Proceedings of the 2006 American Control Conference
Minneapolis, Minnesota, USA, June 14-16, 2006

Observer-based robust fuzzy control for vehicle lateral dynamics

A. El Hajjaji *Member, IEEE*, M. Chadli, M. Oudghiri and O. Pagès *Member IEEE*

Abstract— **In this paper, the robust fuzzy control for Four Wheels Steering (4WS) vehicle dynamics is studied via a Takagi-Sugeno (T-S) uncertain fuzzy model when the road adhesion conditions change and the sideslip angle is unavailable for measurement. After giving the nonlinear model of the vehicle, its representation by a T-S uncertain fuzzy model is discussed. Next, based on the uncertain fuzzy model of the 4WS Vehicle a fuzzy controller and a fuzzy observer are developed. The closed loop stability conditions of a vehicle with the fuzzy controller and the observer are parameterized in terms of Linear Matrix Inequality (LMI) problem which can be solved very efficiently using the convex optimization techniques. The numerical simulation of the vehicle handling with and without the use of the developed observer and controller has been carried out. The simulation results obtained indicate that considerable improvements in the vehicle handling can be achieved whenever the vehicle is governed by the proposed fuzzy observer and fuzzy controller.**

I. INTRODUCTION

There is a continuing effort in the automobile industry to achieve the active control systems which improve the stability and the performance of vehicles in dangerous situations [1] [2] [3] [4]. Some of the systems have already been commercialized and been installed in passenger cars (ABS, ESP, TCS ...). However, these systems are still not optimal and they can be improved using an advanced estimation and control design methods [5] [6] [7] [8]. The vehicle safety improvement in terms of stability and comfort achieved by active control systems continues to be a subject of active research [10] [11]. The ultimate goal is still to produce vehicles that anyone can drive "safely", "pleasantly" and as one wishes''.

In this paper, a robust active control with an estimation of the sideslip angle is developed to improve stability and performances of a 4WS vehicle lateral dynamics. The proposed algorithm is based on the TS Fuzzy representation largely used in control and estimation problems of nonlinear systems these last years [13][14][15][17]. The paper is organized as follows: in section 2, we present the vehicle nonlinear model and its representation by a T-S fuzzy model [18]. The main idea is to approximate the system by a convex combination of linear models. Section 3 presents active control objectives and design methodology of the observer and the controller based on a TS vehicle model. In section 4, simulation results are given to highlight the effectiveness of the design procedure of the observer and of the controller and confirm the good performance of the vehicle in dangerous situations (unstable behavior and low road adhesion). Section 5 concludes this paper.

A. El Hajjaji, M. Chadli, M. Oudghiri and O. Pagès are with the *Centre de Robotique, d'Electrotechnique et d'Automatique, EA 3299* (corresponding author to provide phone: 33322827684; fax: 33322827663; e-mail: {ahmed.hajjaji, mohammed.chadli, mohammed.oudghiri, opages} @u-picardie.fr).

II. VEHICLE MODEL DESCRIPTION

The two-dimensional model with nonlinear tire characteristics of the four wheels vehicle behavior can be described by differential equations (cf. fig. 1)[9][10]:

$$\begin{pmatrix} \dot{\beta} \\ \dot{r} \end{pmatrix} = \begin{pmatrix} \dfrac{2F_f + 2F_r}{mU} - r \\ \dfrac{2a_f F_f - 2a_r F_r}{I_z}\cos(\beta) \end{pmatrix}. \tag{1}$$

A Where β denotes the side slip angle, r is the yaw velocity, F_f is the cornering force of the two front tires, F_r is the corning force of the two rear tires. U is the vehicle velocity, I_z is the yaw moment of inertia, m is the vehicle mass. The parameters of the vehicle are given in the following table:

Parameters	I_z	m	a_f	a_r	U
Values	3000	1500	1.3	1.2	20

Table 1: Vehicle parameters

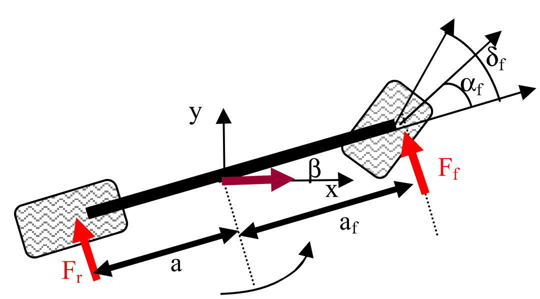

Fig. 1: Vehicle model

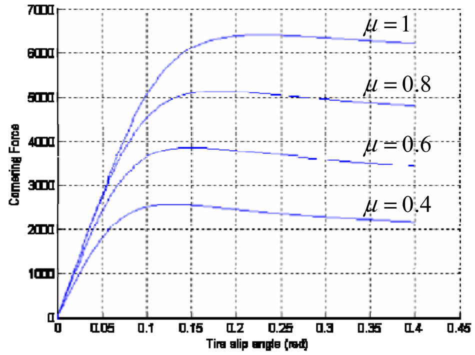

Fig. 2 : Cornering Forces

Using the magic formula [8], the cornering forces F_f and F_r are given as functions of tire slip angles by the following expressions:

$$\begin{cases} F_f = D_f(\mu)\sin\left[C_f(\mu)\tan^{-1}\left\{B_f(\mu)(1-E_f(\mu))\alpha_f + E_f(\mu)\tan^{-1}(B_f(\mu)\alpha_f)\right\}\right] \\ F_r = D_r(\mu)\sin\left[C_r(\mu)\tan^{-1}\left\{B_r(\mu)(1-E_r(\mu))\alpha_r + E_r(\mu)\tan^{-1}(B_r(\mu)\alpha_r)\right\}\right] \end{cases} \quad (2)$$

With

$$\begin{cases} \alpha_f = -\beta - \tan^{-1}\left(\dfrac{a_f}{U} r\cos(\beta)\right) + \delta_f \\ \alpha_r = -\beta + \tan^{-1}\left(\dfrac{a_r}{U} r\cos(\beta)\right) + \delta_r \end{cases} \quad (3)$$

Where δ_f is the front steer angle, δ_r is the rear steer angle, α_f is the slip angle of the front tires and α_r is the slip angle of the rear tires (see figure 1).

Coefficients D_i C_i B_i E_i (i = f,r) depend on the tire characteristics, road adhesion coefficient and the vehicle operational conditions. Figure 2 shows the cornering force characteristics for some road adhesion coefficients (μ).

To obtain the TS fuzzy model, we have modelized the front and rear lateral forces (2) by the following rules:

If $|\alpha_f|$ is M_1 then $\begin{cases} F_f = C_{f2}(\mu)\alpha_f \\ F_r = C_{r2}(\mu)\alpha_r \end{cases}$

If $|\alpha_f|$ is M_2 then $\begin{cases} F_f = C_{f1}(\mu)\alpha_f \\ F_r = C_{r1}(\mu)\alpha_r \end{cases}$

The overall forces are obtained by :

$$\begin{cases} F_f = \mu_1(|\alpha_f|)C_{f1}(\mu)\alpha_f + \mu_2(|\alpha_f|)C_{f2}(\mu)\alpha_f \\ F_r = \mu_1(|\alpha_f|)C_{r1}(\mu)\alpha_r + \mu_2(|\alpha_f|)C_{r2}(\mu)\alpha_r \end{cases} \quad (4)$$

Where μ_j (j = 1,2) is the i^{th} bell curve membership function of fuzzy set M_j. The membership function parameters and consequence parameters of rules are obtained using an identification method based on the Levenberg-Marquadt algorithm [18].

For road friction coefficient $\mu = 0.7$, membership functions μ_1 and μ_2 are given in figure 3.

The front and rear Pacejka forces (2) compared to estimated front and rear forces described by equation (4) are shown in figure 4.

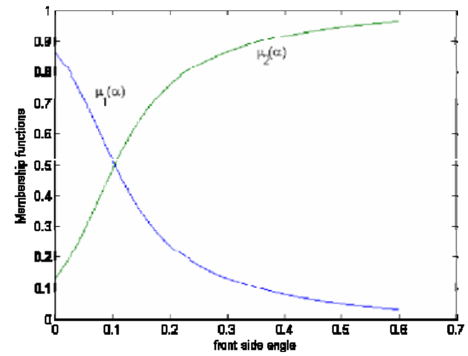

Fig. 3 : Membership functions

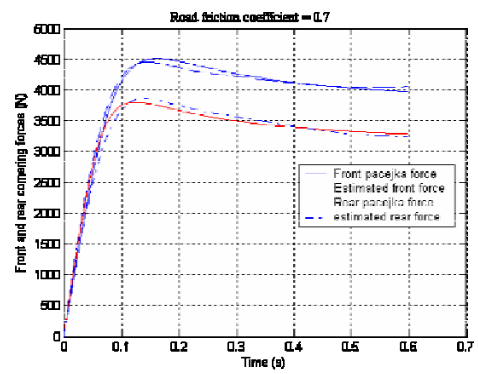

Fig. 4: comparison of the estimated and Pacejka forces

Using the above approximation idea of nonlinear lateral forces by TS rules and by considering that

$$\alpha_f \cong \beta + \frac{a_f r}{U} - \delta_f \qquad \alpha_r \cong \beta + \frac{a_r r}{U} - \delta_r$$

Nonlinear model (1) can be represented by the following TS fuzzy model:

If $|\alpha_f|$ is M_1 then $\begin{cases} \dot{x} = A_1 x + B_{f1}\delta_f + B_{r1}\delta_r \\ y = C_i x \end{cases}$

If $|\alpha_f|$ is M_2 then $\begin{cases} \dot{x} = A_2 x + B_{f2}\delta_f + B_{r2}\delta_r \\ y = C_i x \end{cases}$

Where

$$A_i = \begin{pmatrix} -2\dfrac{C_{fi} + C_{ri}}{mU} & -2\dfrac{C_{fi}a_f - C_{ri}a_r}{mU^2} - 1 \\ -2\dfrac{C_{fi}a_f - C_{ri}a_r}{I_z} & -2\dfrac{C_{fi}a_f^2 + C_{ri}a_r^2}{I_z U} \end{pmatrix}$$

$$B_{fi} = \begin{pmatrix} \frac{2C_{fi}}{mU} \\ \frac{2a_f C_{fi}}{I_z} \end{pmatrix},\quad B_{ri} = \begin{pmatrix} \frac{2C_{ri}}{mU} \\ -\frac{2a_r C_{ri}}{I_z} \end{pmatrix} \quad x = \begin{pmatrix} \beta \\ r \end{pmatrix} \quad y = r$$

$C_i = \begin{pmatrix} 0 & 1 \end{pmatrix}$.

We note that the stiffness coefficients C_{fi} ,C_{ri} are not constant and vary according to the road adhesion. To take into account these variations, we assume that these coefficients vary as follows:

$$\begin{cases} C_{fi} = C_{fi0}(1+d_i f_i) \\ C_{ri} = C_{ri0}(1+d_i f_i) \end{cases} \qquad \|f_i\| \le 1 \tag{5}$$

Where d_i indicates the deviation magnitude of the stiffness coefficient from its nominal value. After some manipulations, the TS fuzzy model can be written as:

If $|\alpha_f|$ is M_1 then

$$\begin{cases} \dot{x} = (A_{10} + \Delta A_1)x + (B_{f10} + \Delta B_{f1})\delta_f + (B_{r10} + \Delta B_{r1})\delta_r \\ y = Cx \end{cases}$$

If $|\alpha_f|$ is M_2 then

$$\begin{cases} \dot{x} = (A_{20} + \Delta A_2)x + (B_{f20} + \Delta B_{f2})\delta_f + (B_{r20} + \Delta B_{r2})\delta_r \\ y = Cx \end{cases}$$

Where

$$A_{i0} = \begin{pmatrix} -2\frac{C_{fi0} + C_{ri0}}{mU} & -2\frac{C_{fi0}a_f - C_{ri0}a_r}{mU^2} - 1 \\ -2\frac{C_{fi0}a_f - C_{ri0}a_r}{I_z} & -2\frac{C_{fi0}a_f^2 + C_{ri0}a_r^2}{I_z U} \end{pmatrix}$$

$$B_{fi0} = \begin{pmatrix} \frac{2C_{fi0}}{mv} \\ \frac{2a_f C_{fi0}}{I_z} \end{pmatrix},\quad B_{ri0} = \begin{pmatrix} \frac{2C_{ri0}}{mv} \\ -\frac{2a_r C_{ri0}}{I_z} \end{pmatrix}$$

$$\Delta A_i = D_i F_i E_{1i} \quad \Delta B_{fi} = D_i F_i E_{f2i} \quad \Delta B_{ri} = D_i F_i E_{r2i}$$

$$D_i = \begin{pmatrix} d_i & 0 \\ 0 & d_i \end{pmatrix} \quad F_i = \begin{pmatrix} f_i & 0 \\ 0 & f_i \end{pmatrix}$$

$$E_{f2i} = \begin{pmatrix} \frac{2C_{fi0}}{mv} \\ \frac{2a_f C_{fi0}}{I_z} \end{pmatrix} \quad E_{r2i} = \begin{pmatrix} \frac{2C_{ri0}}{mv} \\ -\frac{2a_r C_{ri0}}{I_z} \end{pmatrix}$$

$$E_{1i} = \begin{pmatrix} -2\frac{C_{fi0} + C_{ri0}}{mU} & -2\frac{C_{fi0}a_f - C_{ri0}a_r}{mU^2} \\ -2\frac{C_{fi0}a_f - C_{ri0}a_r}{I_z} & -2\frac{C_{fi0}a_f^2 + C_{ri0}a_r^2}{I_z U} \end{pmatrix}$$

The defuzzified output of this T–S fuzzy system is :

$$\begin{cases} \dot{x} = \sum_{i=1}^{2} \mu_i(|\alpha_f|)\left[(A_{i0} + \Delta A_i)x + (B_{fi0} + \Delta B_{fi})\delta_f + (B_{ri0} + \Delta B_{ri})\delta_r\right] \\ y = \sum_{i=1}^{2} \mu_i(|\alpha_f|)C_i x \end{cases} \tag{6}$$

Where

$$\mu_1(|\alpha_f|) = \frac{1}{\left(1 + abs\left(\frac{|\alpha_f| - c_1}{a_1}\right)\right)^{2b_1}}$$

And

$$\mu_2(|\alpha_f|) = \frac{1}{\left(1 + abs\left(\frac{|\alpha_f| - c_2}{a_2}\right)\right)^{2b_2}}$$

With a_1=0.5077, b_1=0.4748, c_1=3.1893, a_2=5.3907 b_2=0.4356, c_2=0.5633;

III. OBSERVER AND CONTROLLER DESIGN AND ANALYSIS

Currently, the sideslip sensor is still very expensive and cannot be installed in the vehicle. This problem can be overcome by introducing the observer theory. If each local linear model is observable, the TS fuzzy observer can be used.

A. TS Fuzzy observer structure

The observer has the same structure as the TS fuzzy model of the vehicle.

If $|\alpha_f|$ is M_1 then $\begin{cases} \dot{\hat{x}} = A_1\hat{x} + B_{r1}\delta_r + B_{r1}\delta_f + G_1(y - \hat{y}) \\ \hat{y} = C_i\hat{x} \end{cases}$

If $|\alpha_f|$ is M_2 then $\begin{cases} \dot{\hat{x}} = A_2\hat{x} + B_{r2}\delta_r + B_{r2}\delta_f + G_2(y - \hat{y}) \\ \hat{y} = C_2\hat{x} \end{cases}$

The overall fuzzy observer is represented as follows:

$$\begin{aligned} \dot{\hat{x}} &= \sum_{i=1}^{2} \mu_i(|\alpha_f|)\left[A_i\hat{x} + B_{ri}\delta_r + B_{fi}\delta_f + G_i(y - \hat{y})\right] \\ \hat{y} &= \sum_{i=1}^{q} \mu_i(|\alpha_f|)C_i\hat{x} \end{aligned} \tag{7}$$

Where G_1 and G_2 are the constant observer gains to be determined

B. TS Fuzzy controller

Like the fuzzy observer, the TS fuzzy controller can be represented by :

If $|\alpha_f|$ is M_1 then $\delta_r = -K_1\hat{x}$

If $|\alpha_f|$ is M_2 then $\delta_r = -K_2\hat{x}$

The overall fuzzy controller is represented as follows:

$$\delta_r = -\sum_{i=1}^{2} \mu_i(|\alpha_f|)K_i\hat{x} \tag{8}$$

Where K_1 and K_2 are the constant feedback gains to be determined

C. Design and stability analysis

The objective is to determine feedback gains K_1, K_2 and observer gains G_1, G_2 such as the closed loop system with fuzzy observer (7) and fuzzy controller (8) is asymptotically stable. To do this, we define:

$e(t) = x(t) - \hat{x}(t)$

From systems (2)- (4) and (5), we have

$$\dot{x}(t) = \sum_{i=1}^{2}\sum_{j=1}^{2}\mu_i(|\alpha_f|)\mu_j(|\alpha_f|)\left[(A_i - B_{ri}K_j)x + B_{ri}K_j e + B_{fi}\delta_f\right] + \sum_{i=1}^{2}\sum_{j=1}^{2}\mu_i(|\alpha_f|)\mu_j(|\alpha_f|)\left[(\Delta A_i - \Delta B_{ri}K_j)x + \Delta B_{ri}K_j x\right]$$

$$\dot{\hat{x}} = \sum_{i=1}^{2}\sum_{j=1}^{2}\mu_i(|\alpha_f|)\mu_j(|\alpha_f|)(A_i - B_{ri}K_j)\hat{x} + \sum_{i=1}^{2}\sum_{j=1}^{2}\mu_i(|\alpha_f|)\mu_j(|\alpha_f|)G_iC_j e$$

$$\dot{e} = \sum_{i=1}^{2}\sum_{j=1}^{2}\mu_i\mu_j(A_i - G_iC_j + \Delta B_{ri}K_j)e + \sum_{i=1}^{2}\sum_{j=1}^{2}\mu_i\mu_j(\Delta A_i - \Delta B_{ri}K_j)x$$

The augmented system can be expressed as the following form:

$$\dot{\tilde{x}} = \sum_{i=1}^{2}\sum_{j=1}^{2}\mu_i(|\alpha_f|)\mu_j(|\alpha_f|)\left[(\tilde{A}_{ij} + \Delta\tilde{A}_{ij})\tilde{x} + \tilde{B}\delta_f\right]$$

Where

$$\tilde{A}_{ij} = \begin{pmatrix} A_i - B_{ri}K_j & B_{ri}K_j \\ 0 & A_i - G_iC_j \end{pmatrix}, \tilde{B} = \begin{pmatrix} B_{fi} \\ 0 \end{pmatrix}$$

$$\Delta\tilde{A}_{ij} = \begin{pmatrix} \Delta A_i - \Delta B_{ri}K_j & \Delta B_{ri}K_j \\ \Delta A_i - \Delta B_{ri}K_j & \Delta B_{ri}K_j \end{pmatrix}, \; \tilde{x} = \begin{pmatrix} x \\ e \end{pmatrix}$$

The main result for the global asymptotic stability of the T-S fuzzy model, with parametric uncertainties and unavailable state variable, are summarized in the following theorem:

Theorem If there exist symmetric and positive definite matrices P_1 and P_2, some matrices K_i and G_i, and scalars ε_{ij}, (i,j=1,2), such that the following LMIs are satisfied, then T-S fuzzy system (6) is globally asymptotically stabilizable via T-S fuzzy controller (8) based on fuzzy observer (7) :

$$\begin{pmatrix} \Phi_{ii} & * & * \\ E_{Ai}Q - E_{Bi}M_i & -(\varepsilon_{ii}^{-1}+1)^{-1}I & * \\ D_i^T & 0 & -(\varepsilon_{ii}+1)^{-1}I \end{pmatrix} < 0 \quad (9a)$$

$$\begin{pmatrix} \Psi_{ij} & * & * & * & * \\ E_{Ai}Q - E_{Bi}M_j & -(\varepsilon_{ij}^{-1}+1)^{-1}I & * & * & * \\ E_{1j}Q - E_{Bj}M_i & 0 & -(\varepsilon_{ij}^{-1}+1)^{-1}I & * & * \\ D_i^T & 0 & 0 & -\varepsilon_{ij}^{-1} & * \\ D_j^T & 0 & 0 & 0 & -\varepsilon_{ij}^{-1} \end{pmatrix} < 0 \quad (9b)$$

i<j

$$\begin{pmatrix} T_{ii} & * & * \\ E_{Bi}K_i & -(\varepsilon_{ii}^{-1}+1)^{-1}I & * \\ D_i^T P & 0 & -(\varepsilon_{ii}+1)^{-1}I \end{pmatrix} < 0 \quad (10a)$$

$$\begin{pmatrix} \Theta_{ij} & * & * & * & * \\ E_{Bi}K_j & -(\varepsilon_{ij}^{-1}+1)^{-1}I & * & * & * \\ E_{Bj}K_i & 0 & -(\varepsilon_{ij}^{-1}+1)^{-1}I & * & * \\ (P_2D_{ii})^T & 0 & 0 & -(\varepsilon_{ij}+1)^{-1}I & * \\ (P_2D_j)^T & 0 & 0 & 0 & -(\varepsilon_{ij}+1)^{-1}I \end{pmatrix} < 0 \quad (10b)$$

With

$$\Phi_{ii} = QA_i^T + A_iQ - M_i^TB_i^T - B_iM_i + I$$

$$\Psi_{ij} = QA_i^T + A_iQ + QA_j^T + A_jQ - M_j^TB_i^T - B_iM_j - M_i^TB_j^T - B_jM_i + D_iD_i^T + D_jD_j^T + 2I$$

$$T_{ii} = A_i^TP + PA_i - C_i^TN_i^T - N_iC_i + K_i^TB_i^TB_iK_i$$

$$\Theta_{ij} = A_i^TP_2 + P_2A_i + A_j^TP_2 + P_2A_j - C_i^TN_j^T - N_iC_j - C_j^TN_i^T - N_jC_i + K_i^TB_j^TB_jK_i + K_j^TB_i^TB_iK_j$$

And $Q = P_1^{-1}$, $M_i = K_iP_1^{-1}$, and $N_i = P_2G_i$, where $*$ denotes the transposed element in the symmetric positions.

Proof: Proof details are not given in this paper for lack of space. We give some indications in the following:

Let $V = \begin{pmatrix} x \\ e \end{pmatrix}^T \begin{pmatrix} P_1 & 0 \\ 0 & P_2 \end{pmatrix} \begin{pmatrix} x \\ e \end{pmatrix}$ Lyapunov function.

By calculating the Lyapunov function derivative and using the following well know lemma [12].

After some manipulations, we can prove that if the conditions given in the above theorem are satisfied the Lyapunov function derivative is negative. More information, see [19].

Remark: We note that the separation principle can not be used directly uncertainties.

4- Simulation results

To show the effectiveness of the proposed observer/control algorithm, we have carried out two series of simulations.

In the design, the nominal stiffness coefficients considered are:

Nominal stiffness coefficients	C_{f10}	C_{f20}	C_{r10}	C_{r20}
values	60712	4812	60088	3455

And $D_1 = D_2 = \begin{pmatrix} 0.4 & 0 \\ 0 & 0.4 \end{pmatrix}$,

By solving the LMIs, we obtain the following controller and observer gains:

$K_1 = \begin{pmatrix} 0.7086 & -1.4146 \end{pmatrix}$, $K_2 = \begin{pmatrix} 1.5491 & -2.9217 \end{pmatrix}$

$$G_1 = \begin{pmatrix} 118.9563 \\ 243.4051 \end{pmatrix} \qquad G_2 = \begin{pmatrix} -83.5390 \\ 246.4051 \end{pmatrix}$$

All the simulations are realized on the nonlinear model given in (1) with vehicle speed 20 m/s. The first series (figures (5) and (6)) is realized to compare the vehicle dynamics behaviour with TS fuzzy rear control based on a fuzzy observer to its behaviour with the linear rear control based on linear observer and without rear control for the front steering angle given in figure 3. The road friction coefficient is fixed to 0.9. We can see that by using the proposed fuzzy control based on fuzzy observer, the results are more better than these with linear control or without control. The estimated state variables using nonlinear fuzzy observer are also compared to measured state variables in figure 7 and 8 for the same front steering angle given in figure 3. We remark that estimated state variables correctly follow the measured state variables with nonlinear observer.

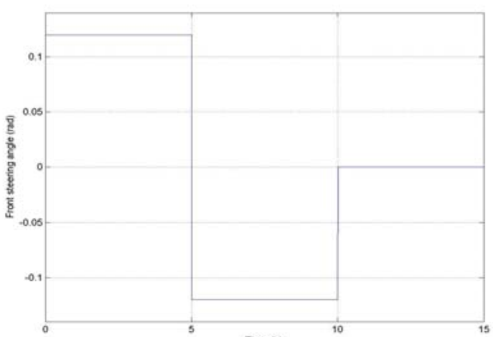

Fig. 4 : Front steering angle

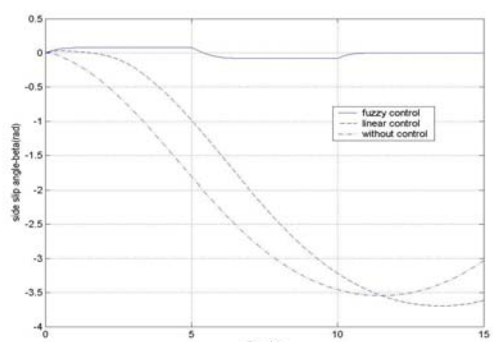

Fig. 5: sideslip angle responses

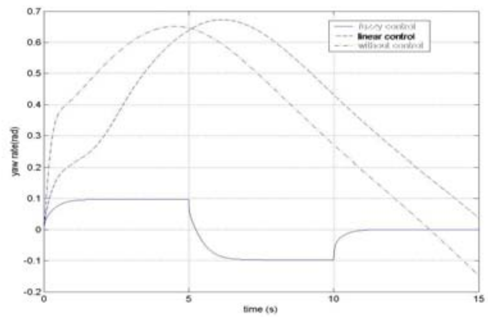

Fig. 6: Yaw rate responses

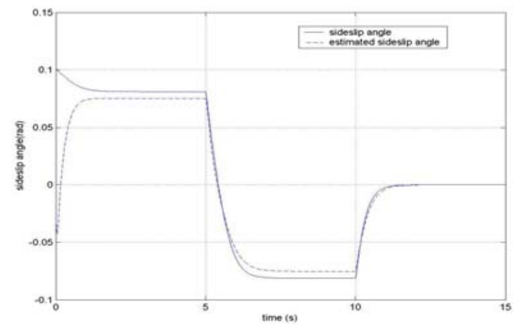

Fig. 7: sideslip angles

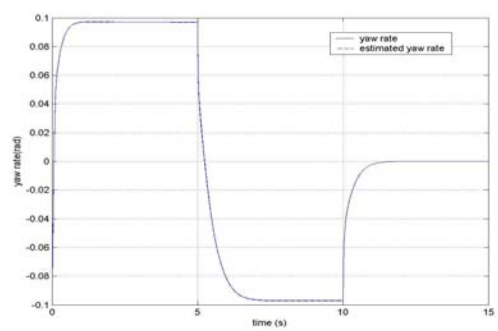

Fig. 8: Yaw rate

front steer angle δ (rad)	road friction : μ									
	1	**0,9**	**0,8**	**0,7**	**0,6**	**0,5**	**0,4**	**0,3**	**0,2**	**0,1**
0,02	+	+	+	+	+	+	+	+	+	+
0,03	+	+	+	+	+	+	+	+	+	-
0,04	+	+	+	+	+	+	+	+	+	-
0,05	+	+	+	+	+	+	+	+	-	-
0,06	+	+	+	+	+	+	+	+	-	-
0,07	+	+	+	+	+	+	+	-	-	-
0,08	+	+	+	+	+	+	-	-	-	-
0,09	+	+	+	+	+	-	-	-	-	-
0,1	+	+	+	+	+	-	-	-	-	-
0,11	+	+	+	+	-	-	-	-	-	-
0,12	+	+	+	+	-	-	-	-	-	-
0,13	+	+	+	-	-	-	-	-	-	-
0,14	+	+	-	-	-	-	-	-	-	-
0,15	+	-	-	-	-	-	-	-	-	-

TABLE 3

front steer angle (rad)	road friction : μ									
	1	0,9	0,8	0,7	0,6	0,5	0,4	0,3	0,2	0,1
0,02	+	+	+	+	+	+	+	+	+	+
0,03	+	+	+	+	+	+	+	+	+	+
0,04	+	+	+	+	+	+	+	+	+	+
0,05	+	+	+	+	+	+	+	+	+	-
0,06	+	+	+	+	+	+	+	+	+	-
0,07	+	+	+	+	+	+	+	+	+	-
0,08	+	+	+	+	+	+	+	+	+	-
0,09	+	+	+	+	+	+	+	+	+	-
0,1	+	+	+	+	+	+	+	+	-	-
0,11	+	+	+	+	+	+	+	+	-	-
0,12	+	+	+	+	+	+	+	+	-	-
0,13	+	+	+	+	+	+	+	-	-	-
0,14	+	+	+	+	+	+	+	-	-	-
0,15	+	+	+	+	+	+	-	-	-	-
0,16	+	+	+	+	+	+	-	-	-	-
0,17	+	+	+	+	+	+	-	-	-	-
0,18	+	+	+	+	+	+	-	-	-	-
0,19	+	+	+	+	+	-	-	-	-	-
0,2	+	+	+	+	+	-	-	-	-	-
0,25	+	+	+	+	-	-	-	-	-	-

TABLE 4

front steer angle δ (rad)	road friction : μ									
	1	0,9	0,8	0,7	0,6	0,5	0,4	0,3	0,2	0,1
0,02	+	+	+	+	+	+	+	-	-	-
0,03	+	+	+	+	+	-	-	-	-	-
0,04	+	+	+	-	-	-	-	-	-	-
0,05	-	-	-	-	-	-	-	-	-	-

TABLE 2

'+': Vehicle is stable
'-': Vehicle is unstable

The second series of simulations are realized to show the importance of a rear fuzzy control based on a fuzzy observer on the stability improvement of the vehicle dynamics. table2, table3, and table 4 summarize the influence of driver steering (δ_f) and road adhesion on the stability of vehicle dynamics respectively without rear control, with linear rear control based on a linear observer and with fuzzy rear control based on a fuzzy observer respectively. It is easy to see from tables 2, 3, 4 that the vehicle stability is considerably more improved by the proposed control/observer algorithm than by the linear control or without control.

IV. CONCLUSION

Active control with sideslip estimation for a 4WS vehicle is proposed. The proposed algorithm is based on the TS uncertain fuzzy representation. The new stabilization conditions are given in terms of LMIs. By vehicle simulations, we know that based on the LMIs, the controller based on an observer designing method for stability improvement is very efficient and practical. We have also shown the contribution of the proposed algorithm compared to linear control.

REFERENCES

[1] E. Lim and , J. K. Hedrick.: Lateral and longitudinal vehicle control coupling for automated vehicle operation, Proceeding of ACC California, 1999, 3676-3680.

[2] Ono E., Hosoe, S. and Asano, K.: Robust stabilisation of vehicle Dynamics by Gain-sheduled *Hoc* control, IEEE-OCA, Hawai USA, 1999,22-27.

[3] E.Ono, S.Hosoe, H.Tuan and S. Doi, Bifurcation in Vehicle Dynamics and Robust Front Wheel Steering Control ,IEEE Transactions on Control Systems Technology, Vol. 6(3), (1999), 412-420.

[4] K. Park , S. Heo and I. Baek, " Controller design for improving lateral vehicle dynamic stability, JSAE Vol 22, p-481-486 (2201).

[5] S. Choi and D. Cho, Control of wheel slip Ratio using sliding mode controller with impulse modulation, Vehicle system Dynamics journal, Vol 32, 1999,275-285.

[6] Tai M. and M. Tomizuka. 2000. "Robust Longitudinal Velocity Tracking of Vehicles using Traction and Brake Control". In Proceedings of the 6^{th} International Workshop on Advanced Motion Control (March/April 2000), 305-310. .

[7] O. Pages, A. El Hajjaji and A. Ciocan, Wheel sleep control by using robust TS Fuzzy approach, Proceeding of AVCS'2004 Vol 1 p. 34-39 Bergeggi-Italy

[8] M. Lin A. A. popov, S. Mcwilliam, ''Stability and performance studies of driver-vehicle systems with electronic chassis control '', Vehicle system dynamics, Vol 41(2) pp.477-486,2004.

[9] A.Hajjaji, A. Ciocan, D. Hamad, ''Four Wheel steering control by fuzzy approach'', Journal of intelligent and robotics systems Vol 41(2-3) pp.141-156, 2005.

[10] B. Catino, S. Santini and M. di Bernardo, "MCS adaptive control of vehicle dynamics :an application of bifurcation techniques to control system design, "IEEE-CDC 2003 p. 2252-2257, Hawaii-USA

[11] A. Ganguli, R. Rajamani, ''Fault diagnostics for GPS-based lateral vehicle dynamics '', Vehicle system dynamics, Vol 39(2) pp.99-120,2003.

[12] B. chen, Y. Chen and C. Lin ," Nonlinear Fuzzy H Fuidance law with saturation of actuators against maneuvering targets", IEEE Trans. On CST Vol 10(2) p.769-779(2002).

[13] L. Xioodong, Z. Qingling "New approaches To H controller designs based on fuzzy observers for T-S fuzzy systems via LMI, Automatica Vol 39 p.1571-1582(2003)

[14] K. Kim, J. Joh, R. Langri and K. Kwon, LMI-Based Design of Takagi-Sugeno Fuzzy Controllers for Nonlinear Dynamic systems using fuzzy estimators, Inter. Jour, of Fuzzy Systems, Vol. 1(2) pp.133-144(1999).

[15] K. Kirakidis, Robust stabilisation of the Takagi Sugeno fuzzy model via Bilinear Matrix Inequalities, IEEE Trans, on Fuzzy Systems Vol. 9(2) 2001, 269-277.

[16] K. Tanaka and O. Wang, Fuzzy Regulators and Fuzzy observers: A linear Matrix Inequality Approach, Proceeding of 36 th IEEE CDC(1998),1315-1320.

[17] Korba P., R. Babuska, H. B. Verbruggen and P. M. Frank "Fuzzy Gain Scheduling: Controller and Observer Design Based on Lyapunov Method and Convex Optimization". IEEE Transactions on Fuzzy Systems, Volume 11, n°3 (June 2003), 285-298. .

[18] C. Lee, W. Lai and Y. Lin, "A TSK Type Fuzzy neural network systems for dynamic systems identification". In Proceedings of the *IEEE-CDC*, p 4002-4007 Hawaii –USA(2003) .

[19] M. Chadli & A. El Hajjaji, Comment on observer-based robust fuzzy control of nonlinear systems with parametric uncertainties, Journal of Fuzzy Sets and Systems to appear