In [1]:
```
'''
    Author: A.Shrikant
'''
```
Out[1]:
```
'\n    Author: A.Shrikant\n'
```
In [2]:
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import seaborn as sns
import os
import warnings
warnings.filterwarnings("ignore")
```
In [3]:
```
os.getcwd()
```
Out[3]:
```
'C:\\Users\\user\\Documents\\Statistics_and_ML'
```
In [4]:
```
df = pd.read_csv('dataset/boston.csv')
```
In [ ]:
```
# Attribute Information
# Input features in order:
# 1) CRIM: per capita crime rate by town
# 2) ZN: proportion of residential land zoned for lots over 25,000 sq.ft.
# 3) INDUS: proportion of non-retail business acres per town
# 4) CHAS: Charles River dummy variable (1 if tract bounds river; 0 otherwise)
# 5) NOX: nitric oxides concentration (parts per 10 million) [parts/10M]
# 6) RM: average number of rooms per dwelling
# 7) AGE: proportion of owner-occupied units built prior to 1940
# 8) DIS: weighted distances to five Boston employment centres
# 9) RAD: index of accessibility to radial highways
# 10) TAX: full-value property-tax rate per $10,000 [$/10k]
# 11) PTRATIO: pupil-teacher ratio by town
# 12) B: The result of the equation B=1000(Bk - 0.63)^2 where Bk is the proportion of blacks by town
# 13) LSTAT: % lower status of the population

# Output variable:
# 1) MEDV: Median value of owner-occupied homes in $1000's [k$]
```
In [5]:
```
df.head()
```
Out[5]:

	CRIM	ZN	INDUS	CHAS	NOX	RM	AGE	DIS	RAD	TAX	PTRATIO	B	LSTAT	MEDV
0	0.00632	18.0	2.31	0	0.538	6.575	65.2	4.0900	1	296.0	15.3	396.90	4.98	24.0
1	0.02731	0.0	7.07	0	0.469	6.421	78.9	4.9671	2	242.0	17.8	396.90	9.14	21.6
2	0.02729	0.0	7.07	0	0.469	7.185	61.1	4.9671	2	242.0	17.8	392.83	4.03	34.7
3	0.03237	0.0	2.18	0	0.458	6.998	45.8	6.0622	3	222.0	18.7	394.63	2.94	33.4
4	0.06905	0.0	2.18	0	0.458	7.147	54.2	6.0622	3	222.0	18.7	396.90	5.33	36.2

In [6]:
```
df.shape
```
Out[6]:
```
(506, 14)
```
In [7]:
```
df.info()
```
```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 506 entries, 0 to 505
Data columns (total 14 columns):
 #   Column   Non-Null Count  Dtype  
---  ------   --------------  -----  
 0   CRIM     506 non-null    float64
 1   ZN       506 non-null    float64
 2   INDUS    506 non-null    float64
 3   CHAS     506 non-null    int64  
 4   NOX      506 non-null    float64
 5   RM       506 non-null    float64
 6   AGE      506 non-null    float64
 7   DIS      506 non-null    float64
 8   RAD      506 non-null    int64  
 9   TAX      506 non-null    float64
 10  PTRATIO  506 non-null    float64
 11  B        506 non-null    float64
 12  LSTAT    506 non-null    float64
 13  MEDV     506 non-null    float64
dtypes: float64(12), int64(2)
memory usage: 55.5 KB
```

No missing values detected.

In [8]:
```
df.describe()
```

Out[8]:

	CRIM	ZN	INDUS	CHAS	NOX	RM	AGE	DIS	RAD	TAX	PTRATIO	B	LSTAT	MED
count	506.000000	506.000000	506.000000	506.000000	506.000000	506.000000	506.000000	506.000000	506.000000	506.000000	506.000000	506.000000	506.000000	506.00000
mean	3.613524	11.363636	11.136779	0.069170	0.554695	6.284634	68.574901	3.795043	9.549407	408.237154	18.455534	356.674032	12.653063	22.5328
std	8.601545	23.322453	6.860353	0.253994	0.115878	0.702617	28.148861	2.105710	8.707259	168.537116	2.164946	91.294864	7.141062	9.1971
min	0.006320	0.000000	0.460000	0.000000	0.385000	3.561000	2.900000	1.129600	1.000000	187.000000	12.600000	0.320000	1.730000	5.0000
25%	0.082045	0.000000	5.190000	0.000000	0.449000	5.885500	45.025000	2.100175	4.000000	279.000000	17.400000	375.377500	6.950000	17.0250
50%	0.256510	0.000000	9.690000	0.000000	0.538000	6.208500	77.500000	3.207450	5.000000	330.000000	19.050000	391.440000	11.360000	21.2000
75%	3.677083	12.500000	18.100000	0.000000	0.624000	6.623500	94.075000	5.188425	24.000000	666.000000	20.200000	396.225000	16.955000	25.0000
max	88.976200	100.000000	27.740000	1.000000	0.871000	8.780000	100.000000	12.126500	24.000000	711.000000	22.000000	396.900000	37.970000	50.0000

Handling Outliers:

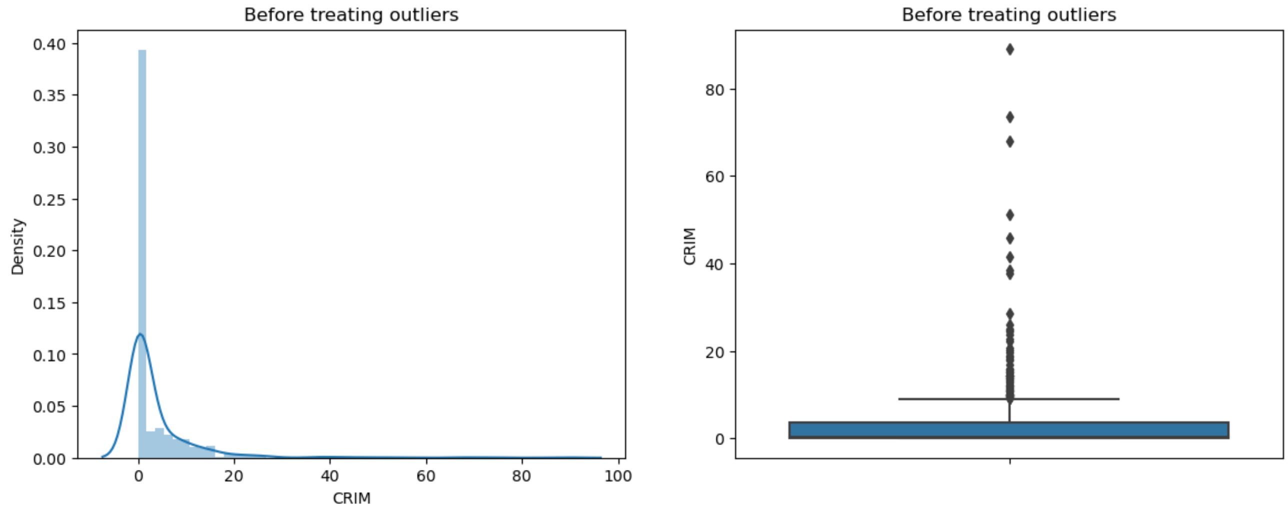
In [9]:
```
def handle_outliers_using_emperical_rule(col):
    upper_cutoff = col.mean() + 3*col.std()
    lower_cutoff = col.mean() - 3*col.std()
    return np.where(col > upper_cutoff, upper_cutoff, np.where(col < lower_cutoff, lower_cutoff, col))
```

In [10]:
```
def handle_outliers_using_iqr(col):
    q1 = np.quantile(col, .25)
    q3 = np.quantile(col, .75)

    iqr = q3 - q1

    upper_limit = q3 + iqr * 1.5
    lower_limit = q1 - iqr * 1.5

    print(f'q1: {q1}')
    print(f'q3: {q3}')
    print(f'iqr: {iqr}')

    return np.where(col > upper_limit, upper_limit, np.where(col < lower_limit, lower_limit, col))
```

In [11]:
```
def draw_distplot_and_boxplot(col, outliers_treated = False):

    word = "Before"

    if outliers_treated:
        word = "After"

    plt.figure(figsize=(14,5))

    plt.subplot(1,2,1)
    sns.distplot(col)
    plt.title(f'{word} treating outliers')

    plt.subplot(1,2,2)
    sns.boxplot(y=col)
    plt.ylabel(col.name)

    plt.title(f'{word} treating outliers')
```

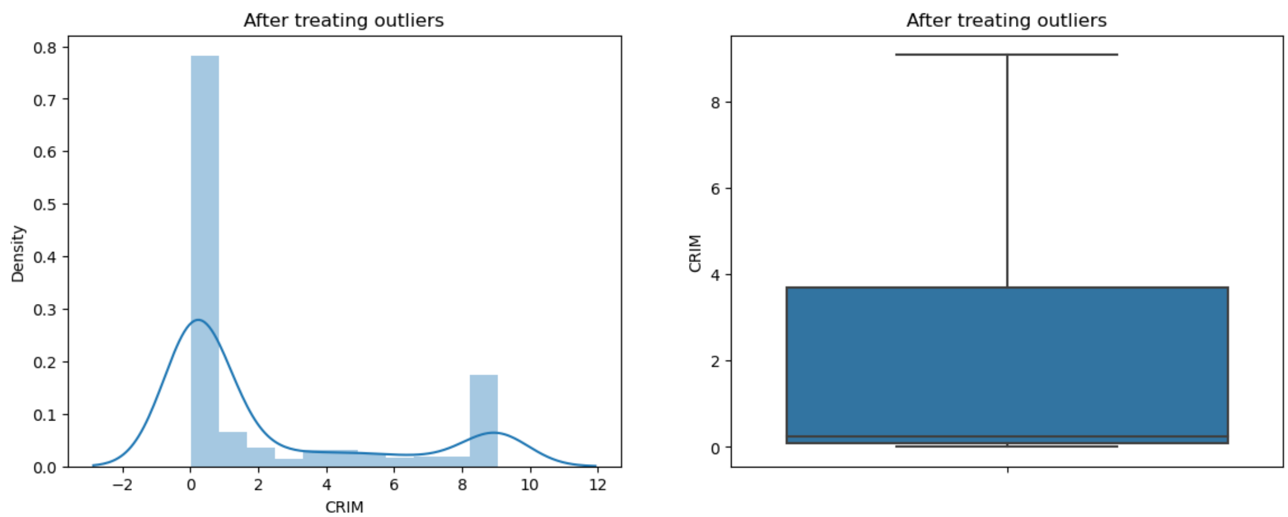
Handling outliers for 'CRIM'

In [12]:
```
draw_distplot_and_boxplot(df['CRIM'])
```



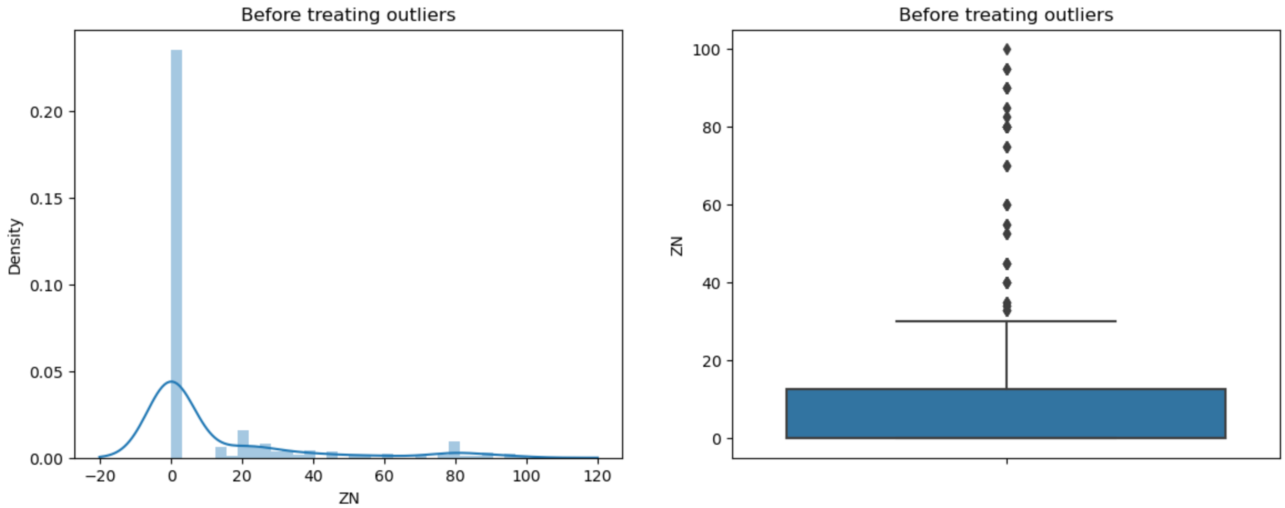
In [13]:
```
# Since 'CRIM' is not normally distributed so we use the IQR based approach to treat outliers.

col_name = 'CRIM'

df[col_name] = handle_outliers_using_iqr(df[col_name])
draw_distplot_and_boxplot(df[col_name], outliers_treated=True)
```

```
q1: 0.082044999999999999
q3: 3.6770825
iqr: 3.5950375
```



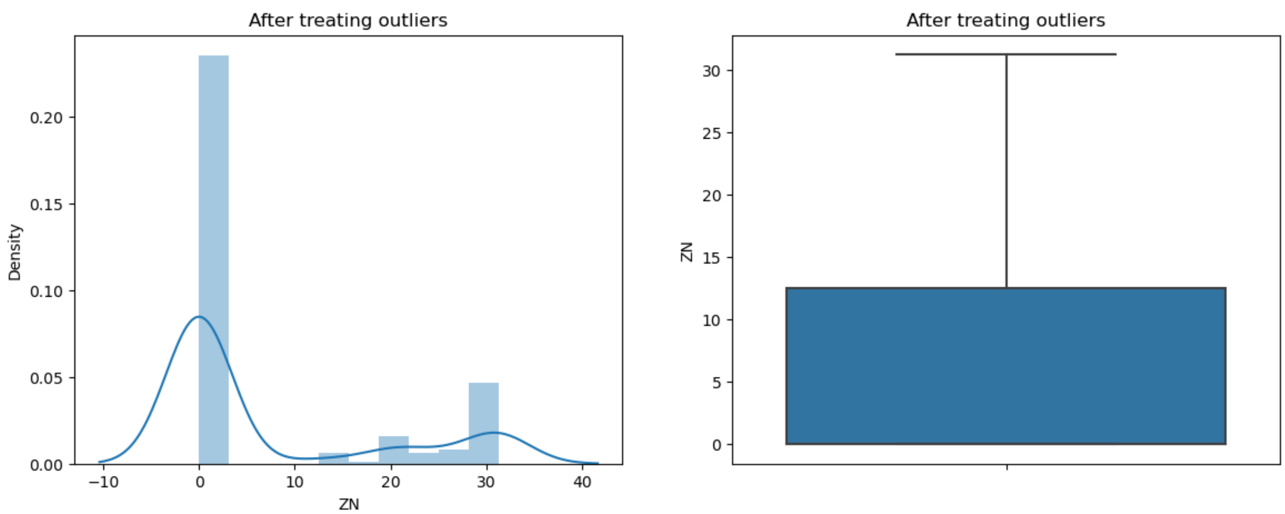
Handling outliers for 'ZN'

In [14]:
```
draw_distplot_and_boxplot(df['ZN'])
```

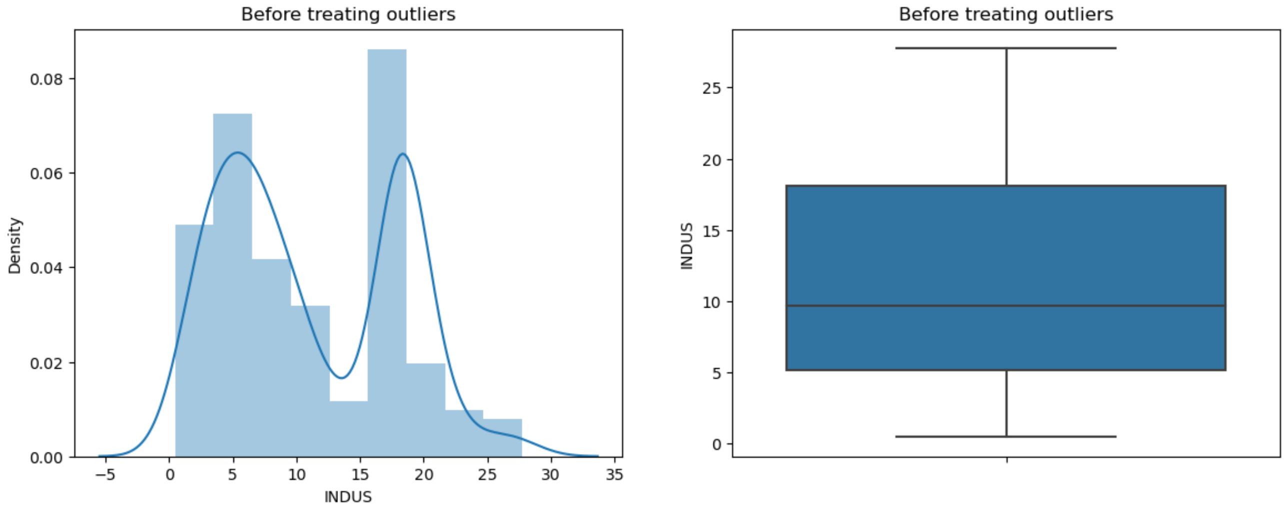


In [15]:
```
# Since 'ZN' is not normally distributed so we use the IQR based approach to treat outliers.

col_name = 'ZN'

df[col_name] = handle_outliers_using_iqr(df[col_name])
draw_distplot_and_boxplot(df[col_name], outliers_treated=True)
```

```
q1: 0.0
q3: 12.5
iqr: 12.5
```

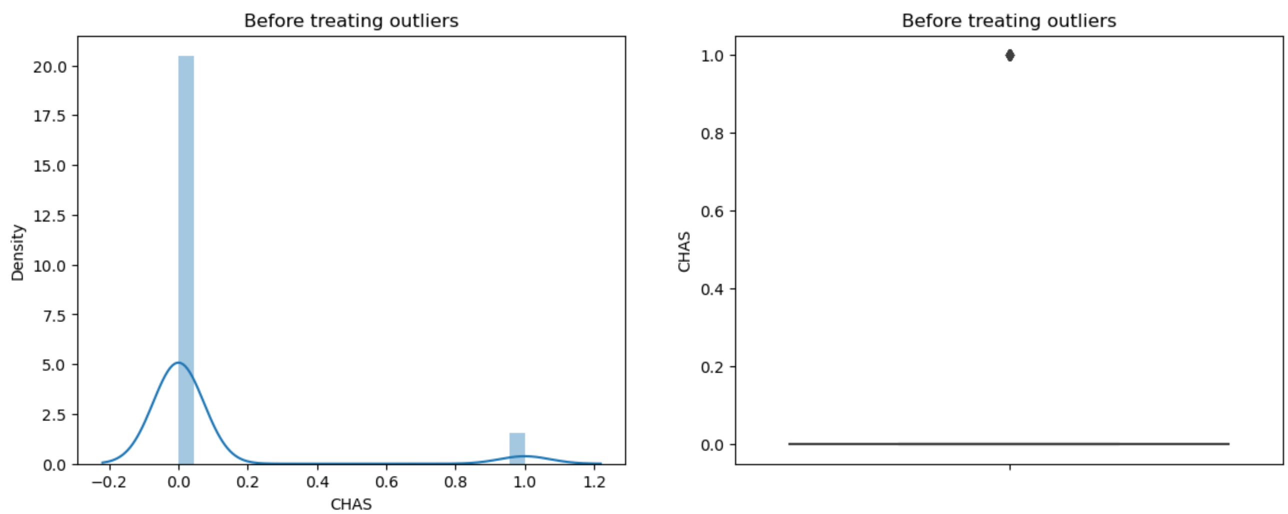


```
In [16]: draw_distplot_and_boxplot(df['INDUS'])
```



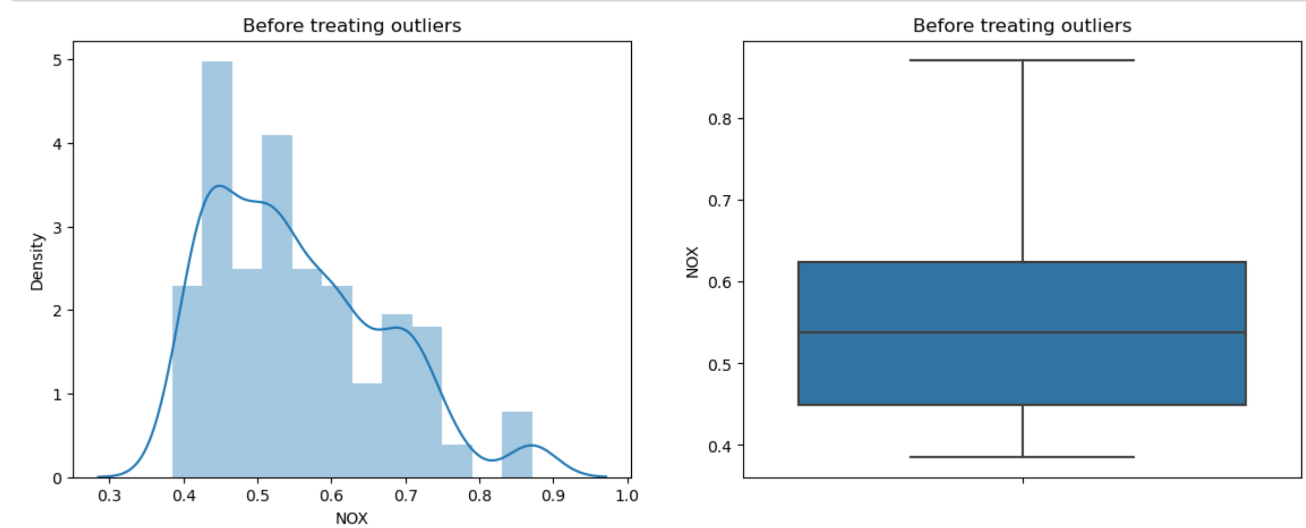
No outliers detected for 'INDUS'.

```
In [17]: draw_distplot_and_boxplot(df['CHAS'])
```



Range for 'CHAS' is very narrow so we chose not to treat the outliers.

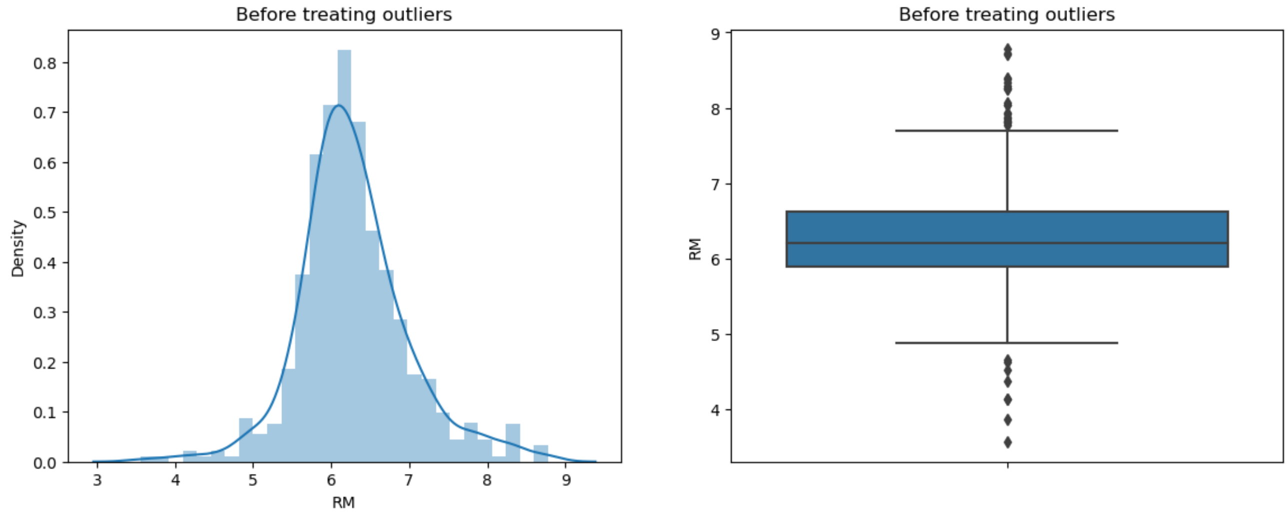
```
In [18]: draw_distplot_and_boxplot(df['NOX'])
```



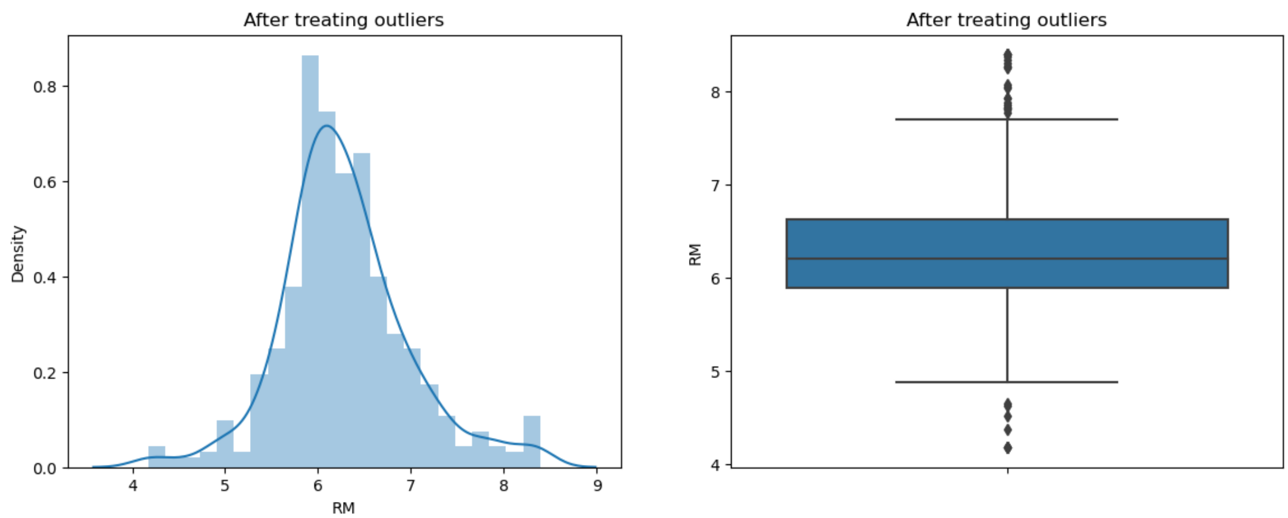
No outliers detected for 'NOX'.

Handling outliers for 'RM'

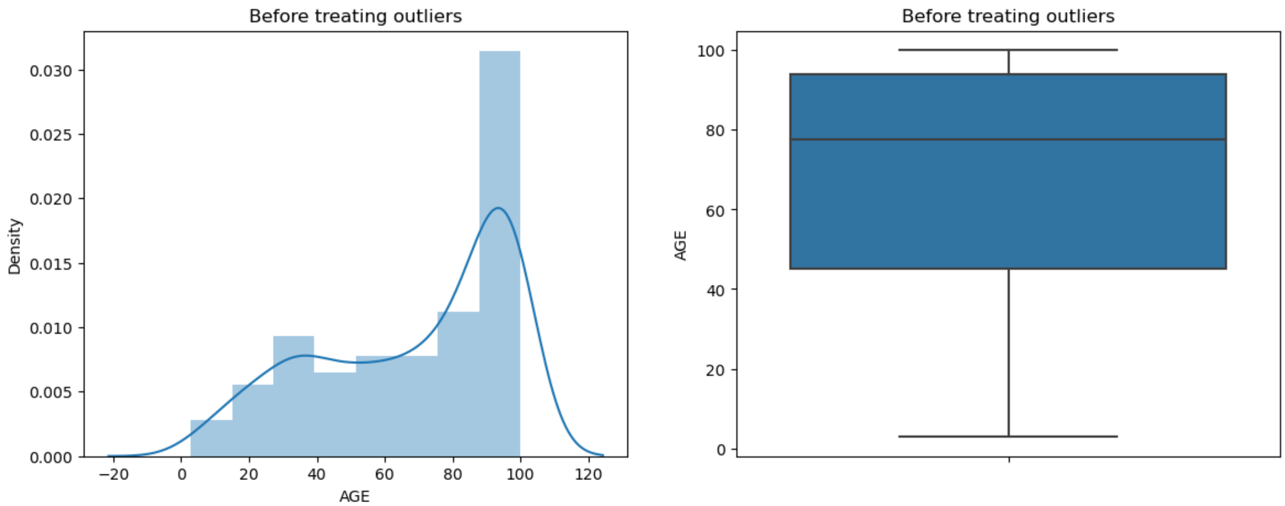
In [19]:
```
draw_distplot_and_boxplot(df['RM'])
```



In [20]:
```
# Since 'RM' is normally distributed so we use the Emperical Rule based approach to treat outliers.

col_name = 'RM'

df[col_name] = handle_outliers_using_emperical_rule(df[col_name])
draw_distplot_and_boxplot(df[col_name], outliers_treated=True)
```

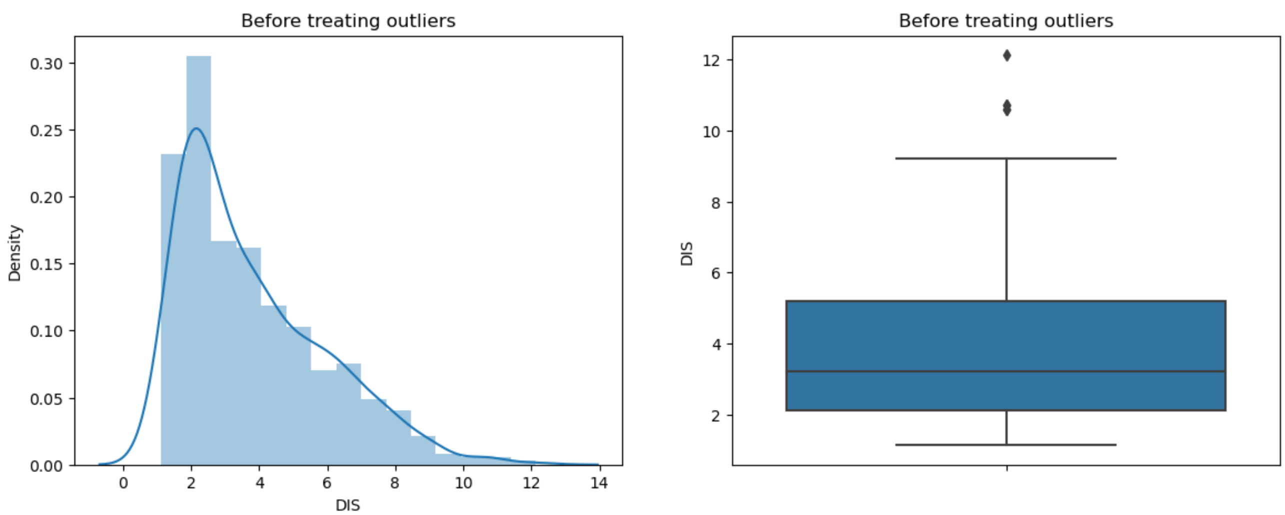


In [21]:
```
draw_distplot_and_boxplot(df['AGE'])
```



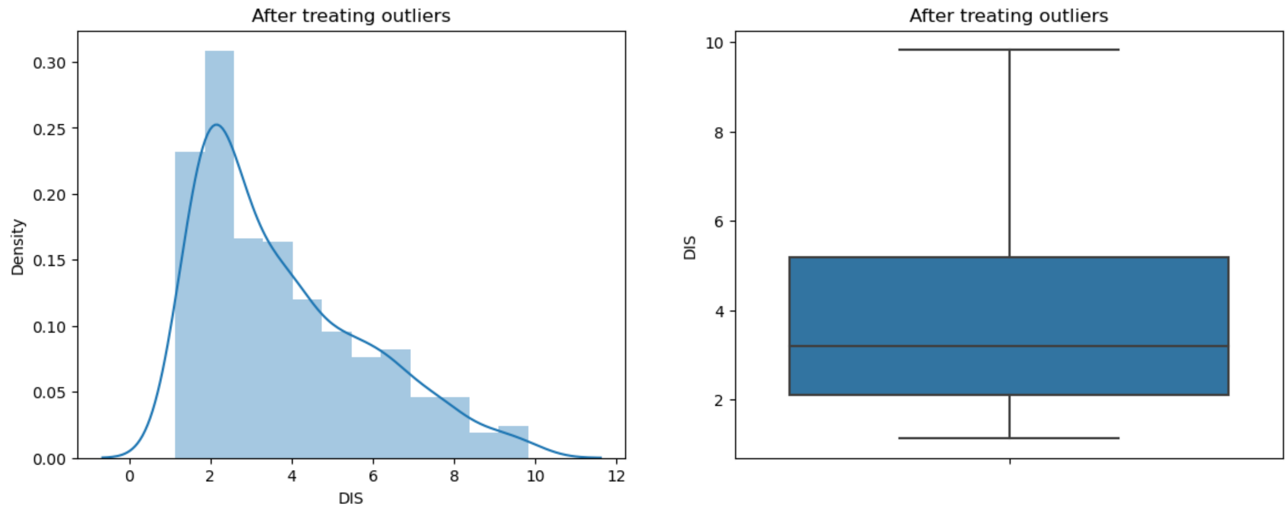
No outliers detected for 'AGE'.

In [22]:
```
draw_distplot_and_boxplot(df['DIS'])
```

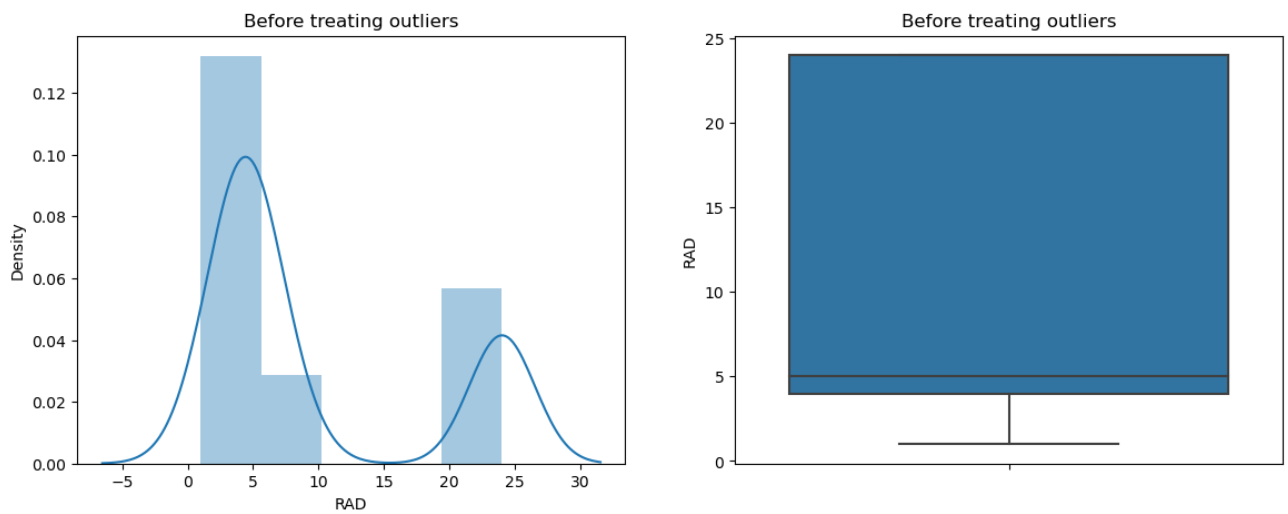


In [23]:
```
# Since 'DIS' is not normally distributed so we use the IQR based approach to treat outliers.

col_name = 'DIS'

df[col_name] = handle_outliers_using_iqr(df[col_name])
draw_distplot_and_boxplot(df[col_name], outliers_treated=True)
```

```
q1: 2.100175
q3: 5.1884250000000005
iqr: 3.0882500000000004
```

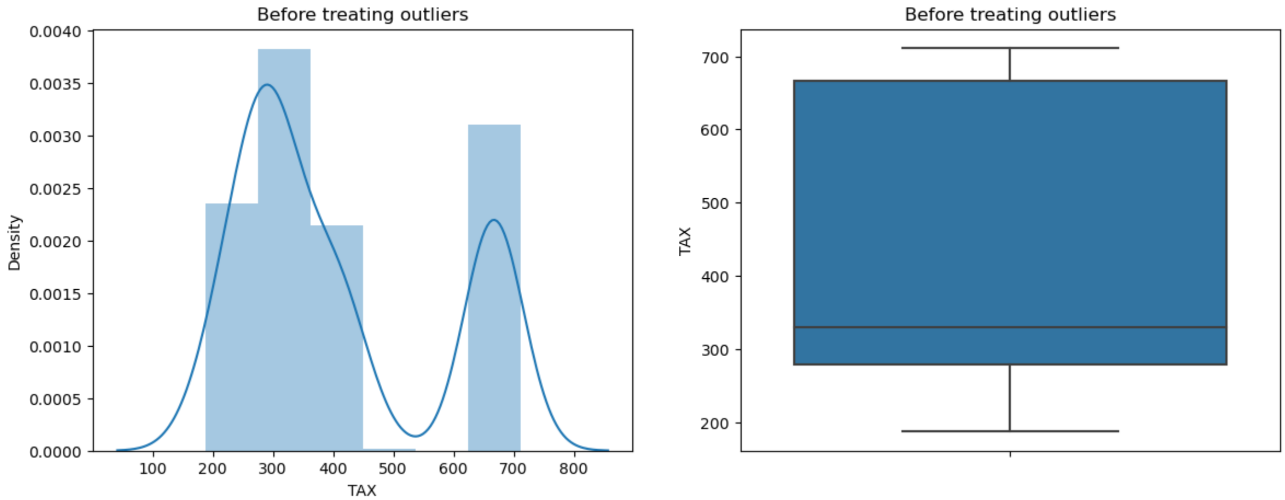


In [24]:
```
draw_distplot_and_boxplot(df['RAD'])
```



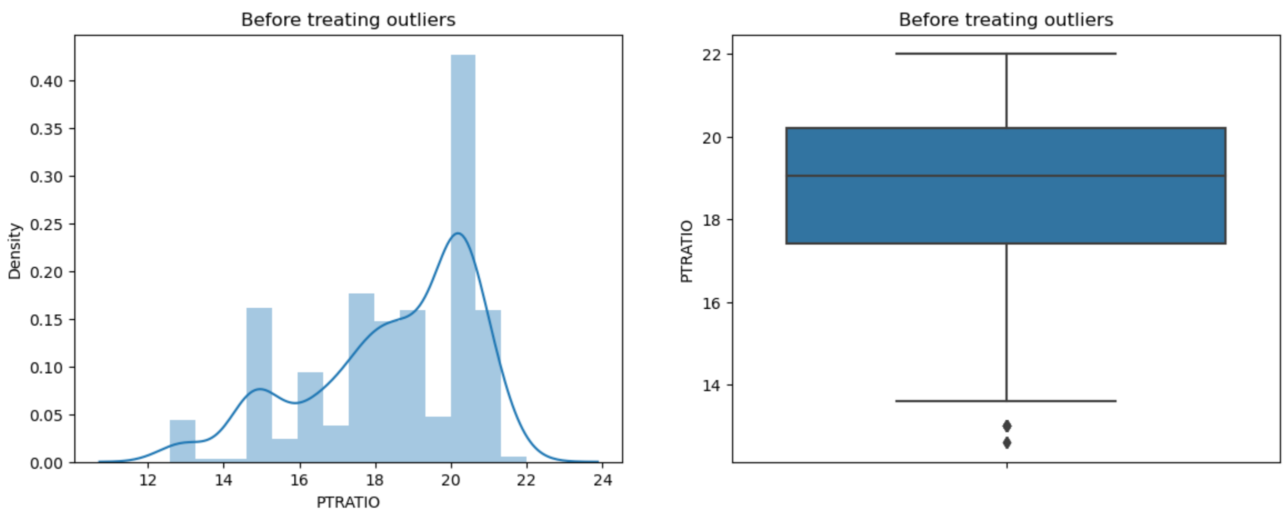
No outliers detected for 'RAD'.

In [25]:
```
draw_distplot_and_boxplot(df['TAX'])
```



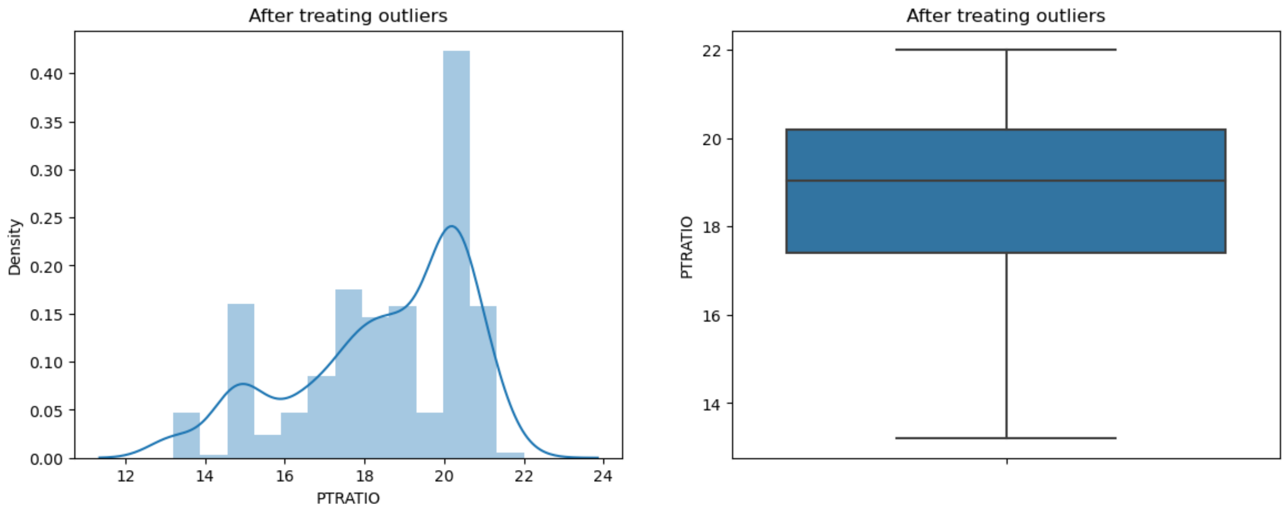
No outliers detected for 'TAX'.

In [26]:
```
draw_distplot_and_boxplot(df['PTRATIO'])
```

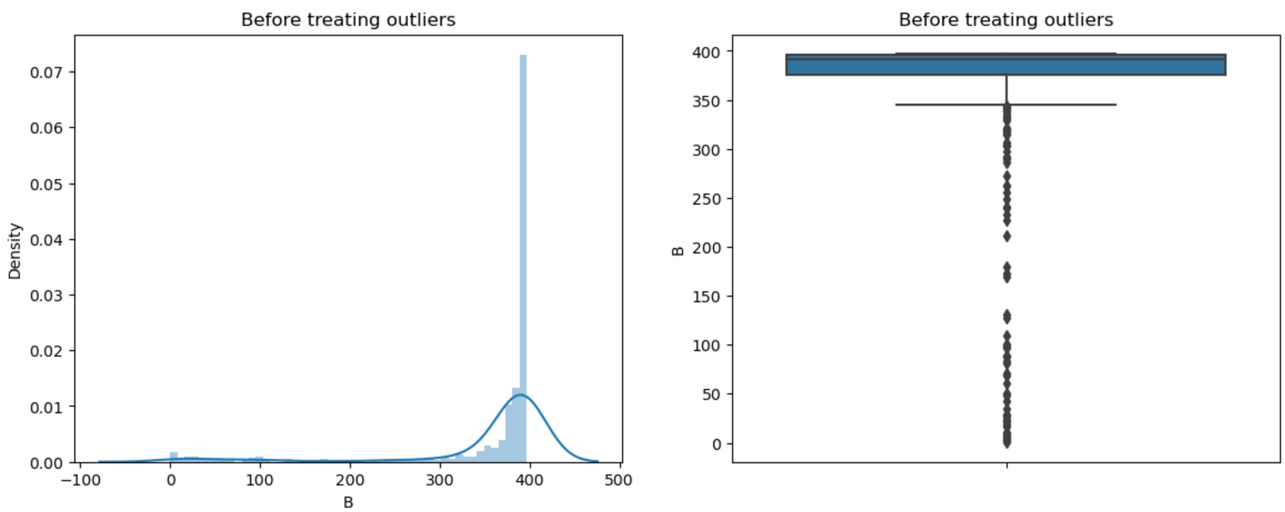


In [27]:
```python
# Since 'PTRATIO' is not normally distributed so we use the IQR based approach to treat outliers.

col_name = 'PTRATIO'

df[col_name] = handle_outliers_using_iqr(df[col_name])
draw_distplot_and_boxplot(df[col_name], outliers_treated=True)
```

```
q1: 17.4
q3: 20.2
iqr: 2.8000000000000007
```

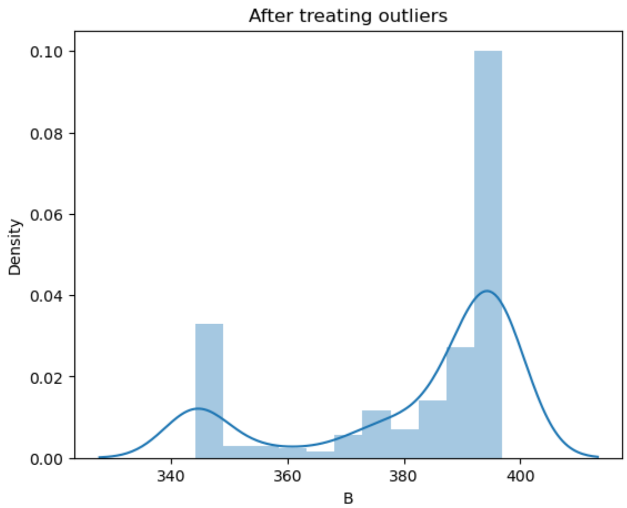


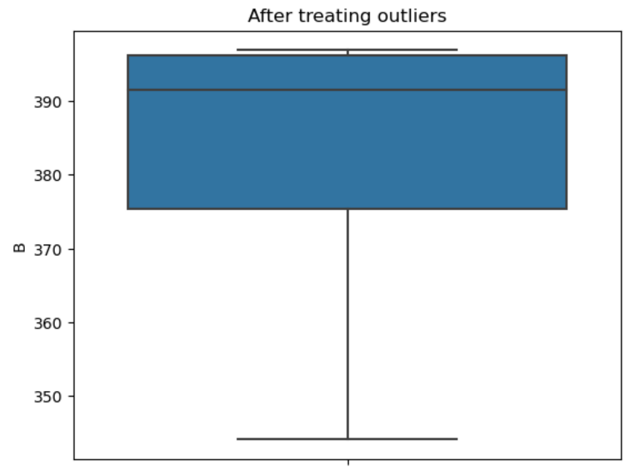
In [28]:
```python
draw_distplot_and_boxplot(df['B'])
```



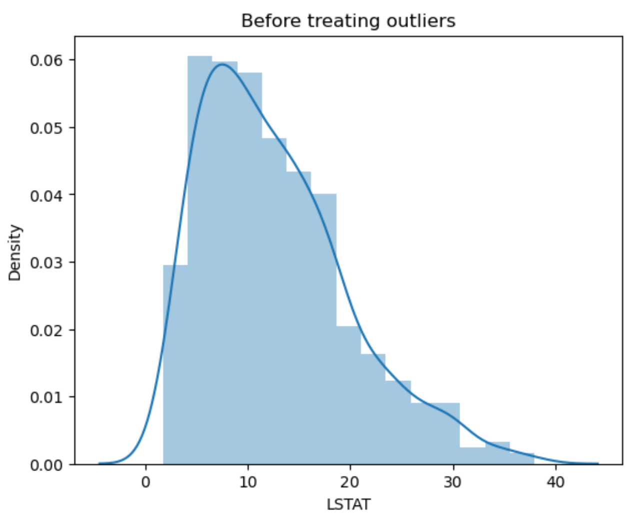
In [29]:
```python
# Since 'B' is not normally distributed so we use the IQR based approach to treat outliers.

col_name = 'B'

df[col_name] = handle_outliers_using_iqr(df[col_name])
draw_distplot_and_boxplot(df[col_name], outliers_treated=True)
```

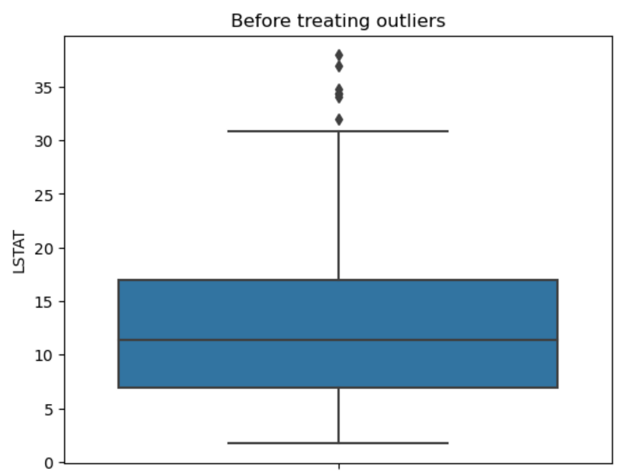
```
q1: 375.3775
q3: 396.225
iqr: 20.847500000000025
```





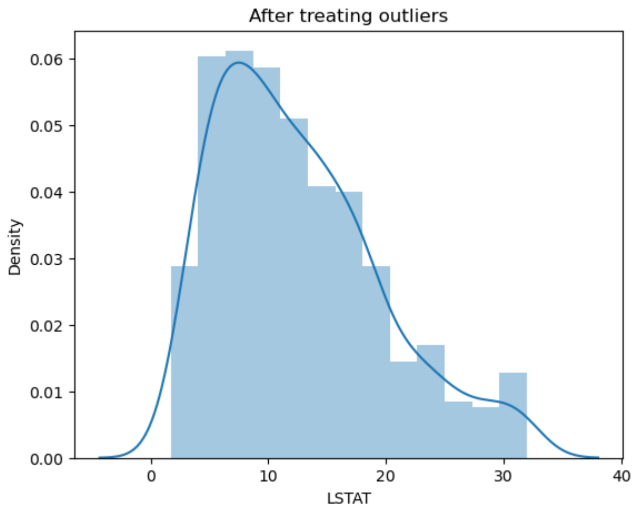
In [30]:
```python
draw_distplot_and_boxplot(df['LSTAT'])
```

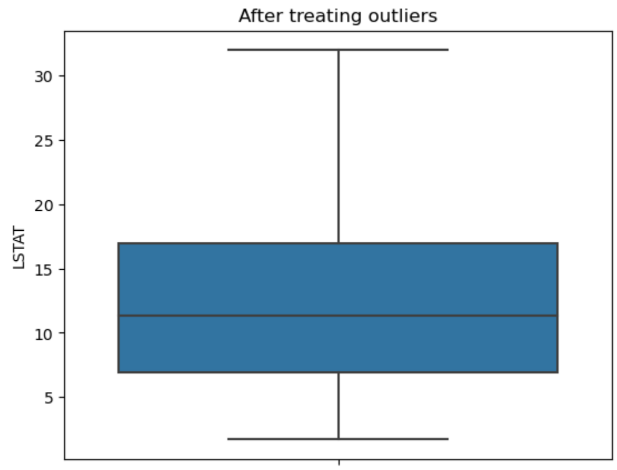




In [31]:
```
# Since 'LSTAT' is not normally distributed so we use the IQR based approach to treat outliers.

col_name = 'LSTAT'

df[col_name] = handle_outliers_using_iqr(df[col_name])
draw_distplot_and_boxplot(df[col_name], outliers_treated=True)
```

```
q1: 6.949999999999999
q3: 16.955000000000002
iqr: 10.005000000000003
```





No Encoding is required since no categorical variables are there.

Separating the independent and dependent variables from the dataframe:

In [32]:
```
x = df.drop('MEDV', axis=1)
y = df.iloc[:, -1:]
```

In [33]:
```
x
```

Out[33]:

	CRIM	ZN	INDUS	CHAS	NOX	RM	AGE	DIS	RAD	TAX	PTRATIO	B	LSTAT
0	0.00632	18.0	2.31	0	0.538	6.575	65.2	4.0900	1	296.0	15.3	396.90	4.98
1	0.02731	0.0	7.07	0	0.469	6.421	78.9	4.9671	2	242.0	17.8	396.90	9.14
2	0.02729	0.0	7.07	0	0.469	7.185	61.1	4.9671	2	242.0	17.8	392.83	4.03
3	0.03237	0.0	2.18	0	0.458	6.998	45.8	6.0622	3	222.0	18.7	394.63	2.94
4	0.06905	0.0	2.18	0	0.458	7.147	54.2	6.0622	3	222.0	18.7	396.90	5.33
...	...	...	...	...	...	...	...	...	...	...	...	...	...
501	0.06263	0.0	11.93	0	0.573	6.593	69.1	2.4786	1	273.0	21.0	391.99	9.67
502	0.04527	0.0	11.93	0	0.573	6.120	76.7	2.2875	1	273.0	21.0	396.90	9.08
503	0.06076	0.0	11.93	0	0.573	6.976	91.0	2.1675	1	273.0	21.0	396.90	5.64
504	0.10959	0.0	11.93	0	0.573	6.794	89.3	2.3889	1	273.0	21.0	393.45	6.48
505	0.04741	0.0	11.93	0	0.573	6.030	80.8	2.5050	1	273.0	21.0	396.90	7.88

506 rows × 13 columns

In [34]:
```
y
```

Out[34]:

	MEDV
0	24.0
1	21.6
2	34.7
3	33.4
4	36.2
...	...
501	22.4
502	20.6
503	23.9
504	22.0
505	11.9

506 rows × 1 columns

Feature Scaling:

In [35]:
```
from sklearn.preprocessing import StandardScaler
```

In [36]:
```
sc = StandardScaler()
sc_x = pd.DataFrame(sc.fit_transform(x), columns = x.columns)
```

In [37]:
```
sc_x
```

Out[37]:

	CRIM	ZN	INDUS	CHAS	NOX	RM	AGE	DIS	RAD	TAX	PTRATIO	B	LSTAT
0	-0.670290	0.918420	-1.287909	-0.272599	-0.144217	0.421834	-0.120013	0.148015	-0.982843	-0.666608	-1.477181	0.786988	-1.088749
1	-0.663949	-0.579471	-0.593381	-0.272599	-0.740262	0.198153	0.367166	0.572202	-0.867883	-0.987329	-0.309941	0.786988	-0.495302
2	-0.663955	-0.579471	-0.593381	-0.272599	-0.740262	1.307841	-0.265812	0.572202	-0.867883	-0.987329	-0.309941	0.573183	-1.224272
3	-0.662420	-0.579471	-1.306878	-0.272599	-0.835284	1.036229	-0.809889	1.101820	-0.752922	-1.106115	0.110265	0.667741	-1.379766
4	-0.651339	-0.579471	-1.306878	-0.272599	-0.835284	1.252647	-0.511180	1.101820	-0.752922	-1.106115	0.110265	0.786988	-1.038819
...	...	...	...	...	...	...	...	...	...	...	...	...	...
501	-0.653278	-0.579471	0.115738	-0.272599	0.158124	0.447978	0.018673	-0.631298	-0.982843	-0.803212	1.184126	0.529057	-0.419694
502	-0.658523	-0.579471	0.115738	-0.272599	0.158124	-0.239041	0.288933	-0.723719	-0.982843	-0.803212	1.184126	0.786988	-0.503861
503	-0.653843	-0.579471	0.115738	-0.272599	0.158124	1.004275	0.797449	-0.781754	-0.982843	-0.803212	1.184126	0.786988	-0.994596
504	-0.639091	-0.579471	0.115738	-0.272599	0.158124	0.739925	0.736996	-0.674679	-0.982843	-0.803212	1.184126	0.605753	-0.874765
505	-0.657876	-0.579471	0.115738	-0.272599	0.158124	-0.369763	0.434732	-0.618530	-0.982843	-0.803212	1.184126	0.786988	-0.675048

506 rows × 13 columns

In [38]:
```
y
```

Out[38]:

	MEDV
0	24.0
1	21.6
2	34.7
3	33.4
4	36.2
...	...
501	22.4
502	20.6
503	23.9
504	22.0
505	11.9

506 rows × 1 columns

In [39]:
```
df1 = pd.concat([sc_x, y], axis=1)
```

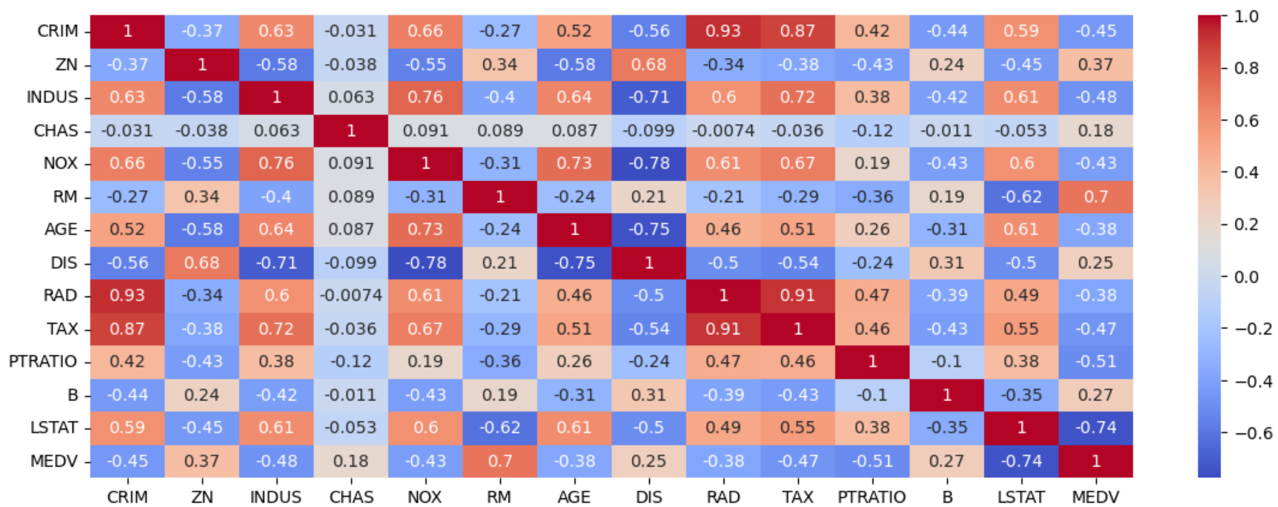
In [40]:
```
df1
```

Out[40]:

	CRIM	ZN	INDUS	CHAS	NOX	RM	AGE	DIS	RAD	TAX	PTRATIO	B	LSTAT	MEDV
0	-0.670290	0.918420	-1.287909	-0.272599	-0.144217	0.421834	-0.120013	0.148015	-0.982843	-0.666608	-1.477181	0.786988	-1.088749	24.0
1	-0.663949	-0.579471	-0.593381	-0.272599	-0.740262	0.198153	0.367166	0.572202	-0.867883	-0.987329	-0.309941	0.786988	-0.495302	21.6
2	-0.663955	-0.579471	-0.593381	-0.272599	-0.740262	1.307841	-0.265812	0.572202	-0.867883	-0.987329	-0.309941	0.573183	-1.224272	34.7
3	-0.662420	-0.579471	-1.306878	-0.272599	-0.835284	1.036229	-0.809889	1.101820	-0.752922	-1.106115	0.110265	0.667741	-1.379766	33.4
4	-0.651339	-0.579471	-1.306878	-0.272599	-0.835284	1.252647	-0.511180	1.101820	-0.752922	-1.106115	0.110265	0.786988	-1.038819	36.2
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
501	-0.653278	-0.579471	0.115738	-0.272599	0.158124	0.447978	0.018673	-0.631298	-0.982843	-0.803212	1.184126	0.529057	-0.419694	22.4
502	-0.658523	-0.579471	0.115738	-0.272599	0.158124	-0.239041	0.288933	-0.723719	-0.982843	-0.803212	1.184126	0.786988	-0.503861	20.6
503	-0.653843	-0.579471	0.115738	-0.272599	0.158124	1.004275	0.797449	-0.781754	-0.982843	-0.803212	1.184126	0.786988	-0.994596	23.9
504	-0.639091	-0.579471	0.115738	-0.272599	0.158124	0.739925	0.736996	-0.674679	-0.982843	-0.803212	1.184126	0.605753	-0.874765	22.0
505	-0.657876	-0.579471	0.115738	-0.272599	0.158124	-0.369763	0.434732	-0.618530	-0.982843	-0.803212	1.184126	0.786988	-0.675048	11.9

506 rows × 14 columns

In [41]:
```
plt.figure(figsize = (14, 5))
sns.heatmap(df1.corr(), annot=True, cmap='coolwarm')
plt.show()
```



In [42]:
```
from statsmodels.stats.outliers_influence import variance_inflation_factor
```

In [43]:
```
df_vif_1 = pd.DataFrame()
number_of_idvs = len(sc_x.columns)
df_vif_1['VIF'] = [variance_inflation_factor(sc_x, i) for i in range(number_of_idvs)]
df_vif_1['Features'] = sc_x.columns
```

In [44]:
```
df_vif_1
```

Out[44]:

	VIF	Features
0	9.378575	CRIM
1	2.416138	ZN
2	4.014900	INDUS
3	1.076203	CHAS
4	4.454974	NOX
5	1.938093	RM
6	3.107826	AGE
7	4.091150	DIS
8	12.644869	RAD
9	8.692795	TAX
10	1.894748	PTRATIO
11	1.345893	B
12	3.149192	LSTAT

In [45]:
```
# Dropping the variable 'RAD' since it has highest vif much greater than 5.

col_names_to_drop_1 = ['RAD']
sc_x1 = sc_x.drop(columns=col_names_to_drop_1)
df2 = df1.drop(columns=col_names_to_drop_1)
```

In [46]:
```
df_vif_2 = pd.DataFrame()
number_of_idvs = len(sc_x1.columns)
df_vif_2['VIF'] = [variance_inflation_factor(sc_x1, i) for i in range(number_of_idvs)]
df_vif_2['Features'] = sc_x1.columns
```

In [47]:
```
df_vif_2
```

Out[47]:

	VIF	Features
0	5.075773	CRIM
1	2.415039	ZN
2	3.853575	INDUS
3	1.058986	CHAS
4	4.438928	NOX
5	1.925925	RM
6	3.102010	AGE
7	4.091150	DIS
8	5.739788	TAX
9	1.811751	PTRATIO
10	1.343688	B
11	3.089874	LSTAT

In [48]:
```
'''
    Using the OLS approach we first came to know that the regression co-efficents of these columns 'TAX', 'B', 'CRIM',
    'ZN', 'AGE', 'INDUS' are not significant.

    That is how we came to the decsion of removing these variables 'TAX', 'B', 'CRIM', 'ZN', 'AGE', 'INDUS'.
'''
col_names_to_drop_2 = ['TAX', 'B', 'CRIM', 'ZN', 'AGE', 'INDUS']
sc_x2 = sc_x1.drop(columns=col_names_to_drop_2)
df3 = df2.drop(columns=col_names_to_drop_2)
```

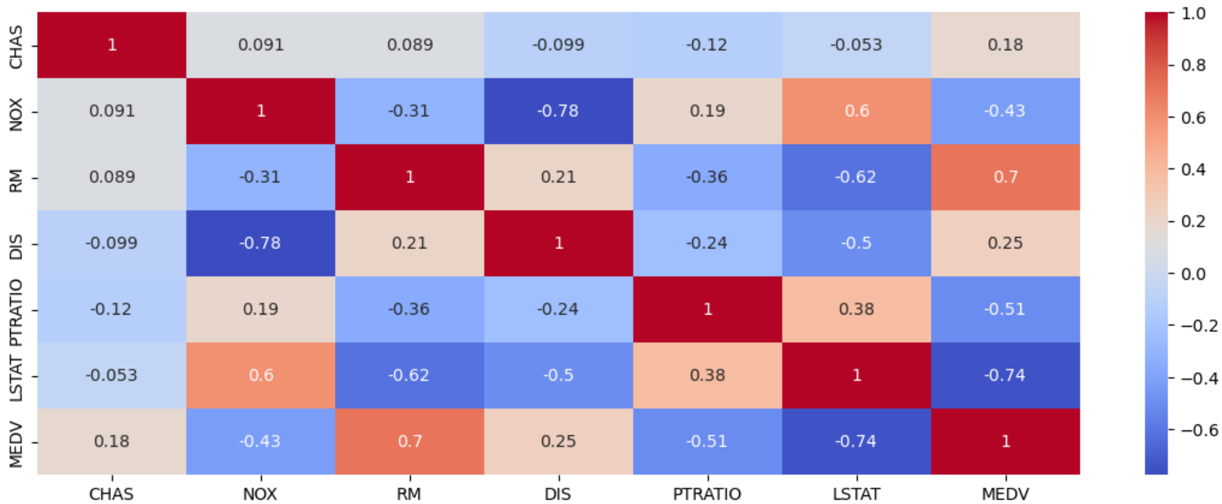
In [49]:
```
df_vif_3 = pd.DataFrame()
number_of_idvs = len(sc_x2.columns)
df_vif_3['VIF'] = [variance_inflation_factor(sc_x2, i) for i in range(number_of_idvs)]
df_vif_3['Features'] = sc_x2.columns
```

In [50]:
```
df_vif_3
```

Out[50]:

	VIF	Features
0	1.045092	CHAS
1	2.985758	NOX
2	1.721501	RM
3	2.629700	DIS
4	1.241742	PTRATIO
5	2.431918	LSTAT

In [51]:
```
plt.figure(figsize = (14, 5))
sns.heatmap(df3.corr(), annot=True, cmap='coolwarm')
plt.show()
```



Splitting the data into train and test datasets:

In [52]:
```
from sklearn.model_selection import train_test_split

x_train, x_test, y_train, y_test = train_test_split(sc_x2, y, test_size=0.2, random_state=101)
```

Building the Multiple Linear Regression based model:

In [53]:
```
from sklearn.linear_model import LinearRegression
```

In [54]:
```
linear_model_1 = LinearRegression()
linear_model_1.fit(x_train, y_train)
```

Out[54]: LinearRegression()

Testing the Multiple Linear Regression based Model:

In [55]:
```
y_pred_test = pd.DataFrame(linear_model_1.predict(x_test), columns=y_test.columns, index=y_test.index)
y_pred_train = pd.DataFrame(linear_model_1.predict(x_train), columns=y_train.columns, index=y_train.index)
```

In [56]:
```
y_pred_test
```

Out[56]:

	MEDV
195	38.611196
4	28.927656
434	17.851627
458	16.929009
39	28.836229
...	...
227	32.235591
405	12.099323
69	21.483005
231	33.438851
104	21.718087

102 rows × 1 columns

In [57]:
```
y_test
```

Out[57]:

	MEDV
195	50.0
4	36.2
434	11.7
458	14.9
39	30.8
...	...
227	31.6
405	5.0
69	20.9
231	31.7
104	20.1

102 rows × 1 columns

Calculating the Metrics for our MLR based model:

In [58]:
```
from sklearn.metrics import mean_absolute_error, mean_squared_error, r2_score
```

In [59]:
```
# MAE(Mean Absolute Error) for the DV(MEDV) on test data.
mean_absolute_error(y_test, y_pred_test)
```

Out[59]: 4.273255066580405

In [60]:
```
# MAE(Mean Absolute Error) for the DV(MEDV) on train data.
mean_absolute_error(y_train, y_pred_train)
```

Out[60]: 3.1831739816783937

In [61]:
```
# MSE(Mean Squared Error) for the DV(MEDV) on test data.
mse_test = mean_squared_error(y_test, y_pred_test)
mse_test
```

Out[61]: 34.78358077076526

In [62]:
```
# MSE(Mean Squared Error) for the DV(MEDV) on train data.
mse_train = mean_squared_error(y_train, y_pred_train)
mse_train
```

Out[62]: 20.42030377106413

In [63]:
```
# RMSE(Root Mean Squared Error) for the DV(MEDV) on test data.
np.sqrt(mse_test)
```

Out[63]: 5.897760657297417

In [64]:
```
# RMSE(Root Mean Squared Error) for the DV(MEDV) on train data.
np.sqrt(mse_train)
```

Out[64]: 4.51888302250281

In [65]:
```
r2_score_test = r2_score(y_test, y_pred_test)
r2_score_test
```

Out[65]: 0.6899615851528227

In [66]:
```
r2_score_train = r2_score(y_train, y_pred_train)
r2_score_train
```

Out[66]: 0.7355481593025808

In [67]:
```
# Adjusted R2 score for the test data.
# Adjusted R2 score formula:
# adj_r2_score = 1 - ((1-r2_score)*(n-1)/(n-k-1)) where:
# n = number of rows in the test dataset, k = number of IDVs.
k = len(x.columns)
n = len(x_test)
adj_r2_score_test = 1 - ((1-r2_score_test)*(n-1)/(n-k-1))
adj_r2_score_test
```

Out[67]: 0.6441604556867624

In [68]:
```
# Adjusted R2 score for the train data.
# n = number of rows in the training dataset, k = number of IDVs.
k = len(x.columns)
n = len(x_train)
adj_r2_score_train = 1 - ((1-r2_score_train)*(n-1)/(n-k-1))
adj_r2_score_train
```

Out[68]:
```
0.726733097946
```

Applying Lasso Regression:

In [69]:
```
from sklearn.linear_model import Lasso
```

In [70]:
```
lasso = Lasso(alpha = 0.1)
lasso.fit(x_train, y_train)
```

Out[70]:
```
Lasso(alpha=0.1)
```

In [71]:
```
lasso.coef_
```

Out[71]:
```
array([ 0.82901359, -1.48097014,  2.95027706, -1.9996023 , -1.89294518,
       -4.2453222 ])
```

In [72]:
```
lasso.intercept_
```

Out[72]:
```
array([22.36113033])
```

In [73]:
```
y_pred_test_lasso = lasso.predict(x_test)
y_pred_train_lasso = lasso.predict(x_train)
```

In [74]:
```
r2_score(y_test, y_pred_test_lasso)
```

Out[74]:
```
0.6880207013685942
```

In [75]:
```
r2_score(y_train, y_pred_train_lasso)
```

Out[75]:
```
0.7340607269918954
```

Applying Ridge Regression:

In [76]:
```
from sklearn.linear_model import Ridge
```

In [77]:
```
ridge = Ridge(alpha=0.3)
ridge.fit(x_train, y_train)
```

Out[77]:
```
Ridge(alpha=0.3)
```

In [78]:
```
ridge.coef_
```

Out[78]:
```
array([[ 0.89435824, -1.91159864,  2.96143107, -2.47424559, -1.99874223,
        -4.2877707 ]])
```

In [79]:
```
ridge.intercept_
```

Out[79]:
```
array([22.35822674])
```

In [80]:
```
y_pred_test_ridge = ridge.predict(x_test)
y_pred_train_ridge = ridge.predict(x_train)
```

In [81]:
```
r2_score(y_test, y_pred_test_ridge)
```

Out[81]:
```
0.6899765010803762
```

In [82]:
```
r2_score(y_train, y_pred_train_ridge)
```

Out[82]:
```
0.7355476634117355
```

In [83]:
```
x_test
```

Out[83]:

	CHAS	NOX	RM	DIS	PTRATIO	LSTAT
195	-0.272599	-1.146264	2.310046	0.901696	-1.897388	-1.375486
4	-0.272599	-0.835284	1.252647	1.101820	0.110265	-1.038819
434	-0.272599	1.367490	-0.111223	-0.755299	0.810609	0.364911
458	-0.272599	1.367490	0.023856	-0.484034	0.810609	0.516126
39	-0.272599	-1.094434	0.450883	0.782095	-0.076493	-1.182901
...	...	...	...	...	...	...
227	-0.272599	-0.437921	1.275887	-0.274818	-0.496700	-0.891884
405	-0.272599	1.194724	-0.873771	-1.140652	0.810609	1.479051
69	-0.272599	-1.258562	-0.580372	1.312583	0.203644	-0.545231
231	-0.272599	-0.437921	1.637552	-0.054382	-0.496700	-1.050232
104	-0.272599	-0.299707	-0.170775	-0.659155	1.137436	-0.040230

102 rows × 6 columns

In [84]:
```
from statsmodels.regression.linear_model import OLS
import statsmodels.regression.linear_model as smf
```

In [85]:
```
x_test2 = x_test.copy()
x_test2['const'] = np.ones(x_test.shape[0])

x_train2 = x_train.copy()
x_train2['const'] = np.ones(x_train.shape[0])
```

In [86]:
```
reg_model = smf.OLS(endog = y_train, exog = x_train2, hasconst=True).fit()
```

In [87]:
```
reg_model.summary()
```

Out[87]:

OLS Regression Results

Dep. Variable:	MEDV	**R-squared:**	0.736
Model:	OLS	**Adj. R-squared:**	0.732
Method:	Least Squares	**F-statistic:**	184.0
Date:	Fri, 28 Jul 2023	**Prob (F-statistic):**	2.35e-111
Time:	13:27:04	**Log-Likelihood:**	-1182.6
No. Observations:	404	**AIC:**	2379.
Df Residuals:	397	**BIC:**	2407.
Df Model:	6		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
CHAS	0.8943	0.235	3.813	0.000	0.433	1.355
NOX	-1.9170	0.399	-4.808	0.000	-2.701	-1.133
RM	2.9609	0.301	9.839	0.000	2.369	3.552
DIS	-2.4828	0.374	-6.637	0.000	-3.218	-1.747
PTRATIO	-1.9996	0.257	-7.766	0.000	-2.506	-1.493
LSTAT	-4.2925	0.372	-11.540	0.000	-5.024	-3.561
const	22.3582	0.227	98.504	0.000	21.912	22.804

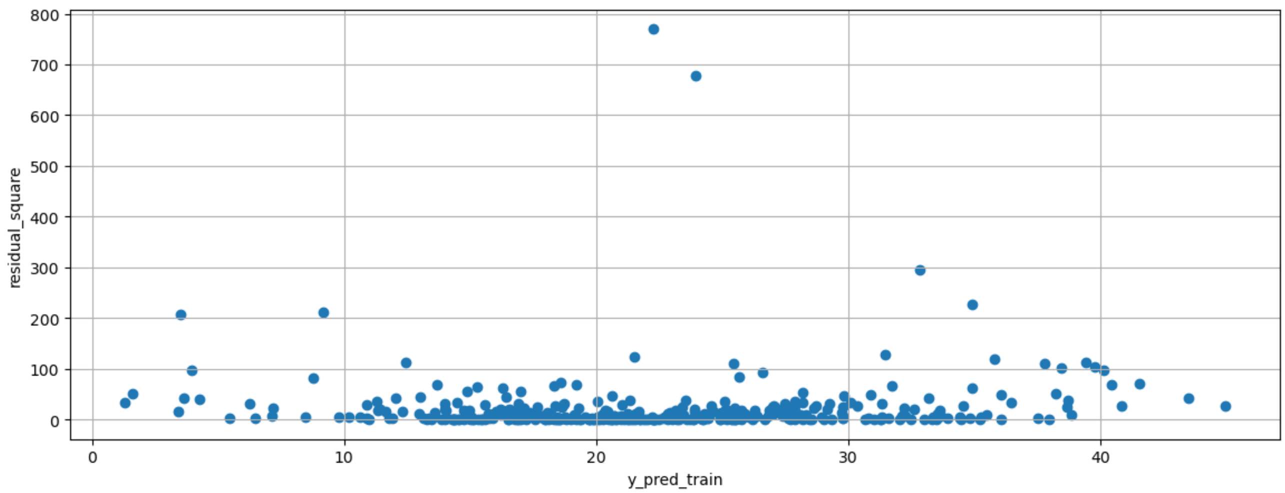
Omnibus:	160.593	**Durbin-Watson:**	1.875
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	857.342
Skew:	1.626	**Prob(JB):**	6.77e-187
Kurtosis:	9.353	**Cond. No.**	3.63

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

Checking Assumptions of Linear Regression:

Homoscedasticity check for the residual error:

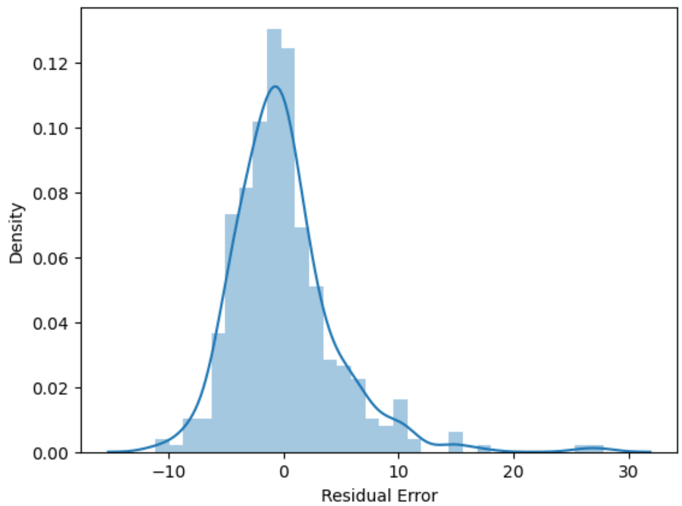
In [88]:
```
plt.figure(figsize = (14,5))
residual_square = (y_train - y_pred_train)**2
plt.scatter(y_pred_train, residual_square)
plt.xlabel('y_pred_train')
plt.ylabel('residual_square')
plt.grid()
```



Normality check for the residual error:

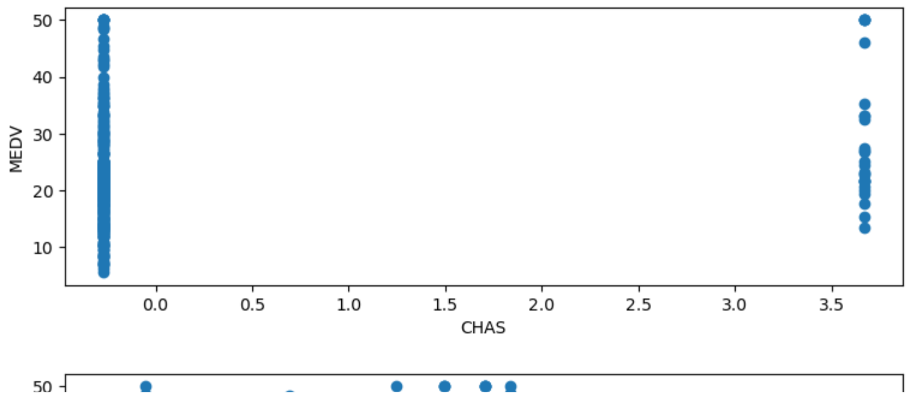
In [89]:
```
sns.distplot(y_train-y_pred_train)
plt.xlabel('Residual Error')
```

Out[89]: `Text(0.5, 0, 'Residual Error')`



Checking relationships b/w DV and IDVs:

In [90]:
```
for col in x_train.columns:
    plt.figure(figsize = (9,3))
    plt.scatter(x_train[col], y_train)
    plt.xlabel(col)
    plt.ylabel(y.columns[0])
```



Autocorrelation check for the residual error:

From the Regression Summary report **Durbin-Watson** which tests the first order autocorrelation(lag = 1) comes as **1.875** which is close to 2 thus indicating almost no autocorrelation b/w the residuals.

Applying Polynomial regression:

About Polynomial Regression:

Polynomial Regression allows us to find a function that fits the non-linear relationship b/w DV and IDVs.

Let the degree of the Polynomial Regression be 2 and let there be 2 IDVs x1, x2 and one DV y.

Equation for the population regression line for polynomial case:

y = beta_0 + x1 * beta_1 + x2 * beta_2 + x1^2 * beta_3 + x1 * x2 * beta_4 + x2^2 * beta_5 + epsilon, epsilon is the random error occuring during measurement of the DV y.

Equation for the sample regression line for polynomial case:

y_hat = beta_0_hat + x1 * beta_1_hat + x2 * beta_2_hat + x1^2 * beta_3_hat + x1 * x2 * beta_4_hat + x2^2 * beta_5_hat

In [91]:
```
from sklearn.preprocessing import PolynomialFeatures
'''
    Using heuristic approach we came to degree=3 as the hyperparameter value for degree.
'''
poly = PolynomialFeatures(degree = 2)
```

In [92]:
```
x_train
```

Out[92]:

	CHAS	NOX	RM	DIS	PTRATIO	LSTAT
288	-0.272599	-1.293115	0.044191	1.708769	-0.870216	-0.714991
72	-0.272599	-1.224009	-0.318927	0.727059	0.343713	-1.011715
471	-0.272599	-0.196047	-0.080721	-0.331112	0.810609	0.036804
176	-0.272599	-0.386091	-0.384288	-0.110773	-0.870216	-0.356926
320	-0.272599	-0.532942	0.205415	0.365839	0.530471	-0.772053
...	...	...	...	...	...	...
63	-0.272599	-0.878475	0.693446	2.029750	0.577161	-0.443946
326	-0.272599	-0.532942	0.039834	0.789253	0.530471	-0.921842
337	-0.272599	-0.342899	-0.565847	0.885543	0.810609	-0.292731
11	-0.272599	-0.265154	-0.400265	1.181376	-1.523871	0.093866
351	-0.272599	-1.241285	0.427643	2.919572	-0.076493	-1.015994

404 rows × 6 columns

In [93]:
```
x_train_trans = poly.fit_transform(x_train)
x_test_trans = poly.fit_transform(x_test)
```

In [94]:
```
x_train_trans.shape
```

Out[94]:
```
(404, 28)
```

In [95]:
```
x_test_trans.shape
```

Out[95]:
```
(102, 28)
```

In [96]:
```
linear_model_2 = LinearRegression()
linear_model_2.fit(x_train_trans, y_train)
```

Out[96]:
```
LinearRegression()
```

In [97]:
```
y_pred_train_trans = linear_model_2.predict(x_train_trans)
y_pred_test_trans = linear_model_2.predict(x_test_trans)
```

In [98]:
```
r2_score(y_train, y_pred_train_trans)
```

Out[98]:
```
0.8693269284200376
```

In [99]:
```
r2_score(y_test, y_pred_test_trans)
```

Out[99]:
```
0.7880534925286308
```

In [100]:

```
print(y_pred_test_trans)
```

```
[[44.671875  ]
 [29.90136719]
 [16.51855469]
 [14.796875  ]
 [27.99609375]
 [31.2109375 ]
 [45.05859375]
 [13.55273438]
 [36.13867188]
 [ 9.703125  ]
 [24.07617188]
 [12.13867188]
 [19.04296875]
 [19.45214844]
 [25.08007812]
 [22.96679688]
 [10.49511719]
 [30.95703125]
 [29.30664062]
 [24.76855469]
 [10.2265625 ]
 [18.64746094]
 [26.05273438]
 [29.3671875 ]
 [32.58496094]
 [17.28320312]
 [29.67382812]
 [16.79199219]
 [35.18945312]
 [36.84082031]
 [20.3828125 ]
 [19.46386719]
 [37.64746094]
 [42.92382812]
 [31.17480469]
 [20.8046875 ]
 [13.54101562]
 [17.61523438]
 [ 7.62988281]
 [30.59960938]
 [21.29003906]
 [22.83398438]
 [40.5546875 ]
 [12.71875   ]
 [16.89355469]
 [24.61328125]
 [32.44335938]
 [14.07910156]
 [25.82714844]
 [30.11523438]
 [34.24902344]
 [42.86132812]
 [20.79980469]
 [25.5390625 ]
 [32.71386719]
 [13.67578125]
 [19.99707031]
 [16.13476562]
 [20.92382812]
 [22.92382812]
 [31.86816406]
 [10.88085938]
 [35.80273438]
 [23.41308594]
 [10.57910156]
 [24.44140625]
 [22.95800781]
 [17.54296875]
 [13.03320312]
 [18.33789062]
 [23.23046875]
 [22.50488281]
 [17.20507812]
 [19.65722656]
 [25.82910156]
 [17.42382812]
 [43.70117188]
 [26.90332031]
 [30.41992188]
 [12.82714844]
 [14.96386719]
 [17.23339844]
 [30.1640625 ]
 [12.80273438]
 [29.22460938]
 [21.296875  ]
 [20.32128906]
 [31.9453125 ]
 [21.48632812]
 [21.6796875 ]
 [10.94921875]
 [12.72167969]
 [26.74609375]
 [35.22558594]
 [ 9.54882812]
```

```
 [39.54785156]
 [11.47851562]
 [32.87011719]
 [11.46191406]
 [20.01757812]
 [34.95996094]
 [21.06445312]]
```

In [101]:
```python
print(y_test)
```

```
     MEDV
195  50.0
4    36.2
434  11.7
458  14.9
39   30.8
..    ...
227  31.6
405   5.0
69   20.9
231  31.7
104  20.1

[102 rows x 1 columns]
```

Conclusion:

Using Polynomial regression of degree 2 gives us the best accuracy for both test and train data.

- **Train Accuracy: 86.93%**
- **Test Accuracy: 78.81%**

Variables that are significant in predicting the DV 'MEDV' are :

- CHAS
- NOX
- RM
- DIS
- PTRATIO
- LSTAT