$; repeat over time.
Size: market cap (right skewed)
Value: Book/Market Equity ratio
Fama-French 3 Factor Model: $\mathbb{E}[r_{it}] - r_f = \alpha_i + \beta_i\left(\mathbb{E}[r_{Mt}] - r_f\right) + s_i\text{SMB}_t + h_i\text{HML}_t + \varepsilon_{it}$
SMB: Small-minus-Big (size factor)
HML: High-minus-Low (value factor)
Momentum: trend in returns
WML: Winners-minus-Losers (momentum factor, Carhart Model)

Econometrics

Model: $y_t = x_t\beta + u_t$
OLS: $\hat{\beta} = (\mathbf{X}^\mathsf{T}\mathbf{X})^{-1}\mathbf{X}\mathbf{y}$
Std. Error: $\sqrt{\text{Var}\left(\hat{\beta}\,\middle|\,\mathbf{X}\right)}$ very messy
u_i uncorrelated and homoskedastic: std. error $\sigma_u\,(\mathbf{X}^\mathsf{T}\mathbf{X})^{-1/2}$
u_i heteroskedastic: use White Standard Errors
u_i serially correlated: use Newey-West Standard Errors
t-stat: $\left|\frac{\hat{\beta}}{\text{Var}(\hat{\beta}|\mathbf{X})}\right| > 2$ significant
Panel data: time-series and cross-sectional data
Fama-MacBeth Regression: i) run cross-section regression for each t; ii) estimate $\hat{\beta}_{FM} = \frac{1}{T}\sum_t \hat{\beta}_t$
Adjust with Newey-West std. errors for time-series correlation
Accounts for cross-sectional correlation within time periods

Cross Section [2]

Systematic risk: positive risk premium (theory)

Idiosyncratic risk: 0 risk premium (theory)
Idiosyncratic vol and negative returns correlated
Betting against Beta (BAB) strategy
Five Factor Model: add RMW & CMA
RMW: Robust-minus-Weak (profit factor)
CMA: Conservative-minus-Aggressive (investment factor)
Factors must have economic meaning

Performance Evaluation

Information Ratio: $IR = \frac{\mathbb{E}[R - R_b]}{\sigma(R - R_b)}$
R_b benchmark, $R_b = 0$ cash benchmark common
Sharpe Ratio: IR with benchmark r_f
High Water Mark: $HWM_t = \max_{s \leq t} P_s$
Drawdown: $DD_t = \max\left\{\frac{HWM_t - P_t}{HWM_t}, 0\right\}$
Maximum Drawdown: $MDD_T = \max_{t \leq T} DD_t$

Time Series

Common model $r_{t+1} = \alpha + \beta x_t + \varepsilon_t$, correlated error terms need Newey-West adjustment
OOS R^2: $R_{OS}^2 = 1 - \frac{\sum_{t=1}^{T}(r_t - \hat{r}_t)^2}{\sum_{t=1}^{T}(r_t - \bar{r}_t)^2}$ where $\bar{r}_t = \frac{1}{t-1}\sum_{j=1}^{t-1} r_j$ and $\hat{r}_t$ predicted
Need historical data before start of forecast $t = 1$
$R_{OS}^2 > 0$: predictor performs better than historical avg OOS,
$R_{OS}^2 < 0$: implies otherwise.
Cum. SSE Diff.: $\sum_{t=1}^{T}(r_t - \bar{r}_t)^2 - \sum_{t=1}^{T}(r_t - \hat{r}_t)^2$

Market Structure

Exchanges: 13 (NYSE, NASDAQ, CBOE)
Open/Close auctions, continuous limit order books (CLOBs)
Best Bid, Best Ask/Offer, "mid"
Bid-ask spread = best offer - best bid
Determinants: adverse selection, fixed costs, inventory costs, market power
NBBO (Natl Best Bid & Offer): across exchanges
Effective cost/spread: diff between execution price and price before (mid)
Realized cost: diff between execution price and price after (mid)
VWAP: Volume-Weighted Average Price
Implementation Shortfall: performance on paper - real
Dark pools: 15% equity volume, trade within NBBO
Payment for Order Flow: retail execution

2 Fixed Income

Treasury Market [1]

Federal Reserve: 12 FOMC members, 8 annual meetings, 5 regional presidents
Dual Mandate: price stability & maximum sustainable employment
Inflation Target: 2%, achieve through monetary policy
Holding period return: $\frac{P_t + D_t}{P_{t-1}} - 1$
Modified Duration: $-\frac{1}{P} \cdot \frac{dP}{dy}$
Price sensitivity to yield: $\frac{\Delta P}{P} \approx -\text{MD} \cdot \Delta y$
DV01: Dollar Value per 01 bp change in yield
Convexity: $\frac{\Delta P}{P} \approx -\text{MD} \cdot \Delta y + \frac{1}{2} \cdot C \cdot (\Delta y)^2$
PCA: orthogonal factors explaining data
PC1: level; PC2: slope; PC3: curvature
Bond excess returns: $y_t^{(n)}$ continuously compounded yield of zero-coupon bond with maturity n periods at time t
Carry: $y_t^{(n)} - y_t^{(1)}$
Capital gain: $-(n-1)\left(y_{t+1}^{(n-1)} - y_t^{(n)}\right)$

Treasury Market [2]

Repurchase Agreement: (overnight) loan secured by Treasury bond; interest rate: repo rate.
Bond Return over financing cost: $\approx [\text{YTM}_t(T-t) - \text{Repo}_t] \times \Delta t - \text{MD}_t \times [\text{YTM}_{t+\Delta t}(T - t - \Delta t) - \text{YTM}_t(T-t)]$
Strategies: duration-neutral positions immune to level shifts
On-the-run: newly issued, most liquid, special repo rate
Off-the-run: old securities, worse liquidity, often cheaper Run trade: $\text{PnL} = \Delta\text{YTM} \times \Delta t - \Delta\text{Repo} \times \Delta t - \overline{D} \times \Delta\text{YTM}$

Corporate Bond Market

Investment grade: rating BBB- or above
High yield/speculative grade/junk: BB+ or below
Loss given default (LGD) = 1 - recovery rate
Expected default loss = Default probability $\times$ LGD
Credit spread = Risk-neutral default probability $\times$ LGD

3 Forwards, Futures, Swaps

Forwards & Futures

Spot price S, Forward price F, maturity T
Dividend yield y, Risk-free rate r
Price: $F = S\exp((r - y)T)$
Margin provides leverage through clearinghouse
Cash vs physical settlement

Interest Rate Swaps

Notional, Maturity, Floating & Fixed rates
Payer: pays fixed rate,
Receiver: receives fixed rate.
Swap spread: swap rate - Treasury par yield same maturity

Credit Default Swaps

CDS + Risky Bond = Risk-free Bond
Corp Bond Yield - CDS Spread = Treasury Yield
CDS spread = Risk-neutral default probability $\times$ LGD

4 Commodities

Commodity Futures

Storage cost c implies
$F = S\exp[(r - (y - c))T]$
Net convenience yield $\hat{y} = y - c$
Contango: futures prices > spot,
Backwardation: futures prices < spot,
Basis: spot price - futures prices.
ExRet of rolling commodity futures = Commodity futures return + Roll return
Roll yield $= \ln\left(F_{T_1}\right) - \ln\left(F_{T_2}\right)$

Commodity Returns

Correlations with financial assets increased over time
Producers hedge, speculators demand exposure

5 Portfolio

Theory

Solve using Lagrange multipliers
Black-Litterman Model: express views in portfolio
1) Expected ExRet $\vec{R} \sim \mathcal{N}\left(\vec{\pi}, \tau\Sigma\right)$
2) View $\mathbf{P}^\mathsf{T}\vec{R} = \vec{Q} + \vec{\varepsilon}$; $\vec{\varepsilon} \mid \vec{R} \sim \mathcal{N}(0, \mathbf{\Omega})$
3) $\mathbb{E}[\vec{R}] = \left[(\tau\Sigma)^{-1} + \mathbf{P}\mathbf{\Omega}^{-1}\mathbf{P}^\mathsf{T}\right]^{-1} \times \left[(\tau\Sigma)^{-1}\vec{\pi} + \mathbf{P}\mathbf{\Omega}^{-1}\vec{Q}\right]$,
$\text{Var}(\vec{R}) = \left[(\tau\Sigma)^{-1} + \mathbf{P}\mathbf{\Omega}^{-1}\mathbf{P}^\mathsf{T}\right]^{-1}$

Black-Litterman Example

$\vec{\mu}$: expected ExRet; Σ: cov matrix

Investor maximizes $\vec{w}^\top \vec{\mu} - \frac{1}{2} \delta \vec{w}^\top \Sigma \vec{w}$

Soln: $\vec{\mu} = \delta \Sigma \vec{w}^*$, or $\vec{w}^* = (\delta \Sigma)^{-1} \vec{\mu}$

Equilibrium: $\vec{\mu}_{eq} = \delta \Sigma \vec{w}_{eq}$

Adding view: create L/S portfolio P with expected return Q and variance Σ; calculate new expected ExRet and cov matrix following formulas for $\vec{R}$ above

δ: risk-aversion; τ inv prop to confidence in consensus

Robust Construction

Equal Weighting portfolios: perform well OOS

Risk Parity portfolios: equate risk of assets

Factor cov matrix F from factor model, $\hat{\Sigma} = \vec{B}^\top \operatorname{Var}(\vec{F}) \vec{B} + \mathbf{U}$.

Shrinkage: target theoretical cov matrix $\mathbf{\Phi}$, $\alpha \in [0,1]$,

$\hat{\Sigma} = \alpha \mathbf{\Phi} + (1 - \alpha) \Sigma$,

optimal $\alpha \approx \frac{1}{T}$.

Dynamic Considerations

Rebalancing induces transaction costs

Long-run weights not same as short-term

Long-run weight = Short-run weight + Opportunistic weight

6 Options

Options [1]

Black-Scholes-Merton: spot price S_0, maturity T, strike price K, interest rate r, volatility σ

$C = S_0 \Phi(d_1) - K e^{-rT} \Phi(d_2),$

$d_1 = \dfrac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}},$

$d_2 = \dfrac{\ln\left(\frac{S}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$

Put-Call Parity: $C - P = S_0 - K e^{-rT}$

Derivation from price movement:

$\frac{dS_t}{S_t} = r\,dt + \sigma\,dB_t$

$S_T = S_0 \exp\left(\left(r - \frac{\sigma^2}{2}\right)T + \sigma B_T\right)$

Value $\mathbb{E}^{\mathbb{Q}}\left[e^{-rT} \max(S_T - K, 0)\right]$

BSM with continuous dividend yield δ:

$C = S_0 e^{-\delta T} \Phi(d_1) - K e^{-rT} \Phi(d_2),$

$d_1 = \dfrac{\ln\left(\frac{S}{K}\right) + \left(r - \delta + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}},$

$d_2 = \dfrac{\ln\left(\frac{S}{K}\right) + \left(r - \delta - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$

Put-Call Parity: $C - P = S_0 e^{-\delta T} - K e^{-rT}$

Implied Volatility: $\hat{\sigma}$ from BSM

VIX^2: risk-neutral variance

Options [2]

Risk-neutral distribution: $\mathbb{Q}$

$\frac{\partial^2 C}{\partial K^2} = e^{-rT} q(K)$

Grid of strikes $K_1, K_2, \ldots$ with $\Delta K = K_{n+1} - K_n$ constant

$q(K_n) \approx e^{rT} \dfrac{C_{n+1} - 2C_n + C_{n-1}}{(\Delta K)^2}$

Capital Structure:

Equity is call on asset struck at debt, Debt plus equity call on asset struck at more senior debt, etc.

Prepayment options: use risk-neutral pricing.

7 Foreign Exchange

Basics

USD, EUR, JPY, GBP main currencies

Quotation: Base/Pricing Currency

Forward & Carry Trade

Interest Rate Parity: $F = S\left(\frac{1+r_p}{1+r_b}\right)^T$

Carry trade: use low interest rate currency to fund saving in high interest rate currency

Carry return: $\frac{1 + r_{investment}}{1 + r_{funding}} \cdot \frac{S_0}{S_T} - 1$

Continuous ret $z = r_{investment} - r_{funding} - \ln\left(\frac{S_T}{S_0}\right)$

Positive expected return, high Sharpe, *negative* skewness