Body Roll Motion Optimal Control

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Abstract — An Anti-Rollover control algorithm for reducing the risk of rollover which is based on the optimal control theory has been proposed in this paper. Vehicle parameters of a 1997 Jeep Cherokee which have been published by Vehicle Research and Test Center (VRTC) have been used to construct the 3DOF model developed from the Lagrangian dynamics. Simulation show that the controller is capable of reducing the likelihood of rollover of vehicle during various standard test maneuvers Also we compare the result of simulation with other control strategies like as PID controller and P controller.

I. INTRODUCTION

Rollovers are the second most dangerous type of crash occurring on highways. During the eight years, 1991 through 1998, Fatality Analysis Reporting System (FARS) data showed that averages of 9,237 people were fatally injured each year in light vehicle rollover crashes. Comparing to other types of vehicles, Sport Utility Vehicles (SUV) had the highest rollover rates. [1], [2] Rollover prevention can be achieved by employing rollover warning or anti-rollover systems.

In this paper, an active stabilizer system which consists of two actuators has been proposed to generate torques between the front/rear axle and the vehicle body when the vehicle is turning.

II. VEHICLE MODELS

Vehicle models typically consist of two components, a Chassis model which describes the dynamics of the vehicle, and a tire model which describes the forces generated at the contact point between the tire and the road.

A. *Tire Models*

All road vehicles interact with the road surface via tires. More specifically, the tires are responsible for generating those forces which are required to alter the vehicle's speed and course according to the driver's inputs. The physical mechanisms which tires functions are complicated, and modeling is therefore difficult. A simple linear approximation of Magic Formula can be used in this paper. [10]

$$F_y = C_{F\alpha}\alpha + C_{F\gamma}\gamma \qquad (1)$$

$C_{F\alpha}$ Is the lateral slip stiffness, α is slip angle, $C_{F\gamma}$ is camber stiffness, γ is camber angle.

B. *Nonlinear modeling of vehicle*

Vehicle parameters of a 1997 Jeep Cherokee which have been published by Vehicle Research and Test Center (VRTC) have been used to construct the 3DOF model developed from the Euler-Lagrange method, including a constant but nonzero pitch angle θ_r. (Fig 1)

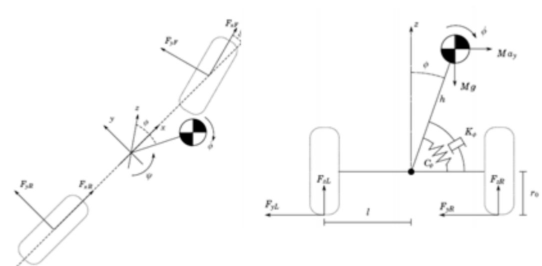

Fig 1: The two-track model, showing suspension modeled as a torsional spring and damper

So a nonlinear 3DOF model (2) is used and matched with, field-tests data to represent enough accuracy of simulation. (Fig 2, 3) [1], [3], [8]

Table 1 Parameters of the two–track model

Symbol	Description
m_{tot}	Total Vehicle mass
m	Rolling sprung mass
h	Height of CG above roll axis
I_{zz}	Moment of inertia about x–axis
I_{yy}	Moment of inertia about y–axis
I_{zz}	Moment of inertia about z–axis
I_{xz}	Product of inertia for x and z axes
K_ϕ	Total roll stiffness
C_ϕ	Total roll damping
θ_r	Angle between roll axis and x–axis

$$
\begin{aligned}
&m_{tot}[\dot{v} + \dot{\psi}u] + mh(\ddot{\phi} - \dot{\psi}\phi) = F_{yT} \\
&I_{zz} - mh(\dot{u} - \dot{\psi}v) + (I_{zz}\theta_r - I_{xz})\ddot{\phi} = M_T \\
&(I_{xz} + mh^2)\ddot{\phi} + mh(\dot{v} - \dot{\psi}u) + (I_{zz}\theta_r - I_{xz})\ddot{\psi} \\
&-(I_{yy} + mh^2 - I_{zz})\dot{\psi}^2\phi + (K_\phi - mgh)\phi - C_\phi\dot{\phi} = 0
\end{aligned} \tag{2}
$$

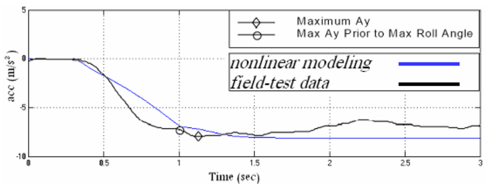

Fig 2: compare lateral acceleration on field-test and simulation

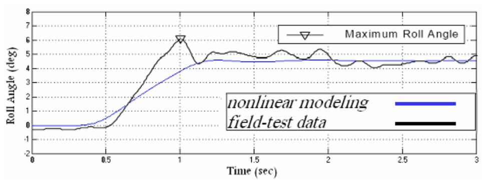

Fig 3: compare body roll motion on field-test and simulation

C. *Linearization of vehicle model*

To use this model in our controller design, it's also linearized. Linear models (3) use a number of assumptions and approximations, these include:

- Constant longitudinal velocity
- Small steering angles
- Linear tire forces
- Simple approximations of tire slip values

These approximations imply that linear models be useful for designing control systems intended for use under standard maneuver and normal driving. (Fig 4) [4], [5], [9]

$$
\begin{aligned}
&m_{tot}[\dot{\psi}u] + mh(\ddot{\phi}) = F_{yT} \\
&I_{zz}\dot{r} + (I_{zz}\theta_r - I_{xz})\ddot{\phi} = M_T \\
&mhu\dot{\psi} + (I_{zz}\theta_r - I_{xz})\ddot{\psi} + (I_{xx} + mh^2)\ddot{\phi} \\
&+ (K_\phi - mgh)\phi - C_\phi\dot{\phi} = 0
\end{aligned} \tag{3}
$$

The tire forces acting on the vehicle are obtained: (4)

$$
\begin{aligned}
&F_{yT} = -(C_{\alpha f} + C_{\alpha r})\beta - [m_{tot}U + \frac{(aC_{\alpha f} - bC_{\alpha r})}{U}]\dot{\psi} \\
&+ (C_{\alpha r}\frac{\partial\delta_r}{\partial\phi} + C_{\gamma f}\frac{\partial\gamma_f}{\partial\phi})\phi + C_{\alpha f}\delta \\
&M_T = -(aC_{\alpha f} - bC_{\alpha r})\beta + [\frac{(a^2C_{\alpha f} + b^2C_{\alpha r})}{U}]\dot{\psi} \\
&-(bC_{\alpha r}\frac{\partial\delta_r}{\partial\phi} - aC_{\gamma f}\frac{\partial\gamma_f}{\partial\phi})\phi + aC_{\alpha f}\delta
\end{aligned} \tag{4}
$$

Table 2 specifications data of the case study vehicle

Symbol	Description
m_{tot}	Total Vehicle mass , 1987.935 Kg
m_R	Rolling sprung mass , 1663 Kg
h	Height of CG above roll axis , 0.306 m
I_{zz}	Moment of inertia about x–axis , 602.8220 $Kg - m^2$
$C_{\gamma f}$	Camber thrust coefficient at the front axle , 2038.8 N/rad
I_{zz}	Moment of inertia about z–axis , 2703.7 $Kg - m^2$
I_{xz}	Product of inertia for x and z axes , 89.9914 $Kg - m^2$
K_ϕ	Total roll stiffness , 56957 N-m/rad
C_ϕ	Total roll damping , 3495.7 N-m-sec/rad
θ_r	Angle between roll axis and x–axis , 0.0873 rad
$C_{\alpha r}$	Front tire cornering stiffness , 59496 N/rad
$C_{\alpha f}$	Rear tire cornering stiffness , 109400 N/rad
$\frac{\partial\delta_r}{\partial\phi}$	Partial derivative of the roll induced steer at the rear axle, 0.07.
$\frac{\partial\gamma_f}{\partial\phi}$	Partial derivative of the camber thrust at the front axle, 0.8

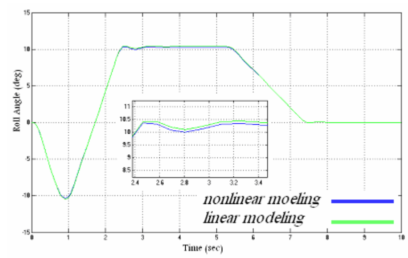

Fig 4: compare body roll motion on linear and nonlinear modeling in fishhook maneuver

The governing equations for Roll, yaw and lateral motions of the vehicle model, in state space form are derived as: (5)

$$
\begin{aligned}
&\dot{X} = AX + B\delta_{steer} \Leftrightarrow \quad x = [\beta \quad r \quad \dot{\phi} \quad \phi]^T \\
&Y = CX + D\delta_{steer}
\end{aligned} \tag{5}
$$

For the vehicle model the side slip angle, yaw rate, roll rate and roll angle are considered as the four state variables and steering angle is control input.

In the further parts Roll moment is control input, which must be determined from control low. Moreover, the vehicle steering angle is considered as the external disturbance.

III. TEST MANEUVERS

In order to obtain a common measure, a number of standardized maneuvers have been developed. The National Highway Traffic Safety Administration (NHTSA) has developed various standard maneuvers, including the so-called fishhook and J-turn maneuvers, which are described here.

A. Fishhook

The fishhook maneuver is an important test maneuver in the context of rollover. It attempts to maximize the roll angle under transient conditions and is performed as follows, with a start speed of 80 km/h:

- The steering wheel angle is increased at a rate of 720deg/sec up To 6.5 d_{stat}, where d_{stat} is the steering angle which is necessary To achieve 0.3g
- This value is held for 250ms
- The steering wheel is turned in the opposite direction at a rate of 720 deg/sec until it reaches -6.5 d_{stat}
- No brake or accelerator commands are given during the maneuver. (Fig 5)

B. J-Turn

The J-turn is a simple step in the steering wheel angle driving the vehicle towards the physical limits. This maneuver can cause a roll over of vehicles with critical load.

- The steering wheel angle is increased at a rate of 1000deg/sec until it reaches 8 times the value of d_{stat}. (Fig 6)

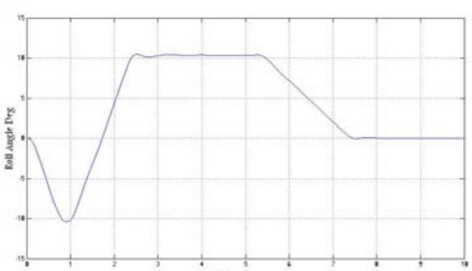

Fig 5: Vehicle Roll angle during the fishhook maneuver

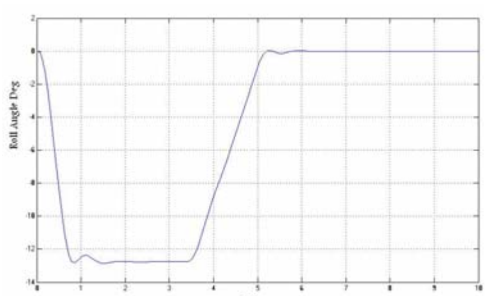

Fig 6: Vehicle Roll angle during the fishhook maneuver

IV. OPTIMAL CONTROL DESIGN

We used an active stabilizer system which consists of two actuators to generate torques between the front/rear axle and the vehicle body, and control body roll motion of vehicle with an anti-rollover control algorithm, based on linear tracking optimal control.

The control law consists of four state variable feedback terms being those of the side slip angle, yaw angle, yaw rate and roll angle.(6)

$$U_t^* = FX_t + V_t \qquad \textbf{(6)}$$

In above control law, according to the current state of technology, direct measurement of yaw angle, yaw rate and roll angle are quit feasible. However due to the impracticality of direct measurement of side slip angle, estimation of this state variable is most desirable.

We defined performance index in the following form: (7)

$$J_2 = \int_{t_0}^{t_f} \frac{1}{2}\left(U_t^T R_t U_t + (X_t - r_d)^T Q_t (X_t - r_d)\right) \qquad \textbf{(7)}$$

r_d Is the desired roll angle and R_t is weight factor.

Minimization of the performance index must be sought in order to obtain the optimum body roll motion to ensure, vehicle roll angle doesn't exceed from defined boundaries and the vehicle has enough body roll motion to create feeling about vehicle maneuver in driver.

It is important to note that the control effort must satisfy some physical constrains due to actuation system and energy consumption in vehicle.

By proper selection of weighting factor in (5) and the appropriate design of the control law, in order to minimize the performance index we achieve a good body roll behavior and also satisfy the physical limit of dynamic system.

To determine the values of feedback control gain, which are based on the defined performance index and the vehicle dynamic model, a LQR problem has been formulated, which its analytical solution is obtained.

The Hamiltonian function, in the expanded form, is therefore given by: (8)

$$H = \frac{1}{2}(X_t - X_d)^T Q (X_t - X_d) + \frac{1}{2} U_t^T R U_t + P^T (AX_t + BU_t) \qquad \textbf{(8)}$$

After written the state an co-state equation, generally we can form a system of nonlinear ordinary differential equations which can be converted into a nonlinear algebraic system of equations by assuming that the solution of the equations converge rapidly to a constant values.(9) (10)

$$0 = -K_t A - A^T K_t - Q + K_t B R^{-1} B^T K_t \qquad \textbf{(9)}$$

$$0 = -[A^T - K_t B R^{-1} B^T] S_t + Q_t X_d \qquad \textbf{(10)}$$

The system of equations given above can be solved analytically in order to determine the corresponding value of feedback gains. (11)

$$U_t^* = -R^{-1}B^T K X_t - R^{-1}B^T S_t \longrightarrow U_t^* = F X_t + V_t \qquad \textbf{(11)}$$

The numerical simulations of the vehicle roll motion based on standard tests like as J-turn and fishhook maneuver, with and without the optimal roll motion controller were carried out. (Fig 7, 8)

Simulation results indicate that the controller is capable of reducing the likelihood of rollover of vehicle during standard test maneuvers. [6], [7]

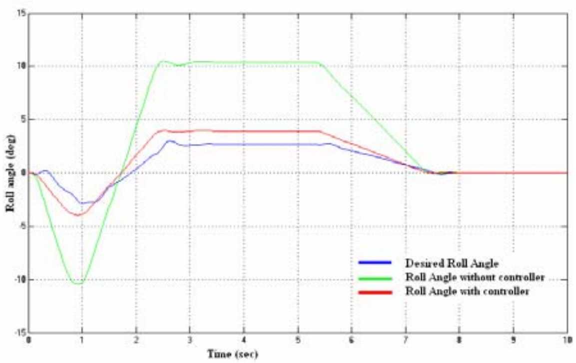

Fig 7: Body roll motion with optimal tracking controller in fishhook maneuver

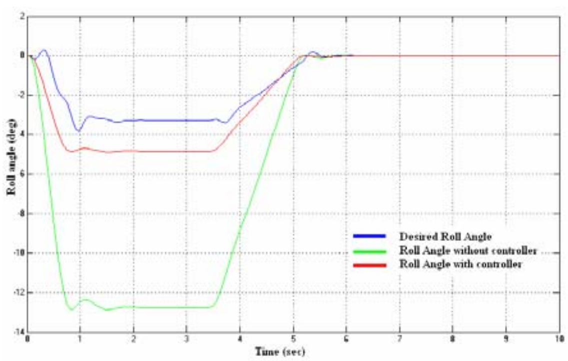

Fig 8: Body roll motion with optimal tracking controller in J-turn maneuver

V. COMPARE RESULT WITH OTHER CONTROL SYSTEMS

In the last, we compared the result of simulations with other control strategies like as regulator optimal and PID, to ensure performance of our controller. (Fig 9, 10)

Simulations indicate that the performance of optimal tracking controller due to simple structure and good behavior of roll motion and less moment for controlling of roll in maneuver is slightly better than other control systems.

Fig 11 and Fig 12 indicate the moment that use for controlling of roll motion, and so we can use weaker actuator that require less energy than other controller like as PID controller and optimal regulator controller.

In Fishhook maneuver, Actuator that control with PID controller, should produce about 8600 (N.m), Actuator that control with optimal regulator controller, should produce about 7100 (N.m), but in optimal regulator controller we use only about 6200 (N.m) to controlling roll motion in Fishhook maneuver so more than 25% less energy consuming .(Fig 11)

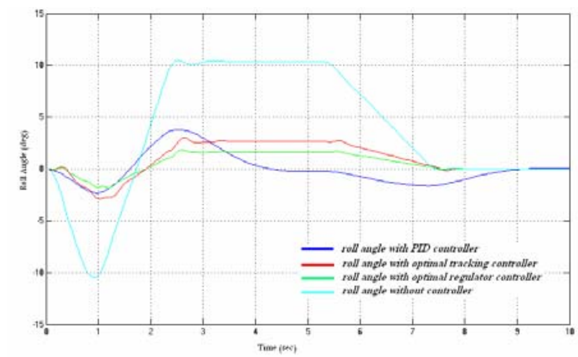

Fig 9: Body roll motion with various controller in fishhook maneuver

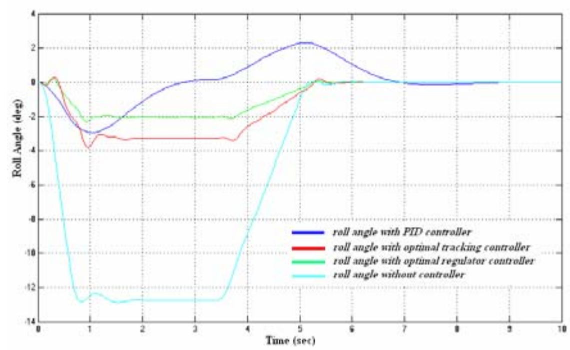

Fig10: Body roll motion with various controller in J-turn maneuver

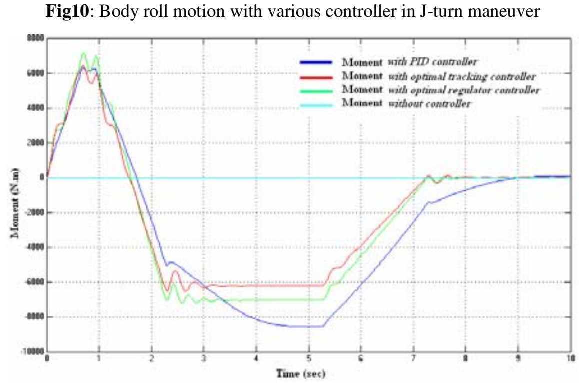

Fig 11: Moment that use for controlling Body roll motion with various controllers in fishhook maneuver

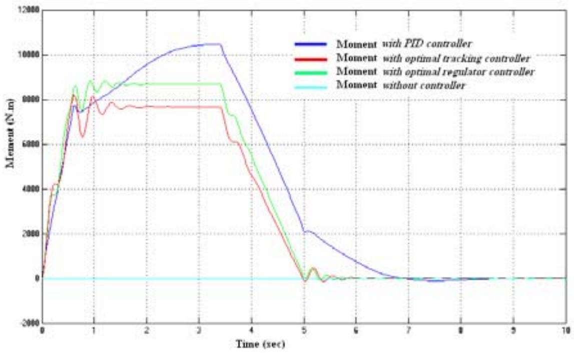

Fig 12: Moment that use for controlling Body roll motion with various controllers in J-turn maneuver

VI. CONCLUSION

A 3DOF yaw-roll model was constructed by using the Cherokee parameters and tuned to match the test data.

We used an anti-rollover control algorithm, based on linear tracking optimal control and also we used an active stabilizer system which is consists of two actuators to generate torques between the front/rear axle and the vehicle body to control body roll motion of vehicle.

We design an optimal tracking controller and defined a desired roll motion that controller try to obey it and get that roll angle. Simulation results indicate that our optimal tracking controller is capable of reducing the likelihood of rollover of vehicle during standard test maneuvers with less usage of energy compare with other controller.

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