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Path Planning for Bearing-only Simultaneous Localisation and Mapping

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Abstract

Simultaneous localisation and mapping (SLAM) is the process of estimating the pose of a mobile robot and the locations of landmarks by using sensors. When SLAM is cast as an information extraction procedure, its quality can be defined as the amount of uncertainty contained in the resultant estimation. Due to the characteristic of the bearing-only sensor and the geometry of the environment, the estimation uncertainty relies critically on the amount of information obtained from measurements and the efficiency of information extraction by the estimator. These quantities are dependent on the relative position between the robot and the landmarks, i.e., the path of the robot motion. Therefore, a well planned path of motion for the robot can significantly improve the SLAM quality. A genetic algorithm is adopted in this research to design a near-optimal one-step-ahead robot path subject to a multiple of planning objectives. The use of genetic algorithm together with a Pareto set, is proved to be efficient in reducing the estimation uncertainty and improving the quality of SLAM by simulation results.

I. Introduction

When a mobile robot is deployed to perform an assigned task such as navigating in an unknown environment or workspace, it is crucial for the robot to know its position, i.e., to localise itself. The robot location can be obtained by odometer measurements but localisation errors tend to accumulate over time. A user supplied and stored map can be pre-loaded into the robot but this method limits the robot to operate in pre-surveyed areas. Therefore, the robot has to build a map by itself especially in an unknown workspace, i.e., to solve the simultaneous localisation and mapping (SLAM) problem. This is usually achieved by making measurements to landmarks scattered across the workspace. It is noted, however, that the success in navigation greatly depends on the quality of SLAM. This work is hence motivated by the need to improve the SLAM quality by planning a path for the robot to favourable locations to make better measurements. The background of mobile robot SLAM can be found in [1] and the references therein. Mathematical foundations applicable to mobile robot localisation and mapping are provided in [2]. Derivations of the extended Kalman filter (EKF) and its equivalent information filtering techniques, which will be adopted in this work, can be referred in [3].

The estimation of robot pose and landmark locations can be viewed as an information processing procedure. When the robot moves, its predicted pose is obtained through the process model that governs its transition from one pose to another. Since modelling is not exact, the system uncertainty will increase while the robot moves. At the moment that a measurement is made, the information is fed to the estimator or filter that produces estimates with reduced uncertainties. The SLAM problem can be alternatively viewed as the process in compensating for the increase in uncertainty by the gain of information from measurements. Hence, the filter output uncertainty is a natural metric in measuring the quality of the SLAM process. Fig. 1 depicts this concept.

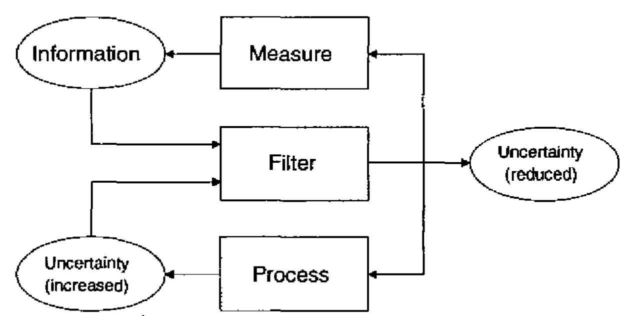


Fig. 1. Simultaneous localisation and mapping process

Due to the limited amount of information available from bearing-only measurements and in order to maximise the information, an appropriately planned path for the robot is required. The effect of robot manoeuvre on target tracking quality was studied in [4]. It proposed to maximise the bearing-rate for a bearing-only measurement. Exploration strategies were compared in [5]. The concentric path was found promising among others. Comparisons between different path designs using information-theoretic approach were conducted in [6], it is found that the quality of resultant estimation obtained depends on the optimisation objective function employed and is rather problem specific. Investigation into the effects of the Fisher information matrix in target tracking quality was conducted in [7]. However, approaches to the upper information bound may be difficult for non-linear systems such as SLAM. The frontier-based approach was proposed in [8] for path planning, where a probability grid was used to represent explored and un-explored areas. A drawback in using probability-grids is that large storage is needed.

These research works described above reveal that path planning for the mobile robot for the purpose of SLAM in unknown environments is very challenging and trade-off between effectiveness and efficiency has to be made. Flexible and efficient approaches are very desirable in improving the SLAM quality. The use of evolutionary computing techniques, e.g., genetic algorithms (GA) [9] and [10], enables the search for a near-optimum solution without regard to the problem domain. A GA was applied in [11] with the focus on map building where path planning for the robot is not discussed. Path generation using evolution computing techniques was also found in [12]. Although position estimation for a mobile robot was enhanced, it is only applicable in a known environment. The work in [13] proposed to design the robot path by multi-sine curves, but the application is limited to range-and-bearing measurements to known landmarks.

In this research, we adopt the multiple hypothesis approach proposed in [14] for an operative bearing-only SLAM using an EKF in an unknown workspace. The quality of SLAM is enhanced by planning a near-optimal robot path using the genetic algorithm with a Pareto set [15] via a population of potential control commands in speed and turn-rate. The commands obtained from the optimal design are confined, by the kinematics of the robot, which reduces the search space and makes the search more efficient.

This paper is organised as follows. In section II, the system model is presented. Then in section III, the estimation process using the EKF and its equivalent information filter is described. Considerations in planning the robot path are discussed in section IV. Implementation and simulation results are presented in section V and VI respectively. A conclusion and directions for further work end the paper in section VII.

II. System Modelling

The mobile robot Simultaneous Localisation and Mapping (SLAM) process consists of a mobile robot and stationary landmarks (point features) in a workspace. Sensors provide measurements between the robot and landmarks to localise the robot and build a map of the workspace.

A process model governing the movement of the robot subject to a control command is given by

$$\mathbf{x}_v(k+1) = \mathbf{f}(\mathbf{x}_v(k), \mathbf{u}(k), \mathbf{n}_v(k)) \tag{1}$$

The robot state is expressed as

$$\begin{aligned} x_v(k+1) &= x_v(k) + v(k)T\cos(\phi_v(k) + \gamma(k)T) + n_x(k) \\ y_v(k+1) &= y_v(k) + v(k)T\sin(\phi_v(k) + \gamma(k)T) + n_y(k) \\ \phi_v(k+1) &= \phi_v(k) + \gamma(k)T + n_\phi(k) \end{aligned} \tag{2}$$

where x_v, y_v, ϕ_v are the robot location and orientation, v and γ are the speed and turn-rate commands, T is the time step, k is the time index and n_x, n_y, n_ϕ are the process noises of $N(0, \mathbf{Q})$.

The landmarks are assumed stationary, that is

$$\mathbf{x}_{fi}(k+1) = \mathbf{x}_{fi}(k) \tag{3}$$

Measurements obey the measurement model for bearing-only measurements, they are obtained from

$$\theta(k) = \mathbf{h}(\mathbf{x}(k)) + n_\theta(k) \tag{4}$$

in particular

$$\theta_i(k) = \arctan\left(\frac{y_{fi}(k) - y_v(k)}{x_{fi}(k) - x_v(k)}\right) - \phi_v(k) + n_\theta(k) \tag{5}$$

where θ_i is the measurement to the i-th landmark, n_θ is the measurement noise of $N(0, R)$.

The system model is depicted in Fig. 2. The robot is located in x_v, y_v in the world co-ordinate. Its orientation is given by the angle ϕ_v with reference to the x-axis. Assume that a landmark is located in x_f, y_f and the bearing from the robot to the landmark is θ. The overall system state is obtained from the aggregation of the robot and landmark states

$$\begin{aligned} \mathbf{x} &= [\mathbf{x}_v^T \quad \mathbf{x}_{fi}^T]^T \\ &= [x_v \quad y_v \quad \phi_v \quad x_{fi} \quad y_{fi}]^T \end{aligned} \tag{6}$$

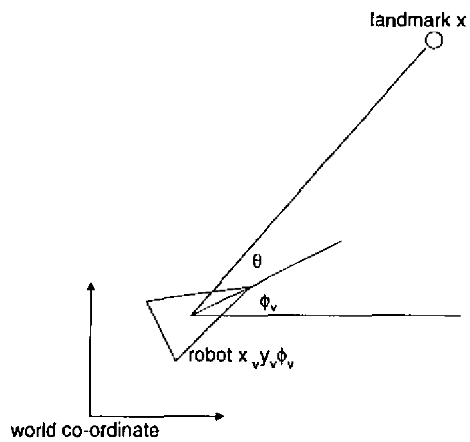


Fig. 2. System Modelling

III. Estimation using Kalman Filter

Kalman filter iterates through the prediction, observation and update steps. The filter is initialised with the initial state estimate $\hat{\mathbf{x}}(0)$ and initial error covariance $\mathbf{P}(0)$ for the robot only, assuming that no landmarks were measured before.

In the prediction step, we have

$$\hat{\mathbf{x}}(k+1|k) = \mathbf{f}(\hat{\mathbf{x}}(k|k), \mathbf{u}(k)) \tag{7}$$

and

$$\mathbf{P}(k+1|k) = \nabla\mathbf{f}_x\mathbf{P}(k|k)\nabla\mathbf{f}_x^T + \nabla\mathbf{f}_u\Sigma\nabla\mathbf{f}_u^T \tag{8}$$

where $\nabla\mathbf{f}_x$ is the Jacobian $\frac{\partial \mathbf{f}}{\partial(x_v, y_v, \phi_v)}$, $\nabla\mathbf{f}_u$ is the Jacobian $\frac{\partial \mathbf{f}}{\partial(v,\gamma)}$ and $\Sigma = diag(\sigma_v^2, \sigma_\gamma^2)$, σ_v and σ_γ are the standard deviations of noises of speed and turn-rate measured from the odometer. Note that the process noise covariance $\mathbf{Q}$ is lumped into $\mathbf{P}(k+1|k)$ through the control command.

Based on the measurements, we calculate the innovation as the difference between the real-life measurement from the i-th landmark and an expected measurement derived from the estimated system states

$$\nu_i = \theta_i - \hat{\theta}_i = \theta_i - \mathbf{h}(\hat{\mathbf{x}}_v, \hat{\mathbf{x}}_{fi}) \tag{9}$$

and its covariance

$$S = \nabla\mathbf{h}\mathbf{P}(k+1|k)\nabla\mathbf{h}^T + R \tag{10}$$

Note that the innovation ν and covariance S are scalars with the bearing-only measurement. The Kalman gain is given by

$$\mathbf{K} = \mathbf{P}(k+1|k)\nabla\mathbf{h}^T S^{-1} \tag{11}$$

Finally, the system states and covariance are updated by

$$\hat{\mathbf{x}}(k+1|k+1) = \hat{\mathbf{x}}(k+1|k) + \mathbf{K}\nu \tag{12}$$

$$\mathbf{P}(k+1|k+1) = \mathbf{P}(k+1|k) - \mathbf{K}S\mathbf{K}^T \tag{13}$$

IV. Path Planning

An optimum path is to be planned such that the amount of information obtained is maximised and the error covariance at the next time step is minimised.

By inverting the error covariance matrix in the Kalman filter, we obtain the information filter (IF) [3] and the evolution of the uncertainty is given by

$$\mathbf{P}^{-1}(k+1|k+1) = \mathbf{P}^{-1}(k+1|k) + \sum_{i=1}^{m} \nabla\mathbf{h}_i^T \mathbf{R}^{-1} \nabla\mathbf{h}_i \tag{14}$$

where m is the number of measurements. The summation gives the total amount of information from measurements.

For a sensor giving the bearing-only measurement, the measurement Jacobian is given as

$$\nabla\mathbf{h}_i = r^{-1}[\sin\alpha \quad -\cos\alpha \quad -r \quad 0 \cdots \quad 0 \quad -\sin\alpha \quad \cos\alpha] \tag{15}$$

where $\alpha = \phi_v + \theta_i$ and $r = \sqrt{(x_{fi} - x_v)^2 + (y_{fi} - y_v)^2}$ is the distance between the robot and the landmark.

Using the Frobenius norm, the amount of information is given in general as

$$||\nabla\mathbf{h}^T R^{-1} \nabla\mathbf{h}|| = R^{-1}\sqrt{\frac{2+r^2}{r^2\sigma_\theta^2}} \tag{16}$$

where σ_θ is the standard deviation of the sensor noise.

The amount of information is inversely proportional to the distance between the robot and the landmark. An implication is that a path should be planned to drive the robot towards the landmark to maximise the information content.

In order to minimise the covariance $\mathbf{P}(k+1|k+1)$, From equ. (13), it is clear that we need to minimise $\mathbf{P}(k+1|k)$ and/or to maximise $\mathbf{KSK}^T$. The magnitude of $\mathbf{P}(k+1|k)$ can be found from the norms of $\nabla\mathbf{f}_x$ and $\nabla\mathbf{f}_u$, see (8). Because these two terms are in quadratic forms, we have

$$||\nabla\mathbf{f}_x|| \propto v^2T^2 \tag{17}$$

and

$$||\nabla\mathbf{f}_u|| \propto T^2(2 + v^2T^2) \tag{18}$$

where ∇f_x and ∇f_u depend only on the speed command. The minimum is attained when $v \to 0$ or $T \to 0$. Obviously, when the robot is required to navigate, $v \neq 0$ and for finite processing time, $T \neq 0$. Hence, minimising $\mathbf{P}(k+1|k)$ seems not practicable.

Alternatively, we may choose to maximise the term $\mathbf{KSK}^T$ such that $\mathbf{P}(k+1|k+1)$ will be a minimum. Expanding the term

$$\mathbf{KSK}^T = \mathbf{P}\nabla\mathbf{h}^T(\nabla\mathbf{h}\mathbf{P}\nabla\mathbf{h}^T + R)^{-1}\nabla\mathbf{h}\mathbf{P}^T \tag{19}$$

we see that $\mathbf{KSK^T}$ is a function of the measurement Jäcobian $\nabla\mathbf{h}$. The term $\mathbf{KSK}^T$ can be interpreted as the reduction of uncertainty and should be maximised by planning a path for the robot to follow.

V. Implementation

The analysis presented in the above section has indicated that path planning, i.e., to obtain the optimal command in speed v and turn-rate γ can be considered as a multi-objective optimisation procedure.

The scenario adopted here is that a preliminary path is already available (e.g., derived from goal seeking assignments), then the path is adjusted in order to have the maximum possible reduction in the system uncertainty. In practice, the robot cannot be driven with abrupt changes in the command due to the kinematics and dynamic limitations. This will be taken into account in designing appropriate objective functions.

A. Objective Functions

The objective functions are identified as

1. maximise the information, $J_1 = ||\nabla\mathbf{h}^T R^{-1}\nabla\mathbf{h}||$
2. minimise the error covariance, $J_2 = ||\mathbf{P}(k+1|k+1)||$
3. minimise the change in velocity command, $J_3 = |\Delta v|$
4. minimise the change in turn-rate command, $J_4 = |\Delta\gamma|$

The magnitude or the norm of the information matrix is a natural metric to measure the quality of localisation and mapping. The error covariance norm is also a justifiable metric because the covariance contains all the uncertainties in the robot, landmarks and the cross coupling or correlation between landmarks. The Frobenius norm is used as the metric in this work.

B. Algorithm

The robot path is planned by using a genetic algorithm where a one-step ahead prediction is applied. The fitness is determined by conducting an EKF iteration. This path planning method can be described in the following steps.

1. randomly generate a set of potential solution commands v^i and γ^i

2. for each potential command, predict the next robot location and error covariance by the EKF prediction equation
3. make pseudo measurements to registered landmarks and calculate the information content $||\nabla \mathbf{h}^T R^{-1} \nabla \mathbf{h}||$
4. perform EKF update to obtain the error covariance magnitude $||\mathbf{P}(k+1|k+1)||$
5. obtain a Pareto set by ranking the objective function from the potential solutions
6. perform genetic operations through a number of generations
7. select the most suitable solution in the final Pareto set and issue it to drive the robot
8. repeat step 2

C. Planning Strategy

We also propose a strategy that the effect of path planning is gradually switched in through an exponential time function. This is motivated by the need to register landmarks in the early exploration period in order to obtain an established SLAM. The control commands are given by

$$v^i = v_i^-(1 - s_w) + v_d \tag{20a}$$

$$\gamma^i = \gamma_i^-(1 - s_w) + \gamma_d \tag{20b}$$

where v_i^- and γ_i^- are the set of randomly generated potential commands, the superscript i refers to the i-th command.

They are weighted by s_w and added to the default commands v_d and γ_d using the weighting factor given by

$$s_w = \exp(\frac{-t_n}{N_w}) \tag{21}$$

where t_n is the current time step, N_w is the exponential time constant and N_w is determined to be 100 in the simulation with total 1000 time steps. The commands are than processed through a genetic algorithm.

D. Genetic Algorithm with Pareto Set

Because there are only two design variables and their values are limited by the robot kinematics, the potential solution space is relatively small. Therefore we will have an efficient implementation of the genetic algorithm. The GA adopted here is a standard GA but mutation is not used. The speed and turn-rate are represented in real numbers. Total crossover is chosen here in order to obtain a high selection pressure. The number of chromosomes used is 30 and generations iterated are limited to 3. We noticed that this is a very small-scale GA but it works fine in the simulations. Its success is also contributed from a restricted search space.

Since the optimisation problem considered here has multi-objectives, the Pareto set [15] approach is employed. By using the Pareto set, various objective functions need not to be scaled or normalised. In addition, there is no need for user assigned weights as in the weighted sum approach. A potential solution $\mathbf{x}_i$ is said to be dominated (inferior) in maximisation, for example, when

$$J(\mathbf{x}_i) \le J(\mathbf{x}_j) \tag{22}$$

where J is one of the objective function and $i \neq j$.

VI. Simulation

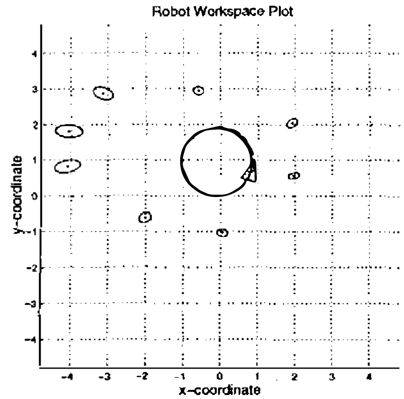
Simulations are conducted for the case with a pre-defined path and the case with a planned path. The pre-defined path is chosen to be a circle while the planned path is based on the circular path, but is adjusted to improve the quality of localisation and mapping.

A. Case 1: Pre-defined Path

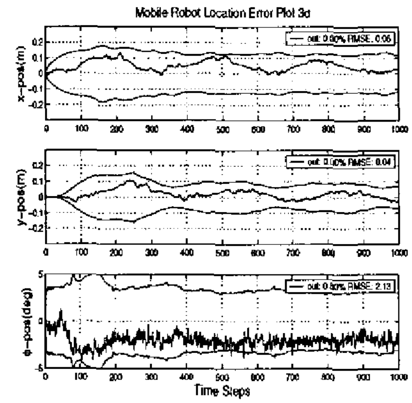
Fig. 3(a) shows the pre-defined circular path of the robot and the estimation uncertainties of the mapped landmarks. It is observed that some landmark estimations contain relatively large uncertainties. It is because the robot is quite far away from those landmarks and the small amount of information content obtained when measurements are made. The robot pose estimation error and uncertainties are acceptable as shown in Fig. 3(b) and are within the 3σ error bound. Fig. 3(c) plots the estimation errors of landmark locations which contain some steady state errors and an extended convergence period. Fig. 3(d) depicts the norm of the error covariance which maintains a residual towards the end of simulation. The pre-defined circular path control command, speed and turn-rate, are plotted in Fig. 3(e). The control is held constant after the initial transition period. These results will be further compared to those obtained from the planned path simulation in the next section.

B. Case 2: Planned Path

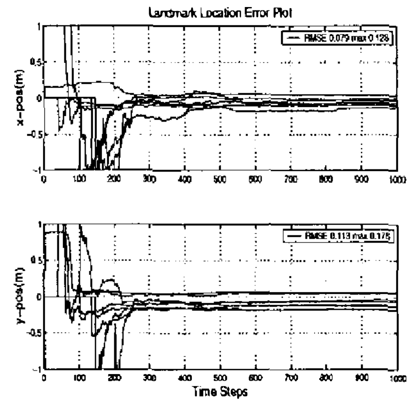
The path followed by the robot, driven by the control obtained from the genetic algorithm, is shown in Fig. 4(a). It is observed that the path deviates from a circle when path planning is taking into effect and the landmarks are being mapped with smaller uncertainties (landmarks on the left). Fig. 4(b) plots the estimation errors for the robot pose. The root-mean-square error (RMSE) in the x-pos reduces from 0.06m to 0.03m, y-pos reduces from 0.04m to 0.02m and the orientation error reduces from 2.13° to 0.96°. The estimations are consistent (within the 3σ error bound) irrespective of the adjusted path. It is also observed that the uncertainties maintain a small reducing trend which is a result from an reduced estimation error. The landmark location estimation errors are illustrated in Fig. 4(c). The RMSE reduces from 0.079m to 0.039m in x-pos and from 0.113m to 0.034m in y-pos. Fig. 4(d) shows the norm of the error covariance. Reduction in convergence time is clearly observed (before 1000 iterations) and the steady state value (sum of the last 5% iteration) is also reduced from $6.00m^2$ to $3.10m^2$. The control commands for the planned path are plotted in Fig. 4(e). The speed command shows an increasing trend. It is due to the need to drive the robot towards the landmarks by tracing circles with larger radius. The gradual switch-in strategy is found operative from the plot for the control speed. The plot for turn-rate also shows that path planning is operating but a trend is not clear. This is expected as the turn toward landmarks depends on the location of the landmarks within the workspace. In general, improvement on the estimation quality is achieved.



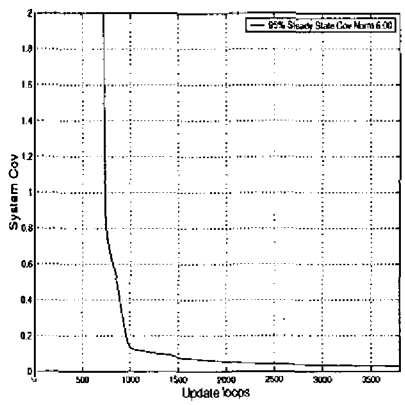
(a) Path followed by robot



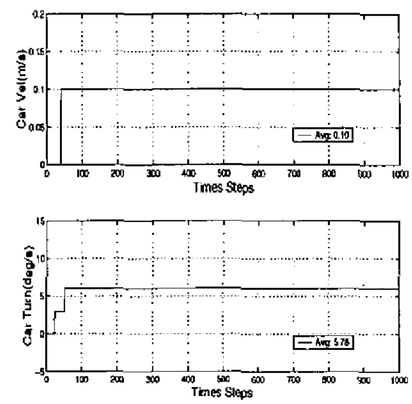
(b) Robot pose estimation error



(c) Landmark location estimation error



(d) System error covariance norm



(e) Control command

Fig. 3. Simulation results from case 1

VII. Conclusion

The planning of the robot path to enhance the quality of bearing-only SLAM is presented in this paper. The SLAM process is implemented using an extended Kalman filter, its special iterative updating structure is revealed and used in deriving the optimization strategy. A genetic algorithm with a Pareto set is used to search for a near-optimal control consists of speed and turn-rate. Using the derived control, the robot is driven to favourable locations to make measurements to landmarks. The information content obtained from measurements is then maximised. The robot and landmark estimation uncertainties resulted from an extended Kalman filter is also minimised. Simulation results had shown that the proposed approach is effective. Future work may include the investigation of further improvement on SLAM quality by using multiple-step-ahead planning, and a combined optimisation including obstacle avoidance and goal seeking.

Acknowledgment

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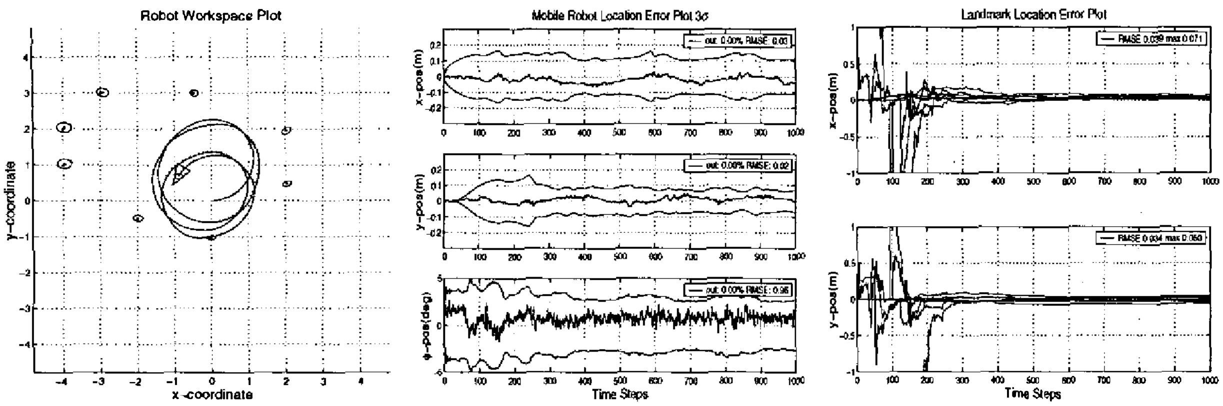
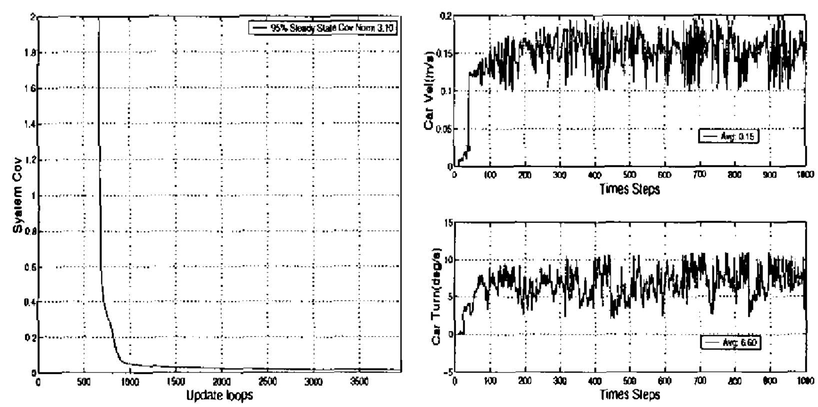


(a) Path followed by robot

(b) Robot pose estimation error

(c) Landmark location estimation error



(d) System error covariance norm

(e) Control command

Fig. 4. Results from planned path – case 2

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