

# N14C47 Supply Chain Planning & Management Planning

## Formula Sheet

### FORECASTING

#### Stationary series:

*Moving average (of order N):*  $F_{t+1} = (1/N) \sum_{i=t-N+1}^t D_i$

*Simple exponential smoothing:*

$$F_{t+1} = \alpha D_t + (1-\alpha) F_t \text{ or } F_{t+1} = F_t - \alpha e_t \text{ with } e_t = F_t - D_t$$

*Multiple-step-ahead forecasts (with moving averages or simple exponential smoothing):* the one-step-ahead and multiple-step-ahead forecasts are the same (i.e.,  $F_{t,t+k} = F_{t,t+1}$ )

#### Series with (linear) trend:

*Holt's double exponential smoothing:*

$$S_t = \alpha D_t + (1-\alpha)(S_{t-1} + G_{t-1}) \text{ and } G_t = \beta(S_t - S_{t-1}) + (1-\beta)G_{t-1}$$

$$F_{t,t+k} = S_t + kG_t$$

**Errors and accuracy:** ( $e_1, \dots, e_n$  the forecast errors observed over  $n$  periods)

*Error (one-step-ahead forecasts):*  $e_t = F_t - D_t$

*Mean absolute deviation:*  $MAD = (1/n) \sum_{i=1}^n |e_i|$

*Mean squared error:*  $MSE = (1/n) \sum_{i=1}^n e_i^2$

*Mean absolute percentage error:*  $MAPE = \left[ (1/n) \sum_{i=1}^n |e_i / D_i| \right] \times 100$

### INVENTORY CONTROL (DETERMINISTIC)

#### Economic order quantity:

*Cost function for EOQ:*  $G(Q) = \frac{K\lambda}{Q} + \lambda c + \frac{hQ}{2}$  (with  $h = Ic$ )

*Economic order quantity:*  $EOQ = Q^* = \sqrt{\frac{2K\lambda}{h}}$

*Optimal cost (ignoring the term  $\lambda c$ ):*  $G(EOQ) = \sqrt{2K\lambda h}$

#### Production order quantity:

*Cost function for POQ (ignoring the constant term  $\lambda c$ ):*

$G(Q) = \frac{K\lambda}{Q} + \frac{hQ}{2}(1 - \lambda/P)$  (with  $h = Ic$ )

*Production order quantity:*  $POQ = \sqrt{\frac{2K\lambda}{h'}} \text{ with } h' = h(1 - \lambda/P)$

*Optimal cost (ignoring the term  $\lambda c$ ):*

$G(POQ) = \sqrt{2K\lambda h(1 - \lambda/P)}$

### Quantity discount models:

$$\text{Cost functions all-units discounts: } G_j(Q) = \lambda c_j + \frac{K\lambda}{Q} + \frac{Ic_j Q}{2}$$

*Cost functions incremental discounts:*

$G_j(Q) = \lambda C_j(Q)/Q + K\lambda/Q + I[C_j(Q)/Q]Q/2$  with  $C_j(Q)/Q$  the per unit (average) purchase price when ordering in batches of size  $Q$

## INVENTORY CONTROL (STOCHASTIC)

### The ( $R$ = reorder point, $Q$ = order quantity) model:

If demand during lead time ( $L$ ) is normally distributed with mean  $\lambda L$  and variance  $\sigma^2$ :

Type 1 service level model:

$$Q = EOQ \text{ and } R = \lambda L + z\sigma \text{ with } F(R) = F(z) = \alpha$$

( $z, F(z)$ ) can be read from the standard normal distribution table

Type 2 service level model:

$$Q = EOQ \text{ and } R = \lambda L + z\sigma \text{ with } n(R) = Q(1 - \beta)$$

$$\text{and } n(R) = \sigma L(z)$$

( $z, L(z)$ ) can be read from the standard normal distribution table

*Scaling of lead time demand:* Assume the periodic demand has mean  $\lambda$  and variance  $\sigma^2$  and let  $L$  be the lead time in periods, then the mean demand during lead time is  $\lambda L$  and the variance of demand during lead time is  $\sigma^2 L$

### Costs ( $R, Q$ ) model:

Holding cost:  $h(Q/2 + R - \lambda L)$

Ordering cost:  $K\lambda/Q$

Penalty cost:  $P\lambda n(R)/Q$

## Lotsizing MRP

Silver – Meal:

$$\begin{aligned} C(1) &= K \\ C(j) &= (j-1)[C(j-1) + hd_j] / j \quad \text{for } j > 1 \end{aligned}$$

Least unit cost:

$$\begin{aligned} C(1) &= K/d_1 \\ C(j) &= [(d_1 + d_2 + \dots + d_{j-1})C(j-1) + (j-1)hd_j]/(d_1 + d_2 + \dots + d_j) \end{aligned}$$