Centre of Mass

$$\boxed{x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}}$$

$$\boxed{y_{cm} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2}}$$

{Centre of Mass will always lie on the line of Symmetry}

$$\boxed{x_{cm} = \frac{\int x\,dm}{\int dm}}, \quad \boxed{y_{cm} = \frac{\int y\,dm}{\int dm}}$$

(1.) Half Ring :- $\boxed{y_{cm} = \frac{2R}{\pi}}$

(2.) Segment of a Ring :-

$$\boxed{y_{cm} = \frac{R\sin\theta}{\theta}}$$

(3.) Half disc :- $\boxed{y_{cm} = \frac{4R}{3\pi}}$

(4.) Sector of a disc :-

$$\boxed{y_{cm} = \frac{2R\sin\theta}{3\theta}}$$

(5.) Hollow Hemisphere :-

$$\boxed{y_{cm} = \frac{R}{2}}$$

(6.) Solid Hemisphere :-

$$\boxed{y_{cm} = \frac{3R}{8}}$$

ESTIMATE

(7) Hollow cone : $\boxed{y_{cm} = \frac{h}{3}}$

(8) Solid Cone : $\boxed{y_{cm} = \frac{h}{4}}$

—x—x—x—x—

Velocity of Centre of Mass

$$\boxed{\vec{V}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}} \bigstar$$

$$\boxed{\vec{V}_{cm} = \frac{\vec{P}_1 + \vec{P}_2}{m_{total}}} \bigstar$$

Accn. of Centre of Mass

$$\boxed{\vec{a}_{cm} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2}{m_1 + m_2}}$$

$$\boxed{\vec{a}_{cm} = \frac{\vec{F}_{net\,ext}}{m_{total}}}$$

If $F_{net\,ext} = 0$ & $V_{cm} = 0$,
then,
{Centre of Mass will Remain at rest},
& $\boxed{m_1 \Delta x_1 = -m_2 \Delta x_2}$

---x---x---x---

For two particle System:-

distance of centre of mass from one particle,

$$\Rightarrow \boxed{\frac{(\text{Opposite Mass}) \times (\text{Distance b/w Particles})}{(\text{Total Mass})}}$$

Motion of Centre of Mass

(1.) Jumping from a Cart:-

a) If velocity is given w.r.t the ground [of Man]

ESTIMATE

:- $$\boxed{V = \frac{mu}{M}}$$

m → mass of man

M → mass of cart

b) If velocity is given w.r.t. the cart.

$$\boxed{V_M = \frac{mu}{m+M}}$$

[$V_M = \frac{\text{Opposite mass} \times \text{Relative velocity}}{\text{Total Mass}}$]

2 Firing of Bullets :-

MUZZLE VELOCITY :- Velocity of bullet w.r.t. gun

$$\boxed{V_{recoil} = \frac{mu}{M}}$$

[We have to consider relative velocity in horizontal direction]

3. EXPLOSIONS :-

Concept :- Bomb explodes due to internal forces and hence its momentum remains conserved.

COLLISIONS

ELASTIC COLLISION

$\boxed{K.E._I = K.E._F}$

$\boxed{P_I = P_F = P_{DC}}$

INELASTIC COLLISION

$\boxed{K.E._i > K.E._f}$

$\boxed{P_i = P_f = P_{DC}}$

P_{DC} → During collision

Coefficient of Elasticity

$$\boxed{e = \frac{\text{velocity of Separation}}{\text{Velocity of apprach}}}$$ ★

→ OR Coefficient of Restitution

(1.) For perfectly elastic collision,

$$\boxed{e = 1}$$

& $\boxed{\text{Vel. of Separn.} = \text{Vel. of Approach}}$

(2.) For perfectly inelastic collision,

$$\boxed{e = 0}$$

& $\boxed{\text{Velocity of Separation} = 0}$ *

PERFECTLY ELASTIC COLLISION

$$\boxed{V_1 = \frac{(m_1 - m_2)u_1 + 2m_2u_2}{m_1 + m_2}}$$ *

$$\left.\begin{aligned} &\boxed{V_1 = \frac{P_i + em_2(u_2 - u_1)}{m_1 + m_2}} \\ &\boxed{V_2 = \frac{P_i + em_1(u_1 - u_2)}{m_1 + m_2}} \end{aligned}\right\} \text{Imp.}$$

Siddhi :

$$\boxed{\Delta K.E._{loss} = \frac{1}{2}\mu(1-e^2)V_{app}^2}$$

Also, $\boxed{E = \frac{P^2}{2\mu}}$

Energy

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

Rotation

Equations of angular motion are :-

i) $\omega = \omega_0 + \alpha t$

ii) $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

iii) $\omega^2 = \omega_0^2 + 2\alpha\theta$

Moment of Inertia

$$\boxed{I = \Sigma m r^2}$$

Angular momentum

$$\vec{L} = m(\vec{r} \times \vec{v})$$

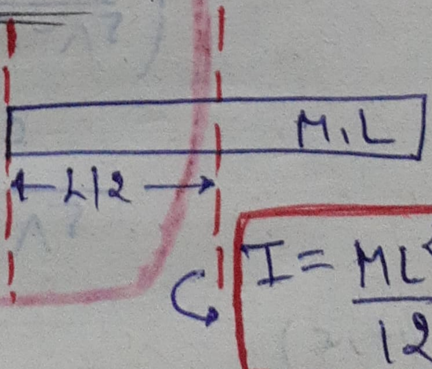
$$L = mvr\sin\theta\,\hat{n}$$

Radius of Gyration

$$I = MK^2 \Rightarrow \boxed{K = \sqrt{\frac{I}{M}}}$$

Moment of Inertia

1.) For ROD →



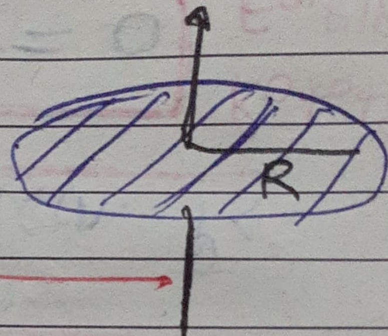
$$\boxed{I = \frac{ML^2}{3}}$$

$$\boxed{I = \frac{ML^2}{12}}$$

2) For Ring → (⊥r to Ring)

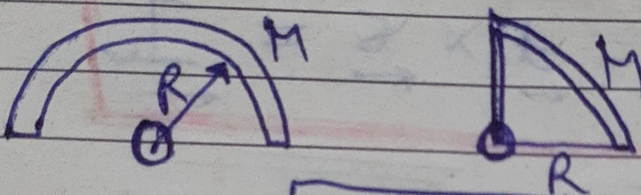
$$\boxed{I = MR^2}$$

(3) For Disc →



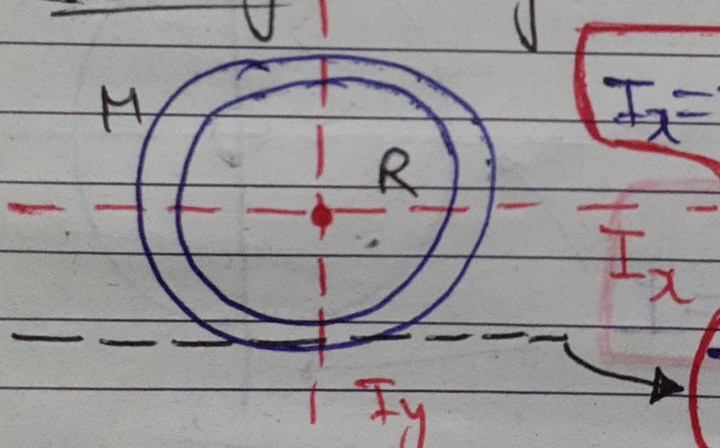
$$\boxed{I = \frac{1}{2}MR^2}$$

(4) For half, quadrant of Ring →



$$\boxed{I = MR^2}$$

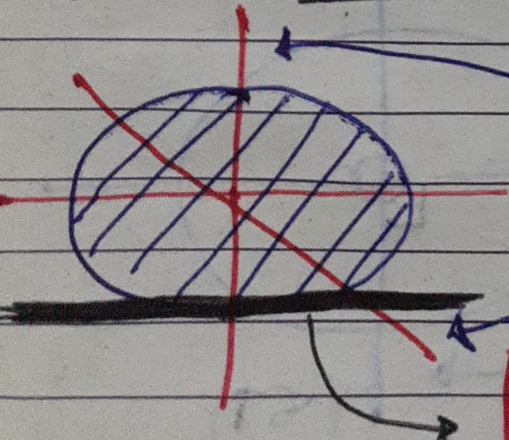
(5) For ring (along the ring) :-



$$\boxed{I_x = I_y = \frac{MR^2}{2}}$$

$$\boxed{I = \frac{3MR^2}{2}}$$

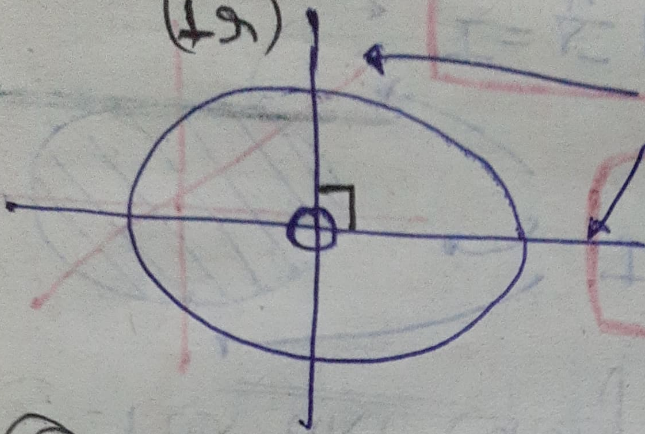
(6) For disc (along the disc) :-



$$\boxed{I = \frac{MR^2}{4}}$$

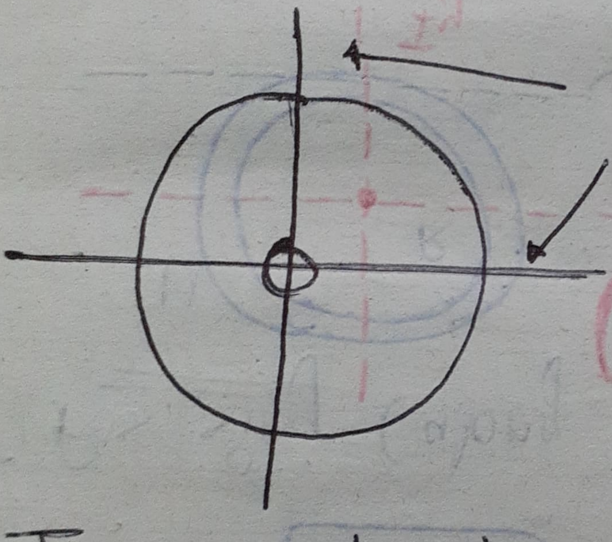
$$\boxed{I = \frac{5}{4}MR^2}$$

(7) Hollow Sphere



$$\boxed{I = \frac{2}{3} MR^2}$$

(8) Solid Sphere



$$\boxed{I = \frac{2}{5} MR^2}$$

Torque about a point

$$\boxed{\vec{\tau} = \vec{r} \times \vec{F} = rF\sin\theta\, \hat{n}}$$

or $\boxed{\tau = r_{\perp} F = r \times F_{\perp}}$

When angular accn. is constant, $\boxed{\tau = 0}$ Rotational Equilibrium

Torque Dynamics

$$\boxed{\tau_{hinge} = I_{hinge}\,\alpha}$$

or

$$\boxed{\tau_{cm} = I_{cm} \times \alpha}$$ = Moment of Inert × Angu. Accn.

For pure rolling,

$\boxed{V = R\omega}$, $\boxed{a = R\alpha}$

Cases

1.) $V > R\omega \rightarrow$ (Forward Sleeping)

2.) $V < R\omega \rightarrow$ (Backward Sleeping)

3.) $V = R\omega \rightarrow$ (Pure Rolling / No slipping)

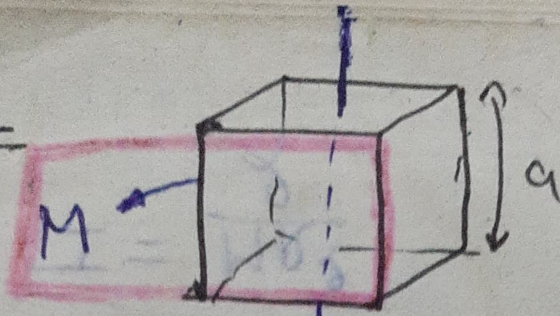
$$\boxed{K.E._{total} = \frac{1}{2} I\omega^2}$$

Angular Momentum

$\boxed{J = I \times \omega}$

Moment of inertia × Angular velocity

Cube



$$\boxed{I_{centre} = \frac{1}{6} Ma^2}$$

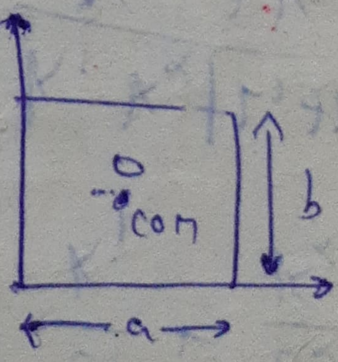
Parallel Axes Theorem

$$\boxed{I_{new} = I_{COM} + MR^2}$$

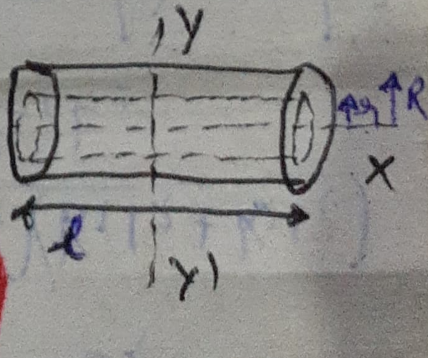
R → shifted distance

Moment of Inertia of rectangular sheet about an axis passing through Centre of mass is

$$\boxed{I_0 = \frac{M}{12}(a^2 + b^2)}$$



Moment of Inertia of a Hollow Cylinder



$$\boxed{\text{About } XX',\ I = \frac{M}{2}(R^2 + r^2)}$$

$$\boxed{\text{About } YY',\ I = M\left(\frac{l^2}{12} + \frac{R^2 + r^2}{4}\right)}$$

Moment of Inertia of Solid Cylinder & Hollow Cone about axis :-

$$\boxed{I = \frac{MR^2}{2}}$$