

On the Vehicle Stability Control for Electric Vehicle Based on Control Allocation

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Abstract—this paper proposes a vehicle stability control (VSC) scheme for a four-wheel-drive electric vehicle based on the control allocation techniques. The wheels of the electric vehicle are driven by the electric motor individually, which makes it more convenient to control tires forces and moments for stabilizing vehicle motion. But the over-actuated system is complex to control. The control allocation technique is well-known used in marine craft and aircraft to resolve over-actuated problems. By introduction of control allocation to electric vehicle control, a large degree of modularization of the different levels of control is obtained. A control analysis layer is designed using a sliding mode control, which specifies a desired yaw moment to satisfy the vehicle stability. And then the control allocation distributes this yaw moment among the individual wheels by commanding appropriate wheel longitudinal slip. Simulation results in the Carmaker[®] environment have shown that the proposed control scheme takes advantages of electric vehicle and enhances the vehicle stability.

Keywords—Electric Vehicle; VSC; Control Allocation

I. INTRODUCTION

Declining fossil resources as a main energy source for today's mobility and the greenhouse effect caused by CO₂ strongly demand for a rapid progress towards more efficient energy consumption. For this reason, the automotive industry researches intensely for alternative and complementary drive concepts. As a result, new resource vehicle developed rapidly, especially the electric vehicle and hybrid vehicle. However, the integration of electric drive concepts is very complex when it comes to meet today's standard of security, comfort and agility for the whole vehicle dynamics in all operating points of longitudinal, lateral and vertical dynamics. In this paper, the electric vehicle lateral dynamics is investigated for enhancement of the vehicle stability.

As known to all, the Vehicle Stability Control (VSC) system is an active safety system for stabilizing the vehicle dynamic behavior in emergency situations such as spinning, drift out, and roll over. In the past 20 years, the vehicle stability control system has become very active, attracting intensive research efforts from both the academic and industry [1-3]. And in recent years, vehicle stability control for electric vehicle based on brake torque distribution [4] has been investigated.

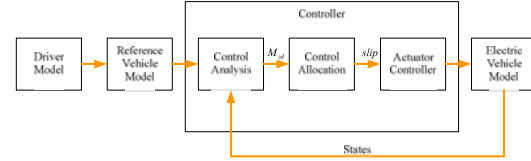


Figure 1. Control schematic diagram

In this paper, a four-wheel-drive electric vehicle model is established, and vehicle stability control system is designed based on the control allocation techniques. The control schematic diagram is shown in Fig.1.

A control allocation approach is generally used when different combinations of effectors' commands can produce the same result and when the number of effectors available exceeds the number of states being controlled. A key feature of control allocation is that of reconfiguration. In the event of an effector failure is detected, the control effort is redistributed among the remaining active effectors to minimizing the track error.

Different methods of control allocation have been developed for aerospace vehicles[7-8], marine vessels[9], and other fields where this proves to be a valuable safety aspect. This is the first time to use the control allocation approach in electric vehicle for stability control.

In this paper, the nonlinear vehicle model and tire model are established, and the effectiveness of brake and drive force to yaw moment have been analyzed. And based on this theory a VSC controller is designed using Sliding Mode Control (SMC). And the rest of this paper shows the simulations in CarMaker[®] environment. Conclusions are summarized in section VI.

II. SYSTEM MODELING

A. Linear Vehicle Model

In this paper, a linear vehicle model with two degree-of-freedom is applied to estimate the driver's intention, computing the desired motion of the vehicle. The model equations can be written as,

$$\begin{aligned} m(\dot{v}_y + v_x r) &= -\frac{(C_{af} + C_{ar})}{v_x} v_y - \frac{(aC_{af} - bC_{ar})}{v_x} r + \frac{C_{af}}{v_x} \delta_f \\ I_z \dot{r} &= -\frac{(aC_{af} - bC_{ar})}{v_x} v_y - \frac{(a^2 C_{af} + b^2 C_{ar})}{v_x} r + aC_{ar} \delta_f \end{aligned} \quad (1)$$

Where v_x is a longitudinal velocity, v_y is a lateral velocity, and r is yaw rate. δ_f is front wheel steering angle input. a and b are the distances from center of mass of vehicle to front and rear axle. And m represents vehicle mass, I_z is moment of inertia about vertical axis. C_{af} and C_{ar} are the slip stiffness of front tire and rear tire respectively.

B. Nonlinear Vehicle Model

The vehicle model used in this paper is a four-wheel-drive electric vehicle, only considering the planar motion: longitudinal, lateral, and yaw. And the vehicle is modeled as a rigid body with three-degree-of-freedom. The bob, pitch and roll motions are ignored. Fig. 2 shows the vehicle diagram with planar motion.

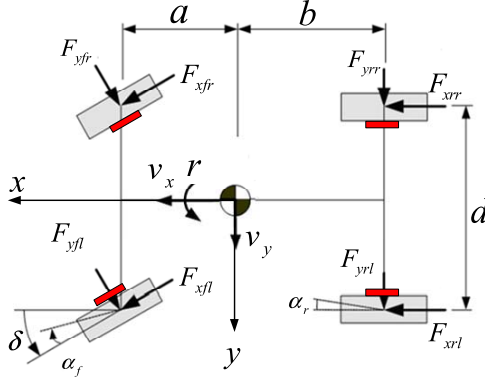


Figure 2. Vehicle planar dynamic motion

Assume that the driver controls the front wheel steering angle by using the steering wheel, while the controller can use the four longitudinal wheel slips for stabilizing the yaw motion. For electric vehicle, assume that not only the braking forces but also the driving forces are available as control inputs. So three actuators available are applied to control the yaw moment: differential driving and braking of the front and rear tires and the steering angle of the front tires.

The equation of motion for the vehicle dynamics can be written as.

$$m(\dot{v}_x - v_y r) = \sum F_{xi} \quad (2)$$

$$m(\dot{v}_y + v_x r) = \sum F_{yi} \quad (3)$$

$$I_z \dot{r} = \sum M_{zi} \quad (4)$$

Where,

$$\begin{aligned} \Sigma F_{xi} &= (F_{xfl} + F_{xfr}) \cos \delta - (F_{yfl} + F_{yfr}) \sin \delta + (F_{xrl} + F_{xrr}) \\ \Sigma F_{yi} &= (F_{yfl} + F_{yfr}) \cos \delta + (F_{xfl} + F_{xfr}) \sin \delta + F_{yrl} + F_{yrr} \\ \Sigma M_{zi} &= (-F_{xfl} \cdot \cos \delta + F_{yfl} \cdot \sin \delta - F_{xrl} + F_{xrr} + \\ &\quad F_{xfr} \cdot \cos \delta - F_{yfr} \cdot \sin \delta) \cdot \frac{d}{2} - (F_{yrl} + F_{yrr}) \cdot b \\ &\quad + ((F_{yfl} + F_{yfr}) \cdot \cos \delta + (F_{xfl} + F_{xfr}) \cdot \sin \delta) \cdot a \end{aligned} \quad (5)$$

Where, v_x , v_y , r , m , I_z , a and b have the same meaning as (1). F_x and F_y are longitudinal and lateral tire forces. fr , fl , rr and rl stand for front right, front left, right rear and rear left wheel. The front and rear tire slip angles, α_f and α_r are the average slip angles of left and right wheels. δ is steered angle. d is the track width.

The slip angle for each tire can be calculated as,

$$\alpha_{fl} = \arctan \left(\frac{v_y + a \cdot r}{v_x - \frac{B}{2} \cdot r} \right) - \delta \quad (6)$$

$$\alpha_{fr} = \arctan \left(\frac{v_y + a \cdot r}{v_x + \frac{B}{2} \cdot r} \right) - \delta \quad (7)$$

$$\alpha_{rl} = \arctan \left(\frac{v_y - b \cdot r}{v_x - \frac{B}{2} \cdot r} \right) \quad (8)$$

$$\alpha_{rr} = \arctan \left(\frac{v_y - b \cdot r}{v_x + \frac{B}{2} \cdot r} \right) \quad (9)$$

The longitudinal slip rate for each tire is given as,

$$\lambda_i = \frac{\omega_i R_e - V_i}{\max(V_i, \omega_i R_e)} \quad (10)$$

It is necessary to consider load transfer in virtue of longitudinal and lateral acceleration of vehicle for analyzing the vehicle behavior of braking and driving situation. The lateral wind and road gradient are neglected, so the vertical load of each wheel can be calculated as follows,

$$F_{zfl} = \frac{mgb}{2L} - \frac{F_x h}{2L} - \frac{F_y h}{d} \frac{b}{L} \quad (11)$$

$$F_{zfr} = \frac{mgb}{2L} - \frac{F_x h}{2L} + \frac{F_y h}{d} \frac{b}{L} \quad (12)$$

$$F_{zrl} = \frac{mga}{2L} + \frac{F_x h}{2L} - \frac{F_y h}{d} \frac{a}{L} \quad (13)$$

$$F_{zrr} = \frac{mga}{2L} + \frac{F_x h}{2L} + \frac{F_y h}{d} \frac{a}{L} \quad (14)$$

C. Tire Model

The tire model needs to describe the dependencies of the tire force on the slip/slip angle, friction coefficient as well as the interaction between longitudinal and lateral forces. The tire model applied is the simplified Dugoff model[12], taking the expressions,

$$\begin{aligned} F_x &= f C_x \lambda \\ F_y &= f C_y \alpha \end{aligned} \quad (15)$$

$$f = \begin{cases} 1, & F_R \leq \frac{\mu_H F_z}{2} \\ (2 - \frac{\mu_H F_z}{2 F_R}) \frac{\mu_H F_z}{2 F_R}, & F_R > \frac{\mu_H F_z}{2} \end{cases} \quad (16)$$

$$F_R = \sqrt{(C_x \lambda)^2 + (C_y \alpha)^2} \quad (17)$$

Where, C_x and C_y are the tire longitudinal and lateral stiffness respectively. F_z is the tire normal load and μ_H is the maximum friction coefficient for a given road surface condition. The tire adhesion ellipse is shown in Fig. 3.

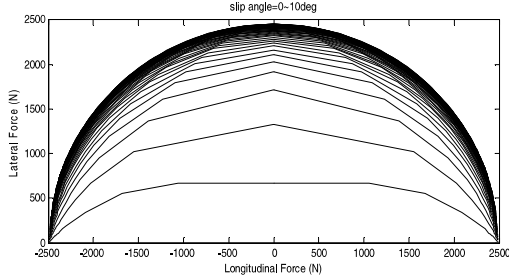


Figure 3. Tire adhesion ellipse

III. CONTROL LAW ANALYSIS

This section focuses on vehicle dynamics analysis, attempting to exploit the full potential of the braking commands. Using a 3-DOF vehicle model in accordance with the friction circle calculate a more accurate estimate of differential braking limits.

In order to analyze the effectiveness of every tire braking force to yaw moment, the simulations is made using the nonlinear vehicle model and tire model above. The vehicle parameters used are shown in Table I.

TABLE I. VEHICLE PARAMETERS

Parameter	Magnitude	Unit
m	1095	kg
I_z	1580	kg·m ²
h_g	0.469	m
a	0.946	m
b	1.526	m
L	2.472	m
d	1.425	m
Re	0.285	m

I_w	0.87	kg·m ²
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As the previous research [15], we can know that for the electric vehicle four levels positive and negative yaw moments can be generated, shown in Table II. We can see that for conventional vehicle which generates active yaw moment only using braking force only two lower levels active yaw moment can be applied.

TABLE II. ACTIVE YAW MOMENT GENERATED

Level	Yaw moment(kN·m)			
	Conventional vehicle		Electric vehicle	
	Positive	Negative	Positive	Negative
□	0.7-1.0	0.8-1.2	0.8-1.0	0.8-1.2
□	1.7	2.0	1.5-2.0	1.8-2.3
□			2.5-3.0	2.9-3.4
□			3.5-4.0	4.0-5.0

From these simulations, we can find that the electric vehicle which makes full use of the brake and drive forces can heighten the braking and driving commands for active yaw moment more than one times.

At the same time, from the analysis above, according to the principle of maximizing the yaw moment, we can choose which actuators used is more efficient for every levels. And the results are shown in Table III and Fig. 4, Fig. 5.

TABLE III. ACTUATORS CHOSEN

Level	Positive yaw moment	Negative yaw moment
□	Brake F_{xfl}	Brake F_{xfr}
□	Brake F_{xfl} ; Drive F_{xfr}	Brake F_{xfr} ; Drive F_{xfl}
□	Brake F_{xfl} and F_{xrl} ; Drive F_{xfr}	Brake F_{xfr} and F_{xrr} ; Drive F_{xfl}
□	Brake F_{xfl} and F_{xrl} ; Drive F_{xfr} and F_{xrr}	Brake F_{xfr} and F_{xrr} ; Drive F_{xfl} and F_{xrl}

In the same way, we can obtain the similar results on other roads, such as wet road and ice road. Based on these analyses we researched the control law of vehicle stability control for 4-wheel-drive electric vehicle. Ulteriorly, the controller based on the control law is designed.

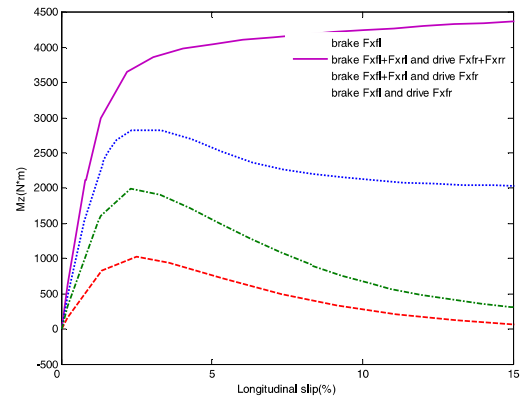


Figure 4. Positive yaw moments

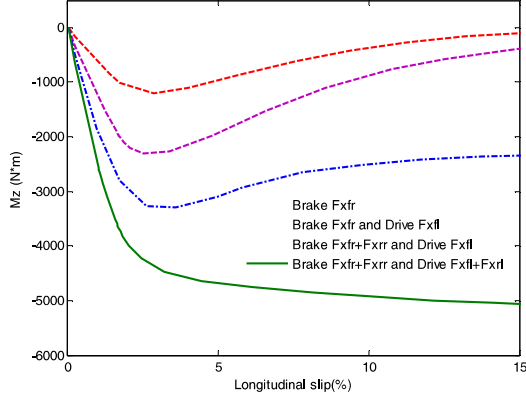


Figure 5. Negative yaw moments

IV. CONTROLLER DESIGN BASED ON CONTROL ALLOCATION

A. Controller Configuration

In this paper, hierarchically vehicle stability control system is designed. The flowchart of the controller is shown in Fig.6. The reference vehicle model computes the desired vehicle motion based on the driver commands. The yaw control analysis layer compares the actual vehicle motion and the desired vehicle motion, and provides the desired yaw moment. The layer of yaw control allocation estimates the desired yaw moment and detect whether these yaw moment can be generated and how to generate these yaw moment. Videlicet, the task of this module is to determine the longitudinal slip rate for each tire. The ABS/TCS system which is assumed to be available is to manipulate braking torque and wheel driving to track the desired longitudinal slip rate calculated by control allocation, and to generate an actual yaw moment and exert itself to satisfy the desired yaw moment. The actual vehicle model affected by this yaw moment provides corresponding actual vehicle motion and feed it back to the vehicle state observer, which estimate the actual motion of the vehicle.

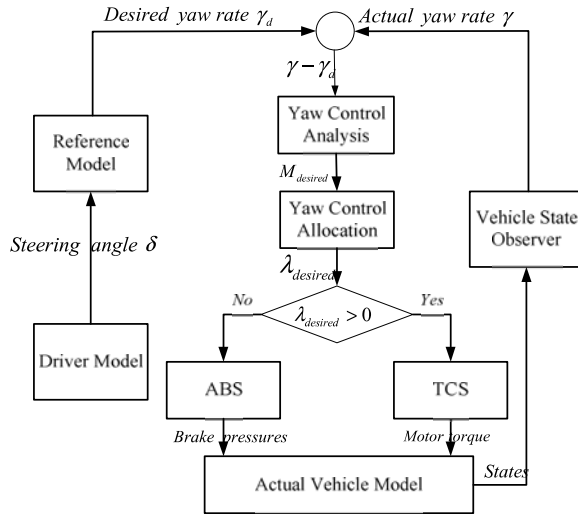


Figure 6. The flowchart of controller

B. Yaw Control Analysis

The yaw control analysis layer compares the actual vehicle motion and the desired vehicle motion, and provides the desired yaw moment basing on these differences and current vehicle motion.

As the vehicle is a nonlinear system, a sliding mode controller (SMC) is designed based on the simplified vehicle model above for its robustness. As VSC focuses on controlling the lateral and yaw motion of the vehicle, the system state is set as $[v_y \ r]^T = [x_1 \ x_2]^T$, and the system equation can be rewritten in state space as,

$$\dot{x} = \begin{pmatrix} -v_x x_2 \\ 0 \end{pmatrix} + \begin{pmatrix} \frac{1}{M} & 0 \\ 0 & \frac{1}{I_z} \end{pmatrix} \begin{pmatrix} F_{ydesired} \\ M_{zdesired} \end{pmatrix} \quad (18)$$

Set the system output as $y = [x_1 \ x_2]^T$ and the input vector as $u = [F_{ydesired} \ M_{zdesired}]^T$, which is also the desired lateral force and active yaw moment sent to the control allocation to make the vehicle actuators generate the matched ones. It is apparent that the matrix $\text{diag}[\frac{1}{M} \ \frac{1}{I_z}]$ is invertible and the system has a vector relative degree $[1 \ 1]^T$, so there is no zero dynamics involved.

To ensure system robustness to model error and parameter uncertainties, such as variation of the vehicle mass caused by load changes etc., a multiple-input-output SMC is applied in this system. As the relative order for each output channel is 1, the sliding surface which will make the output track the desired value can be defined as,

$$s = \lambda(r_d - r) \quad (19)$$

Where λ is positive. The attractive equations are

$$\dot{s} = \lambda \dot{r}_d - \lambda \dot{r} = \lambda \dot{r}_d - \lambda \left(\frac{1}{I_z} M_{zdesired} \right) = -\eta \text{sgn}(s) \quad (20)$$

For the Lyapunov function candidate, $V = \frac{1}{2} s^2$ by choosing η to be sufficiently large, the inequalities $\dot{V} = S\dot{S} = S\eta \text{sgn}(S) \leq K|S|$, ($K > 0$) can be guaranteed and the sliding surfaces are attractive.

In practice, to avoid chattering effects caused by the frequent switching around the sliding surface, a continuous approximation with a thickness of ϕ around the surface is used to smooth out the control discontinuity. The approximation function here used is a simple linear saturation defined as,

$$\text{sat}\left(\frac{s}{\phi}\right) = \begin{cases} s/\phi & \text{if } |s| < \phi \\ \text{sgn}(s/\phi) & \text{if } |s| \geq \phi \end{cases} \quad (21)$$

The attractiveness equations can be rewritten as:

$$\dot{s} = \lambda \dot{r}_d - \lambda \dot{r} = \lambda \dot{r}_d - \lambda \left(\frac{1}{I_z} M_{z_{desired}} \right) = -\eta \text{sat}(s) \quad (22)$$

Thus, the control law can be derived as:

$$M_{z_{desired}} = I_z \left[\dot{r}_d - \frac{\eta}{\lambda} \text{sat}(s) \right] \quad (23)$$

The next task is to design the control allocation layer, which determines the 4 wheels tire forces to satisfy the desired yaw moment.

C. Yaw Control Allocation

A control allocation approach is generally used when different combinations of effectors' commands can produce the same result and when the number of effectors available exceeds the number of states being controlled.

The general control allocation problem is well stated in [13] as the computation of an optimal set of effectors commands u that will produce some desired overall control effect u_d . In other words, given a desired response determine u so that $Bu = u_d$ subject to $u^- \leq u \leq u^+$,

where u^+ and u^- are upper and lower bounds placed on the effectors and B is a matrix defining the effectiveness of the effectors. If multiple solutions exist, choose one that will minimize the predetermined cost function. If there are no solutions, find u so that Bu approximates u_d as well as possible.

In this control system, a typical linear redundant control allocation problem can be formulated as,

$$Bu = u_d \quad (24)$$

Where $u_d = M_{z_d}$ is the desired yaw moment, u is the vector consisting of the longitudinal slip of each tire. Here, B is the matrix defining the effectiveness of the actuators.

Based on the control analysis above, the vector u is calculated.

V. EVALUATIONS OF VSC IN CARMAKER®

In this section, the simulation evaluation of the VSC based on control allocation described above is presented. A commercial vehicle dynamics simulation package, CarMaker®, is used to provide a virtual test platform of a passenger car. The models in CarMaker® are much more complete and complex than those used for control design. Therefore, their use provides a realistic test platform for evaluating the robustness of the controller. Fig. 7 shows the overall structure for the simulation studies for the VSC system. Fig. 8 shows the GUI of CarMaker®.

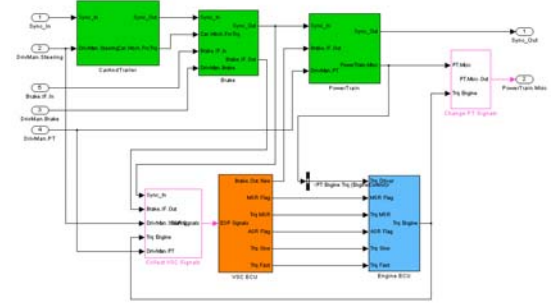


Figure 7. The overall structure of VSC controller in CarMaker®



Figure 8. The GUI of CarMaker®

As known to all, most of the accidents on the highway road are caused by two maneuvers. One is the vehicle running on the high speed to go through the curve road, which is the typical test: step test. The other is the sudden cut-in situation of a previous vehicle or other obstacles. The driver performs the single lane change maneuver, which is typical maneuver: lane change. In this paper, these two tests are conducted to demonstrate the effect of the controller.

A. Steering wheel step input test

A step test has been conducted on normal road at 80km/h. The steering angle is fixed after ten seconds, shown in Fig. 9.

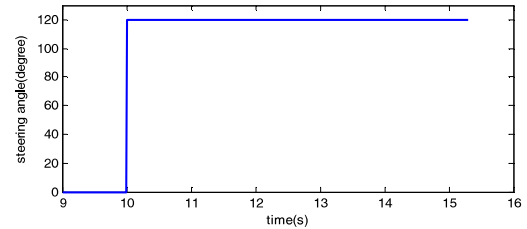


Figure 9. Steering angle input

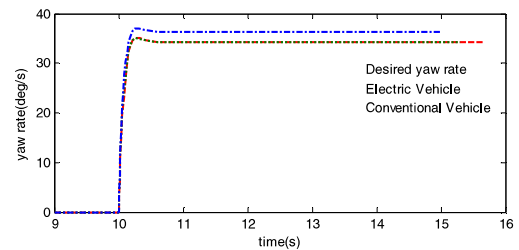


Figure 10. Yaw rate response

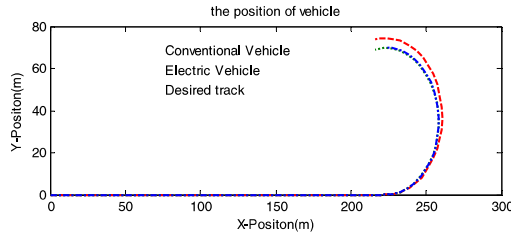


Figure 11. The track of the vehicle

Fig. 10 shows the yaw rate responses of the vehicle. Absolutely, the electric vehicle with VSC ON is more stable than the conventional vehicle. The tracks of the two style vehicle are shown in Fig. 11, which demonstrate that the electric vehicle can follow the desired track, but the conventional vehicle can't run on the desired track.

B. Lane change test

Next, a single lane change test is conducted still on normal road at 80km/h.

The driver changes a lane promptly when the vehicle speeds are about 80km/h. In Fig. 12, a driver steers the wheel to make the vehicle track the desired maneuver.

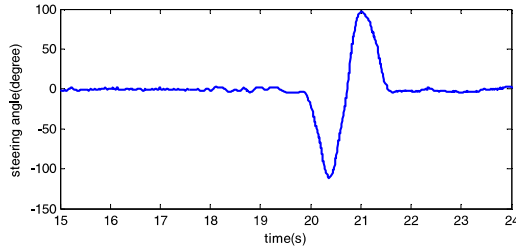


Figure 12. Steering angle input

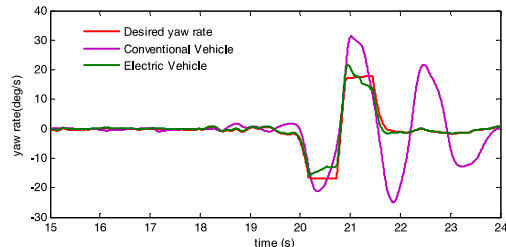


Figure 13. Yaw rate response

From Fig.13, desired value of yaw rate is limited about peak steering wheel angle, but the yaw rate follows the desired value for electric vehicle with VSC. The behavior of the controlled electric vehicle is more stable than the conventional vehicle respect to yaw rate in Fig. 13. It can be seen that the controller based on the control law exhibits superior handling performance. The driver will feel better to stabilize the electric vehicle with the VSC ON.

VI. CONCLUSIONS

A nonlinear vehicle model is established for analyzing the control law of yaw moment control for four-wheel-drive electric vehicle. Based on this control law, the vehicle stability controller is designed using the sliding

mode control to design the yaw control analysis layer for commutating the desired yaw moment. And then based on the control allocation technique, the desired yaw moment is estimated and the control effect is distributed to four tires. Simulation results in CarMaker[®] environment have shown that the proposed controller reduces driving effort and enhances stability of the electric vehicle.

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