Development and Experimental Evaluation of a Tilt Stability Control System for Narrow Commuter Vehicles

Samuel Kidane, Rajesh Rajamani, Lee Alexander, Patrick J. Starr, and Max Donath

***Abstract*—This paper concentrates on development and experimental investigation of a compound control system designed for tilt stability of a narrow commuter vehicle. The control system is a combination of three different types of control schemes: Steering Tilt Control (STC) system, Direct Tilt Control (DTC) system and Tilt Brake system. These schemes utilize different types of actuators and offer complementary advantages over different ranges of operating speeds. The design of the control system is discussed. Then the combined STC-DTC system is first validated by running standard vehicle maneuver experiments such as turns and lane changes. The performance of the Tilt Brake algorithm is then verified for different low speed maneuvers. The feasibility of a stand alone DTC system is also experimentally investigated. Finally different experimental results are presented to demonstrate the effect of desired tilt angle definition on the handling of a narrow tilting vehicle.**

***Index Terms*— Narrow vehicle, roll control, roll dynamics, rollover prevention, tilt control.**

NOMENCLATURE

L	Wheel base of vehicle (1.52 m).
L_f	Distance from center of gravity to front wheels (0.58 m).
L_r	Distance from center of gravity to rear wheels (0.72 m).
h	Height of center of gravity of vehicle (0.6 m).
W	Track width (1 m).
m	Mass of vehicle (200 kg).
I	Tilt moment of inertia of vehicle (80 kg-m2).
C_f	Front wheel cornering stiffness (10 000 N/rad).
C_r	Rear wheel cornering stiffness (6500 N/rad).
λ_f	Front wheel camber stiffness (1000 N/rad).
λ_r	Rear wheel camber stiffness (2000 N/rad).
V	Longitudinal vehicle velocity (0–35 m/s).
F_f	Lateral force on front wheels.
F_r	Lateral force on rear wheels.
δ	Steering input to front wheels.
T	Tilt torque from tilt actuator.
e_1	Lateral position error.
e_2	Heading angle error.
y	Lateral displacement of vehicle.
ω	Angular rotation of wheels.
$I_{\text{wheel\_}\psi}$	Yaw moment of inertia of front wheel.
$I_{\text{wheel\_rot}}$	Rotational moment of inertia of front wheel.
M_G	Gyroscopic moment due to rotating front wheels.
R	Radius of curvature of turn.
ψ	Vehicle heading angle.
θ	Vehicle tilt angle.
θ_{des}	Vehicle desired tilt angle.
θ_{ss}	Vehicle steady state tilt angle.

Manuscript received February 19, 2009. Manuscript received in final form October 27, 2009. First published December 18, 2009; current version published October 22, 2010. Recommended by Associate Editor C. Bohn. This work was supported in part by the National Science Foundation under Contract CMS-0411455 and by funds from the ITS-Institute, University of Minnesota.

The authors are with the Department of Mechanical Engineering, University of Minnesota, Minneapolis, MN 55455 USA (e-mail: skidane@me.umn.edu; rajamani@me.umn.edu; alex@me.umn.edu; pjstarr@me.umn.edu; donath@me.umn.edu).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TCST.2009.2035819

I. INTRODUCTION

TRAFFIC congestion is a growing problem all over the world. According to studies carried out in 2001 by Texas Transportation Institute the urban traffic volume in the U.S. is expected to double by the year 2010 [1]. In the same study it has been also established that the increase in traffic demand every year outpaces the increase in capacity due to new highway lane construction. In addition to this the United States Department of Transportation has reported that the average number of occupants of a single vehicle in the U.S. is about 1.58 [2], which clearly indicates that the current vehicle size is highly under utilized.

The above stated facts make a narrow commuter vehicle having a sitting capacity of one (or two in tandem) very attractive for tackling traffic congestion. This vehicle can be designed to operate in a half-width highway lane, thus increasing the traffic capacity. Due to the reduced projected front area, narrow vehicles also have superior aerodynamic characteristics resulting in high fuel efficiency. Recently, Volkswagen was able to achieve an incredible fuel efficiency of 285 mpg in its *narrow* “1-Liter” concept car [3].

When track width of a vehicle is narrowed, it increases the ratio of center of gravity height to track width. This particular geometric property poses stability problems for these types of vehicles while cornering. Hence such a narrow vehicle needs to be equipped with an inherent tilting mechanism that enables the vehicle to tilt into turns while cornering. The added tilt degree of freedom should not require any stabilizing input from the driver. This brings about the need for an active control system that ensures the tilt stability (appropriate leaning during cornering) without affecting the handling of the vehicle.

A few prototype narrow tilting vehicles (NTVs) have been developed by the automotive industry in the last 50 years. The Ford *Gyron* is one of the earliest NTV vehicles developed by Ford in the 1950s [4]. This vehicle employed a 180 lbs gyroscope that stabilized the vehicle while cornering and used a retractable wheel pods during parking. The unnecessary added weight of the gyroscope and the need for retractable wheels were some of the limitation of this vehicle. General Motors had also developed a three-wheeled vehicle named *Lean Machine* in the 1970s [4]. This vehicle had a nontilting rear engine pod with a rotating body module. A foot pedal operated by the driver controlled the tilt-stabilizing actuator. The limitations of the *Lean Machine* were the fact that a significant portion of the vehicle weight was non-tilting and the tilt actuator had to be manually operated by the driver, which required learned skills. Another interesting NTV concept was introduced by Mercedes-Benz circa 1997. This machine named *F-300 Life-Jet* has an active tilt control system that uses a hydraulic actuator for tilt stability of the vehicle [4]. This vehicle has been designed with a track width of 1.56 m, which is about the same as an average sedan. The track width of this vehicle therefore does not serve the purpose of building narrow vehicles to efficiently utilize existing highway infrastructures.

The most significant recent development in the area of NTVs is the *Carver* by Brink Dynamics. The Carver is the first leaning vehicle that is commercially availably (currently only in Europe). This vehicle has one steered front wheel and two driven rear wheels. Like the Lean Machine, this vehicle has a rotating body module and a non-tilting rear engine pod. A hydraulic based dynamic vehicle control (DVC) system translates the driver input torque into actual steering torque of the front wheel and a tilt torque of the tilt actuator [5]. Even though the Carver's performance has been well established by Brink Dynamics, the authors of this paper feel that there is not enough technical detail available in literature on development of the Carver system. One of the more technical papers on the Brink Dynamics web site [6] presents only steady state analysis; even though it is well known that most of the control system design difficulties arise in the transient phase. From the papers posted on Brink Dynamics web site it is evident that the vehicle predominantly employs a direct tilt control (DTC) system. Steering tilt control (STC) system is briefly mentioned in a few of the papers but the details are not discussed. The particular DTC algorithm implemented on the Carver is based on driving the steering torque of the front wheels to zero [7]. Depending on characteristics of the vehicle like the *trail*, this is likely to result in a vehicle tilt angle that is different from the "ideal" tilt angle achieved in bicycles and motorcycles, lending itself to a nonzero steady-state tilt torque in excess of 100 N·m [6]. This non-ideal tilt angle will also mean that the lateral acceleration felt by the driver will not be completely eliminated. Apart from this, the fact that the DTC acts after initiation of the turn results in high transient torque. The resulting transient torque is as high as 1000 N·m [8].

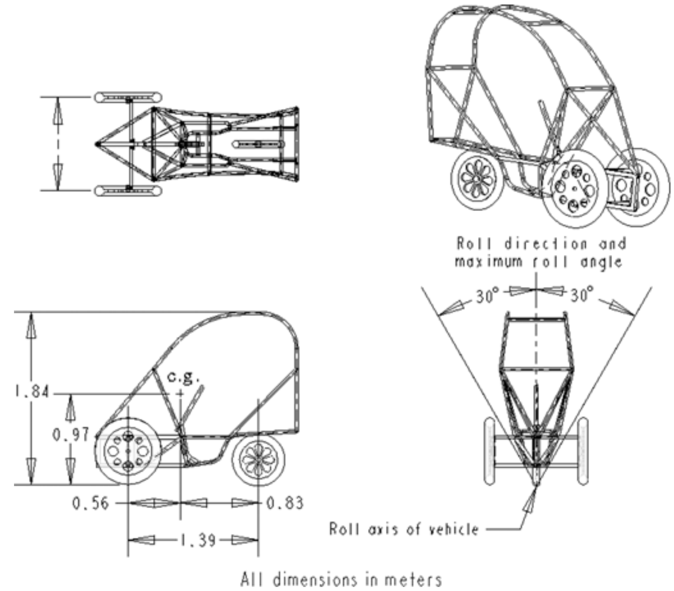


Fig. 1. Overall dimensions of vehicle structural elements.

The control system proposed in this paper utilizes a combined DTC and STC controller that results in minimal transient torque and zero steady state tilt torque. This has been demonstrated by successfully implementing the proposed compound algorithm using a small tilt servomotor actuator having only 40 N·m torque capacity. At extremely low speed and idling, the compound SDTC system is assisted by a *tilt brake* system that locks the vehicle in the upright position. For these maneuvers there is no danger of vehicle rollover and hence the vehicle can be kept upright. The tilt brake is engaged when the vehicle slows down below a certain low speed threshold and is released when the vehicle accelerates past a certain high-speed threshold. This tilt-brake needs to be engaged and disengaged in a manner that ensures smooth ride for the driver while avoiding overshoots in tilt torque and steering input requirements.

II. Prototype Design and Fabrication

The NTV research group in the mechanical engineering department of the University of Minnesota has designed and built a second-generation NTV prototype vehicle. The prototype has one driven rear wheel and two non-driven front wheels. The tilt mechanism is designed in such a way that the center of gravity of the vehicle tilts about a roll axis defined by the intersection of the ground plane and the symmetry plane of the vehicle (see Fig. 1). This second generation NTV has several major improvements over the first generation. A description of the first generation NTV can be found in [13]. This second generation NTV seats one person and has all the needed controls onboard. The joints of the tilting mechanism have been improved in such a way that stiction is reduced significantly. This has improved the self-stabilizing characteristic of the vehicle. For the safety of the rider and increased torsional stiffness of the vehicle, a roll cage has also been incorporated in the design. The vehicle weighs

Fig. 2. Prototype NTV.

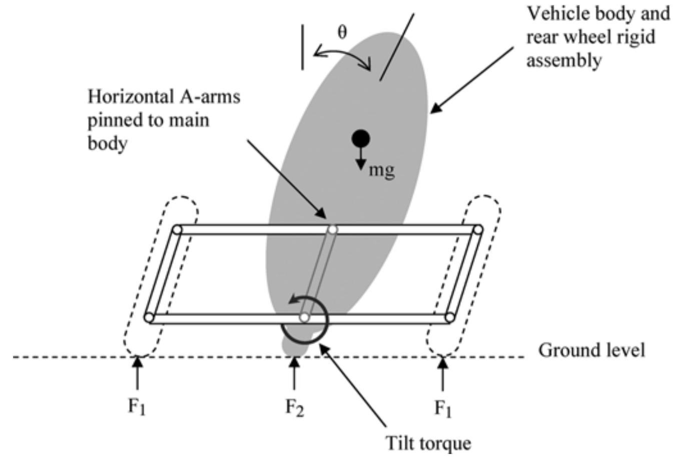


Fig. 4. Vehicle lean and overturning moments.

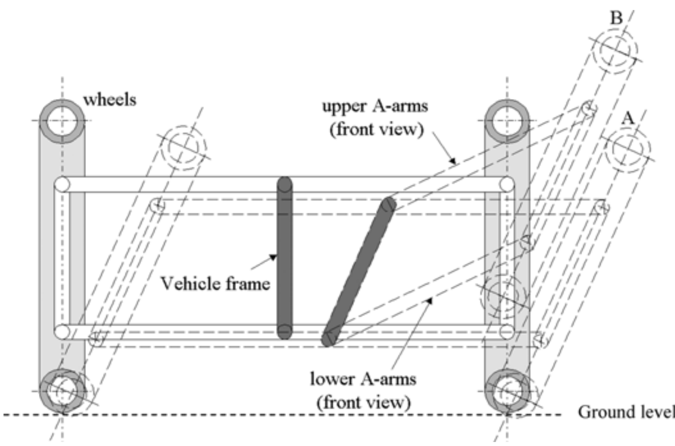


Fig. 3. Lean and bump mechanism (front view of vehicle).

roughly 450 lbs without the rider. The overall dimensions of this second generation vehicle are shown in Fig. 1.

A photograph of the assembled prototype vehicle is shown in Fig. 2. Parts of a Yamaha scooter chassis, including the rear wheel assembly, the steering column assembly, and the 125 cc engine, were used as part of the prototype NTV. A steel frame that constitutes the roll cage was added to the existing scooter chassis, as seen in Fig. 2. In addition to this, the frame serves as a mount for the newly designed steering mechanism, tilting actuator, batteries and other machinery. The tilting front wheels have been adjusted to have zero camber (vehicle in upright position) and a slight $(\sim 0.5^\circ)$ toe-in angle.

This NTV can tilt up to $\pm 35^\circ$ with an additional $\pm 10^\circ$ bump capability. Fig. 3 shows the concept behind the lean and bump mechanism. For the experiment results presented in this paper the additional bump degree of freedom has been constrained (both wheels tilt to the same tilt angle as the vehicle). The frame of the vehicle indicated in Fig. 3 corresponds to the rigid structure that constitutes of the front rectangular frame and everything attached to it except the A-arm and front wheels. The leaning mechanism is a simple extension of the four bar linkage. Leaning of the vehicle occurs when the angle is changed between the frame of the vehicle and the A-arms. In Fig. 3, configuration "A" shows the vehicle in lean, while configuration "B" shows the vehicle doing the same lean with its right wheel going over a bump. For a DTC operation mode the tilt actuator exerts a torque to change the angle between the vehicle frame and the lower A-arms.

In addition to the lean and the bump mechanism the front assembly also includes a steering mechanism that can steer the front wheels $\pm 30^\circ$. The steering axis of the front wheels is inclined 9.5° from the vertical. This will result in a positive trail of 0.06 m. Trail is a geometric parameter defined as the distance between the center of the contact patch of the front wheel and the point of intersection of steering axis with the ground. The NTV is designed so that it is capable of making longitudinal accelerations of up to ± 0.7 g's. For safe operation of the vehicle, the existing seat of the scooter has been replaced by a racing seat with seat belts. The driver is further protected by the 1 1/4″ roll cage tubing that encloses the vehicle. The vehicle has one dc servomotor each for steering the front wheels and tilting the vehicle as well as an electromagnetic brake actuator for the tilt brake.

Fig. 4 further shows the tilt mechanism of the vehicle and the overturning moments. The net moment acting at the center of gravity of the vehicle for a tilt angle of θ is given below (where W is track width and h is c.g. height)

$$M = F_1 \left(\frac{W}{\ } \cdots \right. \qquad (1)$$

[illegible]

Therefore, if there were no lateral forces counterbalancing this moment (a static tilt) a torque equal to M needs to be applied to keep the vehicle from tipping over.

III. Control System

The overall control system goals for the narrow tilting vehicle are: *stabilize the tilt mode of the vehicle, allow the driver to follow desired lateral trajectory* and *minimize tilt actuation torque*. The proposed compound tilt stabilizing control system, as mentioned in the introduction consists of a DTC, STC, and a tilt brake system working together. A DTC system, as the name

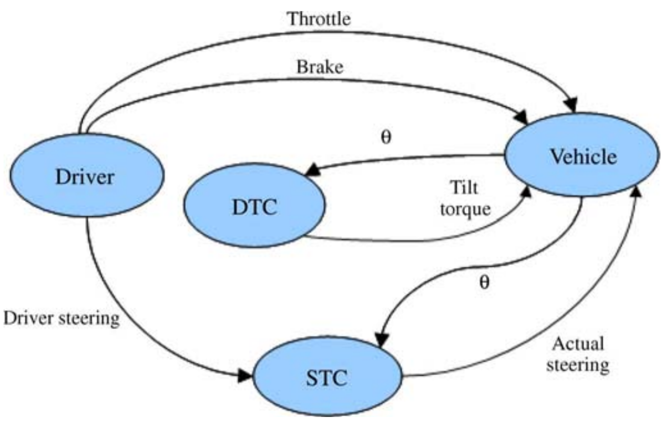


Fig. 5. SDTC—combined system.

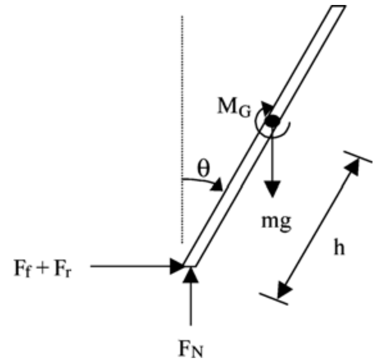


Fig. 6. Bicycle model free body diagram.

implies relies on a direct tilt actuation system to stabilize the tilt mode of the vehicle. The advantage of this control scheme is that it utilizes an additional control input (in addition to the steering input) and makes the control system design a little simpler. Its disadvantage is that the tilt actuator tends to require high torque, especially for high speed transient operations.

On the other hand a control system that emulates the steering maneuvers carried out to stabilize a bicycle or motorcycle is termed STC system. An STC system is a steer-by-wire system where the driver steering input and other measured dynamic quantities are used to calculate the appropriate steering angle to stabilize the tilt of the vehicle while allowing the driver to track his/her trajectory. An STC system does a good job of stabilizing a tilting vehicle at high speed but its performance deteriorates at lower vehicle speeds. Hence if the STC and DTC system can be run in parallel, the STC system can be assisted by the DTC system at low vehicle speed and the DTC system can be assisted by the STC system at higher vehicle speed. Fig. 5 shows a schematic representation of an SDTC (integrated STC/DTC) system.

A. *DTC System*

In a DTC system a tilt actuator mounted on the chassis of the vehicle solely controls the tilt stability of the vehicle. In designing this control system a "desired tilt angle" that stabilizes the tilt mode of the vehicle is first determined. Once this appropriate desired tilt angle is determined, a control law having the form given by (2) can be defined to command the tilt actuator to drive the vehicle to this desired tilt angle

$$T = -k_1(\theta - \theta_{\text{des}}) - k_2\dot{\theta}. \tag{2}$$

A well designed DTC system will have minimal transient torque requirement with zero steady-state torque. Given the control law defined above, the performance of a DTC system is fully governed by the choice of the desired tilt angle. The following paragraphs discuss several choices for the desired tilt angle.

DTC-I: A common way of calculating the desired tilt angle of a leaning vehicle is based on balancing the centrifugal force that occurs during cornering to the weight of the vehicle. In this method it is assumed that the only acceleration experienced by the vehicle during cornering is centripetal acceleration. Under this assumption a fully coordinated turn can be achieved if the resultant of the centrifugal force and the weight of the vehicle is aligned with the body of the vehicle. Under small angle approximations this desired tilt angle is given by (3). This desired tilt angle (θ_{des}) is used by many researchers in designing a DTC system [4], [5], [9]–[11]

$$\theta_{\text{des}} = \frac{\dot{\psi}V}{g} = \frac{V^2}{gR}. \tag{3}$$

This approach simply assumes the sum of the lateral forces is equal to the centripetal acceleration of the vehicle and fails to account for the lateral translational acceleration of the vehicle and also neglects the acceleration due to gyroscopic moments of the rotating wheels. Neglecting the translational acceleration $(\ddot{y})$ results in a significant increase in transient tilt torque requirements while neglecting the acceleration due to gyroscopic moments results in a nonzero steady-state torque requirement.

DTC-II: A much better approximation of the desired tilt angle (θ_{des}) can be achieved by considering all the forces and moments acting on a tilting vehicle during cornering. For this purpose the bicycle model free body diagram presented in Fig. 6 is considered. In this figure, the quantities F_f, F_r, F_N, m, g, h, M_G, and θ are, respectively, the lateral force on front wheels, the lateral force on rear wheel, the normal force from the pavement, the vehicle mass, the gravitational acceleration, the height of the center of gravity of the vehicle, the gyroscopic moment due to wheel rotation and the tilt angle of the vehicle.

Based on this free body diagram the net moment acting at the center of gravity of the vehicle is given by (4). In order for the vehicle to be stable, this net moment should go to zero at steady state. This is the basis for determining the desired tilt angle

[illegible] (4)

[illegible]. (5)

Upon substitution of the forces and moments in (5), linearizing about $\theta = 0$ and dropping higher order terms, the following differential equation in θ results

[illegible] $_{\psi} - I_{wheel\_rot})\omega\dot{\psi}$. (6)

At steady state we wish to have a net zero moment acting at the center of gravity of the vehicle. Hence, we set angular acceleration of the vehicle to zero and solve for the tilt angle θ. The tilt angle so obtained is the desired tilt angle of the vehicle [see (7)]. Even though the lateral translation acceleration $(\ddot{y})$ could be zero during steady-state cornering, it is retained to improve the transient performance of the DTC system

$$\theta_{\mathrm{des}} = \frac{(\ddot{y} + \dot{\psi}V)}{g} - \underbrace{\frac{2(I_{\mathrm{wheel\_}\psi} - I_{\mathrm{wheel\_}rot})\omega\dot{\psi}/(hm)}{g}}_{a_{\mathrm{gyro}}}. \quad (7)$$

This result simply states that, in order for the vehicle to be stable, the vehicle needs to lean in such a way that the resultant of net horizontal accelerations (lateral translation acceleration, centripetal acceleration and acceleration due to gyroscopic moments of rotating wheels) and net vertical acceleration (gravitational acceleration) makes an angle θ_{des} with the vertical. Note that if the gyroscopic term a_{gyro} in (7) were ignored then $g\theta_{\mathrm{des}}$ would be equal to the sum of the centripetal acceleration $\dot{\psi}V$ and the lateral translation acceleration $\ddot{y}$.

Inclusion of the lateral position acceleration term $\ddot{y}$ significantly reduces the transient torque requirement of the tilt actuator. Inclusion of the gyroscopic term a_{gyro} ensures zero steady state tilt effort from the tilt actuator. During implementation, this improvement in tilt actuator requirement can be realized by using a system of accelerometers to read the pure lateral acceleration of the vehicle [the quantity in the numerator of (7)] as opposed to using the yaw rate reading to determine lateral acceleration (*DTC—I*). An accelerometer mounted on the vehicle will read all the acceleration components given in (7) plus the angular acceleration of the vehicle $(\ddot{\theta})$. Care should be taken to filter out the angular acceleration component of the vehicle $(\ddot{\theta})$ which has a destabilizing effect on the system. Since the accelerometer mounted on the vehicle tilts with the vehicle, the lateral acceleration needs to be properly resolved from the z and y acceleration readings of the sensor.

To demonstrate the difference in performance between the *DTC—I* [simple approach, (3)] and the *DTC—II* [*proposed approach*, (7)], computer simulations were carried out for a DTC system based on each of these approaches. The same operating conditions were used for both types of DTC systems. For these simulations the vehicle travels at 5 m/s (11 mi/h) on a straight path for the first 50 s then the driver initiates a left turn of 8 m (26 ft) radius in the form of a slightly smoothed out step input. Note that these are the types of operating conditions envisioned for the DTC system. The STC system will be used at higher vehicle speeds.

The simulation model equations for the simulations have been presented before and can be found in [11]–[13].

Fig. 7 shows the resulting simulation torque requirement comparison between the "simple" (DTC-I) and "proposed" (DTC-II) methods of calculating desired tilt angle. It can be observed that the "proposed approach" results in a significantly reduced transient tilt torque requirement and a zero steady-state tilt torque requirement. The zero steady-state tilt torque requirement for the proposed method shows that the desired tilt angle results in total balance of the lateral force moments by the moment due to the weight of the vehicle.

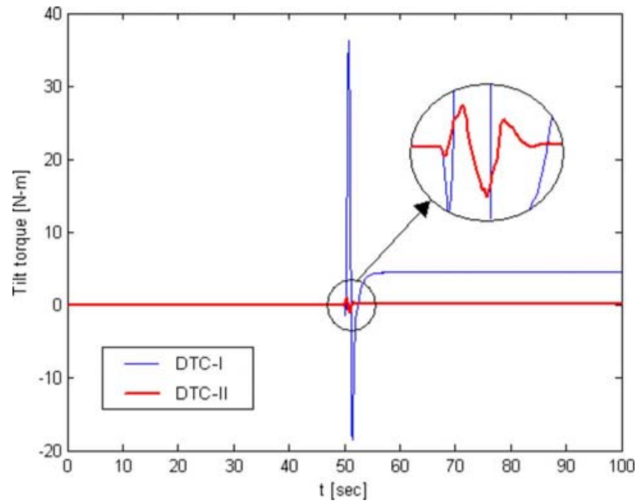


Fig. 7. Tilt actuator torque requirement comparison.

Fig. 8 compares the desired trajectory tracking and tilt angle tracking performances of the "simple" and "proposed" methods. For both methods it is observed that the lateral position error and heading angle error are of comparable magnitude and are within an acceptable range. On the other hand, a nonzero steady-state tilt angle error is observed for the "simple" approach. This nonzero tilt angle error is the cause of the nonzero steady-state torque requirement noted in Fig. 7.

DTC-III: The DTC methods discussed in *I* and *II* will perform adequately if the DTC system is the only tilt stabilizing control input of the vehicle. But a problem will arise if a DTC system based on these desired tilt angles is to be integrated with an STC system. For an STC/DTC integrated system there will be instances when both control systems will be operated at the same time (more will be said about parallel operation of these controllers in later sections). During these instances we would wish for these controllers to exert control effort in the same direction. It turns out a DTC system designed based on Sections I or II exerts a control effort that opposes the control efforts of the STC system during transient maneuvers. In other words the two control systems will try to tilt the vehicle in opposite directions during the transient phase.

The conflict in control efforts will occur when the STC system counter-steers the vehicle to achieve a tilt into the turn. During this period the vehicle will experience a lateral acceleration in the opposite direction of the turn. Since a DTC system based on *a) and b)* uses lateral acceleration to determine the desired tilt direction, at this instance it will try to tilt the vehicle in the opposite direction of the turn, hence destabilizing the system.

A way around this problem is to define the desired tilt angle of the DTC system to be equal to the desired tilt angle defined for the STC system. For STC system the desired tilt angle is defined as being proportional to the driver steering input [see (8)] (this will be discussed in detail in the STC system design section)

$$\theta_{\mathrm{des}} = ks(\mathrm{driver\_input}). \quad (8)$$

This ensures that both the STC and DTC systems exert control efforts to tilt the vehicle to the same tilt angle at all times. A

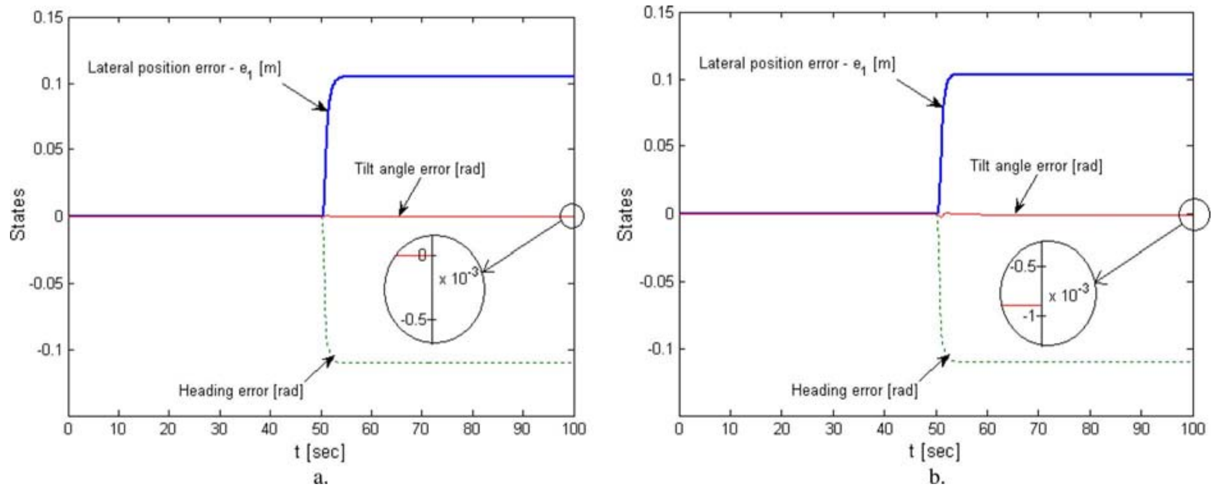


Fig. 8. Lateral position, heading, and tilt angle errors. a. DTC-II; b. DTC-I.

slight disadvantage of this approach is that the transient torque requirement of the DTC system is no longer optimized.

DTC Summary: If tilt actuator size is not an issue, a stand alone DTC system can be used as the only control system in stabilizing an NTV. In such cases the proposed *DTC—II* system should be used to ensure efficient utilization of the actuation effort. On the other hand if there is a need to minimize the tilt actuator as much as possible then an integrated DTC + STC (SDTC) system should be used for tilt stability of a tilting vehicle. For an integrated system, *DTC—III* control scheme should be used since it ensures conflict free operation with an STC system.

B. STC System

Before going into the details of STC systems, it is important to look at the steady state steering requirement of an NTV for a perfectly coordinated turn. The steady-state steering needed to negotiate a certain yaw rate will have the form given by (9). This equation is derived from relationship of steady state steering input to front and rear wheel slip angles of a vehicle. For detailed derivation of this equation please refer to authors' prior work in [12]

$$\delta_{ss} = \frac{L}{R} + \frac{V^2 m}{LR}\left(\frac{L_r}{2C_f} - \frac{L_f}{C_r}\right) + \theta_{ss}\left(\frac{\lambda_r}{C_r} - \frac{\lambda_f}{C_f}\right). \quad (9)$$

It can be observed that the steady-state front wheel steering angle is a function of the yaw rate (V/R) and the tilt angle of the vehicle. From this relation it can also be inferred that, in order to achieve a certain steady state tilt angle under a certain yaw rate (fixed V and R), a unique steady state steering input is required.

It is interesting to note how a bicycle rider negotiates curves without tipping over. While making the turn, the rider also simultaneously leans the bicycle into the curve at the right angle. Making a turn and leaning into the turn demand opposing steering inputs; yet the rider is able to accomplish these tasks by using only the steering input. The opposing requirement of these two actions is explained by an example of a bicycle rider making a right turn. In order to make a right turn the rider needs to generate lateral forces directed into the instantaneous center of rotation of the turn. The slip angles of the wheels that result in these lateral forces are achieved by steering the front wheels

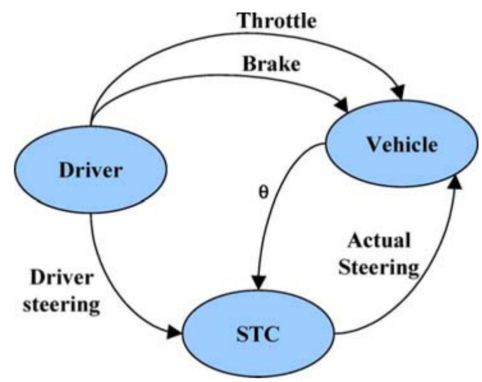


Fig. 9. Vehicle control system based on STC system.

of the bicycle to the right. On the other hand, the only way the rider can generate a titling moment to the right is if he/she is able to generate lateral forces that are directed in the opposite direction of the center of rotation. The slip angles that result in these lateral forces require the wheels to be steered left. From the above discussion it is apparent that an STC system designed by simply superimposing the stability requirement of the lateral dynamics (trajectory tracking), and the stability requirements of the tilt dynamics is not going to stabilize the vehicle. This has been shown by Gohl *et al.* in [13].

A clever way of solving the above control problem is to use the driver's steering input to generate the desired tilt angle of the vehicle [13]. In this type of control scheme, the driver steering input is no longer directly linked to the steering mechanism. Rather, the control system takes this driver input and appropriately scales it to determine the desired tilt angle. Once the desired tilt angle is determined, the controller (i.e., called STC in Fig. 9) calculates the actual steering input needed to generate (i.e., track) that desired tilt angle. Fig. 9 shows the schematic representation of the entire STC system.

Based on the discussion in the previous paragraph, the desired tilt angle equals to a scaling factor ks times the driver input [see (10)]

$$\theta_{\text{des}} = ks(\text{driver\_input}). \quad (10)$$

Using this desired tilt angle, an STC law having the form of (11) can be defined. This control law is designed to drive the vehicle to the desired tilt angle. The rational behind this control law is based on the fact that a tilted vehicle must undergo a certain yaw rate to stay stable at that tilted configuration. Hence by just tracking a certain desired tilt angle derived from the driver input we are able to generate a yaw rate that enables the driver to track his/her desired trajectory. The exact yaw rate generated for a certain driver input and vehicle velocity (the sharpness of the radius followed) depends on the scaling factor ks. This scaling factor can be experimentally tuned to suit the driver's vehicle handling preference

$$\text{[illegible]} \underbrace{\text{[illegible]}}_{a} + \underbrace{\delta_{ss}}_{b}. \tag{11}$$

It is worthwhile to discuss how this controller works in more detail. At the first initiation of a turn by the driver, the tilt controller portion of (11) (indicated by "a") will be more dominant and steers the vehicle in the opposite direction of the intended turn—termed *counter steering*. This will cause the vehicle to tilt in the direction of the intended turn. When the vehicle approaches the desired tilt angle, the "a" portion of the controller will decrease and the "b" portion of the controller becomes more dominant. The "b" portion of the controller takes care of the steady state steering needed to complete the steady-state portion of the turn. In order to eliminate the steady-state tilt angle error (i.e., achieve a perfectly coordinated turn) the steady state steering term "b" needs to be calculated based on (9). For a certain desired tilt angle θ_{des} and desired vehicle yaw rate ($\dot{\psi} = V/R$) (9) can be rewritten as follows:

$$\delta_{ss} = \dot{\psi}\left[\frac{L}{V} + \frac{Vm}{L}\left(\frac{L_r}{2C_f} - \frac{L_f}{C_r}\right)\right] + \theta_{\text{des}}\left(\frac{\lambda_r}{C_r} - \frac{\lambda_f}{C_f}\right). \tag{12}$$

If all the parameters of the vehicle in (12) were known exactly, (12) could have been directly substituted for the feedforward term "b" in the control law given by (11). But this is nearly impossible since some of the above parameters are dependent on driving conditions, weight of the driver, ..., etc., and vary continuously. A steady-state steering input calculated with inaccurate parameters will result in a nonzero steady-state vehicle tilt angle error. This in turn will result in an undesirable nonzero steady-state tilt torque requirement if a DTC system is used in parallel.

The following section analyzes different STC system designs based on different steady-state steering input strategies δ_{ss} in order to deal with this problem.

1) STC—I: For a perfectly coordinated turn there exists a relationship between the steady state yaw rate of the vehicle and the steady state tilt angle of the vehicle. This was determined in the DTC system design section [see (7)]. In this equation, setting the lateral translation acceleration to zero and substituting $\mathrm{V/r_{wheel}}$ for the angular rotation of the wheels and rearranging, yields the following relation:

$$\theta_{\text{des}} = \dot{\psi}V\frac{\left(hm - \frac{2(I_{\text{wheel\_}\psi} - I_{\text{wheel\_rot}})}{r_{\text{wheel}}}\right)}{hmg}. \tag{13}$$

Solving for the yaw rate from the previous equation and substituting in (12) results in steady-state steering input given by (14). In this equation "c" and "d" are lumped parameters that are made up of purely vehicle based parameters. Equation (15) captures these terms in a simpler arrangement based on the terms "c" and "d"

$$\delta_{ss} = \left[\frac{1}{V^2}\underbrace{\left[\frac{Lhmg}{hm - \frac{2(I_{\text{wheel\_}\psi} - I_{\text{wheel\_rot}})}{r_{\text{wheel}}}}\right]}_{c} + \underbrace{\left[\frac{hmg\left(\frac{L_r}{2C_f} - \frac{L_f}{C_r}\right)}{L\left(h - \frac{2(I_{\text{wheel\_}\psi} - I_{\text{wheel\_rot}})}{mr_{\text{wheel}}}\right)} + \left(\frac{\lambda_r}{C_r} - \frac{\lambda_f}{C_f}\right)\right]}_{d}\right] \times \theta_{\text{des}} \tag{14}$$

$$\delta_{ss} = \left(\frac{c}{V^2} + d\right)\theta_{\text{des}}. \tag{15}$$

Substituting the above steady-state steering in the control law given in (11) gives the complete form of the steering control input [see (16)]. In this control law, since the steady-state steering is purely a function of the desired tilt angle, which in turn is proportional to the driver's steering input, the steady state steering is fully dictated by the driver input. This has the added benefit of giving the driver a more natural vehicle handling experience

$$\text{[illegible]} \frac{\text{[illegible]}}{V^2} + \hat{d}\Big)\theta_{\text{des}}. \tag{16}$$

In (16), estimates of the parameters $\hat{c}$ and $\hat{d}$ are used instead of the actual parameters "c" and "d" since the values of these lumped parameters will not be known exactly. Theoretically an adaptation law can be defined for these parameters based on a zero steady state tilt angle error criterion. The problem is we only have one adaptation criterion and two parameters to estimate. If the road curvature provided adequate excitation it would be possible to determine both parameters.

2) STC—II: Another way of eliminating the steady state tilt angle error is to incorporate an integral component in the control law suggested by (11)

$$\text{[illegible]} \underbrace{\text{[illegible]}}_{a} + \underbrace{k_i\int(\theta - \theta_{\text{des}})dt}_{b}. \tag{17}$$

This ensures that the tilt angle error goes to zero at steady state resulting in the right steady-state steering input without requiring knowledge of any vehicle parameters. Since the steady-state steering is now a function of the desired tilt angle (a measure of the driver steering input) the problem associated with STC—II mentioned above is avoided.

Effect of Speed Variation on STC Performance: Based on (11), at steady state a certain steering input by the driver corresponds to a certain tilt angle of the vehicle. For a coordinated turn, neglecting the effect of road bank angle and gyroscopic moments, and making small angle approximations the tilt angle of the vehicle at steady state is equal to the lateral acceleration of the vehicle in g's [see (18)]

$$\theta = \frac{a_{\text{lat}}}{g} = \frac{\dot{\psi} V}{g} = \frac{V^2}{gR}. \tag{18}$$

Therefore if the vehicle velocity increases while going on a turn at a fixed tilt angle (fixed lateral acceleration) the radius of curvature needs to increase to maintain the same lateral acceleration. In other words if the vehicle accelerates while going around a turn the vehicle will start going on a wider turn and the driver will need to steer sharper (lean the vehicle further) to maintain the original radius of curvature. This phenomenon is analogous to the behavior of an under-steered car. The fact that the STC system results in an under steered NTV handling is a desired characteristic since most people are used to driving under-steered cars.

If need be, a neutral-steering STC system that gives a fixed radius of curvature for a fixed steering input regardless of the vehicle speed can also be designed. This can be done by modifying the desired tilt angle definition according to (19)

$$\theta_{\text{des}} = ks(\text{Driver\_input})V^2. \tag{19}$$

At steady state the vehicle tilt will be equal to the desired tilt angle. Therefore

$$\theta_{\text{des}} = ks(\text{Driver\_input})V^2 = \frac{V^2}{gR}. \tag{20}$$

The V^2 term will cancel out from both sides of (20) and the driver input will be inversely proportional to the radius of curvature of the turn. In this new definition of the desired tilt angle, for the same driver steering input, acceleration/deceleration of the vehicle will respectively result in an increase/decrease in the tilt angle of the vehicle and increase/decrease in the yaw rate of the vehicle without affecting the radius of curvature being followed by the driver. If the desired tilt angle is defined only as a function of the velocity ($\theta_{\text{des}} = ks(\text{Driver\_input})V$), the vehicle will have a fixed yaw rate corresponding to a fixed driver's input regardless of the speed of the vehicle.

C. Tilt Brake System

The last component of the integrated control scheme is the Tilt brake system. At very low speed, since the lateral acceleration is very low there is no danger of vehicle rollover. Hence the vehicle can be locked in the upright position. This will eliminate tilt actuator torque requirement that would have been needed to track a nonzero desired tilt angle. Besides this advantage, a tilt lock will restrict unnecessary tilting of the vehicle and make some maneuvers such as parking easier for the driver.

The tilt brake system used on the second-generation vehicle utilizes electromagnetic brakes that can be turned on or turned off using analog reference voltage. The Tilt brake algorithm, in its simplest form is: *lock tilt brake below* $V_{\min}$ *and unlock tilt brake above* $V_{\min}$. When the vehicle is in the locked configuration the driver's steering input will be directly passed to the steered wheels. Once the tilt brake is disengaged the steering will be governed by the STC control law.

IV. Experiments

The proposed compound stability control algorithm was implemented on the second generation NTV built by the research group at the University of Minnesota. The vehicle is instrumented with a system of three actuators (tilt servomotor, steering servomotor, and tilt brake electromagnets) and five sensors (two incremental encoders for measuring the steering angle and tilt angle of the vehicle, one absolute encoder for measuring driver's steering input, one hall effect sensor for measuring speed of the vehicle and a XBow IMU.

In the experimentation phase of the project the overall performance of the SDTC algorithm was verified by carrying out standard vehicle maneuvers such as lane changes and turns. For the experiments the combined SDTC system was designed in such a way that the STC and DTC systems run in parallel for all speeds of the vehicle. The SDTC system was supplemented by the tilt brake system for vehicle speed lower than 4 mi/h (1.8 m/s).

A. SDTC Experiments

To validate the proposed SDTC system under different vehicle maneuvers and operating speeds two sets of experiments were designed. The first set of experiments was designed to show the tilt stability of the system and capability of the driver to follow his/her desired trajectory for different driving paths. For this purpose standard vehicle maneuvers such as lane change maneuvers and turning maneuvers were chosen. A second set of experiments was also carried out to gauge the performance of the tilt brake algorithm during low speed acceleration and deceleration phases of the vehicle.

1) Turning Maneuver: Turning maneuver experiments were performed to simulate a vehicle making a right turn at an intersection. For the turning maneuver experiment, the vehicle starts from rest and travels on a straight path for the first 5 s while accelerating. At 5 s a ramp steering input is introduced. The steering input ramp increases up to 10 s and stays constant for the rest of the trajectory. The desired tilt angle for this experiment is defined to be equal to the steering input of the vehicle (both measured in radians). This corresponds to the proportionality constant ks in (8) being equal to one. The desired tilt angle tracking for this experiment is plotted in Fig. 10. It can be observed that the desired tilt angle is tracked very accurately. In this experiment the vehicles tilts to almost 15° and is seen to stay stable at that tilt angle. The lateral acceleration corresponding to this tilt is about 2.6 m/s^2. This experiment was repeated a number of times with different vehicle velocity profiles to ensure repeatability of the tracking accuracy. For each experiment the vehicle was able to track the desired tilt angle with comparable degree of accuracy.

Tracking a desired tilt angle accurately does not necessarily imply accurate desired trajectory tracking. Hence the next question to ask is, was the driver able to generate the desired turn during the above maneuver? Fig. 11 shows a plot of the driver steering input and the yaw rate generated as measured by the XBow IMU.

It can be observed that the yaw rate of the vehicle increased in phase with the driver's steering input showing the driver is generating the desired turn. Another interesting observation is, the yaw rate of the vehicle still keeps increasing even though the

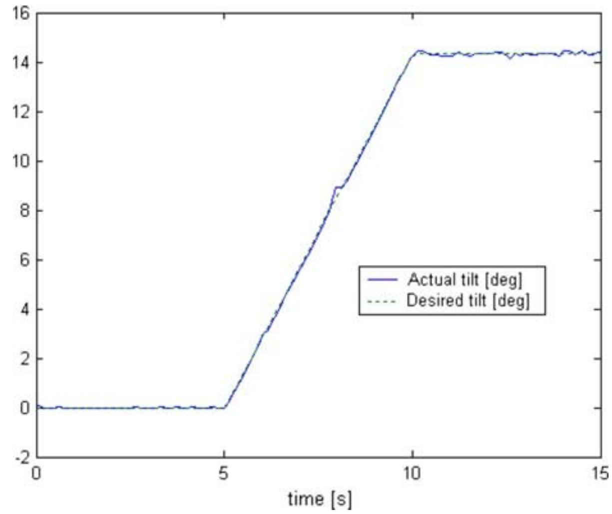


Fig. 10. Desired tilt angle tracking.

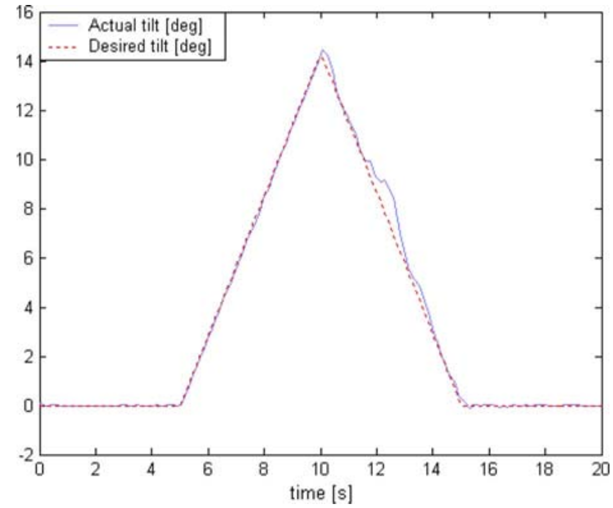


Fig. 12. Desired tilt angle tracking.

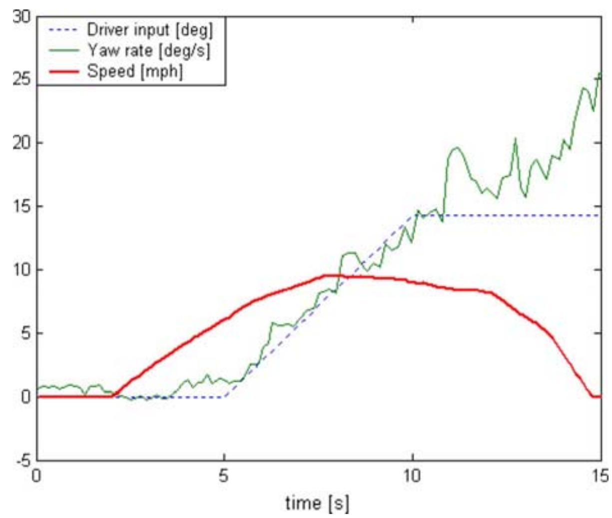


Fig. 11. Yaw rate of vehicle.

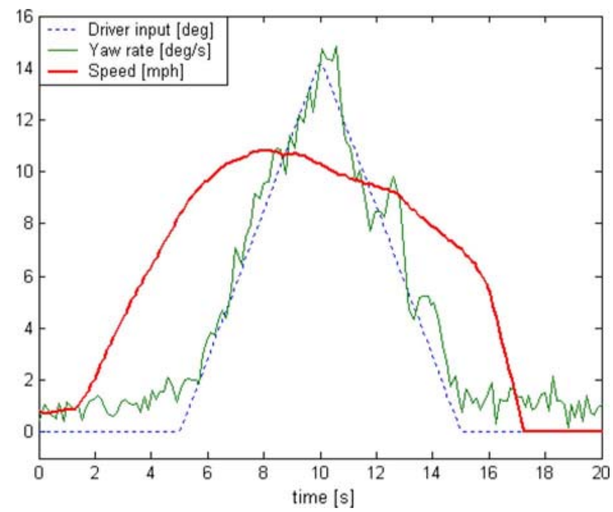


Fig. 13. Yaw rate of vehicle.

driver steering input stays constant after 10 s. This happens due to the drop in the speed of the vehicle. Increase in the yaw rate while the speed of the vehicle drops implies the radius of curvature of the turn is also decreasing. This is the under-steering characteristic discussed in the control system section.

2) Lane Change Maneuver: A lane change maneuver is another common maneuver carried out by vehicles on the road. To simulate this maneuver on the NTV, the vehicle was driven on a straight path for the first 5 s. At 5 s a ramp driver steering input is introduced that increases up to 10 s. At 10 s the driver input ramp is reversed and the steering input is decreased until it reaches zero (which will correspond to 15 s). The driver input is maintained at zero for the rest of the trajectory. The desired tilt angle for this experiment is defined to be equal to the steering input of the vehicle (both measured in radians). The desired tilt angle tracking for this experiment is plotted in Fig. 12. Similar to the turn maneuver experiments, the lane change maneuver experiments also show very accurate desired tilt angle tracking. The maximum tilt of the vehicle in this experiment (14°) corresponds to about 2.45 m/s^2.

Fig. 13 shows a plot of the yaw rate generated and the speed of the vehicle along the desired tilt angle. It can be observed the driver is achieving a yaw rate in the same direction and proportional to his/her steering input.

3) Tilt Brake Experiments: Once the general stability and tracking performance of the control algorithm is established some more experiments were carried out to gauge the low speed performance of the vehicle. The important aspect of the low speed operation is the engaging and disengaging of the tilt brake without disrupting the tilt stability of the vehicle or trajectory tracking ability of the driver.

For these experiments the tilt brake switch speed $V_{\min}$ was set to 4 mi/h (1.8 m/s). Depending on the steering input of the driver, the vehicle could either be undergoing zero lateral acceleration (going straight) or undergoing lateral acceleration (turning) when the 4 mi/h (1.8 m/s) threshold is crossed during acceleration of deceleration. To capture these scenarios, a sinusoidal driver steering input that varies as a function of time was defined. The vehicle is considered to be going "straight" during the instances the driver steering input is close to zero, since at

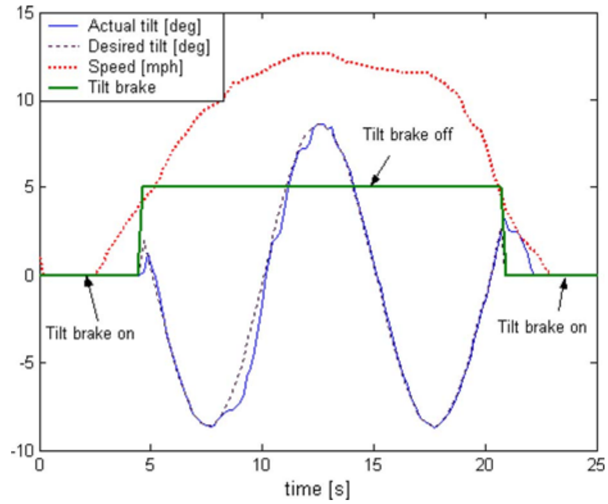


Fig. 14. Desired tilt angle tracking and tilt brake status.

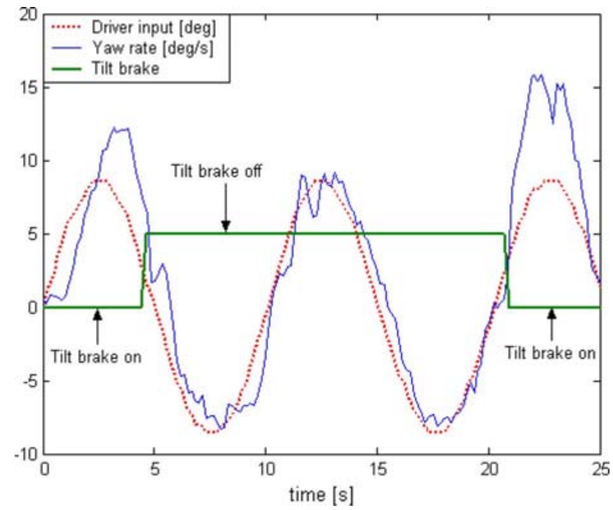


Fig. 15. Yaw rate of vehicle and tilt brake status.

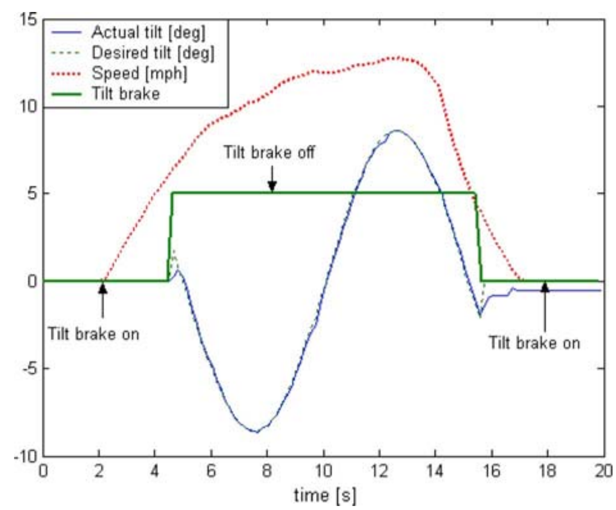


Fig. 16. Desired tilt angle tracking and tilt brake status.

this instance the desired tilt is zero and there is no yaw rate generated. For this experiment, any steering input less than $|2.5°|$ is considered as going "straight". When the vehicle is decelerating past 4 mi/h (1.8 m/s) a tilt angle less than $|2.5°|$ is considered as "zero" tilt.

a) Tilt brake experiment I—Common case: This experiment represents the simple case where the vehicle is going on a straight path when the vehicle accelerates past the 4 mi/h (1.8 m/s) mark. The following figure shows the desired tilt angle tracking performance and the disengaging and engaging of the tilt brake. It can be observe that the tilt brake is released at about 5 s when the vehicle reaches 4 mi/h (1.8 m/s). At the instance the vehicle reaches 4 mi/h (1.8 m/s) the tilt angle is smaller than 2.5°, there fore the vehicle is considered to be going on a "straight" path and tilt brake is released immediately.

During deceleration, when the vehicle reaches 4 mi/h (1.8 m/s) again, the tilt angle of the vehicle happens to be less than 2.5°. At this instance the tilt brake control system engages the tilt brakes. In the above experiment it can be observed that the operation of the tilt brake does not disrupt the accurate desired tilt angle tracking of the vehicle.

Fig. 15 shows plots of the driver steering input and the tilt brake status alongside the yaw rate of the vehicle for the above experiment. The yaw rate of the vehicle is seen to be in phase with the driver steering input regardless of the status of the tilt brake. This shows that the engaging and disengaging of the tilt brake is not disrupting the desired trajectory capability of the driver. The increase in the yaw rate observed at the end of the trajectory is due to the under steering characteristic discussed earlier associated with drop in vehicle speed.

b) Tilt brake experiment II—General case: The second set of experiments were designed to show the performance of the tilt brake when the vehicle is already undergoing a turn when it accelerates past the 4 mi/h (1.8 m/s) mark. Fig. 16 shows the desired tilt angle tracking performance and the disengaging and engaging of the tilt brake for this experiment. It can be observed that the tilt brake release does not occur at 4 mi/h (1.8 m/s) but is postponed until about 6 mi/h at which point the vehicle is close enough to the zero tilt position.

During deceleration, when the vehicle reaches 4 mi/h (1.8 m/s) again, since the tilt of the vehicle is higher than the "zero" tilt threshold, the desired tilt angle is reset to zero. Shortly after this the vehicle tilt angle gets close enough to the zero tilt position and the tilt brake is engaged. In the above experiment it can also be observed that the operation of the tilt brake does not disrupt the accurate desired tilt angle tracking of the vehicle.

Fig. 17 shows plots of the driver steering input and the tilt brake status alongside the yaw rate of the vehicle for the same experiment. Here also it can be observed that the driver's yaw rate intentions are not disrupted by the operation of the tilt brakes.

4) Neutral Steering: In the first set of experiments it was noted that the NTV handled like an under steered vehicle, i.e., the turn got tighter with drop in vehicle speed for a fixed driver input (constant lateral acceleration for a constant steering input). It was discussed in previous sections it is possible to modify the desired tilt angle to obtain a neutral steering behavior in the NTV. Numerous experiments were carried out with the desired tilt angle of the vehicle modified as per (19) to investigate this expected neutral steering behavior.

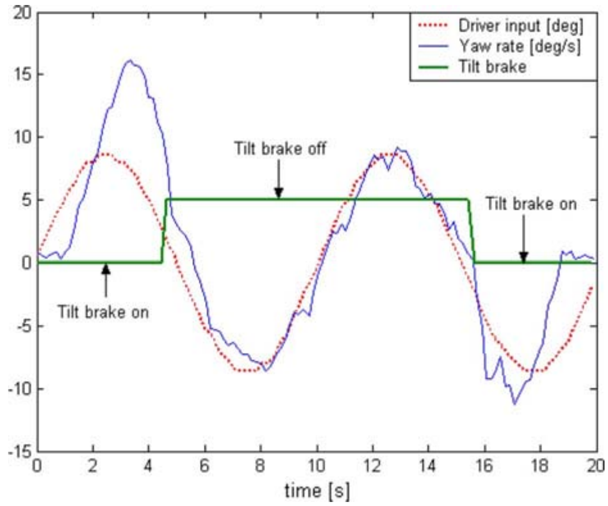


Fig. 17. Yaw rate of vehicle and tilt brake status.

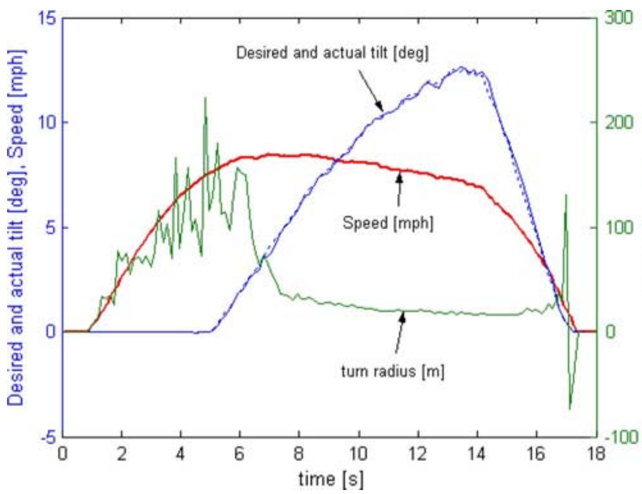


Fig. 18. Turn radius.

For these experiments the vehicle desired trajectory was defined in the same way as for the turn maneuver experiment discussed at the beginning of this section. Fig. 18 shows the experiment results run with the "neutral steering" desired tilt angle definition. In this plot the radius of curvature of the turn is plotted alongside the speed and tilt angle tracking curves. The radius of curvature of the vehicle was calculated by dividing the velocity of the vehicle obtained from the speed observer by the yaw rate readings from the XBow IMU. The yaw rate obtained from the XBow IMU was noisy in nature which resulted in a noisy calculated *turn radius* curve. During the period where the driver's steering input is constant $(t > 10\text{ s})$, it can be observed that the turn radius remains somewhat constant around 66 ft (20 m) even though the speed of the vehicle drops from about 8 to 2 mi/h. Hence it can be concluded that a neutral steering behavior was successfully achieved for the NTV with the modified desired tilt angle definition.

B. DTC Experiments ("Free Steering" DTC)

This section illustrates how the DTC control system can be used to achieve both stable tilt and lateral control. While the STC system can clearly control the tilt angle at a desired value for any steering input from the driver, the DTC system also can control the tilt angle at any desired value and create a corresponding stable steering angle. The DTC system is inherently stable and it is far easier to design than the STC control system. Of course, the DTC system requires a tilt actuator while the STC system only needs steer-by-wire operation.

Three types of DTC systems were described in the *Control System* section. Of the three DTC control systems, we were able to successfully implement a standalone DTC-III stability system. To implement the DTC-III system as a standalone control algorithm, it was operated in a "Free Steering" mode. In this version of DTC-III system, the driver's steering input is interpreted as a certain desired tilt angle of the vehicle [see (2)]. This is very similar to an STC system except that there is no steering control input to the steered wheels from the STC controller or the driver. The front wheels are "free" and are steered by the natural forces from the pavement. In this control system, a certain left steering input by the driver results in the tilt actuator tilting the vehicle to the left. When the vehicle starts leaning to the left the wheels are steered to the left due to the combined effect of the normal force of the wheels and positive trail. This same phenomenon can be demonstrated on a bicycle. If one grabs a bicycle by its horizontal bar and leans it to one side the front wheels immediately steer in the direction of the tilt. In the "Free steering" DTC system, when the tilt motor starts leaning the vehicle the wheels will automatically steer towards the direction of tilt creating lateral force that initiates a turn. The final steering angle of the front wheels is determined by the balance of moments due to the normal force of the pavement and lateral force causing the turn. The forces acting on the front wheels that are responsible for the steering dynamics of the wheels are discussed at the end of this section.

A series of experiments were carried out with the DTC system described in the above paragraph. For all the experiments the vehicle was driven on a straight path for the first 5 s. At 5 s a ramp driver steering input is introduced that increases up to 10 s. At 10 s the driver input ramp is reversed and the steering input is decreased until it reaches zero (which corresponds to 15 s). The driver input is maintained at zero for the rest of the trajectory. Two experiment results that were carried out for two different steering input profiles are shown in Fig. 19(a) and (b) below (the experiments are referred to as "a" and "b", respectively).

The above two experiment plots are presented here to show the general trend observed in the performance of the "Free steering" DTC system. These experiments were carried out under similar experiment conditions except for the maximum desired tilt angle. In experiment "a" the maximum steering input corresponds to a maximum tilt angle of 10°, while in experiment "b" the maximum steering input corresponds to a maximum tilt angle of 11°.

The desired tilt angle tracking performance of the "Free steering" DTC system in experiment "a" is observed to be very accurate with anticipated vehicle yaw rate. While in experiment "b" the desired tilt angle tracking breaks down after 9 s followed by some oscillation. This result was observed for all experiments with desired tilt angle exceeding approximately 10°; while for experiments with desired tilt angle below ap-

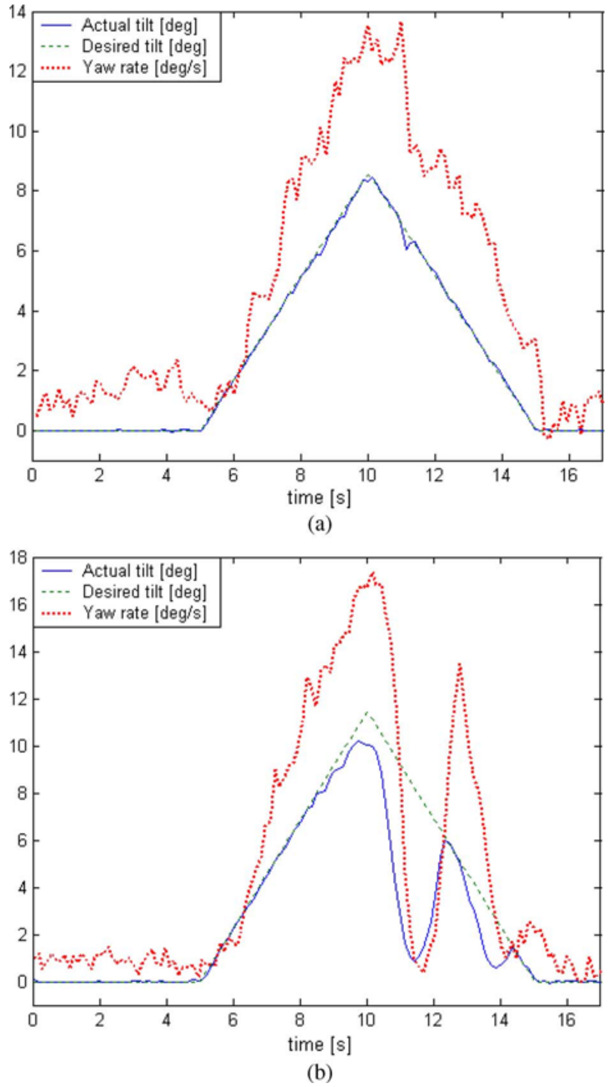


Fig. 19. "Free steering" DTC. (a) Desired tilt less than 10°. (b) desired tilt greater than 10°.

proximately 10° very accurate desired tilt angle tracking was achieved. The 10° tilt angle limit was observed to slightly vary depending on the operating speed of the vehicle.

To explain this limitation of the "Free steering" DTC system at higher vehicle tilt angles, a steady-state analysis of the tire forces acting on the steered wheels were carried out. The two main forces acting on the steered tires are the normal reaction of the pavement F_{Nf} and the lateral tire forces F_f. The gyroscopic forces can be neglected since the tilt rate of the vehicle is small compared to the rotational speed of the wheels and the yaw rate of the vehicle (which further more is zero at steady state). These two forces and their corresponding components are shown in Fig. 20.

The component of the normal force F_{Nf} perpendicular to the wheel plane is responsible for steering the front wheels into the turn $(F_{Nf}\sin\theta)$, and the normal component of the lateral force $F_f(F_f\cos\theta)$ is responsible for straightening the front wheels. When the DTC system first starts to tilt the vehicle, the only force acting on the wheels is the normal force which steers the front wheels into the turn. At this point lateral force start to

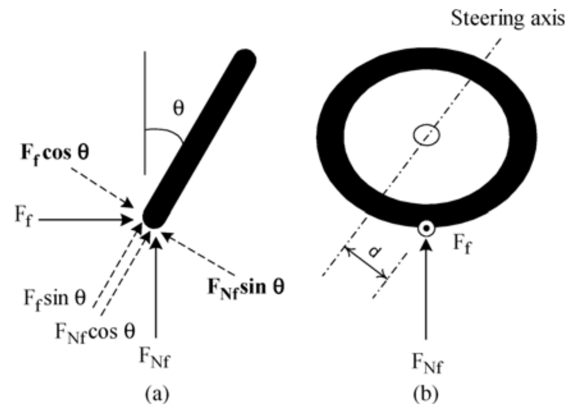


Fig. 20. Forces acting on a tilted tire. a. Front view; b. side view.

build and the vehicle starts to go on a turn. The front wheels will be steered into the turn until the moment equilibrium due to normal and lateral forces is satisfied. Equation (21) shows this equilibrium after canceling out the moment arm term d. Here the coupling between the steering and tilt degree of freedom is ignored and the front wheels' tilt angle is assumed to be equal to the vehicle tilt angle. In reality the front wheels are tilted more than the vehicle tilt angle for any nonzero steering angle, this more general case is discussed later

$$F_f \cos\theta = F_{Nf} \sin\theta. \tag{21}$$

At steady state, the lateral force on the front tires can be rewritten as (22) where m_f is the mass fraction of the vehicle carried by the front wheels and a_{lat} is the lateral acceleration of the vehicle

$$F_f = m_f a_{\text{lat}}. \tag{22}$$

Substituting for the lateral force from (22) and the term $m_f g$ for the normal force in (21) yields (23)

$$m_f a_{\text{lat}} \cos\theta = m_f g \sin\theta \Rightarrow a_{\text{lat}} = g \tan\theta. \tag{23}$$

This is an interesting result because (23) is the condition for a perfectly coordinated turn. For a lateral acceleration of a_{lat} the vehicle needs to tilt to an angle of $\tan^{-1}(a_{\text{lat}}/g)$ to undergo a coordinated turn. There fore if we assume the front wheel tilt angle is the same as the vehicle tilt angle, the final steering angle on the front wheels corresponding to the moment balance on the front wheels results in just the right magnitude of lateral acceleration that balances the vehicle at the tilt angle θ.

Unfortunately the tilt angle of the front wheels is different from the tilt angle of the vehicle due to the steering degree of freedom. The actual tilt angle of the front wheels θ' as a function of the steering angle δ and the rake angle ε is given by (24) [14]

$$\theta' = \theta + \delta \sin\varepsilon. \tag{24}$$

Using the actual tilt angle of the front wheels in (24)

$$\tan(\theta + \delta\sin\varepsilon) = \frac{a_{\text{lat}}}{g}. \tag{25}$$

It can be observed that as the steering angle increases the lateral acceleration generated grows more than the tilt angle of the vehicle by $\delta sin(\varepsilon)$ amount for small tangent angles. Since the steering angle of the front wheels increases with the tilt of the vehicle it can be concluded that the lateral force/acceleration grows at a higher rate than the tilt angle of the vehicle.

The "Free steering" DTC results discussed earlier are in good agreement with this analytical observation. For small vehicle tilt angles the front steering angle is relatively small resulting in θ' being approximately equal to θ. For these small angles accurate tracking of the desired tilt angle is achieved. But when the tilt of the vehicle is increased further (more than 10° in the experiments) the tilt angle of the front wheels starts to be significantly higher than the tilt angle of the vehicle resulting in a higher lateral forces and causing the tilt motor to saturate. The direction of the tilt torque generated by the motor at the saturation point is into the turn, i.e. trying to keep the vehicle from coming back to the upright position. This is another indication that the tilting moment due to lateral forces is exceeding the tilting moment due to the weight of the vehicle.

In conclusion, the "Free steering" DTC system shows good performance for small tilt angles but the performance deteriorates for higher vehicle tilt angles. If hardware modifications such as reducing the trail can be made to reduce the difference between the tilt angle of the vehicle and the tilt angle of the front wheels, "Free steering" DTC system might be a possible control system for stabilizing an NTV.

V. Conclusion

A compound control system composed of DTC, STC, and tilt brake was developed for stabilizing the tilt mode of a tilting vehicle. For this purpose a DTC system that can be easily integrated with an STC system was proposed. An improved STC system was also developed that ensured zero steady-state tilt angle tracking error.

The compound control system (SDTC) was experimentally verified by running standard vehicle maneuvers such as turns and lane changes. The tilt brake algorithm was also verified by running different possible decelerating and accelerating scenarios. Different vehicle handling characteristics such as under steering and neutral steering were successfully achieved by defining the desired tilt angle in different forms. Finally a stand alone "Free Steering" DTC system was experimentally investigated and its performance was theoretically analyzed.

References

[1] Texas Transportation Institute, "Texas transportation institute report, urban mobility study," 2001. [Online]. Available: http://www.mobility.tamu.edu

[2] BTS—United States Department of Transportation, Washington, DC, "Behavioral distinctions: The use of light-duty trucks and passenger cars," 2000. [Online]. Available: http://www.bts.gov/publications/journal_of_transportation_and_statistics/

[3] Volkswagen, U.K., "Volkswagen reveals world's most economical road car," 2002. [Online]. Available: http://www.volkswagen.co.uk/new_devs/one_litre

[4] R. Hibbard and D. Karnopp, "Twenty first century transportation system solutions—A new type of small, relatively tall and narrow active tilting commuter vehicle," *Veh. Syst. Dyn.*, vol. 25, pp. 321–347, 1996.

[5] A. Van den Brink, "Slender comfort vehicles—The forgotten challenge," 2002. [Online]. Available: http://www.brinkdynamics.com/_downloads/

[6] J. P. Pauwelussen, "The dynamic behavior of man-wide vehicle with an automatic active tilting mechanism," in *Proc. EAEC—Veh. Syst. Technol. for the Next Century*, Barcelona, 1999, pp. 50–58.

[7] C. R. Van den Brink and H. M. Kroonen, "Slender comfort vehicles: Offering the best of both worlds," in *Proc. Auto Technol.*, 2004., pp. 56–59.

[8] H. M. Kroonen and C. R. van den Brink, "DVCTM—The banking technology driving the CARVER vehicle class," presented at the AVEC, Anhem, The Netherlands, 2004.

[9] D. Karnopp and R. Hibbard, "Optimum roll angle behavior for tilting ground vehicle," in *Proc. DSC*, 1992, vol. 44, pp. 29–37.

[10] S.-G. So and D. Karnopp, "Methods of controlling the lean angle of tilting vehicles," in *Proc. DSC*, 1993, vol. 52, pp. 311–319.

[11] R. Rajamani, J. Gohl, L. Alexander, and P. Starr, "Dynamics of narrow tilting vehicles," *Math. Comput. Model. Dyn. Syst.*, vol. 9, no. 2, pp. 209–231, 2003.

[12] S. Kidane, L. Alexander, R. Rajamani, P. Starr, and M. Donath, "Road bank angle consideration in modeling and tilt stability controller design for narrow commuter vehicles," presented at the Amer. Control Conf., Minneapolis, MN, Jun. 2006.

[13] J. Gohl, R. Rajamani, L. Alexander, and P. Starr, "Active roll mode control implementation on a narrow tilting vehicle," *Veh. Syst. Dyn.*, vol. 42, no. 5, pp. 347–373, Nov. 2004.

[14] V. Cossalter, *Motorcycle Dynamics*, 1st ed. Greendale, WI: Race Dynamics, Sep. 2002.

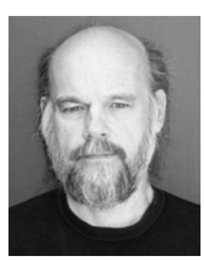
Samuel Kidane received the B.Sc. degree from Addis Ababa University, Ethiopia, in 1997, the M.S. from Louisiana State University, Baton Rouge, in 2002, and the Ph.D. degree from University of Minnesota, Minneapolis, in 2006, all in mechanical engineering.

He worked on structural analysis of advanced grid stiffened composite panels during his Masters studies; while his Ph.D. research emphasis was on the dynamics and control of narrow tilting vehicles. Currently, he is a Process Engineer with the Corporate Research Labs, 3M, Saint Paul, MN.

Rajesh Rajamani received the M.S. and Ph.D. degrees from the University of California at Berkeley, in 1991 and 1993, respectively, and the B.Tech. degree from the Indian Institute of Technology, Madras, India, in 1989.

He is currently a Professor with the Department of Mechanical Engineering, University of Minnesota, Minneapolis. His active research interests include sensors and control systems for automotive and biomedical applications. He has authored over 70 journal publications and received 4 patents. He is the author of *Vehicle Dynamics and Control* (Springer Verlag, 2005).

Dr. Rajamani has served as Chair of the IEEE Technical Committee on Automotive Control and on the editorial boards of the IEEE Transactions on Control Systems Technology and the IEEE/ASME Transactions on Mechatronics. Among several honors, he was a recipient of the CAREER Award from the National Science Foundation, the 2001 Outstanding Paper Award from the journal IEEE Transactions on Control Systems Technology, and the 2007 O. Hugo Schuck Award from the American Automatic Control Council.



Lee Alexander received the Bachelor of Mechanical Engineering and the M.S.M.E degrees from the University of Minnesota, Minneapolis, in 1996 and 1999, respectively.

Since 1999, he has been a Research Fellow with the Department of Mechanical Engineering, University of Minnesota. His research activities include GPS-based automatic control of vehicles, various driver assistive technologies, road friction measurement and the design of small commuter vehicles.

Patrick J. Starr is Professor Emeritus with the Department of Mechanical Engineering, University of Minnesota, Minneapolis. His current interests lie in four areas: creation of models to describe and analyze the behavior of sociotechnical systems in terms of resource use, technological options, and economics; development of strategies for sensitivity analysis of large-scale dynamic models; creating a description of the design process as managing an open system of information; and utilization of modeling, simulation, and expert systems to define and manage recurring problems in manufacturing. In addition, he has for many years served as the advisor for the University of Minnesota solar vehicle and SAE formula car and mini-baja car projects.

Max Donath received the Ph.D. degree from Massachusetts Institute of Technology, Boston, in 1978.

He joined the University of Minnesota in 1978. He is Director of the Intelligent Transportation Systems Institute and Professor of Mechanical Engineering at the University of Minnesota, Minneapolis. The ITS Institute under his direction since 1997, and congressionally designated as a National University Transportation Center, pursues research in many areas including: human performance and behavior, driver interfaces, sensors, traffic and vehicle controls, and methodologies for congestion and crash mitigation. His own efforts have been directed towards keeping the driver in the loop, using sensing technologies, control systems and improved human-machine interfaces to reduce driver error, and thus prevent crashes before they happen. He has led several major national initiatives on road-vehicle safety systems, most recently on new approaches to reducing the high incidence of fatalities at rural unsignalized intersections. He presently also leads efforts developing new systems to reduce lane departure and teenage driver fatalities.