EE2703: ASSIGNMENT 6B

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1 Introduction

We analyse LTI systems in continuous time using Laplace transforms to find the output of the system to a given input with the help of a python library, namely `scipy.signal` toolbox.

Time Response of Spring Oscillator

Our goal is to find the response of a spring oscillator, governed by the equation:

$$\ddot{x} + 2.25x = f(t)$$

where,

$$x(t) = \text{Displacement of spring}$$
$$f(t) = \text{Force applied on the spring}$$

We consider the case that the force applied on the spring is given by:

$$f(t) = e^{-at}cos(\omega t)u(t)$$

We shall do a case by case analysis for the following values of a and ω:

Theory:

The Laplace transform of $f(t) = e^{-at}cos(\omega t)u(t)$ is given as:

$$\mathcal{L}\{f(t)\} = \frac{s+a}{(s+a)^2+\omega^2}$$

From the property of Laplace transforms, we know:

$$x(t) \longleftrightarrow \mathcal{X}(s)$$
$$\implies \dot{x}(t) \longleftrightarrow \; s\mathcal{X}(s) - x(0^-)$$
$$\implies \ddot{x}(t) \longleftrightarrow \; s^2\mathcal{X}(s) - sx(0^-) - \dot{x}(0^-)$$

From the above equations, we get, for $a = 0.5$ and $\omega = 1.5$:

$$\mathcal{F}(s) = \mathcal{L}\{f(t)\} = \frac{s+0.5}{(s+0.5)^2+2.25}$$

So, the equation of the spring oscillator can be written as:

$$s^2\mathcal{X}(s) - sx(0^-) - \dot{x}(0^-) + 2.25\mathcal{X}(s) = \frac{s+0.5}{(s+0.5)^2+2.25}$$

Given that the initial Conditions $x(0)$ and $\dot{x}(0)$ are 0, we get:

$$s^2\mathcal{X}(s) + 2.25\mathcal{X}(s) = \frac{s+0.5}{(s+0.5)^2+2.25}$$

or,

$$\mathcal{X}(s) = \frac{s+0.5}{((s+0.5)^2+2.25)(s^2+2.25)}$$

2 Question : 1,2

2.1 Response for two different α values

Code:

```
# Question : 1,2
def ResX(a, w):
    XsNum = np.poly1d([1, a])
    XsDen = np.polymul([1, 2*a, a**2 + 2.25],[1,0,2.25])
    Xs = sp.lti(XsNum, XsDen)
    t, X = sp.impulse(Xs, None, np.linspace(0, 50, 500))
    return t, X
t1, x1 = ResX(0.5, 1.5)
t2, x2 = ResX(0.05, 1.5)

# Plotting the time response for two different values of decay constant
figure(1)
xlabel(r'$t$')
ylabel(r'$x(t)$')
plot(t2,x2,'b',label = 'Damping coeff = 0.5')
plot(t1,x1,'m',label = 'Damping coeff = 0.05')
title(r"System with Decay constant =0.5 & 0.05")
legend()
show()
```

Plot:

To find the x(t), and plotting it (for $0 < t < 50s$):

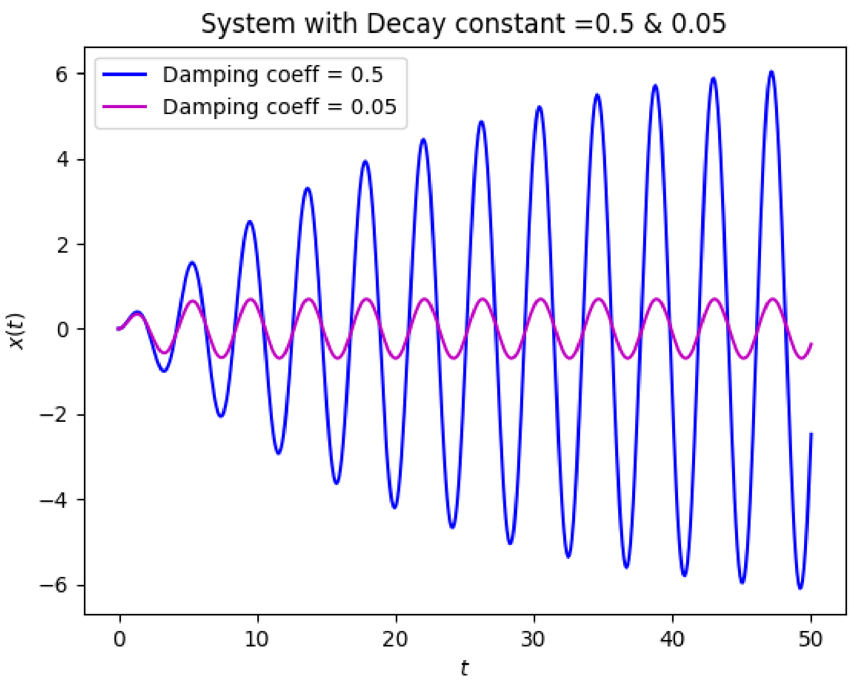

Figure 1: $x(t)$ for $a = 0.5$,$a = 0.05$ and $\omega = 1.5$

3 Question : 3

3.1 Response for different frequencies

Computing the response for various frequencies

Code:

```
# Question : 3
def f(t,w):
    return (cos(w*t))*exp(-0.05*t)

# Simulating it for different values of 'w'
figure(2)
xlabel(r'$t$')
ylabel(r'$x(t)$')
k1 = 0
for i in range(5):
    k = 1.4 + 0.05*k1
    H = sp.lti([1],[1,0,2.25])
    t=linspace(0,50,500)
    u=f(t,k)
    t,y,svec=sp.lsim(H,u,t)
    k1 = k1 + 1
    plot(t,y,label = 'w =  {:.2f}'.format(k))
xlabel(r"$t \to$")
ylabel(r"$x(t) \to$")
title(r"Response of LTI system to various frequencies")
legend()
show()
```

Plot:

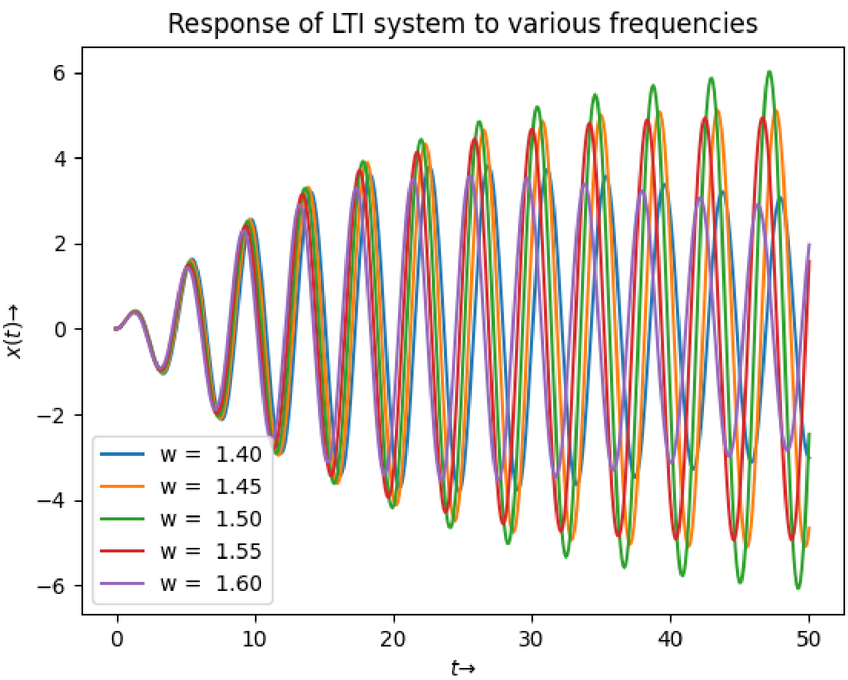

Figure 2: $x(t)$ for $a = 0.05$ and varying ω

From the given equation, we can see that the natural response of the system has the frequency $\omega = 1.5\ rad/s$. Thus, as expected, the maximum amplitude of oscillation is obtained when the frequency of $f(t)$ is $1.5\ rad/s$, as a case of resonance.

4 Question : 4

4.1 Coupled Spring Response:

The coupled equations to solve

$$\ddot{x} + (x - y) = 0$$
$$\ddot{y} + 2(y - x) = 0$$

Given the Initial Conditions $x(0) = 1$ and $\dot{x}(0) = y(0) = \dot{y}(0) = 0$, we can write the above differential equation in the Laplace domain as:

$$s^4\mathcal{X}(s) - s^3 + 3(s^2\mathcal{X}(s) - s) = 0$$
$$\implies \mathcal{X}(s) = \frac{s^2+3}{s^3+3s}$$
$$\implies \mathcal{Y}(s) = \frac{2}{s^3+3s}$$

Solving for $x(t)$ and $y(t)$ is now very simple - use `scipy.signal.impulse` with the above $\mathcal{X}(s)$ and $\mathcal{Y}(s)$. We get the following graph for $x(t)$ and $y(t)$.

Code:

```
# Question : 4
# Defining the transfer function of one spring
H1 = sp.lti([1,0,2],[1,0,3,0])
tt1,X1 = sp.impulse(H1,None,linspace(0,20,500))

# Defining the transfer function of other spring
H2 = sp.lti([2],[1,0,3,0])
tt2,Y1 = sp.impulse(H2,None,linspace(0,20,500))

# Plotting the time Response of coupled spring system
figure(3)
xlabel(r'$t---->$')
plot(tt2,Y1,'r',label = 'y(t)')
plot(tt1,X1,'g',label = 'x(t)')
title(r"Coupled System Response ")
legend()
show()
```

Plot:

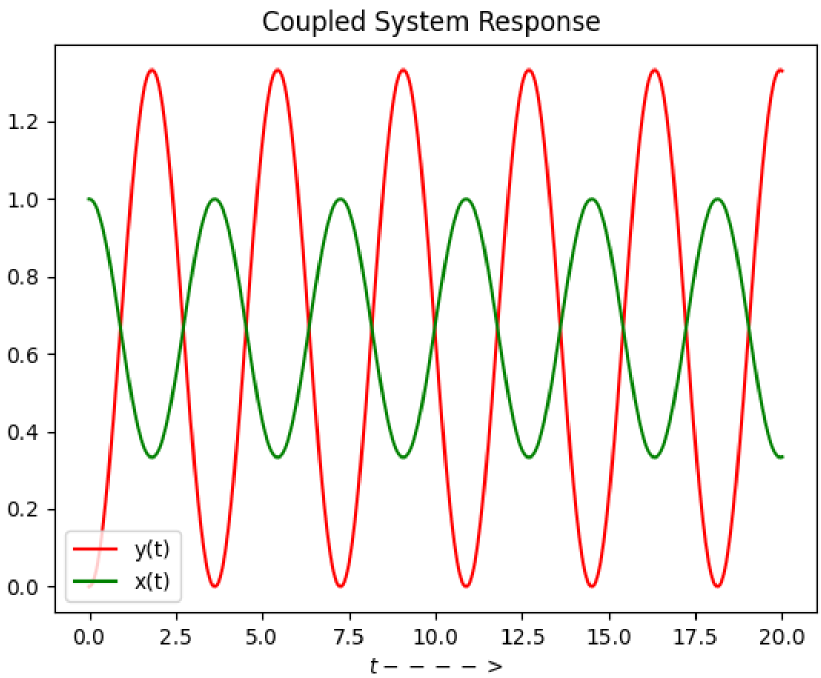

Figure 3: $x(t)$ and $y(t)$ for the coupled spring problem

We can see that $x(t)$ and $y(t)$ are sinusoids of the same frequency, but of different phase and magnitude.

5 Question : 5

5.1 Two-port Network Response

The transfer function of the given two-port network can be written as:

$$\frac{V_o(s)}{V_i(s)} = \mathcal{H}(s) = \frac{1}{10^-12s^2 + 10^-4s + 1}$$

Code:

```
# Question : 5
tx = linspace(0,500 ,1000 )
Hs_num = poly1d([1])
Hs_den = poly1d([10**(-12),10**(-4),1] )
HH = sp.lti(Hs_num,Hs_den) # Defining the transfer function of two port network
w,S,vs = HH.bode() # Bode

figure(4)
subplot(2,1,1)
semilogx(w, S)
xlabel(r"$\omega \ \to$")
ylabel(r"$\|H(jw)\|\ (in\ dB)$")
title("Bode plot of the given RLC network")

subplot(2,1,2)
xlabel(r"$\omega \ \to$")
ylabel(r"$\angle H(jw)\ (in\ ^o)$")
semilogx(w, vs)
```

```
show()
```

Plot:

The Bode magnitude and phase plots can be found using the method . The plots are shown below:

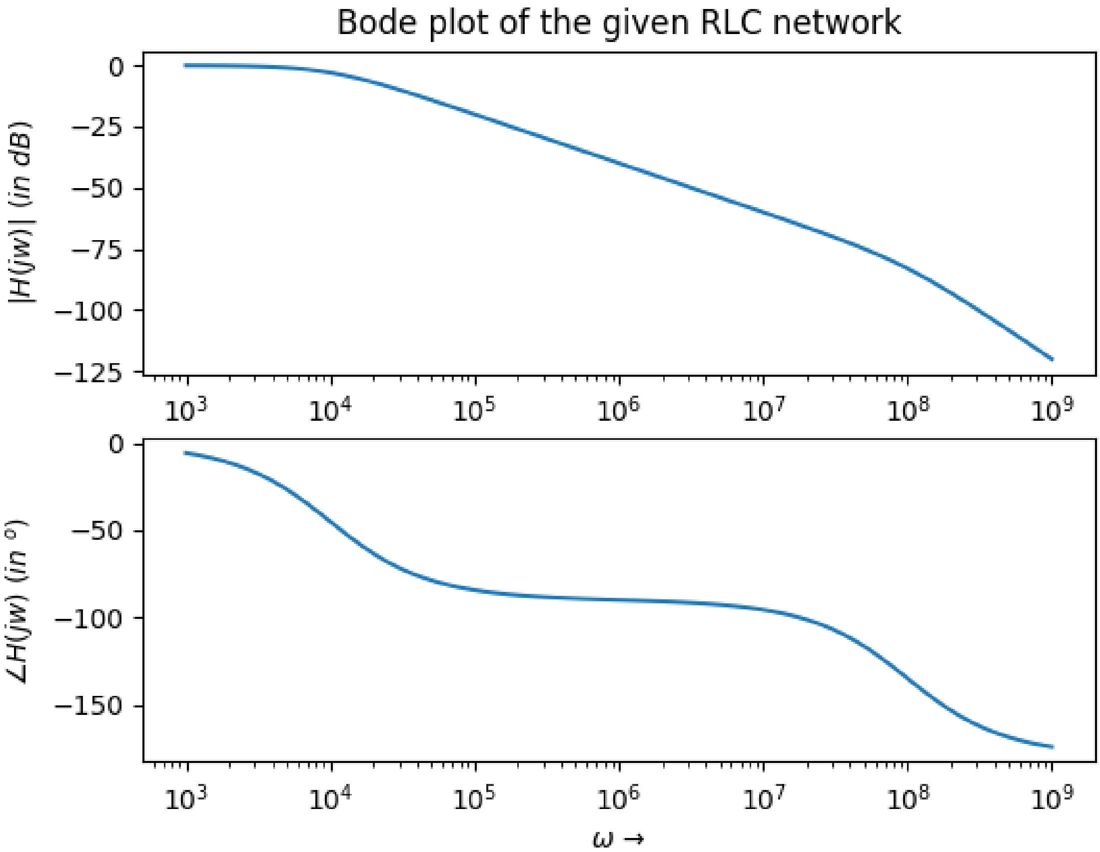

Figure 4: Bode Plots of the RLC Network's Transfer function

6 Question: 6

We excite the system in Question 5 with two sinusoids, one whose frequency is below the 3-dB bandwith and one whose frequency is higher.

Code:

```
# Question : 6
tx1 = linspace(0,10**(-2),10**6)
v = cos(10**(3)*tx1) - cos(10**(6)*tx1) # Input signal
t3,v1,svec = sp.lsim(HH,v,tx1)

figure(5)
# Plotting the Steady State Response of output signal
# Plotting the output signal (Zoomed in)
plot(t3,v1,'b',label ='Vo(t)')
title("Steady state Response")
ylabel(r"$V_{o}(t)   --->$",size=15)
xlabel(r"$t  --->$",size=15)
legend()
show()
```

```
tx2 = linspace(0,30*10**(-6),3001)
v = cos(10**(3)*tx2) - cos(10**(6)*tx2) # Input signal
t4,v2,svec = sp.lsim(HH,v,tx2)

# Plotting the Transient Response of output signal
figure(6)
plot(t4,v2,'b',label ='Vo(t)')
title("Transient Response")
ylabel(r"$V_{o}(t)--->$",size=15)
xlabel(r"$t--->$",size=15)
legend()
show()
```

Plots:

We plot the time domain response in two parts, one for the first 30 μs, to observe transient effects, and one for 10 msec, to observe the steady state response.

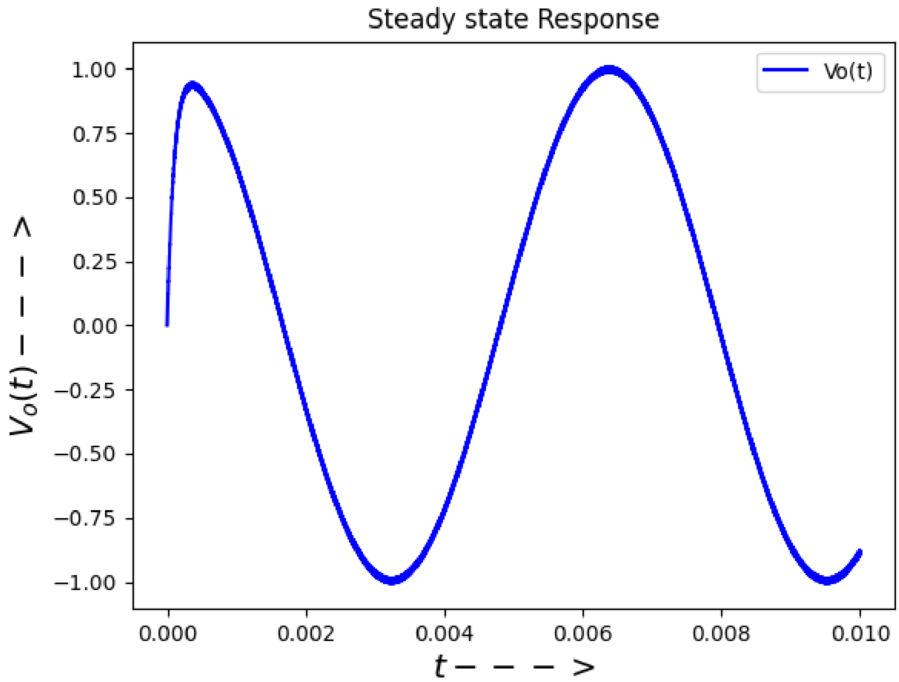

Figure 5

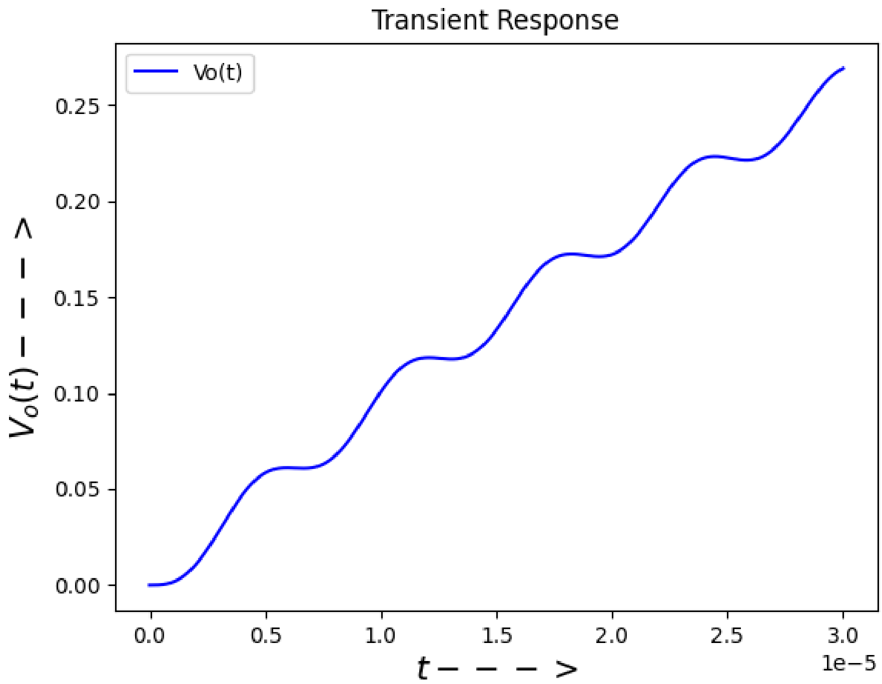

Figure 6

7 Conclusions

- To conclude, we analysed the solution of various continuous time LTI systems using Laplace transforms with help of `scipy.signal` toolbox.
- We have used scipy 's signal processing library to analyze a wide range of LTI systems. Specifically we analysed forced oscillatory system, single sping ,coupled spring and Filters.