Sri Sapthagiri PU College

M1

Mathematics

Time: 23m:00s **Total Questions: 23** **Marks: 92**

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1. If U is the universal set with 100 element; A and B are two set such that n(A) = 50, n (B) = 60, n (A ∩B) = 20 then n (A' ∩ B') =
   (A) 90
   (B) 40
   (C) 10
   (D) 20
   Answer: C
   $n\left(U\right)=100$
   $n\left(A\right)=50$
   $n\left(B\right)=60$
   $n\left(A\cap B\right)=20$
   $n\left(A\cup B\right)=n\left(A\right)+n\left(B\right)-n\left(A\cap B\right)$
   $=50+60-20$
   $=110-20$
   $=90$
   $n\left(A'\cap B'\right)=n\left(\left(A\cup B\right)'\right)$
   $=n\left(U\right)-n\left(A\cup B\right)$
   $=100-90$
   $=10$

2. The domain of the function f : R → R defined by $f(x)=\sqrt{x^2-7x+12}$ is
   (A) $(-\infty,3)\cap(4,\infty)$
   (B) $(-\infty,3)\cup(4,\infty)$
   (C) $(3,4)$
   (D) $(\infty,3)\cap(4,\infty)$
   Answer: B
   $f:R\rightarrow R$
   $f\left(x\right)=\sqrt{x^2-7x+12}$
   $x^2-7x+12\geq 0$
   $\left(x-4\right)\left(x-3\right)\geq 0$
   $\Rightarrow\ x\in\left(-\infty,3\right]\cup\left[4,\infty\right)$

3. The set $A=\{x:|2x+3|<7\}$ is equal to the set
   (A) $B=\{x:-3<x<7\}$
   (B) $C=\{x:-13<2x<4\}$
   (C) $D=\{x:0<x+5<7\}$
   (D) $E=\{x:-7<x<7\}$
   Answer: C
   Given, set $A=\{x:|2x+3|<7\}$
   Now, $|2x+3|<7$
   $\Rightarrow -7<2x+3<7$
   $\Rightarrow -7-3<2x<7-3$
   $\Rightarrow -10<2x<4$
   $\Rightarrow -5<x<2$
   $\Rightarrow 0<(x+5)<7$

4. If A and B are finite sets and, $A\subset B$ then
   (A) $n(A\cup B)\ =\ n(A)$
   (B) $n(A\cap B)\ =\ n(B)$
   (C) $n(A\cup B)\ =\ n(B)$
   (D) $n(A\cap B)\ =\ \phi$
   Answer: C
   We have, $A\subset B$
   $\therefore A\cap B=A\Rightarrow n(A\cap B)=n(A)\ldots$(i)
   Again, we know that
   $n(A\cup B)=n(A)+n(B)-n(A\cap B)$
   $\Rightarrow n(A\cup B)=n(A)+n(B)-n(A)$ [from Eq. (i)]
   $\Rightarrow n(A\cup B)=n(B)$

5. Write the set builder form $A=-1,1$
   (A) A = {x : x is a real number}
   (B) A = (x : x is an integer)
   (C) A = {x : x is a root of the equation x^2 = 1}
   (D) A = {x : x is a root of the equation x^2 + 1 = 0}
   Answer: C
   $-1,1$ are the roots of the equation $x^2-1=0$ Hence, set builder form of A can be written as $A=\{x:x$ is a root of the equation $x^2=1\}$

6. The domain of the function $f(x)=\sqrt{\cos x}$ is
   (A) $\left[0, \frac{\pi}{2}\right]$
   (B) $\left[0, \frac{\pi}{2}\right] \cup\left[\frac{3 \pi}{2}, 2 \pi\right]$
   (C) $\left[\frac{3 \pi}{2}, 2 \pi\right]$
   (D) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$
   Answer: B,D,C
   Given, $f(x)=\sqrt{\cos x}$
   i.e., $\cos x \geq 0$
   But $-1 \leq \cos x \leq 1$
   $\therefore 0 \leq \cos x \leq 1$
   i.e., x lies in Ist or IVth quadrant
   $\Rightarrow 0 \leq x \leq \frac{\pi}{2}$ or $\frac{3 \pi}{2} \leq x \leq 2 \pi$
   $\therefore x \in\left[0, \frac{\pi}{2}\right] \cup\left[\frac{3 \pi}{2}, 2 \pi\right]$
   Also, $\cos (-x)=\cos x$
   Hence, $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ is also the domain of the function.

7. In a class of 60 students, 25 students play cricket and 20 students play tennis, and 10 students play both the games. Then, the number of students who play neither is
   (A) 0
   (B) 25
   (C) 35
   (D) 45
   Answer: B
   Let student play cricket $=C$
   Student play tennis $=T$
   and total number of students $=S$
   $\therefore n(S)=60, n(C)=25$, $n(T)=20$
   and $n(C \cap T)=10$
   Now, $n(C \cup T)=n(C)+n(T)-n(C \cap T)$
   $=25+20-10=35$
   $\therefore$ The number of students who play neither game
   $=n(C \cap T)^{\prime}=n(S)-n(C \cup T)$
   $=60-35=25$

8. If $X=\{4^n-3n-1 : n \in N\}$ and $Y=\{9(n-1) : n \in N\}$,where N is the set of natural numbers, then $X \cup Y$ is equal to
(A) N
(B) Y-X
(C) X
(D) Y
Answer: D
$X=\{(1+3)^n-3n-1, n \in N\}$
$=3^2({}^nC_2+{}^nC_3 \cdot 3+\ldots+3^{n-2}), n \in N\}$
$=\{$ Divisible by $9\}$
$Y=\{9(n-1), n \in N\}$
$=($ All multiples of $9\}$
So, $X \subseteq Y$
i.e., $X \cup Y=Y$

9. If $X=\{4^n-3n-1 : n \in N\}$ and $Y=\{9\,(n-1) : n \in N\}$, then $X \cup Y$ is equal to
(A) X
(B) Y
(C) N
(D) none of these
Answer: B
Here $X \subseteq Y \therefore X \cup Y=Y$

10. Let Z be the set of integers. If $A=\{x \in Z : 2^{(x+2)(x^2-5x+6)}\}=1$ and $B=\{x \in Z : -3<2x-1<9\}$, then the number of subsets of the set $A \times B$, is :
(A) 2^{18}
(B) 2^{10}
(C) 2^{15}
(D) 2^{12}
Answer: C
$A=\{x \in z : 2^{(x+2)(x^2-5x+6)}=1\}$
$2^{(x+2)(x^2-5x+6)}=2^0 \Rightarrow x=-2,2,3$
$A=\{-2,2,3\}$
$B=\ x \in Z : -3<2x-1<9$
$A \times B$ has is 15 elements so number of subsets
of $A \times B$ is 2^{15}

11. Let $X=\{1,2,3,4,5\}$ The number of different ordered pairs (Y, Z) that can formed such that $Y \subseteq X, Z \subseteq X$ and $Y \cap Z$ is empty, is :
(A) 5^2
(B) 3^5
(C) 2^5
(D) 5^3
Answer: B
Every element has 3 options. Either set Y or set Z or none
so number of ordered pairs $=3^5$

12. Let $\underset{i=1}{\overset{50}{U}} X_i = \underset{i=1}{\overset{n}{U}} Y_i = T$ where each X_i contains 10 elements and each Y_i contains 5 elements. If each element of the set T is an element of exactly 20 of sets X_i 's and exactly 6 of sets Y_i 's, then n is equal to :

(A) 45

(B) 15

(C) 50

(D) 30

Answer: D

$n(X_i) = 10. \underset{i=1}{\overset{50}{U}} = T$,

$\Rightarrow n(T) = 500$

each element of T belongs to exactly 20

elements of $X_i \Rightarrow \frac{500}{20} = 25$ distinct elements

so $\frac{5n}{6} = 25$

$\Rightarrow n = 30$

13. In a certain town, 25% of the families own a phone and 15% own a car 65% families own neither a phone nor a car and $2,000$ families own both a car and a phone. Consider the following three statements :

(a) 5% families own both a car and a phone.

(b) 35% families own either a car or a phone.

(c) $40,000$ families live in the town.

Then,

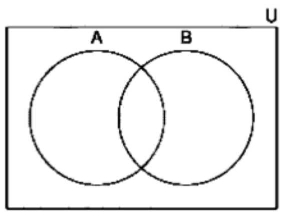
(A) Only (a) and (b) are correct

(B) Only (a) and (c) are correct

(C) Only (b) and (c) are correct

(D) All (a), (b) and (c) are correct

Answer: D



$n(P) = 25$

$n(C) = 15$

$n\left(P' \cup C'\right) = 65\%$

$\Rightarrow n(P \cup C)' = 65\%$

$n\left(P \cup C\right) = 35\%$

$n\left(P \cap C\right) = n\left(P\right) + n\left(C\right) - n\left(P \cup C\right)$

$25 + 15 - 35 = 5\%$

$x \times 5\% = 2000$

$x = 40,000$

14. Let $S=\{x \in R: x \geq 0$ and $2|\sqrt{x}-3|+\sqrt{x}(\sqrt{x}-6)+6=0\}$. Then S :
   (A) is an empty set
   (B) contains exactly one element
   (C) contains exactly two elements
   (D) contains exactly four elements

   Answer: C

   $2|\sqrt{x}-3|+\sqrt{x}(\sqrt{x}-6)+6=0$
   $2|\sqrt{x}-3|+(\sqrt{x}-3+3)(\sqrt{x}-3-3)+6=0$
   $2|\sqrt{x}-3|+(\sqrt{x}-3)^2-3=0$
   $(\sqrt{x}-3)^2+2|\sqrt{x}-3|-3=0$
   $(|\sqrt{x}-3|+3)(|\sqrt{x}-3|-1)=0$
   $\Rightarrow|\sqrt{x}-3|=1,|\sqrt{x}-3|+3 \neq 0$
   $\Rightarrow \sqrt{x}-3= \pm 1$
   $\Rightarrow \sqrt{x}=4,2$
   $x=16,4$

15. Let S = {1,2,3, ..., 100}. The number of non- empty subsets A of S such that the product of elements in A is even is :
   (A) $2^{50}(2^{50}-1)$
   (B) $2^{100}-1$
   (C) $2^{50}-1$
   (D) $2^{50}+1$

   Answer: A

   S = {1,2,3------100}
   = Total non empty subsets-subsets with product of element is odd
   $=2^{100}-1-1[(2^{50}-1)]$
   $=2^{100}-2^{50}$
   $=2^{50}(2^{50}-1)$

16. In a class of 140 students numbered 1 to 140, all even numbered students opted mathematics course, those whose number is divisible by 3 opted Physics course and those whose number is divisible by 5 opted Chemistry course. Then the number of students who did not opt for any of the three courses is :
    (A) 102
    (B) 42
    (C) 1
    (D) 38
    Answer: D
    Let $n(A)$ = number of students opted Mathematics = 70,
    $n(B)$ = number of students opted Physics = 46,
    $n(C)$ = number of students opted Chemistry = 28,
    $n(A \cap B) = 23,$
    $n(B \cap C) = 9,$
    $n(A \cap C) = 14,$
    $n(A \cap B \cap C) = 4,$
    Now $n(A \cup B \cup C)$
    $= n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C)$
    $-n(A \cap C) + n(A \cap B \cap C)$
    $= 70 + 46 + 28 - 23 - 9 - 14 + 4 = 102$
    So number of students not opted for any course
    = Total - $n(A \cup B \cup C)$
    $= 140 - 102 = 38$

17. Two newspapers A and B are published in a city. It is known that 25% of the city populations reads A and 20% reads B while 8% reads both A and B. Further, 30% of those who read A but not B look into advertisements and 40% of those who read B but not A also look into advertisements, while 50% of those who read both A and B look into advertisements. Then the percentage of the population who look into advertisement is :
    (A) 12.8
    (B) 13.5
    (C) 13.9
    (D) 13
    Answer: C
    Let population = 100
    $n(A) = 25$
    $n(B) = 20$
    $n(A \cap B) = 8$
    $n(A \cap \bar{B}) = 17$
    $n(\bar{A} \cap B) = 12$
    $\frac{30}{100} \times 17 + \frac{40}{100} \times 12 + \frac{50}{100} \times 8$
    $5.1 + 4.8 + 4 = 13.9$

18. If $f(x)+2f\left(\frac{1}{x}\right)=3x, x\neq 0$, and $S=\{x\in R: f(x)=f(-x)\}$; then S :
   (A) is an empty set.
   (B) contains exactly one element.
   (C) contains exactly two elements.
   (D) contains more than two elements.
   Answer: C
   $f(x)+2f\left(\frac{1}{x}\right)=3x$
   Replace x by $\frac{1}{x}, f\left(\frac{1}{x}\right)+2f(x)=\frac{3}{x}$
   $\Rightarrow \frac{3x-f(x)}{2}=\frac{\frac{3}{x}-2f(x)}{1}$
   $\Rightarrow 3x-f(x)=\frac{6}{x}-4f(x)$
   $\Rightarrow f(x)=\frac{2}{x}-x$
   $f(x)=f(-x)$
   $\Rightarrow \frac{2}{x}-x=-\frac{2}{x}+x$
   $\Rightarrow \frac{4}{x}=2x$
   $\Rightarrow x=\pm\sqrt{2}$

19. Let $x_1, x_2, \ldots, x_n$ be n observations, and let $\bar{x}$ be their arithmetic mean and σ^2 be the variance.
   Statement-1 : Variance of $2x_1, 2x_2, \ldots, 2x_n$ is $4\sigma^2$.
   Statement-2 : Arithmetic mean $2x_1, 2x_2, \ldots, 2x_n$ is 4 $\bar{x}$.
   (A) Statement-1 is false, Statement-2 is true
   (B) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for Statement-1
   (C) Statement-1 is true, statement-2 is true; statement-2 is not a correct explanation for Statement-1
   (D) Statement-1 is true, statement-2 is false
   Answer: D
   If each observation is multiplied by k, mean gets multiplied by k and variance gets multiplied by k^2. Hence the new mean should be $2\bar{x}$ and new variance should be $k^2\sigma^2$. So statement-1 is true and statement-2 is false.

20. Let A, B and C be sets such that $\Phi \neq A \cap B \subseteq C$,. Then which of the following statements is not true?

(A) If $(A - C) \subseteq B$, then $A \subseteq B$

(B) $(C \cup A) \cap (C \cup B) = C)$

(C) If $(A - B) \subseteq C$, then $A \subseteq C$

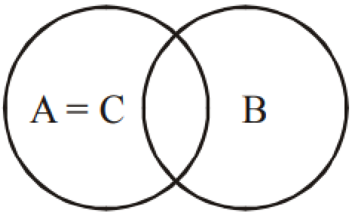
(D) $B \cap C \neq \Phi$

Answer: A

For $A = C, A - C = \Phi$

$\Rightarrow \Phi \subseteq B$

But $A \nsubseteq B$



$\Rightarrow$ option 1 is NOT true

$Let\ x \in (Cx \in (C \cup A) \cap (C \cup B))$

$\Rightarrow x \in (C \cup A) and\ x \in (C \cup B)$

$\Rightarrow (x \in C\ or\ x \in A)\ and\ (x \in C\ or\ x \in B)$

$\Rightarrow x \in C\ or\ x \in (A \cap B)$

$\Rightarrow x \in C\ or\ x \in C\ (as\ A \cup B$

$subseteqC$

$\Rightarrow x \in C$

$\Rightarrow (C \cup A) \cap (C \cup B) \subseteq C \qquad (1)$

$Now\ x \in C \Rightarrow x \in (C \cup A) and\ x \in (C \cup B)$

$\Rightarrow x \in (C \cup A) \cap (C \cup B)$

$\Rightarrow C \subseteq (C \cup A) \cap (C \cup B) \qquad (2)$

$\Rightarrow from(1)and(2)$

$C = (C \cup A) \cap (C \cup B)$

$\Rightarrow$ option 2 is true

Let x $\in$A and $x \notin B$

$\Rightarrow x \in (A - B)$

$\Rightarrow x \in C \qquad (as\ A - B \subseteq C)$

Let x $\in$ $A\ and\ x \in B$

$\Rightarrow x \in (A \cap B)$

$\Rightarrow x \in C \qquad (asA \cap B \subseteq C$

Hence x $\in$ A $\Rightarrow$ x $\in$ C

$\Rightarrow A \subseteq C$

$\Rightarrow$ Option 3 is true

as $C \supseteq (A \cap B)$

$\Rightarrow B \cap C \supseteq (A \cap B)$

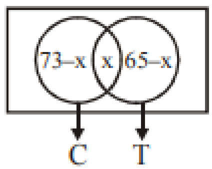
as $A \cap B \leq \Phi$

$\Rightarrow B \cap C \leq \Phi$

$\Rightarrow$ Option 4 is true

21. A relation on the set $A=\{x:|x|<3, x \epsilon Z\}$ where Z is the set of integers is defined by $R=\{(x, y): y=|x|, x \neq-1\}$. Then the number of elements in the power set of R is :
(A) 32
(B) 16
(C) 8
(D) 64
Answer: B
$A=\{x:|x|<3, x \in Z\}$
$A=\{-2,-1,0,1,2\}$
$R=\{(x, y): y=|x|, x \neq 1\}$
$R=\{(-2,2),(0,0),(1,1),(2,2)\}$
R has four elements
Number of elements in the power set of R
$=2^4=16$

22. A survey shows that 73% of the persons working in an office like coffee, whereas 65% like tea. If x denotes the percentage of them. who like both coffee and tea, then x cannot be:
(A) 63
(B) 38
(C) 54
(D) 36
Answer: D
$C \rightarrow$ person like coffee
$T \rightarrow$ person like Tea



$n(C)=73$
$n(T)=65$
$n(C \cup T) \leq 100$
$n(C)+n(T)-n(C \cap T) \leq 100$
$73+65-x \leq 100$
$x \geq 38$
$73-x \geq 0 \Rightarrow x \leq 73$
$65-x \geq 0 \Rightarrow x \leq 65$
$38 \leq x \leq 65$

23. A survey shows that 63% of the people in a city read newspaper A whereas 76% read newspaper B. If $x\%$ of the people read both the newspapers, then a possible value of x can be:
(A) 65
(B) 37
(C) 29
(D) 55
Answer: D
Finding the value of x : Let the people read news A is $n(A)=63\%$ the people read news B is $n(B)=76\%$ the people read both the newspapers $n(A \cap B)=x$ We know that $\max(n(A) \& n(B)) \leq n(A \cup B) \leq 100$
$\Rightarrow 76 \leq 63+76-x \leq 100$
$\Rightarrow -63 \leq -x \leq -39$
$\Rightarrow 63 \geq x \geq 39$