BUSI4496
Supply Chain Planning & Management

Lecture 5 Self Study- Inventory Management
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Why Inventory Management Matters

Inventory: the balance between cost and service

- Inventory is a **buffer** between supply and demand.
- Too little → stockouts, lost sales, penalties.
- Too much → high holding cost, risk of obsolescence.
- SCM objective: right stock, right place, right time, right cost.
- Models help us **structure decisions**.

We're not just doing math, we're learning to manage inventory strategically.

Case study- AeroPart Ltd.

A Single Case, Multiple Models

Aim

We'll use this case to:

- Review the key ideas behind **EOQ**, **POQ** models,
- Explore how **stochastic demand (during lead time) and different service levels and types** affect safety stock and reorder points
- Reflect on **strategic inventory choices** and trade-offs.
- Practice exam question

You'll notice that the core case remains the same**,** what changes is the set of assumptions. This helps you connect the dots between models, rather than treating them as separate topics.

AeroPart Ltd. Company Background

AeroPart Ltd. is a mid-sized manufacturer in the aerospace supply chain, producing precision components used in aircraft assembly. The company's reputation depends on delivering high-quality parts on time to Tier-1 customers.

A critical element in AeroPart's production process is a smart sensor, which must be installed in the components before shipment. Any shortage of this sensor can halt production and delay deliveries. These sensors can be purchased from external suppliers or produced in-house using CNC machines. The company operates 50 weeks per year, with an annual demand of 7,500 units.

While AeroPart has long operated in a stable market, increasing supply chain volatility and fluctuating demand during lead time are now forcing the company to rethink its inventory strategy.

The operations team is tasked with answering a critical question:

How can we ensure continuous supply at the lowest total cost under different scenarios?

Scenario 1: External Supplier

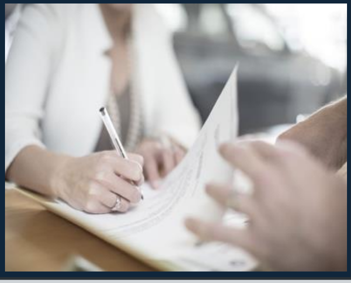
- AeroPart buys a key sensor from an external supplier. Annual demand is steady, the lead time is fixed, and costs are well understood.

The company has annual demand of 7,500 units for this item, with each unit costing £22. The ordering cost is £1,000 per order, and the annual holding cost is determined by an interest rate of 20%. The lead time for replenishment is 2 weeks, and the company operates 50 weeks per year.

Task 1:

When and how much should AeroPart order each time to minimise total inventory cost, while ensuring continuous supply?

Scenario 1: Baseline Case



Given:
- Demand (λ) : 7,500 units/year
- Unit cost (c): £22
- Ordering cost (K): £1,000 per order
- Annual interest rate (i): 20%
- Lead time (τ): 2 weeks
- Operating year: 50 weeks

Assumptions:
- Demand and lead time are constant
- No shortages
- Instant replenishment upon order arrival

Objective:

Determine order quantity for stable demand, reorder point to minimise total annual inventory cost

Formulas

Order Quantity $$Q^* = EOQ = \sqrt{\frac{2K\lambda}{h}}$$

$$h = i \times c$$

Reorder point $$R = \lambda\, \tau$$

Optimal cost $$G(Q*) = \sqrt{2K\lambda h}$$

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Scenario 1: The EOQ model

At this point, demand and reorder points are perfectly predictable. AeroParts simply wants to minimise cost by balancing order and holding cost. This is the most fundamental inventory model.

We know:
$K = 1{,}000$ (ordering cost)
$\lambda = 7{,}500$units/year (annual demand)
$h = i \times c = 0.2 \times 22 = £4.4$per unit per year

Substituting into the EOQ formula: $Q^* = \sqrt{\frac{2(1000)(7500)}{4.4}} \approx 1{,}848$ *units*

This means AeroPart should order about **1,848 units** each time.

We found the economic order quantity, now let's find the reorder point: $R = \lambda\, \tau;\quad \lambda = \frac{7500}{50},\ \tau = 2$

$\Rightarrow R$=7500/50$*$2= 300 units

This means AeroPart should reorder when inventory hits 300 units.

And, finally the total inventory cost of this planning $G(Q*) = \sqrt{2K\lambda h} = \sqrt{2(1000)\,(7500)\,(4.4)}$ = £8124

This means AeroPart inventory cost by this scenario is £81[illegible]4.

📝 **Tip Box: Check units carefully and keep EOQ and R conceptually separate.**

Scenario 2: In-House Production

Historically, AeroPart has relied on a single external supplier. However, as global supply chains become more volatile, the operations team is considering bringing part of the production in-house.

A feasibility study identified two CNC machines with different production rates and setup costs. Producing internally means the company can control lead times but must plan production cycles efficiently to minimise total costs.

Task 2:
If AeroPart chooses to produce in-house, what production lot size and cycle would minimise total cost for each machine option? Which option is more financially feasible for the company: purchasing externally or producing in-house using Machine A or Machine B?

Option	Setup cost (£)	Production rate (units/year)	Demand rate (units/year)	Holding cost (£/unit/year)
Machine A	1,800	12,000	7,500	4.4
Machine B	2,400	9,000	7,500	4.4

Scenario 2: In-House Production

Strategic goal:

- Reduce dependency on external suppliers
- Increase flexibility
- Control cost and lead time
- Maintain or improve service level

📝 Tip Box: P must be greater than demand

Formulas:

$$Q^*_{POQ} = \sqrt{\frac{2K\lambda}{h\left(1 - \frac{\lambda}{P}\right)}}$$

h'= $h\left(1 - \frac{\lambda}{P}\right)$

$$T = \frac{Q^*_{POQ}}{\lambda}$$

Optimal cost $\boldsymbol{G(Q*)}$ = $\sqrt{(\boldsymbol{2K\lambda h'})}$

Scenario 2a: Machine A

Option	Setup cost (£)	Production rate (units/year)	Demand rate (units/year)	Holding cost (£/unit/year)
Machine A	1,800	12,000	7,500	4.4
Machine B	2,400	9,000	7,500	4.4

- Setup cost $K = £1{,}800$,
- Production rate $P = 12{,}000$ (units/year)
- Demand rate $\lambda = 7500$ (units/year)
- Adjustment $1 - \lambda/P = 1 - \frac{7500}{12000} = 0.375$
- Effective holding cost $\boldsymbol{h'} = h(1 - \lambda/P) = 4.40 \times 0.375 = 1.65$

Production Quantity

- $Q*_A = \sqrt{\frac{2(1800)(7500)}{1.65}} = \sqrt{16{,}363{,}636} \approx \boxed{4{,}045 \text{ units}}$

Cycle length

- $T_A = \frac{Q*_A}{\lambda} = \frac{4045}{7500} = 0.539$ years $\approx$ $\boxed{27.0 \text{ weeks}}$

Cost

- $\boldsymbol{G(Q*)} = \sqrt{2(1800)(7500)(1.65)} \approx \boxed{£6{,}675 \text{ per year}}$

Scenario 2b: Machine B

Option	Setup cost (£)	Production rate (units/year)	Demand rate (units/year)	Holding cost (£/unit/year)
Machine A	1,800	12,000	7,500	4.4
Machine B	2,400	9,000	7,500	4.4

- Setup cost $K = £2{,}400$,
- Production rate $P = 9{,}000$ (units/year)
- Demand rate $\lambda = 7500$ (units/year)
- Adjustment $1 - \frac{\lambda}{P} = 1 - \frac{7500}{9000} = 0.1667$
- Effective holding $h' = 4.40 \times 0.1667 \approx 0.7333$

Production Quantity

- $Q_B = \sqrt{\frac{2(2400)(7500)}{0.7333}} = \sqrt{49{,}090{,}909} \approx \boxed{7{,}006 \textit{ units}}$

Cycle length

- $T_B = \frac{Q_B}{\lambda} = \frac{7006}{7500} = 0.934 \textit{ years} \approx \boxed{46.7 \textit{ weeks}}$

Cost

- $\boldsymbol{G(Q*)} = \sqrt{2(2400)(7500)(.733)} \approx \boxed{£5136 \text{ per year}}$

Decision?

To meet the annual demand of 7500 units, which of the scenarios is more cost efficient?

Scenario	Q^* Order/Production Quantity (units)	R Reorder Point (units)	T Cycle length/ Production time (weeks)	$G(Q^*)$ Total Optimal Cost (£)
(1)Buy from external supplier	1,848	300	–	8,124
(2a) In-house production with Machine A	4,045	–	27	6,675
(2b) In-house production with Machine B	7,006	–	47	5,136

Scenario 3: Demand Uncertainty during Lead Time

Recently, AeroPart has begun to experience volatile orders from Tier-1 customers, especially during lead times. While the company’s annual demand remains stable (7500 units) , the timing and order size within the lead time have become less predictable. This variability poses a risk of stock-outs, which can lead to costly delays and disrupt delivery commitments.

To better understand this risk, the operations team analysed historical data and found that demand during lead time follows a normal distribution: $N(\mu = 300 \;\, \sigma = 95)$ This means that, on average, 300 units are needed during each lead time period, but the actual demand can fluctuate around this value.

To manage this uncertainty, AeroPart decides to set clear service level targets that will guide how much safety stock the company should hold to avoid stock-outs.

- **Type I service level (α = 98%)**
  This represents the probability of not running out of stock during lead time. A 98% service level means that in 98 out of 100 replenishment cycles, AeroPart expects to have enough stock to cover lead time demand. Only in about 2 cycles might a stock-out occur.
- **Type II service level (β = 98%)**
  This represents the proportion of total demand met immediately from stock. A 98% fill rate means that 98% of customer demand will be fulfilled without delay, and only a small fraction of 2% may face a short wait.

Task 3:
How can AeroPart set an appropriate reorder point and safety stock level to maintain the desired service level under uncertain demand during lead time?

Scenario 3- (R, Q) model

- We keep **Q from EOQ** for cost efficiency, but calculate **when to reorder (R)** to include a **safety stock (SS)** buffer.
- Lead time remains deterministic: **τ = 2 weeks** (50-week Year)
- Annual demand remains stable: $\lambda =$**7,500 units** (150/week).
- Assumption for the case:

Demand variation during lead time follows the normal distribution: N($\lambda\tau$= 300, σ = 95)

$$R = \lambda\tau + z\,\sigma$$

R= Expected demand during lead time + Safety Stock

📝 **Tip Box: Identify service type (Type I or II). Don't confuse z and Q.**

📝 **Tip Box: Use EOQ for Q and add safety stock for R.**

Scenario 3a- Type-1 Service

- Type 1 service level: probability of no stock-out during lead time (α = 98%)
- Use the **z-value** from normal distribution table

$$SS = z\,\sigma\;,R = \lambda\tau + z\,\sigma$$

- AeroParts numbers (α = **0.98**, $z = 2.05$): SS= $\sigma\;z$
  - $\lambda\tau = 150 \times 2 = 300$
  - $SS = 2.05 \times 95 \approx 195$
  - $\boxed{R_\alpha \approx 300 + 195 = 495\ \mathit{units}}$

Standardized Normal Probability Distribution F(z) (cumulative) and Partial Expectations L(z)

z	F(z)	L(z)	z	F(z)	L(z)	z	F(z)	L(z)	z	F(z)	L(z)	z	F(z)	L(z)
0.00	0.5000	0.3989	0.50	0.6915	0.1978	1.00	0.8413	0.0833	1.50	0.9332	0.0293	2.00	0.9772	0.0085
0.01	0.5040	0.3940	0.51	0.6950	0.1947	1.01	0.8438	0.0817	1.51	0.9345	0.0286	2.01	0.9778	0.0083
0.02	0.5080	0.3890	0.52	0.6985	0.1917	1.02	0.8461	0.0802	1.52	0.9357	0.0280	2.02	0.9783	0.0080
0.03	0.5120	0.3841	0.53	0.7019	0.1887	1.03	0.8485	0.0787	1.53	0.9370	0.0274	2.03	0.9788	0.0078
0.04	0.5160	0.3793	0.54	0.7054	0.1857	1.04	0.8508	0.0772	1.54	0.9382	0.0267	2.04	0.9793	0.0076
0.05	0.5199	0.3744	0.55	0.7088	0.1828	1.05	0.8531	0.0757	1.55	0.9394	0.0261	2.05	0.9798	0.0074
0.06	0.5239	0.3697	0.56	0.7123	0.1799	1.06	0.8554	0.0742	1.56	0.9406	0.0255	2.06	0.9803	0.0072
0.07	0.5279	0.3649	0.57	0.7157	0.1771	1.07	0.8577	0.0728	1.57	0.9418	0.0249	2.07	0.9808	0.0070
0.08	0.5319	0.3602	0.58	0.7190	0.1742	1.08	0.8599	0.0714	1.58	0.9429	0.0244	2.08	0.9812	0.0068
0.09	0.5359	0.3556	0.59	0.7224	0.1714	1.09	0.8621	0.0700	1.59	0.9441	0.0238	2.09	0.9817	0.0066
0.10	0.5398	0.3509	0.60	0.7257	0.1687	1.10	0.8643	0.0686	1.60	0.9452	0.0232	2.10	0.9821	0.0065
0.11	0.5438	0.3464	0.61	0.7291	0.1659	1.11	0.8665	0.0673	1.61	0.9463	0.0227	2.11	0.9826	0.0063
0.12	0.5478	0.3418	0.62	0.7324	0.1633	1.12	0.8686	0.0659	1.62	0.9474	0.0222	2.12	0.9830	0.0061
0.13	0.5517	0.3373	0.63	0.7357	0.1606	1.13	0.8708	0.0646	1.63	0.9484	0.0216	2.13	0.9834	0.0060
0.14	0.5557	0.3328	0.64	0.7389	0.1580	1.14	0.8729	0.0634	1.64	0.9495	0.0211	2.14	0.9838	0.0058
0.15	0.5596	0.3284	0.65	0.7422	0.1554	1.15	0.8749	0.0621	1.65	0.9505	0.0206	2.15	0.9842	0.0056
0.16	0.5636	0.3240	0.66	0.7454	0.1528	1.16	0.8770	0.0609	1.66	0.9515	0.0201	2.16	0.9846	0.0055
0.17	0.5675	0.3197	0.67	0.7486	0.1503	1.17	0.8790	0.0596	1.67	0.9525	0.0197	2.17	0.9850	0.0053
0.18	0.5714	0.3154	0.68	0.7517	0.1478	1.18	0.8810	0.0584	1.68	0.9535	0.0192	2.18	0.9854	0.0052
0.19	0.5753	0.3111	0.69	0.7549	0.1453	1.19	0.8830	0.0573	1.69	0.9545	0.0187	2.19	0.9857	0.0050
0.20	0.5793	0.3069	0.70	0.7580	0.1429	1.20	0.8849	0.0561	1.70	0.9554	0.0183	2.20	0.9861	0.0049

Scenario 3b- Type-2 Service

- Type 2 service level: fraction of demand met immediately from stock (β = 98%)

Use Standard **loss function** $L(z)$:

- $\frac{n(R)}{Q} = 1 - \beta, \;\; n(R) = \sigma L(z), \;\; R = \lambda\tau + z\,\sigma$
- With **Q = 1,846** (EOQ from earlier), **β = 0.98**:
  - n$(R) = 0.02 \times 1846 = 36.92$
  - $L(z) = \frac{36.92}{95} \approx 0.389 \Rightarrow z \approx 0.02$ *)from Normal distribution table)*
  - $SS = 0.02 \times 95 \approx 2$
  - $\boxed{R_\beta \approx 300 + 2 = 302\ \textit{units}}$

Standardized Normal Probability Distribution F(z) (cumulative) and Partial Expectations L(z)

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Summary of solution

Scenario	Model	Q* Order/Production Quantity (units)	R Reorder Point (units)	T cycle length/ Production time (weeks)	*G(Q*) Total Optimal Cost (£)*
(1) Buy from external supplier	EOQ	1,848	300	–	8,124
(2a) In-house production with Machine A	POQ	4,045	–	27	6,675
(2b) In-house production with Machine B	POQ	7,006	–	47	5,136
(3a) demand uncertainty during lead time, (α = 0.98)	(R,Q) Type 1 service	1,848	492	-	
(3b) demand uncertainty during lead time, (β = 99%)	(R,Q) Type 2 service	1,848	302	-	

Key Takeaways

- No single strategy fits all, the best option depends on demand characteristics, cost structure, and supply chain reliability.
- EOQ is simple and cost-efficient when the environment is stable.
- POQ can offer flexibility and control, especially when external supply is uncertain.
- (R, Q) Safety stock and R are essential to protect against uncertain demand during lead time. By setting different service level targets, company is trying to balance cost and reliability.
  - A higher service level requires holding more safety stock, which increases inventory cost but reduces the risk of stock-outs.
  - A lower service level saves on inventory cost but increases the risk of production disruptions.
- Real-world strategies often combine these approaches to balance cost, resilience, and service performance.

Exam Tip Box:
- ✓ Find the core structure, it's probably EOQ + 1 added layer.
- ✓ Always label your parameters (K, λ, h, τ, σ, z).
- ✓ Check units, most common cause of errors.

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Thank you