Units, Dimensions & measurement

Physical Quantities :-

The quantities which can be measured and the various physical phenomenons are expressed in terms these quantities in form of laws are called physical quantities. eg. - mass, length, time temperature, speed, etc.

A physical quantity is expressed in terms of its magnitude and units.

Types of physical quantities on the basis of dependence on each other :-

Fundamental quantities :-

The physical quantities, which are independent of other physical quantities are known as fundamental quantities (MLT) at present (7)

Derived physical quantities

The physical quantities which are expressed in terms of fundamental quantities are called derived physical quantities For eg. - speed, acceleration, etc.

Units :-

The standard choosen for measurements of a quantity which has essentially same nature as that of the quantity is called the unit of the quantity.

Units system:-

There are 4 types of unit systems

1.) FPS (Foot, Pound, seconds)
2.) MKS (Metre, Kilogram, seconds)
3.) CGS (centimetre, gram, seconds)
4.) S.I (Standard International unit system)

In S.I unit system, there are seven fundamental units

Mass (Kg)
Length (m)
Time (s)
Temperature (K)
Current (A)
Mole (amount of substance) (mole)
luminous intensity (candela- cd)

~~Other than 7~~ In S.I unit system there are two other physical quantities :-

Angle → S.I unit (Radians) → rad
Solid angle → Steradian → ~~Sr~~ Sr (Ω)

↓

3d → $\dfrac{\text{area of base}}{(\text{slant height})^2}$

Dimensions and Dimensional formula of physical quantities

speed $= \frac{m}{sec} = m^1 sec^{-1} = L^1 T^{-1}$

Acc. $= \frac{m}{sec^2} = m^1 sec^{-2} = L^1 T^{-2}$

The powers of fundamental units in unit of a physical quantity is called dimensions of physical quantities

⇒ In physics, the units of physical quantities are expressed in seven fundamental units, so that the dimensions of a physical quantity ~~has~~ has seven terms. But in mechanics ~~the~~ all physical quantities are expressed in three fundamental quantities [M, L, T]

In mechanics the mass is represented by M, length by 'L' and time is represented by 'T', ~~so~~ that the dimensions of a physical quantities are expressed in form of $[M^x L^y T^z]$.

$x, y, z \in R \rightarrow$ Real No.s

The above formula is called dimensional formula.

Units and dimensions of some imp. physical quantities :-

S.No	Phy. quantity	S.I. unit	Dimensional formula
1.)	Length	m	$M^0 L^1 T^0$ [
2.)	Mass	Kg	$M^1 L^0 T^0$ [

	Phy quantity	S.I. unit	Dimensions	
3.)	Time	sec.	$M^0L^0T^1$	$[T^1]$
4.)	Surface area	m^2	$M^0L^2T^0$	$[L^2]$
5.)	Volume	m^3	$M^0L^3T^0$	$[L^3]$
6.)	speed	m/sec.	$M^0L^1T^{-1}$	$[L^1T^{-1}]$
7.)	acc.	m/sec^2	$M^0L^1T^{-2}$	$[L^1T^{-2}]$
8.)	linear mom.	$Kg \times \frac{m}{sec}$	$M^1L^1T^{-1}$	
9.)	Force	$Kg \times \frac{m}{sec^2}$	$M^1L^1T^{-2}$	
10.)	Work, $W = F \cdot S$	$N \cdot m$ (Joule), $Kg \times m^2/sec^2$	$M^1L^2T^{-2}$	
11.)	K. E	$Kg \times \frac{m^2}{sec^2}$ (Joule)	$M^1L^2T^{-2}$	
12.)	Pressure	$\frac{N}{m^2}$ (Pascal)	$M^1L^{-1}T^{-2}$	
13.)	Power	$\frac{W}{T}$ ($\frac{Joule}{sec} \Rightarrow$ watt)	$M^1L^2T^{-3}$	
14.)	Torque ($\tau = r \times F$)	$m \times N$	$M^1L^2T^{-2}$	
15.)	Angular displacement ($\Delta\theta$)	radian	$[M^0L^0T^0]$	
16.)	Ang. velocity ($\omega = \frac{\Delta\theta}{t}$)	$\frac{radian}{sec}$	$[M^0L^0T^{-1}]$	
17.)	Ang. momentum ($L = r \times \underset{linear}{P}$)	$Kg \frac{m^2}{sec}$	$[M^1L^2T^{-1}]$	

⇒ In a law of physics all no.s $[\frac{1}{2}, 4\pi, 2e]$ are dimension-less but all constant [G, K] have dimensions

⇒ All Trignometry ratios are dimensionless

$$y = \sin\theta$$

$$\theta = \frac{A \times B}{C \times D}$$

y will be dimensionless as sin function is as radia $\left(\frac{P}{H}\right) \Rightarrow \frac{m}{m} \Rightarrow$ no dimension

$$y = \sin\left[\frac{A \times B}{C}\right]$$

Dimensionless Dimensionless

All exponential funct. and their powers are dimensionless.

$$y = 10^{x}$$
$$y = e^{\frac{P \times Q}{R}}$$

Dimensionless Dimensionless

log. funct. are also dimensionless

Applications of dimension analysis:-

1) To check homogeneity and correctness of an equation.

$$v = u + at$$

all have same dimensions.

In physics same dimensional quantities are added or subtracted so that in a law of physics, the terms on both sides of the equality between them '+' or '-' operation are present, have same dimensions. This is called homogeneity principle.

To compare homogeneity of all terms in an eq.^{n} and check the eq.^{n} is dimensionally correct or incorrect.

⇒ If an eq.^{n} is dimensionally correct the it may or may not be mathematically correct. If an eqⁿ is dimensionally incorrect then it must be mathematically incorrect.

Q– $S = ut + \frac{1}{2}a^2t^2$. check ~~to the~~ correctness of eqⁿ by help of dimensional analysis.

$$S = M^0L^1T^0$$
$$ut = M^0L^1T^0$$
$$a^2t^2 = M^0L^2T^{-2}$$
$$D_o \text{ of } [S] = D_o \text{ of } [ut] \neq D_o \text{ of } [a^2t^2]$$

20) To convert the value of physical quantity from one unit system to other.

M_1 :→ By using units (unit method)

Q– convert 1N into CGS unit

$$N \rightarrow \text{Dyne}$$
$$1N = 1\ \frac{Kg \times m}{sec^2}$$
$$= 1 \times \left[\frac{1\ Kg \times 1\ m}{1\ sec^2}\right]$$
$$= 1 \times \left[\frac{1000g \times 100cm}{1 \times sec^2}\right]$$
$$= 10^5 \text{ dyne}$$

Q– convert 1J into erg

$$1J = F \times S$$
$$= \frac{Kg \times m^2}{sec^2}$$
$$= \left[\frac{1000 \times 10000}{1\ sec^2}\right] \times 1 = 10^7 \text{ erg}$$

M_2 :- Dimensional method

Suppose $n_A [M_A^x L_A^y T_A^z]$ are numerical value and dimensional formula of a physical quantity in "A" unit system and $N_B (M_B^x, L_B^y, T_B^z)$ are numerical value and dimensional formula in unit system B, then

$$n_A [M_A^x, L_A^y T_A^z] = n_B [M_B^x L_B^y T_B^z]$$

Q - Convert 6m/sec. into C.G.S unit by dimensional analysis.

$(S.I)_A$ $(C.G.S)_B$

$$6 [M_A^0 L_A^1 T_A^{-1}] = n_B [M_B^0 L_B^1 T_B^{-1}]$$

$$6 [(1000 M_B)^0 \cdot [100 L_B]^1 \cdot [1 T_B]^{-1}] = n_B [M_B^0 L_B^1 T_B^{-1}]$$

$$6 [\cancel{M_B^0} \cdot 100 \cancel{L_B^1} \cancel{T_B^{-1}}] = n_B [\cancel{M_B^0} \cancel{L_B^1} \cancel{T_B^{-1}}]$$

$$6 \times 100 = n_B$$

$$\therefore n_B = 600$$

3) Finding units and dimensions of unknown quantities in an eq.^{n}

when some unknown physical quantities are present in an eq.^{n} then we compare homogenity of each term in the eq.^{n} and find out units and dimensions of unknown physical quantities.

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Q- when a solid sphere of 'r' is moving with a speed 'v' in a fluid, then the resistive force acting on the sphere is given by 'stock's law' as $F = 6\pi\eta r v$

$\eta \rightarrow$ eta $\rightarrow$ coefficient of viscosity

Find the dimensions and S.I. unit of eta by help of dimensional analysis

$$[M^1L^1T^{-2}] = [\eta][L^1][L^1T^{-1}]$$

$$\boxed{\eta = M^1L^{-1}T^{-1}}$$

$\Rightarrow$ Kg/m sec.

Q.) Deduce relationship b/w physical quantities

Let a physical quantity 'A' that depend upon physical quantity B, C and D. But we don't know in which manner in this situation, we assume.

$$A = B^x C^y D^z \quad -①$$

For eq.^{n} ① we apply homogenity principle and calculate value of x, y, z by comparing powers of M, L and T.

Q- The K.E. of a particle depends on its mass and velocity. Deduce the relation K.E. of particle with mass & velocity by help of dimension analysis.

$m^x V^y \propto$ K.E.

$$K.E. \propto m^x V^y$$

$\therefore$ Do[K.E] = Do[$m^x V^y$]

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$[M^1L^2T^{-2}] = [M^x L^y T^{-y}]$

$x = 1$
$y = 2$ [By comparing power of M, L and T

Limitations of dimensional analysis:-
we can't deduce the relationship between constants of physical laws.
we can't deduce ~~...~~ exponential funct log function
we can't deduce ~~relationship~~ dimensions of a term if it depends on 4 or more than 4 physical quantities

- Out of the followings pairs which one does not have identical dimensions:- (JEE mains 2005

i) angular momentum (mvr), Planck's constant
ii) impulse, momentum
iii) moment of inertia and torque! Fr ⊥ distance
iv) work and Torque

$E = h\nu$ $\qquad$ $I = mr^2$ ← moment of inertia

$E = \frac{hc}{\lambda}$

$[M^1L^1T^0][M^1L^2T^{-2}] = h[M^0L^1T^{-1}]$

$h = \frac{M^1L^3T^{-2}}{M^0L^1T^{-1}}$

$= M^1L^2T^{-1}$

ang momentum $p \times r$

$[M^1L^1T^{-1}][M^0L^1T^0]$

$[M^1L^2T^{-1}]$

Torque → $[M^1L^2T^{-2}]$

$I = [M^1L^2T^0]$

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A.) $L = r \times p \overset{mv}{\cancel{\theta}} = [M^1 L^2 T^{-1}]$

$E = h\nu \Rightarrow h = \frac{E}{\nu} = \frac{[M^1 L^2 T^{-2}]}{T^{-1}}$ $\left(\nu = \frac{1}{T}\right)$

$= [M^1 L^2 T^{-1}]$

B.) Impulse $= [M^1 L^1 T^{-1}]$ & $p = [M^1 L^1 T^{-1}]$

✓ C.) $I = mr^2 \rightarrow [M^1 L^2 T^0]$ & $\tau = r \times F \rightarrow [M^1 L^2 T^{-2}]$

Q- If E, M, J and G resp. denote energy, mass, ang. momentum and gravitational constant then $\frac{E \times J^2}{M^5 G^2}$ has dimensions of

a.) Length

✓ b.) angle

c.) Mass

d.) Time

$$\frac{M L^2 T^{-2} \times (M^1 L^2 T^{-1})^2}{M^5 \times M^{-2} L^6 T^{-4}}$$

$$\frac{\cancel{M} \cancel{L^2} \cancel{T^{-2}} \times \cancel{M^2} \cancel{L^4} \cancel{T^{-2}}}{\cancel{M^3} \cancel{L^6} \cancel{T^{-4}}} = \boxed{M^0 L^0 T^0}$$

$$\frac{F \times R^2}{M^2} = G$$

$$\frac{M^1 L^1 T^{-2} \times L^2}{M^2} \Rightarrow M^{-1} L^3 T^{-2}$$

Q- If $\frac{\alpha}{t^4} = FV + \beta t^2$. Calculate dim formula of α & β. F, v and t has their usual mea.

$$\frac{\alpha}{t^4} = M^1 L^2 T^{-3}$$

$$\alpha = M^1 L^2 T^1$$

$$\beta t^2 = M^1 L^2 T^{-3}$$

$$= M^1 L^2 T^{-5}$$

Errors in measurement :-

The difference between measured value and actual value of a physical value is called error.

In measurement the error comes into account due to some reasons. For example :-

- Error due to instruments
- Error due to imperfection in an experiment.
- Error due to non-seriousness during experiment
- Error (Random - unknown reason such as in a circuit due to unavailability of ideal resisto, ammeters, voltmetres

Types of Errors on the basis of method of error calculation :-

Four types

⇒ Absolute error :-

The magnitude of difference between measure value and actual value of a physical quantity is called absolute error.

In physics we consider the mean of 'n' measured values as actual value.

⇒ Suppose $A_1, A_2 \ldots A_n$ are 'n' measured values of a physical values in n repetitions then

$$\text{actual / mean value } (A_m) = \frac{A_1 + A_2 \ldots + A_n}{n}$$

Absolute error in first measurement :-

$$\Delta A_1 = |A_1 - A_m|$$

$$\Delta A_2 = |A_2 - A_m|$$

$$\Delta A_n = |A_n - A_m|$$

(ii) Mean absolute error :-

The arithmetic mean of absolute errors in 'n' repetitions is called mean absolute error. It is represented by $\overline{\Delta A}$

$$\overline{\Delta A} = \frac{\Delta A_1 + \Delta A_2 + \Delta A_3 - - \Delta A_n}{n}$$

we will consider $\overline{\Delta A}$ as final error (mean absolute error), and Am as final value.

If Am & ΔA are actual/mean value and mean absolute error resp. then the physical quantity A is represented as

$$A = A_m \pm \overline{\Delta A}$$

The real value of phy. quantity lies betwe $A_m - \Delta A$ & $A_m + \Delta A$

(iii) Fractional / Relative error:-

The ratio mean absolute error and mean vali of a physical quantity is called fractional error

$$\text{Fractional error} = \frac{\overline{\Delta A}}{A_m} \qquad [\overline{\Delta A} = \Delta A]$$

Error in per unit mean value.

(iv) % error:-

when the error of a physical quantity is expressed in % then it is called % error.

$$\% \text{ error} = \frac{\overline{\Delta A}}{A_m} \times 100\% \qquad [\text{Errors are alwa} +\text{ve}]$$

Q- A physical quantity by an exp. ~~for~~ in several repetitions. If 1.49, 1.50, 1.32, 1.45, 1.38 & 1.40 are measured values in each repetition, the calculate

(i) actual / mean value = 1.4233 = 1.42

(ii) final / mean ab error = 0.06

(iii) Relative error = 6/142 = 3/71

% error - ~~...~~ 4.22%

Binomial expansion :-

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3$$

$$+ \; x^n$$

* If $x \to 0$ (x approaching to 0)
  In this case consider only, 2
  $(1+x)^n = 1 + nx$ and neglect other valu

$(1+x)^n = 1 + nx +$ (neglect)

Propogations and combinations of errors :-
Suppose two physical quantities $A = A \pm \Delta A$ and $B = B \pm \Delta B$ and a resultant phy. quantity R, such that,

$R = R \pm \Delta R$

$$\begin{array}{ccccc} A & = & A & \pm & \Delta A \\ B & = & B & \pm & \Delta B \\ R & & R & \pm & \Delta R \end{array}$$

mean value — sing

1) Error in result of sum of quantities :-

$R = A + B$ — (1)

put $R = R \pm \Delta R$

$A = A \pm \Delta A$

$B = B \pm \Delta B$

$R \pm \Delta R = A \pm \Delta A + B \pm \Delta B$

$R \pm \Delta R = (A + B) + (\pm \Delta A \pm \Delta B)$

put $A + B = R$ (from (1))

$\cancel{R} \pm \Delta R = \cancel{(A+B)}\cancel{R} + (\pm \Delta A \pm \Delta B)$

$\pm \Delta R = (\pm \Delta A) + (\pm \Delta B)$

In addition of two and more than two phy. quantities, the mean absolute erro in resultant quantity is equal to sum of mean ab. error in physical quantities

(ii) Error of in subt. of phy quantities :-

Set

$$R = A - B$$

Put

$$R = R \pm \Delta R$$
$$A = A \pm \Delta A$$
$$B = B \pm \Delta B$$

$M_1 = 1.5 \pm 0.05$

$M_2 = 1.2 \pm 0.02$

$M = M_1 + M_2$

$M = 2.7 \pm 0.07$

$\pm \Delta M = \pm 0.07$

$$R \pm \Delta R = (A \pm \Delta A) - (B \pm \Delta B)$$
$$R \pm \Delta R = (A - B) + (\pm \Delta A \pm \Delta B)$$
$$\cancel{R} \pm \Delta R = \cancel{R} + (\pm \Delta A \pm \Delta B)$$
$$\pm \Delta R = (\pm \Delta A) + (\pm \Delta B)$$

final error in a quantity is same as in addition & subt. error is always added never subtracted

(iii) Error in product of quantities :-

$$R = A \times B \quad - (1)$$

Put $R = R \pm \Delta R$

$$A = A \pm \Delta A$$
$$B = B \pm \Delta B$$

[May use K_1 K_2 here which are errorless quanti)

($\therefore \pm \Delta R = K_1 \pm \Delta A + K_2 \pm \Delta B$]

$$R \pm \Delta R = (A \pm \Delta A) \times (B \pm \Delta B)$$
$$= A\left(1 \pm \frac{\Delta A}{A}\right) \times B\left(1 \pm \frac{\Delta B}{B}\right)$$
$$= A \times B\left[1 \pm \frac{\Delta B}{B} \pm \frac{\Delta A}{A} \pm \frac{\Delta A \Delta B}{AB}\right] \quad - (2)$$

Tending to 0 (neglect)

$$R \pm \Delta R = R\left[1 \pm \frac{\Delta B}{B} \pm \frac{\Delta A}{A}\right]$$

$$\frac{R \pm \Delta R}{R} = 1 \pm \frac{\Delta B}{B} \pm \frac{\Delta A}{A}$$

$$\cancel{1} \pm \frac{\Delta R}{R} = \cancel{1} \pm \frac{\Delta B}{B} \pm \frac{\Delta A}{A}$$

$$\therefore \pm \frac{\Delta R}{R} = \left(\pm \frac{\Delta B}{B}\right) + \left(\pm \frac{\Delta A}{\Delta A}\right) \quad - (3)$$

The eq.^{n} (3) is multiplied by 100 on both sides

$$\pm \frac{\Delta R}{R} \times 100 = \left(\pm \frac{\Delta B}{B} \times 100\right) + \left(\pm \frac{\Delta A}{A} \times 100\right)$$

Hence fractional and % error in the resultant quant. is equal to fractional and % error in individual quantity.

Assume,

$$R = K \frac{A^{\alpha} B^{\beta}}{C^{\gamma}} \quad - (1)$$

Constant

If we consider errors in A, B, C and R then from (1), we get

$$R \pm \Delta R = \frac{(A \pm \Delta A)^{\alpha} (B \pm \Delta B)^{\beta}}{(C \pm \Delta C)^{\gamma}}$$

$$R \pm \Delta R = \frac{K A^{\alpha} \left(1 \pm \frac{\Delta A}{A}\right)^{\alpha} B^{\beta} \left(1 \pm \frac{\Delta B}{B}\right)^{\beta}}{C^{\gamma} \left(1 \pm \frac{\Delta C}{C}\right)^{\gamma}}$$

$$R \pm \Delta R = \frac{K A^{\alpha} B^{\beta}}{C^{\gamma}} \left(1 \pm \frac{\Delta A}{A}\right)^{\alpha} \left(1 \pm \frac{\Delta B}{B}\right)^{\beta} \left(1 \pm \frac{\Delta C}{C}\right)^{-\gamma}$$

$$R \pm \Delta R = R\left(1 \pm \frac{\Delta A}{A}\right)^{\alpha} \left(1 \pm \frac{\Delta B}{B}\right)^{\beta} \left(1 \pm \frac{\Delta C}{C}\right)^{-\gamma}$$

$\frac{\Delta A}{A}$, $\frac{\Delta B}{B}$ & $\frac{\Delta C}{C}$ are tending to zero.

$$\therefore \left(1 \pm \frac{\Delta A}{A}\right)^{\alpha} \approx 1 \pm \alpha \frac{\Delta A}{A}$$

$$\left(1 \pm \frac{\Delta B}{B}\right)^{\beta} \approx 1 \pm \beta \frac{\Delta B}{B}$$

$$\cancel{\left(1 \pm \frac{\Delta C}{C}\right)^{-\gamma} \approx = 1 - \gamma \pm \frac{\Delta C}{C}}$$

$$= 1 \pm \gamma \frac{\Delta C}{C}$$ [error is always +ve

Putting these values we get

$$R \pm \Delta R = R\left(1 \pm \alpha \frac{\Delta A}{A}\right)\left(1 \pm \beta \frac{\Delta B}{B}\right)\left(1 \pm \gamma \frac{\Delta C}{C}\right)$$

$$R \pm \Delta R = R\left(1 \pm \beta \frac{\Delta B}{B} \pm \alpha \frac{\Delta A}{A} \pm \alpha\beta \frac{\Delta A \Delta B}{AB}\right)\left(1 \pm \gamma \frac{\Delta C}{C}\right)$$

$$R \pm \Delta R = R\left(1 \pm \beta \frac{\Delta B}{B} \pm \alpha \frac{\Delta A}{A}\right)\left(1 \pm \gamma \frac{\Delta C}{C}\right)$$

$$R \pm \Delta R = R\left(1 \pm \gamma \frac{\Delta C}{C} \pm \beta \frac{\Delta B}{B} \pm \gamma\beta \frac{\Delta B \Delta C}{BC} \pm \alpha \frac{\Delta A}{A} \pm \alpha\gamma \frac{\Delta A \Delta C}{AC}\right.$$

$$R \pm \Delta R = R\left(1 \pm \gamma \frac{\Delta C}{C} \pm \beta \frac{\Delta B}{B} \pm \alpha \frac{\Delta A}{A}\right)$$

$$\frac{R \pm \Delta R}{R} = 1 \pm \gamma \frac{\Delta C}{C} \pm \beta \frac{\Delta B}{B} \pm \alpha \frac{\Delta A}{A}$$

$$\cancel{1} \pm \frac{\Delta R}{R} = \cancel{1} \pm \gamma \frac{\Delta C}{C} \pm \beta \frac{\Delta B}{B} \pm \alpha \frac{\Delta A}{A}$$

$$\pm \frac{\Delta R}{R} = \pm \gamma \frac{\Delta C}{C} \pm \beta \frac{\Delta B}{B} \pm \alpha \frac{\Delta A}{A} \quad \text{—— } (x)$$

If eqn (x) is multiplied by 100, we get

$$\pm \frac{\Delta R}{R} \times 100 = \pm \alpha \frac{\Delta A}{A} \times 100 \pm \beta \frac{\Delta B}{B} \times 100 \pm \gamma \frac{\Delta C}{C} \times 100$$

$$\Delta R\% = \alpha \Delta A\% + \beta \Delta B\% + \gamma \Delta C\% \quad \text{— } (y)$$

Special case ①

If $\alpha = \beta = 1$ & $C = 1$ $(\gamma = 0)$

$R = K(A \times B)$

From eq.^{n} ⑧ & ⑨

$$\pm \frac{\Delta R}{R} = \pm \frac{\Delta A}{A} \pm \frac{\Delta B}{B}$$

$$\pm \Delta R\% = \pm \Delta A\% \pm \Delta B\%$$

Special case ②

If $\alpha = \gamma = 1$ & $\beta = 0$

$$R = K\frac{A}{C}$$

From eq.^{n} ⑧ & ⑨, we get

$$\pm \frac{\Delta R}{R} = \pm \frac{\Delta A}{A} \pm \frac{\Delta C}{C}$$

$$\pm \Delta R\% = \pm \Delta A\% \pm \Delta C\%$$

→ $y = x$

Ab. error & frac. error remains same

$y = 2x$

Ab. error will be twice but relative error remains same.

Find the fr. error in Z, if

$$Z = P^{x} Q^{y} R^{2}$$

$$\pm \frac{\Delta Z}{Z} = x\left(\pm \frac{\Delta P}{P}\right) + y\left(\pm \frac{\Delta Q}{Q}\right) + 2\left(\pm \frac{\Delta R}{R}\right)$$

Q- The value of grav. acc. (g) is measured by help of simple pendulum. If % error in length of simple pendulum is 2%. and percentage error in T is 1%. Then find the

% error in 'g'.

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$T = l^{1/2}\, g^{1/2}$$

$$\frac{1}{4\pi^2}\ \frac{T^2}{l} = g$$

$\pm 2(\cancel{2}\%) \pm 1(2\%)$

$\pm 4\% = g$

Q- The inner & outter diameter of a hollow cyl. container are $d_i = 4.45 \pm 0.02$ cm & $d_o = 4.75 \pm .02$ ~~are~~ Find the % error in thickness of cyln

$$\frac{0.02}{\cancel{4.45}_{89}} \times \cancel{100}^{20} \qquad \frac{2}{\cancel{475}_{19}} \times \cancel{100}^{4}$$

$$\frac{40}{89}\% \qquad \frac{8}{19}\%$$

$$0.44\% + 0.42\%$$

$$\pm\boxed{0.86\%} \times \frac{1}{2} \Rightarrow \pm 0.43\%$$

400

3
89
×4
356

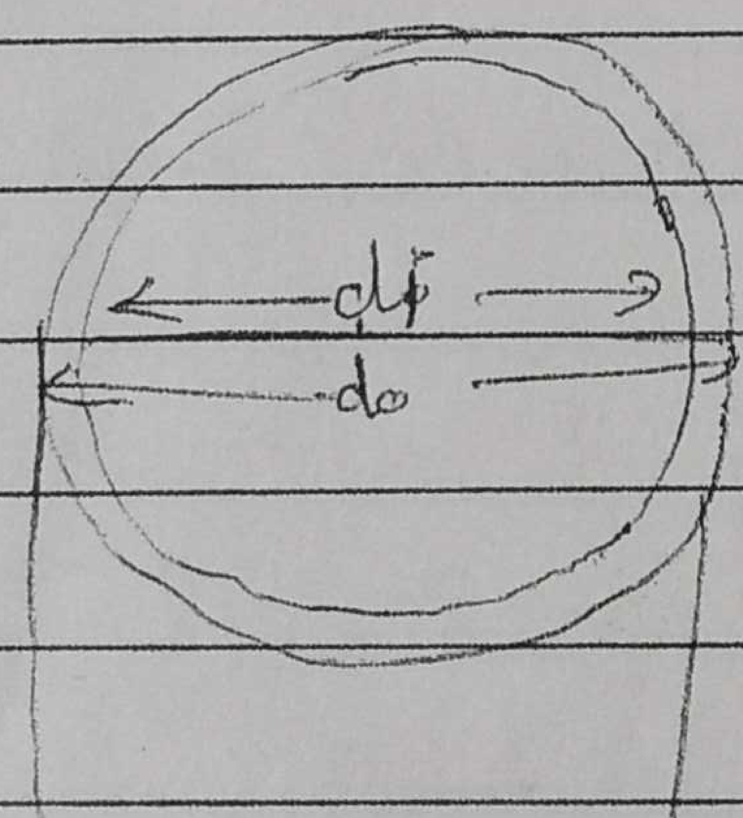
440
4
89
×5
445

80

19
×4

40+36

40



$$t = r_o - r_i$$

$$t = \frac{d_o}{2} - \frac{d_i}{2}$$

$$t = \frac{1}{2}(d_o - d_i)$$

$$t = \frac{1}{2}(4.75 - 4.45)$$

$$= \frac{1}{2}(0.30)\ \text{cm}$$

$$= 0.15\ \text{cm}$$

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$$\pm \Delta t = \frac{1}{2}(\pm \Delta d_o) + \frac{1}{2}(\pm \Delta d_i)$$

$$= \frac{1}{2}(0.02) + \frac{1}{2}(0.02)$$

$$= 0.02$$

$$t = 0.15 \pm 0.02$$

$$\Delta t\% = \frac{0.02}{0.15} \times 100 = 13.33\%$$

Two resistors of resistances $R_1 = 100 \pm 1\,\Omega$ and $R_2 = 200 \pm 1\,\Omega$ are connected in parallel with each other. Find the max. % error in eq. resist

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\cancel{R_{eq}} = \frac{R_1 + R_2}{R_1 R_2}$$

$$\therefore R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

(let $R_1 + R_2 = X$)

$$R_{eq} = \frac{R_1 R_2}{X}$$

$\cancel{\Delta X\% = \Delta R_1\% + \Delta R_2}$

$$\Delta X\% = \frac{\Delta X}{X} \times 100$$

$$= \frac{\Delta R_1 + \Delta R_2}{R_1 R_2}$$

$$\Delta R\% = \Delta R_1\% + \Delta R_2\% + \Delta X\%$$

$$\Delta R\% = \frac{\Delta R_1}{\Delta R_1} \times 100\% + \frac{\Delta R_2}{R_2} \times 100\% + \left(\frac{\Delta R_1 + \Delta}{R_1 + R}\right.$$

$$= 1\% + 0.5\% + \left(\frac{2}{300} \times 100\right)$$

$$= 1.5\% + 0.66$$

$$= 2.166\%$$

Max. error = least count of instrument

Q- Value of g is measured by help of a simple pendulum. If length of simple pendulum is measured by help of a metre scale i.e., $l = 20$ cm and time period is measured by an ordinary stopwatch. If time of hundred oscillations of simple pendulum is equal to 50 sec then find max % error in g

0.5

$100 \times T = 50$

$T = \frac{50}{100} = \frac{1}{2} = 0.5$

$$\cancel{T = 2\pi \sqrt{\frac{l}{g}}}$$

$$\cancel{\frac{T^2}{4\pi^2} \quad g = \frac{4\pi^2 l}{T^2}}$$

$\frac{2 \times 100}{10}$

0.5

$$\cancel{g = \frac{2 \times 1}{1000} \times \frac{5}{10} + 2 \times \frac{1}{[illegible]} \times \frac{200}{1000}}$$

$\frac{\Delta T \times 100}{T}$

$\frac{1 \times 100}{0.5}$

$$\cancel{= 40 + 2002}$$

$$\cancel{= 0.5 + 400}$$

$$\cancel{= 400.5}$$

$$T = 2\pi \sqrt{\frac{l}{g}}$$

$$T^2 = 4\pi^2 \frac{l}{g}$$

$$g = \frac{4\pi^2 l}{T^2}$$

$$\Delta g \% = \Delta l \% + 2 \Delta T \%$$

$l = 20$ cm.

$t_{100} = 50$ sec.

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$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$g = \frac{4\pi^2 l}{T^2}$$

$$t_{100} = 100 \times T$$

$$T = \frac{t_{100}}{100}$$

$$g = \frac{4\pi^2 l \times 10^4}{t_{100}^2} = \frac{4\pi^2 10^4 l}{t_{100}^2}$$

$$\Delta g\% = \Delta l\% + 2\Delta t_{100}\%$$ $(l = 20 \pm 0.1\ cm.)$

$$= 0.5\% + 2 \times 2\%$$ $(t_{100} = 50 \pm 1\ cm.)$

$$= 4.8\%$$

Note :- ① If $y = Kx$ (K constant)

(i) $\pm\Delta y = K(\pm\Delta x)$

(ii) $\frac{\Delta y}{y} = \frac{\Delta x}{x}$

② $z = \alpha x + \beta y$

$$\Delta z = \alpha\Delta x + \beta y$$

$$\frac{\Delta z}{z} = \frac{\alpha\Delta x + \beta y}{\alpha x + \beta y}$$

③ $z = xy$

$$\frac{\Delta z}{z} = \frac{\Delta x}{x} + \frac{\Delta y}{y}$$

$$\Delta z = \left(\frac{\Delta x}{x} + \frac{\Delta y}{y}\right) z$$

$$= \left(\frac{\Delta x}{x} + \frac{\Delta y}{y}\right) xy$$

$$= \Delta x(y) + \Delta y(x)$$

④ If a phy quantity is measured by hel, of an instrument then the more possible error in measurement is equal to least co[unt] accuracy of the instrument.

$l = 20cm \xrightarrow{M.P.A} 0.1cm \xrightarrow{considering\ error} 20 \pm 0.$

$t = 20sec \xrightarrow{Max} 1sec \xrightarrow{error} t = 20 \pm 1sec$

stop watch (L.C = 1sec)

Q- The length & width of a rect. are measured by a metre scale. If $l = 20cm$ & $b = 10cm$. find area of rect. with error consideration

$$\frac{1}{200} \times 100 \qquad \frac{1}{100} \times 100$$

$$0.5\% \qquad 1\%$$

$\Delta A = 1.5\%$

1.5

$A = 200 \pm 3\ cm^2$

$l = 20 \pm 0.1cm.$

$b = 10 \pm 0.1cm.$

$S = l \times b \rightarrow S = 200cm^2$

$$\frac{\Delta S}{S} = \frac{\Delta l}{l} + \frac{\Delta b}{b}$$

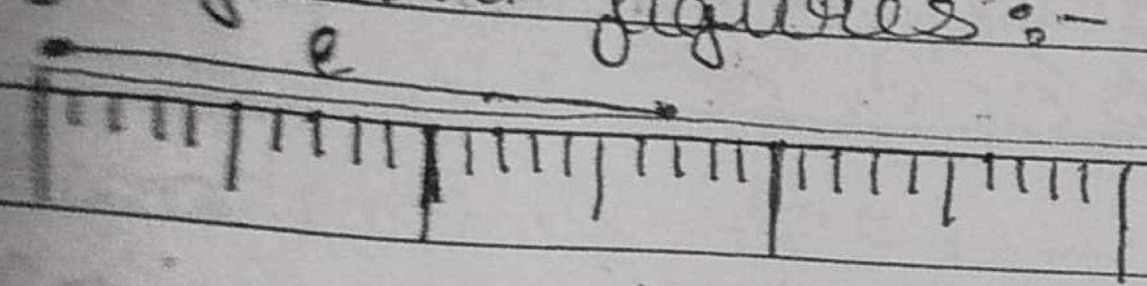
$$\frac{\Delta S}{200} \times 100 = \frac{1}{200} \times 100 + \frac{1}{100} \times 100$$

$$\frac{\Delta S}{\cancel{200}} \times 100 = \frac{3}{\cancel{200}} \times 100$$

$\Delta S = 3$

$S = 200 \pm 3cm^2$

Significant figures :-



l = 1.65 ← least significant digit

most significant digit

uncertain digit

The no. of all certain digits and first uncertain digit in a measurement is called no. of significant figures.

If in a measurement, the no. of significant figures is large then the measurement has high accuracy.

When the value of a physical quantity is conver from one unit system to another, then the no. of significant digits remain unchange

Rules for counting of significant figures:-

1) All non-zero digits are significant.

2) All zeroes are significant if they are present between two non-zero digits.

eg. → 426 → 3

4026 → 4

3.) If in a measurement, there is no decimal poi then the starting and ending zero are non-significant digits

6200 → 2

4.) If in a non decimal no., the ending zero has bar underline or decimal point then the zeroes between last non-~~zero~~ zero digit and zero that have bar / underline / decimal are significant digits and zero that has bar decimal or underline is also significant

$1400\underline{0}00^{\times} \rightarrow 5$

5.) If in a measurement there is a decimal point then the ending zeroes are significant.

eg. – $4.20600 \rightarrow 6$

6.) If the value of a measurement is less than then starting zeroes are non-significant and ending zeroes are significant

$\underbrace{0.00}_{\times}\underbrace{200}_{✓} \rightarrow 3$

7.) All exact values (for eg. – 2 apples, 2 cars or

π? 2, 4, π) have infinite no. of significant digits.

Note:- To avoid the confusion in no. of significant digits in unit conversion. We express the values of physical quantities in scientific notations.

5 Km → 5000 m

5×10^{0} Km → 5×10^{3} m → [This is done to avoid confusion]

$R = n_A \times 10^{n_B}$

$1 \leq n_A < 10$

In scientific notations, the significant figures are present in base no. (n_A)

Q- Find the no. of sig. digits in given value

$2402.044 \rightarrow 7$

$2667.000100 \rightarrow 10$

$0.00100200 \rightarrow 6$

$1.02000 \rightarrow 6$

$160000 \rightarrow 2$

$1267 \times 10^4 \rightarrow 4$

Rounding of the uncertain digits :-
For reducing the complexity in calculation of measured values, we reduce the no. of significant digits by rounding off.

Rules for rounding off :-
1) If the dropping digit is less than 5 then the preceding digit remains unchanged.
eg. - $6.2\,3\,72 \xrightarrow[\text{1 decimal place}]{R.O} 6.2$ [don't round off 3 as it will cause a lot of error]

2) If the dropping digit is more than 5 then the preceding digit is increased by 1.
$6.4672 \longrightarrow 6.47$

3) If the dropping digit is equal to 5 and it is followed by some non-zero digits then preceding digit increases by 1.
$4.26523 \xrightarrow{\text{2 decimal}} 4.27$

4) If the dropping digit is equal to 5 and it is followed by zeroes or 5 is last digit, then the preceding digit is :-
(i) Remains unchanged if it is even.
(ii) increases by 1 if it is odd.
$6.42500 \xrightarrow{2\,D.P} 6.42$
$6.435 \xrightarrow{2\,D.P} 6.44$

Rules for significant digits in arithmetic op. :-
1) In addition and subt. of measurements, the result should have same no. of decimal places as min. no. of decimal places tha

are present in individual terms.

(ii) In multiplication & division, the result should have same no. of significant digits as min. no. of significant digits are present in any individual value.

Q- The length and width of a rectangle are $l = 4.25$ cm and $b = 1.156$ cm, Find :-
(i) Perimeter
(ii) Area

Perimeter $= 2 \times (l + b)$
$= 2 \times (4.25 + 1.156)$
$= 2 \times (5.406)$
$= 2 \times (5.41)$
$= 10.82 = 10.8$
(Round off 10.82 in order to gain 3 sig. digits =

$A = l \times b$
$= (4.25)(1.156)$
$= 4.91300$
$= 4.91\ cm^2$ (Round - off)

Q- The side of a cube is equal to 1.2×10^{-2} m
The volume of cube is?
a) $1.7 \times 10^{-6} m^3$
b.) $1.70 \times 10^{-6} m^3$
c.) $1.72 \times 10^{-6} m^3$
d.) $1.72 \times 10^{6} m^3$

Note :- The final result of mathematical opperations does not depend only on the no. of significant digits present in individual no.s but also depends on its own's value.

$$x = \frac{6.267 \times 7.667 \times 1.21}{6.67 \times 1.0} \quad \begin{matrix}(3)\\(2)\end{matrix}$$

= Find exact answer and then find answer acc. to lowest sig. digits

⇒ wire of length, $l = 6 \pm 0.06$ cm. and radius $r = 0.5 \pm 0.005$ cm. has mass $m = 0.3 \pm 0.003$ g max % error in density is :-

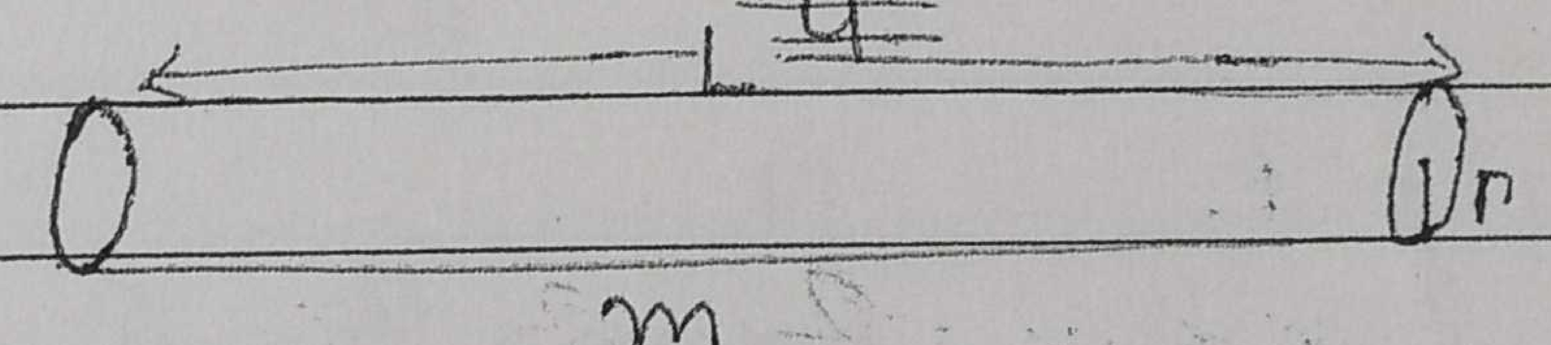
1) 4
2) 2
3) 1
4) 6.8

$$\cancel{m = V \times \rho}$$

$$\cancel{\frac{6}{100} \times \frac{100}{1000} \times 6 = 3 \times \frac{5}{1000} \times \frac{100}{1000} \times \frac{5}{10} + \Delta}$$

$$\cancel{36 = \frac{75}{100} + \frac{\Delta P}{P} \times 100}$$

$$\cancel{36 - \frac{3}{4}}$$



$$\rho = \frac{m}{\pi r^2 l}$$

$$\Delta\rho\% = \Delta m\% + 2\Delta r\% + \Delta l\%$$

$$= \frac{3}{3} \times \frac{100}{100} + 2 \times \frac{5}{5} \times \frac{100}{1000} + \frac{6}{6} \times \frac{1}{}$$

$$= 1 + 2 + 1$$

$$= 4\%$$

Q- The resp. no. of significant figures for the no.s

$23.023 \rightarrow 5$

$0.0003 \rightarrow 1$

$2.1 \times 10^{-3} \rightarrow 2$

a.) 5, 1, 2

b.) 5, 1, 5

c.) 5, 5, 2

d.) 4, 4, 2

Q- Planck's constant h, speed of light c, and gravitational G are used to form a unit of length L and unit of mass M. Then correct option(s) are

a.) $M \propto \sqrt{c}$

b.) $M \propto \sqrt{G}$

c.) $L \propto \sqrt{h}$

d.) $L \propto \sqrt{G}$

$\frac{hc}{\lambda} = E \quad \frac{c}{\lambda} = \frac{m s^{-1}}{m}$

$M^1 L^2 T^{-1} = h M^0 L T^{-1} \quad \frac{G}{h}$

$h = M^1 L^2$

$F = \frac{M_1 M_2 G}{R^2}$

$G \frac{M^2}{L^2} = M^1 L^1 T^{-2}$ N

$\frac{Kg\, m s^{-2} \times m^2}{Kg^2} = G$

$G = M^{-1} L^3 T^{-2}$

$m^3 Kg^{-1} s^{-2} \quad G = M^{-1} L^3 T^{-2}$

$\frac{m^3 s^{-2}}{G} = h = M^1 L^2 T^{-1}$

$c = M^0 L^1 T^{-1}$

$\frac{G}{hc} = M^{-1} L^2 T^0$

$\frac{Gh}{c} = M^0 L^4 T$

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$L \propto h^{x} c^{y} G^{z}$

$[M^0 L^1 T^0] = [M^{x} L^{2x} T^{-x}][M^0 L^{y} T^{-y}][M^{-z} L^{3z} T^{-2z}]$

$= M^{x-z} L^{2x+y+3z} T^{-x-y-2z}$

$x - z = 0$

$x = z$

$5x + y = 1$

$y = 1 - 5x$

$-3x - 1 + 5x = 0$

$2x = 1$

$x = \frac{1}{2},\ z = \frac{1}{2},\ \frac{-3}{2} = y$

$L \propto h^{1/2} c^{-3/2} z^{1/2}$

$M \propto h^{\alpha} c^{\beta} G^{\gamma}$

$[M^1 L^0 T^0] = [M^{\alpha} L^{2\alpha} T^{-\alpha}][M^0 L^{\beta} T^{-\beta}][M^{-\gamma} L^{3\gamma} T^{-2\gamma}]$

$= M^{(\alpha-\gamma)} L^{(2\alpha+\beta+3\gamma)} T^{(-\alpha-\beta-2\gamma)}$

$\alpha - \gamma = 1$

$\alpha = \gamma + 1$

$\cancel{2\alpha+\beta+3}$

$2\alpha + \beta + 3\gamma = 2\gamma + 2 + \beta + 3\gamma$

$0 = 5\gamma + \beta + 2$