Four-Wheel Steering Vehicle Control using Takagi-Sugeno Fuzzy Models

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Abstract— **This paper deals with the problem of vehicle active control. The focus is to improve the vehicle stability and handling. Vehicle dynamics is described by a *10*-DOF (Degree Of Freedom) model which include lateral, longitudinal, yaw and roll dynamics. Parametric variations (due to the variation of road conditions) and also, a variation of vehicle velocity are taken into account in the controller synthesis. The nonlinear vehicle model is approximated by a Takagi-Sugeno (T-S) fuzzy model with structured parametric uncertainties. State variables are not always available. Thus, fuzzy observer-based algorithm will be proposed to ensure control with performance specifications. Robust pole placement LMI (Linear Matrix Inequalities) conditions are obtained using Lyapunov and H_∞ approach, which ensure desired performances of the closed loop system.**

I. Introduction

MANY systems have been developed and installed in vehicles like ABS and ESP to improve safety and performances during driving. However, in some critical driving situations (variation of road state, emergency braking, skid in cornering…), these systems are still not optimal and can be improved using advanced control methods [1], [2], [3], [4], [5], [6], [7]. However, most of this works are based on simplified vehicle dynamics models and or assume that the longitudinal velocity of the vehicle is constant. Recent research works, which take into account the variation of the longitudinal vehicle velocity, have been based on simplified vehicle models [8], [9]. In this work, we describe the vehicle dynamics by a *10*-DOF mathematical model which is a time varying parameter-dependent model. So, we will approximate it by an uncertain T-S fuzzy model. Sensors information is necessary to achieve control. However, some sensors are still very expensive (like the side slip angle sensor). Thus, observer-based pole placement method is proposed to satisfy desired control performances (fast decay, good damping …). However, because of uncertainties, the separation property is no more applicable. An alternative solution has been found in [13] and is used in this work to guarantee good estimation error convergence to the equilibrium point zero. The paper is organized as follows: in section *II*, we present the vehicle dynamics mathematical model. The controller design strategy is given in section *III* whereas, in section *IV*, simulation results of the developed controller, applied to the nonlinear vehicle model, are given to show the effectiveness of our approach.

II. Vehicle Model Description

In this section, we describe a four-wheel steering (4WS) vehicle by a *10*-DOF model including lateral, longitudinal, yaw and roll dynamics. We suppose that front and rear cornering forces are given as follows:

$$F_{yfi} = C_f \alpha_f, \quad F_{yri} = C_r \alpha_r, \quad i = l, r.$$

Where α_f, α_r are front and rear tire slip angles given by:

$$\alpha_f = \beta + \frac{d_f}{V_x} r - \delta_f - R_f \phi, \quad \alpha_f = \beta - \frac{d_f}{V_x} r - \delta_r - R_r \phi.$$

And front and rear longitudinal forces are given by:

$$F_{xfi} = C_{\sigma f} \sigma_{xf}, F_{xri} = C_{\sigma r} \sigma_{xr}, \quad i = l, r.$$

Where σ_f, σ_r are front and rear longitudinal slip ratio given by:

$$\sigma_{xi} = \frac{V_x - r_{eff} \omega_i}{V_x}, i = 1, 2, 3, 4.$$

During braking maneuver, and by considering small angles, the following dynamic equations are obtained:

- Longitudinal dynamics

$$m_v \dot{V}_x = m_v V_y r + 2C_{\sigma f} + 2C_{\sigma r} - \frac{C_{\sigma f}}{V_x} R_t (\omega_1 + \omega_2) - \frac{C_{\sigma r}}{V_x} R_t (\omega_3 + \omega_4) \quad (1)$$

- Lateral dynamics

$$\begin{aligned} m_v \dot{V}_y = & -m_v V_x r - m_s h_s \dot{p} + (2C_f + 2C_r) \frac{V_y}{V_x} + \left(2C_f \frac{d_f}{V_x} - 2C_r \frac{d_r}{V_x} \right) r \\ & -2C_f \delta_f - 2C_r \delta_r - (2C_f R_f + 2C_r R_r) \phi \end{aligned} \quad (2)$$

- Yaw dynamics

$$\begin{aligned} J_{zz} \dot{r} - J_{xz} \dot{p} = & (2d_f C_f - 2d_r C_r) \frac{V_y}{V_x} + \left(2C_f \frac{d_f^2}{V_x} + 2C_r \frac{d_r^2}{V_x} \right) r \\ & -2d_f C_f \delta_f + 2d_r C_r \delta_r + 2(d_r C_r R_r - d_f C_f R_f) \phi \\ & + \frac{d_t C_{\sigma f} r_{eff}}{V_x} (\omega_2 - \omega_1) + \frac{d_t C_{\sigma r} r_{eff}}{V_x} (\omega_4 - \omega_3). \end{aligned} \quad (3)$$

- Roll dynamics

$$\begin{aligned} & J_{xx} \dot{p} - J_{xz} \dot{r} = -m_s h_s \dot{V}_y - m_s h_s V_x r - C_\phi p - (k_\phi - m_s g h_s) \phi \\ & \dot{\phi} = p. \end{aligned} \quad (4)$$

- Wheel dynamics

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$$J_t\dot{\omega}_i = -\tau_{bi} + R_t C_{\sigma f} - \frac{C_{\sigma f} R_t^2}{V_x}\omega_i, i = 1,2,3,4. \quad (5)$$

- Actuator dynamics

$$\dot{\delta}_f = \frac{-k_{sf}}{c_{sf}}\delta_f + \frac{k_{sf}}{c_{sf}}\delta_{sf} - \frac{k_{sf}}{c_{sf}}\delta_c, \quad \dot{\delta}_r = \frac{-k_{sr}}{c_{sr}}\delta_r + \frac{k_{sr}}{c_{sr}}\delta_{sr}. \quad (6)$$

Where $C_\phi = c_{\phi f} + c_{\phi r}$, $k_\phi = k_{\phi f} + k_{\phi r}$ and δ_c is the driver action. Definitions of the vehicle parameters are given in table *I*.

TABLE I
VEHICLE PARAMETER DEFINITIONS

Parameters	Definitions
V_x , V_y	*Longitudinal velocity, lateral velocity*
r , p , β	*Yaw rate, roll rate, side slip angle*
φ , ϕ	*Yaw angle, roll angle*
ω_1, ω_2	*Angular velocity of front left, front right wheel*
ω_3, ω_4	*Angular velocity of rear left, rear right wheel*
m_v , m_s	*Vehicle's total mass, vehicle's sprung mass*
$d_{f,r}$	*Distance from (front, rear) axle to vehicle's gravity center*
$C_{f,r}$	*Cornering stiffness coefficients (front, rear)*
$C_{\sigma f,\sigma r}$	*Longitudinal stiffness coefficients (front, rear)*
R_t	*Effective wheel rolling radius*
h_s	*Height of sprung mass center gravity above roll axis*
J_{xx}	*Sprung mass roll moment of inertia*
J_{zz}	*Principal yaw inertia moment*
J_{xz}	*Sprung mass inertia moment about yaw and roll axes*
J_t	*Effective rotational inertia moment*
$k_{sf,sr}$	*Rotary compliances of steering actuators (front, rear)*
$c_{sf,sr}$	*Rotary viscous damping coefficients*
$\delta_{f,r}$	*Steering angle (front, rear)*
$\delta_{sf,sr}$	*Actuators command (front, rear)*
$c_{\phi f,\phi r}$	*Rotational damping coefficients (front, rear)*
$k_{\phi f,\phi r}$	*Rotational stiffness coefficients (front, rear)*
$\tau_{b1,b2}$	*Brake torque of front wheels (left, right)*
$\tau_{b3,b4}$	*Brake torque of rear wheels (left, right)*

Let us consider the state vector:

$$x = \begin{bmatrix} V_y & r & p & \phi & \delta_f & \delta_r & \omega_1 & \omega_2 & \omega_3 & \omega_4 \end{bmatrix}^t$$

And the control input:

$$u = \begin{bmatrix} \delta_{sf} & \delta_{sr} & \tau_1 & \tau_2 & \tau_3 & \tau_4 \end{bmatrix}^t$$

Where $\tau_i = \tau_{bi} - R_t C_{\sigma f}$, $i = 1,2.$ and $\tau_i = \tau_{bi} - R_t C_{\sigma r}$, $i = 3,4.$

Then, from *(1)*, *(2)*, *(3)*, *(4)*, *(5)* and *(6)*, we can describe the nonlinear vehicle model as follows:

$$\dot{x}(t) = A(V_x(t))x(t) + Bu(t) + B_c\delta_c(t) \quad (7)$$

Where

$$A = \begin{pmatrix} A_{11} & A_{12} & A_{13} & A_{14} & A_{15} & A_{16} & A_{17} & A_{18} & A_{19} & A_{110} \\ A_{21} & A_{22} & A_{23} & A_{24} & A_{25} & A_{26} & A_{27} & A_{28} & A_{29} & A_{210} \\ A_{31} & A_{32} & A_{33} & A_{34} & A_{35} & A_{36} & A_{37} & A_{38} & A_{39} & A_{310} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{k_{sf}}{c_{sf}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{k_{sr}}{c_{sr}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{-C_{\sigma f}R_t^2}{V_xJ_t} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-C_{\sigma f}R_t^2}{V_xJ_t} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-C_{\sigma r}R_t^2}{V_xJ_t} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-C_{\sigma r}R_t^2}{V_xJ_t} \end{pmatrix},$$

$$B = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{k_{sf}}{c_{sf}} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{k_{sr}}{c_{sr}} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{-1}{J_t} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{-1}{J_t} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{-1}{J_t} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{-1}{J_t} \end{pmatrix}.$$

The coefficients $A_{ij}, i, j = 1,...,10$ depend on $V_x(t)$. Then, they are not given in this paper due to the lack of space. Note that the nonlinear vehicle model given by *(7)* is dependent of the longitudinal vehicle velocity. During braking maneuver, the latter parameter influences greatly the vehicle dynamics. What we want to do is to obtain a T-S fuzzy model from *(7)* which is a time varying parameter-dependent model (here $V_x(t)$). The idea is to obtain two linear time invariant (LTI) models from the decomposition of V_x and $1/V_x$. The premise variable of the T-S fuzzy model is $z(t) = V_x(t)$ where $V_x(t) \in [V_{\min}, V_{\max}]$. Let us define two symbols: M_1 and M_2 for the input $z(t)$. The fuzzy meaning of the two symbols are $h_1(z(t))$ and $h_2(z(t))$ which can be found from:

$$\begin{aligned} & z(t) = h_1(z(t))V_{\max} + h_2(z(t))V_{\min}, \\ & \frac{1}{z(t)} = h_1(z(t))\frac{1}{V_{\min}} + h_2(z(t))\frac{1}{V_{\max}}, \\ & h_1(z(t)) + h_2(z(t)) = 1. \end{aligned} \quad (8)$$

Using *(8)*, *(7)* and the recursive least squares method, the membership functions $h_i\left(z(t)\right)$ are given as follows:

$$\begin{pmatrix} h_1\left(z(t)\right) \\ h_2\left(z(t)\right) \end{pmatrix} = \left(\left(\Omega^t \times \Omega\right)^{-1} \times \Omega^t\right) \times \Pi,\ \Omega = \begin{pmatrix} \dfrac{1}{V_{\min}} & \dfrac{1}{V_{\max}} \\ V_{\max} & V_{\min} \\ 1 & 1 \end{pmatrix},$$

$$\Pi = \begin{pmatrix} \dfrac{1}{z(t)} & z(t) & 1 \end{pmatrix}^t,\ h_i\left(z(t)\right) \geq 0,\ i = 1,2.$$

Then, the obtained T-S fuzzy model is given by the following rules:

Fuzzy model rule 1: *If* $z(t)$ *is* M_1 *Then*

$$\dot{x}(t) = A_1 x(t) + Bu(t) + B_c \delta_c(t)$$

Fuzzy model rule 2: *If* $z(t)$ is M_2 *Then*

$$\dot{x}(t) = A_2 x(t) + Bu(t) + B_c \delta_c(t)$$

Thus, the nonlinear model given by *(7)* is equivalent to:

$$\dot{x}(t) = \sum_{i=1}^{2} h_i\left(z(t)\right)\left(A_i x(t) + Bu(t) + B_c \delta_c(t)\right) \tag{9}$$

The matrices $A_i, i = 1,2$ are not given in this paper due to the lack of space.

On the other hand, we know that the road adhesion greatly influences the vehicle dynamics. The cornering stiffness coefficients C_f, C_r and also the longitudinal stiffness coefficients $C_{\sigma f}, C_{\sigma r}$ vary according to the road type. Thus, to take this fact into account, we consider that:

$$C_k = C_{k0}(1 + e\Delta),\quad k = f, r, \sigma f, \sigma r.$$

Where $C_{k0},\ k = f, r, \sigma f, \sigma r.$ are the nominal values, $|\Delta| \leq 1$ and *e* is the magnitude deviation of the different coefficients from their nominal values. After development, we can write *(9)* as follows:

$$\dot{x}(t) = \sum_{i=1}^{2} h_i\left(z(t)\right)\left\{\left(A_{i0} + \Delta A_{i0}\right)x(t) + \left(B + \Delta B\right)u(t) + B_c \delta_c(t)\right\} \tag{10}$$

Where

$$\Delta A_{i0} = H_{ai}\Delta_{ai}(t)E_{ai},\ \ \Delta B = H_b \Delta_b(t) E_b,\ \ i = 1,2. \tag{11}$$

And $\Delta_{ai}(t), \Delta_{bi}(t)$ are unknown functions satisfying:

$$\Delta_{ai}^T(t)\Delta_{ai}(t) \leq I,\ \ \Delta_b^T(t)\Delta_b(t) \leq I,\ \ i = 1,2. \tag{12}$$

I is the identity matrix, $E_b = 0$. $E_{ai}, i = 1,2$ are not given in this paper due to the lack of space and H_{ai}, H_b are constant matrices given as follows:

$$H_{ai} = \begin{pmatrix} e_i & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & e_i \end{pmatrix}_{(10\times 10)},\ H_b = \begin{pmatrix} e & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & e \end{pmatrix}_{(10\times 6)} \tag{13}$$

III. Observer-based fuzzy control

In this paper we assume that the state variables are not measurable. Fuzzy state observer for T-S fuzzy model with parametric uncertainties *(10)* is formulated as follows:

Observer rule i: *If* $z(t)$ *is* M_i *Then*

$$\begin{cases} \dot{\hat{x}}(t) = A_{i0}\hat{x}(t) + Bu(t) - G_i\left(y(t) - \hat{y}(t)\right), \\ \hat{y}(t) = C_i \hat{x}(t),\quad i = 1,2 \end{cases} \tag{14}$$

Where $M_i, i = 1,2$ are linguistic variables defined previously for the T-S model. The fuzzy observer design is to determine the local gains G_i. The output of *(14)* is given as follows:

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{i=1}^{2} h_i\left(z(t)\right)\left\{A_{i0}\hat{x}(t) + Bu(t) - G_i\left(y(t) - \hat{y}(t)\right)\right\} \\ \hat{y}(t) = \sum_{i=1}^{r} h_i\left(z(t)\right)C_i \hat{x}(t) \end{cases} \tag{15}$$

To stabilize this class of systems *(10)*, we use the Parallel Distributed Compensation (PDC) observer-based controller [11] defined as follows:

Controller rule i : *If* $z(t)$ *is* M_i *Then*

$$u(t) = K_i \hat{x}(t),\qquad i = 1,2 \tag{16}$$

The overall fuzzy controller is represented by:

$$u(t) = \sum_{i=1}^{2} h_i\left(z(t)\right)K_i \hat{x}(t) \tag{17}$$

Let us denote the estimation error by:

$$e(t) = x(t) - \hat{x}(t) \tag{18}$$

The augmented system containing both the fuzzy controller and observer is represented as follows:

$$\begin{pmatrix} \dot{x}(t) \\ \dot{e}(t) \end{pmatrix} = \bar{A}\left(z(t)\right) \times \begin{pmatrix} x(t) \\ e(t) \end{pmatrix} + \begin{pmatrix} B_c \\ B_c \end{pmatrix}\delta_c(t) \tag{19}$$

Where

$$\bar{A}\left(z(t)\right) = \sum_{i=1}^{2} h_i\left(z(t)\right)\bar{A}_i,$$

$$\bar{A}_i = \begin{pmatrix} \left(A_{i0} + \Delta A_{i0}\right) + BK_i & -BK_i \\ \Delta A_{i0} & \left(A_{i0} + G_i C_i\right) \end{pmatrix}. \tag{20}$$

The main goal is first, to find the sets of matrices K_i and G_i in order to guarantee the global asymptotic stability of the equilibrium point zero of *(19)* and secondly, to design the fuzzy controller and the fuzzy observer of the augmented system *(19)* separately by assigning both “observer and controller poles” in a desired region in order to guarantee that the error between the state and its estimation converges faster to zero. The faster the estimation error will converge to zero, the better the transient behaviour of the controlled system will be. Sufficient conditions for the global asymptotic stability of the closed-loop augmented system *(19)* are given in the following lemma *1* [13]:

Lemma 1: *The equilibrium point zero of the augmented system described by (19) is globally asymptotically stable if there exist common positive definite matrices* P_1 *and* P_2, *matrices W_i, matrices V_i and positive scalars $\varepsilon_i \succ 0$ such as*

$$\begin{pmatrix} D_i & P_1 E_{ai}^t & B & H_b \\ E_{ai} P_1 & -0.5\varepsilon_i I & 0 & 0 \\ B^t & 0 & -\varepsilon_i I & 0 \\ H_b^t & 0 & 0 & -\varepsilon_i I \end{pmatrix} \leq 0, \ i = 1,2 \tag{21}$$

$$\begin{pmatrix} D_i^* & P_2 H_{ai} & P_2 H_b & K_i^t \\ H_{ai}^t P_2 & -\varepsilon_i^{-1} I & 0 & 0 \\ H_b^t P_2 & 0 & -0.5\varepsilon_i^{-1} I & 0 \\ K_i & 0 & 0 & -\varepsilon_i^{-1} I \end{pmatrix} \leq 0, \ i = 1,2. \tag{22}$$

With

$$D_i = A_{i0} P_1 + P_1 A_{i0}^t + B V_i + V_i^t B^t + \varepsilon_i H_{ai} H_{ai}^t + \varepsilon_i H_b H_b^t,$$

$$D_i^* = P_2 A_{i0} + A_{i0}^t P_2 + W_i C_i + C_i^t W_i^t.$$

***Proof*:** Using lemma *1* in [13] and considering $E_{bi} = E_b = 0$ and $H_{bi} = H_b$ for $i = 1,2$.

Remark 1: Note that the controller and the observer design is a two-step procedure. First, we solve *(21)* for decision variables $(P_1, K_i, \varepsilon_i)$ and secondly, we solve *(22)* for decision variables (P_2, G_i) by using the results from the first step. Furthermore, the controller and observer gains are given by: $G_i = P_2^{-1} W_i$ and $K_i = V_i P_1^{-1}$, respectively, for $i = 1,2$.

Remark 2: The location of the poles associated with the state dynamics and with the estimation error dynamics is unknown. However, since the design algorithm is a two-step procedure, we can impose two pole placements separately, the first one for the state and the second one for the estimation error. To ensure control performances, in the following, we focus on robust pole placement [12].

From *(19)* and *(20)*, Let us define:

$T^s = \sum_{i=1}^{2} h_i(z(t))\big((A_{i0} + \Delta A_{i0}) + BK_i\big)$ and

$$S^s = \sum_{i=1}^{2} h_i(z(t))(A_{i0} + G_i C_i).$$

Lemma 2: *Matrix T^s is D_T-stable if and only if there exist a symmetric matrix $P_1 > 0$, matrices V_i and positive scalars $\mu_i \succ 0$ such as*

$$\begin{pmatrix} E_i & (\beta^t \otimes P_1 E_{ai}^t) \\ (\beta \otimes E_{ai} P_1) & -\mu_i I \end{pmatrix} \leq 0, \ i = 1,2. \tag{23}$$

With

$$E_i = \xi_i + \mu_i (I \otimes H_{ai} H_{ai}^t) + \mu_i (I \otimes H_b H_b^t), \ V_i = K_i P_1,$$

$$\xi_i = \alpha \otimes P_1 + \beta \otimes A_{i0} P_1 + \beta^t \otimes P_1 A_{i0}^t + \beta \otimes B V_i + \beta^t \otimes V_i^t B^t.$$

Where $\otimes$ denotes the Kronecker product.

***Proof*:** Using lemma *2* in [13] and considering $E_{bi} = E_b = 0$ for $i = 1,2$.

Lemma 3: *Matrix S^s is D_S-stable if and only if there exist a symmetric matrix $P_2 > 0$, matrices W_i and positive scalars $\lambda_i \succ 0$ such as*

$$\begin{pmatrix} R_i & I \otimes P_2 H_b \\ I \otimes H_b^t P_2 & -\lambda_i I \end{pmatrix} \leq 0, \ i = 1,2. \tag{24}$$

With

$$R_i = \alpha \otimes P_2 + \beta \otimes P_2 A_{i0} + \beta^t \otimes A_{i0}^t P_2 + \beta \otimes W_i C_i + \beta^t \otimes C_i^t W_i^t$$

$$W_i = P_2 G_i$$

Where $\otimes$ denotes the Kronecker product.

***Proof*:** Using lemma *3* in [13] and considering $E_{bi} = E_b = 0$ and $H_{bi} = H_b$ for $i = 1,2$.

Remark 3: From *(19)*, the estimation error dynamics depends on the state. In lemma *2* and *3*, we have imposed the dynamics of the state and the dynamics of the estimation error. However, if the state dynamics are slow, we will have a slow convergence of the estimation error to the equilibrium point zero in spite of its own fast dynamics. So in this paper, we add an algorithm using the H_∞ approach to ensure that the estimation error converges faster to the equilibrium point zero. We know from *(19)* that:

$$\dot{e}(t) = \sum_{i=1}^{2} h_i(z(t))\big[(A_{i0} + G_i C_i)e(t) + \Delta A_{i0} x(t) + B_c \delta_c(t)\big] \tag{25}$$

Let us denote $\bar{x}(t) = \begin{bmatrix} x(t) & \delta_c(t) \end{bmatrix}^t$ and $W_i = \begin{bmatrix} \Delta A_{i0} & B_c \end{bmatrix}$. The following system is obtained:

$$\begin{pmatrix} \dot{e}(t) \\ e(t) \end{pmatrix} = \sum_{i=1}^{2} h_i(z(t)) \begin{pmatrix} A_{i0} + G_i C_i & W_i \\ I & 0 \end{pmatrix} \begin{pmatrix} e(t) \\ \bar{x}(t) \end{pmatrix} \tag{26}$$

The objective is to minimize the L_2 gain from $\bar{x}(t)$ to $e(t)$ in order to guarantee that the estimation error converges faster to zero. Thus, we define the following H_∞ performance criterion under zero initial conditions:

$$\int_0^\infty \left\{ e^t(t) e(t) - \gamma^2 \bar{x}^t(t) \bar{x}(t) \right\} dt \prec 0 \tag{27}$$

Where γ has to be minimized. The signal $\bar{x}(t)$ is square integrable because of lemma *1*. We give the following lemma to satisfy the H_∞ performance.

Lemma 4: *If there exist symmetric positive definite matrix P_2, matrices W_i and a positive scalar $\gamma \succ 0$ such as:*

$$\begin{bmatrix} Y_i & P_2 H_{ai} & P_2 H_b & 0 & P_2 B_c \\ H_{ai}^t P_2 & -I & 0 & 0 & 0 \\ H_b^t P_2 & 0 & -0.5I & 0 & 0 \\ 0 & 0 & 0 & U_i & 0 \\ B_c^t P_2 & 0 & 0 & 0 & -\gamma^2 I + B_c^t B_c \end{bmatrix} \leq 0, \ i = 1,2 \tag{28}$$

With

$$Y_i = P_2 A_{i0} + A_{i0}^t P_2 + W_i C_i + C_i^t W_i^t + I,$$

$U_i = -\gamma^2 I + E_{ai}^t E_{ai}$, *and* $W_i = P_2 G_i$.

Then, the system given by (19) satisfies the H_∞ performance with a L_2 gain equal or less than γ (27).

Proof: Using lemma *3* in [13] and considering $E_{bi} = E_b = 0$ and $H_{bi} = H_b$ for $i = 1, 2$.

In order to improve the estimation error convergence, we obtain the following convex optimization problem: minimization γ under the LMI constraints *(28)*. From lemma *1*, *2*, *3* and *4* yields the following theorem:

Theorem 1: *The closed-loop uncertain fuzzy system (19) is robustly stabilizable via the observer-based controller (17) with control performances defined by a pole placement constraint in LMI region D_T for the state dynamics, a pole placement constraint in LMI region D_S for the estimation error dynamics and a L_2 gain γ performance (27) as small as possible if first, LMI systems (21) and (23) are solvable for the decision variables $(P_1, K_i, \varepsilon_i, \mu_i)$ and secondly, LMI conditions (22), (24) and (28) are solvable for the decision variables (P_2, G_i, λ_i). Furthermore, the controller and observer gains are $K_i = V_i P_1^{-1}$ and $G_i = P_2^{-1} W_i$, respectively, for $i = 1, 2$.*

Remark 4: Because of uncertainties, we could not use the separation property but we have overcome this problem by designing the fuzzy controller and observer in two steps with two pole placements. We use the H_∞ approach to ensure that the estimation error converges faster to zero although its dynamics depend on the state.

Remark 5: Theorem *1* proposes a two-step procedure: the first step concerns the fuzzy controller design by imposing a pole placement constraint for the poles linked to the state dynamics and the second step concerns the fuzzy observer design by imposing the second pole placement constraint for the poles linked to the error estimation dynamics and by minimizing the H_∞ performance criterion *(27)*. The designs of the observer and the controller are separate but not independent.

IV. Simulation Results

The control design purpose of this example is to design a robust observer-based controller in order to improve the vehicle stability and manoeuvrability when the vehicle is subject to change lane manoeuvre and in presence of parametric variations (variation of the road type). To ensure good performances of the controlled system, we place both the poles linked to the state dynamics and the ones linked to the estimation error dynamics in the LMI region defined by the intersection between the half-left complex plane given by $\alpha = -0.5$ and $\beta = 1$ (in *(23)* and *(24)*) to ensure minimum response time. In simulation, we consider that $e_i = e = 0.1$ in *(13)* thus, the designed controller will ensure stability although the stiffness coefficients vary until *10%* about their nominal values. The driver action as well as the evolution of the control signal is given in figure *1*. Controlled system outputs and their estimations are shown in figure *2*.

To show our control method effectiveness, a superposition of estimation errors, obtained with and without H_∞ approach, is given in figure *3*. On the other hand, comparison of the controlled and uncontrolled system outputs is given in figure *4*. We clearly see that without control, the vehicle is unstable whereas, with our controller, the vehicle maneuverability is guaranteed. The vehicle parameters values are given in table *II*.

Remark 6: Due to the lack of space, figures *2*, *3* and *4*, do not show all of state variables evolutions.

TABLE II
NOMINAL VALUES

Parameters	Values
m_v , m_s	*1067 kg, 900 kg*
$C_{f,r}$, $C_{\sigma f,\sigma r}$	*-15000 N/rad, -6000 N/rad*
R_r , R_f	*0.05m, 0.07m*
R_t , h_s	*0.33 m, 0.55 m*
J_{xx} , J_{zz} , J_{xz} , J_t	*500kg.m², 3000 kg.m², 400 kg.m², 2.1 kg.m²*
$c_{sf,sr}$	*10 N. m. sec/rad , 13 N. m. sec/rad*
$k_{sf,sr}$	*82 N. m/rad*
$c_{\phi f,\phi r}$	*1100 N. m. sec/rad*
$k_{\phi f,\phi r}$	*15450 N. m/rad*
g, V_x	*9.81 m/sec²*, $[20m/s, 40m/s]$

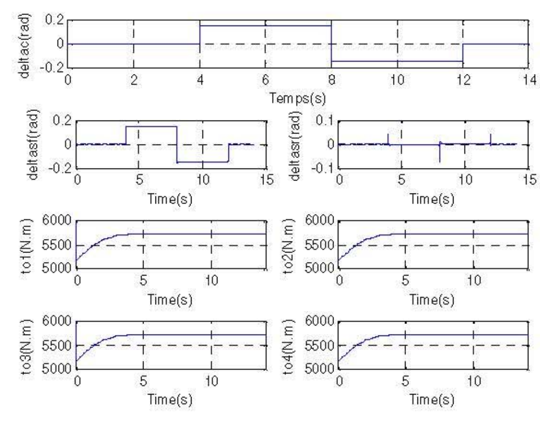

Figure 1. Evolution of the driver action and the control signal

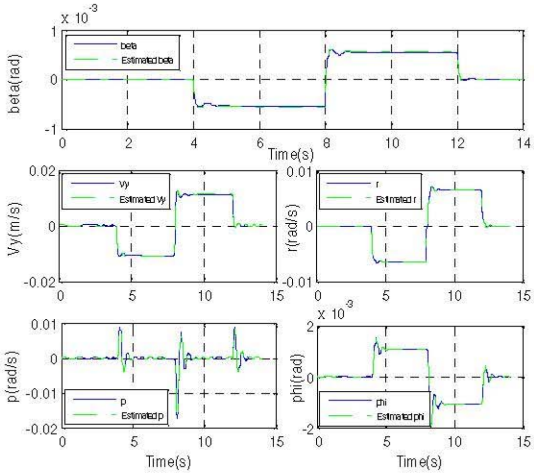

Figure 2. Comparison of estimated and measured states

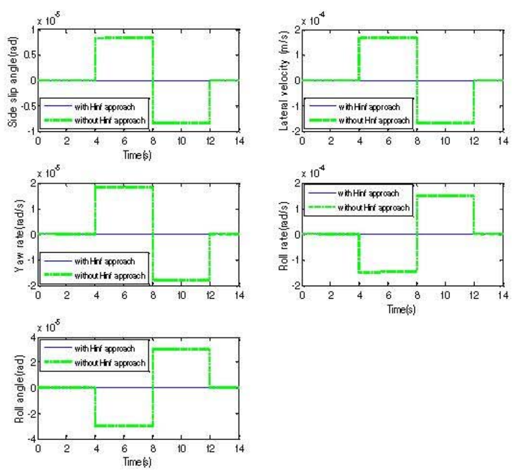

Figure 3. Behavior of estimation errors

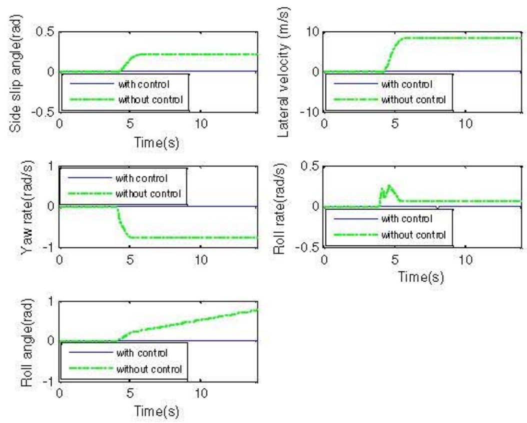

Figure 4. Comparison of controlled and uncontrolled outputs

V. Conclusion

In this paper, a robust observer-based fuzzy control method is proposed in order to improve vehicle stability. A *10*-DOF mathematical model has been used to describe the vehicle dynamics. Variation of the longitudinal vehicle velocity as well as variation of the road adhesion was considered in the controller design. LMI conditions have been developed using T-S fuzzy representation and robust pole placement, in order to guarantee global stability of the closed loop system with desired performances. The effectiveness of our approach has been demonstrated through simulations on the nonlinear vehicle model.

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