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Vehicle yaw control using a robust H_∞ observer-based fuzzy controller design

M. Oudghiri, *Student Member, IEEE*, M. Chadli and A. El Hajjaji
Centre de Robotique d'Electrotechnique et d'Automatique, Université de Picardie Jules Verne
Email: mohammed.oudghiri, mohammed.chadli, ahmed.hajjaji@u-picardie.fr

***Abstract*— A vehicle dynamics control system has been developed in this study for improving vehicle yaw rate dynamics under critical motions. The system consists of a fuzzy robust H_∞ estimated state feedback controller. The variation of the road adhesion conditions is considered and represented by additive parametric uncertainties in the vehicle Takagi-Sugeno fuzzy model. The vehicle lateral velocity is unavailable for measurement and it is estimated by a fuzzy observer. The gains of the fuzzy controller and fuzzy observer are determined in terms of Linear Matrix Inequality ($\mathcal{LMI}$). Simulations have been conducted to evaluate the performance of the closed loop system under various driving conditions and results are presented in this paper.**

***keywords* : Vehicle Dynamics ; Takagi-Sugeno fuzzy model ; H_∞Robust Control ; $\mathcal{LMI}$.**

I. INTRODUCTION

These last years, many efforts are being made in automotive industries to develop new active control systems which improve the vehicle response in critical situations, enhance passenger comfort and reduce the driver's workload. Some of these systems have already been commercialized and becoming a standard equipment in many vehicles (ABS, ASR, VDC, DYC, TCS ...). However, these systems are still not optimal and they are an object of actual research activities from both industrial and academic sides [15], [16], [17], [18].

In this work, a lateral vehicle dynamics control system has been developed. The system consists of a robust controller based on an observer to estimate the lateral vehicle velocity. As such design methodology is based on fuzzy robust H_∞ control technique, a representation of the nonlinear model of lateral vehicle dynamics by an uncertain Takagi-Sugeno (T-S) fuzzy model will be considered [8]. This representation largely used these lasts years (see for example [7], [9], [12], [13] and references therein) allows to describe the vehicle dynamics in large domain, takes account parametric variations and uses Linear Matrix Inequalities (LMI) techniques [11] easy to implement using classical numerical tools.

There have been several recent studies concerning the stability and the synthesis of controllers and observers for T-S models [1], [2], [3], [5], [7], [12]. For example in [2] new stability conditions are obtained by relaxing the stability conditions of the previous works and particularly these derived in [12] allowing the symmetrical off-diagonal matrix blocks to be non-symmetrical. However the majority of these papers deals with a state feedback control design that requires all states of systems to be measured. In many cases, this requirement is too restrictive and some papers have proposed design conditions for robust observer-based controller by considering parametric uncertainties [4], [6], [13], [9], [10]. Recently, a new design method of observer-based H_∞ control for T-S fuzzy systems is given in [2]. Unfortunately the proposed method needs two-step procedure to design controller and then observer. To overcame this drawback, the design method given by [14] has the advantage to be solved in one step. However, in these two last papers, parametric uncertainties are not considered. Consequently the robustness of the closed-loop fuzzy model is not guaranteed. Based on these works, uncertainties are introduced in this paper. The proposed stability conditions of the closed loop system are given in terms of LMI and can be solved in a single-step procedure.

The structure of this paper is organized as follows : the second section describes the used vehicle model in uncertain T-S fuzzy representation. In section 3 the proposed control structure is introduced and its design is described. In section 4, simulation results are presented in order to show the effectiveness of the presented control strategy. Finally, concluding remarks are made in section 5.

Notations : The symbol $*$ denotes the transpose elements in the symmetric positions. $P > 0$ means a positive-definite symmetric matrix P.

II. MODEL STRUCTURE DESCRIPTION

A. Nonlinear Vehicle Model

Control design will be worked out on the basis of two freedom degrees vehicle lateral dynamics model as reported in Figure (1). The employed model is based on the following hypothesis :

– No rear wheel steering angle.
– Road wheel steer angle and vehicle sideslip angle are small enough to linearize their trigonometrical functions.
– Vehicle speed is a known parameters, vehicle longitudinal acceleration is low or equal to zero.

Thus, for the considered model, dynamic equations are the following :

$$
\begin{aligned}
m\dot{v}(t) + mur(t) &= 2F_{yf}(t) + 2F_{yr}(t), \\
J_z\dot{r}(t) &= 2a_f F_{yf}(t) - 2a_r F_{yr}(t) + M_z(t),
\end{aligned} \tag{1}
$$

where u and v are the components of the vehicle velocity along longitudinal and lateral principle axis of the vehicle body, r is the yaw rate, m is the vehicle mass, J_z is the moment of inertia around the vertical axis, a_f and a_r are the distances between the center of gravity and the front and rear axles, respectively, while M_z is the control input, which must be determined from the control law. F_{yf} and F_{yr} are the front and rear tyre lateral forces.

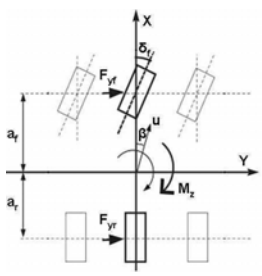

Fig. 1. Bicycle model (2ddl)

B. Takagi-Sugeno Fuzzy Vehicle Model

Using the T-S fuzzy representation, the nonlinear cornering forces can be approximated by [21]. They are given as functions of tire slip angles by the following expressions

$$
\begin{cases} F_{yf}(t) = \sum_{i=1}^{2} h_i(\alpha_f) C_{fi}(\mu)\alpha_f(t) \\ F_{yr}(t) = \sum_{i=1}^{2} h_i(\alpha_r) C_{ri}(\mu)\alpha_r(t) \end{cases} \tag{2}
$$

where C_{fi} and C_{ri} are the front and rear cornering stiffnesses coefficients, which vary according to the road adhesion μ, h_i (i=1,2) are membership functions satisfying the following conditions

$$
\begin{cases} \sum_{i=1}^{2} h_i(\alpha_f) = 1, \\ 0 \le h_i(\alpha_f) \le 1, \forall i = 1, 2. \end{cases} \tag{3}
$$

α_f and α_r are the slip angles of the front and rear tires, respectively. They are defined as follows

$$
\begin{aligned}
\alpha_f &= -\frac{v + a_f r}{u} + \delta_f(t), \\
\alpha_r &= -\frac{v - a_r r}{u},
\end{aligned} \tag{4}
$$

where δ_f is the road wheel steer angle, commanded by the driver.

After replacing the lateral forces F_{yf} and F_{yr} in the nonlinear model (1) by their fuzzy expressions (2), we obtain the fuzzy model of lateral vehicle dynamics as follows

$$
\begin{aligned}
\dot{x}(t) &= \sum_{i=1}^{2} h_i(\alpha_f)(A_i x(t) + BM_z(t) + B_{fi}\delta_f(t)), \\
z(t) &= \sum_{i=1}^{2} h_i(\alpha_f) C_{1i} x(t), \\
y(t) &= \sum_{i=1}^{2} h_i(\alpha_f)(C_{2i} x(t) + D_i \delta_f(t))
\end{aligned} \tag{5}
$$

where $x = \begin{bmatrix} v & r \end{bmatrix}^T$, $y = \begin{bmatrix} a_y & r \end{bmatrix}^T$, $z = r$

$z(t)$ is the controlled output variable and $y(t)$ is the output vector. It consists of measurements of the lateral acceleration a_y and the yaw rate r about the center of gravity.

$$
A_i = \begin{bmatrix} -2\frac{C_{fi}+C_{ri}}{mu} & -2\frac{C_{fi}a_f - C_{ri}a_r}{mu} - u \\ -2\frac{C_{fi}a_f - C_{ri}a_r}{J_z u} & -2\frac{C_{fi}a_f^2 + C_{ri}a_r^2}{J_z u} \end{bmatrix}, \tag{6}
$$

$$
B_{fi} = \begin{bmatrix} 2\frac{C_{fi}}{mu} \\ 2\frac{a_f C_{fi}}{J_z} \end{bmatrix}, B = \begin{bmatrix} 0 \\ \frac{1}{J_z} \end{bmatrix}, D_i = \begin{bmatrix} 2\frac{C_{fi}}{m} \\ 0 \end{bmatrix}, \tag{7}
$$

$$
\begin{aligned}
C_{1i} &= \begin{bmatrix} 0 & 1 \end{bmatrix}, \\
C_{2i} &= \begin{bmatrix} -2\frac{C_{fi}+C_{ri}}{mu} & -2\frac{C_{fi}a_f - C_{ri}a_r}{mu} \\ 0 & 1 \end{bmatrix},
\end{aligned} \tag{8}
$$

C. Uncertain T-S Fuzzy Vehicle Model

Vehicle yaw dynamics may show unexpected dangerous behavior in presence of unusual external conditions such as lateral wind force, variation of the road adhesion coefficient, etc., so to deal with this problem, the active control system has to be robust and appropriate uncertainties have to be introduced as follows

$$
\begin{aligned}
\dot{x}(t) &= \textstyle\sum_{i=1}^{2} h_i(\alpha_f)((A_i + \Delta A_i)x(t) + BM_z(t) \\
&\qquad + B_{fi}\delta_f(t)), \\
z(t) &= \textstyle\sum_{i=1}^{2} h_i(\alpha_f) C_{1i} x(t), \\
y(t) &= \textstyle\sum_{i=1}^{2} h_i(\alpha_f)((C_{2i} + \Delta C_{2i})x(t) + D_i\delta_f(t)),
\end{aligned} \tag{9}
$$

we assume that the uncertainties can be formulated as follows

$$
\Delta A_i = D_{Ai} F_i(t) E_{Ai}, \quad \Delta C_i = D_{Ci} F_i(t) E_{Ci} \tag{10}
$$

where E_{Ai}, E_{Ci}, D_{Ai} and D_{Ci} are known real matrices of appropriate dimensions that characterize the structures of uncertainties, $F_i(t)$ $(i = 1, 2)$ are unknown matrices such that $F(t)_i^T F(t)_i < I$ $(i = 1, 2)$ with I is the identity matrix of appropriate dimension.

III. ROBUST CONTROL STRATEGY

Currently, there are many practical considerations that inhibit production vehicles from using sensors of lateral velocity such as high cost, signal degradation, and lost of signal during certain weather/environment conditions. This problem can be overcome by introducing the observer theory.

A. T-S Observer-Based Controller

We define the fuzzy controller as follows

$$M_z(t) = \sum_{i=1}^{2} h_i(\alpha_f) K_i \hat{x}(t), \tag{11}$$

where $\hat{x}(t) \in \Re^2$ is the estimated state and $K_i \in \Re^{1\times 2}(i = 1, 2)$ are the controller gains to be determined.

Based on T-S fuzzy model of the vehicle (5), the structure of the observer is defined as

$$\begin{aligned} \dot{\hat{x}}(t) &= \sum_{i=1}^{2} h_i(\alpha_f)(A_i\hat{x}(t) + BM_z(t) + B_{fi}\delta_f(t) \\ &- L_i(y(t) - \hat{y}(t))), \\ \hat{y}(t) &= \sum_{i=1}^{2} h_i(\alpha_f)(C_{2i}\hat{x}(t) + D_i\delta_f(t)) \end{aligned} \tag{12}$$

where $\hat{y}(t) \in \Re^2$ is the estimated output. $L_i \in \Re^{2\times 2}(i = 1, 2)$ are the observer gains to be determined.

B. Robust H_∞ Fuzzy Control Technique

In the previous paper [6], asymptotic stability conditions allowing to determine the fuzzy observer and the fuzzy controller (11) are presented. However the obtained conditions are nonlinear and need two-step procedure for resolution. In the following, we propose a new H_∞controller that allows to obtain the stability conditions in terms of LMIs which are less conservatives and can be resolved in a single step.

Firstly let us recall the following lemmas which constitute the essential for the proposed design.

Lemma 1: [6] Given constant matrices X and $Y$of appropriate dimensions for $\forall \epsilon > 0$, the following inequality holds :

$$X^TY + Y^TX \leq \epsilon X^TX + \frac{1}{\epsilon}Y^TY \tag{13}$$

Lemma 2: [2] For a given positive number $\gamma > 0$ if there exist matrices $P > 0$, M_i and Z_{ij} $i < j$, $i, j = 1, 2, \cdots, r$ (r is the number of fuzzy IF-THEN rules) satisfying the following matrix inequalities

$$\begin{pmatrix} \bar{A}_{ii}^T P + P\bar{A}_{ii} & PB_{1i} \\ * & -\gamma^2 I \end{pmatrix} < Z_{ii}$$

$$\begin{pmatrix} (\bar{A}_{ij} + \bar{A}_{ji})^T P + P(\bar{A}_{ij} + \bar{A}_{ji}) & P(B_{1i} + B_{1j}) \\ * & (B_{1i} + B_{1j})^T P \end{pmatrix} < Z_{ij} + Z_{ij}^T$$

$$H_k = \begin{pmatrix} Z_{11} & \cdots & Z_{1r} & V_{1k}^T \\ \vdots & \ddots & \vdots & \vdots \\ * & \cdots & Z_{rr} & V_{rk}^T \\ * & \cdots & V_{rk} & -I \end{pmatrix} < 0, \quad k = 1, \cdots, r$$

where

$\bar{A}_{ij} = A_i + B_{2i}K_j$, $\quad V_{ij} = C_{1i} + D_{1i}K_j$

Then the controller $u(t) = \sum_{i=1}^{r} h_i K_i x(t)$ makes the H_∞ norm of fuzzy system (14)

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^{r} h_i(A_i x + B_{1i}w(t) + B_{2i}u(t)) \\ z(t) &= \sum_{i=1}^{r} h_i(C_{1i}x(t) + D_{1i}u(t)) \end{aligned} \tag{14}$$

less than γ, where $K_i = M_i Z^{-1}, i = 1, \cdots, r$.

Lemma 3: [20] Consider a negative definite matrix $\Omega < 0$.

Given a matrix X of appropriate dimension such that $X^T\Omega X < 0$, then $\exists\alpha \in R$ such that $\alpha > 0$ and $X^T\Omega X \leq -2\alpha X - \alpha^2\Omega^{-1}$.

The proof of the following result is based on the above lemmas (for more detail see annex).

Theorem 1: For a given positive number $\gamma > 0$, some positive scalars α, ϵ_{2ij}, ϵ_{3ij}, ϵ_{4ij}, ϵ_{5ij}. If there exist matrices Z, Y, M_i, J_i, Z_{ij} and a positive scalars ϵ_{1ij}, ϵ_{6ij} with $Z_{ji} = Z_{ij}^T$ satisfying the following $\mathcal{LMI}$

$$\begin{pmatrix} \Theta_{ii} & \Lambda_{ii} \\ * & \Psi_{ii} \end{pmatrix} < Z_{ii} \quad i = 1, 2 \tag{15}$$

$$\begin{pmatrix} \Theta_{ij} + \Theta_{ji} & \Lambda_{ij} + \Lambda_{ji} \\ * & \Psi_{ij} + \Psi_{ji} \end{pmatrix} < Z_{ij} + Z_{ji} \quad i < j \tag{16}$$

$$\begin{pmatrix} Z_{11} & \cdots & Z_{1r} & V_{1k}^T \\ \vdots & \ddots & \vdots & \vdots \\ Z_{r1} & \cdots & Z_{rr} & V_{rk}^T \\ V_{1k} & \cdots & V_{rk} & -I \end{pmatrix} < 0, \; k = 1, 2 \tag{17}$$

where $\Theta_{ij}, \Lambda_{ij}$ and Ψ_{ij} are given in (19), (20) and (21) and

$$V_{ik} = \begin{pmatrix} C_{1i} & 0 \end{pmatrix} \tag{18}$$

Then H_∞ control performance is guaranteed for the fuzzy model (9) via the fuzzy observer-based controller (11) where $K_i = M_i Z^{-1}$ and $L_i = Y^{-1}J_i$

Proof : See Annex.

2

$$\Theta_{ij} = \begin{pmatrix} \begin{array}{c}(ZA_i^T + M_j^T B_i^T) + (ZA_i^T + M_j^T B_i^T)^T + \rho^{-2} B_{1i} B_{1i}^T \\ +\epsilon_{1ii} D_{Ai} D_{Ai}^T + \epsilon_{6ij} D_{Bi} D_{Bi}^T\end{array} & ZE_{Ai}^T & M_j^T E_{Bi}^T & ZD_{Bi} & ZE_{Ai}^T & M_j^T E_{Bi}^T & ZE_c^T \\ E_{Ai} Z & -\epsilon_{1ii} I & 0 & 0 & 0 & 0 & 0 \\ E_{Bi} M_j & 0 & -\epsilon_{2ij}^{-1} I & 0 & 0 & 0 & 0 \\ D_{Bi}^T Z & 0 & 0 & -\epsilon_{2ij} I & 0 & 0 & 0 \\ E_{Ai} Z & 0 & 0 & 0 & -\epsilon_{4ii}^{-1} I & 0 & 0 \\ E_{Bi} M_j & 0 & 0 & 0 & 0 & -\epsilon_{3ij}^{-1} I & 0 \\ E_C Z & 0 & 0 & 0 & 0 & 0 & -\epsilon_{5ii}^{-1} I \end{pmatrix} \tag{19}$$

$$\Lambda_{ij} = \begin{pmatrix} B_{2i} M_j & 0 \\ 0 & 0 \end{pmatrix} \tag{20}$$

$$\Psi_{ij} = \begin{pmatrix} -2\alpha Z & M_j^T E_{Bi}^T & \alpha I & 0 & 0 & 0 & 0 & \\ E_{Bi} M_j & -\epsilon_{3ij}^{-1} I & 0 & 0 & 0 & 0 & 0 & 0 \\ E_{Bi} M_j & 0 & -\epsilon_{6ij} & 0 & 0 & 0 & 0 & 0 \\ \alpha I & 0 & 0 & A_i^T Y + C_{2j}^T J_i^T + Y A_i + J_i C_{2j} & Y D_{Bi} & Y D_{Ai} & Y D_{Bi} & J_i D_c \\ 0 & 0 & 0 & D_{Bi}^T Y & -\epsilon_{3ij} I & 0 & 0 & 0 \\ 0 & 0 & 0 & D_{Ai}^T Y & 0 & -\epsilon_{4ii} I & 0 & 0 \\ 0 & 0 & 0 & D_{Bi}^T Y & 0 & 0 & -\epsilon_{3ij} I & 0 \\ 0 & 0 & 0 & D_c^T J_i^T & 0 & 0 & 0 & -\epsilon_{5ii} I \end{pmatrix} \tag{21}$$

IV. NUMERICAL ILLUSTRATIONS

Consider the uncertain T-S model of the vehicle lateral dynamics given by (6) ∽ (10). The considered nominal stiffness coefficients are :

Nominal stiffness coefficients	C_{f1}	C_{f2}	C_{r1}	C_{r2}
Values	60712	4812	60088	4555

By solving LMI (15)-(16)-(17), we obtain the following controller and observer gains :

$$K_1 = 10^3 \left[\begin{array}{cc} -0.550 & -6.408 \end{array} \right], \quad K_2 = 10^3 \left[\begin{array}{cc} 0.794 & -6.426 \end{array} \right]$$

$$L_1 = \left[\begin{array}{cc} -0.9283 & 4.6052 \\ -2.0011 & 12.2327 \end{array} \right], \quad L_2 = \left[\begin{array}{cc} 0.0034 & 2.5205 \\ -2.5034 & 6.1203 \end{array} \right]$$

All the simulations are realized on the nonlinear model given in (1) with the following consideration :

- Vehicle speed : $20\, ms^{-1} \leq u \leq 30\, ms^{-1}$
- Adhesion coefficient : $0.4 \leq \mu \leq 1$
- Front wheel steer angle (rad) : $-0.06 \leq \delta_f \leq 0.06$

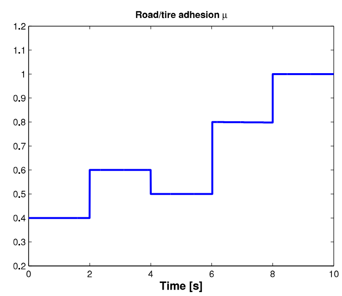

Fig. 2. Variation of the road/tire adhesion

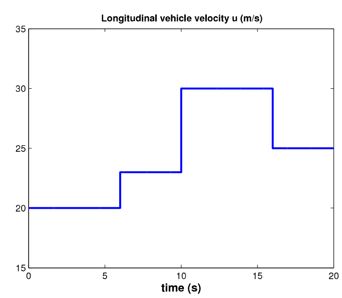

Fig. 3. Variation of longitudinal vehicle speed

In order to show the robustness of the proposed law control , we have considered variations of both road/tire adhesion as given in figure 2 and vehicle speed as given in figure 3. In figures (5, 6, 7), the understeering performance and the good estimation of the vehicle state variables are showed for the considered steering pad maneuvers (figure 4). We denote that in these driver conditions, without direct moment control the vehicle has an unstable behavior [21]. We can see that the proposed robust observer based controller makes possible to improve stability and safety under difficult driving condition and even when the measurement of the sideslip is unavailable.

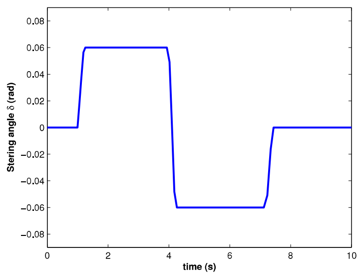

Fig. 4. Profile of the road wheel steer angle

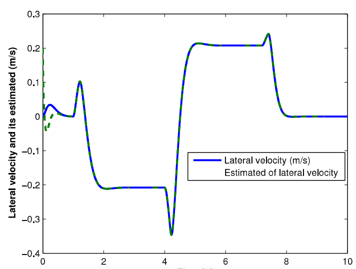

Fig. 5. Time evolution of lateral velocity and its estimated, initial conditions : $v(0) = 0,\ \hat{v}(0) = 0.2$.

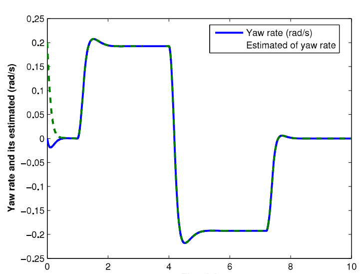

Fig. 6. Time evolution of yaw rate and its estimated, initial conditions : $r(0) = 0,\ \hat{r}(0) = 0.2$.

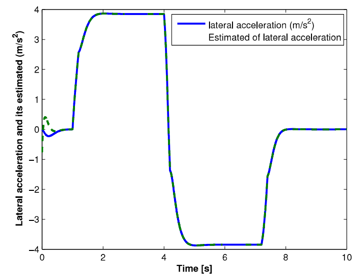

Fig. 7. Time evolution of lateral acceleration and its estimated

V. CONCLUSION

A robust stabilization of lateral vehicle dynamics satisfying the H_∞ performance requirement is investigated when the road adhesion conditions change and the lateral velocity is unavailable for measurement. The proposed observer-based controller are given in LMI formulation and designed using numerical tools (such LMI Toolbox for Matlab). From simulation results, the performances of the designed robust observer-based controller are satisfactory and the capability of this controller are shown under critical situations.

VI. ANNEX

Consider the vehicle fuzzy system given in (9), and define observation error as :

$$e(t) = x(t) - \hat{x}(t) \tag{22}$$

from (9), (11) and (22), we obtain

$$\begin{aligned}\dot{x}(t) = \textstyle\sum_{i=1}^{2} h_i(\alpha_f)((A_i + BK_i + \Delta A_i)x \\ +B_{fi}\delta_f(t) + BK_i e)\end{aligned} \tag{23}$$

$$\begin{aligned}\dot{e}(t) = \textstyle\sum_{i=1}^{2}\sum_{j=1}^{2} h_i(\alpha_f)h_j(\alpha_f)((A_i + L_i C_{2j})e \\ -(\Delta A_i + L_i \Delta C_i)x)\end{aligned} \tag{24}$$

$$\tag{25}$$

Then the augmented system can be expressed as the following form :

$$\begin{pmatrix} \dot{\bar{x}} \\ z \end{pmatrix} = \sum_{i=1}^{2}\sum_{j=1}^{2} h_i(\alpha_f)h_j(\alpha_f)\begin{pmatrix} \bar{A}_{ij} & \bar{B}_i \\ \bar{C}_i & 0 \end{pmatrix}\begin{pmatrix} \bar{x} \\ \delta_f \end{pmatrix} \tag{26}$$

where

$$\bar{x}(t) = \begin{pmatrix} x \\ e \end{pmatrix}$$

$$\bar{A}_{ij} = \begin{pmatrix} A_i + BK_j + \Delta A_i & BK_j \\ -(\Delta A_i + L_i \Delta C_{2j}) & A_i + L_i C_{2j} \end{pmatrix},$$

$$\bar{B}_i = \begin{pmatrix} B_{fi} \\ 0 \end{pmatrix}, \bar{C}_i = \begin{pmatrix} C_{1i} & 0 \end{pmatrix}$$

Choosing

$$P = \begin{bmatrix} X & 0 \\ 0 & Y \end{bmatrix} > 0 \tag{27}$$

and according to Lemma 2, the closed-loop fuzzy system given in (26) is stable with unitary H_∞ disturbance attenuation if

$$\sum_{i=1}^{2}\sum_{j=1}^{2} h_i(\alpha_f)h_j(\alpha_f)(G_{ij} + \Delta G_{ij}) < 0 \tag{28}$$

with

$$G_{ij} = \begin{bmatrix} \nu_{ij}^{11} & XB_{1i}B_{1i}^TX \\ * & \nu_{ij}^{22} \end{bmatrix}, \Delta G_{ij} = \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ * & \Omega_{22} \end{bmatrix} \tag{29}$$

where

$$\begin{aligned}
\nu_{ij}^{11} &= (A_i^T + K_j^T B_i^T)X + X(A_i^T + K_j^T B_i^T)^T + \rho^{-2}XB_{2i}K_j \\
\nu_{ij}^{22} &= (A_i^T + C_{2j}^T L_i^T)Y + Y(A_i^T + C_{2j}^T L_i^T)^T \\
\Omega_{11} &= (E_{Ai}^T F^T D_{Ai}^T + K_j^T E_{Bi}^T F^T D_{Bi}^T)X + \\
&\quad X(E_{Ai}^T F^T D_{Ai}^T + K_j^T E_{Bi}^T F^T D_{Bi}^T)^T \\
\Omega_{12} &= -E_{Ai}^T F^T D_{Ai}^T - K_j^T E_{Bi}^T F^T D_{Bi}^T - E_c^T F^T D_c^T L_i^T)Y \\
\Omega_{22} &= -K_j^T E_{Bi}^T F^T D_{Bi}^T Y - Y D_{Bi} F E_{Bi} K_j
\end{aligned}$$

By applying Lemma 1, we obtain

$$G_{ij} + \Delta G_{ij} < \begin{bmatrix} \Phi_{11} & XB_{2i}K_j \\ * & \Phi_{22} \end{bmatrix} \tag{30}$$

with $\Phi_{11} = (A_i^T + K_j^T B_i^T)X + X(A_i^T + K_j^T B_i^T) + \rho^{-2}XB_{1i}B_{1i}^TX + \epsilon_{1ii}^{-1}E_{Ai}^TE_{Ai} + \epsilon_{1ii}XDD^TX + \epsilon_{2ij}K_j^TE_{Bi}^TE_{Bi}K_j + \epsilon_{2ij}^{-1}D_{Bi}D_{Bi}^T + \epsilon_{4ii}E_{Ai}^TE_{Ai} + \epsilon_{3ij}K_j^TE_{Bi}^TE_{Bi}K_j + \epsilon_{5ii}E_c^TE_c + \epsilon_{6ij}XD_{Bi}D_{Bi}^TX$

$\Phi_{22} = (A_i^T + C_{2j}^TL_i^T)Y + Y(A_i^T + C_{2j}^TL_i^T)^T + \epsilon_{3ij}K_j^TE_{Bi}^TE_{Bi}K_j + \epsilon_{3ij}^{-1}YD_{Bi}D_{Bi}^TY + \epsilon_{4ii}^{-1}YD_{Ai}D_{Ai}^TY + \epsilon_{3ij}^{-1}YD_{Bi}D_{Bi}^TY + \epsilon_{5ii}^{-1}YL_iD_cD_c^TL_i^TY + \epsilon_{6ij}^{-1}K_j^TE_{Bi}^TE_{Bi}K_j$

Pro- and post-multiplying both sides of (28) by $\begin{bmatrix} Z & 0 \\ 0 & Z \end{bmatrix}$ with $Z = X^{-1}$ and denoting $M_i = K_iZ, J_i = YL_i$, yields

$$\begin{bmatrix} Z & 0 \\ 0 & Z \end{bmatrix}(G_{ij}+\Delta G_{ij})\begin{bmatrix} Z & 0 \\ 0 & Z \end{bmatrix} < \begin{bmatrix} Z\Phi_{11}Z & B_{2i}M_j \\ * & Z\Phi_{22}Z \end{bmatrix} \tag{31}$$

with

$Z\Phi_{11}Z = (ZA_i^T + M_j^TB_i^T) + (ZA_i^T + M_j^TB_i^T)^T + \rho^{-2}B_{1i}B_{1i}^T + \epsilon_{1ii}^{-1}ZE_{Ai}^TE_{Ai}Z + \epsilon_{1ii}DD^T + \epsilon_{2ij}M_j^TE_{Bi}^TE_{Bi}M_j + \epsilon_{2ij}^{-1}ZD_{Bi}D_{Bi}^TZ + \epsilon_{4ii}ZE_{Ai}^TE_{Ai}Z + \epsilon_{3ij}M_j^TE_{Bi}^TE_{Bi}M_j + \epsilon_{5ii}ZE_c^TE_cZ + \epsilon_{6ij}D_{Bi}D_{Bi}^T$

$Z\Phi_{22}Z = \epsilon_{3ij}M_j^TE_{Bi}^TE_{Bi}M_j + \epsilon_{6ij}^{-1}M_j^TE_{Bi}^TE_{Bi}M_j + Z\Pi Z$ and

$\Pi = (A_i^T + C_{2j}^TL_i^T)Y + Y(A_i^T + C_{2j}^TL_i^T)^T + \epsilon_{3ij}^{-1}YD_{Bi}D_{Bi}^TY + \epsilon_{4ii}^{-1}YD_{Ai}D_{Ai}^TY + \epsilon_{3ij}^{-1}YD_{Bi}D_{Bi}^TY + \epsilon_{5ii}^{-1}J_iD_cD_c^TJ_i^T$

Using Lemma 3 ($Z\Pi Z \leq -2\alpha Z - \alpha^2\Pi^{-1}$) and Schur complement, we prove the sufficient conditions of theorem 1.

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