15.433 Final Exam Cheat Sheet
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1 Equity

CAPM

$\mathbb{E}[r_{it}] - r_f = \beta_{iM}\left(\mathbb{E}[r_{Mt}] - r_f\right)$

Assumptions: mean-variance agents, one period investment horizon, no information asymmetry, risk-free rate r_f, only financial wealth, no taxes and TC

CAPM doesn't hold empirically

Cross Section [1]

Portfolio sorting: in each period, sort stocks into K portfolios; track realized returns at $t = 1$; repeat over time.

Size: market cap (right skewed)

Value: Book/Market Equity ratio

Fama-French 3 Factor Model: $\mathbb{E}[r_{it}] - r_f = \alpha_i + \beta_i\left(\mathbb{E}[r_{Mt}] - r_f\right) + s_i \text{SMB}_t + h_i \text{HML}_t + \varepsilon_{it}$

SMB: Small-minus-Big (size factor)

HML: High-minus-Low (value factor)

Momentum: trend in returns

WML: Winners-minus-Losers (momentum factor, Carhart Model)

Econometrics

Model: $y_t = x_t\beta + u_t$

OLS: $\hat{\beta} = (\mathbf{X}^\intercal\mathbf{X})^{-1}\mathbf{X}\mathbf{y}$

Std. Error: $\sqrt{\text{Var}\left(\hat{\beta}|\mathbf{X}\right)}$ very messy

u_i uncorrelated and homoskedastic: std. error $\sigma_u(\mathbf{X}^\intercal\mathbf{X})^{-1/2}$

u_i heteroskedastic: use White Standard Errors

u_i serially correlated: use Newey-West Standard Errors

t-stat: $\left|\frac{\hat{\beta}}{\text{Var}(\hat{\beta}|\mathbf{X})}\right| > 2$ significant

Panel data: time-series and cross-sectional data

Fama-MacBeth Regression: i) run cross-section regression for each t; ii) estimate $\hat{\beta}_{FM} = \frac{1}{T}\sum_t \hat{\beta}_t$

Adjust with Newey-West std. errors for time-series correlation

Accounts for cross-sectional correlation within time periods

Cross Section [2]

Systematic risk: positive risk premium (theory)

Idiosyncratic risk: 0 risk premium (theory)

Idiosyncratic vol and negative returns correlated

Betting against Beta (BAB) strategy

Five Factor Model: add RMW & CMA

RMW: Robust-minus-Weak (profit factor)

CMA: Conservative-minus-Aggressive (investment factor)

Factors must have economic meaning

Performance Evaluation

Information Ratio: $IR = \frac{\mathbb{E}[R-R_b]}{\sigma(R-R_b)}$

R_b benchmark, $R_b = 0$ cash benchmark common

Sharpe Ratio: IR with benchmark r_f

High Water Mark: $HWM_t = \max_{s\le t} P_s$

Drawdown: $DD_t = \max\left\{\frac{HWM_t - P_t}{HWM_t}, 0\right\}$

Maximum Drawdown: $MDD_T = \max_{t\le T} DD_t$

Time Series

Common model $r_{t+1} = \alpha + \beta x_t + \varepsilon_t$, correlated error terms need Newey-West adjustment

OOS R^2: $R^2_{OS} = 1 - \frac{\sum_{t=1}^{T}(r_t - \hat{r}_t)^2}{\sum_{t=1}^{T}(r_t - \bar{r}_t)^2}$ where $\bar{r}_t = \frac{1}{t-1}\sum_{j=1}^{t-1} r_j$ and $\hat{r}_t$ predicted

Need historical data before start of forecast $t = 1$

$R^2_{OS} > 0$: predictor performs better than historical avg OOS,

$R^2_{OS} < 0$: implies otherwise.

Cum. SSE Diff.: $\sum_{t=1}^{T}(r_t - \bar{r}_t)^2 - \sum_{t=1}^{T}(r_t - \hat{r}_t)^2$

Market Structure

Exchanges: 13 (NYSE, NASDAQ, CBOE)

Open/Close auctions, continuous limit order books (CLOBs)

Best Bid, Best Ask/Offer, "mid"

Bid-ask spread = best offer - best bid

Determinants: adverse selection, fixed costs, inventory costs, market power

NBBO (Natl Best Bid & Offer): across exchanges

Effective cost/spread: diff between execution price and price before (mid)

Realized cost: diff between execution price and price after (mid)

VWAP: Volume-Weighted Average Price

Implementation Shortfall: performance on paper - real

Dark pools: 15% equity volume, trade within NBBO

Payment for Order Flow: retail execution

2 Fixed Income

Treasury Market [1]

Federal Reserve: 12 FOMC members, 8 annual meetings, 5 regional presidents

Dual Mandate: price stability & maximum sustainable employment

Inflation Target: 2%, achieve through monetary policy

Holding period return: $\frac{P_t + D_t}{P_{t-1}} - 1$

Modified Duration: $-\frac{1}{P}\cdot\frac{\mathrm{d}P}{\mathrm{d}y}$

Price sensitivity to yield: $\frac{\Delta P}{P} \approx -\text{MD}\cdot\Delta y$

DV01: Dollar Value per 01 bp change in yield

Convexity: $\frac{\Delta P}{P} \approx -\text{MD}\cdot\Delta y + \frac{1}{2}\cdot C\cdot(\Delta y)^2$

PCA: orthogonal factors explaining data

PC1: level; PC2: slope; PC3: curvature

Bond excess returns: $y_t^{(n)}$ continuously compounded yield of zero-coupon bond with maturity n periods at time t

Carry: $y_t^{(n)} - y_t^{(1)}$

Capital gain: $-(n-1)\left(y_{t+1}^{(n-1)} - y_t^{(n)}\right)$

Treasury Market [2]

Repurchase Agreement: (overnight) loan secured by Treasury bond; interest rate: repo rate.

Bond Return over financing cost: $\approx [\text{YTM}_t(T-t) - \text{Repo}_t] \times \Delta t - \text{MD}_t \times [\text{YTM}_{t+\Delta t}(T-t-\Delta t) - \text{YTM}_t(T-t)]$

Strategies: duration-neutral positions immune to level shifts

On-the-run: newly issued, most liquid, special repo rate

Off-the-run: old securities, worse liquidity, often cheaper Run trade: $\text{PnL} = \Delta\text{YTM} \times \Delta t - \Delta\text{Repo} \times \Delta t - \overline{D} \times \Delta\text{YTM}$

Corporate Bond Market

Investment grade: rating BBB- or above

High yield/speculative grade/junk: BB+ or below

Loss given default (LGD) = 1 - recovery rate

Expected default loss = Default probability $\times$ LGD

Credit spread = Risk-neutral default probability $\times$ LGD

3 Forwards, Futures, Swaps

Forwards & Futures

Spot price S, Forward price F, maturity T

Dividend yield y, Risk-free rate r

Price: $F = S\exp((r-y)T)$

Margin provides leverage through clearinghouse

Cash vs physical settlement

Interest Rate Swaps

Notional, Maturity, Floating & Fixed rates

Payer: pays fixed rate,

Receiver: receives fixed rate.

Swap spread: swap rate - Treasury par yield same maturity

Credit Default Swaps

CDS + Risky Bond = Risk-free Bond

Corp Bond Yield - CDS Spread = Treasury Yield

CDS spread = Risk-neutral default probability $\times$ LGD

4 Commodities

Commodity Futures

Storage cost c implies $F = S\exp[(r-(y-c))T]$

Net convenience yield $\hat{y} = y - c$

Contango: futures prices > spot,

Backwardation: futures prices < spot,

Basis: spot price - futures prices.

ExRet of rolling commodity futures = Commodity futures return + Roll return

$\text{Roll yield} = \ln\left(F_{T_1}\right) - \ln\left(F_{T_2}\right)$

Commodity Returns

Correlations with financial assets increased over time

Producers hedge, speculators demand exposure

5 Portfolio

Theory

Solve using Lagrange multipliers

Black-Litterman Model: express views in portfolio

1) Expected ExRet $\vec{R} \sim \mathcal{N}\left(\vec{\pi}, \tau\Sigma\right)$

2) View $\mathbf{P}^\intercal\vec{R} = \vec{Q} + \vec{\varepsilon}$; $\vec{\varepsilon}\,|\,\vec{R} \sim \mathcal{N}(0, \mathbf{\Omega})$

3) $\mathbb{E}\left[\vec{R}\right] = \left[(\tau\Sigma)^{-1} + \mathbf{P}\mathbf{\Omega}^{-1}\mathbf{P}^\intercal\right]^{-1} \times \left[(\tau\Sigma)^{-1}\vec{\pi} + \mathbf{P}\mathbf{\Omega}^{-1}\vec{Q}\right]$,

$\text{Var}\left(\vec{R}\right) = \left[(\tau\Sigma)^{-1} + \mathbf{P}\mathbf{\Omega}^{-1}\mathbf{P}^\intercal\right]^{-1}$

Black-Litterman Example

$\vec{\mu}$: expected ExRet; Σ: cov matrix
Investor maximizes $\vec{w}^\intercal \vec{\mu} - \frac{1}{2}\delta \vec{w}^\intercal \Sigma \vec{w}$
Soln: $\vec{\mu} = \delta\Sigma\vec{w}^*$, or $\vec{w}^* = (\delta\Sigma)^{-1}\vec{\mu}$
Equilibrium: $\vec{\mu}_{\text{eq}} = \delta\Sigma\vec{w}_{\text{eq}}$
Adding view: create L/S portfolio P with expected return Q and variance Σ; calculate new expected ExRet and cov matrix following formulas for $\vec{R}$ above
δ: risk-aversion; τ inv prop to confidence in consensus

Robust Construction

Equal Weighting portfolios: perform well OOS
Risk Parity portfolios: equate risk of assets
Factor cov matrix F from factor model, $\hat{\Sigma} = \vec{B}^\intercal \operatorname{Var}\left(\vec{F}\right)\vec{B} + \mathbf{U}$.
Shrinkage: target theoretical cov matrix $\mathbf{\Phi}$, $\alpha \in [0,1]$,
$\hat{\Sigma} = \alpha\mathbf{\Phi} + (1-\alpha)\Sigma$,
optimal $\alpha \approx \frac{1}{T}$.

Dynamic Considerations

Rebalancing induces transaction costs
Long-run weights not same as short-term
Long-run weight = Short-run weight + Opportunistic weight

6 Options

Options [1]

Black-Scholes-Merton: spot price S_0, maturity T, strike price K, interest rate r, volatility σ
$C = S_0\Phi(d_1) - Ke^{-rT}\Phi(d_2)$,
$d_1 = \frac{\ln\left(\frac{S}{K}\right)+\left(r+\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$,
$d_2 = \frac{\ln\left(\frac{S}{K}\right)+\left(r-\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$
Put-Call Parity: $C - P = S_0 - Ke^{-rT}$
Derivation from price movement:
$\frac{dS_t}{S_t} = r\,dt + \sigma\,dB_t$
$S_T = S_0\exp\left(\left(r - \frac{\sigma^2}{2}\right)T + \sigma B_T\right)$
Value $\mathbb{E}^{\mathbb{Q}}\left[e^{-rT}\max(S_T - K, 0)\right]$
BSM with continuous dividend yield δ:
$C = S_0e^{-\delta T}\Phi(d_1) - Ke^{-rT}\Phi(d_2)$,
$d_1 = \frac{\ln\left(\frac{S}{K}\right)+\left(r-\delta+\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$,
$d_2 = \frac{\ln\left(\frac{S}{K}\right)+\left(r-\delta-\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$
Put-Call Parity: $C - P = S_0e^{-\delta T} - Ke^{-rT}$
Implied Volatility: $\hat{\sigma}$ from BSM
VIX^2: risk-neutral variance

Options [2]

Risk-neutral distribution: $\mathbb{Q}$

$\frac{\partial^2 C}{\partial K^2} = e^{-rT}q(K)$
Grid of strikes $K_1, K_2, \ldots$ with $\Delta K = K_{n+1} - K_n$ constant
$q(K_n) \approx e^{rT}\frac{C_{n+1}-2C_n+C_{n-1}}{(\Delta K)^2}$

Capital Structure:

Equity is call on asset struck at debt,
Debt plus equity call on asset struck at more senior debt, etc.
Prepayment options: use risk-neutral pricing.

7 Foreign Exchange

Basics

USD, EUR, JPY, GBP main currencies
Quotation: Base/Pricing Currency

Forward & Carry Trade

Interest Rate Parity: $F = S\left(\frac{1+r_p}{1+r_b}\right)^T$
Carry trade: use low interest rate currency to fund saving in high interest rate currency
Carry return: $\frac{1+r_{\text{investment}}}{1+r_{\text{funding}}}\cdot\frac{S_0}{S_T} - 1$
Continuous ret $z = r_{\text{investment}} - r_{\text{funding}} - \ln\left(\frac{S_T}{S_0}\right)$
Positive expected return, high Sharpe, *negative* skewness