EE2703: ASSIGNMENT-8

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Introduction

In this assignment, we want to explore how to obtain the DFT, and how to recover the analog Fourier Tranform for some known functions by the proper sampling of the function.

ASSIGNMENT TASKS:

1 Spectrum of $sin(5t)$

We calculate the DFT of $f(t)$ by the method mentioned in the above section. Then, we plot the phase and magnitude of the DFT and the following is obtained for $sin(5t)$:

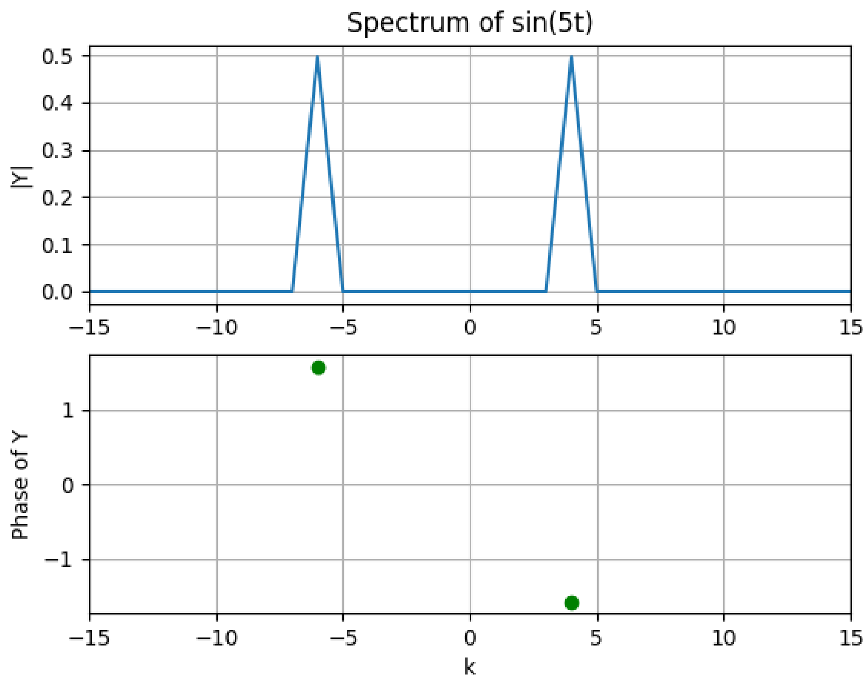

Figure 1: Spectrum of $sin(5t)$

This is expected, because:

$$sin(5t) = \frac{1}{2j}(e^{5jt} - e^{-5jt}) \tag{1}$$

So, the frequencies present in the DFT of $sin(5t)$ are $\omega = \pm 5\ rad/sec$, and the phase associated with them is $\phi = \pm\frac{\pi}{2}\ rad/sec$ respectively. This is exactly what is shown in the above plot.

2 Amplitude Modulation with $(1 + 0.1cos(t))cos(10t)$

We have,

$$(1 + 0.1cos(t))cos(10t) = \frac{1}{2}(e^{10jt} + e^{-10jt}) + 0.1 \cdot \frac{1}{2} \cdot \frac{1}{2}(e^{11jt} + e^{-11jt} + e^{9jt} + e^{-9jt}) \quad (2)$$

Writing $(1+0.1cos(t))cos(10t)$ in a different form as shown in (2), we observe that the frequencies present in the signal are $\omega = \pm 10\ rad/sec$, $\omega = \pm 11\ rad/sec$ and $\omega = \pm 9\ rad/sec$. Thus we expect the DFT also to have non-zero magnitudes only at these frequencies.

Plotting the DFT using the `numpy.fft` package, we get:

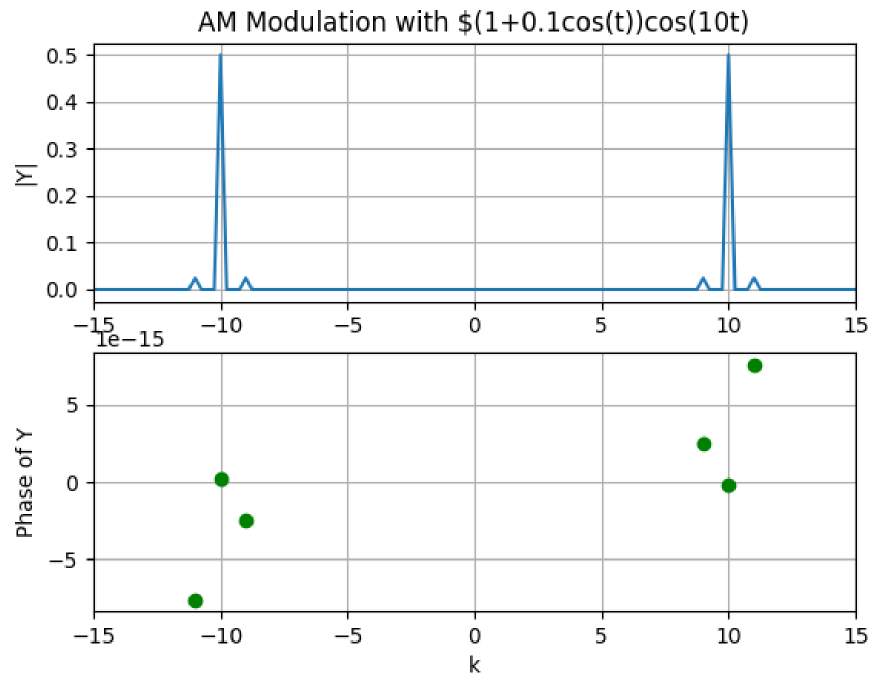

Figure 2: DFT of $(1 + 0.1cos(t))cos(10t)$

Figure (2) is in line with the theory we have discussed above.

3 Spectra of $sin^3(t)$ and $cos^3(t)$

DFT Spectrum of $sin^3(t)$:

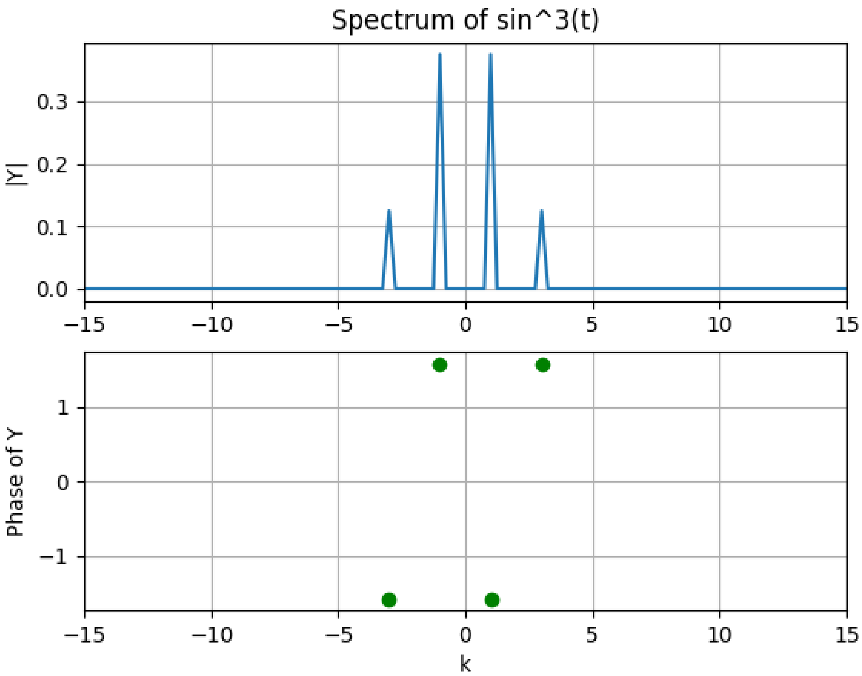

Figure 3: Spectrum of $sin^3(t)$

DFT Spectrum of $cos^3(t)$:

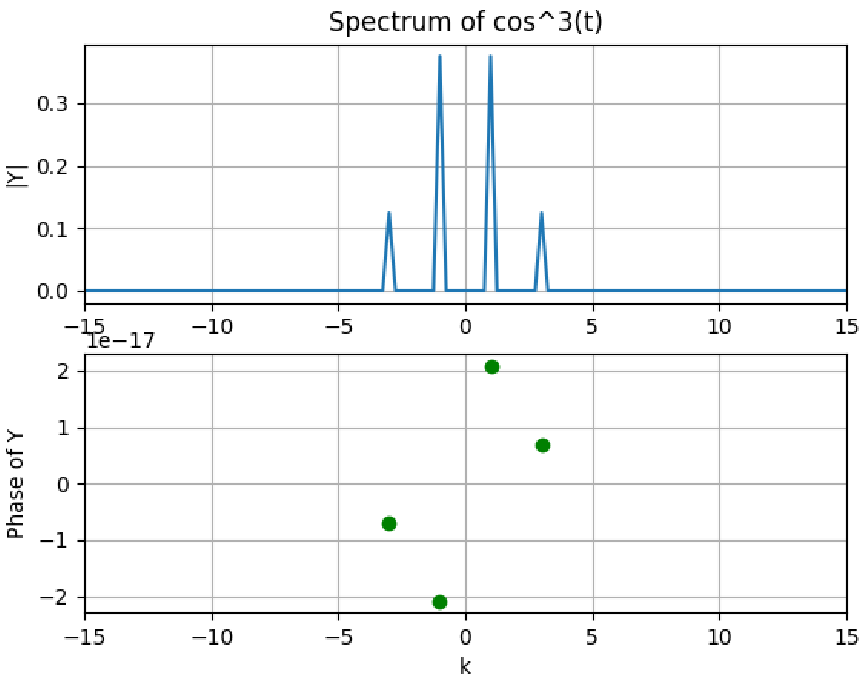

Figure 4: Spectrum of $cos^3(t)$

The above 2 figures are expected because:

$$sin^3(t) = \frac{3}{4}sin(t) - \frac{1}{4}sin(3t) \quad (3)$$

$$cos^3(t) = \frac{3}{4}cos(t) + \frac{1}{4}cos(3t) \quad (4)$$

So, we expect peaks $\omega = \pm 1\ rad/sec$ and $\omega = \pm 3\ rad/sec$.

4 Frequency Modulation with $cos(20t + 5cos(t))$

The DFT of $cos(20t + 5cos(t))$ can be seen below:

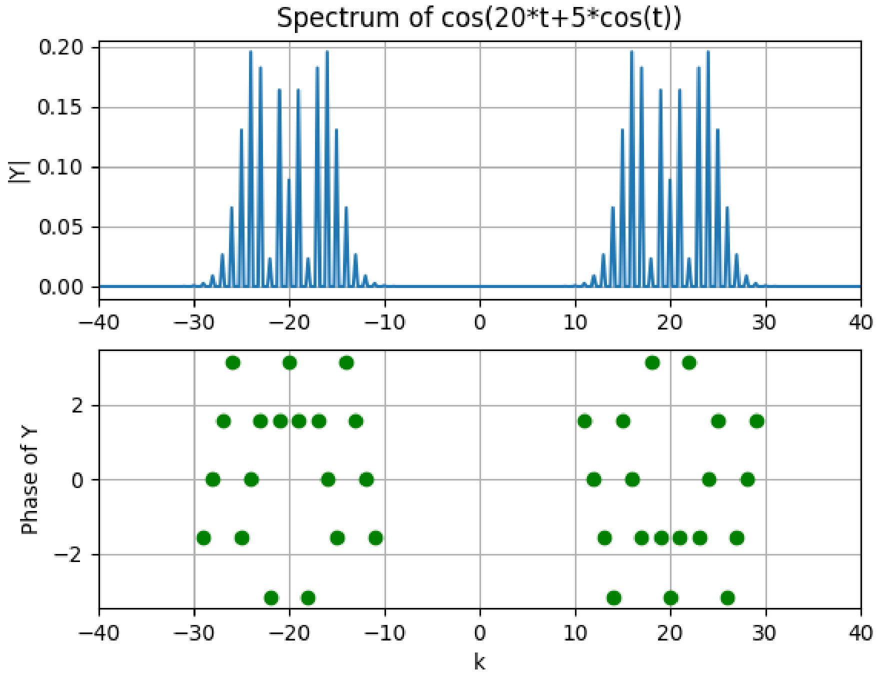

Figure 5: DFT of $cos(20t + 5cos(t))$

When we compare this result with that of the Amplitude Modulation as seen in Fig (2), we see that there are more side bands, and some of them have even higher energy than $\omega = \pm 20\ rad/sec$.

5 DFT of Gaussian

The DFT of a gaussian is also a gaussian, as shown below:

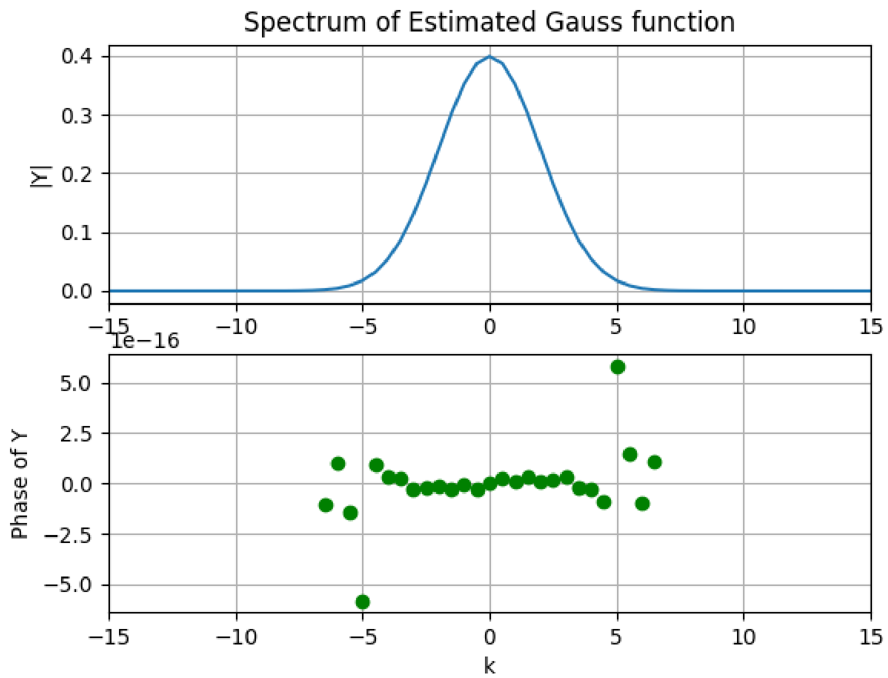

Figure 6: Estimated Gaussian Spectrum-1

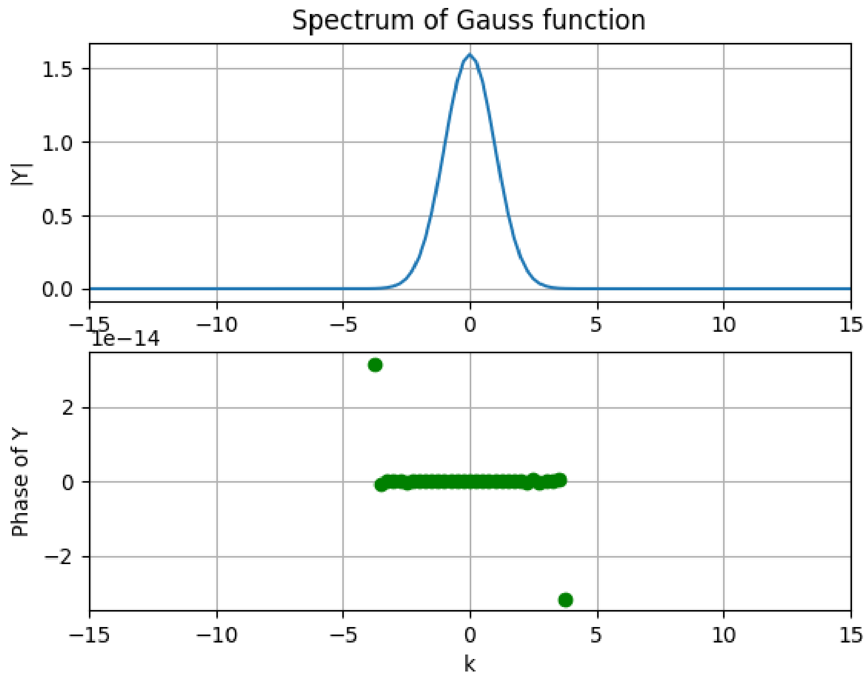

Figure 7: Estimated Gaussian Spectrum-2

We see that Gaussian function has multiple frequency components because it is not periodic. So, its DFT spectrum has a peak at =0 and then it gradually reduces on both sides. If we increase the range of timescale, we will get more number of points in the frequency domain and a smoother graph.

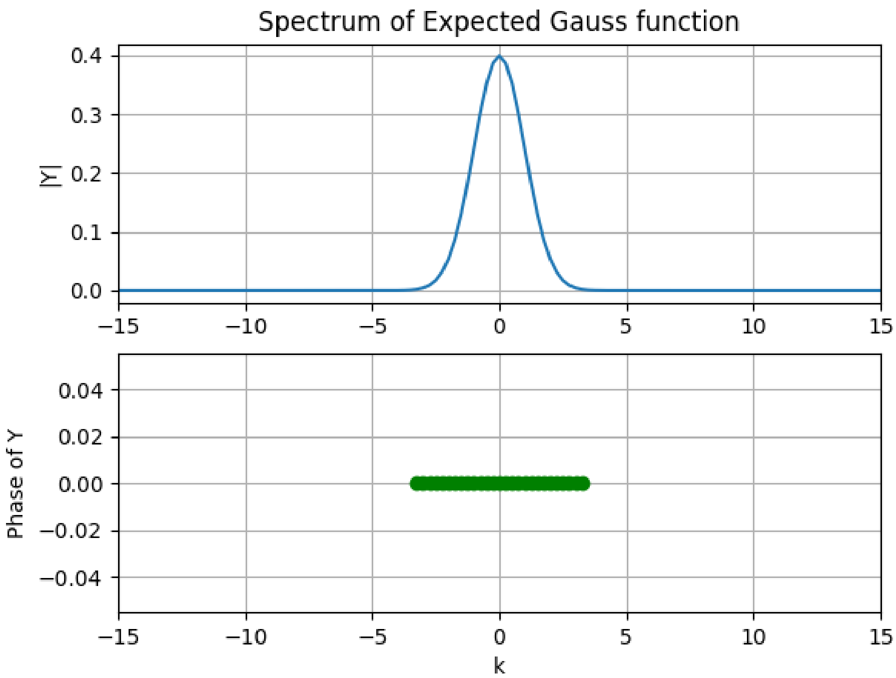

Figure 8: Expected Gaussian Spectrum

6 Conclusion

We have thus found out the DFT's of various signals using the `numpy.fft` package. We need to use the `numpy.fft.fftshift()` and `numpy.fft.ifftshift()` methods to fix distortions in the phase response.