# Contribution to the Integrated Control Synthesis of Road Vehicles

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***Abstract*—A nonlinear vehicle model with 22 motion degrees of freedom, used for synthesis of the system autopilot, was described in the paper. It was demonstrated how a controller can be designed on the basis of such relatively complex dynamic model, ensuring simultaneous motion stability of the vehicle in longitudinal, lateral and vertical directions, as well as the stability of roll, pitch, and yaw dynamics of the vehicle about corresponding axes. Vehicle automatic control was realized at two hierarchical levels: tactical and executional. Proposed scheme of the distributed hierarchy control enables control of entire vehicle dynamics as a multibody dynamic system. Control has been synthesized in such a way that the system satisfies set criteria of dynamic behavior. The synthesized controller improves system motion caused by action of casual, external perturbations, and internal inertial and centrifugal forces which appear as a consequence of an inadequately adapted ride velocity to the road geometry. Also, necessary information for estimation of unknown time-variant parameters of the dynamic model and of tire-road interaction were briefly given in the paper. Simulation results were presented and analyzed for one example of characteristic trajectory with perturbation of type of an uneven and slippery road, as well as a wind gust.**

***Index Terms*—Centralized control, distributed hierarchy control strategy, multibody dynamic systems, road vehicles, vehicle autopilot.**

## I. Introduction

THE ROAD vehicle represents a complex, expressedly nonlinear multibody dynamic system, consisting of rigid bodies and elastic elements. Such system possesses a great number of degrees of freedom (DOF's). Some of these motions, quality of which is essential for the safety and ride comfort, need to be controlled. Others, for example elastic modes of the vehicle chassis or of the particular elements of subsystems, should not be controlled. Vehicle stability, quality of dynamic behavior and its maneuvering capabilities, predominantly depend on the system design and on the performance of its active control subsystems. The choice of the best control strategy, within the limits of a technical feasibility, represents a complex task solving which demands knowledge of the vehicle dynamic behavior in different ride conditions. For these reasons, there appears a necessity of knowing the most accurate model. However, the conventional approach to control problem solving is to adapt the model complexity to the conditions of the application of the selected control procedure. Very often the linear optimal regulator with respect to the state variables was used [1], solving the typical LQ-problem by satisfying the set performance index. This regulator, as well as similar procedures which minimize the criterion function in the frequency domain or in the space of state coordinates, use linear [1], [2] or bilinear [3] quarter car model, i.e., suspension system model. Somewhat more complex model is the half car model or single-track-model, popularly called the “bicycle model” of the vehicle [4], [5]. This model approximates the vehicle dynamics with restricted number of DOF's [6]. It describes the vehicle dynamics in the longitudinal and lateral direction of motion, the dynamics of vehicle yawing as well as rotations of the front and rear tires about their vertical axes. The common characteristic of all aforementioned simplified models is that none of them fully describes the entire vehicle dynamics, but only the dynamics in the particular motion directions. Accordingly, the controllers for longitudinal and lateral motion are synthesized separately, as well as the controller of the vertical dynamics of wheel suspension. The latter is minimizing vibrations, i.e., undesirable vertical bobbing of the vehicle due to the variation of road-surface profile. These independently synthesized controllers are then coupled into one unique control system. Our approach represents the synthesis of system controller, based on the relatively complex model of an entire road vehicle.

The purpose of this paper is to demonstrate how a relatively complex, nonlinear, complete vehicle model can be implemented for the synthesis of the dynamic control system. One such model is capable to describe the nonlinear vehicle behavior in a sufficiently broad range of its state values and its input, control variables. On the other hand, linearized models are good approximations of the system only in some relatively narrow zones of linear dependence from their state variables. Out of these zones they are just rough approximations of their real behavior. Beside that, the simplified, decoupled models lose information about the cross-section interactions of the particular values, so that they have only limited practical applicability.

By the chosen approach of the *model based control* it is understood that real-time estimation of time-variant system parameters should be ensured, as well as the estimation of the interaction parameters of the vehicle and the road surface. The algorithm for the estimation of the stated parameters is based on known equations of the system model. Which control strategy, i.e., which control scheme will be chosen, depends on the control task which has to be realized. For road vehicles, several criteria which system should satisfy during its motion

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can be established. These are: motion stability, ride quality, ride comfort, criterion of minimal suspension system deflection and tire deflection, and the criterion of high level maneuvering capabilities of the vehicle. By *motion stability* we mean the stability of the prescribed, i.e., desired vehicle trajectory and the realization of the desired chassis orientation. *Ride quality* during driving refers to the elimination of sudden, jerky changes of the lateral acceleration of the vehicle and sudden changes of the yaw rate about the vertical vehicle axis. Good quality behavior, in that sense, means continual, smooth changes of velocities and accelerations of the given state values. *Ride comfort* represents elimination of unpleasant vibrations, i.e., minimization of vehicle body heave motion relative to the road surface. *Minimal suspension system deflection* and *tire deflection* relate to the dilations (compression or extensions) of springs of shock absorbers and to the dilations of the tire pneumatics. Large deformation amplitudes raise the danger of vehicle instability or tire defects due to large dynamic loads. By *maneuvering capabilities* of vehicle we mean the technical ability of the vehicle to change the driving course in a rather broad range, quickly and easily, with minimal changes of the control magnitudes. Satisfying the nominated criteria of vehicle control depends on the equipment of the vehicle that is at our disposal. In that sense, we can speak about completely automatic controlled vehicle. Such vehicles possess active control systems,[1] sensor systems, and, more recently, a system for communication with the environment. This system ensures information about the road geometry, as well as about the exact vehicle position on the road. By semiautomatic vehicle control, the existence of some of the active control systems which correct the commands of the operator (driver) is understood. Automatic vehicle control can possess three hierarchical levels: 1) strategic control level; 2) tactical control level; and 3) executive control level. The *strategic control level* enables the controller, based on information about the terrain topography and the momentary position of the vehicle in space, to determine the desired, optimal trajectory in the presence of obstacles. By this control level the use of the vision system, telecommunication and artificial intelligence elements is understood. The *tactical control level* determines the way, i.e., the "tactics" of the realization of the prescribed vehicle trajectory and the dynamics of the relative attitude deflection of vehicle body with respect to the horizontal road surface during motion. The *executive control level* is realized by the controllers at the actuators level. Directly, they produce the control forces and torques at the driving/braking subsystems and at the active suspension system. The synthesis of vehicle control will be treated in this paper at the tactical and executive control levels exclusively.

## II. Model of Vehicle Dynamics

A nonlinear vehicle model with 22 DOF's, with possibility of autonomous four-wheel driving (4WD) and four-wheel steering (4WS) was used for control synthesis, parameters estimation and simulation. This model is illustrated in Fig. 1. The mentioned model describes motion of the vehicle mass center (MC) in three coordinate directions and three rotations of the vehicle about its main axes of inertia. Also, the model describes dynamics of the vehicle suspension system (VSS) on all four wheels in the vertical direction, as well as the tire dynamics in the same direction. Each wheel possesses, beside vertical deflection, two extra DOF's: rotation about the horizontal axis with angular velocity $\omega_i$, $i = 1, \cdots, 4$, and rotation about the vertical axis with respect to the road surface. The second rotation represents a change of the ground steering angle $\delta_i$, $i = 1, \cdots, 4$. Vehicle body model is determinated by rigid body dynamics, while the model of the suspension system is described by the behavior of a mechanical oscillatory system. They are represented by their functional and symbolical schemes in Fig. 1. Tire model is determinated by a nonlinear, functional dependence between the longitudinal and lateral tire-road interaction forces, respectively, and a tire sideslip ratio, i.e., the corresponding tire side slip angle. The used vehicle model is based on the modified model by Peng and Tomizuka [7], who have used the original models by Lugner [8] and Sakai [9], [10]. The model from [7] has been structurally rearranged and extended by relations describing the vertical dynamics of tire pneumatic behavior. The mentioned model is expressed in a compact vector form, suitable for simulations and synthesis of control laws.

The defined model is valid in the case when the following assumptions can be adopted: 1) vehicle body represents a rigid body in a mechanical sense, supported by the road surface with four elastic subsystems, representing the VSS; 2) VSS possesses elastic properties only in vertical direction (perpendicular to the road surface), while it is considered that it is absolutely rigid in the longitudinal and lateral directions; and 3) deviation of the vehicle chassis position (orientation) with respect to the road surface is small due to its referent value when the vehicle is at rest. This means that approximations $\Phi \approx \sin(\Phi)$ and $\cos(\Phi) \approx 1$ are valid in case of small angles of pitch and roll; and 4) the road surface is practically horizontal, i.e., the angle of road side elevation is small (not more than a few degrees). Then the model of vehicle dynamics can be presented in the form

$$H(q)\ddot{q} + h(q, \dot{q}) = \tau + F_w(q, \dot{q}) \tag{1}$$

where the following symbols were used: $q = [x\ y\ z\ \Phi\ \theta\ \varepsilon]^T$ is a (6 × 1) vector of the global state variables, describing the position and orientation of the vehicle body MC with respect to the coordinate system fixed to the ground; $x$, $y$, $z$ are the positions of the vehicle MC along three coordinate directions in [m]; $\Phi$, $\theta$, $\varepsilon$ are the corresponding angles of roll, pitch, and yaw of the vehicle body in [rad]; $H(q)$ is a (6 × 6) inertia matrix, expressed in [kg] and [kgm$^2$], respectively, $h(q, \dot{q})$ is a (6 × 1) vector of gravitational and centrifugal forces acting in the vehicle MC, expressed in [N] and [Nm], respectively; $\tau$ is a (6 × 1) vector of driving forces and torques referred to the vehicle MC expressed in [N] and [Nm], respectively; $F_w(q, \dot{q})$ is a (6 × 1) vector of the external forces and torques, acting on the vehicle body during its motion along the road. Elements of this vector take into account forces and torques of tire rolling, resistance, aerodynamic resistance forces during motion, as

[1] Active suspension system, active driving/braking system, antilock braking system (ABS-system), etc.

Fig. 1. Functional-symbolic scheme of a road vehicle model with 22 DOF's of motion.

well as the damping torque of the yaw rate during cornering. The system possesses three control variables for each of the four wheels (see Fig. 1): *tire ground steering angle* $\delta_i$, *tire angular velocity* $\omega_i$, and *the active damping force* $P_{di}$ in the viscous cylinder of the VSS, where $i = 1, \cdots, 4$.

The vector of driving forces and torques $\tau$ in (1) has the elements shown in (2) at the bottom of the page, where $M_X$, $M_Y$, and $M_Z$ are the corresponding torques acting in the vehicle body MC (Fig. 1) about the axes of the coordinate system originating from this point. The elements of vector $\tau$ are functions of control variables $\tau = \tau(\delta, \omega, P_d)$, where vectors $\delta = [\delta_1 \;\; \delta_2 \;\; \delta_3 \;\; \delta_4]$, $\omega = [\omega_1 \;\; \omega_2 \;\; \omega_3 \;\; \omega_4]$, and $P_d = [P_{d1} \; P_{d2} \; P_{d3} \; P_{d4}]$ represent output signals of the vehicle actuators.

The longitudinal and lateral forces (with respect to the vehicle motion direction) in the joints of VSS (Fig. 1), are calculated from relations

[illegible] (3)

[illegible] (4)

[illegible] (2)

where $F_{xi}$ and $F_{yi}$ are longitudinal and lateral tire forces of interaction between the $i$th wheel and the road surface. They are nonlinear functions of the tire slip ratio $s_i$ and corresponding tire slip angle $\alpha_i$ [7]. These forces depend on the tire parameters and on the conditions of the road surface. They are calculated [7] by means of following relations in the cases of traction and braking.

*During Braking:*

[illegible] (5)

[illegible] (6)

*During Traction:*

[illegible] (7)

[illegible] (8)

Here: $\nu_{di}$ is the dynamic sliding friction coefficient of a wheel; $F_{zi}$ is tire load; $k_{ti}$, $h_{ti}$, and $q_{ti}$ are intermediate model parameters depending on tire slip angle and on the parameters of tire-road interaction. They are defined in [7].

Dynamic model of the active vehicle suspension system (AVSS) and tire pneumatics, considered as vehicle subsystems (Fig. 1), is a nonlinear function of the local state variables (see Fig. 1): suspension system deflection $x_1^i = z_s^i - z_t^i$, sprung mass velocity $x_2^i = \dot{z}_s^i$, tire pneumatic deflection $x_3^i = z_t^i - z_r^i$, and unsprung mass velocity $x_4^i = \dot{z}_t^i$ $(i = 1, \cdots, 4)$. Dynamic behavior of the AVSS in vertical direction can be described by two scalar differential equations

[illegible] (9)

where $P_{bi}$, $P_{ci}$, $T_{bi}$, $T_{ic}$, $M_{si}$, $M_{ti}$, and $g$ are damping force of the VSS, elastic force of the VSS, damping force of tire pneumatic, elastic force of tire pneumatic, sprung mass, tire mass, and gravity acceleration, respectively.

Payload $P_i$ of the AVSS in the suspension joint (Fig. 1) and the tire load $F_{zi}$ can be calculated from the relations

[illegible] (10)

## III. Control Strategy

The vehicle controller has to ensure motion stability, i.e., realization of the prescribed trajectory and velocity. Also, it has to reduce the relative attitude deflection of the vehicle chassis to minimum, relative to its referent position. The controller has the task of ensuring satisfactory ride quality, too. Sudden changes of the conditions of longitudinal and lateral accelerations, i.e., "jerky" motion, have to be avoided. Concerning ride comfort, it is essential that the designed vehicle controller ensures the minimization of the vertical vibrations and the minimization of tire deflections and suspension system deflection. This means that the control unit has a complex task of satisfying a broad spectrum of criteria simultaneously. A promising strategy, leading to this goal, could be the strategy of *distributed hierarchy control,* a version of which was realized in [11]. Our solution is based on the knowledge of the entire vehicle dynamics, so we shall call this control strategy *centralized dynamic control*. The notion of "hierarchical" strategy was introduced because it is being realized on two levels, *tactical* and *executive,* divided with respect to the dynamic modules: *vehicle body—active subsystems*. It is "distributed" because the global control signals from a higher tactical level are distributed as referent signals on the lower, executive level. The control strategy consists of the following. The controller at the tactical control level, based on information about the error of the global state values $q$ and $\dot{q}$, calculates the vector of control forces and torques which should act in the vehicle body MC in order to realize the desired motion. The control forces and torques calculated in this way are realized in indirect way, through the action of the vehicle actuators at the executive control level. These actuators operate within the AVSS, and also within the active driving/braking and active steering systems and can be installed on all four wheels.

### A. Synthesis of a Vehicle Autopilot Control

Problems of traffic and transport efficiency imperatively impose development of *automated highway systems* and *automated cruising vehicles*. Here, the role of a driver as an operator would be only to supervise the system functions and to react only in cases of certain vehicle failures, or in cases when vehicle approaches some unexpected obstacle on the road. Means of automatic control of longitudinal and lateral vehicle motion were presented within the PATH project [7], [12], intended for the traffic on specially equipped highways. Practical solutions of the sensor systems and the communications system between the vehicle and road were proposed in [12], since it is understood that high automatization of the vehicle includes the existence of the corresponding equipment, not only in the vehicle, but also along the highway. Operating of the vehicle autopilot demands information about exact vehicle position, its velocity, distances from fixed or moving obstacles, changes of trajectory geometric parameters, etc. This paper treats only the development of the control strategy and controller synthesis which should ensure the motion stability and quality under various riding conditions on an automated highway system.[2] Hereby it has been assumed that the vehicle trajectory has been known *a priori,* or that the path geometry can be estimated in real time. It has also been assumed that the vehicle position with respect to the desired trajectory can be measured at any time instant with sufficient accuracy.

*1) Tactical Control Level:* If in (1) we denote a $(6 \times 1)$ position and orientation vector of the vehicle body MC by

[2] Under the notion of automated highway system (AHS), a road or its section equipped with corresponding sensor and communications equipment will be understood. Such AHS enables vehicle riding in a fully automated traffic regime.

$q$ then $q_p = [x_p\ y_p\ z_p\ \Phi_p\ \theta_p\ \varepsilon_p]^T$ designates the programmed trajectory, along which we wish a vehicle to ride with prescribed velocity profile $\dot{q}_p$. The trajectory tracking error with respect to the position and velocity is defined by relations: $\Delta q = q - q_p$ and $\Delta\dot{q} = \dot{q} - \dot{q}_p$, respectively. External and internal disturbances of various nature act upon the vehicle tending to deviate its position from the desired trajectory. The vehicle controller should ensure stability of the system motion along the programmed path, without the manual operator action. The transition regime of the vehicle, in the case of perturbed motion, can be various in character. It can be influenced by imposing a desired behavior character, as it will be shown in the text to follow. The vector of control signals $\tau$ can be calculated on the basis of information about the deviations of the global state variables $\Delta q$ and $\Delta\dot{q}$ and based on the knowledge of the dynamic vehicle model (1). The form of such control law can be expressed in the following way:

[illegible] (11)

$$\Pi(\Delta q, \Delta\dot{q}) = -K_V\Delta\dot{q} - K_P\Delta q$$

(PD-regulator)

[illegible]

(PID-regulator)

where $\hat{H}$, $\hat{h}$, and $\hat{F}_w$ are corresponding estimated matrices and vectors of (1); the matrices $K_V$, $K_P$, and $K_I$ are 6 × 6 matrices. They represent the coefficient matrices of the vector polynomial $\Pi$, defining the character of curve families of desired transient responses. At the same time, they are matrices of velocity, position, and integral control gains of the PD or PID regulator. The calculated vector $\tau$ represents the vector of driving forces and torques which should act in the vehicle MC, in order to realize the desired programmed motion $q_p$ in the longitudinal $x$, lateral $y$, and vertical $z$ directions, as well as the stability of the chassis orientation with respect to the programmed values of the roll angle $\Phi$, pitch angle $\theta$, and yaw angle $\varepsilon$ about the main axes of inertia.

The values of gain matrices $K_P$, $K_V$, and $K_I$ are determined from the characteristic vector equation of the system, so that they satisfy the demands of dynamic system behavior along various coordinate directions in frequency domain. The procedure for determining the unknown control gains is the following. If we assume that the state variables can be measured with a desired accuracy degree and that the dynamic parameters of (1) can be determined sufficiently accurately by estimation, then the characteristic equation of the system can be written in the closed-loop form. From it, the unknown gains $K_P$, $K_V$, and $K_I$ can be determined in such way, that the system motion $q(t)$ will converge toward its nominal, prescribed value $q_p(t)$. In the case that the above stated assumption holds, it can be taken that $\hat{H}(q) \simeq H(q)$, $\hat{h}(q, \dot{q}) \simeq h(q, \dot{q})$, and $\hat{F}_w(q, \dot{q}) \simeq F_w(q, \dot{q})$. By substituting $\tau$ from the equation of (1) into the control law (11) and, introducing the vector polynomial $\Pi$ in the form of the PD or PID controller, two possible relationships are obtained

[illegible]

[illegible] (12)

By adopting the gain matrices $K_P = \mathrm{diag}\{k_P^i\}$, $K_V = \mathrm{diag}\{k_V^i\}$, and $K_I = \mathrm{diag}\{k_I^i\}$ $(i = 1, \cdots, 6)$ in diagonal form and applying the Laplace operator onto the previous relations, we arrive to the characteristic equation of the closed-loop system, which can for example take two of its forms

$$f_{\mathrm{PD}}(s) = s^2 + K_V s + K_P$$

or

$$f_{\mathrm{PID}}(s) = s^3 + K_V s^2 + K_P s + K_I \tag{13}$$

where $s$ is the (6 × 1) vector of the Laplace operands. By finding the "zeroes" of the vector characteristic equations $f_{\mathrm{PD}}(s)$ and $f_{\mathrm{PID}}(s)$, values of the desired gains $K_P$, $K_V$, and $K_I$ are determined. The diagonal members of these matrices represent the control gains $k_P^i$, $k_V^i$, and $k_I^i$ along the specific directions of system motion. By setting various values for these gains we can influence the character of the transient process, as well as on the total dynamic system behavior (response velocity, overshoot value, etc.). The gains $k_P^i$, $k_V^i$, and $k_I^i$ are determined in such way, that the angular frequency of closed-loop system $\omega_f$ in the $i$th direction lies within the boundaries of its natural frequency $\omega_f \le \omega_n$, as well as that the system is relatively damped $(\zeta = 0.80 - 0.90)$. Also, it has to be taken care not to prescribe some frequency value near or equal to the resonant system frequency. In the case of PD controller, in accordance with the previous notes, it can be written that the characteristic equation of the system in the $i$th direction is equal to expression: [illegible]. Then the control gain values, in the corresponding coordinate direction $i$, are determined from relations: $k_V^i = 2\zeta\omega_f$ and $k_P^i = \omega_f^2$. Gains for the PID controller are determined similarly as for the PD controller, with the remark that care has to be taken of the frequency $\omega_f$ and relative damping coefficient $\zeta$ of the dominant pair of poles, while the third pole is placed on the real axis to the left of the set dominant poles. In that way we have performed the synthesis of the vehicle controller on the tactical level, ensuring it the desired dynamic behavior. However, the real system trajectory will converge to the programmed one only if the accuracy of the dynamic parameters is high. Since this is not the case, robust control has to be synthesized (based on estimation of the practical stability) which includes ensuring the system stability in conditions of in advance specified parametric variation in a certain range. The problem of practical stability of a road vehicle modeled by a "full" dynamic model, will be considered in our next paper.

The control strategy proposed in this paper is represented by its control block-scheme in Fig. 2. The idea of hierarchical control at two levels is clearly emphasized in this figure. The vehicle possesses three active control systems,[3] by means

[3] These are: active wheel steering (AWS), active wheel driving (AWD) or braking (AWB), and active suspension system (ASS).

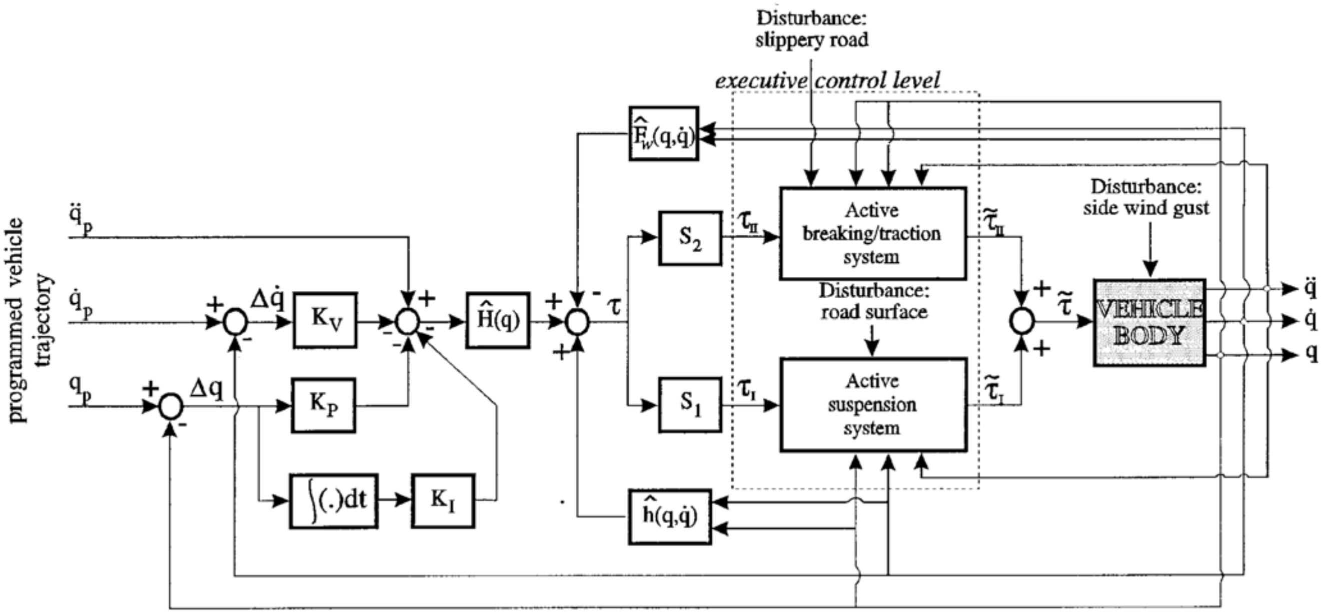



Fig. 2. Block-scheme of a road vehicle control—the tactical control level.

of which it realizes motions along six main directions. By considering Fig. 2, the calculated control signal $\tau$ is divided into two control signals $\tau_{\mathrm{I}} = [0 \;\; 0 \;\; \tau_3 \;\; \tau_4 \;\; \tau_5 \;\; 0]^T$ and $\tau_{\mathrm{II}} = [\tau_1 \;\; \tau_2 \;\; 0 \;\; 0 \;\; 0 \;\; \tau_6]^T$. They were obtained by multiplying the $\tau$ vector by the diagonal ($6 \times 6$) matrices of selectivity $S_1$ and $S_2$. Input, referent signals for the lower executive control level are calculated on the basis of $\tau_{\mathrm{I}}$ and $\tau_{\mathrm{II}}$ signals. The mentioned reference signals are: active damping force $P_{di}^r(t)$ in the $i$th viscous cylinder of the ASS, the reference angular velocity of the $i$th wheel $\omega_i^r(t)$ and the reference ground steering angle $\delta_i^r(t)$ of the $i$th wheel. They have to be realized at the outputs of the vehicle control actuators, in order that the controlled motion be ensured $q(t) \to q_p(t)$ in some finite time interval $t$. The driving loads $\tau_{\mathrm{I}}$ and $\tau_{\mathrm{II}}$ are distributed among the four suspension system joints (see Fig. 1), generating the longitudinal, lateral, and vertical force components $F_{Ai}$, $F_{Bi}$, and $P_i$ (where $i = 1, \cdots, 4$). The mentioned forces are further conveyed via the suspension system of each of the wheels to the vehicle actuators. It is desirable that the driving forces $\tau_{\mathrm{II}}$ should be distributed if possible *uniformly* in all the four joints of the ASS. This is for the reason of more economic energy consumption in the driving actuators, less dynamic load of the vehicle subsystems and more uniform wear of the tire pneumatics. The uniformity criterion of load distribution can be formulated mathematically in the form of the following relations:

$$
\begin{aligned}
F_{A1}^r &= F_{A2}^r = \frac{1}{2}\,\frac{\tau_1}{1+(l_1/l_2)} \\
F_{A3}^r &= F_{A4}^r = \frac{1}{2}\,\frac{\tau_1}{1+(l_1/l_2)}\,\frac{l_1}{l_2} \\
F_{B1}^r &= F_{B2}^r = \left(\frac{1}{2} - \frac{l_1}{l_1+l_2}\right)\tau_2 + \frac{1}{l_1+l_2}\,\tau_6 \\
F_{B3}^r &= F_{B4}^r = \frac{1}{2}\,\frac{1}{l_1+l_2}\,(\tau_2 l_1 - \tau_6)
\end{aligned}
\tag{14}
$$

which determine the load distribution in the individual ASS joints. These forces cause the motion in longitudinal and lateral direction. The parameters $l_1$ and $l_2$ represent the spans of vehicle between the MC and the front and rear wheel axes, respectively. The forces $F_{Ai}^r(t)$ and $F_{Bi}^r(t)$ represent the reference force values in the SS joints, based on which the variables $\omega_i^r$ and $\delta_i^r$ are determined in the further procedure. The variables $\omega_i$ and $\delta_i$ (for $i = 1, \cdots, 4$) should be varied iteratively in each time sample $t$, within the limits of realizable values (e.g., between their minimum and maximum values). Based on the knowledge of the nonlinear tire model, defined by relations (3)–(4), (5)–(6), and (7)–(8), the forces $F_{Ai}(t)$ and $F_{Bi}(t)$ are successively calculated, which correspond to the chosen pair of variables $\omega_i(t)$ and $\delta_i(t)$. Using the method of the *least square error* the best combination of $\omega_i(t)$, $\delta_i(t)$ $(i = 1, \cdots, 4)$ is detected $(\omega_i(t) \to \omega_i^r(t),\ \delta_i(t) \to \delta_i^r(t))$, which ensures the least sum $(S_i \to S_i^{\min})$ of the square errors [illegible]$^2$. The obtained values $\omega_i^r(t)$, $\delta_i^r(T)$ represent the reference values of the angular velocity of wheel $\omega_i(t)$ and the ground steering angle $\delta_i(t)$ $(i = 1, \cdots, 4)$.

The reference values of the viscous damping active force $P_{di}^r$ are determined based on the known value of vector $\tau_{\mathrm{I}}$ using relations (2) and (10). The unknown values are calculated by solving the defined system of equations with respect to unknown variables $P_{di}$ $(i = 1, \cdots, 4)$ in each sampling instant.

The output signals at the lowest hierarchical level represent controlled variables. These outputs of the system actuators $(P_{di}(t),\ \omega_i(t),\ \delta_i(t),\ i = 1, 4)$ cause the real driving forces and torques $\tilde{\tau}$ (see Fig. 2) which act in the vehicle body MC, grouped in vectors $\tilde{\tau}_{\mathrm{I}}$ and $\tilde{\tau}_{\mathrm{II}}$. Their vector sum $\tilde{\tau}$ corresponds to the vector of the driving forces and torques, defined by relation (2). In an ideal case, vectors $\tau$ and $\tilde{\tau}$ are equal, and they differ from one another only to an extent determined by the technical limitations of the actuators, the time lag of the actuators and the inaccuracy of the methods for measuring the vehicle subsystem variables $\delta$, $\omega$, $P_d$, etc.

*2) Executive Control Level:* This control level generates the control signals at the vehicle actuators in such way that they realize the forces and torques at their outputs. These actions are transferred to the vehicle chassis causing the corresponding forces and torques at the vehicle MC. In that way, they indirectly influence the system motion. The ASS produces an active damping force $P_{di}$ (Fig. 1) in the $i$th hydrocylinder of the viscous damper, so that the character of the system behavior is changed in the vertical direction. Suitable synthesized positional controllers distributed on all four wheels minimize amplitudes of the vertical accelerations in the SS joints. In that way they essentially influence the improvement of the ride comfort quality. Also, they influence the diminishing of the amplitudes of the vertical payload forces $P_i$ in SS joints, as well as the amplitudes of the suspension system deflections $e_i = z_s^i - z_t^i$ and the tire pneumatics defections $e_{ti} = z_t^i - z_r^i$ (Fig. 1). In that way, the negative dynamic effects on the wheels and in the SS joints are lowered, and their life service is prolonged.

Lateral motion stability, yawing stability about the vertical axis as well as the longitudinal motion stability can be influenced directly by means of distributed control of driving/braking actuators or by means of changing the ground steering angles on all four wheels. The global control forces and torques $\tau$, calculated at hierarchically higher control level, are realized by means of variation of the longitudinal $F_{xi}$ and lateral $F_{yi}$ tire interaction forces between the wheels and the road surface. These forces, by their intensity and direction, influence the change of the riding direction and the vehicle velocity, aimed at achieving the desired programmed motion $q_p(t)$. Since the forces $F_{xi}$ and $F_{yi}$ are functions of the tire rotation velocity $\omega_i$ and of their ground steering angles $\delta_i$ $(i = 1, \cdots, 4)$, the control problem reduces to determining the control vectors $\delta(t) i \omega(t)$ in every sampling instant $t$.

The calculated variables, $P_d(t)$, $\omega(t)$, and $\delta(t)$, are realized by using hydromechanical and electromechanical actuators of different kind. Accordingly, the real control magnitudes of the system are not those previously stated. Conventionally, these are input solenoid current of a hydrocylinder servovalve and input voltage in the armature winding (induct) of an electric motor. Which kind of servo actuator will be applied depends on concrete vehicle construction and subsystems design. Proposed control scheme of a vehicle autopilot can be realized by choosing the following actuators. A 4WD vehicle can be designed as a *serial hybrid drive-train* [13] with internal combustion engine and supplemental four electric motors powering all four wheels. An active 4WS can be realized by four linear actuators (hydrocylinders) that rotate four mounting brackets resulting in steering all car wheels. It has similarly been done in Ford 4WS vehicle with power steering, brakes, and automatic transmission [14].

Controlling a hydrocylinder means realizing the desired force on the piston which overcomes some external load. The applied cylinders within the active suspension system and active steering system are double-acting and represent control objects with typical aperiodic system response. Their dynamic model is nonlinear [15]. However, the dynamic object characteristics can be fairly well approximated by the transfer function [16]

$$G_{ob}(s) = \frac{K}{T_{ob}s + 1} e^{-\tau_{ob} s} \tag{15}$$

where $K$ represents the object static gain, $T_{ob}$ is the time constant and $\tau_{ob}$ is the transport object delay in hydrocylinder. For a more exact approximation of the object dynamic characteristics with a periodic step response, the procedure illustrated in Fig. 3(a) can be used. Let us assume that the values of $g_i$ $(i = 1, 2, \cdots, n)$ are measured on the object step response. Then the object discrete transfer function can be approximated [17] by

$$G_{ob}(z^{-1}) = g_1(z^{-1}) + g_2(z^{-2}) + \cdots + g_{n-1}(z^{-(n-1)}) + \frac{g_n z^{-n}}{1 - pz^{-1}} \tag{16}$$

or by

$$G_{ob}(z^{-1}) = \frac{g_1(z^{-1}) + g_2(z^{-2}) + \cdots + g_n(z^{-n})}{1 - pz^{-1}}$$

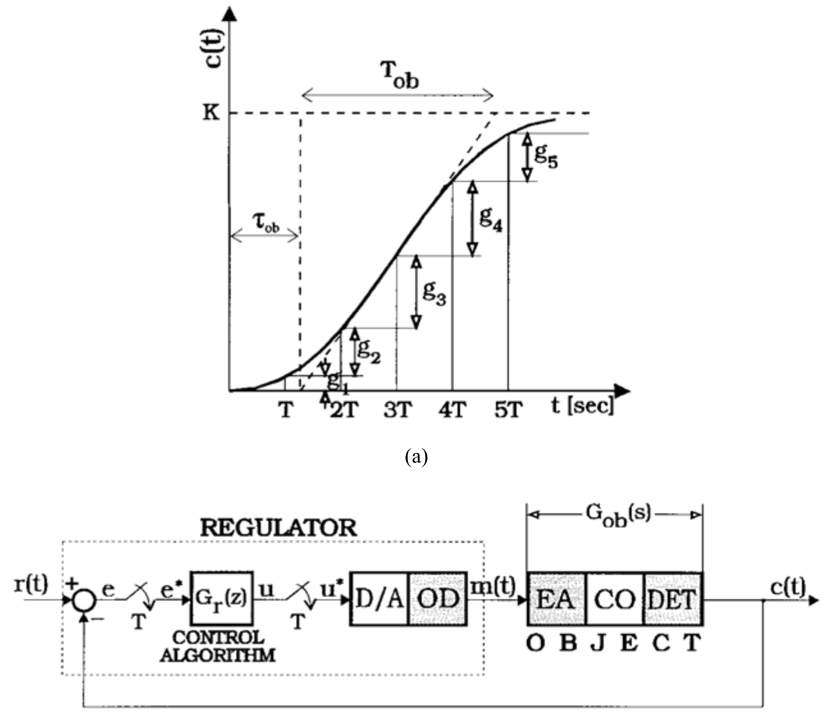
$$p = 1 - \frac{g_n}{K - \sum_{i=1}^{n-1} g_i} \tag{17}$$

where $p$ is a positive constant smaller than one, called the attenuating factor.

Since the discrete transfer function of object $G_{ob}(z^{-1})$ has been determined, the digital PID regulator, illustrated in Fig. 3(b), can be synthesized.

It is assumed that the digital regulator possesses in its input part an A/D converter generating the error samples $e^* = e(kT)$ of the controlled variable $c(t)$. The regulator possesses a microprocessor $G_r(z)$ which, based on the signals of error samples $e(kT)$ $(k = 1, 2, \cdots)$, realizes the control algorithm described by the transfer function of positional PID regulator [18] $G_r(z) = K_p + (K_i/(1 - z^{-1})) + K_d(1 - z^{-1})$ where $K_p$, $K_i$, and $K_d$ are the corresponding factors of the proportional-integral-differential actions. The synthesized PID regulator possesses at its output a D/A converter and an output device (OD) which transforms the signal at the D/A converter output into a desired standard signal, demanded by the end-effector actuator (EA). The EA, control object (CO) and the detector of the controlled variable (DET) altogether represent the control object $(G_{ob}(s))$ in broader sense. Tuning of the control gains $K_p$, $K_i$, and $K_d$ of the digital PID regulator can be done using some of the well-known procedures, for example [18].

The electric drives of the vehicle wheels can be suitably controlled by means of a microprocessor. The control manner itself depends on the electromotor type. *In this paper we will not engage in choosing the optimal actuator type, but the goal has been only to demonstrate the procedure, how the control signals from the higher hierarchical level are used for the control on a lower, executive control level.* The problem has been treated here exclusively at the level of chosen model,




(a)

(b)

Fig. 3. (a) Form of the unit step response of the hydrocylinder used in the active subsystems of a road vehicle. (b) A control system structure with PID regulator for controlling a hydrocylinder.

sufficient to show the corresponding effects by simulation. Of course, in real implementations more care must be taken of this problem. Since in previous presentation of the tactical control level we have explained how the reference values of the wheels angular velocities $\omega_i^r$ $(i = 1, \cdots, 4)$ are calculated, it remains to show how, in the case of a dc motor,[4] we can control its angular velocity $\omega_i(t)$. In Fig. 4 the structure of a microprocessor system [19] is presented for controlling the angular velocity of a dc motor. The system contains a continual control object (electric motor) and a digital part, realizing the digital control law. In the continual part there are the D/A converter, power amplifier (PA), with output voltage saturation at $\pm U$ and the dc motor. The transfer function $G_m(s)$ of the dc motor from the voltage $U(s)$ at the ends of the rotor winding to angular velocity $\Omega(s)$ of the motor shaft, can be approximated by a linear model and the D/A converter can be treated as a zero order sampling hold of the transfer function $G_h(s)$

$$G_m(s) = \frac{\Omega(s)}{U(s)} = \frac{K_m}{T_m s + 1}$$
$$G_h(s) = K_h \frac{1 - e^{-Ts}}{s} \tag{18}$$

where $K_m$ and $K_h$ are the corresponding gain factors, $T_m$ is the mechanical time constant and $T$ is the conversion time. The transfer function of the feedback elements $H(s)$, taking into account the behavior of the counting type incremental encoder, is

$$H(s) = \frac{B(s)}{\Omega(s)} = \frac{K_n}{2\pi} \frac{1 - e^{-Ts}}{s} \tag{19}$$

where $K_n$ is the number of markers on the encoder disk. In Fig. 4 the control structure with serial compensator of digital integrator type is presented with a proportional action in the local feedback loop. By shifting the $p$-action into the feedback loop the abrupt jump of the control variable $u(kT)$ is avoided in instants when the set value of the controlled variable $\omega_i(t)$ changes stepwise. The proposed control law can be written in the form of three relations, suitable for the programmed realization on the microprocessor

$$- \overline{K}_i e(kT) + u_1(kT - T)$$
$$u(kT) = -\overline{K}_p b(kT) + u_1(kT) \tag{20}$$

where $e(kT)$ and $u_1(kT)$, respectively, are the error samples and output signals of the integral compensator, while $b(kT)$ is the sample of the incremental encoder output in the $k$th sampling instant. When the PA is working in a linear regime with gain denoted by $K_A$, the characteristic equation of the system can be derived relatively easily [19] based on the block-diagram given in Fig. 4. It is of the form

$$+ \overline{K}(T - T_m + AT_m)(\overline{K}_p + \overline{K}_i)$$
$$a_1 = A - \overline{K}TA(\overline{K}_p + \overline{K}_i) - \overline{K}T\overline{K}_p + \overline{K}T_m(1 - A)(2\overline{K}_p + \overline{K}_i)$$
$$a_0 = \overline{K}TA\overline{K}_p - \overline{K}T_m\overline{K}_p(1 - A) \tag{21}$$

[4] For vehicles, realizing higher driving torques on their wheels, it is suitable to use triphase, asynchronous motors, which can be of different design and thus with various control possibilities.