Circle

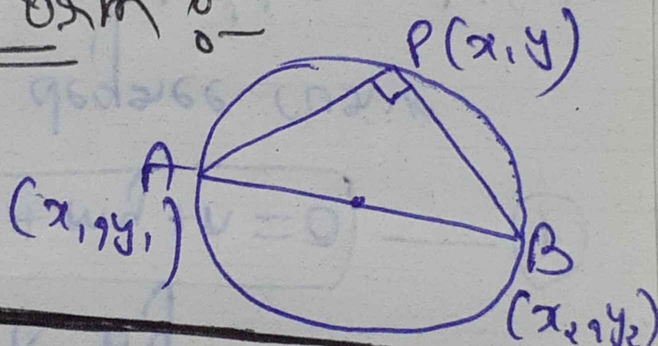
Central Form :-

$$\boxed{(x-\alpha)^2+(y-\beta)^2=r^2}$$

Standard Eqn :-

$$\boxed{x^2+y^2=r^2}$$

Diametric Form :-



$$\boxed{(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0}$$

General Eqn :-

$$\boxed{x^2+y^2+2gx+2fy+c=0}$$

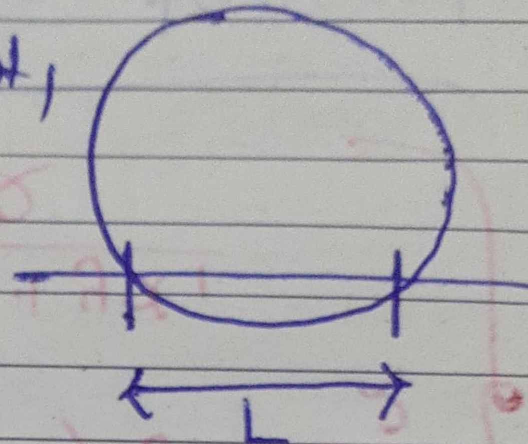
→ Centre $(-g, -f)$

→ Radius $= \sqrt{g^2+f^2-c}$

Length of Intercept on x-axis

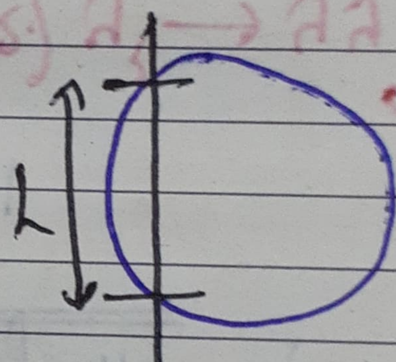
Length of Intercept,

$$\boxed{L = 2\sqrt{g^2 - c}}$$

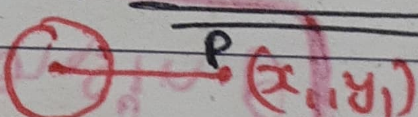


Length of Intercept on y-axis

$$\boxed{L = 2\sqrt{f^2 - c}}$$



Position of a Point w.r.t. a Circle



$$\boxed{S_1 = x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c}$$

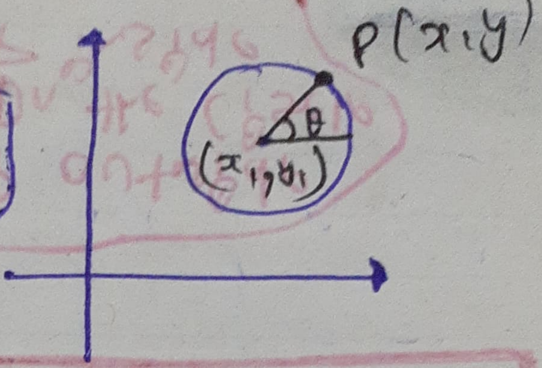
In General,

$S_1 > 0$	,	P outside
$S_1 = 0$	,	On the Circle
$S_1 < 0$	,	P inside

Parametric Equation

Circle is

$$\boxed{(x-x_1)^2+(y-y_1)^2=r^2}$$



$$\boxed{\begin{aligned} x &= x_1 + r\cos\theta \\ y &= y_1 + r\sin\theta \end{aligned}}$$

For writing tangent & Normal at x_1, y_1

Put 1) $x^2 \rightarrow xx_1$ 2) $y^2 \rightarrow yy_1$

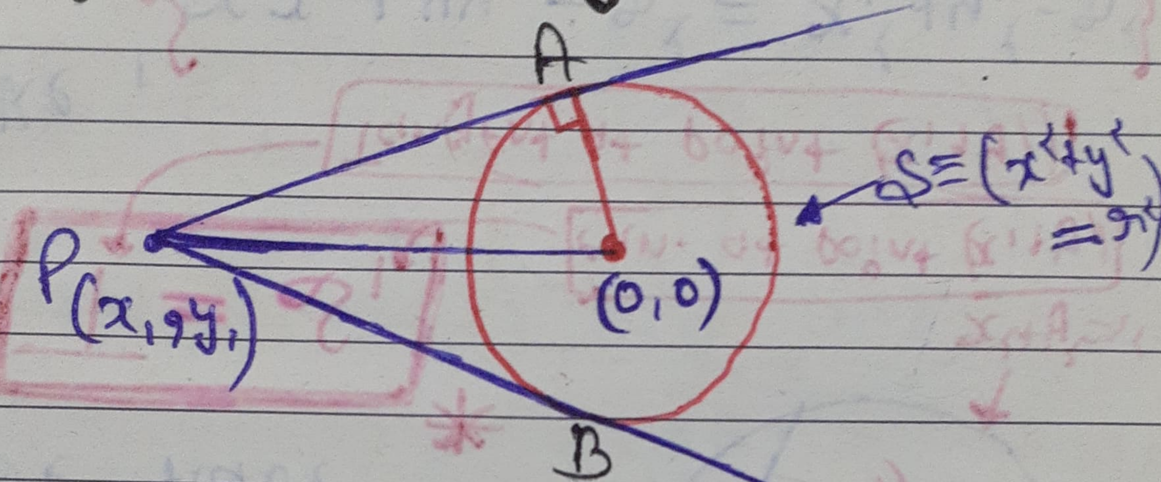
3) $x \rightarrow \frac{x+x_1}{2}$ 4) $y \rightarrow \frac{y+y_1}{2}$

5) $xy \rightarrow \frac{xy_1+yx_1}{2}$

Only Apply When tangent at point (x_1, y_1) where

(x_1, y_1) passes through curve

Length of Tangent



PA = PB = Length of tangent

$\therefore$ $\boxed{PA = \sqrt{S_1} = \sqrt{x_1^2+y_1^2-r^2}}$

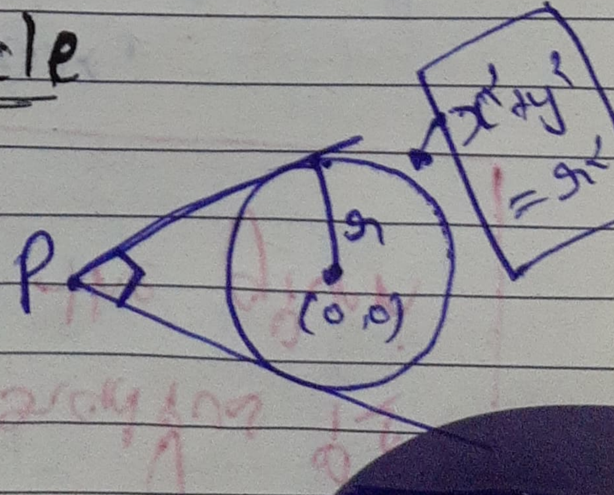
(Only applied for circle)

Power of a Point

$\boxed{\text{Power of a point} = S_1}$

Director Circle

Locus of point of Intersection of two $\perp$ tangents

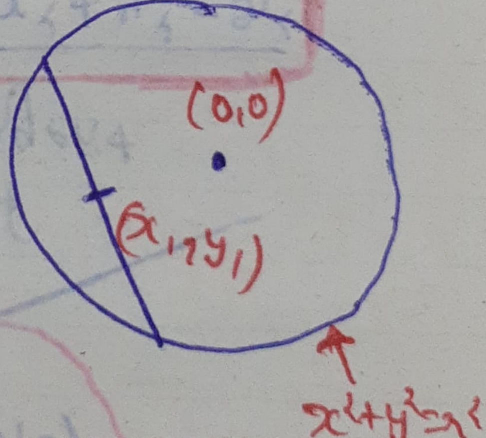


Director circle is concentric with given circle. Its radius is $\sqrt{2}$ times of the given circle

Like in previous,
Eqn. of director circle is
e.g., $\boxed{x^2+y^2=2r^2}$

Chord Having Middle Point

For chord having middle points we have



$$\boxed{T = S_1}$$

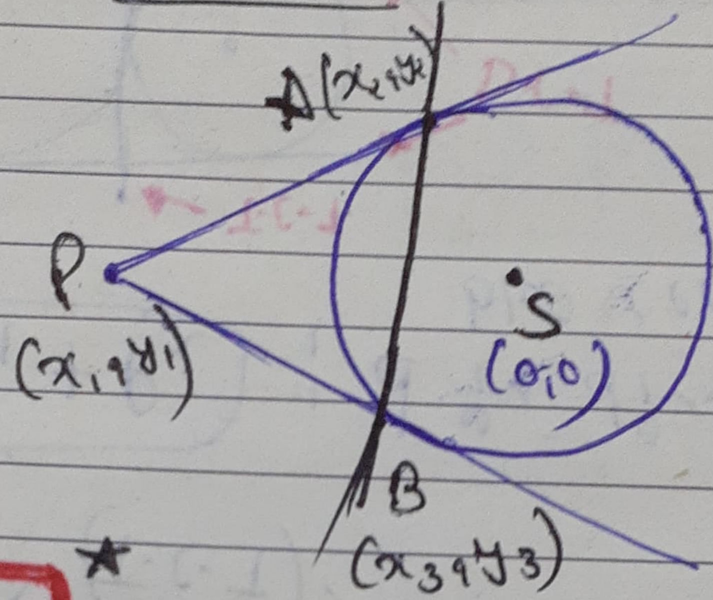
eqn. at point (x_1,y_1)

Tangent at point (x_1,y_1)

Like,
$$\{xx_1+yy_1-r^2 = x_1^2+y_1^2-r^2\}$$

Chord of Contact

AB is called chord of contact



$\boxed{T = 0}$ ★ ← for chord of contact

Tangent from (x_1, y_1)

Like $\boxed{xx_1 + yy_1 = a^2}$

PAIR OF TANGENTS

$\boxed{SS_1 = T^2}$

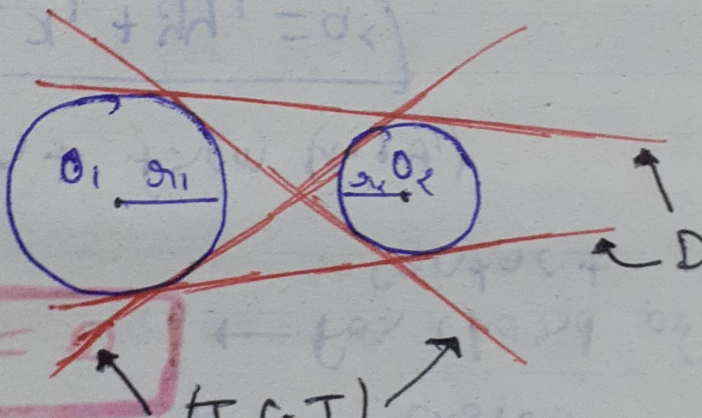
Eqn of Pair of Tangents

No. of Common tangents

- **D.C.T.** (Direct Common Tangent)
  → {having both centres on same side}

- **T.C.T** (Transverse Common Tangent)
  → {having centres on opp. side}

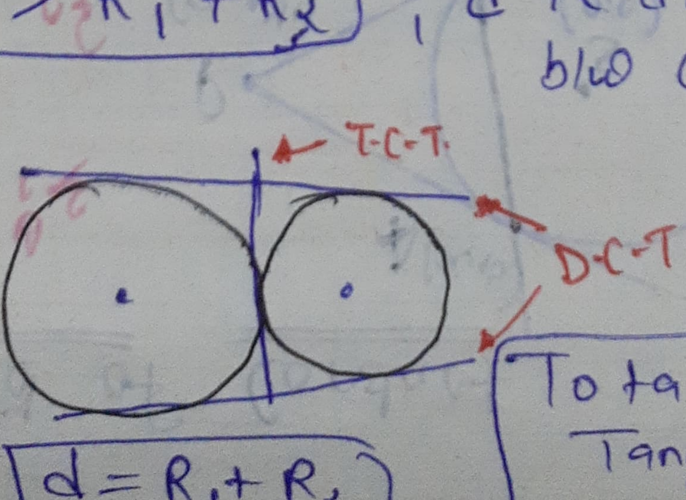
Case 1:-



Total Tangent = 4

$\boxed{d > R_1 + R_2}$; d is distance b/w centres

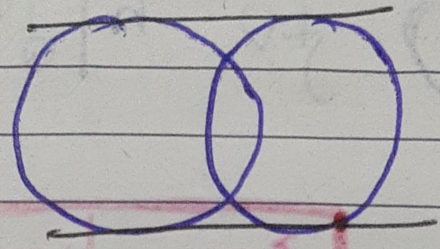
Case 2



$\boxed{d = R_1 + R_2}$

Total Tangent = 3

Case 3 :-



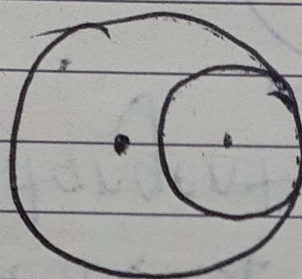
2 D.C.T

$\boxed{d < R_1 + R_2}$

$\boxed{d > |R_1 - R_2|}$

$\boxed{|r_1 - r_2| < d < r_1 + r_2}$

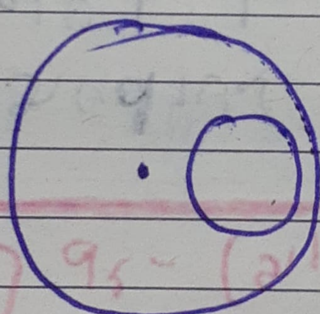
Case 4 :-



$\boxed{d = |R_1 - R_2|}$

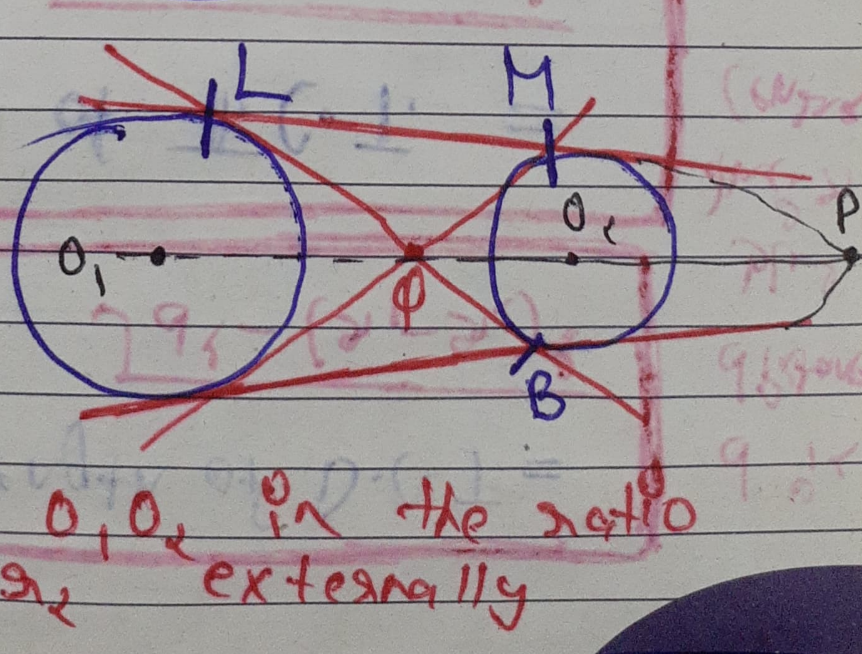
1 common tangent

Case 5 :-



$\boxed{d < |r_1 - r_2|}$

Geometry



- P divides O_1O_2 in the ratio of r_1 / r_2 externally
- ϕ divides O_1O_2 in the ratio of r_1 / r_2 internally.

- T.C.T & D.C.T. meet at the point on line joining centre.

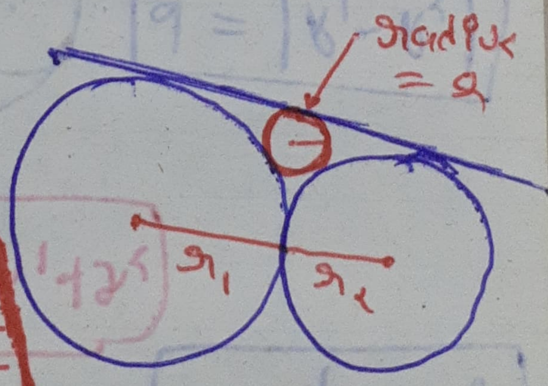
$$\boxed{LM = \text{Length of D.C.T.} = \sqrt{d^2 - (r_1 - r_2)^2}}$$

d is distance b/w their centre

- $$\boxed{\text{Length of T.C.T.} = \sqrt{d^2 - (r_1 + r_2)^2}}$$

* To find radius of that circle which touches two given circles & their common tangent

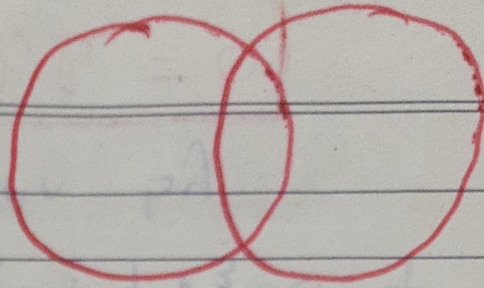
So.



$$\boxed{\frac{1}{\sqrt{r}} = \frac{1}{\sqrt{r_1}} + \frac{1}{\sqrt{r_2}}}$$

Family of Circles

(1.) ESTIMATE



Eqn. of family of circles which passes through the points of intersection of two circles

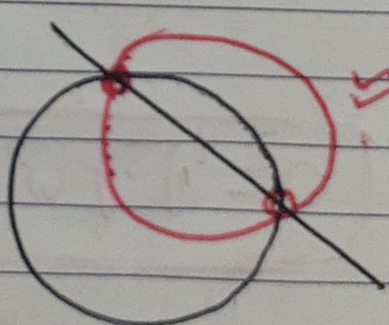
$S_1 = 0$ & $S_2 = 0$ is

$\boxed{S_1 + \lambda S_2 = 0}$, $\lambda \neq -1$

(2.) Eqn. of family of circle passes through point of intersection of a circle

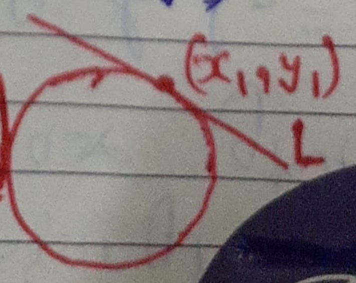
$S_1 = 0$ & Line $L = 0$

$\boxed{S + KL = 0}$



(3.) Eqn. of family of circle touching a line at its fixed point (x_1, y_1) is

$\boxed{(x - x_1)^2 + (y - y_1)^2 + KL = 0}$



4. Eqn of circle circumscribing a triangle whose sides are given by $l_1 = 0$, $l_2 = 0$ & $l_3 = 0$ is given by

$$\boxed{l_1 l_2 + k l_2 l_3 + m l_3 l_1 = 0}$$

provided,

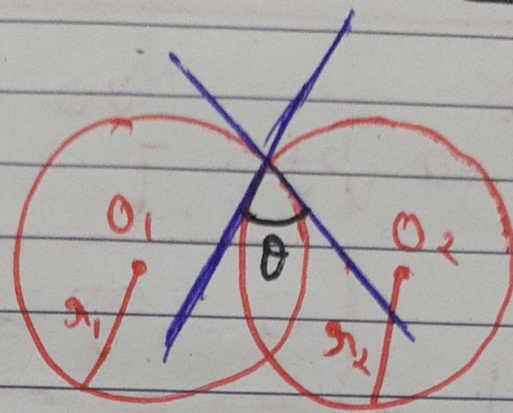
Coefficient of x^2 = Coeff. of y^2

& Coefficient of $xy = 0$

5. Eq^n of a circle circumscribing a quadrilateral whose sides in order are represented by the lines $l_1 = 0$, $l_2 = 0$, $l_3 = 0$ & $l_4 = 0$ is given by

$$\boxed{l_1 l_3 + k l_2 l_4 = 0}$$

ORTHOGONALITY OF TWO CIRCLES



∠ b/w Curves = ∠ b/w tangents

$S_1 \equiv x^2+y^2+2g_1x+2f_1y+c_1=0$

$S_2 \equiv x^2+y^2+2g_2x+2f_2y+c_2=0$

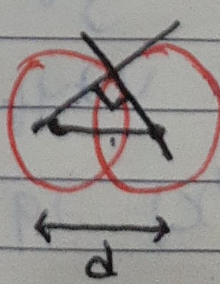
$$\boxed{\cos\theta = \frac{r_1^2+r_2^2-d^2}{2r_1r_2}}$$

If $\theta = 90^\circ$, Circle cut each other orthogonally

Condition of orthogonality is,

$$\boxed{2g_1g_2+2f_1f_2 = c_1+c_2}$$

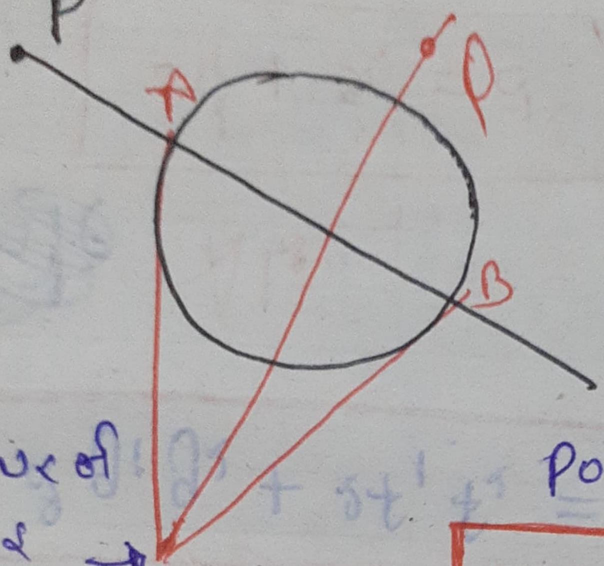
Also,



$$\boxed{r_1^2+r_2^2=d^2}$$

PROBLEMS ON POLE & POLAR

1 (Pole)

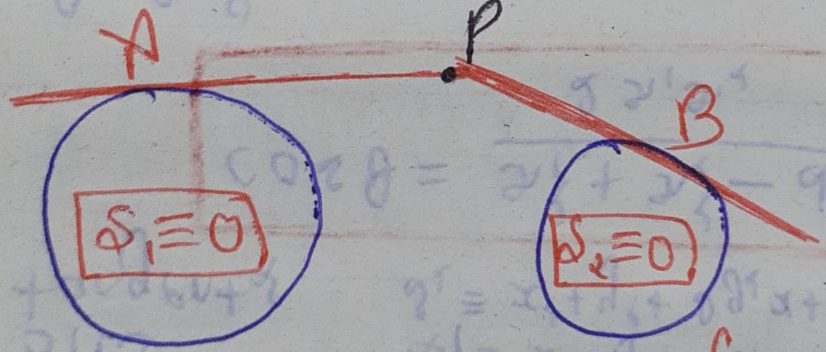


Locus of this is known as polar

If pole be (x_1, y_1) then eqn of polar is

$$\boxed{T \equiv 0}$$

Radical Axis :-



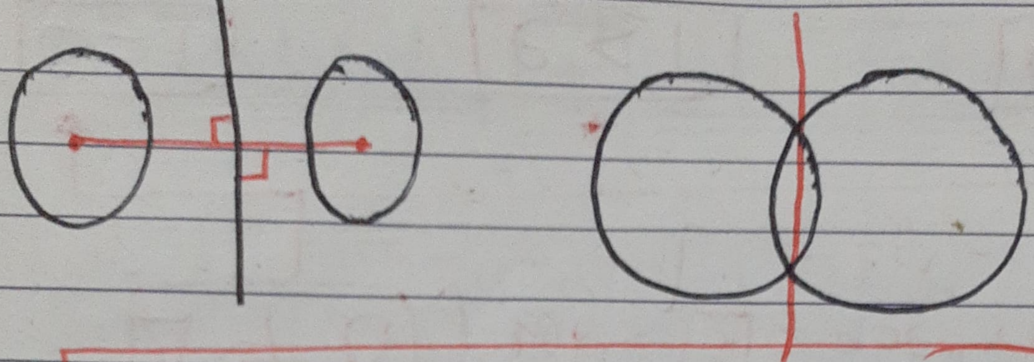
* {PA = PB}

$$\boxed{S_1 - S_2 = 0}$$

Eqn of radical Axis

OR

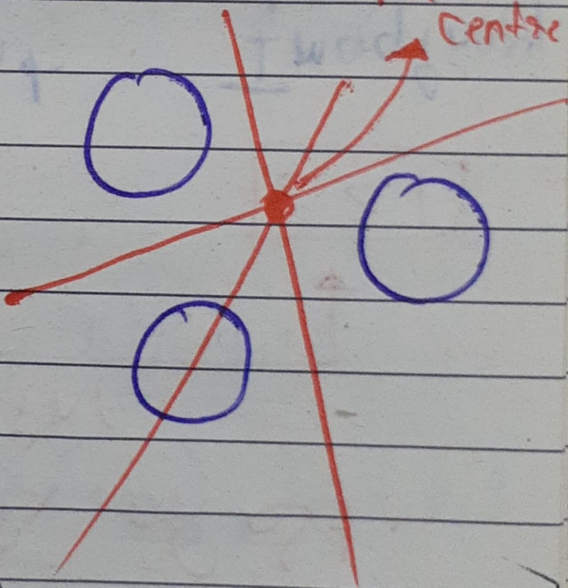
$$\boxed{2(g_1 - g_2)x + 2(f_1 - f_2)y + c_1 - c_2 = 0}$$



Eqn. of Radical Axis = Common Chord = $S_1 - S_2 = 0$

RADICAL CENTRE

Pair of circles & their radical axis meet at a point called Radical Centre



1.) $x^2+y^2=r^2 \rightarrow (r\cos\theta, r\sin\theta$

2.) $(x-x_1)^2+(y-y_1)^2=r^2 \rightarrow (x \pm r\cos\theta, y \pm r\sin\theta)$

Parametric Points

Conic Section

General Eqn :-

$$ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$$

Case 1 :- If focus lies on directrix. [Degenerate conic]

& $\boxed{\Delta = 0}$ } Pair of St. Line

- $\boxed{e > 1}$ distinct Lines
- $\boxed{e = 1}$ Coincident Lines
- $\boxed{e < 1}$ Imaginary Lines

Case 2 :- If focus ~~lies~~ doesn't lies on directrix

[Non-degenerate Conic]

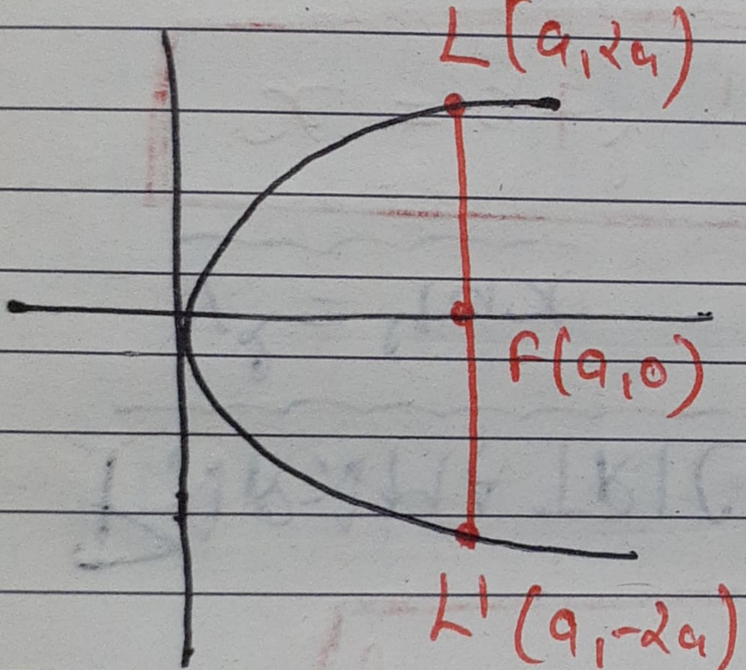
$\boxed{\Delta \neq 0}$ } where $\left\{ \Delta = abc + 2fgh - af^2 - bg^2 - ch^2 \neq 0 \right\}$

- $\boxed{e = 1}$ → Parabola → $h^2 = ab$
- $\boxed{e < 1}$ → Ellipse → $h^2 < ab$
- $\boxed{e > 1}$ → Hyperbola → $h^2 > ab$

Standard Eq^n of Parabola

$$\boxed{y^2 = 4ax}$$

Linear in x i.e. Symmetrical in x



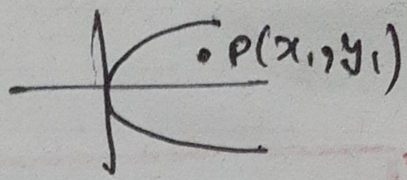
or double ordinate

$\boxed{LL' = \text{Latus Rectum} = 4a}$

Area of triangle whose vertices are (x_i, y_i), $i = 1, 2, 3$ inscribed in Parabola $y^2 = 4ax$

$$\boxed{\Delta = \frac{1}{8a}\left|(y_1 - y_2)(y_2 - y_3)(y_3 - y_1)\right|}$$

Position of a Point Relative to the Parabola



1) For point lies inside the curve

$$\boxed{y_1^2 - 4ax_1 < 0}$$

2) For point Lies On curve

$$\boxed{y_1^2 - 4ax_1 = 0}$$

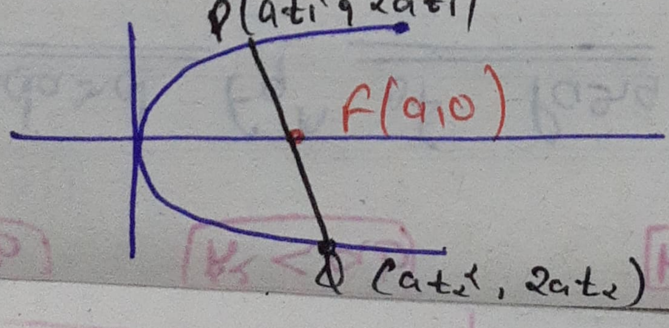
3) For point Lies outside curve

$$\boxed{y_1^2 - 4ax_1 > 0}$$

PARAMETRIC EQN OF $y^2 = 4ax$

$$\boxed{x = at^2 \, , \, y = 2at}$$ ★

Chord Joining Two Points



Eq^n of Chord PQ is,

$$\boxed{2x - (t_1+t_2)y + 2at_1t_2 = 0}$$

If chord PQ is a focal chord, then

$$\boxed{t_1 \cdot t_2 = -1}$$

i.e., eqn. of focal chord is

$$\boxed{2x - (t_1+t_2)y - 2a = 0}$$

Length of focal chord =

$$\boxed{a(t_1 - t_2)^2}$$

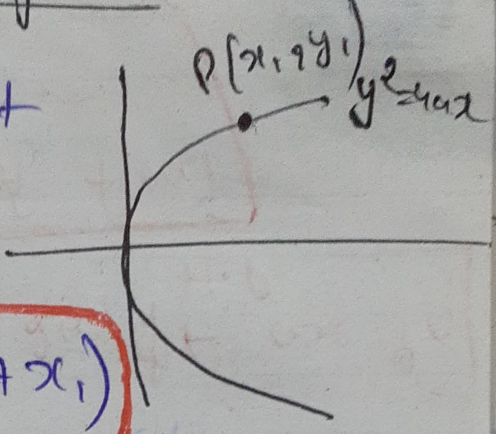
Also, $\frac{1}{PF} + \frac{1}{QF} = \frac{2}{2a}$

i.e., ~~Harm~~ PF, 2a, QF are in H.P.

Harmonic Mean of focal Segment = 2a

Results on Tangents

Eqn. of tangent at (x_1, y_1) is



$$\boxed{yy_1 = 2a(x + x_1)}$$

Eqn of Variable Tangent

Tangent at $(at^2, 2at)$

$$y \times 2at = 2a(x + at^2)$$

$$\boxed{ty = x + at^2 \rightarrow y = \frac{1}{t}x + at}$$

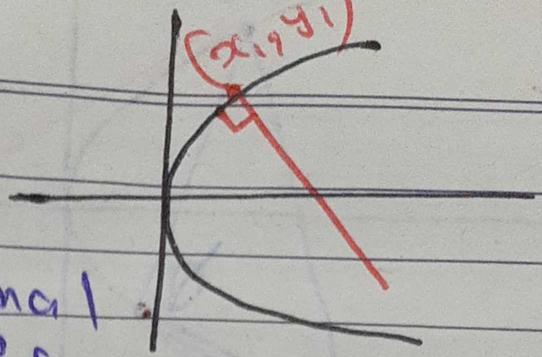
Imp. Point :- Foot of ⊥r drawn from focus to a variable tangent lies on tangent at vertex.

i.e. $\boxed{x = 0}$

$\boxed{y = mx + \frac{a}{m}}$ $\boxed{c = \frac{a}{m} + am}$

Slope form   Condition of tangency

NORMAL



Eqn. of Normal at (x_1, y_1) is

$$\boxed{y - y_1 = \frac{-y_1}{2a}(x - x_1)}$$

& Variable Normal i.e. at $(at^2, 2at)$

$$y - 2at = -t(x - at^2)$$

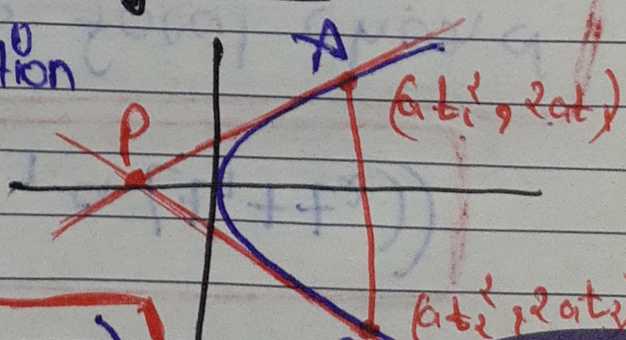
$$\boxed{y = -tx + 2at + at^3}$$

& Eqn of Normal in slope form is

$$\boxed{y = mx - 2am - am^3}$$

Results on Tangents

Point of intersection of tangents at A & B is,



$$\boxed{P \equiv (\underbrace{at_1t_2}_{G.M.}, \underbrace{a(t_1+t_2)}_{A.M.})}$$

If AB is a focal chord,
then $t_1 t_2 = -1$
So, Now,

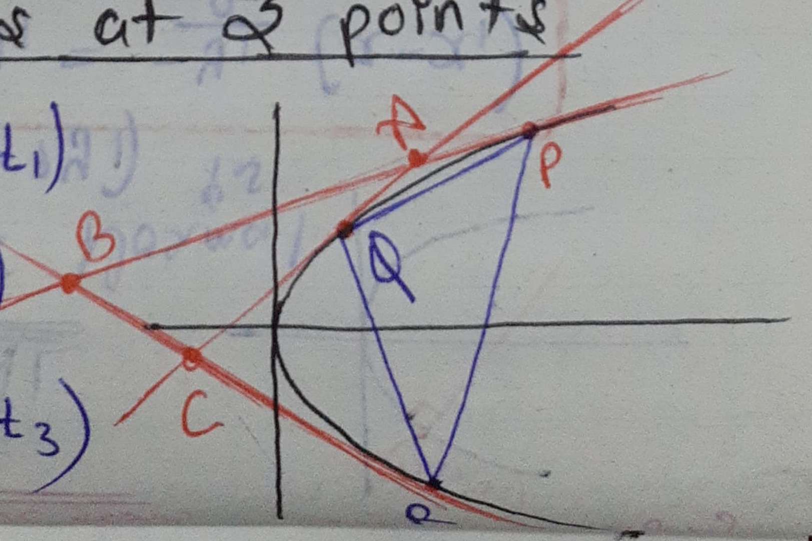
$$\boxed{P \equiv (-a,\ a(t_1+t_2))}$$

If AB is a focal chord then tangent from them meet at $\boxed{x=-a}$ i.e., on directrix

Tangents at end points of focal chord meet on the directrix & are always $\perp$r i.e. eq^n of director circle is $\boxed{x+a=0}$

Tangents at 3 points

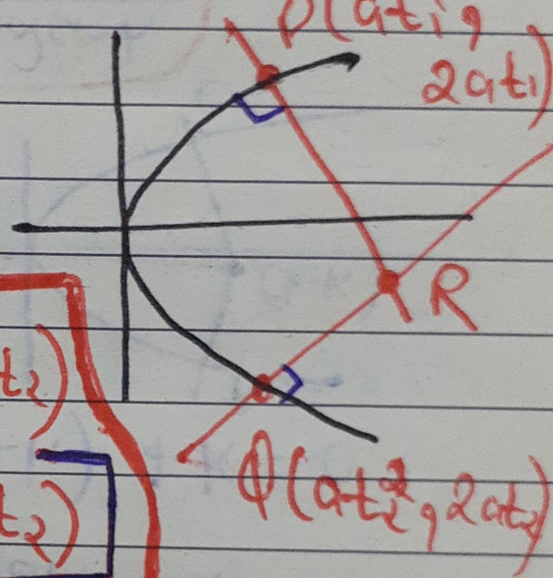
$P(at_1^2, 2at_1)$
$Q(at_2^2, 2at_2)$
$R(at_3^2, 2at_3)$



$$\boxed{\text{Area of } \Delta PQR = 2 \text{ Area of } \Delta ABC}$$

Results on Normal

(1.) So, Normals at P & Q meet at R

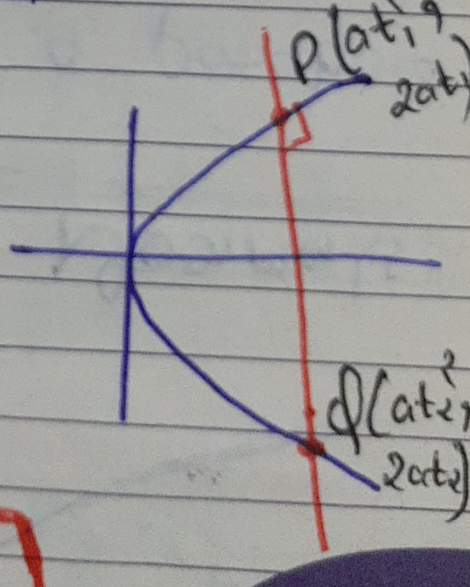


$$\boxed{R\left[2a + a(t_1^2 + t_2^2 + t_1t_2), -at_1t_2(t_1+t_2)\right]}$$

If point of intersection i.e., R lies on the curve $y^2 = 4ax$, then

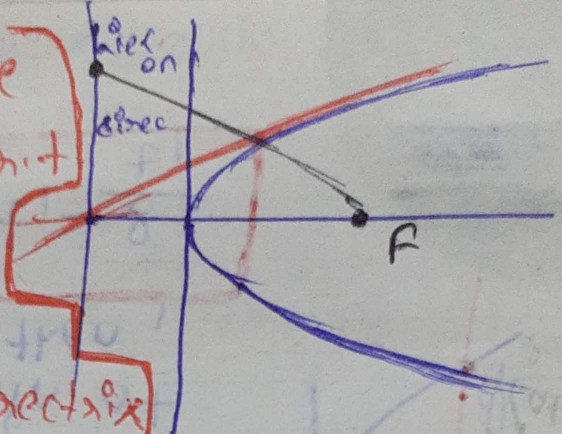
$$\boxed{t_1t_2 = 2}$$

(2.) If normal at $P(t_1)$ cuts the parabola again at $Q(t_2)$, then,



$$\boxed{t_2 = -t_1 - \frac{2}{t_1}}$$

Mirror Image of focus w.r.t a variable tangent lies always on directrix

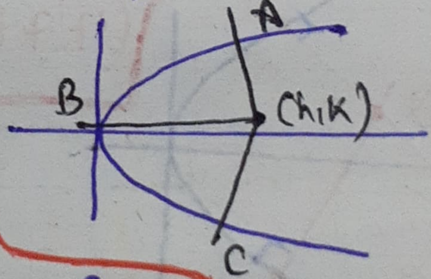


Concept of Three Normals:-

Normals drawn to a parabola passes through a point (h, k), then

$$k = mh - 2am - am^3$$

$$am^3 + m(2a-h) + k = 0$$



$\therefore$

$$m_1 + m_2 + m_3 = 0$$

$$m_1m_2 + m_2m_3 + m_3m_1 = \frac{2a-h}{a}$$

$$m_1m_2m_3 = -\frac{k}{a}$$

A, B, C are known as feet of normals or Conormal Points

ESTIMATE

$A(am_1^2, -2am_1)$, $C(am_3^2, -2am_3)$

$B(am_2^2, -2am_2)$

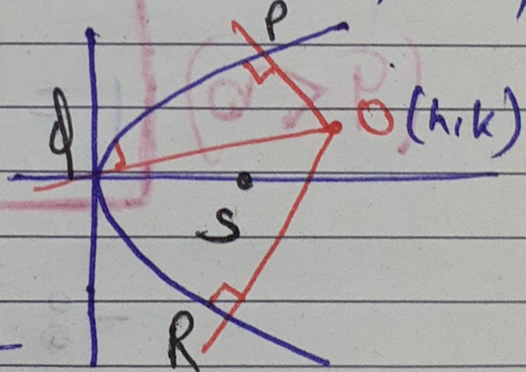
Where m_1, m_2, m_3 are the slopes of three concurrent normals

★ Centroid of the Δ formed by three conormal points lies on the x-axis

Direct Result on Three Normals

(1.) If the normals at 3 points P, Q, R meet in a point O & S can be focus, then,

$$\boxed{SP \cdot SQ \cdot SR = a(SO)^2}$$

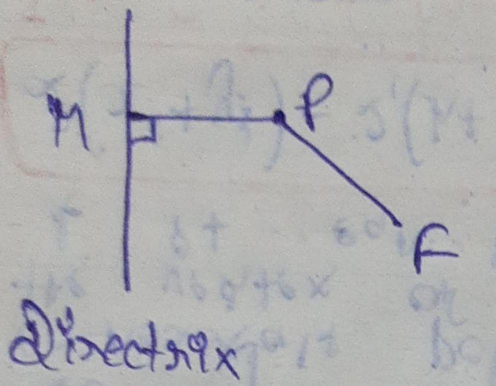


(2.) A circle circumscribing the Δ formed by 3 co-normal points passes through the vertex of the parabola & it eqn is, ← Always passes through vertex

$$\boxed{2(x^2+y^2) - 2(h+2a)x - ky = 0}$$

i.e; feet of normals (P, Q, R) & vertex are concyclic

Ellipse $(e<1)$

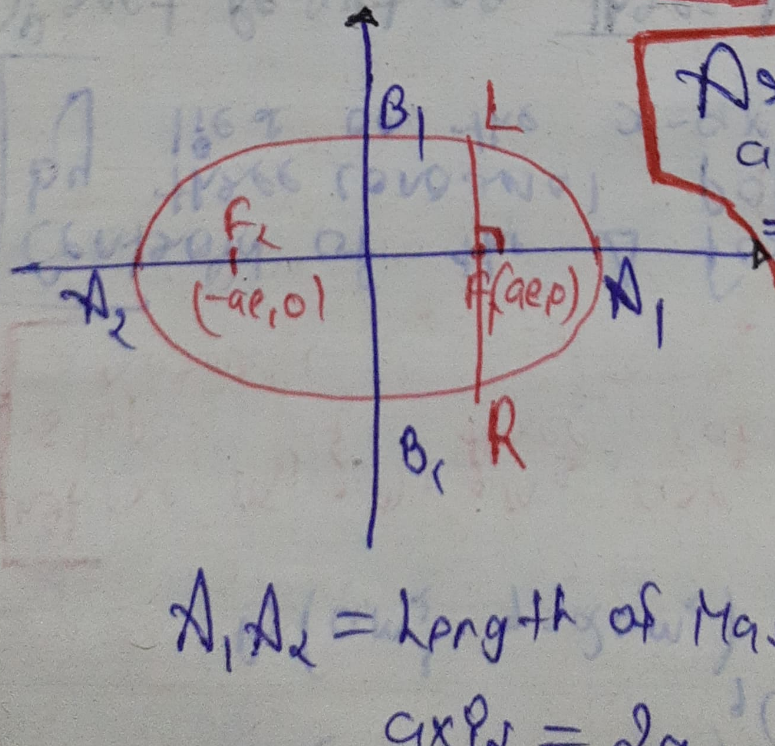


$$\boxed{PF = e(PM)}$$

Standard Eqn :-

$$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1} \quad (a>b)$$

$$\boxed{b^2 = a^2(1-e^2)}$$



Area of an ellipse $= \pi ab$

A_1A_2 = Length of Major axis $= \underline{2a}$

ESTIMATE

B_1B_2 = Length of Minor axis $= 2b$

End points of Latus Rectum

axe :-

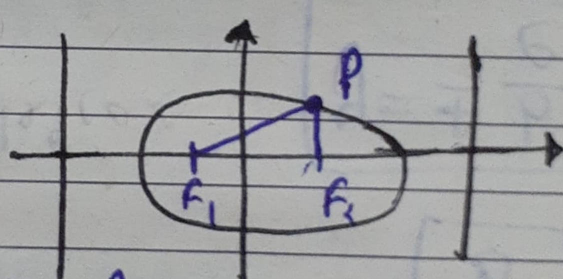
$$\boxed{L\left(ae, \frac{b^2}{a}\right)}$$

$$\boxed{R\left(ae, -\frac{b^2}{a}\right)}$$

Length of Latus Rectum = $\boxed{\frac{2b^2}{a}}$

—x—x—x—

$\boxed{PF_1 + PF_2 = 2a}$



Eq^n of directrix is,

$$\boxed{x = \pm\frac{a}{e}}$$

Concept of Vertical Ellipse

$S \equiv \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$; $\boxed{a < b}$

$$\boxed{a^2 = b^2(1-e^2)}$$

In General,

$$\boxed{e = \sqrt{1 - \left(\frac{\text{Minor}}{\text{Major}}\right)^2}}$$

foci in this case are

$\underbrace{f_1(0, be)}$, $\underbrace{f_2(0, -be)}$

Length of Latus Rectum $= \boxed{\frac{2a^2}{b}}$

&

Eqn of directrices $= \boxed{y = \pm \frac{b}{e}}$

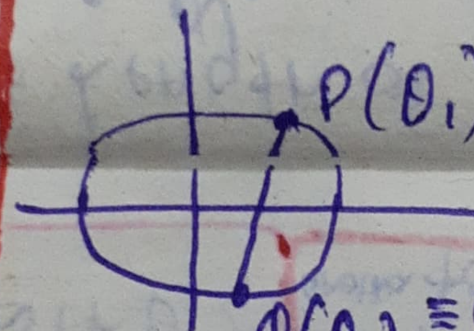
Standand Eqn of Ellipse

(If centre is not at origin)

$$\boxed{\frac{(x-\alpha)^2}{a^2} + \frac{(y-\beta)^2}{b^2} = 1}$$

Chord Joining Two Points

Harmonic mean of focal segment = Semi Latus Rectum



ESTIMATE

$$\frac{x}{a}\cos\left(\frac{\theta_1+\theta_2}{2}\right)+\frac{y}{b}\sin\left(\frac{\theta_1+\theta_2}{2}\right) = \cos\left(\frac{\theta_1-\theta_2}{2}\right)$$ ★

If this is a focal chord passing through $(ae, 0)$

$$\boxed{e = \frac{\cos\left(\frac{\theta_1 - \theta_2}{2}\right)}{\cos\left(\frac{\theta_1 + \theta_2}{2}\right)}}$$

OR

$$\boxed{e = \frac{\sin\theta_1 + \sin\theta_2}{\sin(\theta_1 + \theta_2)}}$$

OR

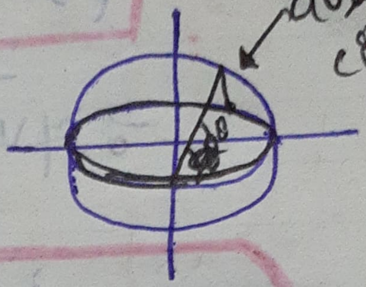
$$\boxed{\frac{e-1}{e+1} = \tan\left(\frac{\theta_1}{2}\right)\tan\left(\frac{\theta_2}{2}\right)}$$

Parametric Points:

$$\boxed{(a\cos\theta, b\sin\theta)}$$

Eqn. of Auxiliary Circle

$$\boxed{x^2+y^2=a^2}$$



Tangent to an Ellipse

$$\boxed{T = \frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1}$$

(At (x_1, y_1))

Variable Tangent (At $a\cos\theta$, $b\sin\theta$)

$$\boxed{\frac{x\cos\theta}{a} + \frac{y\sin\theta}{b} = 1}$$

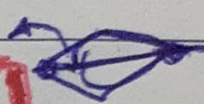
It This tangent cuts the coordinate axes at A & B :-

$$A\left(\frac{a}{\cos\theta}, 0\right)$$

$$B\left(0, \frac{b}{\sin\theta}\right)$$

Ray coming from focus of ellipse after reflection passing through another focus

Area of the Quadrilateral (Rhombus) formed by tangents at the End points of Latus Rectum



$$\boxed{\text{Area of Rhombus} = \frac{2a^2}{e}}$$

Condition of Tangency

$$\boxed{c^2 = a^2m^2 + b^2}$$

Eqn of Tangent in Slope form :-

$$\boxed{y = mx \pm \sqrt{a^2m^2 + b^2}}$$

Eqn of Normal at (x_1, y_1)

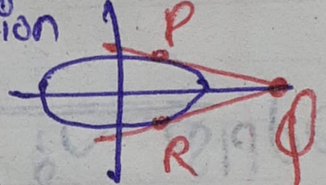
$$\boxed{\frac{a^2x}{x_1} - \frac{b^2y}{y_1} = a^2 - b^2}$$

Variable Normal at θ $(a\cos\theta, b\sin\theta)$

$$\boxed{\frac{ax}{\cos\theta} - \frac{by}{\sin\theta} = a^2 - b^2}$$

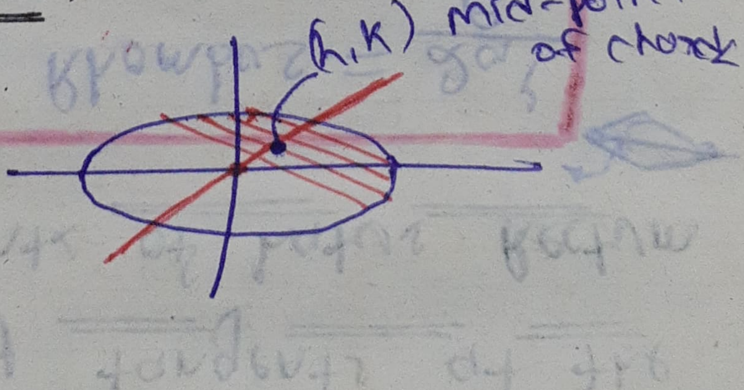
Point of Intersection of tangents at $P(\alpha)$ & $R(\beta)$

Point of Intersection i.e., ϕ is



$$\left(\frac{a\cos\left(\frac{\alpha+\beta}{2}\right)}{\cos\left(\frac{\alpha-\beta}{2}\right)}, \frac{b\sin\left(\frac{\alpha+\beta}{2}\right)}{\cos\left(\frac{\alpha-\beta}{2}\right)} \right)$$

Diameter



Slope of || chords $= m$

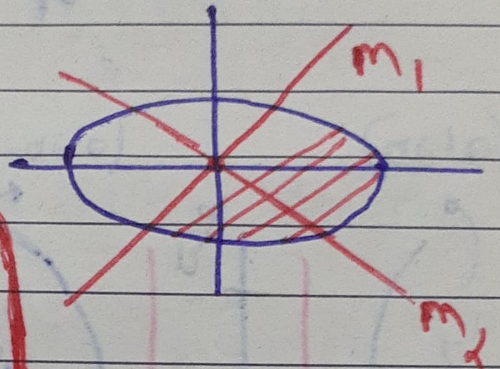
Eqn of diameter is

$$\boxed{y = \left(\frac{-b^2}{a^2 m}\right) x}$$

ESTIMATE Two diameters bisecting each other are known as **Conjugate diameter**

For conjugate diameters,

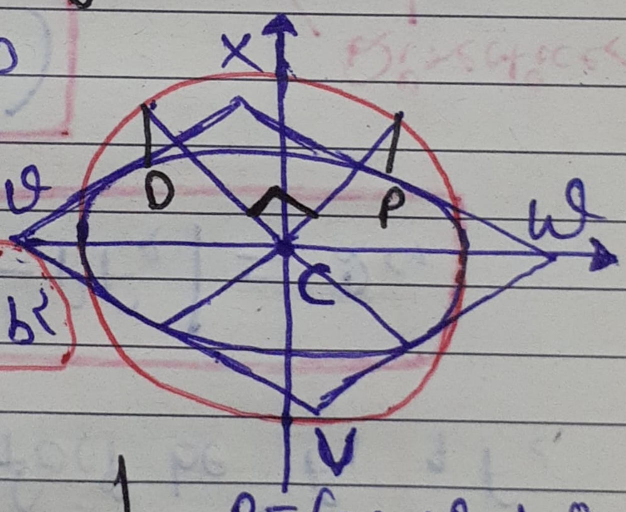
$$\boxed{m_1 m_2 = -\frac{b^2}{a^2} = e^2 - 1}$$



Result on Conjugate Diameters

If ∠ b/w two diameter = 90°

1) $\boxed{CP^2 + CD^2 = a^2 + b^2}$



$P \equiv (a\cos\theta, b\sin\theta)$

$D \equiv (-a\sin\theta, -b\cos\theta)$

2) Area of ||gm formed by the tangents at the end points of conjugate diameters,

$\boxed{\text{Area of } UVWX = 4ab}$

Hyperbola $(e>1)$

Standard eqn. is

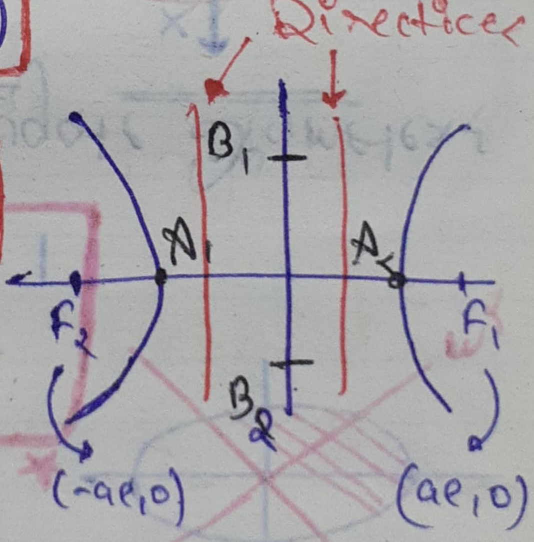
$$\boxed{\frac{x^2}{a^2}-\frac{y^2}{b^2}=1}$$ *

If any point on hyperbola be P & foci be F_1 & F_2

then $\boxed{|PF_1 - PF_2| = 2a}$

$$\boxed{b^2 = a^2(e^2-1)}$$

$$\boxed{\text{foci} \equiv (\pm ae, 0)}$$



1.) A_1A_2 is transverse axis

2) B_1B_2 is conjugate axes

Length of Latus Rectum $= \frac{2b^2}{a}$

Eqn of directrices $\Rightarrow \boxed{x = \pm\frac{a}{e}}$

Parametric Points

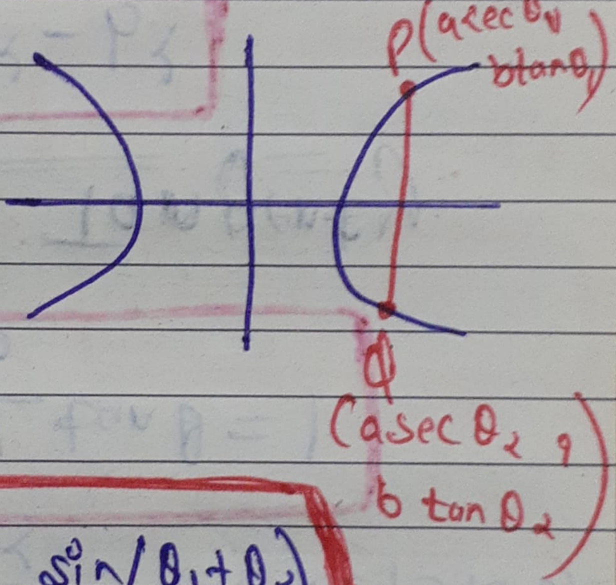
Any point on hyperbola be

$$\boxed{(a\sec\theta), (b\tan\theta)}$$

Chord Joining Two points

After solving, lot, we get

Eqn. of chord as



↓

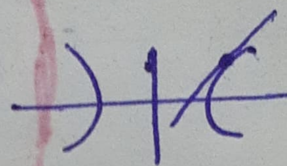
$$\boxed{\frac{x}{a}\cos\left(\frac{\theta_1-\theta_2}{2}\right)-\frac{y}{b}\sin\left(\frac{\theta_1+\theta_2}{2}\right)=\cos\left(\frac{\theta_1+\theta_2}{2}\right)}$$

$$\boxed{e=\frac{\cos\left(\frac{\theta_1+\theta_2}{2}\right)}{\cos\left(\frac{\theta_1-\theta_2}{2}\right)}}$$

$$\boxed{e=\frac{\sin(\theta_1+\theta_2)}{\sin\theta_1+\sin\theta_2}}$$

$$\boxed{\frac{1-e}{1+e} = \tan\left(\frac{\theta_1}{2}\right)\tan\left(\frac{\theta_2}{2}\right)}$$

Tangent :-



$$\boxed{\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1}$$ (At x_1, y_1)

Any variable Tangent at $(a\sec\theta, b\tan\theta)$ is :-

$$\boxed{\frac{x}{a}\sec\theta - \frac{y}{b}\tan\theta = 1}$$

Condition of Tangency

$$\boxed{c^2 = a^2m^2 - b^2}$$

Eqn. of tangent in slope form is

$$\boxed{y = mx \pm \sqrt{a^2m^2 - b^2}}$$

Eqn. of Normal at (x_1, y_1)

ESTIMATE

$$\boxed{\frac{a^2 x}{x_1} + \frac{b^2 y}{y_1} = a^2 + b^2}$$

Variable Normal :-

$$\boxed{ax\cos\theta + by\cot\theta = a^2 + b^2}$$

Eqn of Director Circle :-

1) In Ellipse :- $\boxed{x^2 + y^2 = a^2 + b^2}$

2) In Hyperbola :- $\boxed{x^2 + y^2 = a^2 - b^2}$

Relation b/w the Eccentricities of the two conjugate hyperbola is :-

$$\boxed{\frac{1}{e^2} + \frac{1}{e_1^2} = 1}$$

Diameter in Hyperbola

$$\boxed{y = \left(\frac{b^2}{a^2 m}\right)x}$$

Also, for conjugate diameters :-

$$\boxed{m_1 \cdot m_2 = \frac{b^2}{a^2}}$$

or $\boxed{m_1 \cdot m_2 = e^2 - 1}$

Concept of Asymptotes

- How to find?

Let eqn of Hyperbola is

$S \equiv 0$

So, Eqn of pair of Lines (Asymptotes) are given by

$$\boxed{S + \lambda = 0}$$

To get (λ), we put, $\Delta = 0$

i.e., $\boxed{abc + 2fgh - af^2 - bg^2 - ch^2 = 0}$

for $S \equiv \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

ESTIMATE

Equation of Asymptotes are

$$\boxed{\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0}$$

Angle b/w Asymptotes

$$\boxed{\theta = 2\tan^{-1}\left(\frac{b}{a}\right) = 2\sec^{-1} e}$$

Rectangular Hyperbola

Angle b/w Asymptotes = 90°

or

$\boxed{a = b} \Rightarrow \boxed{e = \sqrt{2}}$ always

Eqn of rectangular hyperbola is

$$\boxed{x^2 - y^2 = a^2}$$

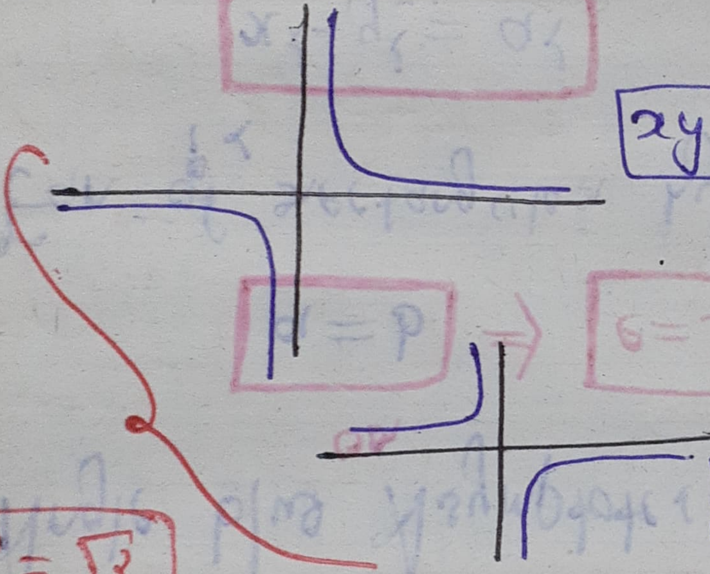
Also, It's Asymptotes are

$x = y$

& $x = -y$

Standard form of Rectangular Hyperbola :-

$\boxed{xy = c^2}$ where $\boxed{c^2 = \frac{a^2}{2}}$



$\boxed{xy = +ve}$

$\boxed{xy = -ve}$

$\boxed{e = \sqrt{2}}$ always

Any point on $xy = c^2$ can be taken as,

$$\boxed{\left(ct , \frac{c}{t}\right)} \; *$$

Length of transverse axis

$$\boxed{\text{length} = 2\sqrt{2}\,c}$$

Foci are :-

$$\boxed{(\sqrt{2}c, \sqrt{2}c) \ \& \ (-\sqrt{2}c, -\sqrt{2}c)}$$

Eqn. of Latus Rectum is :-

$$\boxed{x + y = 2\sqrt{2}c}$$

Eqn. of directrices is :-

$$\boxed{x + y = \pm\sqrt{2}c}$$

For
$xy = c^2$

Eqn. of Tangent at (x_1, y_1) is

$$\boxed{xy_1 + yx_1 = 2c^2}$$

Also, $x_1 y_1 = c^2$

So, $xy_1 + yx_1 = 2x_1 y_1$

$$\boxed{\frac{x}{x_1} + \frac{y}{y_1} = 2}$$

Also, Variable tangent i.e.

eqn of tangent at $\left(ct, \frac{c}{t}\right)$ is

$$\boxed{\frac{x}{t} + ty = 2c}$$

Eqn of Normal at (x_1, y_1)

$$\boxed{y - y_1 = \frac{x_1}{y_1}(x - x_1)}$$

Variable Normal

$$\boxed{y - \frac{c}{t} = t^2(x - ct)}$$