Binomial Theorem

$$\boxed{(x+y)^n = \sum_{r=0}^{n} {}^nC_r\, x^{n-r} y^r}$$

$$\boxed{\frac{1}{(-n)!} = 0}$$

$$\boxed{{}^nC_{n-1} = n}$$

$$\boxed{{}^nC_r = {}^nC_{n-r}}$$

$$\boxed{{}^nC_r = \frac{n!}{r!(n-r)!}}$$

$${}^nC_p = {}^nC_q \Rightarrow \text{either } p = q$$
$$\text{or } p + q = n$$

$$\boxed{{}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1}}$$

$$\boxed{{}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r}$$

ESTIMATE

$$\boxed{\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}}$$

$$\boxed{{}^nC_r = \frac{n}{r}\,{}^{n-1}C_{r-1}}$$

Cases :-

1. $$\boxed{(x+y)^n + (x-y)^n = 2\{{}^nC_0 x^n + {}^nC_2 x^{n-2} y^2 + \dots\}}$$

2. $$\boxed{(x+y)^n - (x-y)^n = 2\{{}^nC_1 x^{n-1} y + {}^nC_3 x^{n-3} y^3 + \dots\}}$$

General Term :-

$$\boxed{T_{r+1} = {}^nC_r x^{n-r} y^r}$$

for $(x+y)^n$

$(r+1)$th term from the beginning

Numerically Greatest Term

In general,

$(x+y)^n$ has numerically greatest value T_{r+1} where

$$\boxed{\frac{n+1}{1+\left|\frac{x}{y}\right|} - 1 \leq r \leq \frac{n+1}{1+\left|\frac{x}{y}\right|}}$$

Formulae :-

$$\boxed{C_0 + C_1 + C_2 + \dots + C_n = 2^n}$$

$$\boxed{C_0 + C_2 + C_4 + \dots = 2^{n-1}}$$

$$\boxed{C_1 + C_3 + C_5 + \dots = 2^{n-1}}$$

$$\boxed{C_0^2 + C_1^2 + C_2^2 + \dots + C_n^2 = {}^{2n}C_n = \frac{(2n)!}{n!\,n!}}$$

$$\boxed{C_0 C_r + C_1 C_{r+1} + C_2 C_{r+2} + \dots + C_{n-r} C_n = {}^{2n}C_{n-r}}$$

ESTIMATE

Multinomial Theorem

$$(x+y+z)^n = \sum \frac{n!}{p!\,q!\,r!} x^p \cdot y^q \cdot z^r$$

where $p+q+r = n$

$$(x+y+z+w)^n = \sum \frac{n! \times x^p y^q z^r w^s}{p!\,q!\,r!\,s!}$$

where $p+q+r+s = n$

To get Sum of Coefficient

$\longrightarrow$ Put all Variables = 1

Sum of Binomial Coefficient

$= 2^n$

No. of terms in $(x+y)^n = {}^{n+1}C_1$

$= n+1$

No. of terms in $(x+y+z)^n$

$= {}^{n+2}C_2$

Formula for General :-

$\{x_1 + x_2 + x_3 + \cdots x_r\}^n \Rightarrow$

No. of terms $= {}^{n+r-1}C_{r-1}$

Binomial Theorem for any Index :-

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 \cdots \infty$$

$n \in Q$ (Rational No.)

$$(1-x)^{-n} = 1 + {}^nC_1 x + {}^{n+1}C_2 x^2 + {}^{n+2}C_3 x^3 + \cdots \infty$$

$${}^nP_r = \frac{n!}{(n-r)!}$$