University of Nottingham
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BUSI4496
Supply Chain Planning & Management

Forecasting Seminar- problems and solutions

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Problem 1

The sales data of the first 6 months for a shop is given in 100 dollars as following:

Month	January	February	March	April	May	June	July
Sales ($100)	180	220	290	310	370	380	

(1a) Using a three-months simple moving average, estimate the sales for April, May, and June.

(1b) Considering the forecast of April calculated in question (1a) as F_{April}, using exponential smoothing method, forecast the sales for May, June and July using $\alpha_1 = 0.1$ and $\alpha_2 = 0.8$.

(1c) Assuming that the slope and the intercept of the regression of the given data is 42 and 145 respectively, estimate the sales in May, June and July by regression method.

(1d) Given the information that the income in July in 100 dollars is 376. Using MAD through May, June and July, determine whether regression gave you a better forecast or the exponential smoothing using $\alpha_2 = 0.8$. (only over these 3 months)

Month	January	February	March	April	May	June	July
Sales ($100)	180	220	290	310	370	380	

(1a) Using a three-months simple moving average, estimate the sales for April, May, and June.

$$F_{April} = \frac{D_{Jan} + D_{Feb} + D_{Mar}}{3}$$

$$\frac{180 + 220 + 290}{3} = 230$$

$$F_{t+1} = \frac{D_t + D_{t-1} + D_{t-2} + \cdots + D_{t-n+1}}{n}$$

$$F_{May} = \frac{310 + 290 + 220}{3} = 273.33$$

$$F_{Jun} = \frac{370 + 310 + 290}{3} = 322.33$$

(1a) Using a three-months simple moving average, estimate the sales for April, May, and June.

$$F_{t+1} = \frac{D_t + D_{t-1} + D_{t-2} + \cdots + D_{t-n+1}}{n}$$

$$(1) F_{April} = \frac{180 + 220 + 290}{3} = 230,$$

$$(2) F_{May} = \frac{220 + 290 + 310}{3} = 273.33,$$

$$(3) F_{June} = \frac{290 + 310 + 370}{3} = 323.33$$

	Month	January	February	March	April	May	June	July
1	Actual Sales ($100)	180	220	290	310	370	380	
1a	Forecasted MA (n=3)				230	273.33	323.33	

Month	January	February	March	April	May	June	July
Sales ($100)	180	220	290	310 ✓	370 ?	380 ?	?

(1b) Considering the forecast of April calculated in question (1a) as F_{April}, using exponential smoothing method, forecast the sales for May, June and July using $\alpha_1 = 0.1$ and $\alpha_2 = 0.8$.

$\boxed{F_{April} = 230}$

✓ $F_t = \alpha \times D_{t-1} + (1-\alpha) \times F_{t-1}$

OR

✓ $F_{t+1} = \alpha D_t + (1-\alpha) F_t$

May April

$\alpha = .1$ $F_{May} = \alpha D_{April} + (1-\alpha) F_{April}$

$F_{May} = 0.1 \times 310 + (1-0.1)\, 230 = 238$

$F_{Jun} = 0.1 \times 370 + (0.9)\, 238 = 251.2$

$F_{July} = 0.1 \times 380 + (0.9)\, 251.2 = 264.08$

Month	January	February	March	April	May	June	July
Sales ($100)	180	220	290	310	370	380	

(1b) Considering the forecast of April calculated in question (1a) as F_{April}, using exponential smoothing method, forecast the sales for May, June and July using α_1 = 0.1 and α_2 = 0.8.

$a = 0.8$

$$F_{May} = \alpha D_{April} + (1-\alpha) F_{April}$$

$$F_{May} = 0.8 \times 310 + \underbrace{(1-0.8)}\, 230 = \boxed{294}$$

$$F_{Jun} = 0.8 \times 370 + 0.2 \times 294 = 354.8$$

$$F_{July} = 0.8 \times 380 + 0.2 \times 354.8 = 374.96$$

(1b) Considering the forecast of April calculated in question (1a) as F_{April}, using exponential smoothing method, forecast the sales for May, June and July using $\alpha_1 = 0.1$ and $\alpha_2 = 0.8$.

For $\alpha = 0.1$

$$F_{April} = 230$$

$$F_t = \alpha \times D_{t-1} + (1-\alpha) \times F_{t-1}$$

$$(1)\ F_{May} = 0.1 \times 310 + 0.9 \times 230 = 238$$

$$(2)\ F_{June} = 0.1 \times 370 + 0.9 \times 238 = 251.2$$

$$(3)\ F_{July} = 0.1 \times 380 + 0.9 \times 251.2 = 264.08$$

For $\alpha = 0.8$

$$(1)\ F_{May} = 0.8 \times 310 + 0.2 \times 230 = 294$$

$$(2)\ F_{June} = 0.8 \times 370 + 0.2 \times 294 = 354.8$$

$$(3)\ F_{July} = 0.8 \times 380 + 0.2 \times 354.8 = 374.96$$

$$F_t = \alpha \times D_{t-1} + (1-\alpha) \times F_{t-1}$$

OR

$$\boldsymbol{F_{t+1} = \alpha D_t + (1-\alpha) F_t}$$

	Month	January	February	March	April	May	June	July
1	Actual Sales ($100)	180	220	290	310	370	380	
1b	Forecasted ES ($\alpha_1 = 0.1$)					238	251.2	264.08
1b	Forecasted ES ($\alpha_2 = 0.8$)					294	354.8	374.96

VERY IMPOERTANT Note: 1200

In **MA** the forecast is the *average* of the most recent $n$data points.

$$F_{t+1} = \frac{D_t + D_{t-1} + D_{t-2} + \cdots + D_{t-n+1}}{n} = \frac{\overset{1500}{D_t}}{3} + \frac{\overset{2000}{D_{t-1}}}{3} + \frac{\overset{1500}{D_{t-1}}}{3}$$

So, each of the past n data points are equally important and have **equal weight = 1/n** & Older data points (beyond the window n) have **no effect** at all.

$$1500 - 1200 = \frac{300}{3} = 100$$

In **ES**, forecast is

$$F_{t+1} = \alpha D_t + (1-\alpha)\alpha D_{t-1} + (1-\alpha)^2\alpha D_{t-2} + (1-\alpha)^3 F_{t-2}$$

So, **past data are not equally important,** the **further back** a data point is, the **smaller its influence** on the current forecast. That influence (or weight) shrinks exponentially using this formula:

$$\boxed{\text{weight} = \alpha(1-\alpha)^n}$$

$$\text{weight} = \boxed{\alpha(1-\alpha)^3} =$$

n = number of **periods ago** the value occurred.

$\alpha = 0.1$ $\quad 0.1(1-0.1)^3 = 10.05\%$

This means: the data point from **n years ago** still has about $\alpha(1-\alpha)^n$ **%** influence on the current forecast. It has **diminishing influence**.

(1c) Assuming that the slope and the intercept of the regression of the given data is 42 and 145 respectively, estimate the sales in May, June and July by regression method.

$y = a + bx$ → Number month/year/day

a ↓ intercep

b → slop

$$y = a + bx$$

x

1

2

3

4

5

$y = 145 + 42x$

$F_{May} = y = 145 + 42 \times 5 =$

$F_{Jun} =$ 6 =

F_{July} 7 =

(1c) Assuming that the slope and the intercept of the regression of the given data is 42 and 145 respectively, estimate the sales in May, June and July by regression method.

$$\boldsymbol{y = a + bx}$$

$$\begin{cases} a = 42 \\ b = 145 \end{cases}$$

$$(1)\ F_{May} = 42 \times 5 + 145 = 355$$

$$(2)\ F_{June} = 42 \times 6 + 145 = 397$$

$$(3)\ F_{July} = 42 \times 7 + 145 = 439$$

	Month	January	February	March	April	May	June	July
1	Actual Sales ($100)	180	220	290	310	370	380	
1c	Forecasted Regression					355	397	439

(1d) Given the information that the income in July in 100 dollars is 376. Using MAD through May, June and July, determine whether regression gave you a better forecast or the exponential smoothing using α_2 = 0.8. (only over these 3 months)

$$MAD = \frac{1}{n}\sum_{t=1}^{n}|F_t - D_t|$$

	Month	January	February	March	April	May	June	July
1	Actual Sales ($100)	180	220	290	310	370	380	376
1a	Forecasted MA (n=3)				230	273.33	323.33	
1b	Forecasted ES (α_1 = 0.1)					238	251.2	264.08
1b	Forecasted ES (α_2 = 0.8)					294	354.8	374.96
1c	Forecasted Regression					355	397	439

$$MAD_{\alpha=0.8} = \frac{|294 - 370| + |354.8 - 380| + |374.96 - 376|}{3}$$

$$MAD_{Reg} = \frac{|355 - 370| + |397 - 380| + |439 - 376|}{3}$$

(1d) Given the information that the income in July in 100 dollars is 376. Using MAD through May, June and July, determine whether regression gave you a better forecast or the exponential smoothing using α_2 = 0.8. (only over these 3 months)

For exponential smoothing with $\alpha = 0.8$:

$$\frac{|294 - 370| + |354.8 - 380| + |374.96 - 376|}{3} = \underline{34.08}$$

For regression:

$$\frac{|355 - 370| + |397 - 380| + |439 - 376|}{3} = \underline{31.66}$$

Regression gave better results.

Problem 2

Product XYZ was introduced at the beginning of 2019. The sales data for this product is given until October as in the following table.

Month	**Sale Amount**
January	138
February	231
March	351
April	421
May	601
June	625
July	711
August	856
September	975
October	1014

(2a) Considering that the slope and intercept for this data set is equal to 100 and 42 respectively, calculate the estimated sales for the next 2 months: November and December of 2019.

(2b) Now assume that the actual demand for November and December of 2019 is 1140 and 1230 respectively. Assuming that $Ssept$ = 900 and $Gsept$ = 100, using a one step ahead double exponential smoothing (Holt's method), with α = 0.2 and β = 0.1 estimate the sales of November and December.

(2c) Using the Slope and Intercept that you found in part 2.b for December, using a two step ahead double exponential smoothing, forecast the sales of February 2020. (α = 0.2, β = 0.1)

(2d) Given the real sales for November and December in part 2.b compare the estimations of the regression (part 2.a) and double exponential smoothing (part 2.b) using MAD, MSE methods.

(2a) Considering that the slope and intercept for this data set is equal to $\boxed{100 \text{ and } 42}$ respectively, calculate the estimated sales for the next 2 months: November and December of 2019.

Month	Sale Amount	Forecasted sale Regression
January	138	
February	231	
March	351	
April	421	
May	601	
June	625	
July	711	
August	856	
September	975	
October	1014	
November		1142
December		1242

$$y = a + bx$$

$$F_{Nov} = 42 + 100 \times 11$$

$$F_{Dec} = 42 + 100 \times 12$$

(2a) Considering that the slope and intercept for this data set is equal to 100 and 42 respectively, calculate the estimated sales for the next 2 months: November and December of 2019.

Month	**Sale Amount**	Forecasted sale Regression
January	138	
February	231	
March	351	
April	421	
May	601	
June	625	
July	711	
August	856	
September	975	
October	1014	
November		1142
December		1242

$$y = a + bx$$

2.a) (Regression Method)
Slope = 100, Intercept = 42 →

$$F_{Nov} = 11 \times 100 + 42 = 1142, F_{Dec} = 12 \times 100 + 42 = 1242$$

Problem & Solution

(2b) Now assume that the actual demand for November and December of 2019 is 1140 and 1230 respectively. Assuming that $Ssept = 900$ and $Gsept = 100$, using a one step ahead double exponential smoothing (Holt's method), with $\alpha = 0.2$ and $\beta = 0.1$ estimate the sales of November and December.

Month	Sale Amount	Forecasted sale Holt's
January	138	
February	231	
March	351	
April	421	
May	601	
June	625	
July	711	
August	856	
September	975	
October	1014	
November	1140	
December	1230	

intercept → slop

$F_{t+1} = S_t + G_t$ **(one step ahead)**

$F_{t,t+k} = S_t + kG_t$ **(k-steps ahead)**

$S_t = \alpha D_t + (1-\alpha)(S_{t-1} + G_{t-1})$

$G_t = \beta(S_t - S_{t-1}) + (1-\beta)G_{t-1}$

t+1 = Nov

t = Oct

$F_{Nov} = S_{Oct} + G_{Oct}$

$\boxed{F_{Dec} = S_{Nov} + G_{Nov}}$

1002.8

$S_{Oct} = 0.2(1014) + (.8)(900+100) =$

t = Oct

t-1 = Sep

$G_{Oct} = 0.1(1002.8 - 900) + 0.9(100) =$

100.28

$F_{Nov} = 1002.8 + 100.28 = 1103.08$

$$F_{t+1} = S_t + G_t$$

$$F_{Nov} = S_{Oct} + G_{Oct}$$

$$F_{Dec} = S_{Nov} + G_{Nov}$$

$$S_t = \alpha D_t + (1-\alpha)(S_{t-1} + G_{t-1})$$

$$G_t = \beta(S_t - S_{t-1}) + (1-\beta)G_{t-1}$$

$t+1 = Dec \quad t = Nov$

$$F_{Dec} = \underline{S_{Nov}} + \underline{G_{Nov}}$$

$$S_{Nov} = \alpha D_{Nov} + (1-\alpha)(S_{Oct} + G_{Oct})$$

$$S_{Nov} = 0.2 \times 1140 + (0.8)(1002.8 + 1002.8) = 1110.464$$

$+$

$$G_{Nov} = 0.1 \times (1110.464) + 0.9(100.28) = 101.018$$

$$F_{Dec} = 1211.482$$

(2b) Now assume that the actual demand for November and December of 2019 is 1140 and 1230 respectively. Assuming that $Ssept$ = 900 and $Gsept$ = 100, using a one step ahead double exponential smoothing (Holt's method), with α = 0.2 and β = 0.1 estimate the sales of November and December.

Month	**Sale Amount**	Forecasted sale Holt's
January	138	
February	231	
March	351	
April	421	
May	601	
June	625	
July	711	
August	856	
September	975	
October	1014	
November	1140	1103.08
December	1230	1211.482

2.b) (Holt's Method One Step ahead)

$$S_{Sept} = 900, G_{Sept} = 100 \text{ (September is assumed to be period 0)}$$

$$S_t = \alpha D_t + (1-\alpha)(S_{t-1} + G_{t-1})$$
$$G_t = \beta(S_t - S_{t-1}) + (1-\beta)G_{t-1}$$

$$S_{Oct} = 0.2 \times 1014 + 0.8 \times (900 + 100) = 1002.8$$
$$G_{Oct} = 0.1 \times (1002.8 - 900) + 0.9 \times 100 = 100.28$$
$$F_{Nov} = 1002.8 + 100.28 = 1103.08$$

$$S_{Nov} = 0.2 \times 1140 + 0.8 \times (1103.08) = 1110.464$$
$$G_{Nov} = 0.1 \times (1110.464 - 1002.8) + 0.9 \times 100.28 = 101.018$$
$$F_{Dec} = 1110.464 + 101.018 = 1211.482$$

(2c) Using the Slope and Intercept that you found in part (2b) for December, using a two step ahead double exponential smoothing, forecast the sales of February 2020. (α = 0.2, β = 0.1)

Month	**Sale Amount**	Forecasted sale Holt's
January	138	
February	231	
March	351	
April	421	
May	601	
June	625	
July	711	
August	856	
September	975	
October	1014	
November	1140	1103.08 ✓
December ✓	1230	1211.482 ✓
January		
February		

$F_{t+1} = S_t + G_t$ **(one step ahead)**

$\boxed{F_{t,t+k} = S_t + kG_t}$ **(k-steps ahead)**

$F_{\underline{Dec}, \underline{Feb}} = \boxed{S_{Dec}} + 2\boxed{G_{Dec}}$

$F_{\boxed{Dec}, \boxed{Feb}}$

(2c) Using the Slope and Intercept that you found in part (2b) for December, using a two step ahead double exponential smoothing, forecast the sales of February 2020. (α = 0.2, β = 0.1)

Month	Sale Amount	Forecasted sale Holt's
January	138	
February	231	
March	351	
April	421	
May	601	
June	625	
July	711	
August	856	
September	975	
October	1014	
November	1140	1103.08
December	1230	1211.482
January		
February		1417.962

$\boxed{F_{Dec}} = S_{Nov} + G_{Nov}$ these are slope and intercept for december that you use in 2 step ahead.

intercept, slop

$F_{t+1} = S_t + G_t$ **(one step ahead)**

$F_{t,t+k} = S_t + kG_t$ **(k-steps ahead)**

2.c) (Holt's Method K-Steps ahead)

$$S_t = \alpha D_t + (1-\alpha)(S_{t-1} + G_{t-1})$$
$$G_t = \beta(S_t - S_{t-1}) + (1-\beta)G_{t-1}$$

$$S_{Dec} = 0.2 \times 1230 + 0.8 \times 1211.482 = 1215.186$$
$$G_{Dec} = 0.1 \times (1215.186 - 1110.464) + 0.9 \times 101.018 = 101.388$$
$$F_{Feb} = \underline{1215.186} + 2 \times \underline{101.388} = 1417.962$$

(2d) Given the real sales for November and December in part 2b compare the estimations of the regression (part 2a) and double exponential smoothing (part 2b) using MAD, MSE methods.

Month	Actual Sale	Forecasted Sale Regression (2a)	Forecasted Sale Holt's Method (2b)
November	1140	1142	1103.08
December	1230	1242	1211.482

$$MAD = \frac{1}{n}\sum_{t=1}^{n}|F_t - D_t|$$

$$MSE = \frac{1}{n}\sum_{t=1}^{n}(F_t - D_t)^2$$

$$MAD_{Reg} = \frac{|1142-1140| + |1242-1230|}{2} = 7$$

$$\rightarrow MAD_{Holts} = \frac{|1211.482-1230| + |1103.08-1140|}{2} = 27.71$$

$$MSE_{Rg} = \frac{(1142-1140)^2 + (1242-1230)^2}{2} = 74$$

$$MSE_{Holt} = \frac{(1211.482-1230)^2 + (1103.08-1140)^2}{2} = 853.04$$

(2d) Given the real sales for November and December in part 2b compare the estimations of the regression (part 2a) and double exponential smoothing (part 2b) using MAD, MSE methods.

$$MAD = \frac{1}{n}\sum_{t=1}^{n}|F_t - D_t|$$

$$MSE = \frac{1}{n}\sum_{t=1}^{n}(F_t - D_t)^2$$

2.d) (Evaluating Errors)

Month	Actual Demand	Regression (2.a)	Holt's Method (2.b)
November	1140	1142	1103.08
December	1230	1242	1211.482

MAD for regression: $\frac{|1140-1142|+|1230-1242|}{2} = 7$

MAD for Holt's method: $\frac{|1103.08-1140|+|1211.48-1230|}{2} = 27.71$

MSE for regression: $\frac{(1140-1142)^2+(1230-1242)^2}{2} = 74$

MSE for Holt's method: $\frac{(1103.08-1140)^2+(1211.48-1230)^2}{2} = 853.04$

Problem 3

Assume that the demand of a product has the following data:

Period

Year	Season	Demand
1	1 Period 1	15
	2 Period 2	24
	3 3	36
	4 4	30
2	1	21
	2	27
	3	51
	4	36

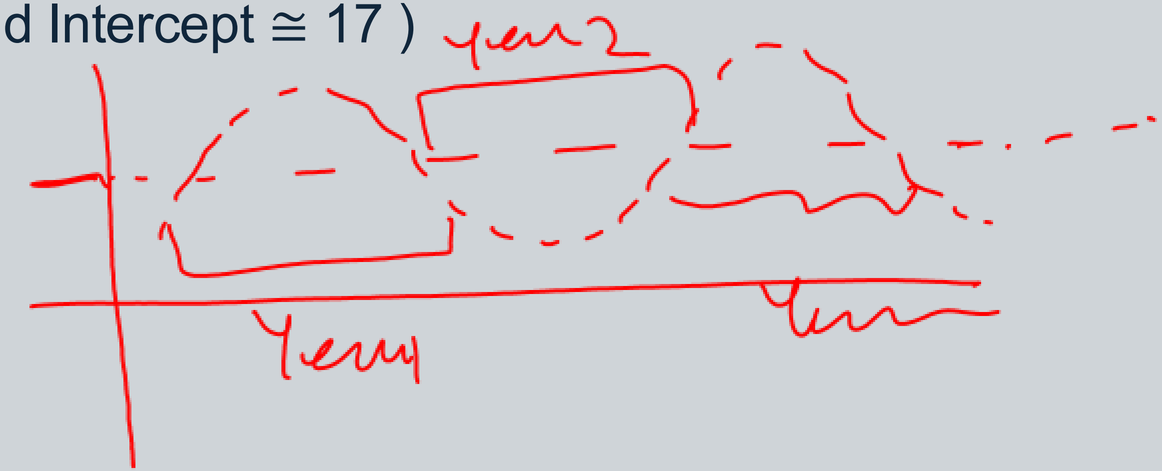
Season 1

Season 2

(3a) Assuming that the data is seasonal but stationary, calculate the seasonal factors for each period of the season.

(3b) With the same assumptions from part 3.a, find the corresponding deseasonalized series of the given data.

(3c) Assuming that the data has both seasonality and trends, Calculate the Adjusted Regression forecast for the first 3 years. (Assume Slope ≅ 3 and Intercept ≅ 17)



(3a) Assuming that the data is seasonal but stationary, calculate the seasonal factors for each period of the season.

$\mu = 30$

STEPS:

1) Compute the sample mean
2) Divide each observation by mean to obtain corresponding seasonal factor
3) Average the factors for like periods to obtain seasonal factors

Year	Season	Demand	Corresponding seasonal factors	Seasonal Factor
season 1	1 Period	15 / 30		(0.5 + 0.7) / 2
	2	24 / 30		(0.8 + 0.9) / 2
	3	36 / 30		
	4	30		
2	1	21		
	2	27		
	3	51		
	4	36		

(3a) Assuming that the data is seasonal but stationary, calculate the seasonal factors for each period of the season.

STEPS:

1) Compute the sample mean
2) Divide each observation by mean to obtain corresponding seasonal factor
3) Average the factors for like periods to obtain seasonal factors

3.a) The average of the data is 30.

Corresponding seasonal factors are as following:

$$Average = 30 \rightarrow \begin{cases} season\ 1: & 0.5, \quad 0.8, \quad 1.2, \quad 1 \\ season\ 2: & 0.7, \quad 0.9, \quad 1.7, \quad 1.2 \end{cases}$$

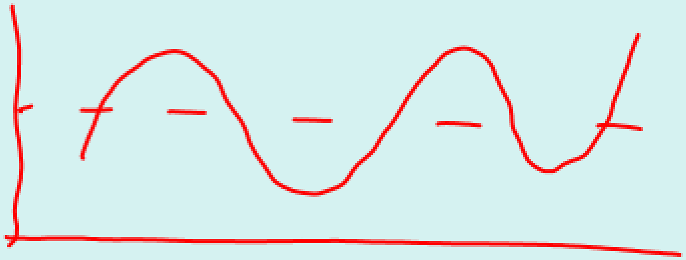
$$\rightarrow seasonal\ factors: 0.6, \quad 0.85, \quad 1.45, \quad 1.1$$

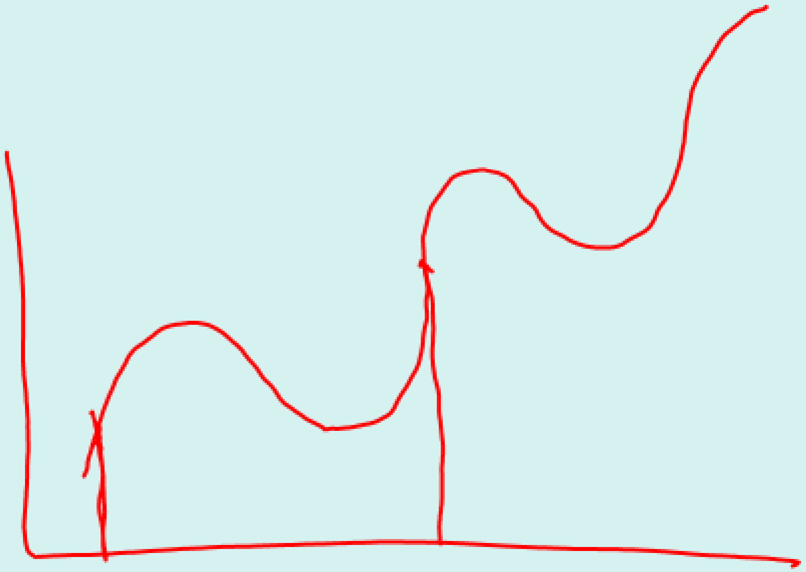
(3b) With the same assumptions from part (3a), find the corresponding deseasonalized series of the given data.

Deseasonalised data= Data/seasonal factor

Year	Season	Demand	Seasonal Factor	Deseasonalized
1	1	15	0.6	15/0.6
	2	24	0.85	24/0.85
	3	36	1.45	
	4	30	1.1	
2	1	21	0.6	
	2	27	0.85	25/
	3	51	1.45	26
	4	36	1.1	

(3b) With the same assumptions from part (3a), find the corresponding deseasonalized series of the given data.





3.b) The deseasonalized data is calculated by dividing the data points over their corresponding seasonal factor.

Year	Season	Demand	Seasonal Factor	Deseasonalized
1	1	15	0.6	25
	2	24	0.85	28.24
	3	36	1.45	24.83
	4	30	1.1	27.27
2	1	21	0.6	35
	2	27	0.85	31.76
	3	51	1.45	35.17
	4	36	1.1	32.73

α

(3c) Assuming that the data has both seasonality and trends, Calculate the Adjusted Regression forecast for the first 3 years. (Assume Slope ≅ 3 and Intercept ≅ 17)

$$y = a + bx$$

Adjusted Regression= Unadjusted regression x Seasonal Factors

$y = 17 + 3x$

Deseasonlised

Year	Season	Demand	Unadjusted Regression	Demand/ Forecast (Reg)	Seasonal Factor	Adjusted Regression
1	1	15	20	15/20		20x0.7
	2	24		24/23		23x0.[illegible]
	3	36				
	4	30				
2	5 1	21	17+5x3			
	6 2	27				
	7 3	51				
	8 4	36				
3	9 1	?		?		?
	10 2	?		?		?
	11 3	?		?		?
	12 1	?		?		?

Year	Season	Demand	Unadjusted Forecast	Demand/ Forecast	Seasonal Factor	Adjusted Regression
1	1	15	20	0.75	0.70	14.06
	2	24	23	1.04	0.91	20.87
	3	36	26	1.38	1.36	35.45
	4	30	29	1.03	0.96	27.73
2	5	21	32	0.66	0.70	22.50
	6	27	35	0.77	0.91	31.76
	7	51	38	1.34	1.36	51.81
	8	36	41	0.88	0.96	39.21
3	9		44		0.70	30.94
	10		47		0.91	42.65
	11		50		1.36	68.17
	12		53		0.96	50.68

(3c) Assuming that the data has both seasonality and trends, Calculate the Adjusted Regression forecast for the first 3 years. (Assume Slope ≅ 3 and Intercept ≅ 17)

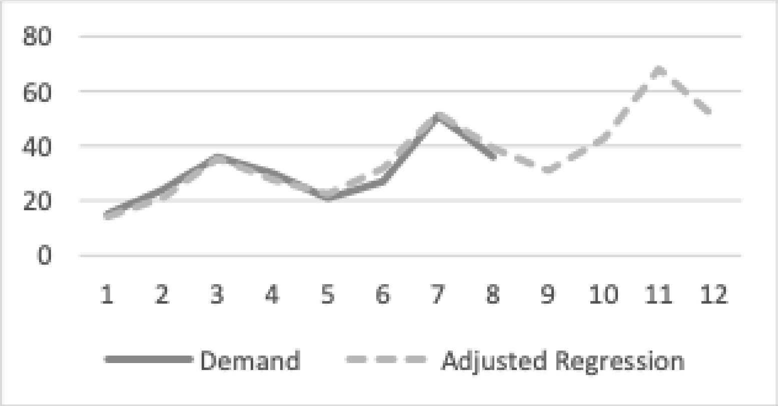
$$y = a + bx$$

Adjusted Regression=
Unadjusted regression x Seasonal Factors

3.c)

Year	Season	Demand	Un adjusted	demand/forecast	Seasonal Factor	Adjusted Regression
1	1	15	20	0.75	0.70	14.06
	2	24	23	1.04	0.91	20.87
	3	36	26	1.38	1.36	35.45
	4	30	29	1.03	0.96	27.73
2	5	21	32	0.66	0.70	22.50
	6	27	35	0.77	0.91	31.76
	7	51	38	1.34	1.36	51.81
	8	36	41	0.88	0.96	39.21
3	9		44		0.70	30.94
	10		47		0.91	42.65
	11		50		1.36	68.17
	12		53		0.96	50.68

Following graph shows the performance of the adjusted forecast.



Problem & Solution

University of Nottingham
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Thank you