The Stability Control of Electric Vehicle Based on Optimal Predictive Control Method

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***Abstract*—A direct yaw moment control (DYC) method based on optimal and predictive control is proposed in this paper. The 8-degree-of-freedom model of electric vehicle and the non-linear Unitire model have been developed. The simulation results verified the validity of control strategy. The control method of electric vehicle put forward in this paper can reduce the control energy in case of satisfying the control requirement.**

Keywords-electric vehicle; direct yaw moment; predictive control; optimal control

I. INTRODUCTION

In order to enhance the vehicle's active safety, a variety of chassis control systems had been developed, such as four-wheel steering system (4WS), active front steering system (AFS), direct yaw moment control system (DYC) and so on. Direct yaw moment control as the method to improve vehicle handling stability has been shown better control effect in adhesion which is close to the limits[1]. Direct yaw moment is generated by changing drive or brake forces of each wheel. For electric vehicles, we can take advantage of the flexible control method of electric motors. A reasonable distribution of force to each drive motor results the required direct yaw moment. There are many ways to calculate the required yaw moment, and the optimal control and sliding-control methods have been frequently applied to this because of their intrinsic robustness [2, 3]. Optimal control can meet the control requirements of the situation while reduce yaw torque value as much as possible, but it needs to solve the Riccati equation, so that it will slow the system down. Sliding mode control can achieve real-time control, but can not optimize the value of control torque input simultaneously.

The direct yaw moment control based on prediction and optimal control has been proposed in this paper. This control method not only meets the requirements of vehicle's stability control, but also can reduce the control energy.

II. ELECTRIC VEHICLE DYNAMICS MODEL

A. Vehicle model

The micro-electric vehicle mentioned in this paper is front-wheel steer and rear-wheel drive architecture. The power train of this type of vehicle consists of two electric motors, which are integrated into each one of the drive wheels and can be controlled independently. In order to make the vehicle model close to the actual situation, considering the non-linear of suspension, tires and body, four-wheel drive and eight-degree-of-freedom (8DOF) model has been established. And the longitudinal velocity, lateral velocity, yaw rate, roll, and rotational speeds of four wheels constitute the degrees of freedom for this model.

Let the front wheel steering angle be denoted by δ. Let the longitudinal tire forces at the front left, front right, rear left and rear right tires be given by F_{x1}, F_{x2}, F_{x3} and F_{x4} respectively. Let the lateral forces at the front left, front right, rear left and rear right tires be denoted by F_{y1}, F_{y2}, F_{y3} and F_{y4} respectively. According to D'Alembert's principle, ignoring the tire resistance and rolling resistance, the balance equations of the vehicle model in all directions can be written as:

$$\sum F_x = m(\dot{v}_x - v_y\gamma), \tag{1}$$

$$\sum F_y = m(\dot{v}_y + v_x\gamma), \tag{2}$$

$$I_x\ddot{\varphi} = \sum M_x + m_s h_s(\dot{v}_y + v_x\gamma), \tag{3}$$

$$I_z\dot{\gamma} = \sum M_z, \tag{4}$$

$$I_w\dot{\omega}_{wi} = T_i - F_{xi}R, \tag{5}$$

where

$$i = 1,2,3,4,$$

$$\sum F_x = (F_{x1} + F_{x2})\cos\delta - (F_{y1} + F_{y2})\sin\delta + F_{x3} + F_{x4},$$

$$\sum F_y = (F_{y1} + F_{y2})\cos\delta + (F_{x1} + F_{x2})\sin\delta + F_{y3} + F_{y4},$$

$$\sum M_z = a - b + c - p + q - h,$$

here

$$a = l_f(F_{y1} + F_{y2})\cos\delta,\ b = l_r(F_{y3} + F_{y4}),$$

$$c = \frac{d}{2}(F_{y1} - F_{y2})\sin\delta,\ p = \frac{d}{2}(F_{x1} - F_{x2})\cos\delta,$$

$$q = l_f(F_{x1} + F_{x2})\sin\delta,\ h = \frac{d}{2}(F_{x3} - F_{x4}),$$

$$\sum M_x = -(K_f + K_r)\varphi - (C_{\varphi f} + C_{\varphi r})\dot{\varphi} + m_s g h_s \sin\varphi.$$

In above formulas, the lengths l_f, l_r and d refer to the longitudinal distance from the center of gravity (c.g.) to the front wheels, longitudinal distance from the c.g. to the rear wheels and the lateral distance between left and right wheels respectively. The symbols v_x, v_y, m and I_z denote the longitudinal velocity, the lateral velocity, the mass and the yaw moment inertia of vehicle respectively. While I_w, T_{di} and R refer to the rotational moment of inertia, drive torque and radius of each wheel. φ is roll angle, I_x is roll moment of inertia, M_s is sprung mass, and h_s is the distance between the roll axis and c.g of vehicle. K_f and K_r represent the roll stiffness of front and back axle respectively. $C_{\varphi f}$ and $C_{\varphi r}$ represent the roll angle damping of front and rear tire respectively.

B. Tire Model

Non-linear characteristic of tire force is the main factor of vehicles' instability, so that the establishment of precise non-linear mechanical model of tire is very important for the vehicle dynamic analysis. Unitire model, proposed by Academician Guo Konghui, is a tire mechanics modeling theory. The Unitire model fully consider tire lateral slip, longitudinal slip, roll slip， turn-slip and other non-steady-state non-linear partial working conditions. It is characteristic of good theoretical boundary, fast operation speed and high simulation accuracy [4]. The tire model in this article, simplified based on Unitire model, is just relevant to lateral slip and longitudinal slip. It can be expressed as:

$$\overline{F} = 1 - \exp(-\Phi - E\Phi^2 - (E^2 + \tfrac{1}{12})\Phi^3) , \tag{6}$$

$$F_x = \overline{F}_x \mu_x F_z , \tag{7}$$

$$F_y = \overline{F}_y \mu_y F_z , \tag{8}$$

where

$$\Phi_x = -\frac{K_x S_x}{\mu_x F_z} , \Phi_y = -\frac{K_y S_y}{\mu_y F_z} , \Phi = \sqrt{\Phi_x{}^2 + \Phi_y{}^2} ,$$

$$\overline{F}_x = \overline{F}\frac{\Phi_x}{\Phi} , \overline{F}_y = \overline{F}\frac{\Phi_y}{\Phi} ,$$

In above formulas, E, K_x and K_y, indicating structure parameters of tire, can be obtained by fitting the experimental data, μ_x, μ_y, S_x, S_y and F_z denote longitudinal, lateral friction coefficient of tire-road, the longitudinal, lateral slip ratio and normal force on tire respectively.

III. ELECTRIC VEHICLE STABILITY CONTROL ALGORITHM

The electric vehicle mentioned in this paper is direct driven by hub-motors. With advantages of motor respond quickly and easy to control, direct yaw moment method is applied to vehicle stability control. Through analysis of vehicle dynamics, the sideslip angle and yaw rate characterize the stability of vehicle from different sides, and also are important parameters to describe the vehicle motion state. The sideslip angle and yaw rate are selectd as the main control variables in this paper. The target values of control variables determine the accuracy of vehicle stability control, so the target values should be defined according to the road information and vehicles' real condition.

A. Determination of Target Value

The steady-state of vehicle with constant velocity is a constant speed circular motion when the input of front wheel is step. The target values of sideslip angle and yaw rate can be obtained from the steady-state cornering performance of 2 DOFs linear vehicle. And they can be expressed as:

$$\gamma_d = \frac{v_x}{L + K v_x{}^2}\delta , \tag{9}$$

$$\beta_d = f_u \gamma_d , \tag{10}$$

here, K is the stability factor and is the important parameters that characterize steady-state responses of cars, K and f_u can be expressed as:

$$K = \frac{m}{L^2}\left(\frac{l_f}{C_r} - \frac{l_r}{C_f}\right) , f_u = \frac{l_r}{v_x} - \frac{l_f m v_x}{L} ,$$

where C_f and C_r are the total cornering stiffness of the front and rear tire respectively. The L refers to the distance of the wheelbase.

If the the road condition is good, the static properties of 2 DOFs linear vehicle can be taken as ideal representation of stability. To the contrary, it is unsafe to obtain above desired yaw rate, if the friction coefficient is unable to provide tire forces to support a high yaw rate. The limitation of the sideslip angle is approximately ±12° on dry asphalt roads, while on packed snow roads this value is about ±4°[5].Owing to the lateral acceleration of the vehicle cannot exceed the maximum friction coefficient of the road, the desired value of the yaw rate is limited according to the tire-road condition using lateral acceleration and longitudinal velocity. Hence the desired values of 2 DOFs linear vehicle can be adjusted as follows:

$$\gamma_{t\arg et} = \begin{cases} \gamma_d & if\left|\gamma_d\right| \le \dfrac{\mu g}{v_x} \\ \dfrac{\mu g}{v_x}\operatorname{sgn}(\gamma_d) & otherwise \end{cases} , \tag{11}$$

$$\beta_{t\arg et} = f_u \gamma_{t\arg et} . \tag{12}$$

Here, μ and g denote friction coefficient of pavement and acceleration of gravity respectively.

B. Direct yaw moment control strategy based on Prediction and Optimization Method

According to 2 DOFs vehicle model, the state equation of yaw rate and sideslip angle can be written as[6]:

$$\begin{bmatrix} \dot{\beta} \\ \dot{\gamma} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\begin{bmatrix} \beta \\ \gamma \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{1}{I_z} \end{bmatrix} M_z + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}\delta , \tag{13}$$

where $a_{11} = -2\frac{C_f + C_r}{mv_x}$, $a_{12} = 2\frac{l_r C_r - l_f C_f}{mv_x^2} - 1$, $a_{21} = 2\frac{l_r C_r - l_f C_f}{I_z}$, $a_{22} = -2\frac{l_r^2 C_r + l_f C_f}{I_z v_x}$, $b_1 = \frac{2C_f}{mv_x}$, $b_2 = \frac{2l_f C_f}{I_z}$.

The signs C_f and C_r refer to the total cornering stiffness of front and rear wheels respectively.

The description of the nonlinear vehicle model can be rewritten as:

$$\dot{x}_1 = f_1(x, \delta), \tag{14}$$

$$\dot{x}_2 = f_2(x, \delta) + \frac{1}{I_z} M_z, \tag{15}$$

here $x = [\beta \quad \gamma]^T$.

The two order Taylor expansion of $\beta(t+h)$ and $\gamma(t+h)$ are expressed as:

$$\beta(t+h) = \beta(t) + h\dot{\beta}(t) + \frac{h^2}{2!}\ddot{\beta}(t), \tag{16}$$

$$\gamma(t+h) = \gamma(t) + h\dot{\gamma}(t) + \frac{h^2}{2!}\ddot{\gamma}(t), \tag{17}$$

here h indicate the predict time. The combination of (14) and (15) deduce:

$$\ddot{\beta} = \frac{\partial f_1}{\partial \beta} f_1 + \frac{\partial f_1}{\partial \gamma}\left(f_2 + \frac{1}{I_z} M_z\right) + \frac{\partial f_1}{\partial \delta}\dot{\delta}, \tag{18}$$

$$\ddot{\gamma} = \frac{\partial f_2}{\partial \beta} f_1 + \frac{\partial f_2}{\partial \gamma}\left(f_2 + \frac{1}{I_z} M_z\right) + \frac{\partial f_2}{\partial \delta}\dot{\delta}. \tag{19}$$

Substituting (18) in (16) and substituting (19) in (17) yield:

$$\beta(t+h) = \beta(t) + hf_1 + \frac{h^2}{2}\left[\frac{\partial f_1}{\partial \beta} f_1 + \frac{\partial f_1}{\partial \gamma}\left(f_2 + \frac{1}{I_z} M_z\right) + \frac{\partial f_1}{\partial \delta}\dot{\delta}\right], \tag{20}$$

$$\gamma(t+h) = \gamma(t) + hf_2 + \frac{h^2}{2}\left[\frac{\partial f_2}{\partial \beta} f_1 + \frac{\partial f_2}{\partial \gamma}\left(f_2 + \frac{1}{I_z} M_z\right) + \frac{\partial f_2}{\partial \delta}\dot{\delta}\right] \tag{21}$$

In order to ensure the vehicle handling stability, yaw rate and sideslip angle as the control variables should track the target value mentioned above. Considering the limitation of electric vehicle driving and braking torque, the control torque should be kept as low as possible. So the control law should obtain a compromise between tracking accuracy and control energy. By the optimal control theory the overall system performance index is as follows:

$$J = \frac{1}{2}\omega_1[\gamma(t+h) - \gamma_d(t+h)]^2 + \frac{1}{2}\omega_2[\beta(t+h) - \beta_d(t+h)]^2 + \frac{1}{2}\lambda[M_z(t)]^2, \tag{22}$$

where $\omega_1 \geq 0$, $\omega_2 \geq 0$ and $\lambda \geq 0$, they are weight factors indicating the relative importance of the corresponding items. The entire system with $\lambda = 0$ just indicates that yaw rate and sideslip angle are controlled and the optimization of the control torque is ignored.

The desired sideslip angle and yaw rate can be expanded in the same manner, they can be expressed as:

$$\gamma_d(t+h) = \gamma_d(t) + h\dot{\gamma}_d(t) + \frac{h^2}{2}\ddot{\gamma}_d(t), \tag{23}$$

$$\beta_d(t+h) = \beta_d(t) + h\dot{\beta}_d(t) + \frac{h^2}{2}\ddot{\beta}_d(t). \tag{24}$$

The expanded performance index can be obtained by substituting (16)-(19), (23) and (24) in (22). The necessary condition for optimality is seen as [7]:

$$\frac{\partial J}{\partial M_z} = 0, \tag{25}$$

which leads to:

$$M_z(t) = \frac{2I_z h^{-2}}{\omega_1\left(\frac{\partial f_1}{\partial \gamma}\right)^2 + \omega_2\left(\frac{\partial f_2}{\partial \gamma}\right)^2 + 4\lambda I_z^2 h^{-4}} \times (\mu_1 + \mu_2),$$

Here

$$\mu_1 = \omega_1 \frac{\partial f_2}{\partial \beta}\left[e_\gamma(t) + h\dot{e}_\gamma(t) + \frac{h^2}{2}\left(\frac{\partial f_2}{\partial \beta} f_1 + \frac{\partial f_2}{\partial \gamma} f_2 + \frac{\partial f_2}{\partial \delta}\dot{\delta} - \ddot{\gamma}_d\right)\right],$$

$$\mu_2 = \omega_2 \frac{\partial f_1}{\partial \gamma}\left[e_\beta(t) + h\dot{e}_\beta(t) + \frac{h^2}{2}\left(\frac{\partial f_1}{\partial \beta} f_1 + \frac{\partial f_1}{\partial \gamma} f_2 + \frac{\partial f_1}{\partial \delta}\dot{\delta} - \ddot{\beta}_d\right)\right],$$

where $e_\gamma(t)$ and $e_\beta(t)$ are the sideslip angle and yaw rate difference between the actual value and the target value at t time, i.e., tracking errors. $M_z(t)$ is the control torque at t time, generated by difference of drive/brake forces of tires.

IV. SIMULATION AND ANALYSIS

Using the previously mentioned 8DOFS model established by Matlab/Sinulink, the effectiveness of designed control strategy based on optimal predictive control method has been verified. The main parameters are: m=1490kg, I_z=2350kg·m^2, l_f=0.986m, l_r=1.596m, C_f=6300N/rad,

C_r=6600N/rad, $C_{\varphi f}$=1756N·s/rad, $C_{\varphi r}$=1756N·s/rad, K_f=38400N·m/rad, K_r=31420N·m/rad.

In this manoeuvre, the vehicle runs on the level dry road at a constant speed of 72 Km/h, and the steering angle input is shown in Fig. 1. It is assumed that the control torque input is restricted by the given maximum value $|M_{ZMAX}|$=2000Nm. Set $\lambda=4\times10^{-9}$. The time responses of side-slip angle and yaw rate are shown in Fig. 2 and Fig. 3. The time response of the required yaw moment is seen in Fig. 4.

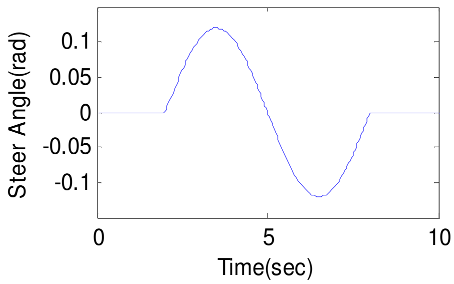

Figure 1. Sinusoidal Front Wheel Steering Angle

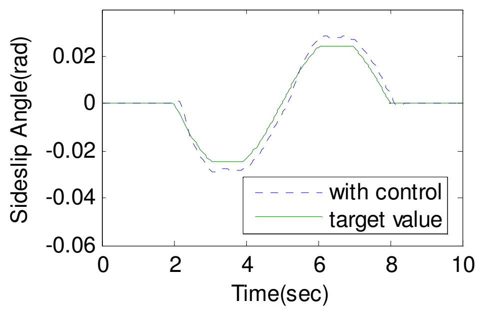

Figure 2. The Time Response of Sideslip Angle

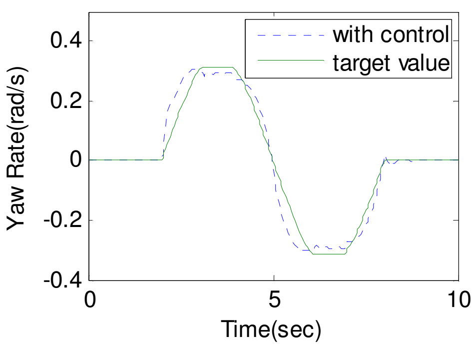

Figure 3. The Time Response of Yaw Rate

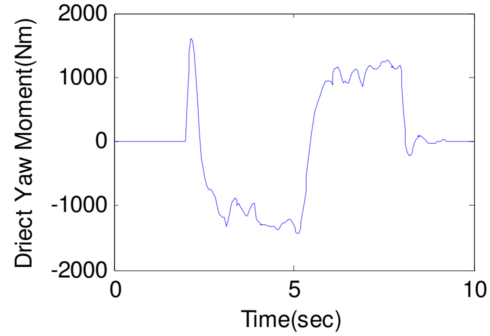

Figure 4. The Time Response of Required Yaw Moment

The simulation results show the control system possess good tracking performance of sideslip angle and yaw rate, so the proposed predictive controller can stabilize the vehicle successfully. And the control torque remains below the given maximum value for a suitable value of weight coefficient.

To illustrate the effect of the proposed control law on limiting input control torque, the sideslip angle responses obtained by λ=0 and $\lambda=4\times10^{-9}$ are compared, as shown in Fig 5. Although a perfect tracking is achieved through zero weight coefficient, the sideslip angle tracking error caused by the specified λ is also acceptable. Fig 6 shows that the required yaw moment is decreased obviously by setting the weight coefficient λ. In order to reduce the control energy, the weight coefficient λ can be increased to same extent; otherwise the sideslip angle and the yaw rate can not follow the target values precisely.

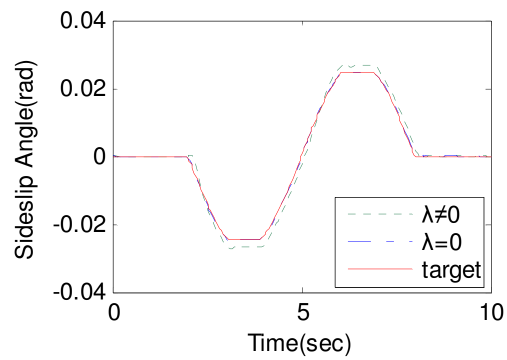

Figure 5. The Time Responses of Sideslip Angle

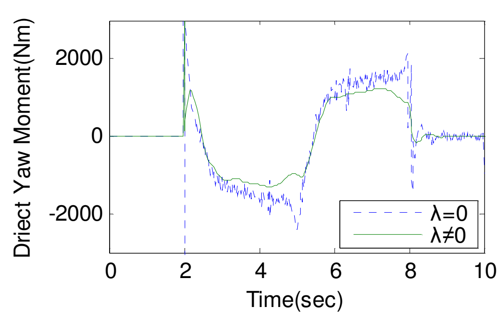

Figure 6. The Time Responses of Requried Yaw Moment

V. CONCLUSIONS

The electric vehicle stability control strategy is proposed based on the optimal predictive theory in this paper. The feasibility of the control method proposed in this paper has been verified by simulation results. The simulation results show that the optimal predictive control method can not only control the vehicle stability but also can reduce the control torque as much as possible.

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