1 Instructions

- Due date: Monday 14th December 8am
- Please drop it in the box for the class ME232 (my name is written on the box) third floor of Etcheverry
- Solve at least one of the two problems. You can get an "A+" with just one problem. Wrong/incomplete answers/results will not be used to remove points.
- Note that for the items marked with "ANSWER", I am expecting a comment/answer/result in your submitted solution.
- Email me if you need any clarification.

Thanks for taking the MPC class!!
Francesco Borrelli

2 Problem 1

Consider the Ball and Plate System (only one axis).

1. The control goal is to bring the ball from any admissible position to the origin of the state space (position and speed of the plate equal to zero and position and speed of the ball equal to zero).

2. Substitute the state feedback discrete time controller given in the past homework $u(k) = Kx(k)$ (remember is sampling $\Delta T = 30ms$) with an MPC controller with same sampling time.

3. Design the MPC controller in order to satisfy the state and input constraints. Recall that they are:

$$
\begin{aligned}
u_{\max} &= -u_{\min} = 10 \text{ V} \\
x_{\max} &= -x_{\min} = 30 \text{ cm} \\
\alpha_{max} &= -\alpha_{min} = 0.26 \text{ rad}
\end{aligned}
$$

   Ball speed and plate angular speed are not constrained (only by the physics of the system). If your software requires constraints on the full state, use large bounds for constraining ball speed and plate angular speed.

4. The weights P, Q and R, the horizon length N and the terminal constraint $\mathcal{X}_f$ are design variables.

5. Start with $P = Q$, and no $\mathcal{X}_f$

6. Simulate the closed loop system (DT controller and CT LTI system in state space form) when the system starts from the initial condition $x(0) = [20, 0, 0, 0]$ (plate and ball at rest and ball at distance 20cm from the origin)

7. (ANSWER) Find a choice of Q and R which guarantees persistent feasibility in the above simulation (and thus state and input constraint satisfaction). We will call this “original tuning”: the desired Q and R, $P = Q$ and no $\mathcal{X}_f$.

8. (ANSWER) Plot the corresponding closed loop behavior which shows convergence to the origin and constraints satisfaction.

9. (ANSWER) Consider the 2 dimensional space of initial ball positions and initial ball speeds (assume that the other two states, plate position and speed, are 0). For the original tuning, plot the set $\mathcal{X}_0$ of initial states for which the MPC is feasible at time 0.

10. In the next questions when the MPC design variables are not specified, use the one in the original tuning.

11. (ANSWER) What is the effect of Q and R on $\mathcal{X}_0$? Try to explain.

12. (ANSWER) What is the effect of N on $\mathcal{X}_0$? Try to explain.

The following are two facultative questions:

1. Use now $\mathcal{X}_f = 0$. What happens to $\mathcal{X}_0$? What is now the effect of N on $\mathcal{X}_0$? Try to explain.

2. Use the original tuning. Grid the set $\mathcal{X}_0$ and study for each grid point the persistent feasibility of the closed loop system as a function of Q and R.

3 Problem 2

Consider the vehicle model (for fuel consumption), assume zero grade

1. Use the simulink files you have already.

2. Take the full average speed profile provided on b-Space (101 highway).

3. Design an MPC controller which regulates the wheel force $F_w(k)$ which minimizes fuel consumption and satisfies the following constraints.

   - The car cannot speed up (or slow down) more than (less than) 20% of the average speed profile.
   - The completion time for full trip is less than α% of the one of the average speed profile.

4. (ANSWER) Provide the fuel consumption (and completion time) when the car travels (the full trip) at the average speed profile. Compare this with the fuel consumption (and completion time) when the car travels at the speed profile computed by the MPC you designed for some α.

5. Note that a critical parameter will be the horizon length N of the MPC. This is the length of the piece of road that the MPC will use to make predictions at each time step. It will affect the complexity of your problem.

6. (ANSWER) Study the effect of N on fuel consumption.