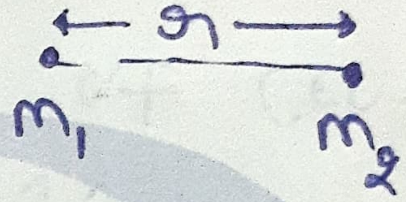
Gravitation

- Force b/w two mass,

$$\boxed{F = \frac{G m_1 m_2}{r^2}}$$



$$G = 6.67 \times 10^{-11}\ N m^2 kg^{-2}$$

& Mass of Earth $= 6 \times 10^{24}$ kg

Gravitational field

$$\boxed{\vec{E} = \frac{\vec{F}}{m_{test}}}$$

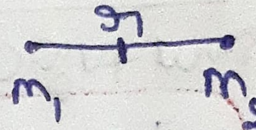
- Gravitational field of a point mass

$$\boxed{E = \frac{Gm}{r^2}}$$

Gravitational field at :-

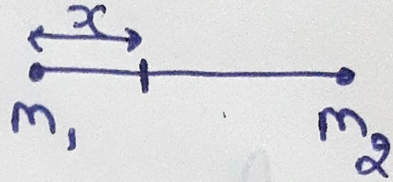
i) Centre of two masses →

$$\boxed{E_{net} = \frac{4G(m_2 - m_1)}{r^2}}$$



ii) At Null Point $(E=0)$ →

Distance of null point from m_1 is

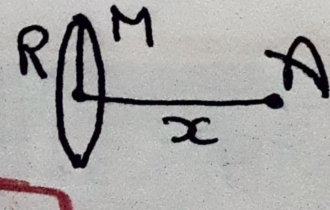


$$\boxed{x = \frac{r\sqrt{m_1}}{\sqrt{m_1} + \sqrt{m_2}}}$$

★ For a Regular Polygon, Gravitational field at Centre of polygon is zero.

Gravitational field due to :-

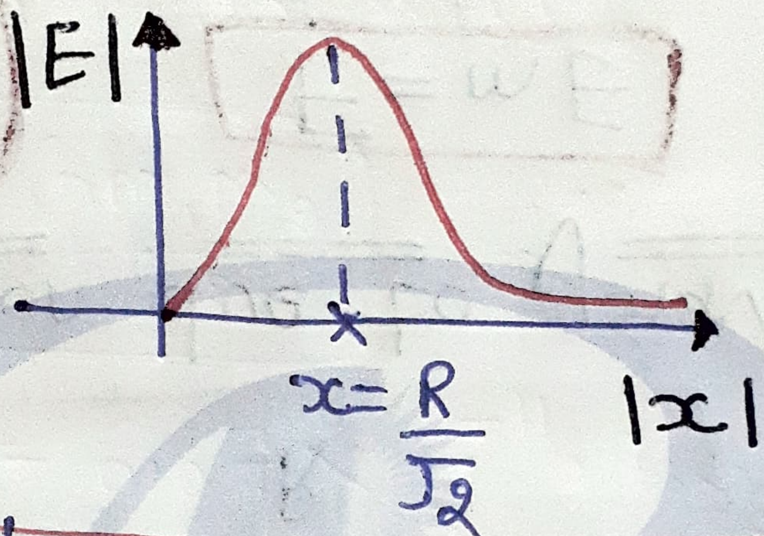
(i) Ring → $\boxed{E_{centre} = 0}$



$$\boxed{E_A = \frac{GMx}{(R^2 + x^2)^{3/2}}}$$

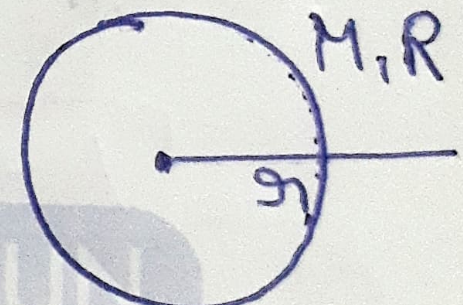
Graph of Field due to

Ring →



• At $x = R/\sqrt{2}$, Field is Maximum

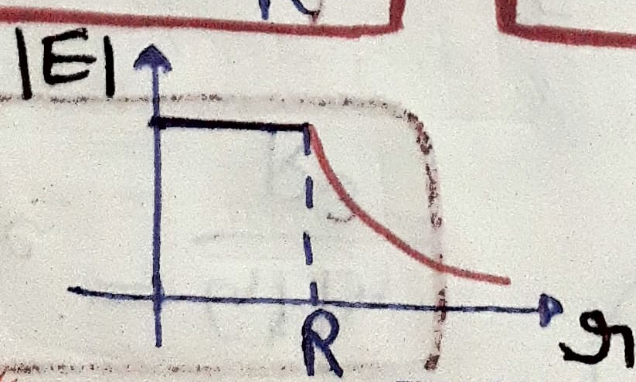
(ii) Hollow Sphere →



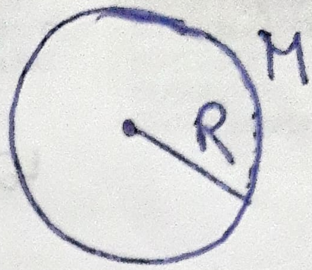
$E_{inside} = 0$

$E_{surface} = \frac{GM}{R^2}$

$E_{outside} = \frac{GM}{r^2}$

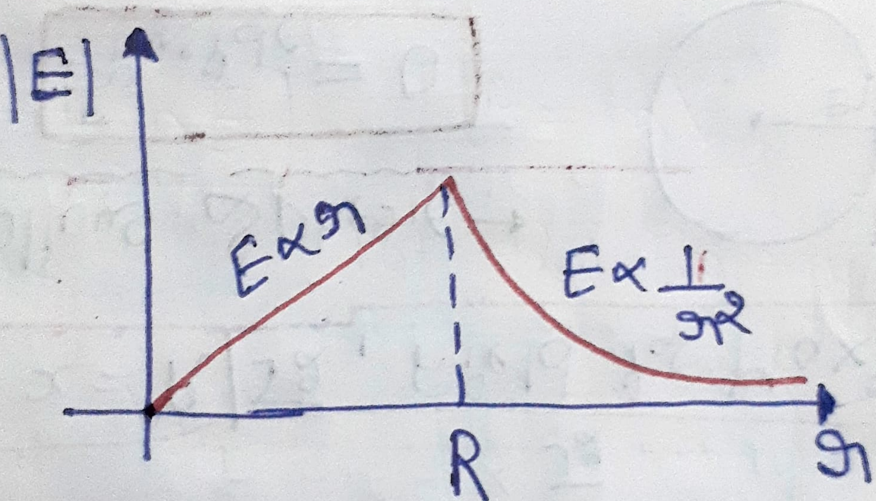


(iii) Solid Sphere →



$$\boxed{E_{inside} = \frac{GMr}{R^3}}$$

$$\boxed{E_{outside} = \frac{GM}{r^2}} \qquad \boxed{E_{surface} = \frac{GM}{R^2}}$$



Acceleration due to gravity

$$\boxed{g = \frac{GM}{R^2}} \qquad \boxed{F = mE}$$

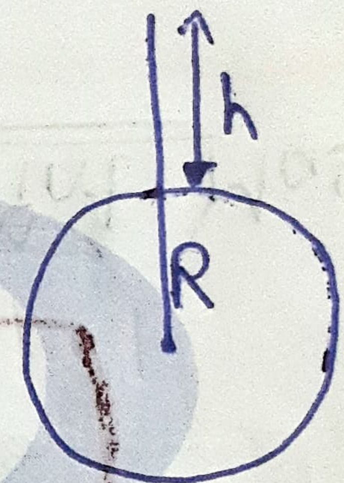
Force

$\therefore$ $\boxed{g = E}$

g at height h →

(i) For large height :-

$$\boxed{g_h = \frac{g}{\left(1+\frac{h}{R}\right)^2}}$$

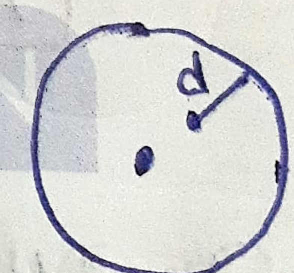


(ii) For small height :-

$$\boxed{g_h = g\left(1-\frac{2h}{R}\right)}$$

g at depth d

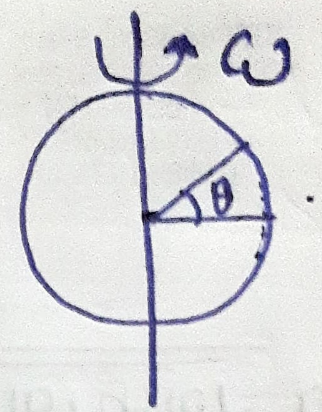
$$\boxed{g_d = g\left(1-\frac{d}{R}\right)}$$



Variation in g due to rotation of Earth :-

$$\boxed{g_\theta = g - \omega^2 R\cos^2\theta}$$

$$\boxed{g_{equator} = g - \omega^2 R}$$



For weightlessness in equator we have to increase the ω by 17 times.

$$\boxed{F = mE} \qquad \boxed{U = mV}$$

Force $\qquad$ Potential Energy

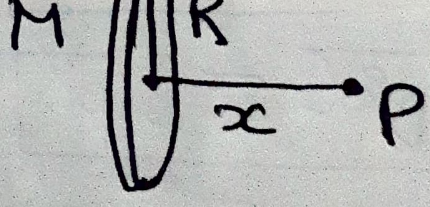
$$\boxed{V_{r_2} - V_{r_1} = -\int_{r_1}^{r_2} \vec{E} \cdot \vec{dr}}$$

Potential due to a Point Mass

$$\boxed{V_r = -\frac{Gm}{r}}$$

Potential :-

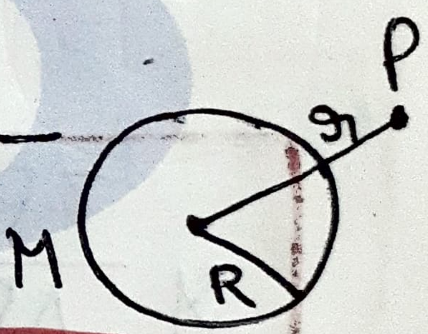
1.) For Ring →



$$\boxed{V_p = \frac{-GM}{\sqrt{R^2+x^2}}} \qquad \boxed{V_{centre} = \frac{-GM}{R}}$$

$$\& \quad \boxed{V_\infty = 0}$$

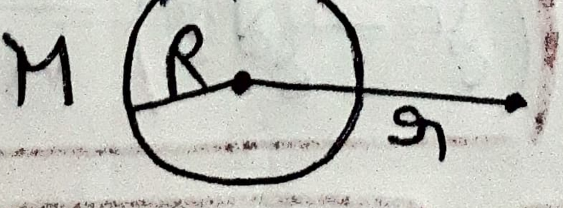
2.) For Hollow Sphere :-



$$\boxed{V_{inside} = -\frac{GM}{R}} \qquad \boxed{V_{centre} = -\frac{GM}{R}}$$

$$\boxed{V_{surface} = -\frac{GM}{R}} \qquad \boxed{V_{outside} = -\frac{GM}{r}}$$

3.) For Solid Sphere :-



$$\boxed{V_{outside} = -\frac{GM}{r}} \quad \boxed{V_{surface} = \frac{-GM}{R}}$$

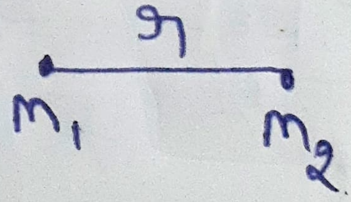
$$\boxed{V_{in} = \frac{-GM}{2R^3}\left(3R^2 - r^2\right)} \quad \boxed{V_{centre} = -\frac{3}{2}\frac{GM}{R}}$$

Relation b/w field & Potential

$$\boxed{E = -\frac{dV}{dr}}$$

$$\boxed{\vec{E} = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k}}$$

Potential Energy



$$\boxed{U_r = -\frac{Gm_1m_2}{r}}$$

Escape Speed

↳ Consider T.E = 0

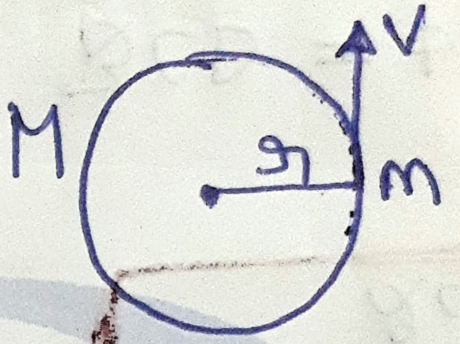
$$\boxed{V_{esc.} = \sqrt{\frac{2GM}{R}}} \quad \boxed{V_{esc.} = \sqrt{2gR}}$$

In general,

$$\boxed{V_{esc.} = 11.2\ Km/s}$$

Orbital Velocity (in Circular Orbits)

$$\boxed{V_0 = \sqrt{\frac{GM}{r}}}$$



$$\boxed{V_{esc.} = \sqrt{2}\, V_0}$$

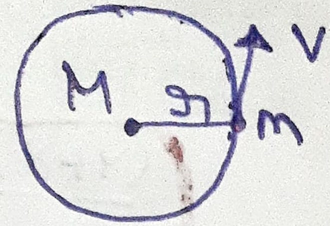
& Time Period is, $\boxed{T = \frac{2\pi r^{3/2}}{\sqrt{GM}}}$

By Kepler's Law of planetary motion, $\boxed{T^2 \propto r^3}$

For a satellite orbiting close to earth, orbital velocity is given by $\rightarrow$

$$\boxed{\begin{aligned} V_0 &= \sqrt{g(R+h)} \\ V_{esc.} &= \sqrt{2g(R+h)} \end{aligned}}$$

$$\boxed{K.E. = \frac{GMm}{2r}}$$

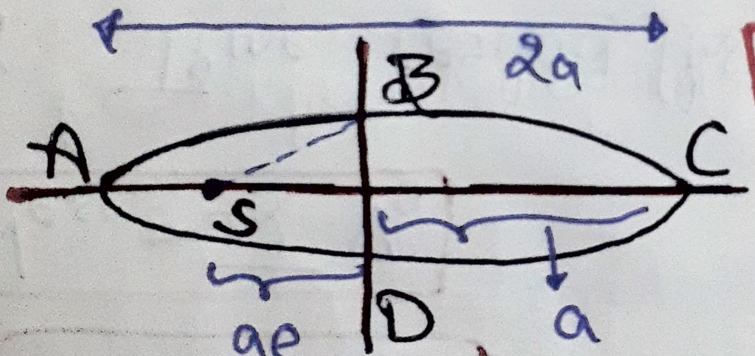


$$\boxed{P.E. = -\frac{GMm}{r}}$$

$$\boxed{\text{Total Energy} = \frac{P.E.}{2} = -K.E. = -\frac{GMm}{2r}}$$

Kepler's Laws

1) Law of Orbits →



$$\boxed{\text{Total Energy} = -\frac{GMm}{2a}}$$

If, Time travel over BCD $= t_1$

Time travel over BAD $= t_2$

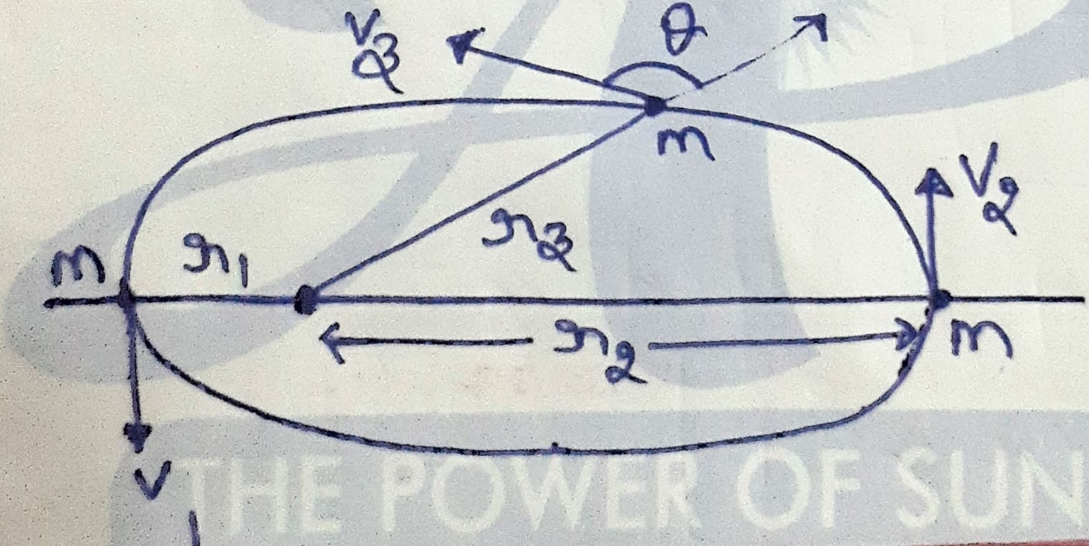
then, $\boxed{t_1 = 3t_2}$

2) Law of Area →

$$\boxed{\text{Total Energy} = \text{Const.}}$$

- **Areal Velocity** →

$$\boxed{\frac{dA}{dt} = \frac{L}{2m} = \text{const.}}$$



$$\boxed{mv_1r_1 = mv_2r_2 = mv_3r_3\sin\theta}$$