

## Worksheet Three: Aggregate Planning

*Please attempt the questions before the session and be prepared to share your solutions.*

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**Question One**

Yog-to-go is a small business that produces high protein yogurts. Their demand fluctuates seasonally due to varying customer preferences. The company needs to create a 6-month aggregate production plan to meet demand while minimising costs. The company currently employs 18 workers, and each worker produces 15 yogurts per day. The hiring cost per worker is £350 and the firing cost is £725 per worker. The inventory holding cost per unit per month is £0.75. The regular production cost is £3.75 per unit. The demand and the number of production days per month are presented in the table below.

Month	Demand	Production Days
January	5,000	21
February	7,500	20
March	6,600	24
April	7,900	23
May	7,750	21
June	8,150	19

Calculate the following for Yog-to-go and show your calculations steps:

**a) Using the minimum constant workforce strategy first:**

First, we note that typically aggregate planning requires us to identify an aggregate unit, based on a common unit. But here we only have one product, demand is given in units of yogurt so the question is trivial. Instead, we set aggregate unit = demand. *Note this is not typical in aggregate planning.*

**Using the minimum constant workforce:** We need to find the minimum number of workers required by comparing the cumulative demand and cumulative production.

**Cumulative demand:** To find cumulative demand, we add demand from the previous period to the demand from the current period

**Cumulative production days:** Following the same process we calculate cumulative production days

**Cumulative production per worker:** multiply the number of units per worker by the cumulative production days

**Number of workers required:** divide cumulative demand by cumulative production per worker

Month	Cumulative demand	Cumulative Production days	Cumulative production per worker	Number of workers required	Number of workers required (round)
January	5000	21	$21 * 15 = 315$	$5000 / 315 = 15.873$	16
February	$5000 + 7500 = 12500$	$21 + 20 = 41$	$41 * 15 = 615$	$12500 / 615 = 20.325$	21
March	$12500 + 6600 = 19100$	$41 + 24 = 65$	$65 * 15 = 975$	$19100 / 975 = 19.59$	20
April	$19100 + 7900 = 27000$	$65 + 23 = 88$	$88 * 15 = 1320$	$27000 / 1320 = 20.455$	21
May	$27000 + 7750 = 34750$	$88 + 21 = 109$	$109 * 15 = 1635$	$34750 / 1635 = 21.254$	22
June	$34750 + 8150 = 42900$	$109 + 19 = 128$	$128 * 15 = 1920$	$42900 / 1920 = 22.344$	23

From the table we can see that **23 is the highest number of workers required**.

We currently employ 18 workers, so we need to **hire 5 workers**.

With 5 workers, we can now calculate our production per worker, and resulting inventory:

**Total Production:** production per worker multiplied by number of workers

**Inventory:** in January, inventory = production **subtract** demand

In remaining months, inventory = inventory from previous month **add** production **subtract** demand

Month	Demand	Production per worker	No of workers	Total production	Inventory
January	5000	315	23	$315 * 23 = 7245$	$7245 - 5000 = 2245$
February	7500	300	23	$300 * 23 = 6900$	$2245 + 6900 - 7500 = 1645$
March	6600	360	23	$360 * 23 = 8280$	$1645 + 8280 - 6600 = 3325$

April	7900	345	23	$345 \times 23 = 7935$	$3325 + 7935 - 7900 = 3360$
May	7750	315	23	$315 \times 23 = 7245$	$3360 + 7245 - 7750 = 2855$
June	8150	285	23	$285 \times 23 = 6555$	$2855 + 6555 - 8150 = 1260$
				44160	14690

b) Calculate the total cost of the plan found in part a).

Costs

Hiring costs: £1,750.00  
 Firing cost: £0.00  
 Production costs: £165,600.00  
 Inventory cost: £11,017.50

Total cost is thus: £178,367.50

### Question Two

A furniture company produces three products: sofa, recliner and footstools. The forecasts over the next 5 months are shown in the table below. Production is as follows: sofas require 5 labour hours, recliners require 4 hours and footstools require 2 hours. There are 20, 18, 23, 15 and 23 working days in the months January through to May respectively.

Month	Sofa	Recliner	Footstools
Jan	10	100	130
Feb	60	90	110
March	75	55	170
April	35	75	135
May	10	50	150

The company currently employs 50 workers that each work 8 hours per day. Workers are paid £1750 per month. Hiring costs are £600 and firing costs are £1750. The inventory holding cost is £2 per aggregate unit of production per month.

- a) Convert the product forecasts into an aggregate forecast, clearly explain how you defined an aggregate unit of production.

Here we note that we have three different items, but they all require labour in their production so we can define our aggregate unit as labour hour. Thus, noting that sofas require 5 labour hours, recliners require 4 labour house and footstools require 1 labour hour we can calculate aggregate demand as follows:

		Demand		Aggregate Demand			
Month	Sofa	Recliner	Footstools	Sofa	Recliner	Footstools	Total aggregate demand
Jan	10	100	130	$5 \times 10 = 50$	$4 \times 100 = 400$	260	710
Feb	60	90	110	$5 \times 60 = 300$	$4 \times 90 = 360$	220	880
March	75	55	170	$5 \times 75 = 375$	$4 \times 55 = 220$	340	935
April	35	75	135	$5 \times 35 = 175$	$4 \times 75 = 300$	270	745
May	10	50	150	$5 \times 10 = 50$	$4 \times 50 = 200$	300	550

- b) How many workers should be working each month to most closely match the aggregate forecasts from part a)? Compute the total cost of the plan.

Next, we want to find the chase strategy, i.e. changing workforce levels on a monthly basis by hiring and firing workers to match demand as close as possible. First we need to find the production per worker, then we find the workers required by dividing the aggregate demand by the production per worker. This is done as follows:

Month	Aggregate demand	Working days	Production per worker	Workers required	Workers rounded	Workers hired	Workers fired
Jan	710	20	$8 \times 20 = 160$	$710/160 = 4.4375$	5	0	45
Feb	880	18	$8 \times 18 = 144$	$880/18 = 6.111$	7	2	0
March	935	23	$8 \times 13 = 184$	$935/184 = 5.082$	6	0	1
April	745	15	$8 \times 15 = 120$	$745/120 = 6.208$	7	1	0

May	550	23	$8 \times 23 = 184$	$550 / 184 = 2.989$	3	0	4
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Next, we need to find the costs associated with the plan. We have the following costs:

Hiring:  $3 \times 600 = \text{£}1800$   
 Firing:  $50 \times 1750 = \text{£}87,500$   
 Production:  $(5+7+6+7+3) \times 1750 = \text{£}49,000$   
 Inventory:  $1661 \times 2 = \text{£}3,322$

Which gives us a total cost of **£141,622**

- c) Assuming that stockouts are not allowed, determine a minimum constant workforce plan using the aggregate forecasts from part (a). Calculate also the cost of the plan and compare with plan (b).

Month	Demand	Cumulative demand	Production per worker	Cumulative production	Workers required	Workers rounded
Jan	710	710	160	160	$810/160 = 4.4375$	5
Feb	880	$710+880 = 1590$	144	$160+144 = 304$	$1590/304 = 5.230$	6
March	935	$1590 + 935 = 2525$	184	$304 + 184 = 488$	$2525/488 = 5.174$	6
April	745	$2525+745 = 3270$	120	$488 + 130 = 608$	$3270/608 = 5.378$	6
May	550	$3270 + 550 = 3820$	184	$608 + 184 = 792$	$3820/792 = 4.823$	5

We note that the minimum number of workers required is 6 workers. We currently employ 50 workers, so we need to fire 44 workers.

Next, we calculate the total production for 6 workers and the corresponding inventory levels for each month.

Month	Total aggregate demand	Production for	inventory
Jan	6300	9600	3300
Feb	10700	8640	1240
March	11400	11040	880
April	7850	7200	230
May	4500	11040	6770
		47520	12420

Finally, we calculate the costs associated with this plan:

Costs:

Hiring: 0  
 Firing:  $44 \times 1750 = \text{£}77,000$   
 Production:  $6 \times 5 \times 1750 = \text{£}52,500$   
 Inventory:  $2197 \times 2 = \text{£}4,395$

Giving us a total cost of **£133,894**

### Question Three

ThreadKind produces a clothing line using responsibly sourced wool and cotton. At present, the company produces an Organic Wool Cardigan and a Cotton Everyday Tee from responsibly sourced fibres. The predicted demand for these two items over a six-month planning horizon is as follows:

Month	Number of Working Days	Organic Wool Cardigan	Cotton Everyday Tshirt
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1	25	1400	700
2	20	1200	600
3	21	400	200
4	24	500	250
5	18	600	300
6	20	800	400

On average, cardigans require one hour to produce, and tshirts require three hours to produce. All workers are skilled in production of both garments. ThreadKind currently have 20 full-time permanent employees. Permanent employees earn £19 per hour and work 8 hours per day (regular time). They can also do overtime, up to 3 hours per day and during this overtime earn £25 per hour. Recruiting extra full-time permanent workers costs £4250 per worker.

It costs ThreadKind £1.50 to hold one aggregate unit of production in inventory for one month. At the start of month one, ThreadKind holds 300 Cardigans and 150 Tshirts in inventory. There are no ending inventory requirements.

- a) Using the individual product forecasts, calculate the corresponding forecasts for aggregate units of production and clearly explain how you defined the aggregation scheme.

Month	Organic Wool Cardigan	Cotton Everyday Tshirt	Net demand - Cardigan	Net demand - Tshirt	Aggregate Demand
January	1400	700	$1400 - 300 = 1100$	$700 - 150 = 550$	2,750
February	1200	600	1200	600	3,000
March	400	200	400	200	1,000
April	500	250	500	250	1,250

May	600	300	600	300	1,500
June	800	400	800	400	2,000

- b) Ignoring the overtime option for the permanent staff, what would be the size of the permanent workforce required (i.e., the minimum constant workforce plan) to satisfy the demand for the coming six months using regular time only? Calculate the monthly production and inventory levels and all the relevant costs for this plan. Visualise your plan in an appropriate diagram.

Month	Aggregate Demand	Cumulative demand	Working days per month	Cumulative working days	cumulative working hrs	Workers required	Workers rounded
January	2750	2750	25	25	$8 \times 25 = 200$	$2750/200 = 13.75$	14
February	3000	$2750+3000 = 5750$	20	$24+20 = 45$	$8 \times 45 = 360$	$5750/360 = 15.972$	16
March	1000	$5750+1000 = 6750$	21	$45+21 = 66$	$8 \times 66 = 528$	$6750/528 = 12.784$	13
April	1250	$6750+11250 = 8000$	24	$66+24 = 90$	$8 \times 90 = 720$	$8000/720 = 11.111$	12
May	1500	$8000+1500 = 9500$	18	$90+18 = 108$	$8 \times 108 = 864$	$9500/864 = 10.995$	11
June	2000	$9500+2000 = 11500$	20	$108+20 = 128$	$8 \times 128 = 1024$	$11500/1024 = 11.230$	12

Looking at the number of workers required for each month we see that the minimum number of workers required to meet the demand across the period is 16 (i.e, the maximum of “Workers rounded”)

Currently, 20 workers are employed but we only need 16 so we must fire 4 workers.

Month	Production (16 workers)	Inventory
January	$16 \times 200 = 3200$	$3200 - 2750 = 450$
February	$16 \times 160 = 2560$	$450 + 2560 - 3000 = 10$
March	$16 \times 168 = 2688$	$10 + 2688 - 1000 = 1698$



April	$16 \times 192 = 3072$	$1698 + 3072 - 1250 = 3520$
May	$16 \times 144 = 2304$	$3520 + 2304 - 1500 = 4324$
June	$16 \times 160 = 2560$	$4324 + 2560 - 2000 = 4884$

**costs**

hiring: 0

firing: We don't know how much it costs to fire workers!

labour:  $19 \times 16384 = \text{£}204,800$ inventory:  $1.5 \times 10002 = \text{£}22,329$ **Total cost:** £227,129 plus firing cost!