ME 190M
Introduction to
Model Predictive Control

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Department of Mechanical Engineering

University of California

Berkeley, USA

Instruction

- **Instructor:** Francesco Borrelli, Room 5139 EH, 643-3871, fborrelli@me.berkeley.edu
  Office Hours: Tu and Th 9.30-11
- **Teaching Assistant:** None
- **Lectures:** Friday 11-12 in Room 1165, Etcheverry Hall
- **Class Notes:** Slides distributed before (sometime after) the class
- **Class Web Site:** bSpace

Grading

- Homework assignments: 100%

- Every 1 week
  - Includes matlab programming and simulation assignments

- Only selected ones (~5) will be graded (will announce it beforehand)

Matlab

- Matlab running the computers in 2109 Etcheverry Hall

- Card key access required

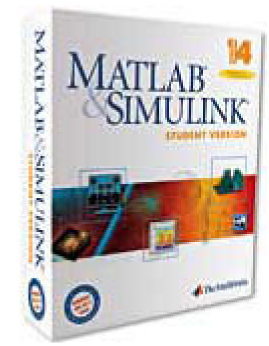
- I will submit class list to so that everyone in the class has access to that room
  - Please enroll in the class ASAP

- **Need** additional toolbox/Software distributed through bSpace

Recommended software purchase

Student Price: **~$ 100.00**

– The Scholar's Workstation



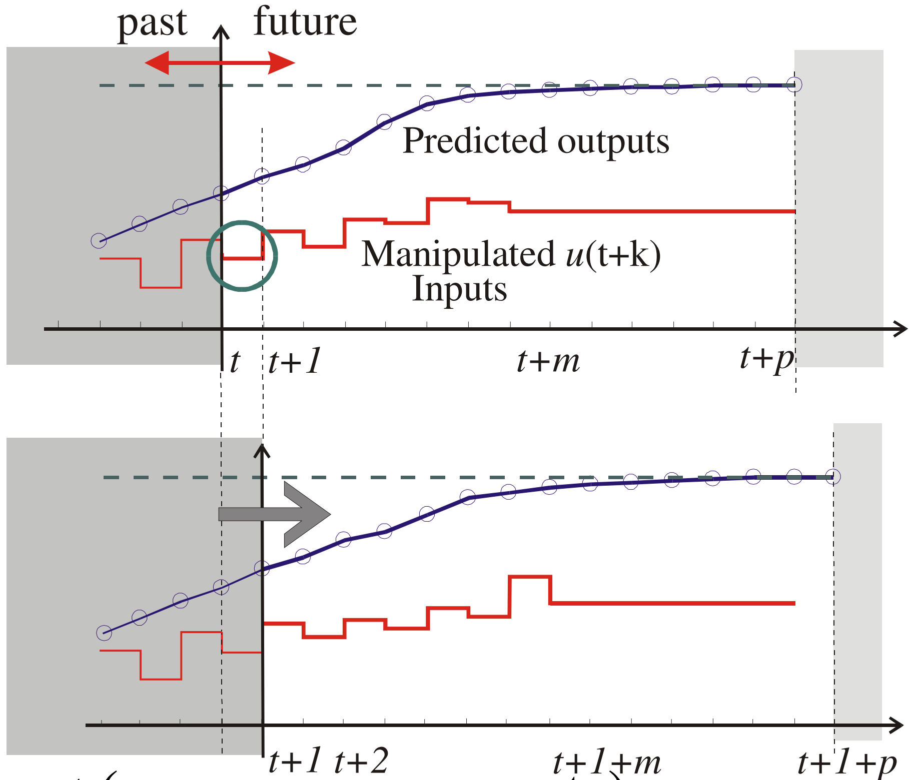
Professors:
Request Instructor Evaluation Copy

- Contains:
  – Matlab, Simulink, Symbolic Math, and two books/manuals

- Professional price is about $5,000.
  – Some minor limitations, but you would be hard pressed to notice them

ME190M Overview

Model Predictive Control



- Optimize at time *t* (new measurements)
- Only apply the first optimal move *u(t)*
- Repeat the whole optimization at time *t +1*
- Optimization using current measurements Feedback

MPC Algorithm

$$\min_{U} \sum_{k=t}^{t+N-1} l(x_k, u_k)$$

$$\text{subj. to } \begin{cases} x_{k+1} = f(x_k, u_k), \; k = t, \ldots, t+N-1 \\ u_k \in \mathcal{U}, \; k = t, \ldots, t+N-1 \\ x_k \in \mathcal{X}, \; k = t, \ldots, t+N-1 \\ x_t = x(t) \end{cases}$$

At time t:

- Measure (or estimate) the current state ***x(t)***
- Find the optimal input sequence $U^* = \{u^*_t, u^*_{t+1}, u^*_{t+2}, \ldots, u^*_{t+N-1}\}$
- Apply only $u(t)=u^*_t$, and discard $u^*_{t+1}, u^*_{t+2}, \ldots$

Repeat the same procedure at time $t+1$

Multivariable, Model Based

Nonlinear, Constraints Satisfaction, Prediction

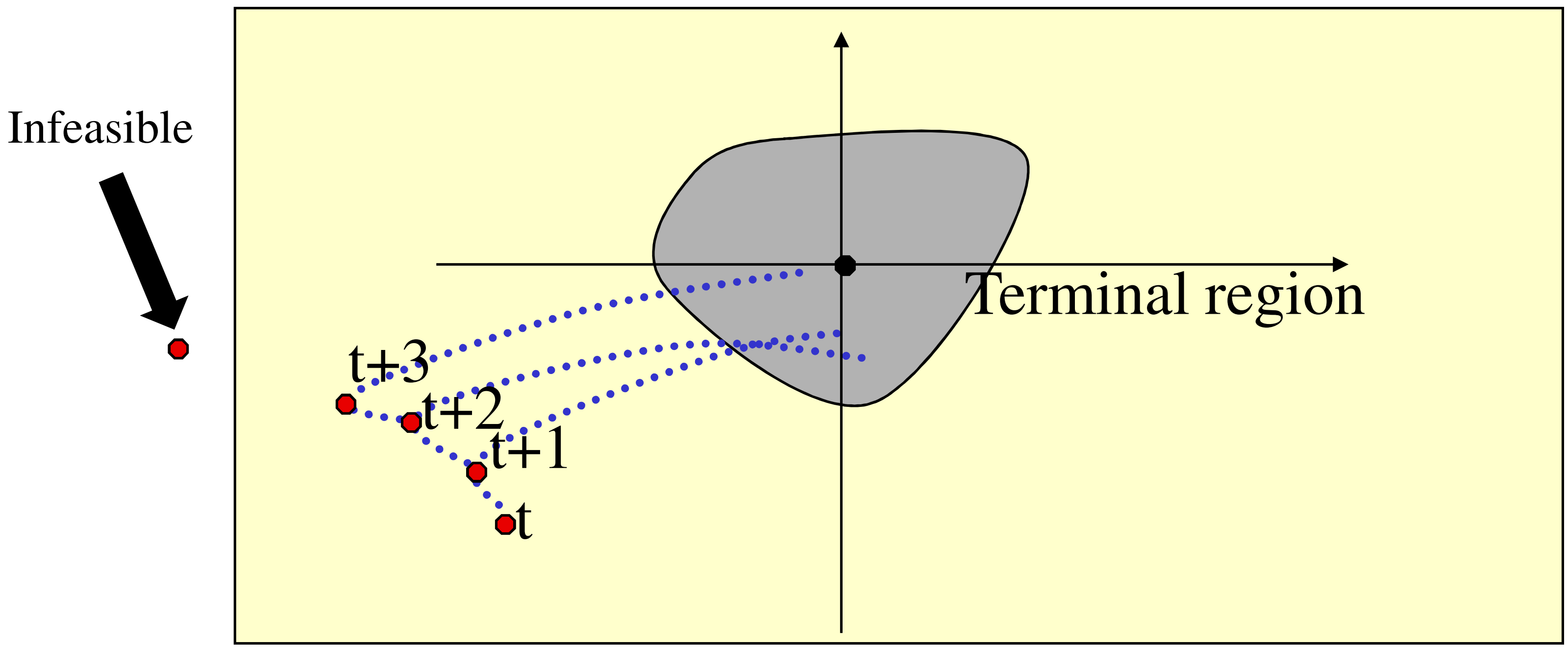
Important Issues in Model Predictive Control

Even assuming perfect model, no disturbances:

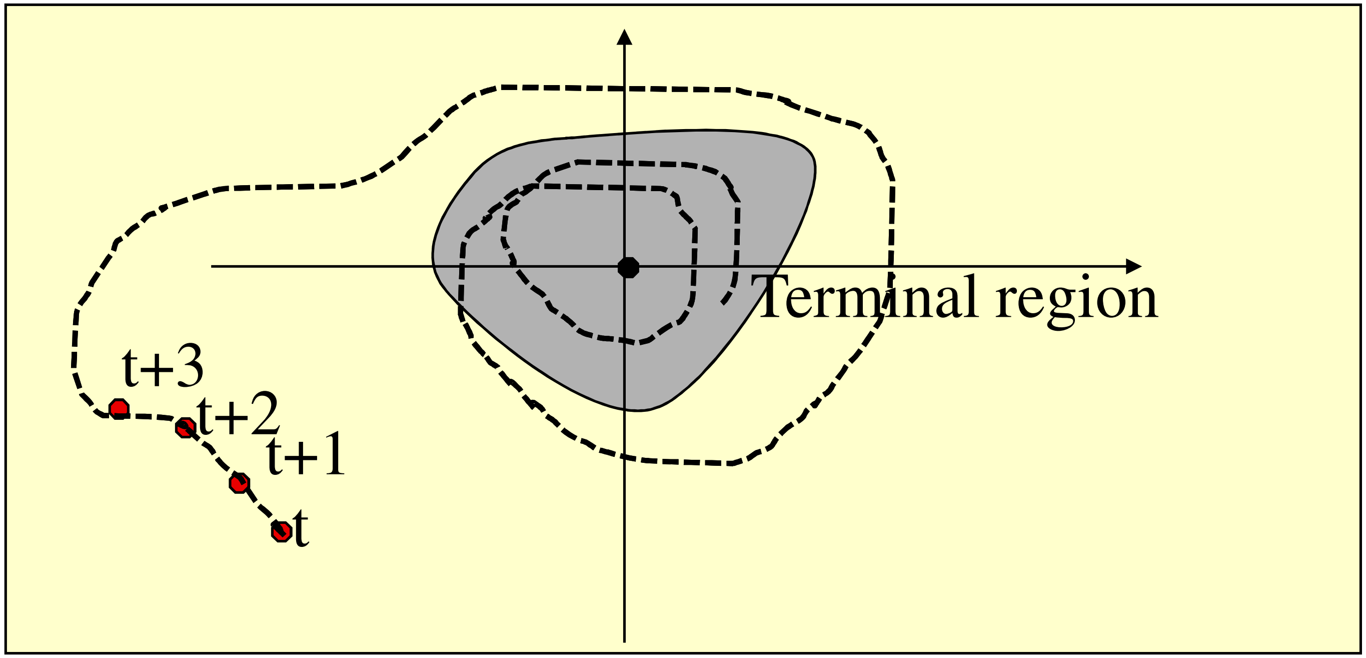
$$\boxed{\begin{array}{c}\text{predicted open-loop trajectories} \\ \neq \\ \text{closed-loop trajectories}\end{array}}$$

- **Feasibility**
  Optimization problem may become infeasible at some future time step.
- **Stability**
  Closed-loop stability is not guaranteed.
- **Performance**
  Goal: $\min \sum_{k=t}^{\infty} l(x_k, u_k)$
  What is achieved by repeatedly minimizing $\min \sum_{k=t}^{t+N} l(x_k, u_k)$
- **Real-Time Implementation**

Feasibility Issues



Stability Issues



Feasibility and Stability Constraints

$$\min_{U} \quad \sum_{k=t}^{t+N-1} l(x_k, u_k) + p(x_{t+N})$$

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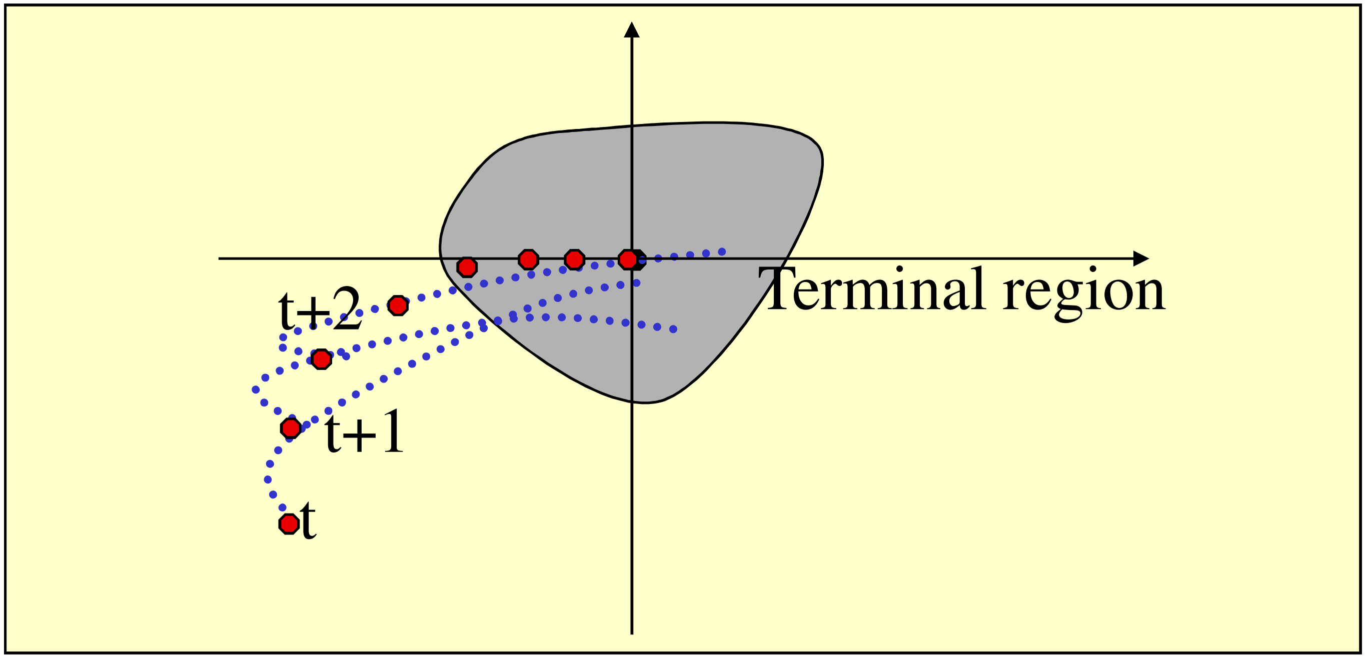
Modified Problem

(Large Body of Literature)

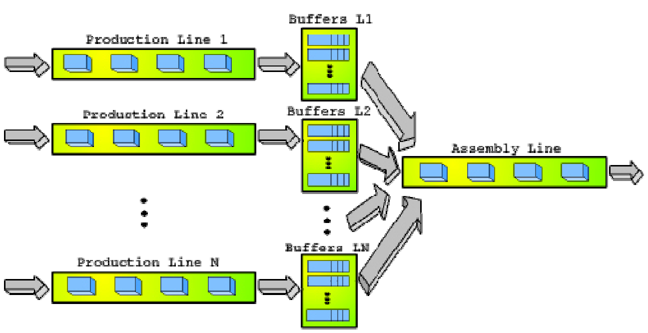
X_f (Robust) Invariant Set

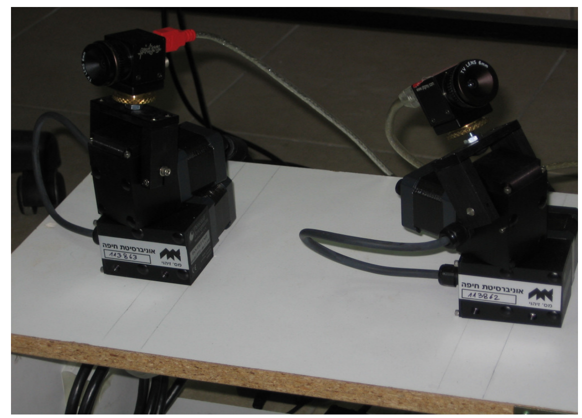
p(x) Control Lyapunov Function

Feasibility and Stability Issues



Real Time Implementation





Class Goals

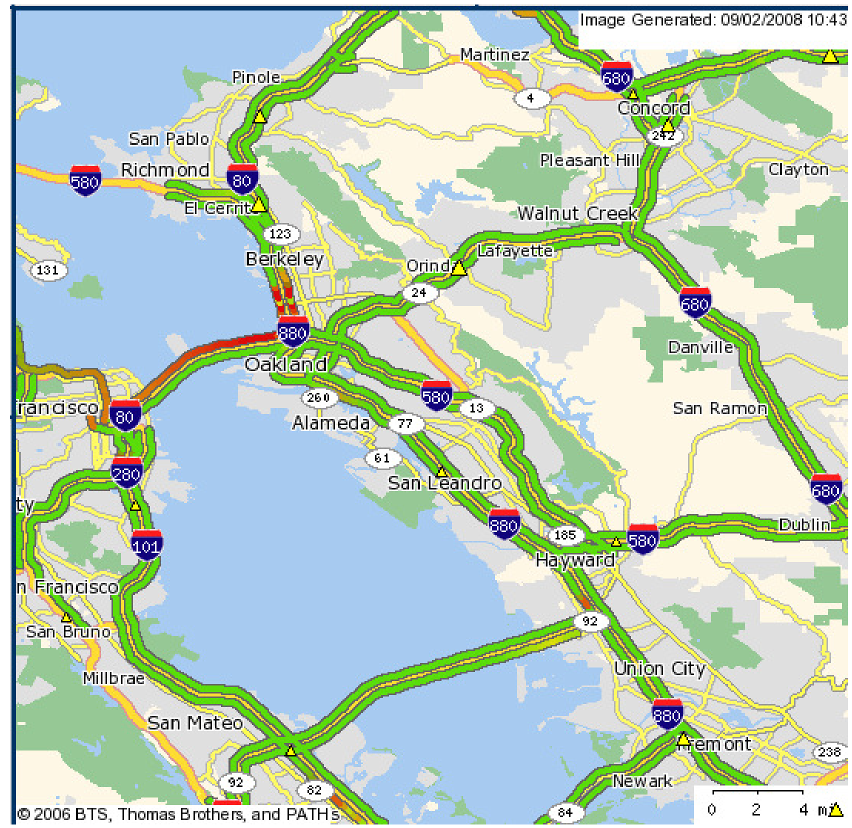
• Design and Implement a “simple” MPC Controller in Matlab (for linear and nonlinear systems)

• Tune it for achieving Desired Performance

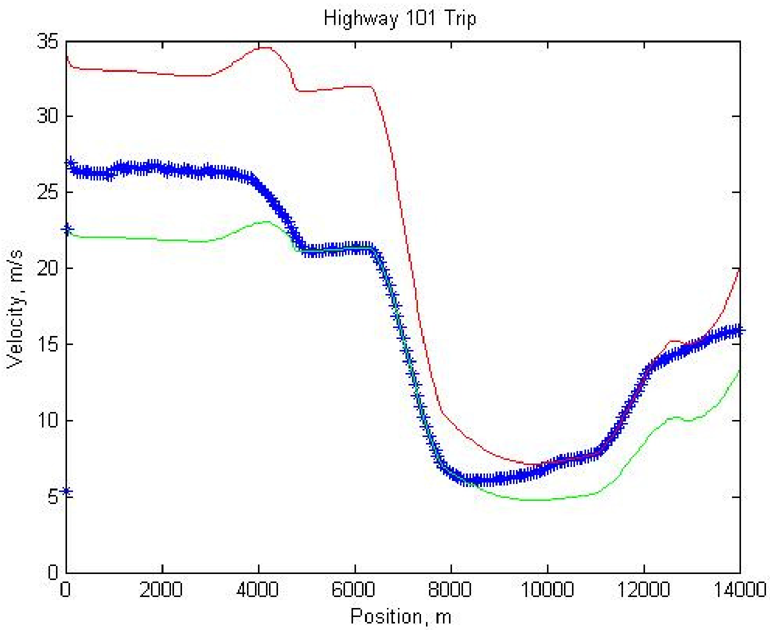
• Understand main issues of Stability and Feasibility

Example 1
Data from PeMS

- California Freeway Performance Measurement System
- Collects real-time data on CA freeways via loop detectors
- Able to communicate average traffic speed at loop location every 5 minutes
- Loops typically positioned every 0.3-3 miles

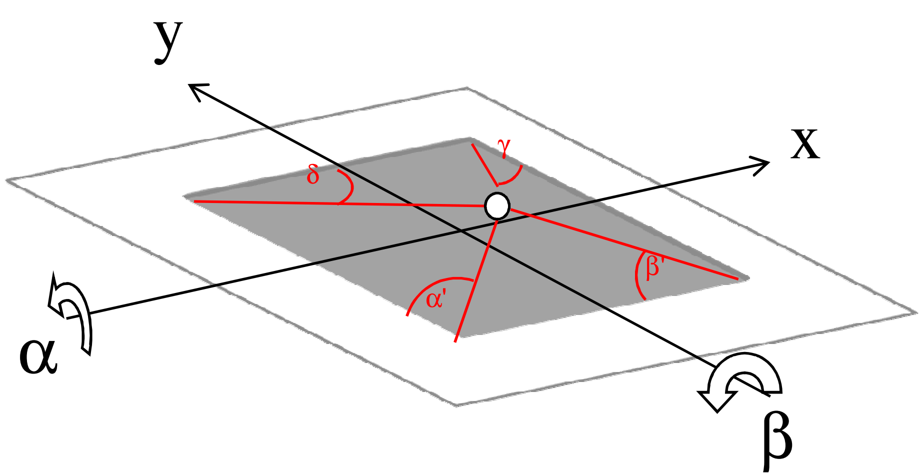


Example 1
Audi SmartEngine



- Design and MPC Controller regulating the desired speed (through an Automatic Cruise Control) in order to reach the destination in the most fuel-efficient way
- Prediction: Max and Min Speed of traffic, Grade
- Constraints: Max and Min Speed (of traffic and of vehicle)

Example 2
Ball and Plate Experiment



- Specification of Experiment:

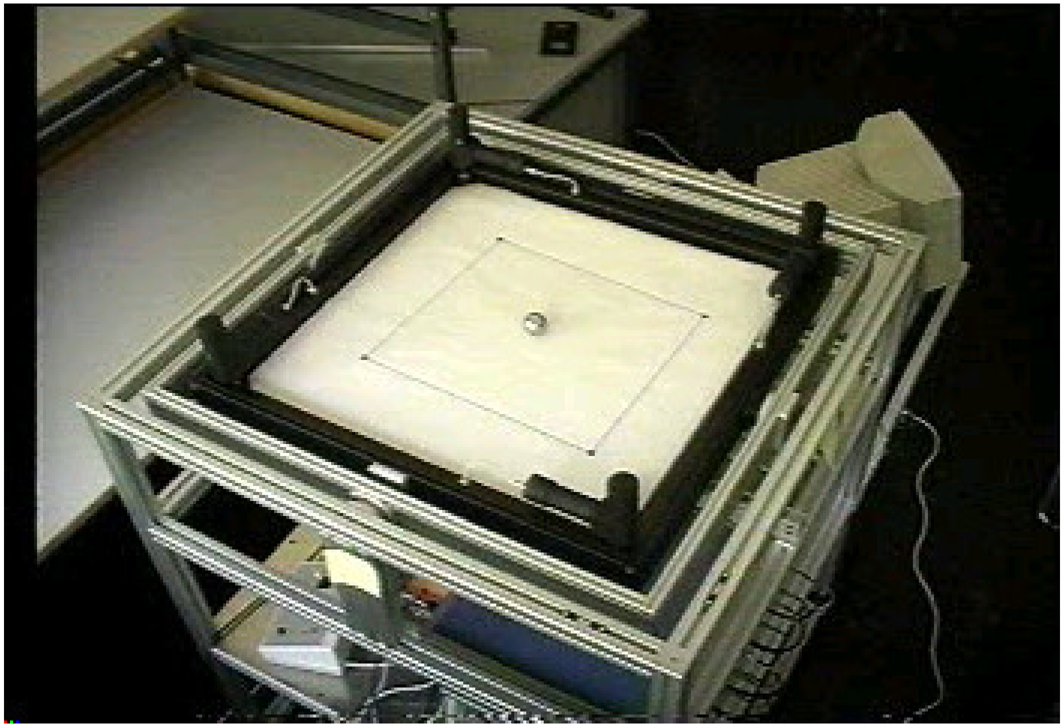
  Angle: -17°… +17° , Plate:-30 cm…+30 cm

  Input Voltage: -10 V… +10 V

  Computer: PENTIUM166

  Sampling Time: 30 ms

Example 2
Ball and Plate Experiment



Summarizing…

Need:

- A discrete-time model of the system

  (Matlab, Simulink)
- A state observer
- Set up an Optimization Problem

  (Matlab, MPT toolbox/Yalmip)
- Solve an optimization problem

  (Matlab/Optimization Toolbox, NPSOL)
- Verify that the closed-loop system performs as desired (avoid infeasibility/stability)
- Make sure it runs in real-time and code/download for the embedded platform

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Class Topics

(Subject to changes)

Week 1/2: Modeling

- Cont. time vs Discrete time , Transfer function vs State Space, Linear vs Nonlinear

Week 3/4/5: Fundamentals of Optimization

- Basis Concept of Optimizations
- Linear Program, Quadratic Program, Nonlinear program.
- Polyhedral and their manipulation
- Piecewise-linear Optimization.

Week 6/7/8: Constrained Optimal Control

- General Formulation of constrained control problems
- Linear 2-norm, Linear 1-norm, nonlinear
- Solution via batch approach and dynamic programming

Week 9/10/11: Predictive Control

- General formulation, Fundamental Properties
- Invariant set and Feasibility
- Soft constraints and tracking

Week 12/13/14/15: Examples and Review

Matlab Oriented

Initial Remarks

- **Continuous-Time versus Discrete-Time**
- **MPC Name**
- **s-functions**

Movie