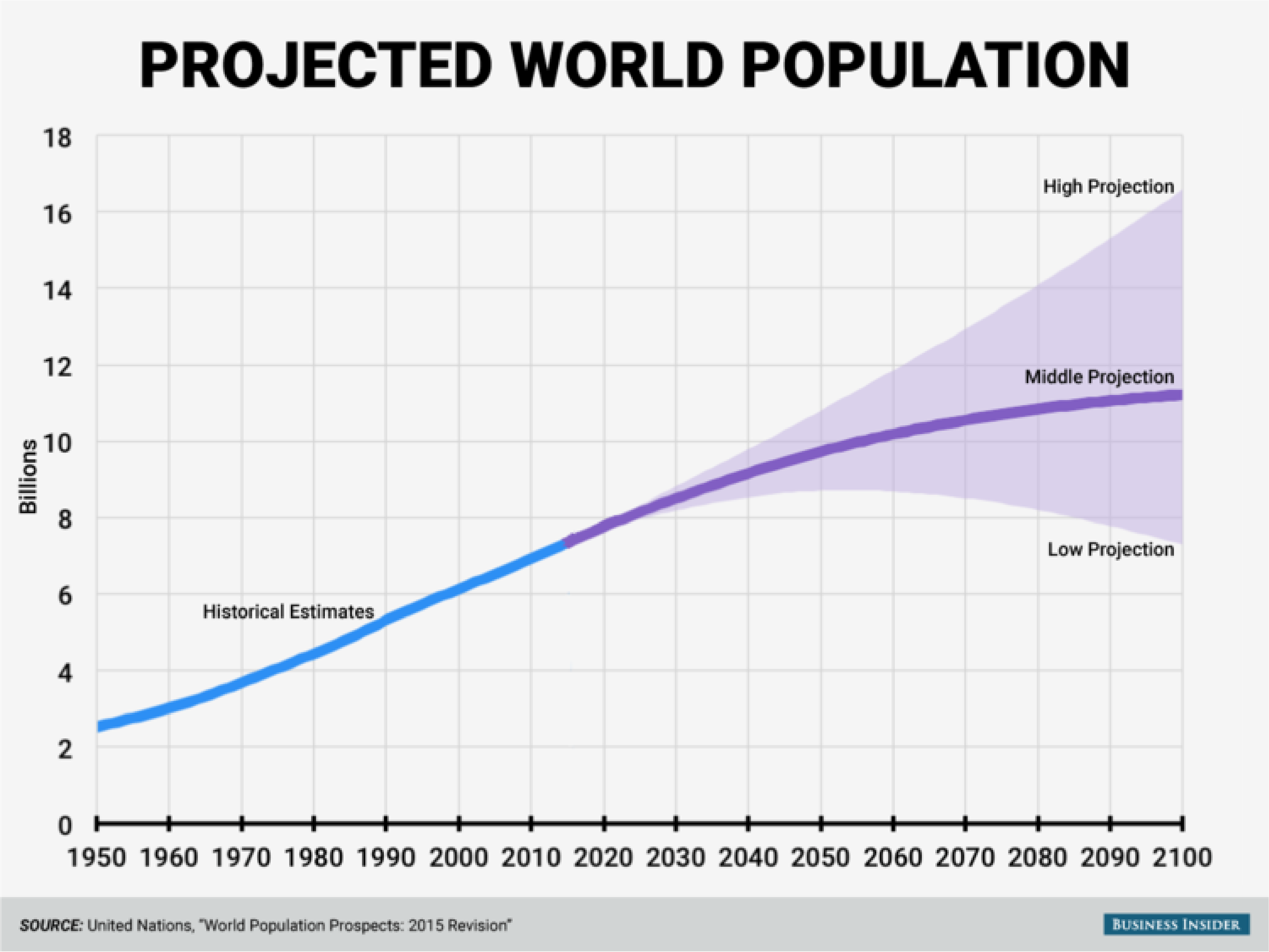
BUSI4496
Supply Chain Planning & Management

Lecture 4: Forecasting

Lecture Outline

- Introduction to forecasting
- Characteristics of time series
- Methods for stationary time series
- Evaluating forecasts
- Methods for time series showing a trend and/or seasonality
- Advanced forecasting methods


PROJECTED WORLD POPULATION
Billions
18
16
14
12
10
8
6
4
2
0
1950 1960 1970 1980 1990 2000 2010 2020 2030 2040 2050 2060 2070 2080 2090 2100
High Projection
Middle Projection
Low Projection
Historical Estimates
SOURCE: United Nations, "World Population Prospects: 2015 Revision"
BUSINESS INSIDER

Introduction to forecasting

Goal of forecasting:
Support better decision-making by making unbiased, objective estimates about future events under uncertainty (Petropoulos et al., 2022).

Application areas

- **Economics/Finance:** GDP, inflation, stock prices.
- **Healthcare:** patient inflow, disease spread.
- **Tourism:** seasonal demand forecasting.
- **Traffic Flow:** congestion prediction for infrastructure planning.
- **Sports:** performance analytics, attendance

Typical forecasting items in supply chains:

- **Demand**: drives most operational decisions
- **Sales**: informs revenue projections and pricing
- **Spare parts**: critical for service and maintenance
- **Returns**: essential for reverse logistics
- **Inventories**: balance service level with cost efficiency

Characteristics of Forecasts

- They are usually wrong.
- A good forecast is more than a single number.
- Aggregate forecasts are more accurate.
- The longer the forecast horizon, the less accurate the forecast will be.
- Forecasts should not be used to the exclusion of known information.

Introduction to forecasting

Forecasting Across Different Time Horizons

Short-term

- Operational (day-to-day) planning
- Scope: Daily/weekly, up to 2 months
- Daily demand and resource requirements
- Example: Workforce scheduling, replenishment of perishable goods

Medium-term

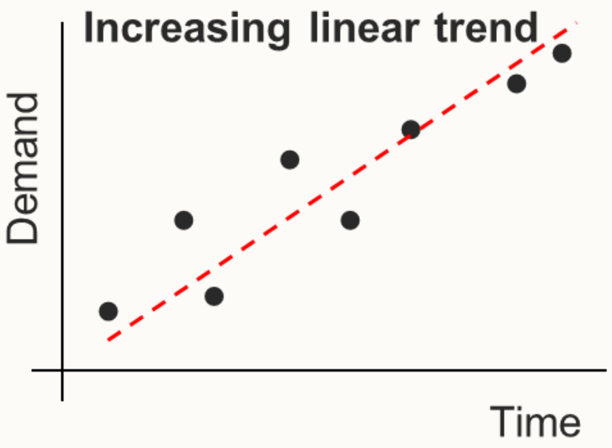
- Tactical planning
- Scope: Weekly/monthly, up to 1-2 years
- Annual production planning
- Example: Sales planning for product groups, Deciding production capacity for the coming year

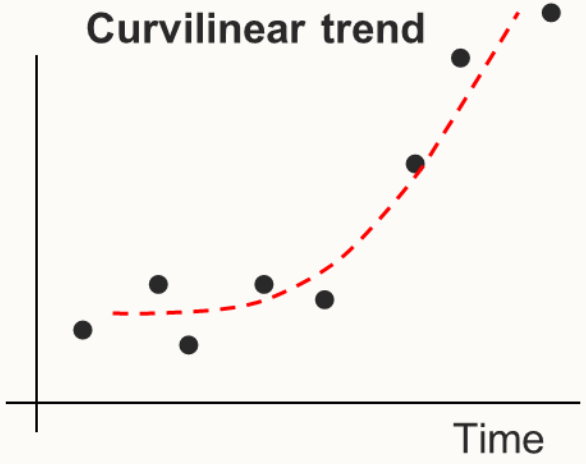
Long-term

- Strategic planning
- Scope: Monthly/yearly, 2 years and beyond
- Strategic decisions
- Example: Major investments, e.g. Introduction of new products, capacity acquisition, new facilities

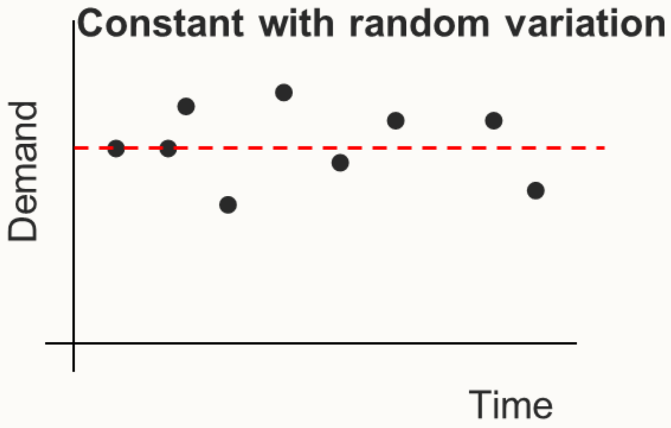
Characteristics of time series

Trend:
Gradual, long-term up or down movement in the item being forecasted (e.g., demand).



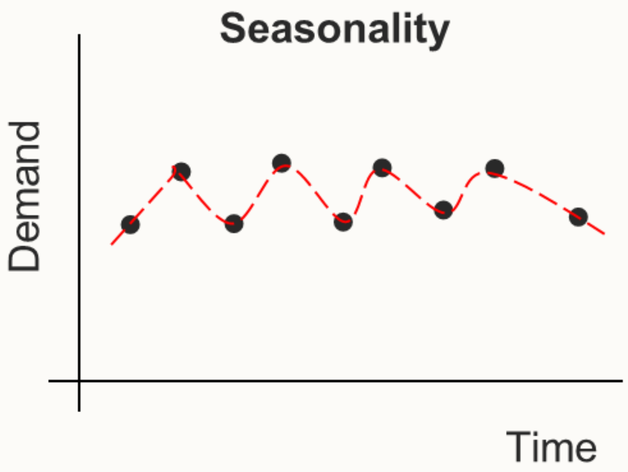


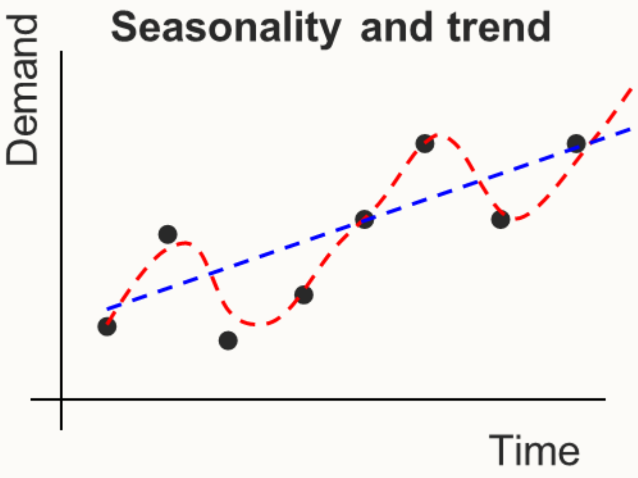
Random variation:
Unpredictable movements in the forecasted item



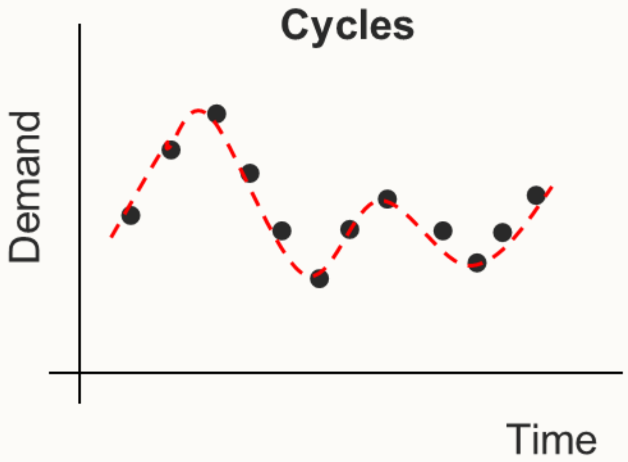
Characteristics of time series

Seasonality:
A pattern in data that repeats itself periodically (e.g., demand for a seaside hotel)





Cycles:
Similar to seasonality with irregular variation of patterns



Methods for stationary time series

Simple moving average (MA)

Exponential smoothing (ES)

Methods for stationary time series

A stationary series has no strong trend or seasonal component; we assume that it is essentially constant over the long-term with short-term, random fluctuations

- **Simple Moving Average (MA)**
  - smooths short-term fluctuations using recent data.
- **Exponential Smoothing (ES)**
  - gives more weight to recent observations, responsive to change.

- Best for **short-term forecasts** when data is stable.
- Not appropriate if **trend** or **seasonality** is present.

Methods for stationary time series

- **Notation and terminology**

Consecutive time periods = 1, 2, …, t

$\boldsymbol{D_t}$: demand in period t

$\boldsymbol{F_t}$: forecast in period t

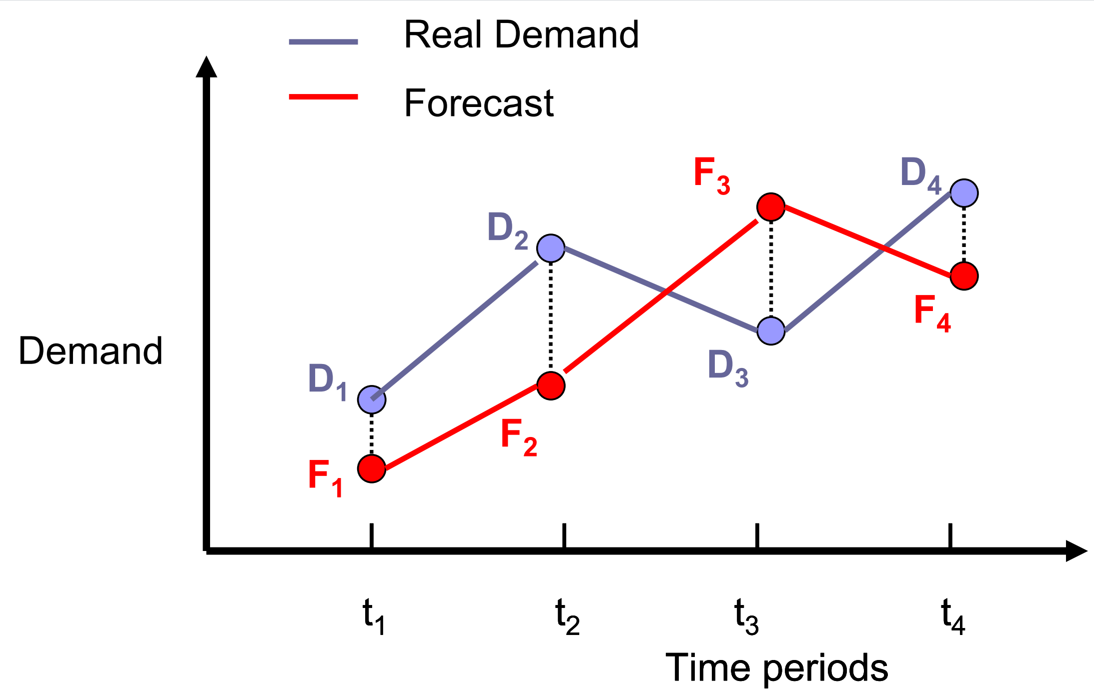
$\boldsymbol{e_t}$: forecast error (deviation), $\boldsymbol{e_t = F_t - D_t}$

$\boldsymbol{F_{t+1}}$: one step ahead forecast made in period t for $t + 1$

$\boldsymbol{F_{t,t+k}}$: k steps ahead forecast made in period t for $t + k$

$\boldsymbol{k}$: forecast horizon (number of periods ahead)

?? Why is it important to define **k** clearly in business forecasting?



Forecasts are made **one period ahead**, then checked after actual demand is observed.

Methods for stationary time series

▪ Simple Moving Average (MA)

▪ The **MA** predicts the next period's value by taking the **arithmetic average of the last *n* observations**.

$$F_{t+1} = \frac{D_t + D_{t-1} + D_{t-2} + \cdots + D_{t-n+1}}{n}$$

▪ All observations within the chosen window (*n*) are **equally weighted**, and **older data is dropped** as time progresses.

Methods for stationary time series

- **Simple Moving Average (MA)**

Characteristics

- Suitable for **stationary series** (no trend/seasonality).
- **Does not model trend or seasonality,** if these are present, MA will lag behind or perform poorly.
- **Multiple-step-ahead forecasts** are the same as one-step-ahead because once future values are forecast, they are repeated (no additional information).

Choice of *n*

- The **only decision** the forecaster makes is how many past periods (n) to include.
- **Shorter n** → more sensitive to recent changes, less smoothing.
- **Longer n** → more smoothing, but slower to react to shifts in level.
- The **"optimal" n** can be found by **minimising forecast error** (e.g., MAD, MSE) using historical data.

Methods for stationary time series

- **Simple Moving Average (MA)**

- Example:

	Demand	MA(3)	MA(6)
Week 1	650		
Week 2	678		
Week 3	720		
Week 4	785		
Week 5	859		
Week 6	920		
Week 7	850		
Week 8	758		
Week 9	892		
Week 10	920		
Week 11	789		
Week 12	844		

Methods for stationary time series

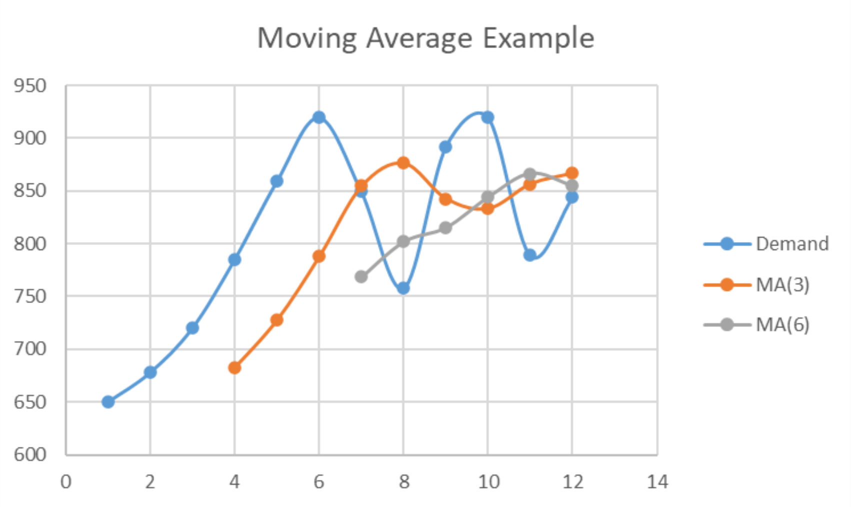
▪ Simple Moving Average (MA)

▪ Example Solution:

OBSERVATIONS:

- MA(6) is smoother than MA(3), and MA(3) is smoother than the actual demand curve.
- MA method has a smoothing effect on the raw data – peaks and troughs have smaller amplitude.
- Large *n* produces a stable forecast but may miss trends and turning points.
- Small *n* reacts quickly to most recent observations but may not be desirable.
- Simple but important in more complex methods.

	Demand	MA(3)	MA(6)
Week 1	650		
Week 2	678		
Week 3	720		
Week 4	785	682.67	
Week 5	859	727.67	
Week 6	920	788.00	
Week 7	850	854.67	768.67
Week 8	758	876.33	802.00
Week 9	892	842.67	815.33
Week 10	920	833.33	844.00
Week 11	789	856.67	866.50
Week 12	844	867.00	854.83



Methods for stationary time series

▪ Exponential Smoothing (ES)

The forecast for the next period is the sum of a proportion of the demand in the current period and a proportion of the forecast in the current period
(= weighted average of the last forecast and current demand value).

$$F_{t+1} = \alpha D_t + (1 - \alpha) F_t$$

D_t: Demand value of the current period t
F_t: Forecasted demand of the current period t
F_{t+1}: Forecasted demand of the next period $t + 1$
α: The smoothing parameter, $0 < \alpha \leq 1$

👉 So, it's like a **"moving average with memory"**, every past observation affects the forecast, but their influence **decreases exponentially over time**.

Methods for stationary time series

- **Exponential Smoothing (ES)**

- Example:

	Demand	$\alpha = 0.1$	$\alpha = 0.6$
Week 1	820	820	820
Week 2	775		
Week 3	680		
Week 4	655		
Week 5	750		
Week 6	802		
Week 7	798		
Week 8	689		
Week 9	775		
Week 10			

Methods for stationary time series

- **Exponential Smoothing (ES)**

- Example Solution:

Exponential Smoothing Example (α = 0.1)
Formula:

$$\boldsymbol{F_{t+1} = \alpha D_t + (1-\alpha)F_t}$$

$$F_2 = 0.1 \times 820 + 0.9 \times 820 = 820$$

$$F_3 = 0.1 \times 775 + 0.9 \times 820 = 815.50$$

$$F_4 = 0.1 \times 680 + 0.9 \times 815.50 = 801.95$$

	Demand	$\alpha = 0.1$	$\alpha = 0.6$
Week 1	820	820	820
Week 2	775	820.00	820.00
Week 3	680	815.50	793.00
Week 4	655	801.95	725.20
Week 5	750	787.26	683.08
Week 6	802	783.53	723.23
Week 7	798	785.38	770.49
Week 8	689	786.64	787.00
Week 9	775	776.88	728.20
Week 10		776.69	756.28

Methods for stationary time series

▪ Exponential Smoothing (ES)

ES includes all past observations.

$$F_{t+1} = \alpha D_t + (1-\alpha)F_t$$

$$F_t = \alpha D_{t-1} + (1-\alpha)F_{t-1}$$

$$F_{t-1} = \alpha D_{t-2} + (1-\alpha)F_{t-2}$$

so

$$F_{t+1} = \alpha D_t + (1-\alpha)\alpha D_{t-1} + (1-\alpha)^2\alpha D_{t-2} + (1-\alpha)^3 F_{t-2}$$

The process can be continued with infinite expansion of F_{t+1}

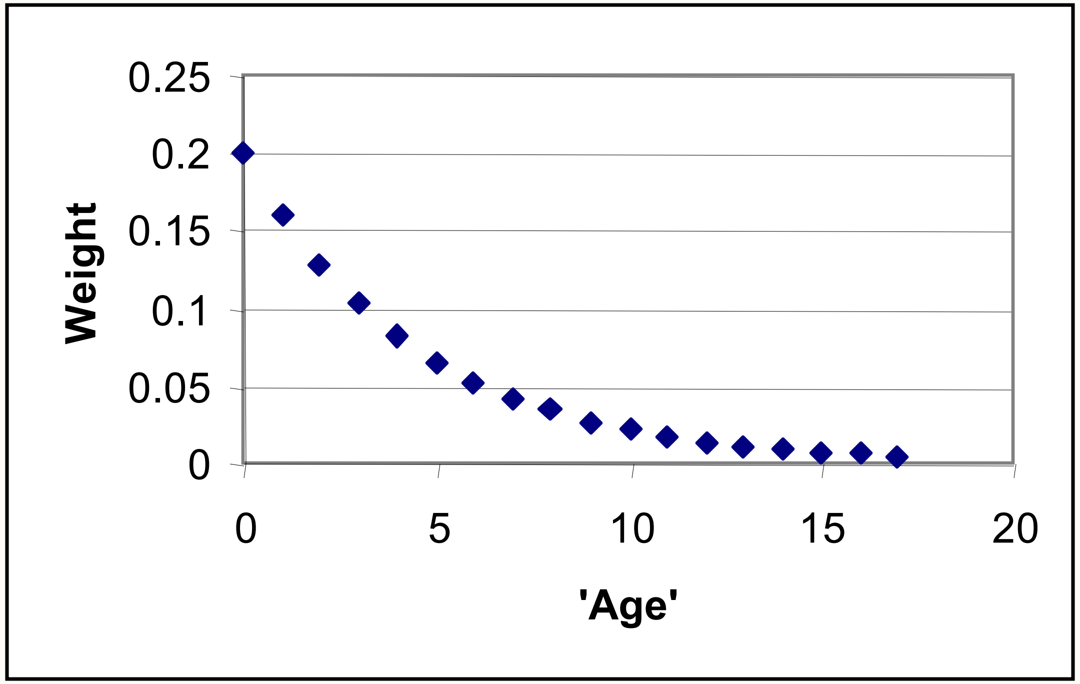
This shows that **all past observations are used**, with **weights that decrease exponentially** over time.

Methods for stationary time series

Exponential Smoothing (ES)

Current period D_t : weight = α
Previous period D_{t-1}: weight = $\alpha(1-\alpha)$
Two periods ago D_{t-2} : weight = $\alpha(1-\alpha)^2$
…
n period ago D_{t-n}: weight = $\alpha(1-\alpha)^n$

Example $\alpha = 0.2$



- An exponential curve can be fitted through the weights –hence ES
- Weights decrease with each previous term and the weights sum to 1
- α controls the rate of decrease
  - •Large α → rapid decay → more responsive
  - •Small α → slow decay → smoother forecasts
- By varying α the dominance of the most recent observation can be changed

Methods for stationary time series

▪ Exponential Smoothing (ES)

Example: (α = 0.4)

$$F_{t+1} = 0.4Dt + 0.24\,D_{t-1} + 0.144D_{t-2} + 0.0864D_{t-3} + 0.0518D_{t-4} + 0.0311D_{t-5}$$

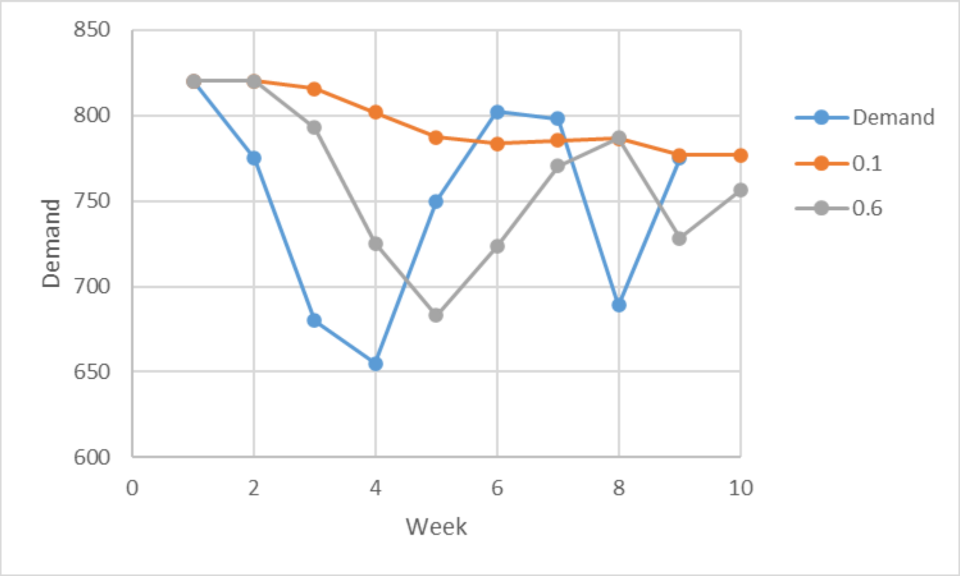
$$F_{t+1} = \alpha D_t + (1-\alpha)\alpha D_{t-1} + (1-\alpha)^2\alpha D_{t-2} + (1-\alpha)^3 F_{t-2}$$

Period back (k)	Weight contribution	Example value
0 (current, D_t)	0.400 (40%)	Dominates the forecast
1 (D_{t-1})	0.240 (24%)	Significant influence
2 (D_{t-2})	0.144 (14.4%)	Moderate influence
3 (D_{t-3})	0.086 (8.6%)	Smaller influence
4 (D_{t-4})	0.052 (5.2%)	Minor influence
5 (D_{t-5})	0.031 (3.1%)	Very small influence

Observation:
A value 6 periods ago contributes **only about 3%** when α = 0.4.
Older data has **diminishing influence**
This calculation shows how quickly older data fades when α is moderately large.

Methods for stationary time series

- **Exponential Smoothing (ES)**



- If α is small, a smoother curve due to more weight on past data
- If α is large, a more responsive curve due to more weight on recent data
- α in ES and n in MA work in a similar way

Methods for stationary time series

▪ Exponential Smoothing (ES)

- Very important practical forecasting method

  - Easy to use, update but requires a starting value – use mean of the data or most recent observation
  - Earlier observations are discounted progressively
  - Small value for a fairly stable series (e.g., $\alpha = 0.1$)
  - Large value for a quick response (e.g., $\alpha > 0.5$)
  - In practice, α around 0.1 to 0.3 is chosen for practical reasons

- Smoothing methods **always** lag behind a trend (if one exists).
- With ES and MA, the forecast for *k* periods ahead is the same as the forecast for one period ahead.
- It can be shown that a value of *n* in MA is roughly equivalent to a smoothing constant of

$n \approx (2 - \alpha) / \alpha$

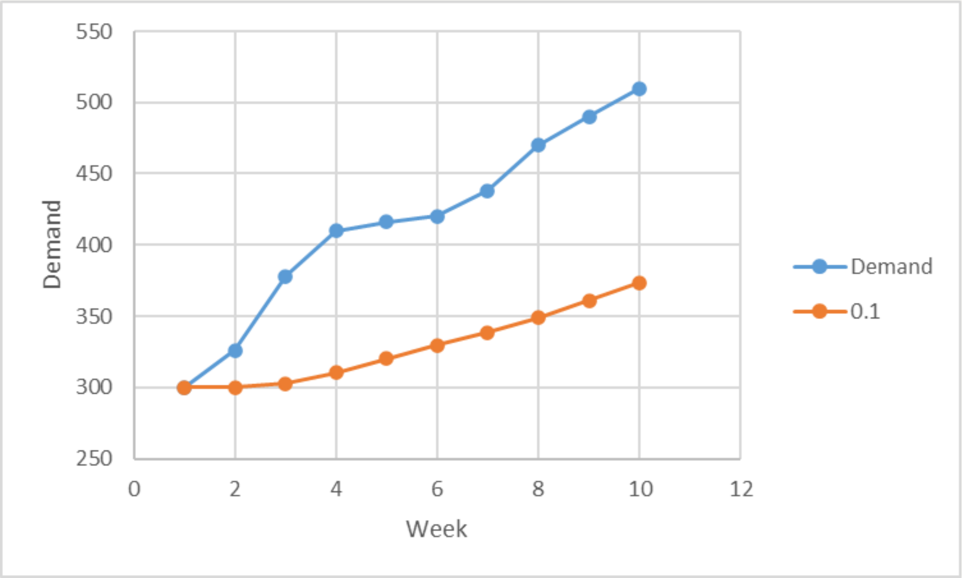
This equation helps you relate α to the number of periods in MA. It's useful when shifting between methods."

Methods for stationary time series

▪ Exponential Smoothing (ES)

- ES lags behind the trend (like MA)

	Demand	$\alpha = 0.1$
Week 1	300	300
Week 2	326	300.00
Week 3	378	302.60
Week 4	410	310.14
Week 5	416	320.13
Week 6	420	329.71
Week 7	438	338.74
Week 8	470	348.67
Week 9	490	360.80
Week 10	510	373.72



Evaluating forecasts in time series

Mean Absolute Deviation (MAD)
Mean Squared Error (MSE)
Mean Absolute Percentage Error (MAPE)

Evaluating forecasts

Monitoring forecast error and bias

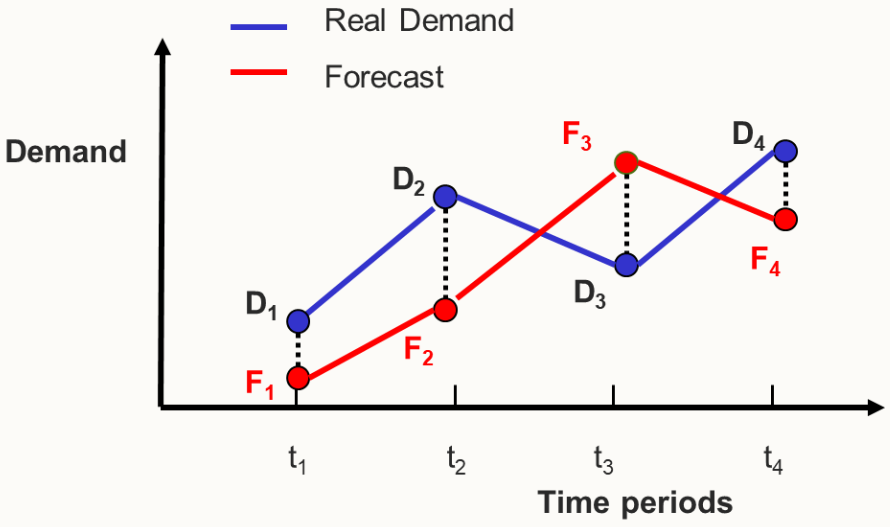
- Measurement and tracking of forecasting errors is important, particularly for time series methods.
- Errors made consistently in one direction imply bias, so it is important to track errors and bias over time (tracking signal).

Deviation ≡ error, $e_t = F_t – D_t$

- For time series, three measures are common:
  - **Mean Absolute Deviation (MAD)**
  - **Mean Squared Error (MSE)**
  - **Mean Absolute Percentage Error (MAPE)**

Evaluating forecasts

▪ Mean Absolute Deviation (MAD)

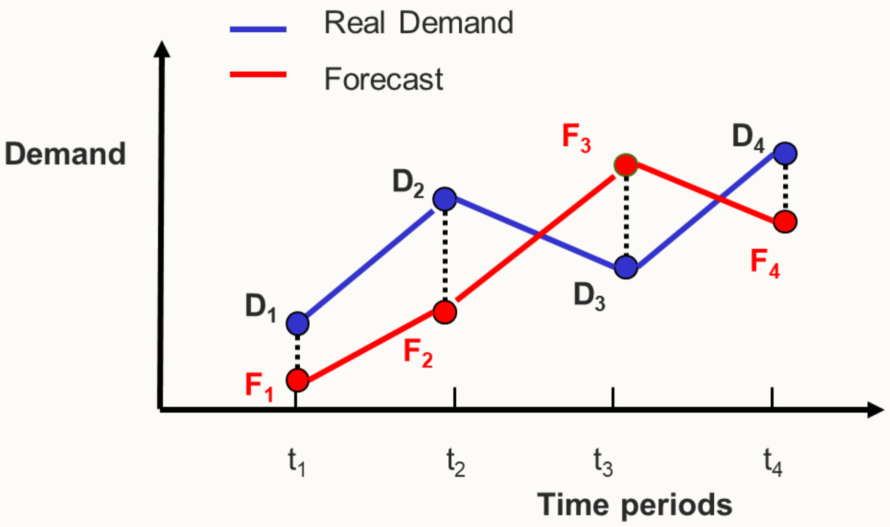


$$MAD = \frac{1}{n}\sum_{t=1}^{n}|F_t - D_t|$$

Period	Actual demand (D_t)	Forecasted demand (F_t)	Deviation $(F_t - D_t)$	Absolute deviation
1	100	90		
2	87	92		
3	89	91		
4	87	91		
5	95	90		
6	85	91		
Total	-	-		
Average	-	-		

Evaluating forecasts

▪ Mean Absolute Deviation (MAD)

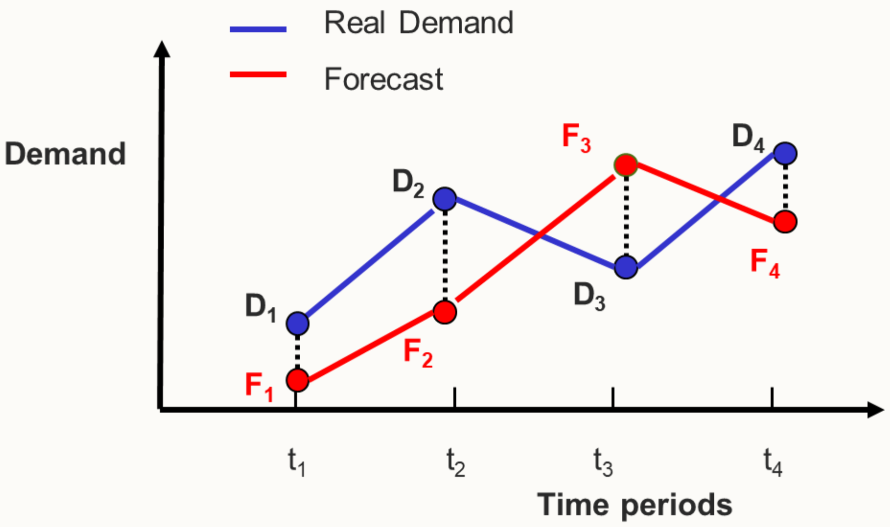


$$MAD = \frac{1}{n}\sum_{t=1}^{n}|F_t - D_t|$$

Period	Actual demand (D_t)	Forecasted demand (F_t)	Deviation $(F_t - D_t)$	Absolute deviation
1	100	90	-10	10
2	87	92	5	5
3	89	91	2	2
4	87	91	4	4
5	95	90	-5	5
6	85	91	6	6
Total	-	-	-	32
Average	-	-	-	MAD=5.33

Evaluating forecasts

▪ Mean Squared Error (MSE)

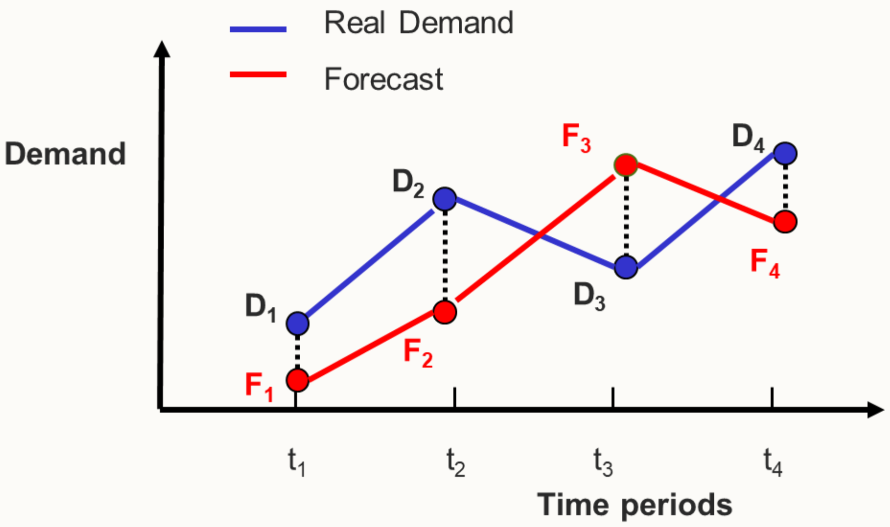


$$MSE = \frac{1}{n}\sum_{t=1}^{n}(F_t - D_t)^2$$

Period	Actual demand (D_t)	Forecasted demand (F_t)	Deviation $(F_t - D_t)$	Squared deviation
1	100	90		
2	87	92		
3	89	91		
4	87	91		
5	95	90		
6	85	91		
Total	-	-		
Average	-	-		

Evaluating forecasts

▪ Mean Squared Error (MSE)

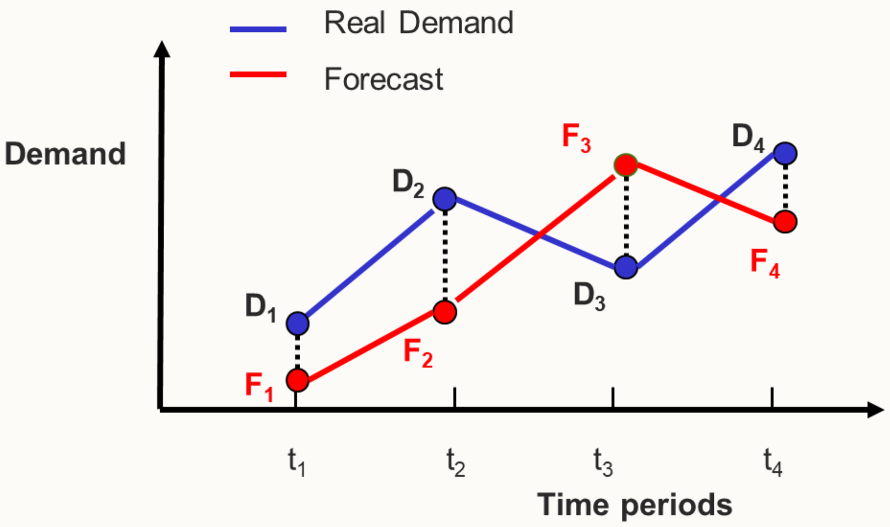


$$MSE = \frac{1}{n}\sum_{t=1}^{n}(F_t - D_t)^2$$

Period	Actual demand (D_t)	Forecasted demand (F_t)	Deviation $(F_t - D_t)$	Squared deviation
1	100	90	-10	100
2	87	92	5	25
3	89	91	2	4
4	87	91	4	16
5	95	90	-5	25
6	85	91	6	36
Total	-	-	-	206
Average	-	-	-	**MSE=34.33**

Evaluating forecasts

- **Mean Absolute Percentage Error (MAPE)**

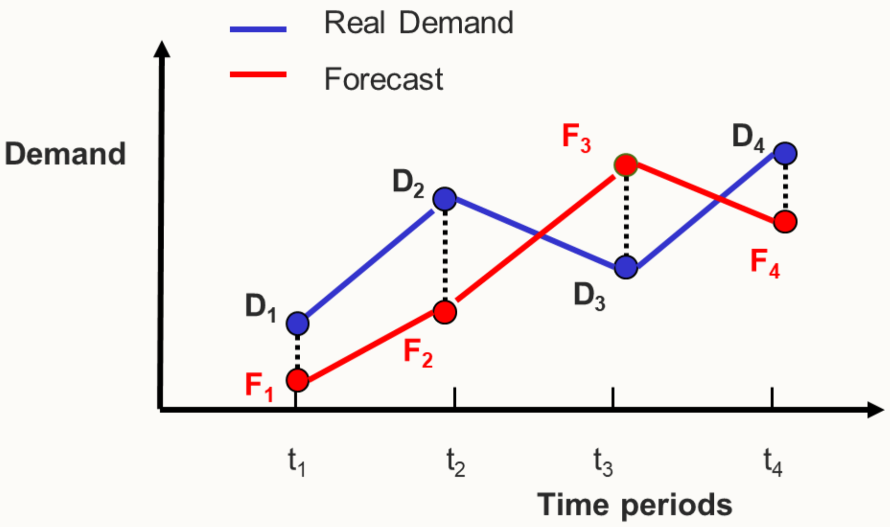


$$MAPE = \frac{1}{n}\sum_{t=1}^{n}\left|\frac{F_t - D_t}{D_t}\right| * 100\%$$

Period	Actual demand (D_t)	Forecasted demand (F_t)	Deviation (F_t – D_t)	Absolute deviation	Percentage absolute deviation
1	100	90			
2	87	92			
3	89	91			
4	87	91			
5	95	90			
6	85	91			
Total	-	-			
Average	-	-	-		**MAPE=5.82%**

Evaluating forecasts

- **Mean Absolute Percentage Error (MAPE)**



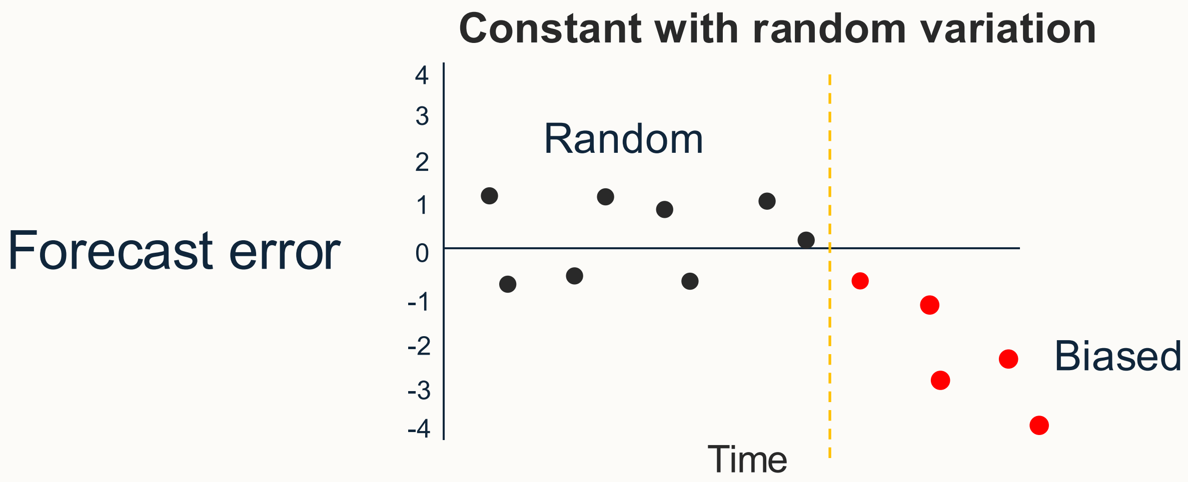
$$MAPE = \frac{1}{n}\sum_{t=1}^{n}\left|\frac{F_t - D_t}{D_t}\right| * 100\%$$

Period	Actual demand (D_t)	Forecasted demand (F_t)	Deviation ($F_t - D_t$)	Absolute deviation	Percentage absolute deviation
1	100	90	-10	10	10.00%
2	87	92	5	5	5.75%
3	89	91	2	2	2.25%
4	87	91	4	4	4.60%
5	95	90	-5	5	5.26%
6	85	91	6	6	7.06%
Total	-	-	-		34.91%
Average	-	-	-		**MAPE=5.82%**

Evaluating forecasts

Monitoring forecast error and bias

- Absolute measures are better for interpretation
- MSE is more sensitive to a single, large error compared to MAD.
- MAPE gives a relative measure of error and is therefore popular but may be misleading when demand levels are very low.
- In an unbiased forecasting method, the errors would fluctuate around zero in a random manner.



Methods for time series with trend and/or seasonality

Linear Regression

Holt's method

Adjusted Linear Regression

Holt-Winters technique

Methods for time series with trend and/or seasonality

- What if a series exhibits trend, seasonality or both?
- **Trend extrapolation** – may be simple and quick
- Series with trend and seasonality may be 'pre-treated' to 'de-trend' and 'de-seasonalise' before applying smoothing methods

1. **Linear Regression**
2. **Holt's method**
3. **Adjusted Regression**
4. **Holt-Winters technique**

Series with a trend (1–2)

Series with trend and seasonality (3–4)

Methods for time series with trend and/or seasonality

▪ Linear regression

- Let $\{x_i, y_i\}$, i = 1, 2, …, n be two data points where **x** is the independent variable and **y** is the dependent variable.
- If a trend relationship exists between **x** and **y**, then we can fit a least squares regression line (= linear trend line) to obtain a linear equation.
- In time series, interpret **x** as the time period (e.g., x = 1 for week 1) and **y** as the forecasted item in the corresponding value of **x** (e.g., demand at week 1).

$$\boldsymbol{y = a + bx}$$

a: intercept (or ‘base’, demand at time 0)

b: slope of the line (or trend)

x: time period

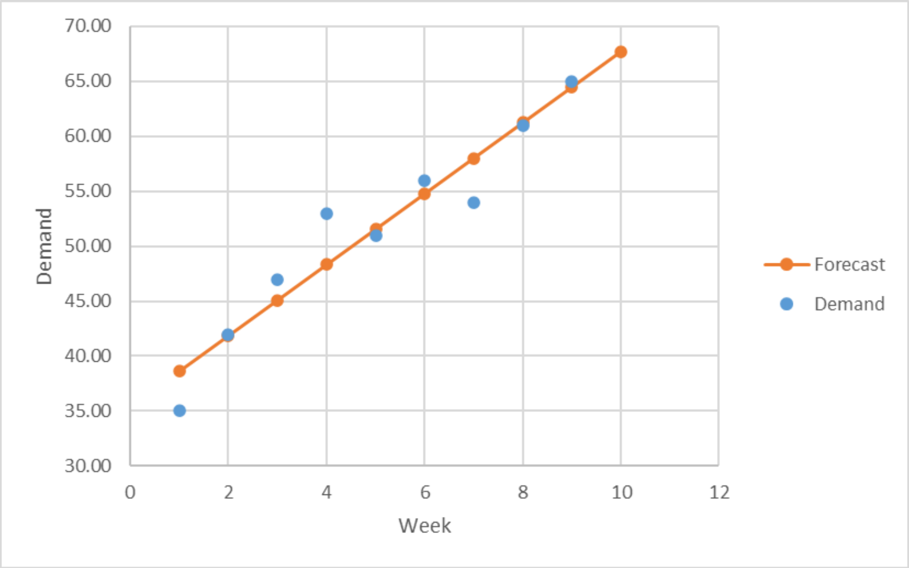
y: forecasted demand at time period x

Methods for time series with trend and/or seasonality

▪ Linear regression

- To predict for the **next period, set the value of X to the next period.**
- To predict for **further periods ahead , set the value of X to the future period required**
- **With a trend method the one step ahead forecast is not the same as the forecast for k periods ahead.**
- $y = 35.390 + 3.233x$

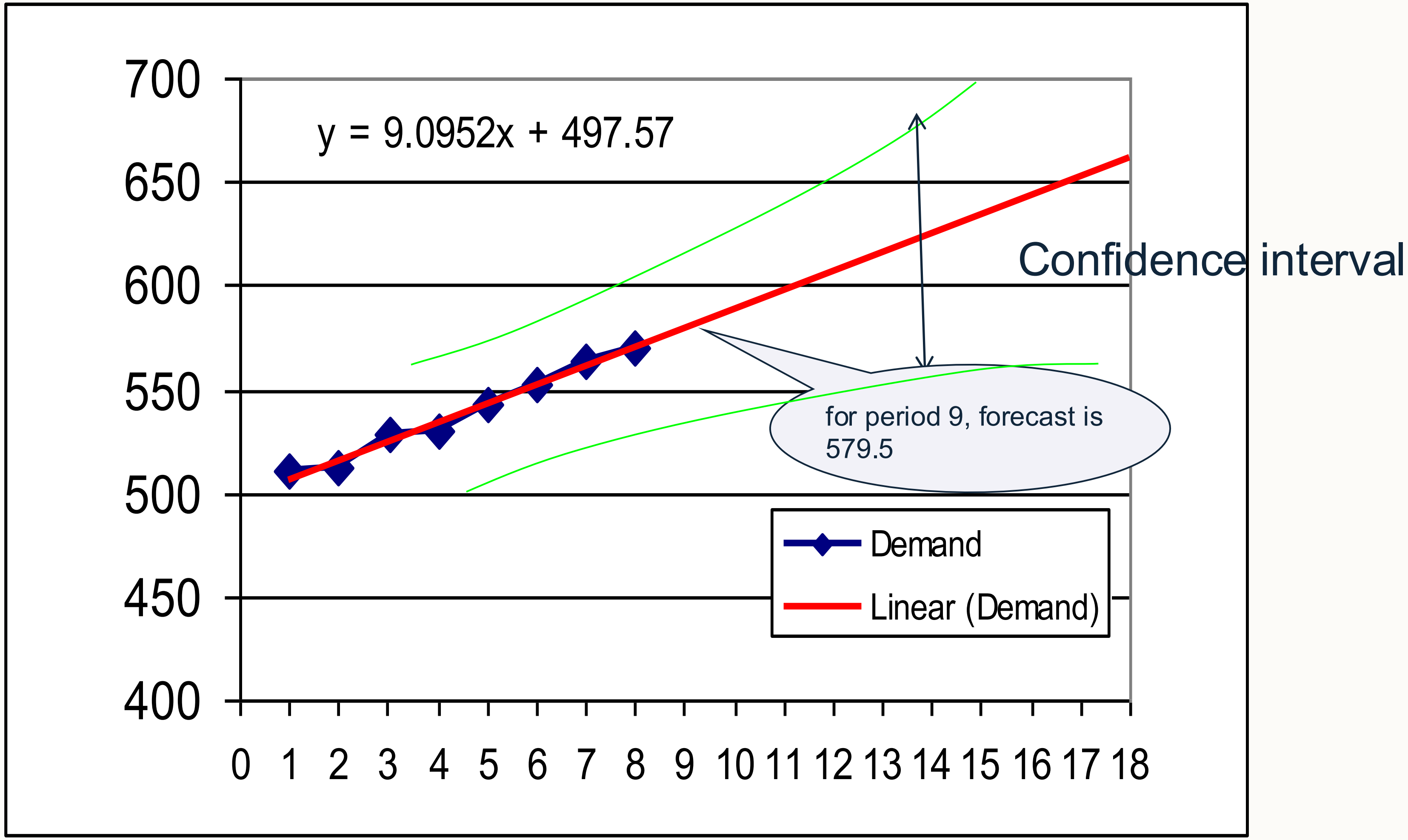
Week	Demand	Forecast
1	35	38.62
2	42	41.86
3	47	45.09
4	53	48.32
5	51	51.56
6	56	54.79
7	54	58.02
8	61	61.26
9	65	64.49
10		67.72



Methods for time series with trend and/or seasonality

- **Linear regression**

We could recalculate the linear projection with every new observation, but Holt's method is better (as it is adaptive)



Methods for time series with trend and/or seasonality

▪ Holt's method

- Assumption: linear trend in series
- Key idea:
  - 'De-trend' time series by **separating *base* from *trend*** effects
  - **Smooth *base*** (intercept) in the usual manner using α
  - **Smooth *trend*** (slope) using β
- This is a form of **double exponential smoothing**

Think of regression as your 'planned route' [base line model] and Holt's as your 'satnav with live updates' [automatic forecaster]. Regression gives you a clear, fixed equation, but if traffic conditions change, you need to recalculate. Holt's updates for you continuously

Methods for time series with trend and/or seasonality

▪ Holt's method

- S_t is **base or intercept** at period t
- G_t is the **slope or trend** at period t
- $S_t = \alpha D_t + (1-\alpha)(S_{t-1} + G_{t-1})$
- $G_t = \beta(S_t - S_{t-1}) + (1-\beta)G_{t-1}$
- $F_{t+1} = S_t + G_t$ **(one step ahead)**
- $F_{t,t+k} = S_t + kG_t$ **(k-steps ahead)**
- For more stability, typically $\beta \leq \alpha$

Methods for time series with trend and/or seasonality

Holt's method

- Observed number of failures: 200, 250, 175, 186, 225, 285, 305, 190
- Assume S_0 = 200, G_0 = 10. Use α=0.1, β=0.1

t	$F_{t-1,t}$ (forecasted)	D_t (actual)	S_t (intercept)	G_t (slope)
0	---	---	200.0	10.0
1		200		
2		250		
3		175		
4		186		
5		225		
6		285		

- Multi-step ahead forecast: $F_{2,5}$=

Methods for time series with trend and/or seasonality

Holt's method

- Observed number of failures: 200, 250, 175, 186, 225, 285, 305, 190
- Assume $S_0 = 200$, $G_0 = 10$. Use $\alpha=0.1$, $\beta=0.1$

t	$F_{t-1,t}$ (forecasted)	D_t (actual)	S_t (intercept)	G_t (slope)
0	---	---	200.0	10.0
1	210.0	200	209.0	9.9
2	218.9	250	222.0	10.2
3	232.2	175	226.5	9.6
4	236.1	186	…	…
5	240.3	225	…	…
6	247.7	285	…	…

- Multi-step ahead forecast:

$F_{2,5}=S_2+(3)G_2=222+(3)(10.2)=252.6$

Methods for time series with trend and/or seasonality

Linear Regression

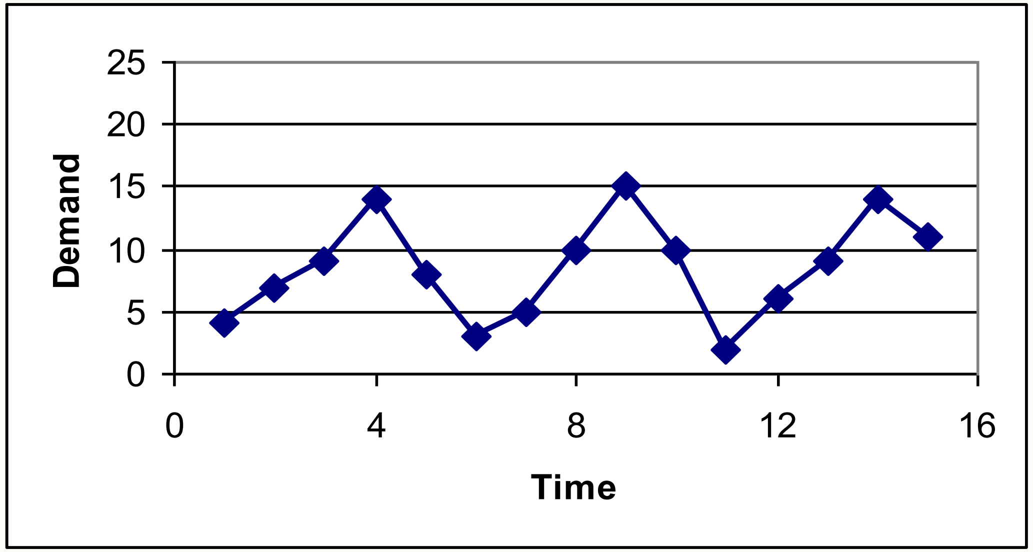
Holt's method

Adjusted Linear Regression

Holt-Winters technique

Methods for time series with trend and/or seasonality

• **Seasonal series** have a pattern that repeats every *N* periods.



- **Representation of seasonality:**
  - Define a set of multipliers c_t for $1 \leq t \leq N$ such that $\Sigma\, c_t = N$.
  - The multiplier c_t denotes the amount that the demand in period t of the season is above or below the overall average, also called the *seasonal factor*.
  - Example: $c_2 = 1.5$, $c_3 = 0.7$.
    - The demand in the second period is 50% above the average demand.
    - The demand in the third period is 30% below average.

• **Key idea:** "de-seasonalise" (and de-trend) the series.

Methods for time series with trend and/or seasonality

	W1	W2	W3
Mon	16.2	17.3	14.6
Tue	12.2	11.3	13.1
Wed	14.2	15	13
Thu	17.3	17.6	16.9
Fri	22.5	23.5	21.9

1) Compute the sample mean

$\mu = 16.44$

0.985	1.052	0.888
0.742	0.687	0.797
0.864	0.912	0.791
1.052	1.071	1.028
1.369	1.429	1.332

2) Divide each observation by μ to obtain seasonal factors

Mon	0.975	16.033
Tue	0.742	12.200
Wed	0.856	14.067
Thu	1.050	17.267
Fri	1.377	22.633

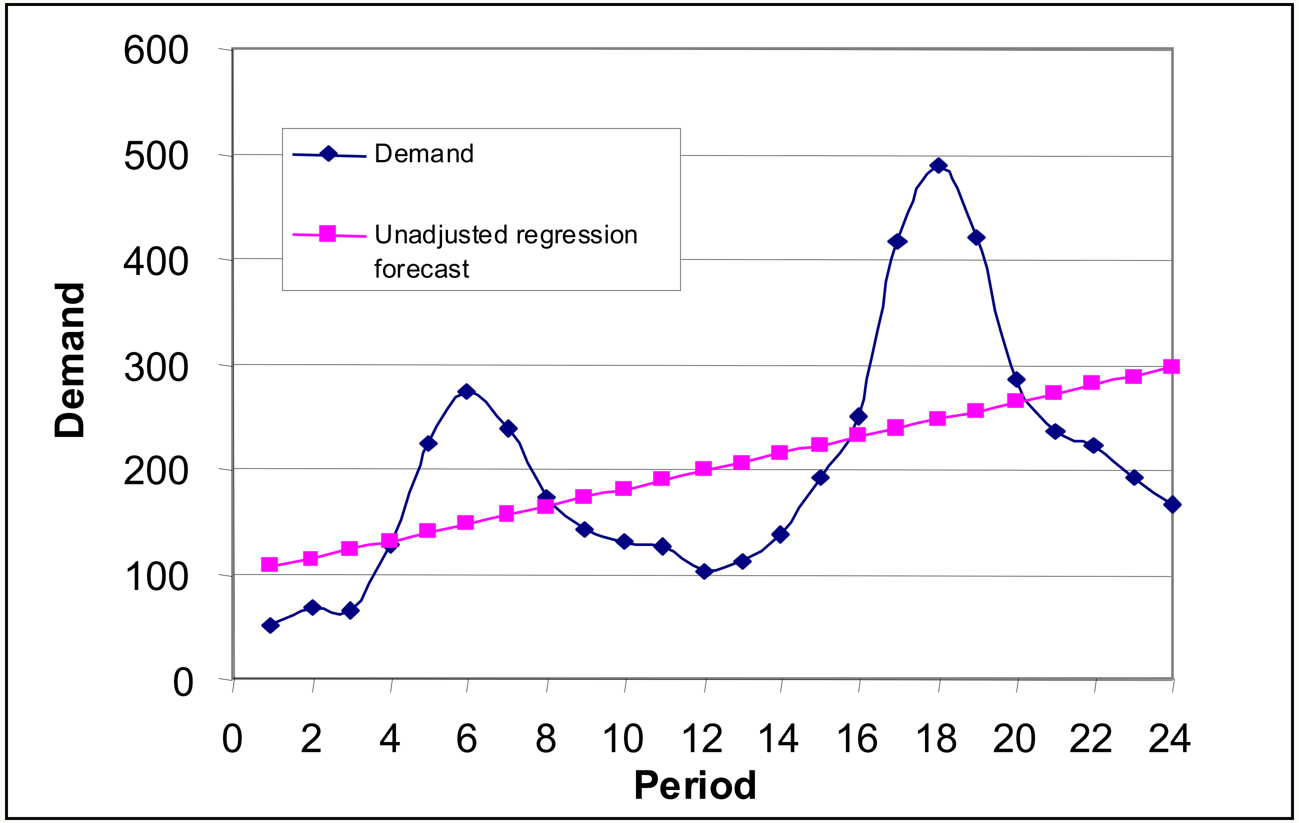
3) Average the factors for like periods (i.e., days of the week)

What is the forecast for Tuesdays?

16.44*0.742 = 12.20

Methods for time series with trend and/or seasonality

- **Seasonal decomposition with (linear) trend and seasonality: Adjusted Regression**



Linear regression: forecasted demand = 98.71 + 8.22*period

Methods for time series with trend and/or seasonality

Seasonal decomposition with (linear) trend and seasonality: Adjusted Regression

Period	Demand	Unadjusted regression forecast	Demand / forecast	Seasonal factor	Adjusted regression forecast
1	51	106.9	0.477	$\boxed{0.511}$	54.6
2	67	115.2	0.582	0.611	70.4
3	65	123.4	0.527	0.694	85.6
4	129	131.6	0.980	1.033	135.9
5	225	139.8	1.609	1.677	234.5
6	272	148.0	1.837	1.906	282.1
7	238	156.3	1.523	1.587	248.0
8	172	164.5	1.046	1.064	175.1
9	143	172.7	0.828	0.847	146.3
10	131	180.9	0.724	0.759	137.3
11	125	189.1	0.661	0.664	125.6
12	103	197.4	0.522	0.540	106.5
13	112	205.6	0.545	$\boxed{0.511}$	105.0
14	137	213.8	0.641	0.611	130.7
15	191	222.0	0.860	0.694	154.0
16	250	230.2	1.086	1.033	237.8
17	416	238.5	1.745	1.677	399.9
18	487	246.7	1.974	1.906	470.1
19	421	254.9	1.652	1.587	404.6
20	285	263.1	1.083	1.064	280.1
21	235	271.3	0.866	0.847	229.8
22	222	279.6	0.794	0.759	212.2
23	192	287.8	0.667	0.664	191.1
24	165	296.0	0.557	0.540	159.7

Seasonal factor periods 1 and 13:

= (0.477+0.545)/2

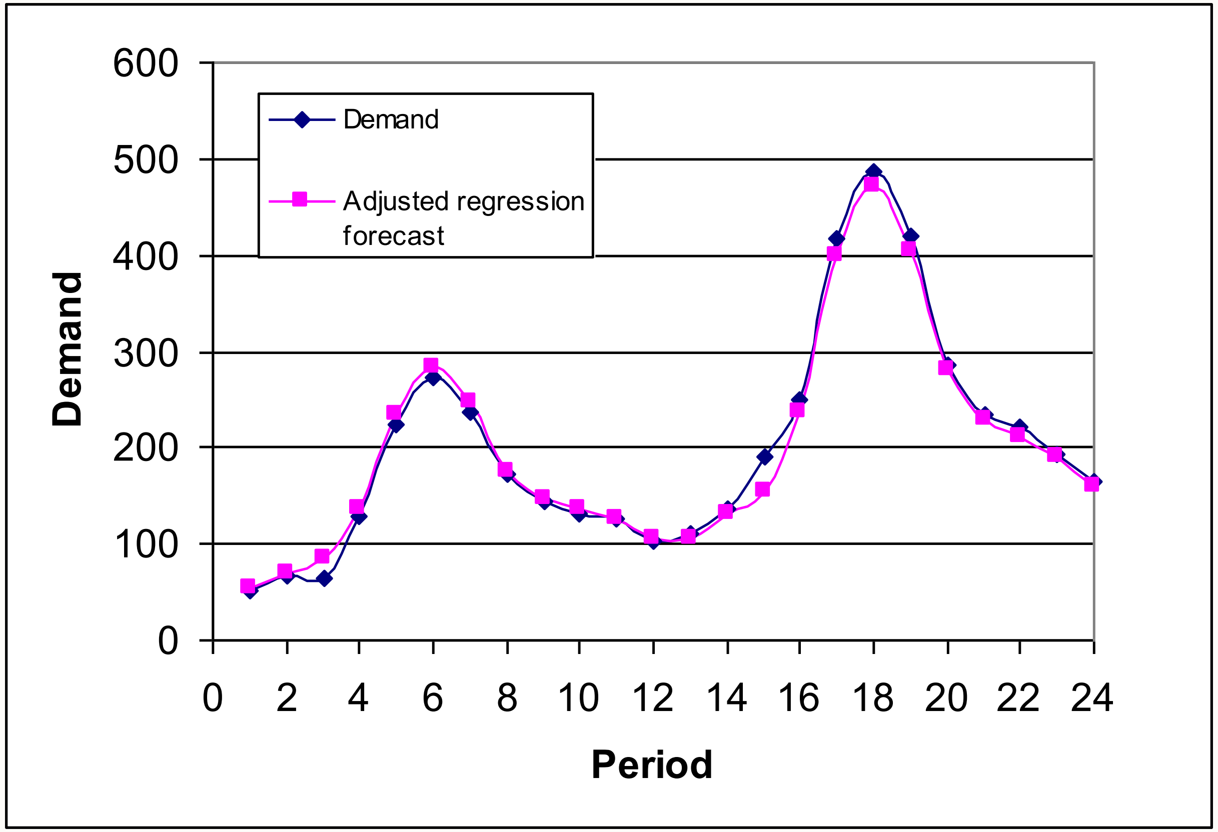
= 0.511

Adjusted forecast:

54.6 =0.511*106.90

Methods for time series with trend and/or seasonality

- Seasonal decomposition with (linear) trend and seasonality: Adjusted Regression



Methods for time series with trend and/or seasonality

▪ Holt-Winter Method

- Key idea:
  - **‘De-trend’ and ‘de-seasonalise’ the time series by separating *base* from *trend* and *seasonality* effects**
  - **Smooth *base*** (intercept) in usual manner using α
  - **Smooth *trend*** (slope) in usual manner using β
  - **Smooth *seasonality*** using γ
  - Assume season n periods and $\Sigma c_t = n$

Methods for time series with trend and/or seasonality

▪ Holt-Winter Method

Formulae for your reference only:

- Smooth the base forecast S_t

$$S_t = \alpha(D_t / c_{t-N}) + (1-\alpha)(S_{t-1} + G_{t-1})$$

- Smooth the trend forecast G_t

$$G_t = \beta(S_t - S_{t-1}) + (1-\beta)G_{t-1}$$

- Smooth the seasonality forecast c_t

$$c_t = \gamma(D_t \,/\, S_t) + (1-\gamma)c_{t-n}$$

- Forecast: $F_{t,t+k} = (S_t + k\,G_t)c_{t+k-n}$ (k-step-ahead; $k \leq n$)

- (Initialisation procedure required for Holt-Winters)

More advanced time series methods

- Weighted moving average: Assign weights (between 0 and 1) to the periods considered in MA.
- Adaptive methods try to update the value of the smoothing parameters based on recent information
- Models with autocorrelation (errors of different time periods are not independent of each other).
  - Box-Jenkins models (aka Autoregressive Integrated Moving Average (ARIMA) models (De Gooijer and Hyndman, 2006)) looks for autocorrelation among values of observed data (many data points required for a meaningful model).
- Data-driven methods (e.g., neural networks, machine learning (Petropoulos et al., 2022))

Difficulties with time series forecasting

- **Note:** a series **may be badly behaved** and not allow forecasting with sufficient accuracy
  - Intermittent and erratic demand: forecast time between consecutive transactions and magnitude of demand transactions
- Replacement and service parts: usually more information available (failure rate of components); mathematically complex
- **Human judgement** may be required to **adjust statistical forecasts (internal factors:** price changes, promotions, introduction of substitute products…; **external factors:** general economic situation, government regulations, competitor actions…)

References and recommended reading

Books:

Keller, G. (2012). *Managerial Statistics* (9th edition), UK: Cengage (Chapter 20)

Nahmias, S. (2009). Production and Operations Analysis (6th edition), Singapore: McGraw Hill. (Chapter 2)

Taylor III, B.W. (2016). *Introduction to Management Science* (12th edition), Harlow: Pearson (Chapter 15).

Articles:

De Gooijer, J.G. and Hyndman, R.J., 2006. 25 years of time series forecasting. *International journal of forecasting*, *22*(3), pp.443-473.

Petropoulos, F., Apiletti, D., Assimakopoulos, V., Babai, M.Z., Barrow, D.K., Taieb, S.B., Bergmeir, C., Bessa, R.J., Bijak, J., Boylan, J.E. and Browell, J., 2022. Forecasting: theory and practice. *International Journal of Forecasting*, *38*(3), pp.705-871.

Thank you