CHAPTER 1

INTRODUCTION, LETTERING AND DIMENSIONING

1.1 Introduction

Drawing is the representation of any object into a two –dimensional sheet. The object may be the real existing object or may be the result of creative idea of an engineer, a technician or a designer.

Drawing can be broadly classified into two groups: artistic drawing and engineering drawing. Drawings prepared by an artist such as paintings, slides etc as a result of his/her imagination are called artistic drawing. Any artist can make an artistic drawing depending upon his/her mood and is not bounded by any rules. Therefore different people may perceive artistic drawing in different ways.

The drawing of engineering objects such as buildings, roads, machines etc. prepared by an engineer or a technician is called engineering drawing. The engineering drawing is prepared by following the standard rules and guidelines, so it gives unique information to all readers. In this regard engineering drawing is also called a universal graphic language.

For example consider a drawing as shown in *Figure 1.1*. By looking at the drawing any one can understand that the given object has a rectangular shape with a circle within it. The dimensions 60×40 give the size of the rectangle and ϕ 20 gives the size of the circle. Similarly, the dimensions 30 and 20 give the location of the circle. Hence the basic information that any engineering drawing can provide are shape, size and location of the object. Despite these information complete engineering drawings also provide information about material, surface finish, allowable tolerances, etc.

The importance of engineering drawing can be highlighted by the fact that a simple sketch can give information about any object more clearly than that a paragraphs of word language can give.

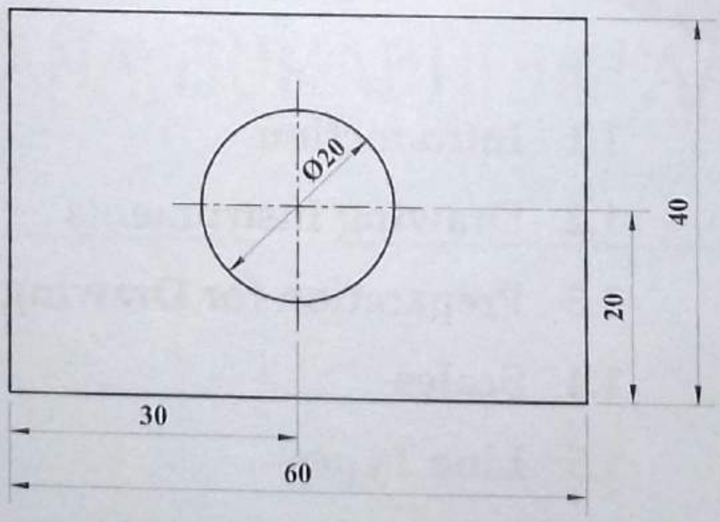


Figure 1.1: An Example of an Engineering Drawing

Engineering drawing can also be further classified into three groups as: freehand sketch, instrumental drawing and computer aided drawing. In freehand sketch, the lines are sketched without using instruments other than pencils and erasers. In instrumental drawing, instruments are used to draw straight lines, circles, and curves concisely and accurately. Thus, the drawings are usually made to scale. In case of computer aided drawing, the drawings are usually made by commercial software such as AutoCAD, solid works etc.

This text book focuses on the instrumental drawing which is the backbone of the computer aided drawing.

1.2 Drawing Instruments

For the better presentation as well as quick and efficient preparation of engineering drawing, the proper use of drawing instruments is very essential. Common drawing instruments used in engineering drawing are explained below.

(a) Drawing Board

The drawing board provides the space to fix the drawing sheet *(Figure 1.2)*. Drawing boards are usually made of soft wood and the left edge of the board is fitted with ebony strip on which T-square can slide. The drawing boards are available in various sizes. The selection of the drawing board depends upon the size of the drawing sheet to be used.

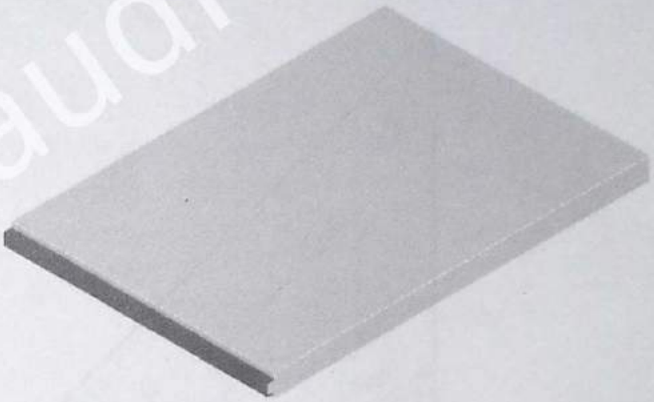


Figure 1.2: Drawing Board

(b) T-Square

The T-square consists of two parts, a long blade and the head joined together at right angle to each other by means of screws, as shown in *Figure 1.3*. It is used to draw horizontal lines and it also provides base for the set squares to draw incline lines. While using T-square, its head should be aligned with the left edge of the drawing board. The working length of the T-square should be equal to the length of the drawing board.

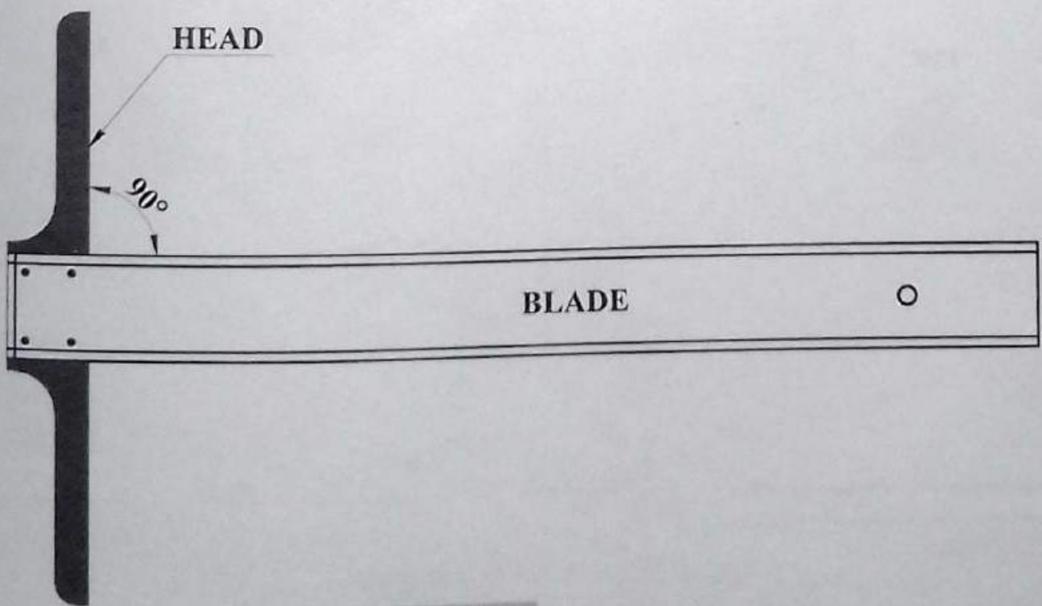
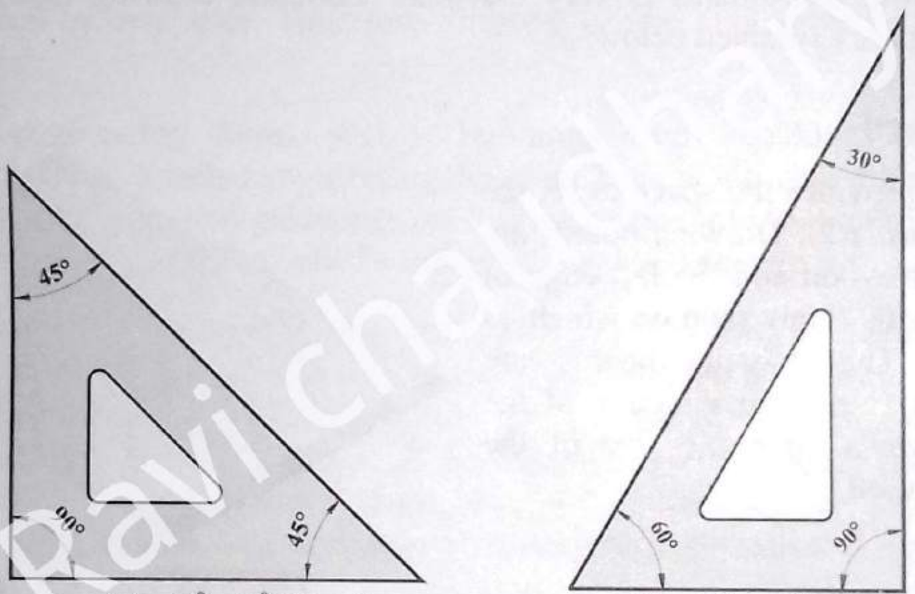


Figure 1.3: T-Square

(c) Set-Squares (Triangles)

Set-squares consist of two types of triangles: 45^0 –45^0 and $30^0 - 60^0$, as shown in *Figure 1.4.* Set-squares are used to draw vertical lines and inclined lines. With $45^0 - 45^0$ triangle we can draw lines inclined at 45^0, 90^0 and 135^0 as shown in *Figure 1.5* whereas with $30^0 - 60^0$ triangle we can draw lines inclined at 30^0, 60^0, 90^0, 120^0 and 150^0 as shown in *Figure 1.5*. By using both triangles together, we can also draw lines inclined at 15^0, 75^0, 105^0 and 165^0 as shown in *Figure 1.5*. Set squares can also be used to draw lines perpendicular and parallel to the given lines, as shown in *Figure 1.6.* Adjustable set-squares are also available which can be used to draw parallel, perpendicular and inclined lines with speed.



(a) 45^0–45^0 Triangle (b) $30^0 - 60^0$ Triangle

Figure 1.4: Set-squares (Triangles)

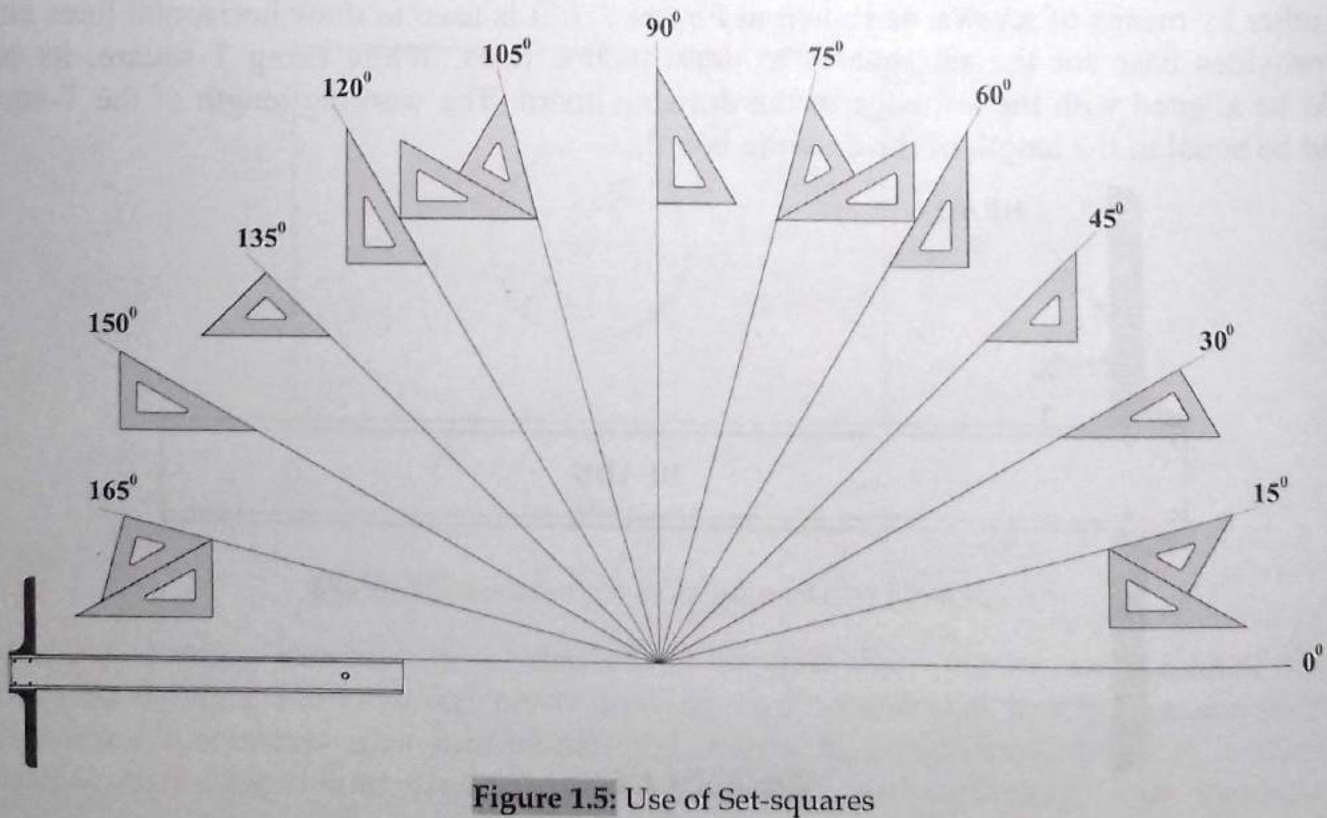


Figure 1.5: Use of Set-squares

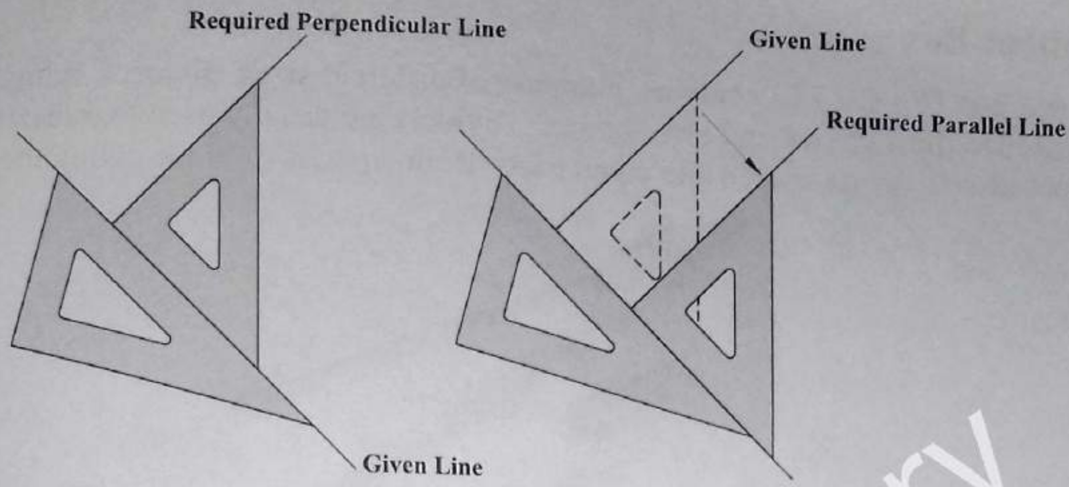


Figure 1.6: Use of Set-squares to Draw Perpendicular and Parallel Lines

(d) Protractor

Protractor *(Figure 1.7)* is used to measure and construct angles which cannot be drawn by using set-squares. Some manufacturer also provide protractor on $45^0 - 45^0$ triangle.

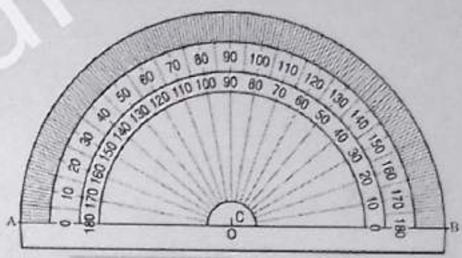


Figure 1.7: Protractor

(e) Drafting Machine

For the preparation of drawing quickly, drafting machine *(Figure 1.8)* can be used. It can perform the functions of T-square, Set-squares and protractor. The drafting machine consists of two arms, fixed on a circular disc in such a way that they can move freely and smoothly. The one end of the movable arm is hinged to the gripping plate. This gripping plate is attached to the edge of the drawing board with the help of screw. The other end of the movable arm is hinged to a disc where two fixed plate are provided. Along with the scales angular graduations from 0^0 to 360^0 are marked which acts as a protractor.

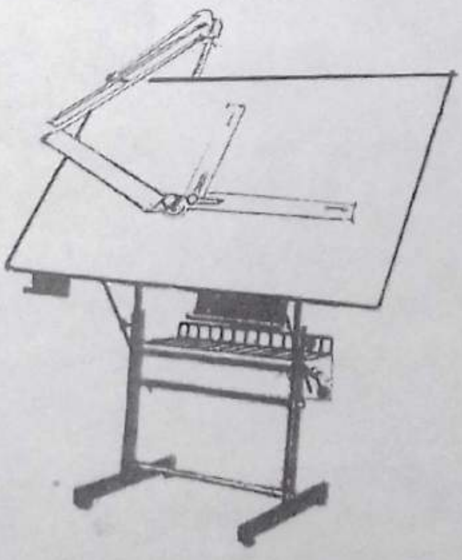


Figure 1.8: Drafting Machine

(f) Instrument Box

The instrument box *(Figure 1.9)* contains: compass of different sizes, dividers, ruling pens, etc. Compass is used to draw circles and circular arcs. Dividers are usually used to transfer distances and sometimes also to divide spaces into equal parts. Ruling pen is used for inking the drawing.

Figure 1.9: Compass and Divider

(g) Scales

All measurements of distances on a drawing are made with the scales. Scales are made in variety of graduations to meet the requirements of many kinds of drawing.

(h) French Curves

French curves are used to draw non-circular irregular curves. These are the patterns of templates having a series of different shaped curved edges as shown in *Figure 1.10*. It should be used very skillfully to obtain the smooth curve.

Some set-squares also have these curves cut in their middle. Flexible curves or splines are also available.

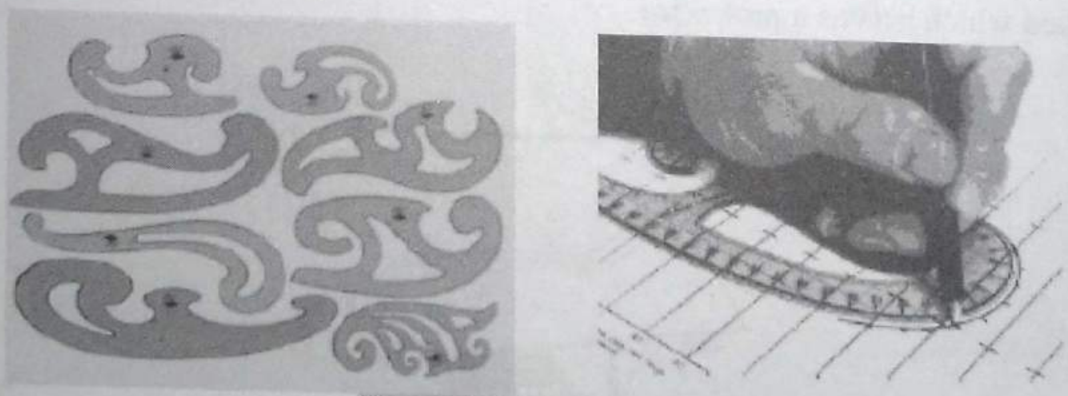


Figure 1.10: French Curves

(i) Pencils

Pencils are used to make drawings on the drawing sheet. The accuracy and appearance of a drawing depends largely upon the proper selection of the pencils. The grade of the pencil is usually shown by a number and letters marked at one of its ends. Letter H denotes hard grade, B denotes soft grade and HB denotes the medium grade. The increase in hardness is specified by the number put in front of the letter H, e.g., 2H, 3H, 4H, etc while the increases in softness is specified by the number put in front of the letter B, e.g., 2B, 3B, 4B, etc *(Figure 1.11)*.

Lead pencils are also common. They can be easily refilled with the new leads.

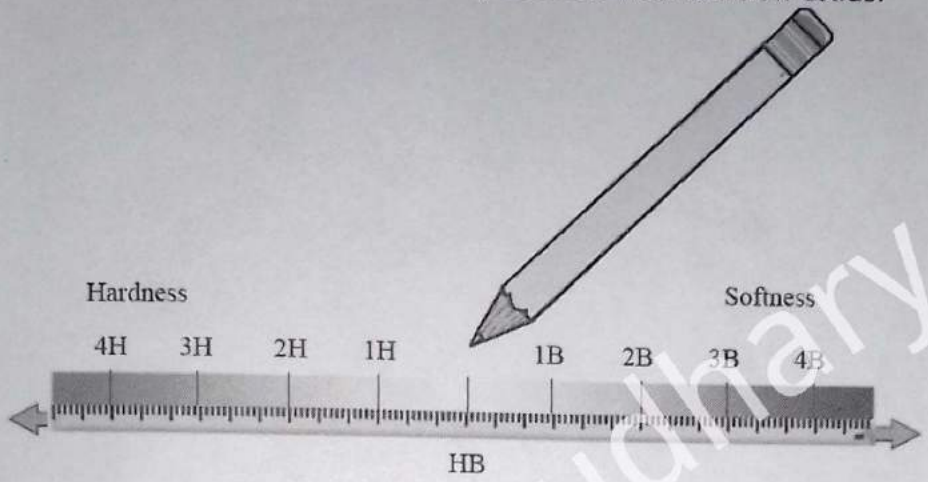


Figure 1.11: Pencil Grades

(j) Sharpener

After using pencil for a while its point becomes dull and it needs sharpening. The sharpener removes the wood and provides the desired shape to the pencil point.

(k) Eraser

Soft rubber is commonly used eraser for the pencil drawings. Frequent use of the eraser should be avoided by careful planning because it may spoil surface of the paper and reduces neatness of the drawing.

(l) Erasing Shield

Erasing shield is made of thin metal sheet with different shapes cut on it *(Figure 1.12)*. It is used to erase some portion of the drawing while protecting the adjacent area.

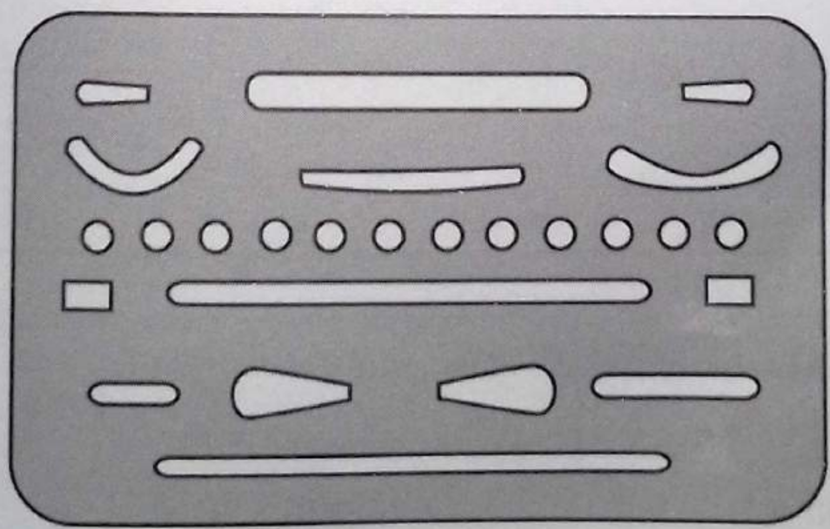


Figure 1.12: Erasing Shield

(m) Templates

For drawing small circles, ellipses, standard symbols. etc quickly, templates can be used *(Figure 1.13)*.

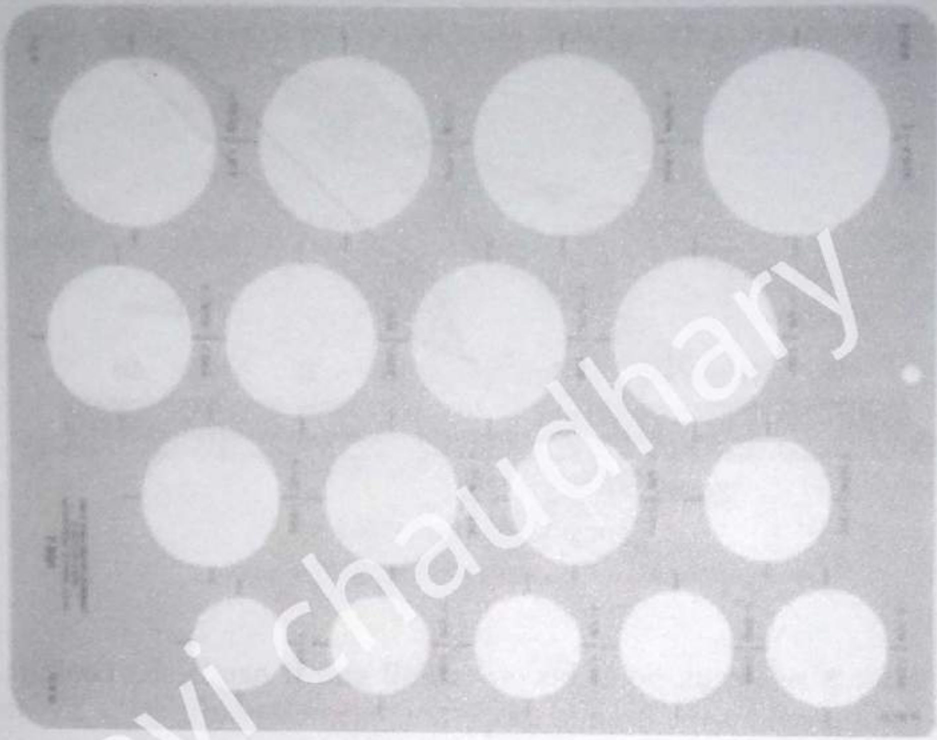


Figure 1.13: Template for Circles

(n) Drawing Paper

All engineering drawings are made on sheets of paper of strictly defines sizes. The standard sizes of drawing papers are given in *Table 1.1* and shown in *Figure 1.14*.

Table1.1: **Sizes of Drawing Sheets**

S. No.	Sheet Designation	Standard Size (mm × mm)	Area (m^2)
1	A_0	1189 × 841	1
2	A_1	841 × 594	0.5
3	A_2	594 × 420	0.25
4	A_3	420 × 297	0.125
5	A_4	297 × 210	0.0625
6	A_5	210 × 148	0.03125

Successive formats (from A_0 to A_5) are obtained by halving along the length and area of the sheet is also halved. The ratio of the sides of the sheet is always $\sqrt{2}$.

Engineering students generally use A_3 sheets for their course work.

(o) Tracing Paper

Tracing papers are thin transparent papers on which drawings are traced by ink. From the traced drawings blue prints are prepared.

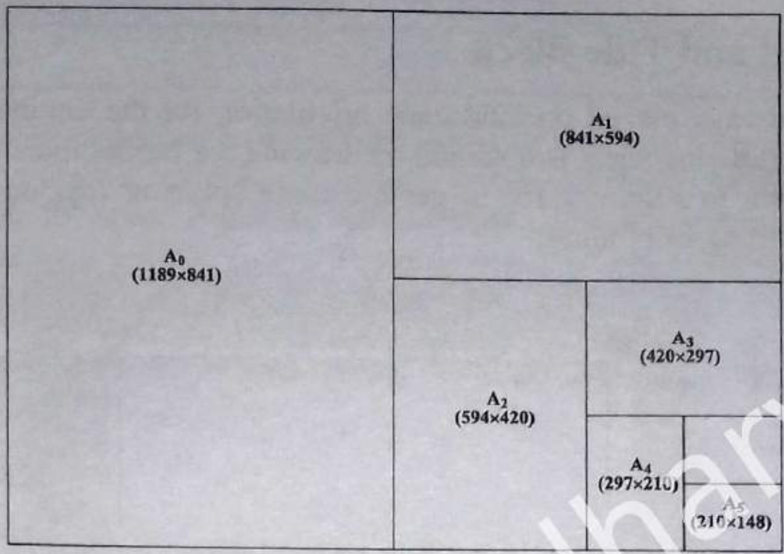


Figure 1.14: Sizes of Drawing Sheets

(p) Drawing Pins, Tapes or Clips

Drawing pins, tapes or clips are used to fix the drawing sheet on proper position on the drawing boards.

Cello tapes are most commonly used for its practical convenience. T-square, set-squares or drafting machines can be moved very easily over the tape.

1.3 Preparation for Drawing

1.3.1 Fixing the Drawing Sheet

Place the drawing sheet approximately at the middle of the drawing board. Tape the upper left corner of the sheet. Align the T-square on the working edge of the board or fix the drafting machine on any suitable positions. Then slide the drawing sheet until its edge is in level with the edge of the T-square or the drafting machine. Then tape the remaining three corners of the sheet as shown in *Figure 1.15*.

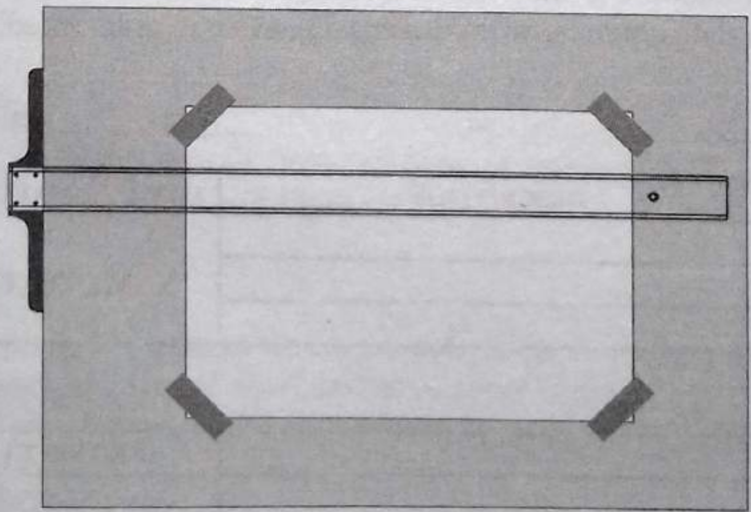


Figure 1.15: Fixing the Drawing Sheet on a Drawing Board

1.3.2 Sheet Layout and Title Block

Drawing sheets are always placed on landscape orientation for the engineering drawing. The working space on the drawing sheet is obtained by drawing the border lines. The margins for A3 drawing sheet is shown in *Figure 1.16*. In general more space is kept on the left side of the drawing sheet for the filing or binding.

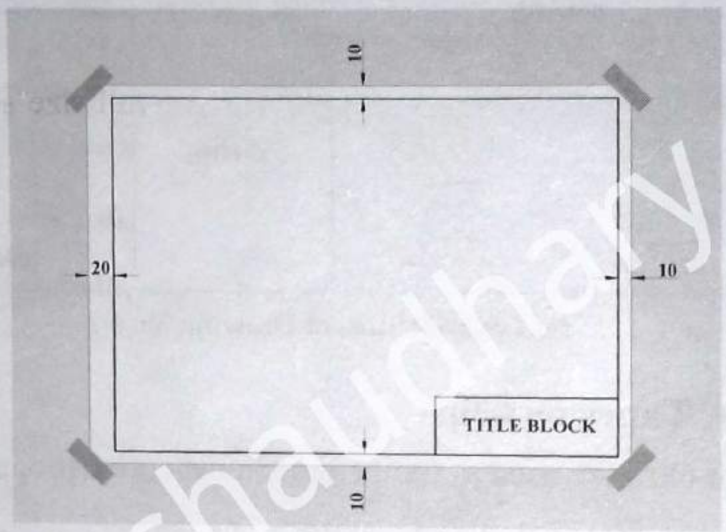


Figure 1.16: Margin for the A_3 Sheet

Title block is an important feature in drawing because it gives all the information of the prepared drawing. It is placed at the right bottom corner of the drawing sheet. Typical title block suitable for A_3 sheet for engineering drawing course work is shown in *Figure 1.17*. All the title blocks should contain at least the following information:

- Title of the drawing
- Drawing number
- Date
- Scale
- Symbols denoting the method of projection
- Name of the institute
- Initials with dates, person who have designed, drawn, checked standards and approved the drawing.

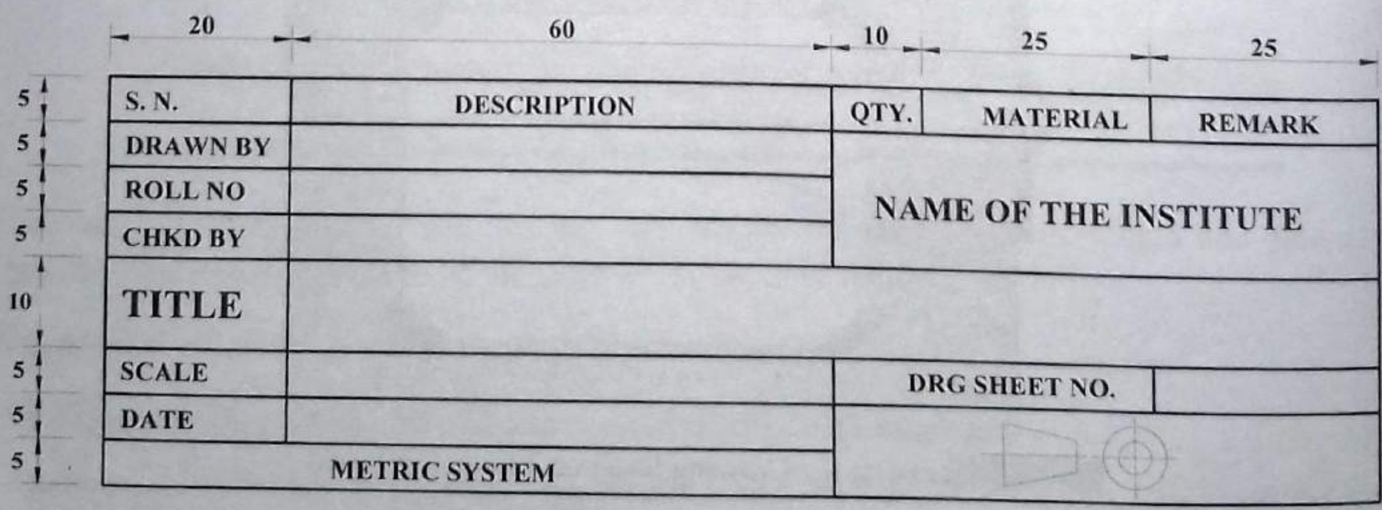


Figure 1.17: Title Block for the A_3 Sheet

1.4 Scales

It is not always possible to represent the actual object in real size into a drawing sheet. For example, if we have to draw a building on a sheet, we should divide its real size by a suitable factor to make it fit into the sheet. But if we have to draw a part of a watch into a sheet, we have to multiply its real size by a suitable factor. The proportion by which we reduce or increase the actual size of the object on the drawing is known as scale.

The following are the scales used in engineering drawings: full-size scale, reducing scale and enlarging scale.

1.4.1 Full Size Scale

If a drawing is made with the size equal to that of the real size, then it is called a full size scale. It is specified by 1:1. It means 1 unit of the drawing is equal to 1 unit of the real size.

1.4.2 Reducing Scale

If the real size of the object is divide by a certain factor and drawn on the drawing sheet, it is called a reducing scale. The commonly used reducing scales are 1:2, 1:5, 1:10, 1:20, 1:50, 1:100, etc. The scale specified by 1:2 means 1 unit of drawing is equal to 2 units of the real size.

1.4.3 Enlarging Scale

If the real size of the object is multiplied by a certain factor and drawn on the drawing sheet, it is called an enlarging scale. The commonly used enlarging scales are 2:1, 5:1, 10:1 etc. The scale specified by 2:1 means 2 units of drawing is equal to 1 unit of the real size.

1.5 Line Types

As explained earlier, shape of any object is explained by the combination of lines. To give complete information about all features of the object different types of lines are used. The following are the description of various types of lines used in engineering.

(a) Visible Outline

Visible outline is drawn to represent the visible edges of the object. It is thick, dark and continuous line as shown in *Figure 1.18* and *Figure 1.19*.

(b) Hidden Line

Hidden line is drawn to represent the edges which are not visible from the particular direction. It is thick, dark and broken and made up of short dashes of approximately equal lengths of about 2 mm spaced at equal distances of about 1 mm as shown in *Figure 1.18* and *Figure 1.19*.

(c) Center Line

Center lines are drawn to represent the diameters of circles, centers of circles or circular arcs. They are also used to represent the axis of the solid of revolution and any symmetrical solids.

Center lines thin, medium darkness and chain type and composed of alternately long and short dashes approximately 1 mm apart. The longer dashes are about 6 to 8 times the short dashes which are about 1.5 mm long as shown in *Figure 1.18* and *Figure 1.19*. Center lines should extend for a short distance beyond the outlines to which they refer. The point of intersection between two center lines must always be indicated. Locus lines, extreme positions of movable parts and pitch circles are also shown by this type of line.

(d) Projection Line

Projection lines are used for projecting or transferring any features from one view to another. Projection lines are thin fain and continuous as shown in *Figure 1.18* and *Figure 1.19*.

(e) Dimension Line

Dimension lines are the lines drawn to give dimensions of the object. They should terminate with arrowhead between the extension lines. Dimension lines are thin and continuous as shown in *Figure 1.18* and *Figure 1.19*.

(f) Extension Line

Extension lines are the lines drawn from the ends of edges of the object used to terminate the dimension lines. They are also thin and continuous and extend about 3 mm beyond the dimension lines as shown in *Figure 1.18* and *Figure 1.19*.

(g) Construction Line

Construction lines are the auxiliary lines drawn to complete any geometrical construction. They are thin, faint and continuous as shown in *Figure 1.18*.

(h) Cutting Plane Line

Cutting plane line is the line used to represent the location of the cutting plane. It is similar to center line with thick lines or arrow heads on both ends, as shown in *Figure 1.18* and *Figure 1.20*.

Line Type	Sample	Features
Visible outline		Thick, dark and continuous
Hidden line		Thick, dark and broken
Center line		Medium thickness and chain
Projection line		Thin, faint and continuous
Dimension line		Thin and continuous with arrowheads at the ends
Extension line		Thin and continuous
Construction line		Thin, faint and continuous
Cutting plane line		Chain type with perpendicular arrowheads
Hatching line or Section line		Thin, continuous and usually inclined at 45^0
Short break line		Thin, continuous and freehand
Long break line		Thin and continuous with zig zag

Figure 1.18: Sample and Features of Different Line Types

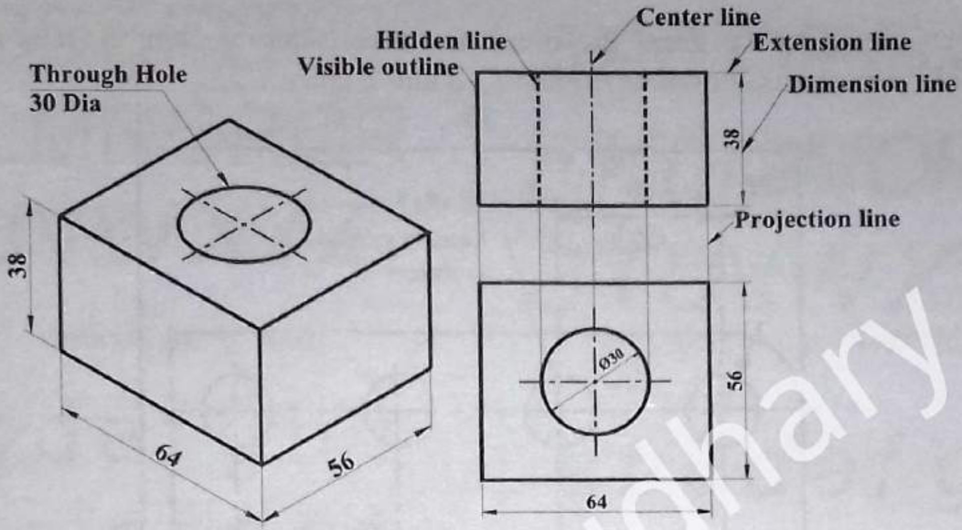


Figure 1.19: Examples of Different Line Types

(i) Section Line or Hatching Line

Hatching lines are the line drawn to represent the surface which is obtained after cutting any object. They are thin and continuous line usually inclined at 45^0 as shown in *Figure 1.18* and *Figure 1.20*.

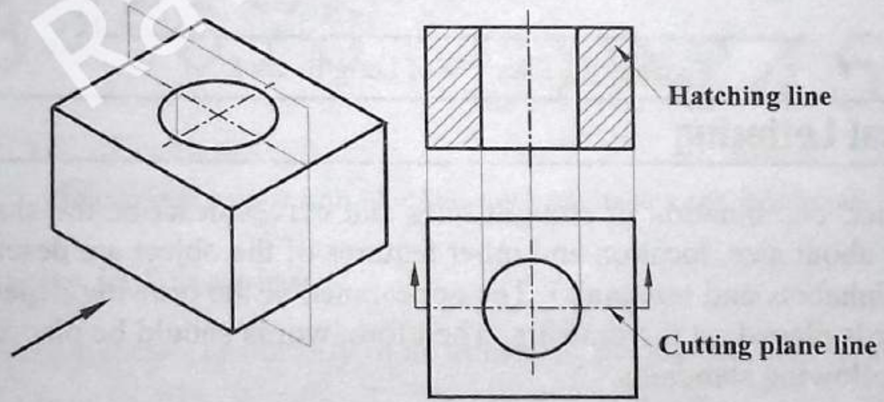


Figure 1.20: Examples of Cutting Plane Line and Hatching Line

(j) Short Break Line

Short break line is used to represent the break of an object for a short length. It is a thin line usually drawn free-hand as shown in *Figure 1.18* and *Figure 1.21*.

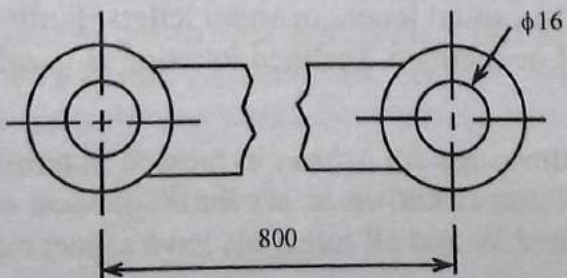


Figure 1.21: Example of Short Break Line

(k) Long Break Line

Long break line is used to represent the break for a considerable length. It is thin line with zig-zag at suitable interval as shown in *Figure 1.18* and *Figure 1.22*.

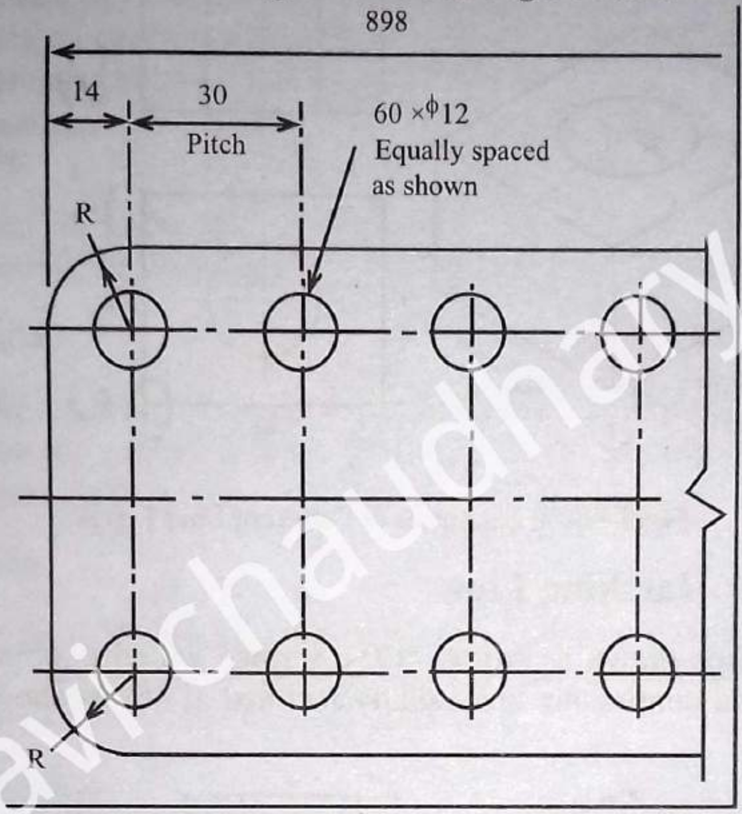


Figure 1.22: Example of Long Break Line

1.6 Technical Lettering

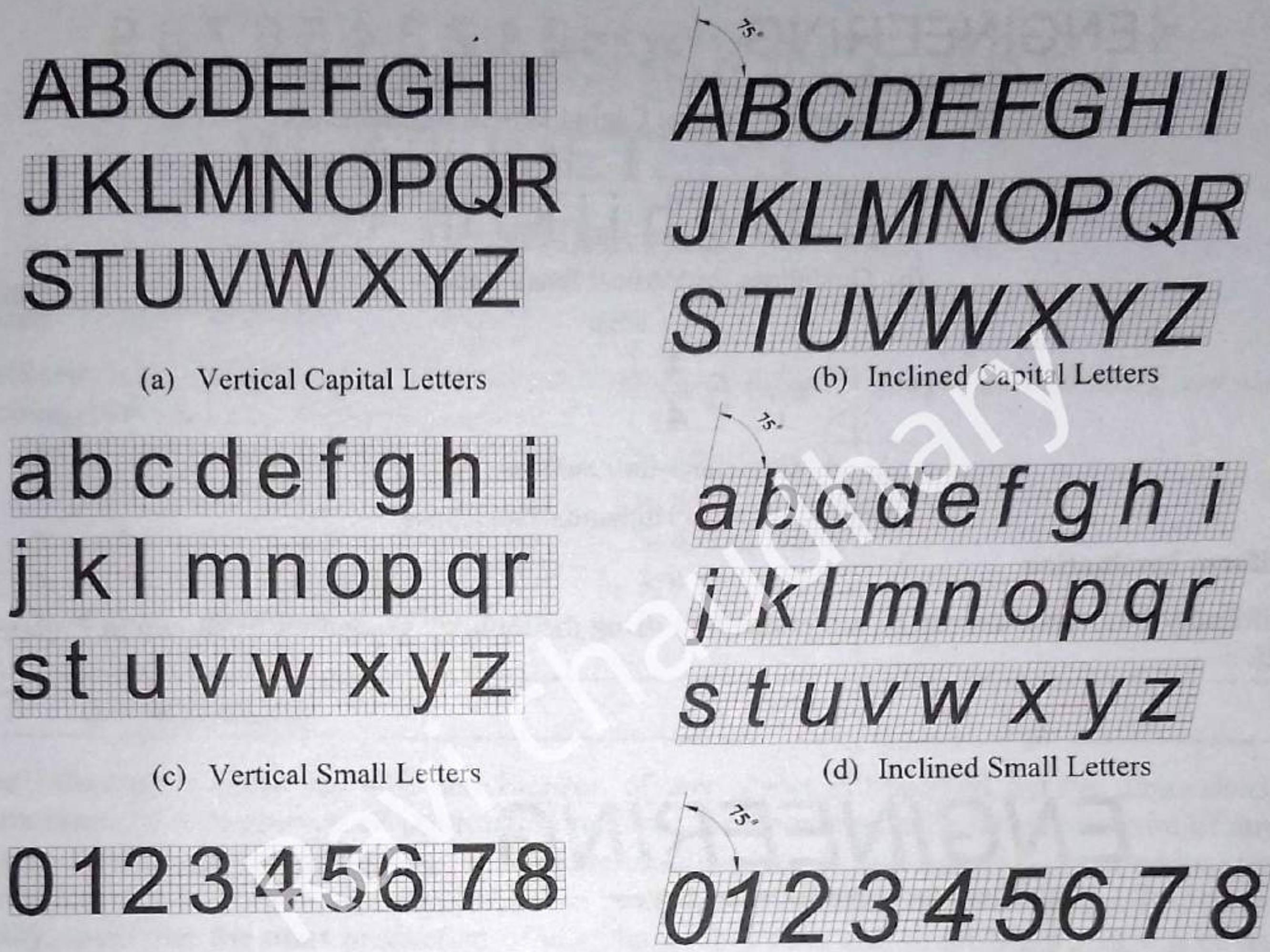
As explained earlier, combination of straight lines and curves describe the shape of the object while information about size, location and other features of the object are described with words (combination of alphabets and numerals). The appearance of the drawing depends largely upon the letters and words placed on the drawing. Therefore, words should be placed on the drawing skillfully and by following standards.

The technique of inserting texts on the drawing by following some standard guidelines is called technical lettering. Special lettering templates are also available. But the use of such devices takes considerable time is usually avoided. Lettering on engineering drawing is usually made freehand.

1.6.1 Proportions for Letters and Numerals

Letters used in drawing may be capital letters or small letters. Further the style of writing letters and numerals may be vertical or inclined. Inclined lettering is usually done at an inclination of 75^0.

Parameters of letters used in drawings are usually expressed in terms of height of the letter. The ratio of height to width of a letter is known as aspect ratio. Most of the alphabets have aspect ratio of 7:5 except I, J, L, M and W and all numerals have aspect ratio of 7:4 except I as shown in *Figure 1.23*.



(a) Vertical Capital Letters (b) Inclined Capital Letters

(c) Vertical Small Letters (d) Inclined Small Letters

(e) Vertical Numerals (f) Inclined Numerals

Figure 1.23: Aspect Ratio for Different Alphabets and Numerals

1.6.2 Uniformity in Lettering

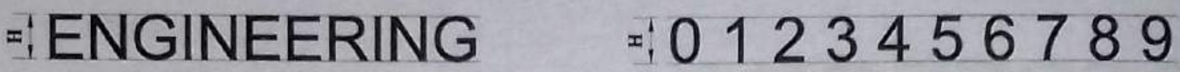
Uniformity in lettering means uniformity in its thickness, height, inclination and spacing.

Uniform Thickness

All letters used in drawing should have uniform thickness. Uniform thickness can be maintained by using single stroke of the pencil. Thickness of the letters is usually 0.1 times the height of the letter.

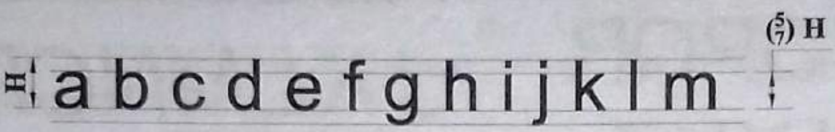
Uniform Height

Uniformity in height can be maintained by using the horizontal guidelines. Guidelines for finished pencil lettering should be drawn very lightly such that they need not be erased, as it is not possible to do so, after the lettering has been completed. Two horizontal guidelines are used for capital letters and numerals and the distance between the two horizontal guidelines is equal to the height of the letter as shown in *Figure 1.24(a)*, whereas four guidelines are used for small letters as shown in *Figure 1.24(b)*. Similarly guidelines required for fraction are shown in *Figure 1.24(c)*.

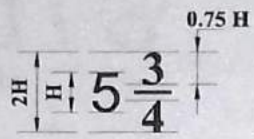


Vertical Capital Letters Vertical Numerals

(a) Guidelines for Vertical Capital Letters and Numerals



(b) Guidelines for Vertical Small Letters

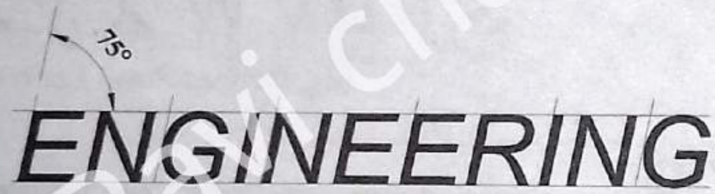


(c) Guidelines for Fractions

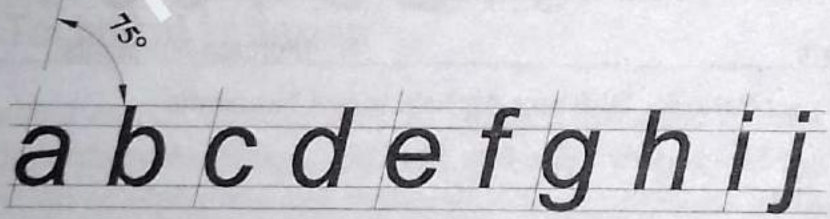
Figure 1.24: Use of Horizontal Guidelines

Uniform Inclination

Uniformity in inclination can be maintained by using the inclined guidelines as shown in *Figure 1.25*.



(a) Guidelines for Inclined Capital Letters



(b) Guidelines for Inclined Small Letters

Figure 1.25: Use of Inclined Guidelines

Spacing between Letters

Space between the letters depends on the contour of the letters at an adjacent side. Good spacing creates approximately equal background area between letters as shown in *Figure 1.26*.

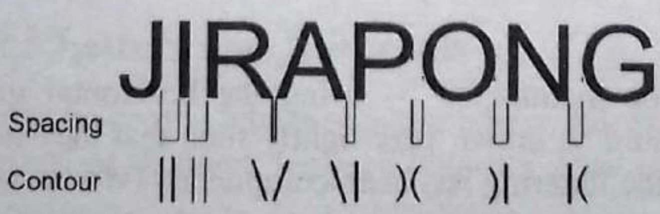


Figure 1.26: Spacing between Letters

Spacing between Words

While writing sentences, the space between words is usually equal to the space required for writing a letter "O" as shown in *Figure 1.27*.

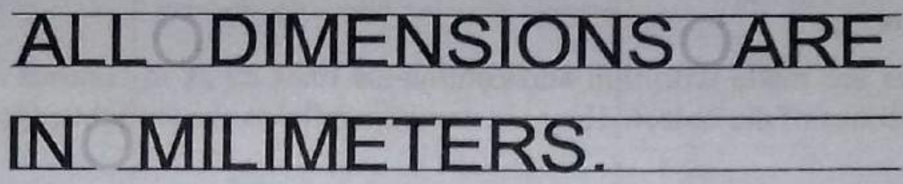


Figure 1.27: Spacing between Words

1.6.3 Size of Letters

Different sizes of letters and numerals are used for different purposes. Following are the recommended sizes for different purposes.

- Main Title and Drawing No.: 8, 10, 12 mm
- Title of Drawing: 6, 8 mm
- Sub-titles Headings: 3, 4, 5 or 6 mm
- Notes such as Legends: 3, 4 or 5 mm
- Alternation entries and Tolerances: 2, 3 mm

1.7 Dimensioning

The information about the size and location of any object is specified by the dimensions. Dimensions should provide all information required for the construction or manufacture of any object. Dimensions in engineering drawing are usually given in millimeters without adding the abbreviation mm. Any drafter should bear in mind that satisfactory dimensioning contribute to utility, facilitates the mass production of interchangeable parts and simplifies inspection of the finished work. For this certain standard guidelines and rules should be followed while placing the dimensions on the drawing of any object.

1.7.1 Terms of Dimensioning

Different terms used in dimensioning are shown in *Figure 1.28*. It consists of dimension lines, extension lines, arrowheads, dimension figure or text, notes, symbols, etc.

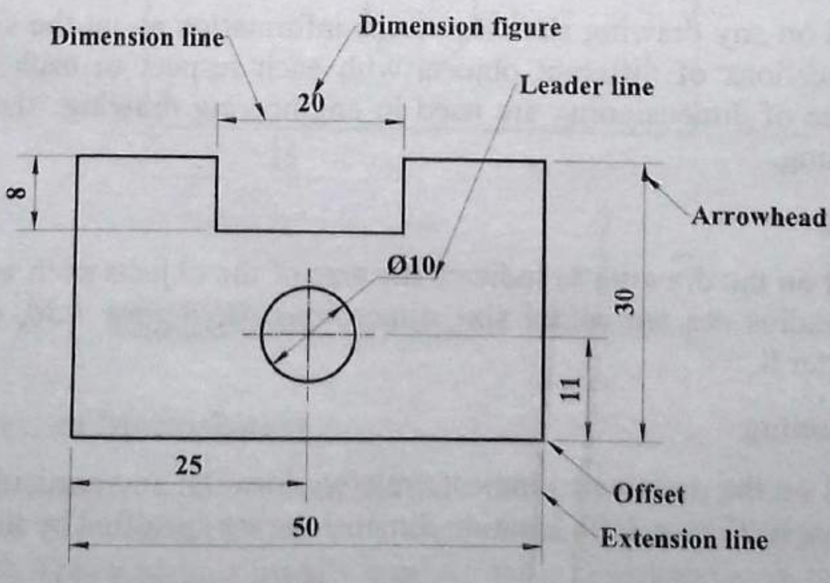


Figure 1.28: Terms of Dimensioning

Dimension Line

Dimension lines are made with thin and continuous lines so as to contrast with the dark and thick visible outlines of the object. They are drawn parallel to the straight edges of the object.

Arrowhead

Arrowheads are used to terminate the dimension lines. They always touch the extension lines. The ratio of the length and width of the arrowheads used in engineering drawing is usually 3:1 as shown in *Figure 1.29*.

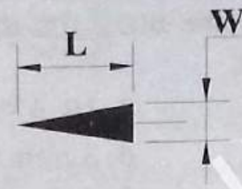


Figure 1.29: Arrowhead Used for Dimensioning

Extension Line

Extension lines are also thin and continuous lines extending from the ends of the visible outlines of the object. There should be a gap of 2-3 mm between the visible outline of the object and the extension line, which is also known as offset. Extension lines should extend 3-4 mm beyond the arrowheads.

Dimension Figure

A number used to indicate the measurement or size of any feature is called dimension figure. Dimension features must be written above or parallel to the dimension lines and should be at the middle of the dimension lines.

Leader Line

Line with arrowhead at its one end used to provide certain explanatory note is called a leader line.

1.7.2 Theory of Dimensioning

Dimensions placed on any drawing should provide information about the size of the objects as well as relative locations of different objects with each respect to each other. For this, the following two types of dimensioning are used in engineering drawing: size dimensioning and location dimensioning.

Size Dimensioning

Dimensions placed on the drawing to indicate the size of the objects such as its length, breadth, height, diameter, radius etc are called size dimensions. In *Figure 1.30*, size dimensions are specified by the letter S.

Location Dimensioning

Dimensions placed on the drawing to indicate relative location any particular feature are called location dimensions. In *Figure 1.30*, location dimensions are specified by the letter L.

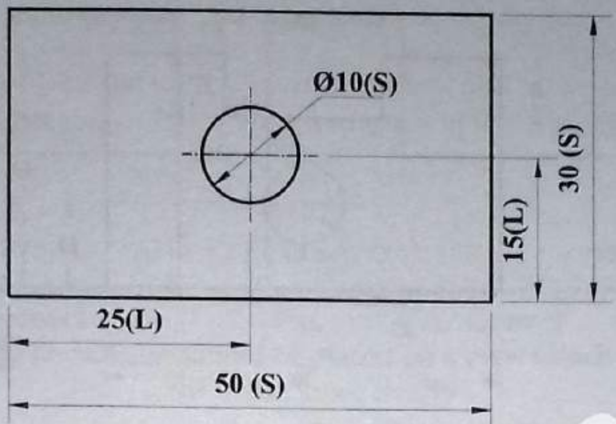


Figure 1.30: Size and Location Dimensioning

1.7.3 System of Dimensioning

The following two systems of dimensioning are usually used in drawings: aligned system of dimensioning and unidirectional system of dimensioning.

Aligned System of Dimensioning

In the aligned system of dimensioning, dimension figure is aligned with the visible outline of the object for which dimension is placed. Therefore, dimensions figures are placed either above or left to the dimension lines such that they are readable from the bottom or right hand side of the drawing sheet as shown in *Figure 1.31*. This system is usually preferred in machine drawing.

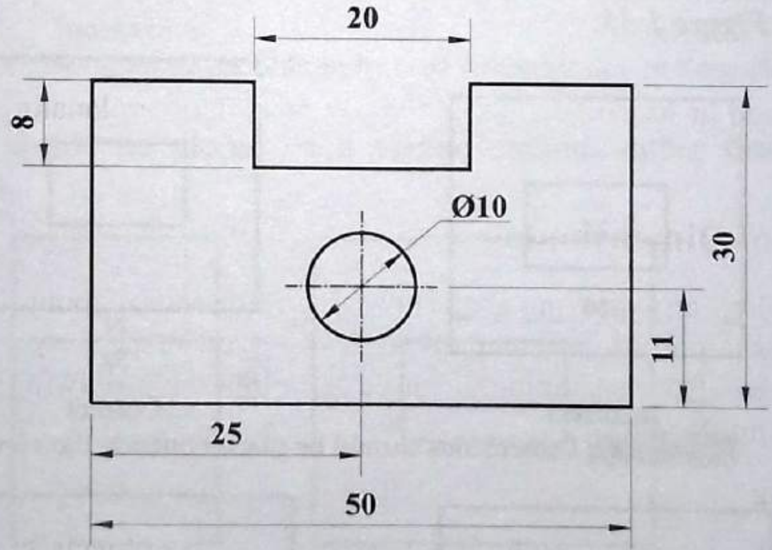


Figure 1.31: Aligned System of Dimensioning

Unidirectional System of Dimensioning

In the unidirectional system of dimensioning, dimension lines are broken at the middle and dimension figures are inserted such that they are readable from the bottom of the sheet, as shown in *Figure 1.32*. This system is usually used for large size drawing in which turning of the drawing is not convenient.

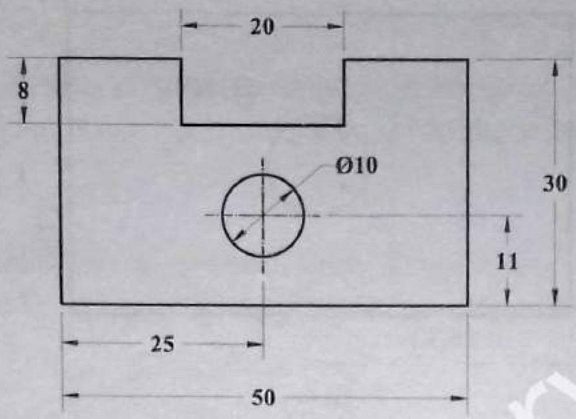


Figure 1.32: Unidirectional System of Dimensioning

1.7.4 General Rules of Dimensioning

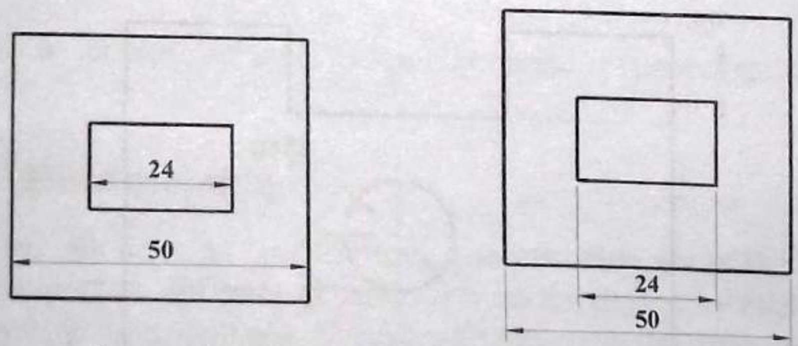
General rules that should be followed during dimensioning of any drawing are explained below.

(a) Dimensions should be complete for the description of the finished work such that no further calculation or assumption of any dimensions is required.

(b) Dimensions should be placed outside the views unless there is sufficient blank space within the views *(Figure 1.33)*.

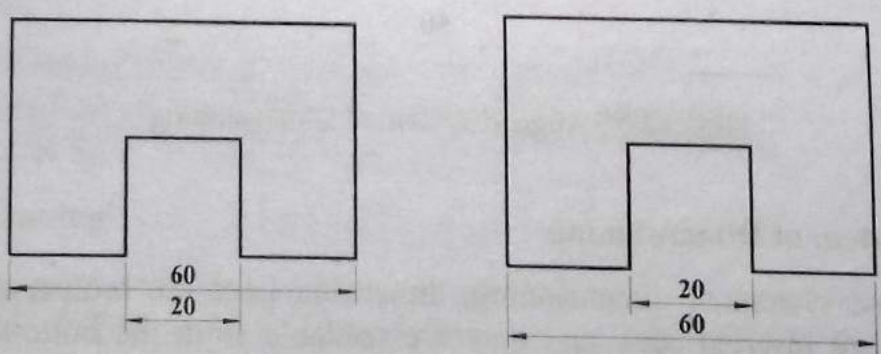
(c) Intersection of the dimension lines should be avoided as far as possible. For this smaller measurements are placed nearer to the object and larger measurements are placed far from the objects as shown in *Figure 1.34*.

(d) Dimensions should be placed on a view which represents the object more clearly. Visible outline should be preferred than hidden edges to place the dimensions as shown in *Figure 1.35*.



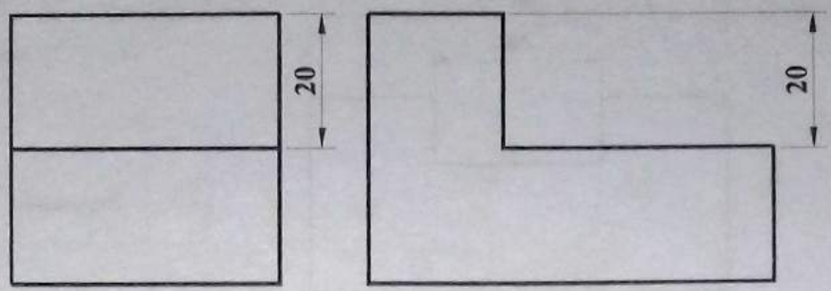
Incorrect   Correct

Figure 1.33: Dimensions should be placed outside the view



Incorrect   Correct

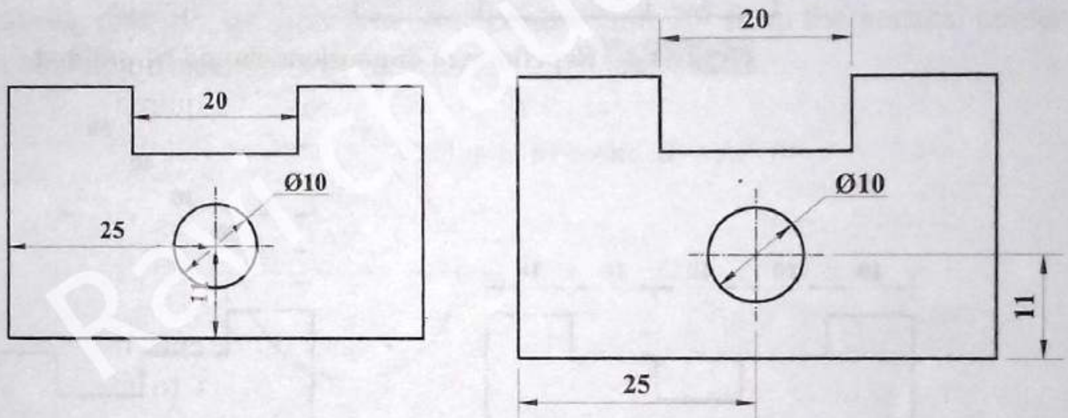
Figure 1.34: Avoid intersection of dimension lines



Incorrect Correct

Figure 1.35: Dimension should be placed on a view which represents the object more clearly

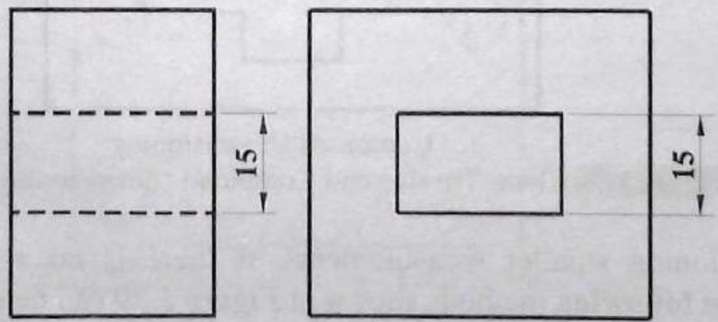
(e) Visible Outlines should not be used as dimensions lines or extension lines. However, center lines can be used as extension lines, but they also should not be used as dimension lines *(Figure 1.36)*.



Incorrect Correct

Figure 1.36: Visible Outlines should not be used as Dimension or Extension Lines

(f) Dimensions should be placed on a visible outlines rather than on hidden lines *(Figure 1.36)*.



Incorrect Correct

Figure 1.36: Dimensions should not be placed on hidden lines

(g) Dimensions placed on one view should not be repeated in other views *(Figure 1.37)*.

(h) Dimensions should be placed in systematic manner. Commonly dimensioning can be done either by chain dimensioning, parallel dimensioning or combined dimensioning as shown in *Figure 1.38*.

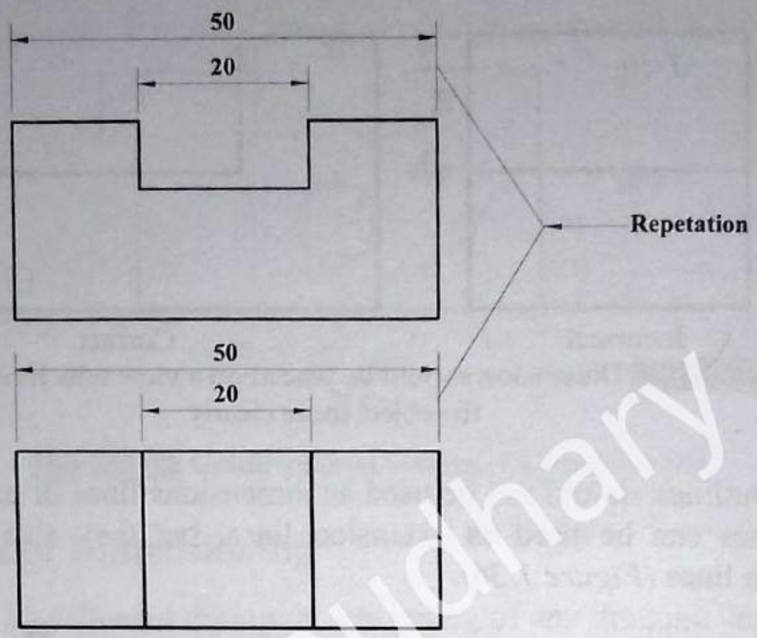
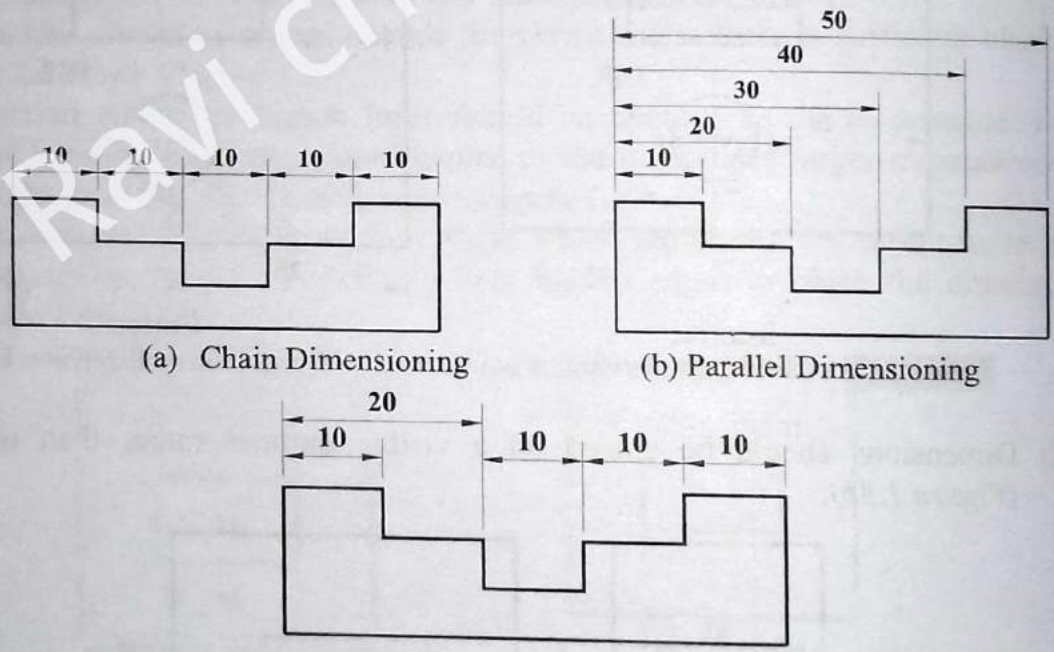


Figure 1.37: Repetition of dimensions should be avoided



(a) Chain Dimensioning

(b) Parallel Dimensioning

(c) Combined Dimensioning

Figure 1.38: Chain, Parallel and Combined Dimensioning

(i) While dimensioning smaller measurements, if there is no sufficient space for the arrowheads, the following methods shown in *Figure 1.39* can be used.

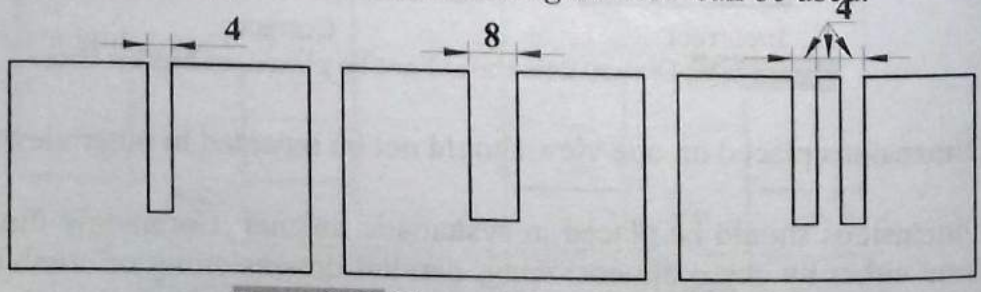


Figure 1.39: Dimensioning of Small Slots

(j) Dimensioning of the circles is done by specifying their diameters with a symbol ϕ, as shown in *Figure 1.40*.

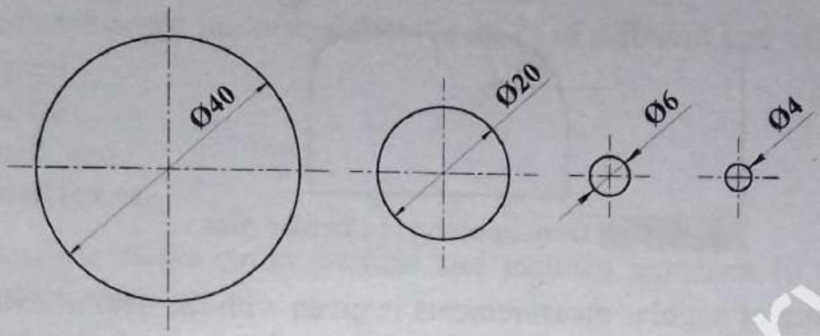


Figure 1.40: Dimensioning of Circles

While drawing diameter as dimension line zone within 30^0 from the vertical center line should be avoided, as shown in *Figure 1.41*.

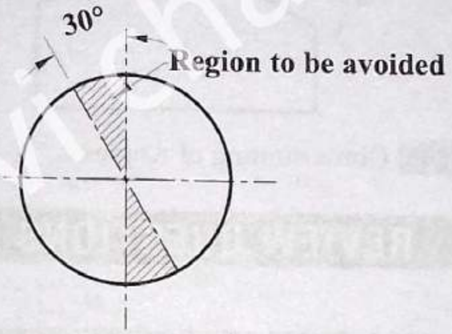


Figure 1.41: Region to be avoided while Dimensioning Circle

When there are number of circles with the same size, dimensions can be given as shown in *Figure 1.42*.

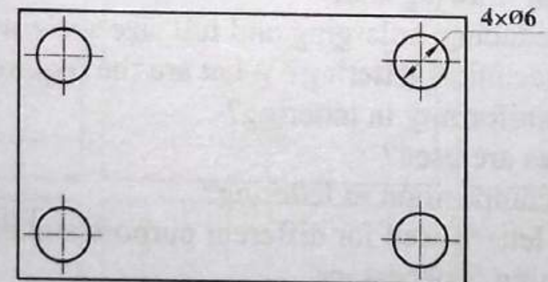


Figure 1.42: Dimensioning of Circles of Same Size

When single view for a cylindrical object is drawn it can be dimensioned as shown in *Figure 1.43*.

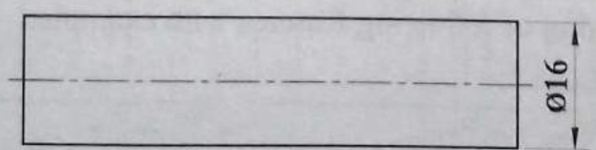


Figure 1.43: Dimensioning of Single View of a Cylinder

(k) Dimensioning of circular arcs is done by specifying their radii with the symbol R, as shown in *Figure 1.44*.

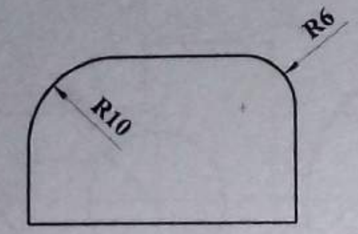


Figure 1.44: Dimensioning of Circular Arcs

(l) Dimensioning of angular measurements is given with the help of extension lines drawn from the ends of angle and dimension line in the form of circular arc, as shown in *Figure 1.45*.

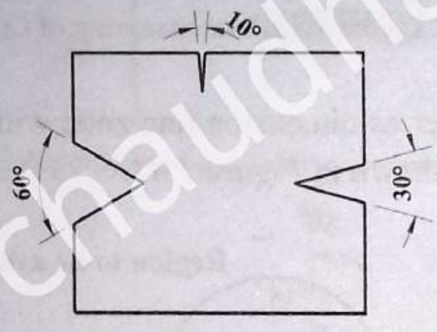


Figure 1.45: Dimensioning of Angles

REVIEW QUESTIONS

1. Define drawing. Differentiate between artistic drawing and engineering drawing.
2. Explain why engineering drawing is called a universal language of engineers.
3. List the instruments used for drawing. Also write down the functions of each instrument.
4. What are the common grades of pencils? Write down the use of each grade of pencils.
5. List the standard sizes of drawing sheet.
6. Define scale. Explain reducing, enlarging and full size scale with examples.
7. What do you mean by technical lettering? What are the requirements of good lettering?
8. What do you mean by uniformity in lettering?
9. Why and how guidelines are used?
10. What do you mean by composition in lettering?
11. Write down the size of letters used for different purpose suitable for A3 drawing sheet.
12. Explain why dimensioning is necessary.
13. Explain the terms of dimensioning with examples.
14. Differentiate between size dimensioning and location dimensioning with examples.
15. Differentiate between aligned and unidirectional system of dimensioning with examples.
16. Explain chain, parallel and combined dimensioning with examples.
17. Explain the dimensioning of following features with examples
    - circles
    - circular arcs
    - small slots
    - angles
18. Explain the general rules of dimensioning with examples.

EXERCISES

1. Draw the following patterns shown in *Figure P1.1* using T-square and set-square or drafter.
2. Write down freehand single stroke alphabets (A to Z) of different size using guidelines in
   - Vertical capital
   - Inclined capital
   - Vertical small, and
   - Inclined small letters.
3. Write down freehand single stroke vertical and inclined numerals (0 to 9) and fractions different size using guidelines.
4. Write the following sentence in freehand, single stroke vertical and inclined uppercase letters in 6 mm height.

   ENGINEERING DRAWING IS THE FUNDATION OF ALL ENGINEERING DESCIPLINES.
5. Draw the following lines with 100 mm length
   - Visible outline
   - Hidden
   - Center
   - Projection
   - Cutting plane
   - Break

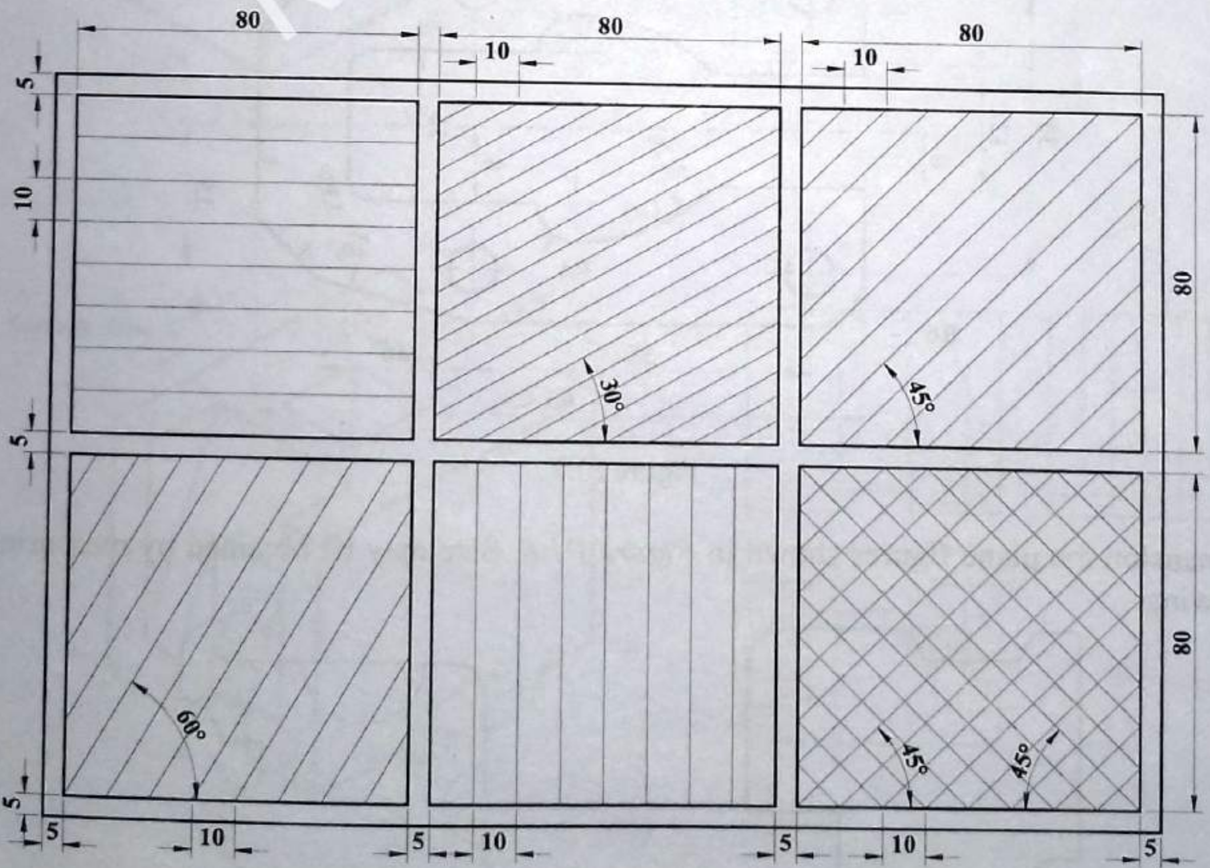


Figure P1.1

6. Copy the plane figure shown in *Figure P1.6* using scale 1:2.

7. Copy the plane figure shown in *Figure P1.7* using scale 2:1.

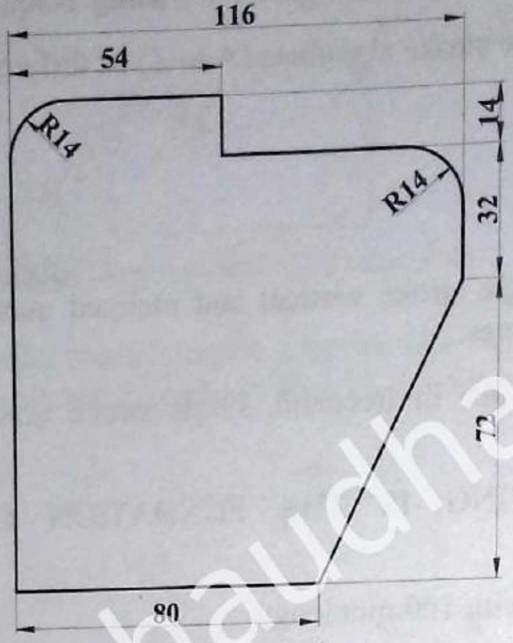


Figure P1.6

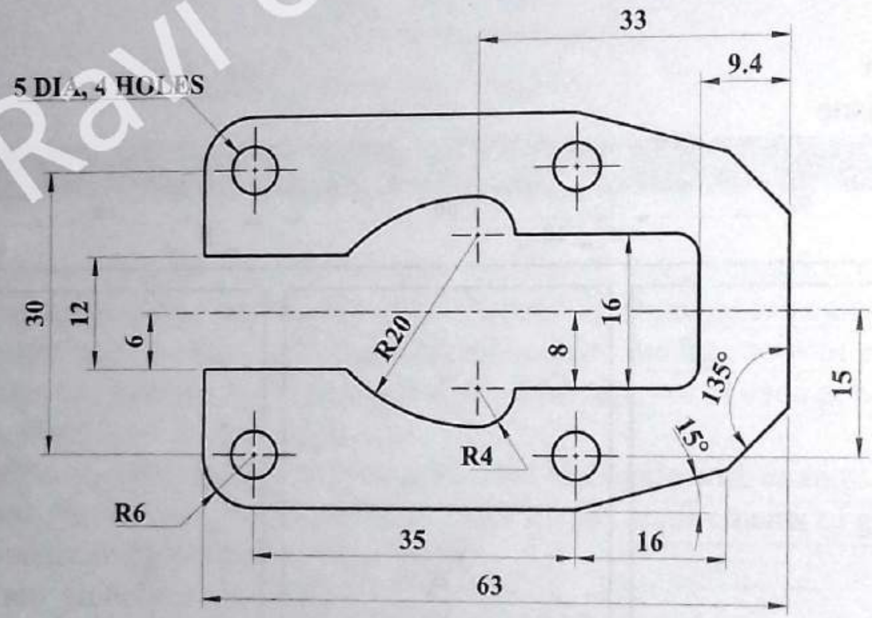
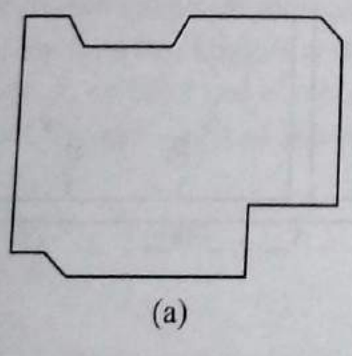
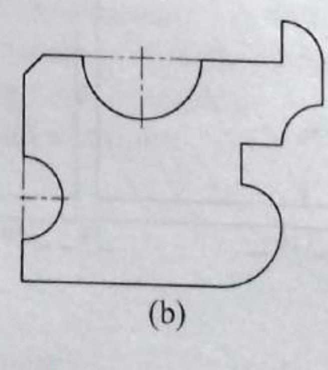


Figure P1.7

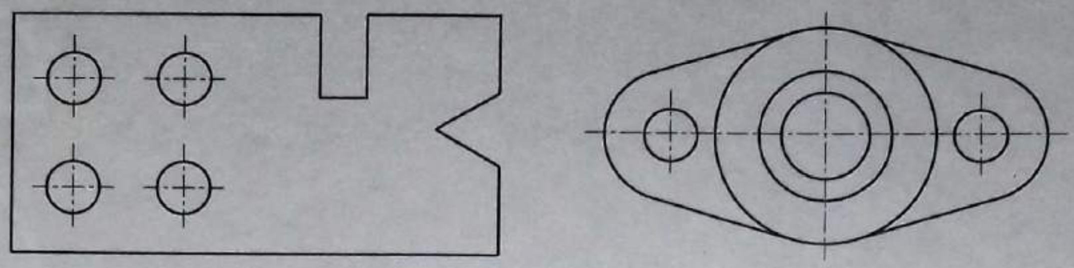
8. Dimension the plane figures shown in *Figure P1.8*. Size may be obtained by measuring the drawing.



(a)



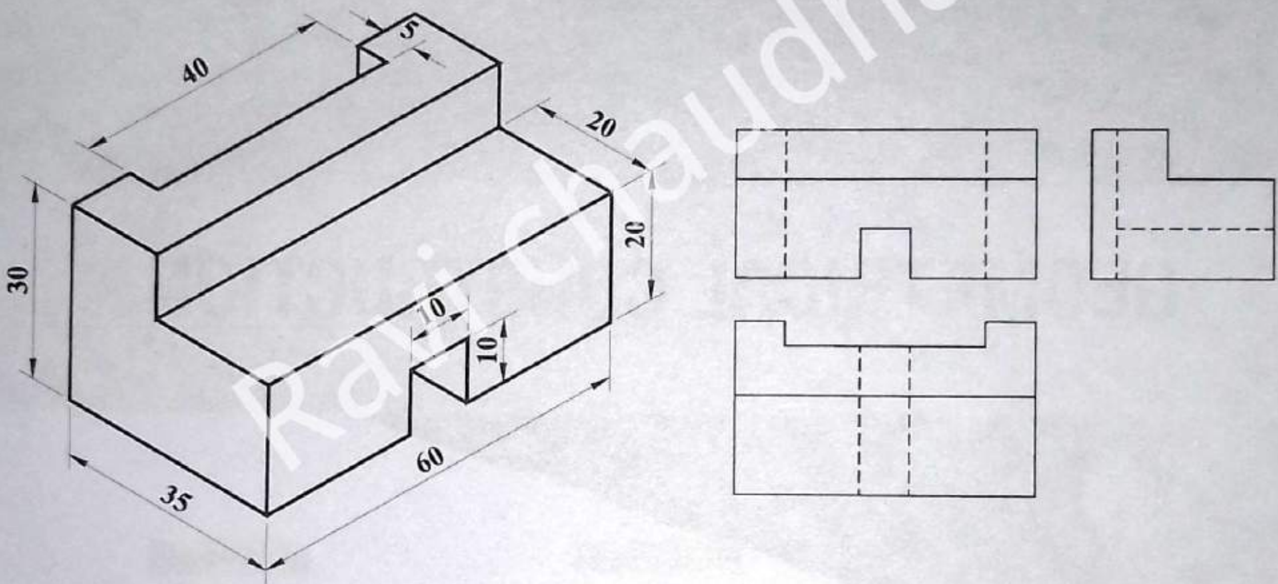
(b)



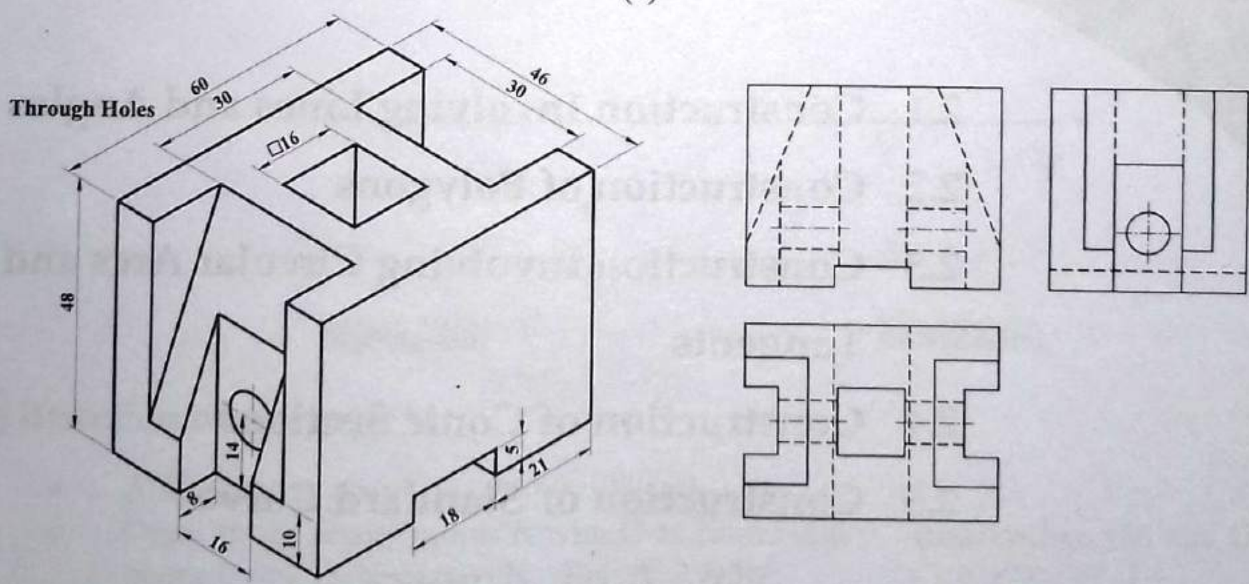
(c) (d)

Figure 1.8

9. Pictorial view and orthographic views for different objects are shown in *Figure P1.9*. Dimension the orthographic views.



(a)



(b)

Figure P1.9

CHAPTER 2

GEOMETRICAL CONSTRUCTION

For accurate graphical representation of any object, different geometrical constructions are used. The geometrical constructions generally used for the composition of any plane geometrical figures are explained below.

2.1 Construction Involving Lines and Angles

2.1.1 Bisection of a Straight Line

- Line AB is given. *(Figure 2.1(a))*
- With A as center and any radius greater than half length of AB, draw arcs on both sides of the line AB. *(Figure 2.1(b))*
- Draw similar arcs of same radius with B as center. *(Figure 2.1(c))*
- Mark intersection of arcs as C and D. *(Figure 2.1(d))*
- Join C and D with a straight line which intersects given line AB at point E. *(Figure 2.1(e))* Point E is the mid-point of the line AB.

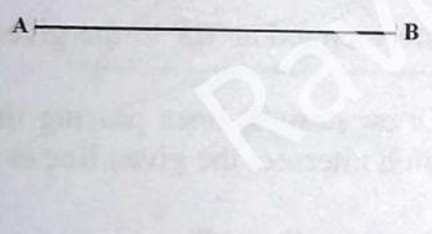


Figure 2.1(a)

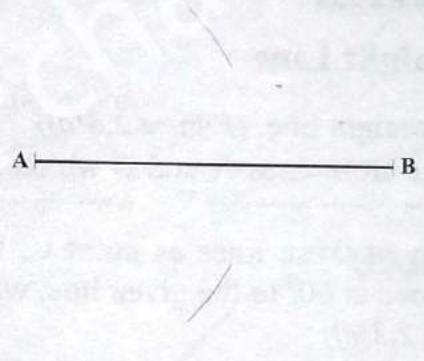


Figure 2.1(b)

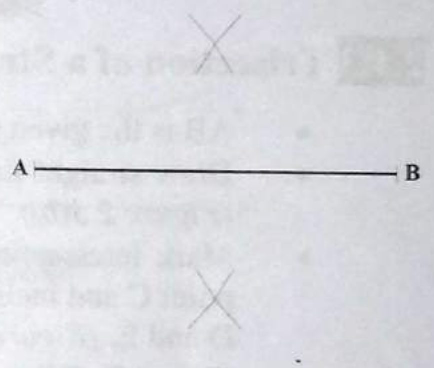


Figure 2.1(c)

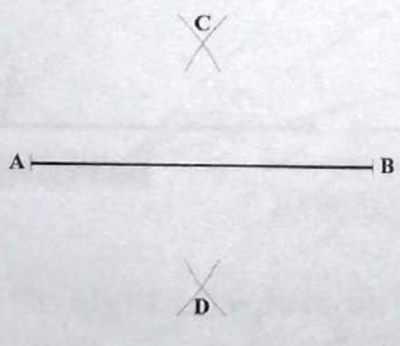


Figure 2.1(d)

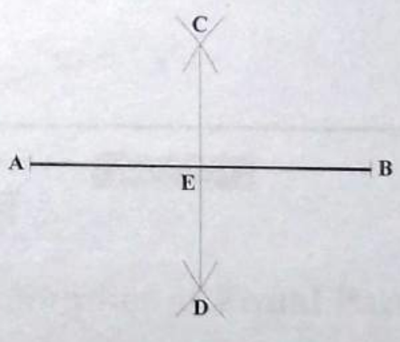


Figure 2.1(e)

2.1.2 Bisection of an Angle

- AOB is the given angle. *(Figure 2.2(a))*
- Draw an arc of any radius R with O as center which intersects line OA and OB at points C and D, respectively. *(Figure 2.2(b))*
- Draw arcs with radii R' greater than half of CD with centers as C and D respectively. *(Figure 2.2(c))*
- Mark intersection of arcs as point E. Join O and E by a straight line, which the required bisector of the given angle. *(Figure 2.2(d))*

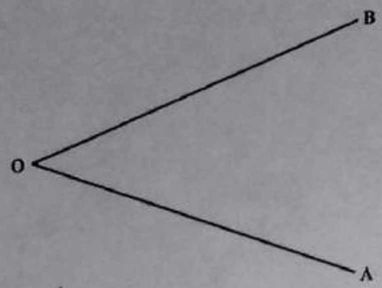


Figure 2.2(a)

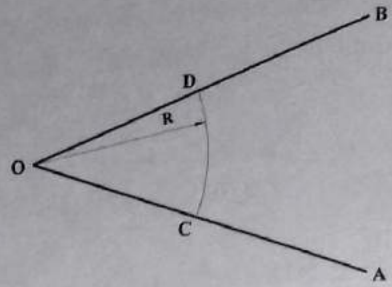


Figure 2.2(b)

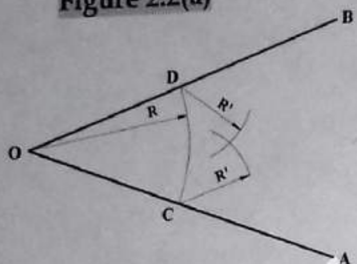


Figure 2.2(c)

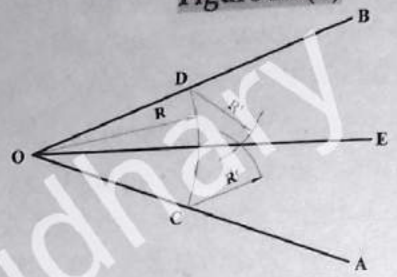


Figure 2.2(d)

2.1.3 Trisection of a Straight Line

- AB is the given straight line. *(Figure 2.3(a))*
- Draw straight line from ends A and B which are inclined at 30^0 to the given line. *(Figure 2.3(b))*
- Mark intersection of these lines as point C. Draw straight lines passing through point C and inclined at 60^0 to the given line, which intersect the given line at points D and E. *(Figure 2.3(c))*
- Then AD, DE and EB each equal one-third of the given line AB.

Figure 2.3(a)

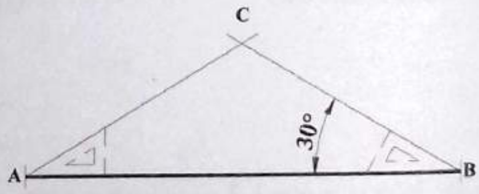


Figure 2.3(b)

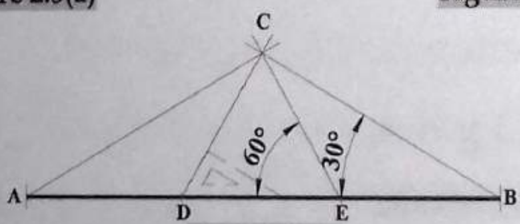


Figure 2.3(c)

2.1.4 Trisection of an Angle

- AOB is the given angle. *(Figure 2.4(a))*
- Mark any point C on line OB. Draw straight line CD passing through point C and perpendicular to OA. Draw straight line CE passing through C and parallel to OA. *(Figure 2.4(b))*
- Place scale passing through point O and mark point F on line CD and point G on line CE such that FG is twice OC. *(Figure 2.4(c))*
- Then angle GOA is equal to one-third of the given angle AOB. *(Figure 2.4(d))*

- Draw bisector OH of the remaining angle GOB to get three equal angles, i.e., ∢GOA = ∢GOH= ∢HOB. *(Figure 2.4(e))*

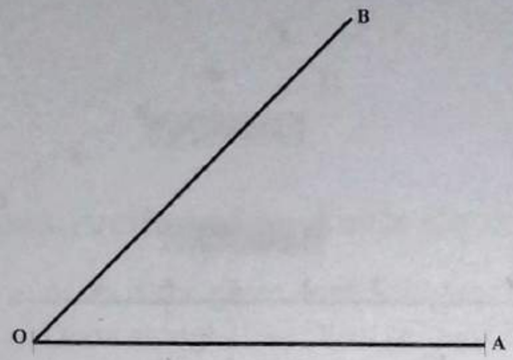


Figure 2.4(a)

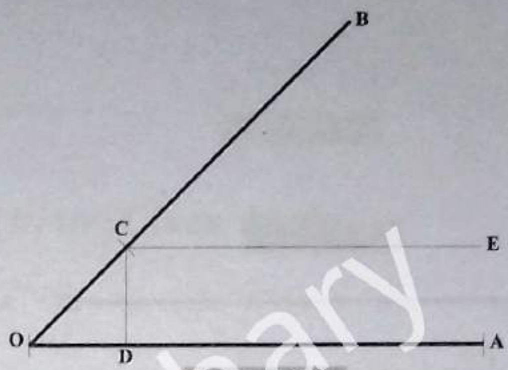


Figure 2.4(b)

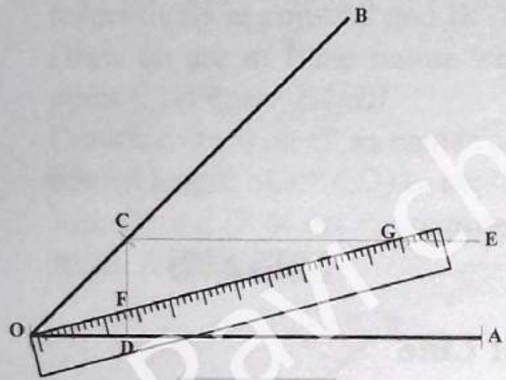


Figure 2.4(c)

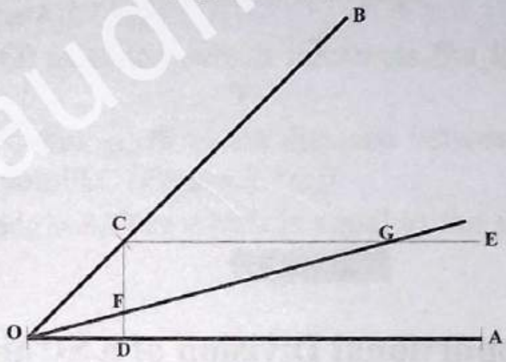


Figure 2.4(d)

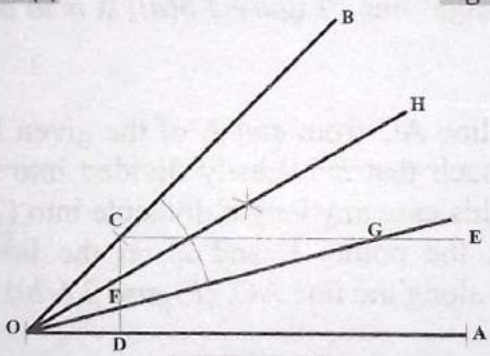


Figure 2.4(e)

2.1.5 Division of a Straight Line into any Number of Equal Parts

- AB is the given straight line. *(Figure 2.5(a))* It is to be divided into any number of equal parts (say 7).
- Draw any straight line AC from end A of the given line at any inclination. Select the length of AC such that it is easily divided into 7 equal parts by a common scale (A divider or compass may be used to mark seven equal segments on line AC). Mark the dividing points on the line AC is 1', 2', 3' .. 6'. *(Figure 2.5(b))*
- Join B and C. *(Figure 2.5(c))*
- Draw straight lines passing through 1', 2', 3' .. 6' and parallel to the line BC, which intersect the given line respectively at 1, 2, 3 … 6. *(Figure 2.5(d))*
- Then A1 = 12 = 23= 34 = 45 = 56 = 6B.

Figure 2.5(a)

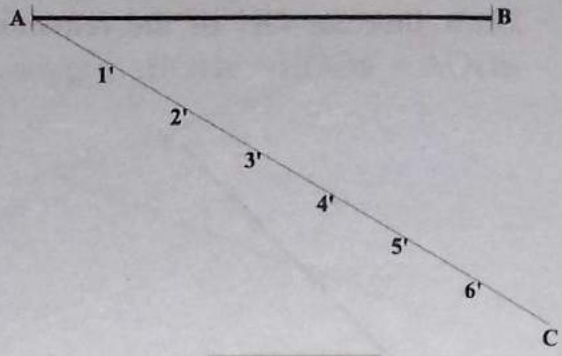


Figure 2.5(b)

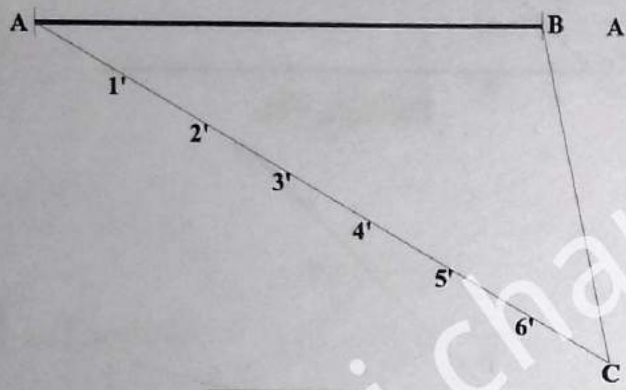


Figure 2.5(c)

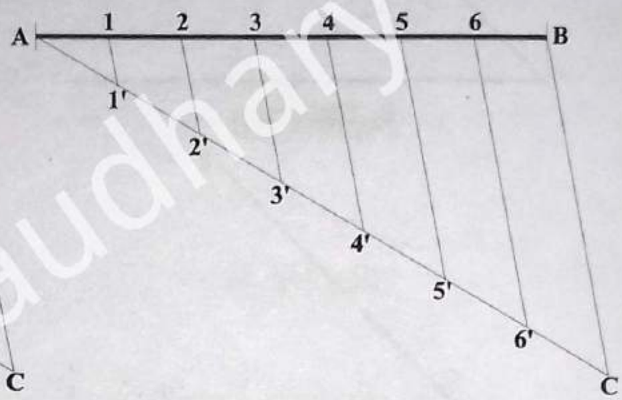


Figure 2.5(d)

2.1.6 Proportional Division of a Straight Line

- AB is the given straight line. *(Figure 2.6(a))* It is to be divided into any proportion (say 2:3:4).

- Draw any straight line AC from end A of the given line at any inclination. Select the length of AC such that it is easily divided into the required proportion by a common scale. In this case any length divisible into (2+3+4= 9) equal parts will be convenient. Mark the points 1' and 2' on the line AC such that they are in proportion of 2:3:4 along the line AC. *(Figure 2.6(b))*

- Join B and C. *(Figure 2.6(c))*

- Draw straight lines passing through 1', and 2' and parallel to the line BC, which intersect the given line respectively at 1 and 2. *(Figure 2.6(d))*

- Then points 1 and 2 divide the given line AB in the proportion of 2:3:4.

Figure 2.6(a)

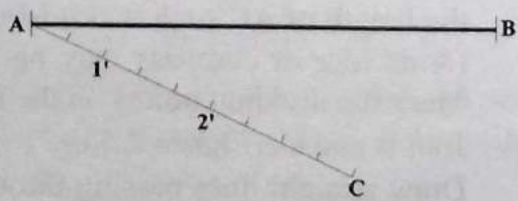


Figure 2.6(b)

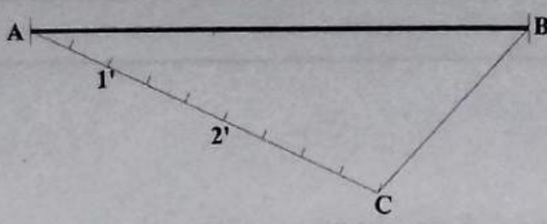


Figure 2.6(c)

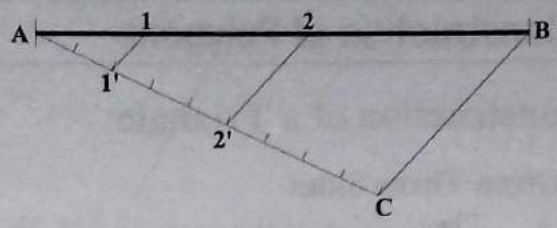


Figure 2.6(d)

2.1.7 Construction of an Angle Equal to the Given Angle

- AOB is the given angle. *(Figure 2.7(a))*
- Draw straight line O'A'. *(Figure 2.7(b))*
- Draw an arc of any radius with O as center which intersect the lines OA and OB respectively at points C and D. *(Figure 2.7(c))*
- Draw an arc of same radius with O' as center which intersects the line O'A' at point C'. *(Figure 2.7(d))*
- Draw an arc with C' as center and radius equal to the distance between C and D (chord length of arc CD) to get the point D'. *(Figure 2.7(e))*
- Join O' and D' to get the required angle A'O'B' which is equal to the given angle AOB. *(Figure 2.7(f))*

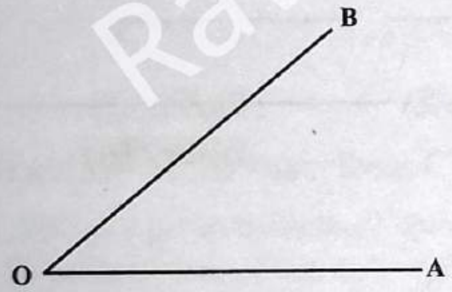


Figure 2.7(a)

Figure 2.7(a)

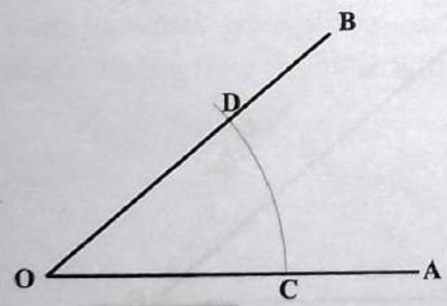


Figure 2.7(c)

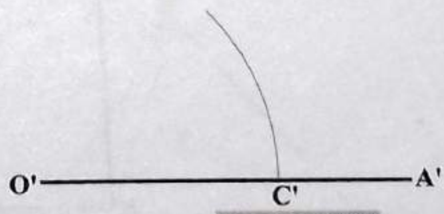


Figure 2.7(d)

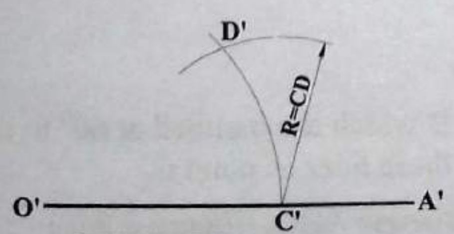


Figure 2.7(e)

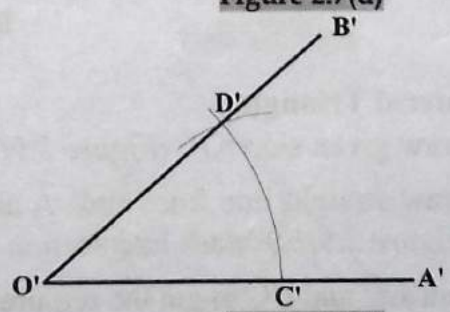


Figure 2.7(f)

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2.2 Construction of Polygons

2.2.1 Construction of a Triangle

(a) Given Three Sides

- Three sides of the triangle AB, BC and CA are given.
- Draw side AB. *(Figure 2.8(a))*
- Draw arcs with A as center and AC as radius and B as center and BC as radius. Intersection of the arcs given vertex C of the triangle. *(Figure 2.8(b))*
- Join AC and BC to get the required triangle ABC. *(Figure 2.8(c))*

Figure 2.8(a)

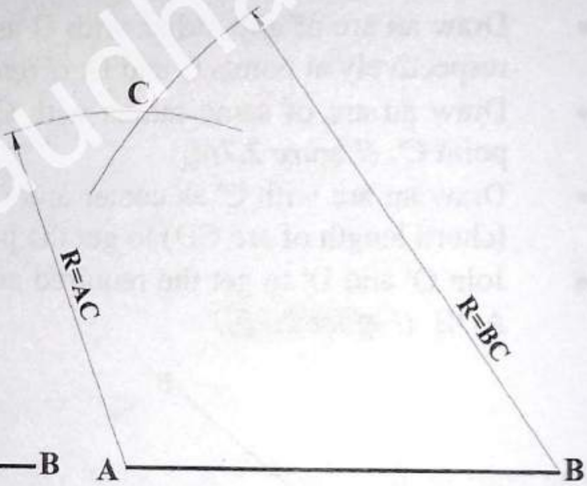


Figure 2.8(b)

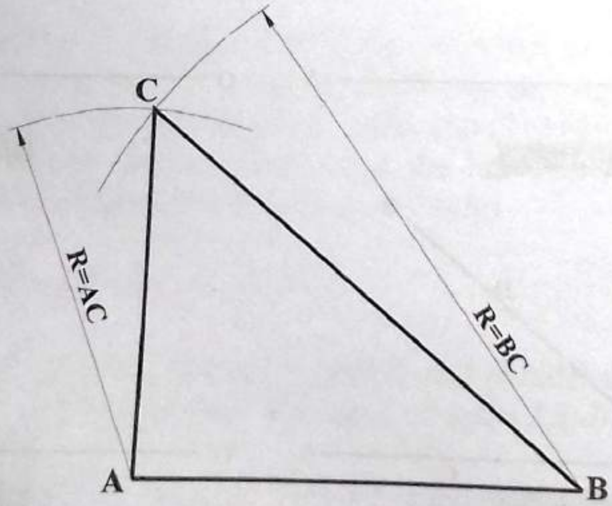


Figure 2.8(c)

(b) Equilateral Triangle

- Draw given side AB. *(Figure 2.9(a))*
- Draw straight line from ends A and B which are inclined at 60^0 to the given line. *(Figure 2.9(b))* Mark intersection of these lines as point C.
- Join AC and BC to get the required triangle ABC. *(Figure 2.9(c))*

Figure 2.9(a)

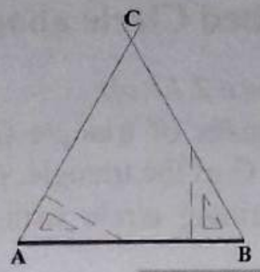


Figure 2.9(b)

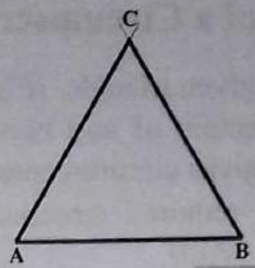


Figure 2.9(c)

Alternative Method

- Draw given side AB. *(Figure 2.10(a))*
- With centers as A and B and radius equal to AB, draw circular arcs intersecting each other at point C. *(Figure 2.10(b))*
- Join C with A and B by straight lines to get the required triangle ABC. *(Figure 2.10(c))*

Figure 2.10(a)

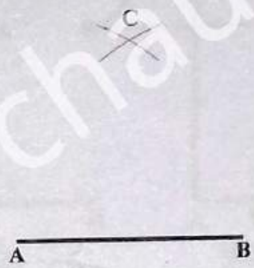


Figure 2.10(b)

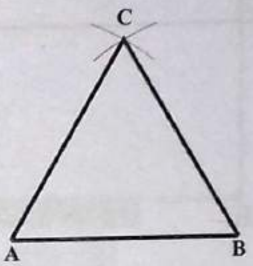


Figure 2.10(c)

2.2.2 Construction of an Inscribed Circle in a Given Triangle

- ABC is a given triangle. *(Figure 2.11(a))*
- Draw bisectors of any two angles of triangle (∢CAB and ∢ABC). Intersection of the bisectors gives in-center O of the triangle. *(Figure 2.11(b))*
- Draw a line OP passing through point O and perpendicular to any one side of the triangle, which gives radius of the in-circle. *(Figure 2.11(c))*
- Draw the required in-circle with O as center and OP as radius. *(Figure 2.11(d))*

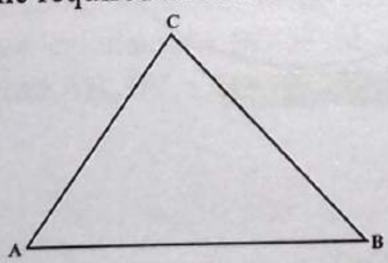


Figure 2.11(a)

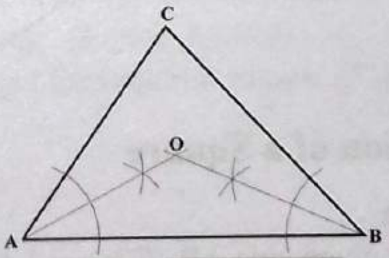


Figure 2.11(b)

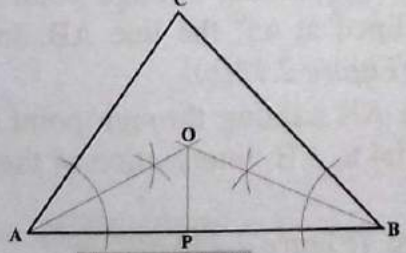


Figure 2.11(c)

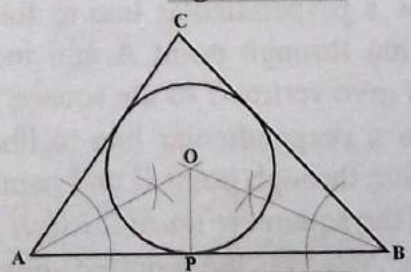


Figure 2.11(d)

2.2.3 Construction of a Circumscribed Circle about a Given Triangle

- ABC is a given triangle. *(Figure 2.12(a))*
- Draw bisectors of any two sides of triangle (AB and BC). Intersection of the bisectors gives circum-center O of the triangle. *(Figure 2.12(b))*
- Draw the required circumscribing circle with O as center and OA as radius. *(Figure 2.12(c))*

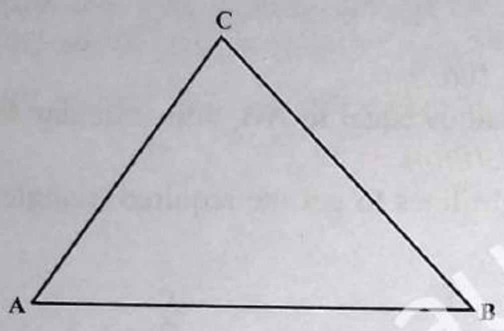


Figure 2.12(a)

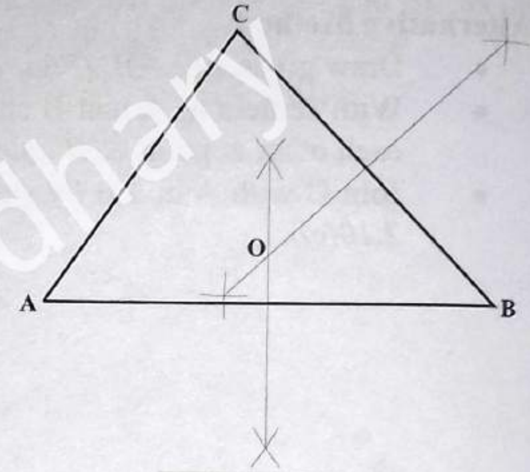


Figure 2.12(b)

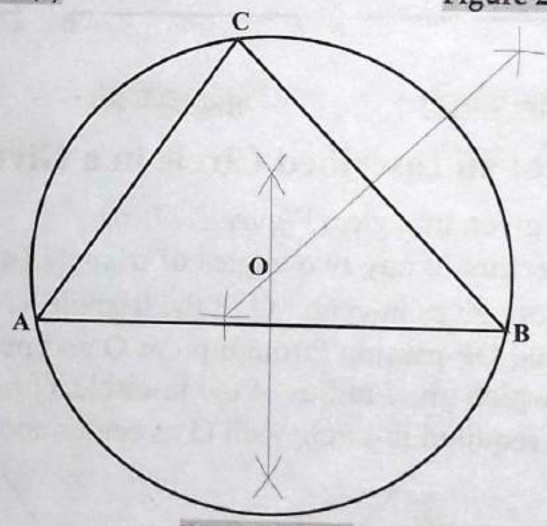


Figure 2.12(c)

2.2.4 Construction of a Square

(a) Given Side

- Draw given side AB. *(Figure 2.13(a))*
- Draw a perpendicular line to line AB passing through point B and another line passing through point A and inclined at 45^0 the line AB. Intersection of these lines give vertex C of the square. *(Figure 2.13(b))*
- Draw a perpendicular line to line AB passing through point B and another line passing through point C and parallel to AB. Intersection of these lines give vertex D of the square. *(Figure 2.13(c))*
- Then, ABCD is the required square. *(Figure 2.13(d))*

Figure 2.13(a)

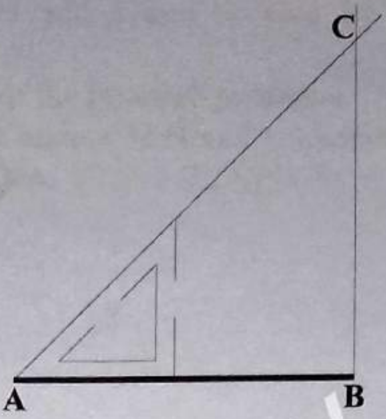


Figure 2.13(b)

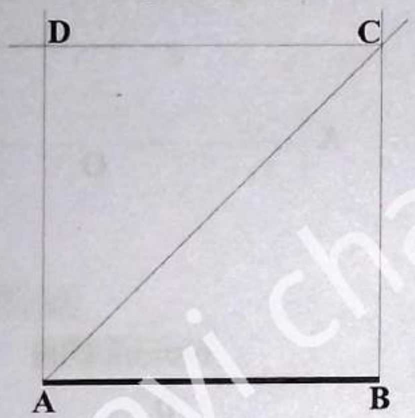


Figure 2.13(c)

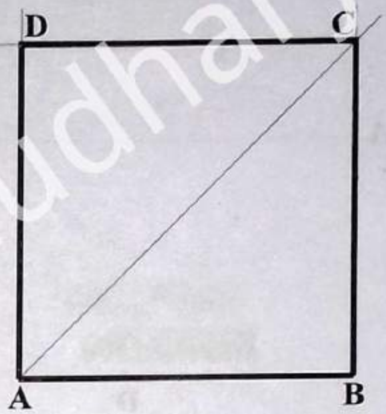


Figure 2.13(d)

(b) Given Diagonal

- Draw given diagonal AC. *(Figure 2.14(a))*
- Draw straight line passing through ends A and C and inclined at 45^0 to the given diagonal AC. Intersection of these inclined lines give other vertices B and D. *(Figure 2.14(b))*
- Then, ABCD is the required square. *(Figure 2.14(c))*

Alternative Method

- Draw given diagonal AC. *(Figure 2.15(a))*
- Mark midpoint O of AC. Draw circle with AC as it diameter. *(Figure 2.15(b))*
- Draw vertical diameter BD of the circle. *(Figure 2.15(c))*
- Draw lines AB, BC, CD and DA to get the required square. *(Figure 2.15(d))*

Figure 2.14(a)

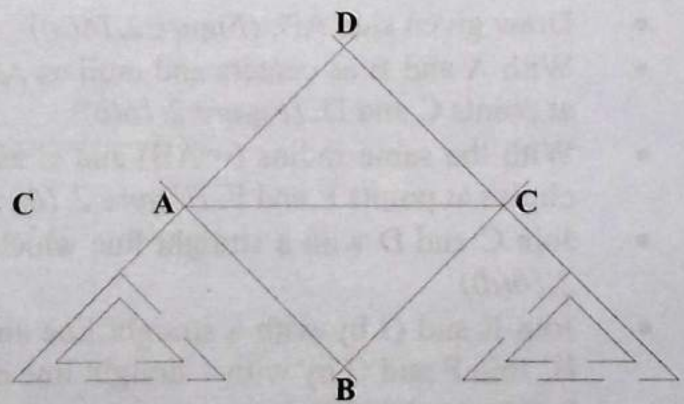


Figure 2.14(b)

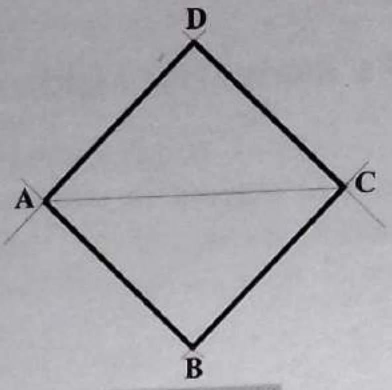


Figure 2.14(c)

Figure 2.15(a)

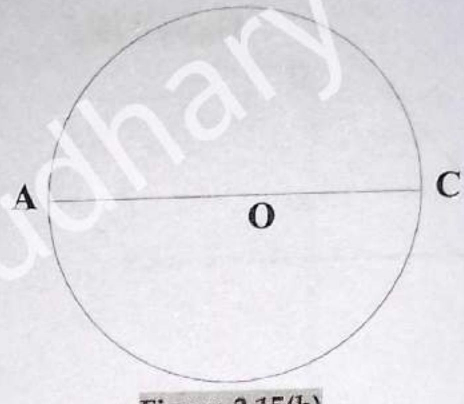


Figure 2.15(b)

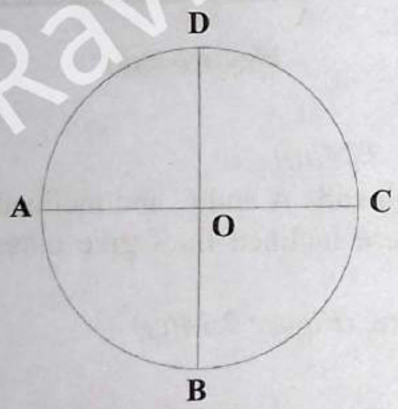


Figure 2.15(c)

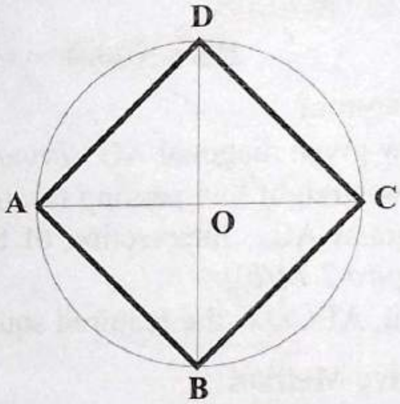


Figure 2.15(d)

2.2.5 Construction of Regular Pentagon

(a) Given Side

- Draw given side AB. *(Figure 2.16(a))*
- With A and B as centers and radii as AB, draw circles which intersect each other at points C and D. *(Figure 2.16(b))*
- With the same radius (= AB) and C as center, draw an arc which intersects the circles at points E and F. *(Figure 2.16(c))*
- Join C and D with a straight line which intersects the arc EF at point G. *(Figure 2.16(d))*
- Join E and G by with a straight line and extend it intersecting the circle at point H. Join F and G by with a straight line and extend it intersecting the circle at point I. *(Figure 2.16(e))*

- Join H and B with a straight lines and I and A also by a straight line. *(Figure 2.16(e))*
- Then, HB and IA are two other sides of the required pentagon. *(Figure 2.16(f))*
- Draw arcs with radii equal to AB and centers as H and I. Intersection of the arcs give the vertex J of the required pentagon. *(Figure 2.16(g))*
- Then ABHJI is the required pentagon.

Figure 2.16(a)

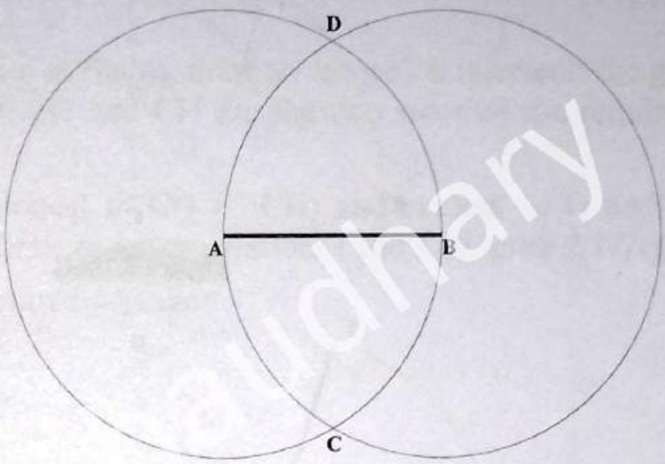


Figure 2.16(b)

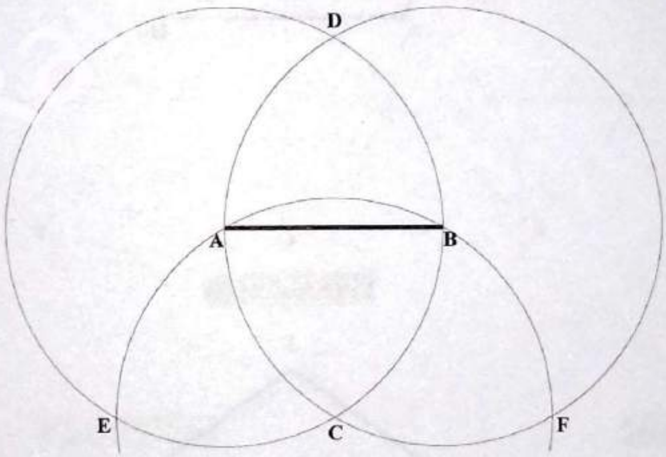


Figure 2.16(c)

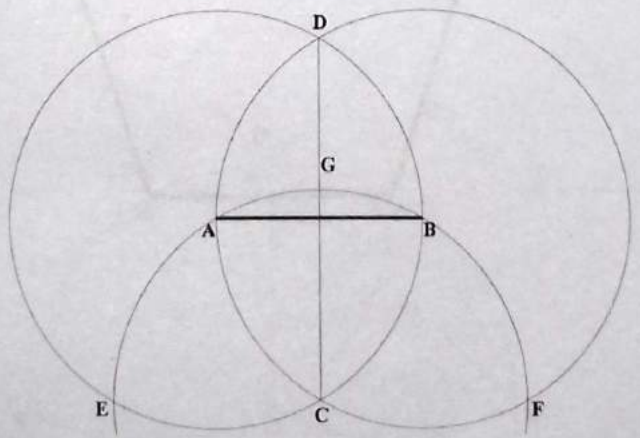


Figure 2.16(d)

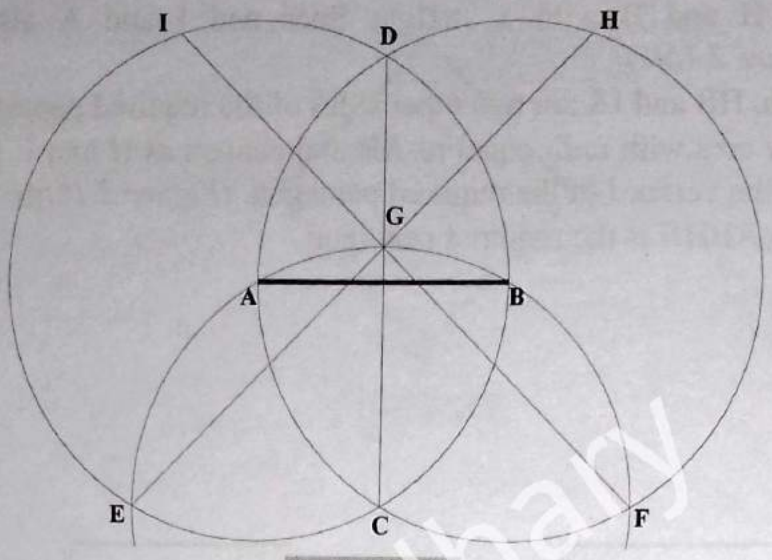


Figure 2.16(e)

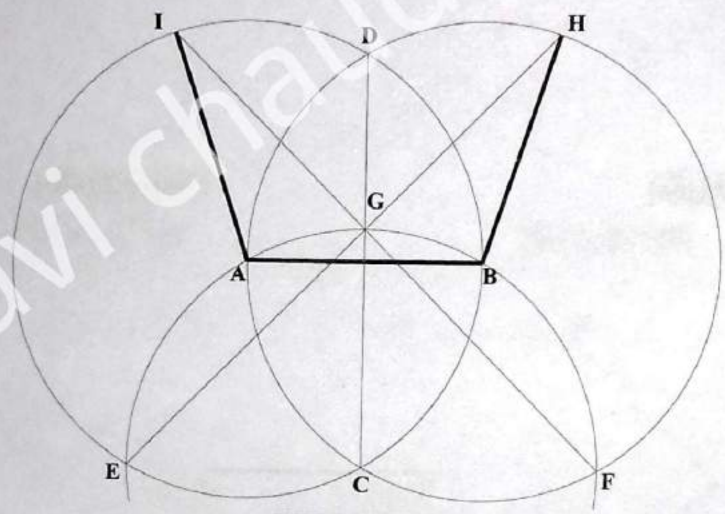


Figure 2.16(f)

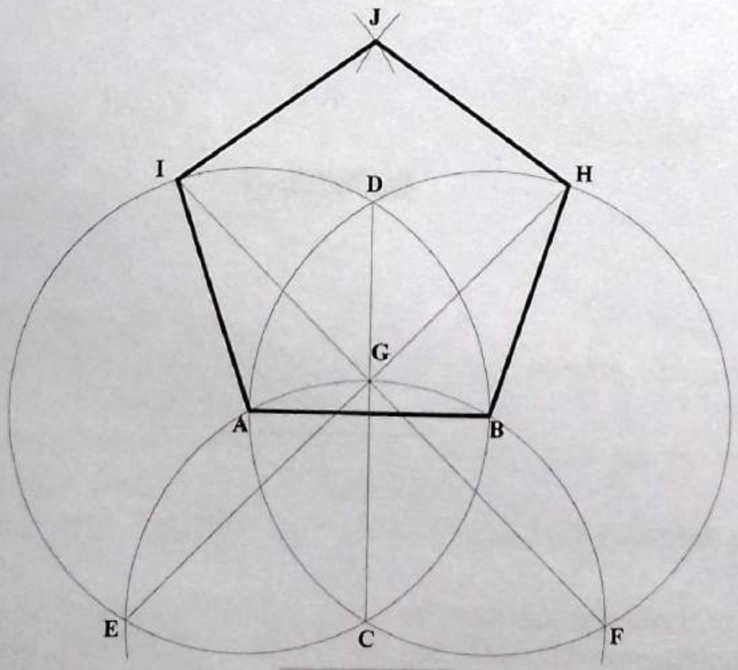


Figure 2.16(g)

(b) Given Circumscribing Circle

- Draw given circumscribing circle with O as its center. Draw its horizontal and vertical diameters AB and CD respectively. *(Figure 2.17(a))*
- Draw perpendicular bisector of OB and mark its midpoint as point E. (*Figure 2.17(b))*
- With E as center and EC as radius, draw an arc which intersects AO at point F. *(Figure 2.17(c))*
- With C as center and CF as radius, draw an arc which intersects the given circle at points G and H. Then CG and CH are the two sides of the required pentagon. *(Figure 2.17(d))*
- Draw arcs with radii equal to CG (= CH) and centers as G and H along the circumference of the circle to get the vertices I and J. *(Figure 2.17(e))*
- Then CGIJH is the required pentagon.

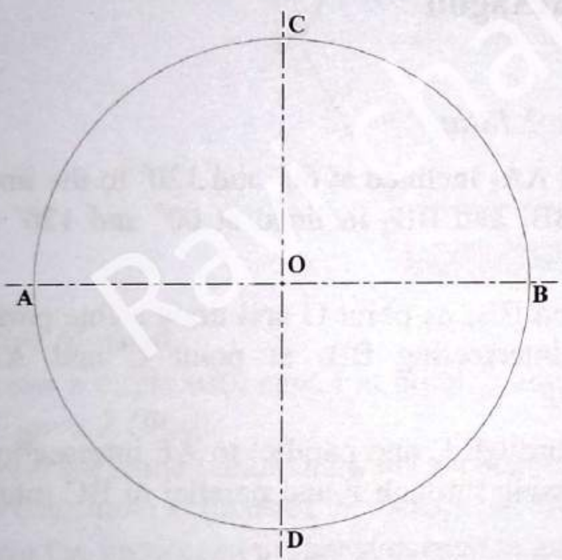


Figure 2.17(a)

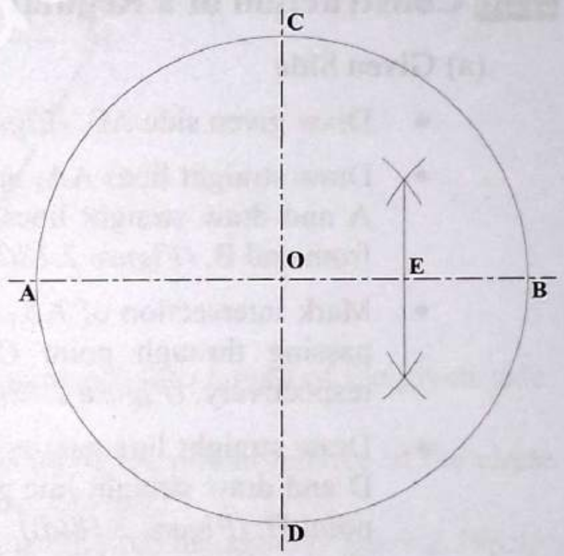


Figure 2.17(b)

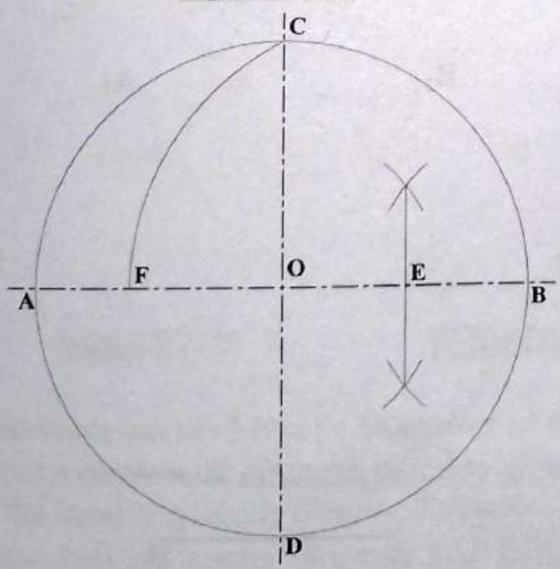


Figure 2.17(c)

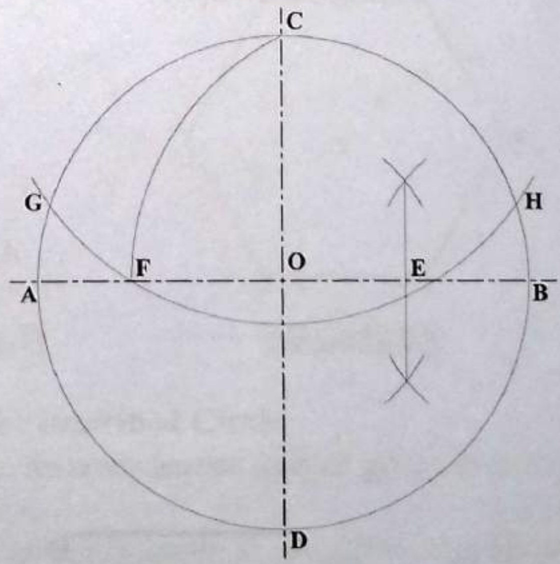


Figure 2.17(d)

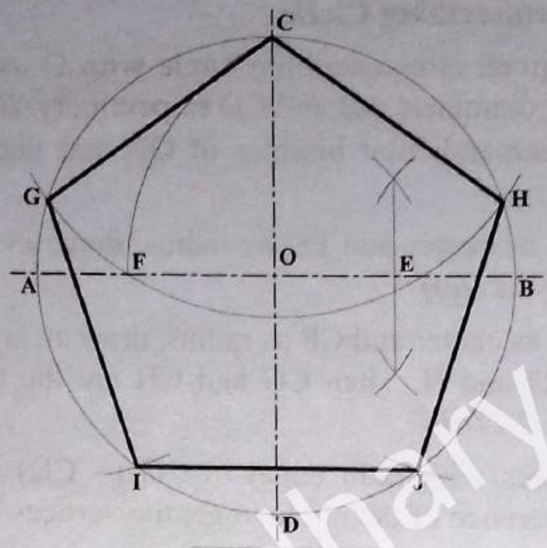


Figure 2.17(e)

2.2.6 Construction of a Regular Hexagon

(a) Given Side

- Draw given side AB. *(Figure 2.18(a))*
- Draw straight lines AA_1 and AA_2 inclined at 60° and 120° to the line AB from end A and draw straight lines BB_1 and BB_2 inclined at 60° and 120° to the line AB from end B. *(Figure 2.18(b))*
- Mark intersection of AA_1 and BB_2 as point O and draw a line parallel to AB and passing through point O intersecting BB_1 at point C and AA_2 at point F respectively. *(Figure 2.18(c))*
- Draw straight line passing through C and parallel to AF intersecting AA_1 at point D and draw straight line passing through F and parallel to BC intersecting BB_2 at point D. *(Figure 2.18(d))*
- Draw sides BC, CD, DE, EF and FA to get the required hexagon. *(Figure 2.18(e))*

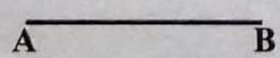


Figure 2.18(a)

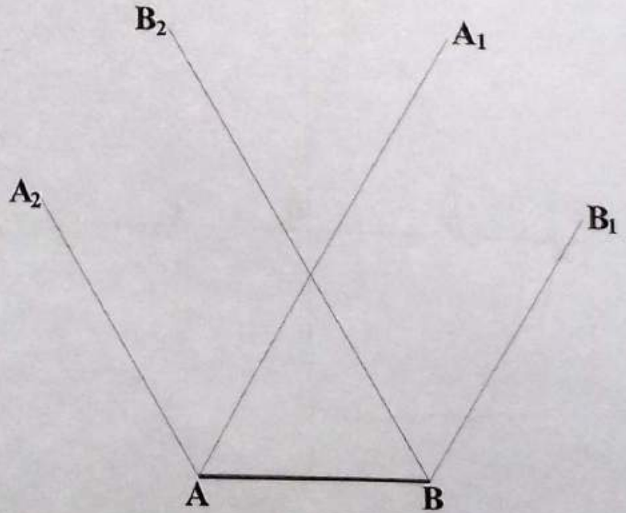


Figure 2.18(b)

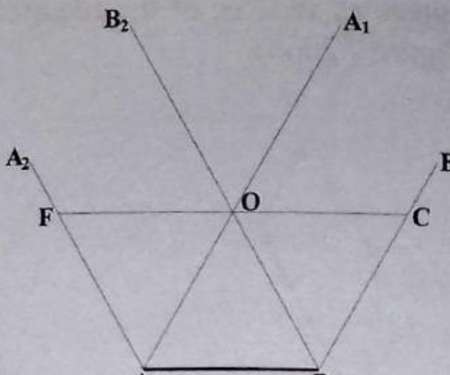


Figure 2.18(c)

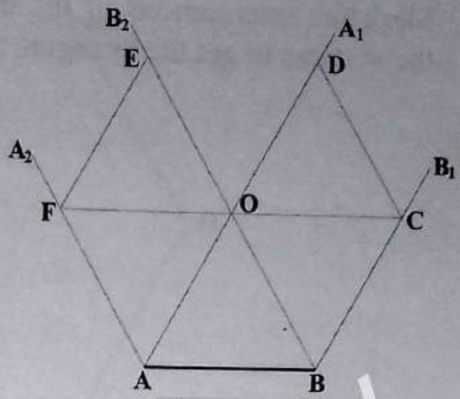


Figure 2.18(d)

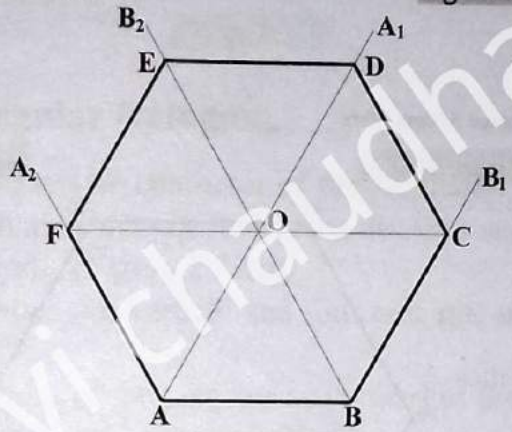


Figure 2.18(e)

Alternative Method

- Draw a circle with center at point O and radius equal to length of the given side. *(Figure 2.19(a))*
- With the same radius mark off six segments along the circumference of the circle to determine vertices of the hexagon. *(Figure 2.19(b))*
- Join the vertices in proper sequence to get the required hexagon. *(Figure 2.19(c))*

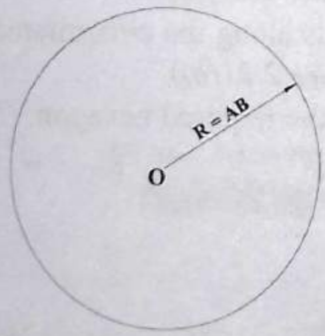


Figure 2.19(a)

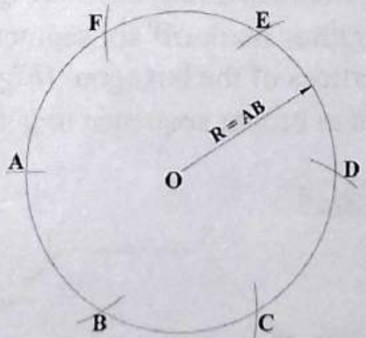


Figure 2.19(b)

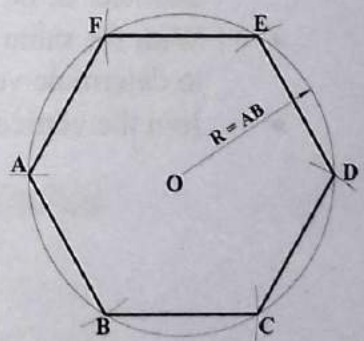


Figure 2.19(c)

(b) Given Distance across Flats or Diameter of the Inscribed Circle

- Draw a circle with diameter equal to given distance across flats or given diameter of the inscribed circle. *(Figure 2.20(a))*
- Draw two horizontal tangents and four tangents inclined at 60^0 to the circle. *(Figure 2.20(b))*

- Mark the intersections of the tangents as vertices of the required hexagon. Join the vertices to get the hexagon. *(Figure 2.20(c))*

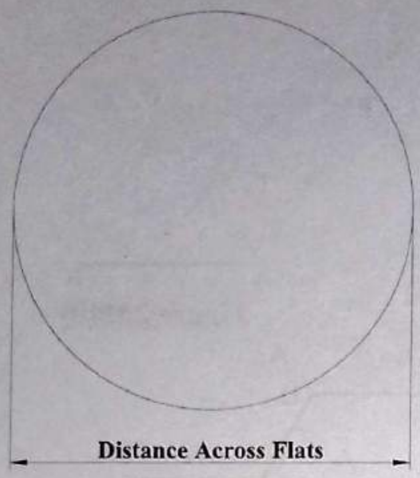


Figure 2.20(a)

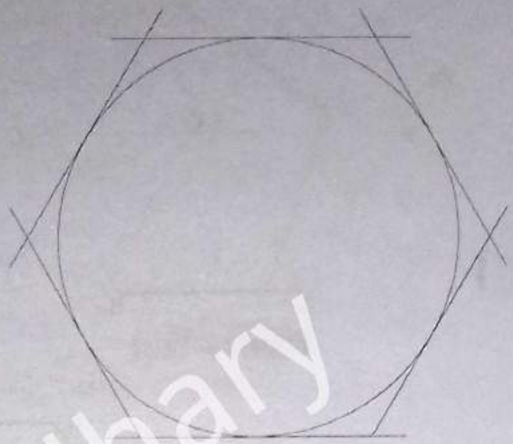


Figure 2.20(b)

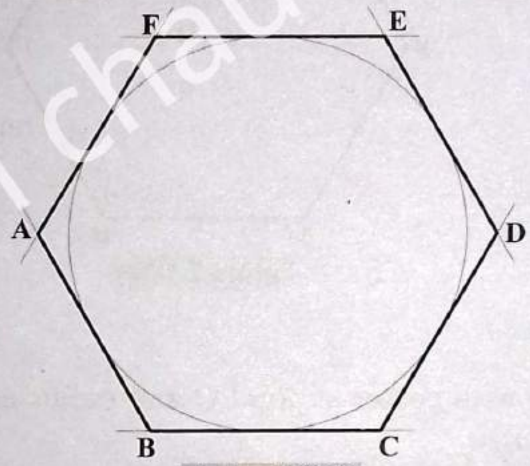


Figure 2.20(c)

(c) Given Distance across Corners or Diameter of the Circumscribing Circle

- Draw a circle with diameter equal to given distance across corners or given diameter of the circumscribing circle. *(Figure 2.21(a))*
- With the same radius mark off six segments along the circumference of the circle to determine vertices of the hexagon. *(Figure 2.21(b))*
- Join the vertices in proper sequence to get the required hexagon. *(Figure 2.21(c))*

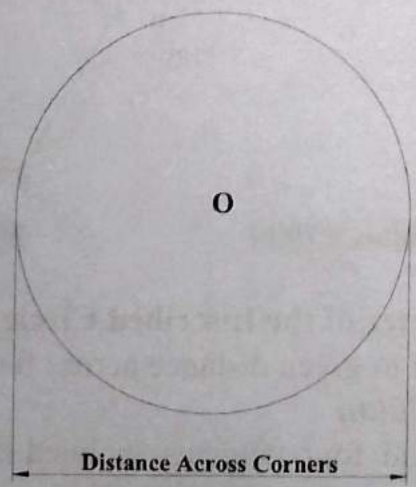


Figure 2.21(a)

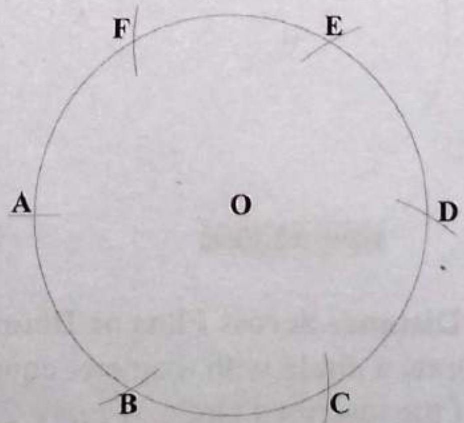


Figure 2.21(b)

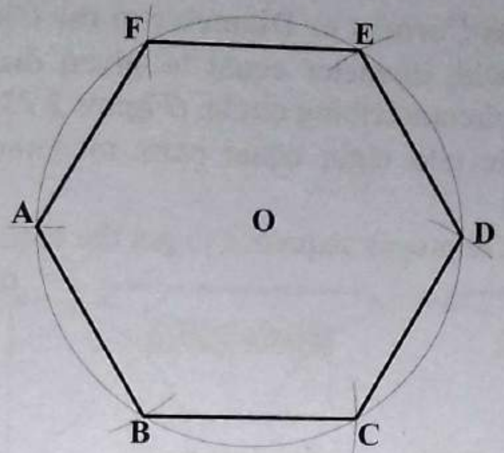


Figure 2.21(c)

2.2.7 Construction of a Regular Octagon

(a) Given Distance across Flats or Diameter of the Inscribed Circle

- Draw a circle with diameter equal to given distance across flats or given diameter of the inscribed circle. *(Figure 2.22(a))*
- Draw two horizontal, two vertical and four tangents inclined at 45^0 to the circle. *(Figure 2.22(b))*
- Mark the intersections of the tangents as vertices of the required octagon. Join the vertices to get the octagon. *(Figure 2.22(c))*

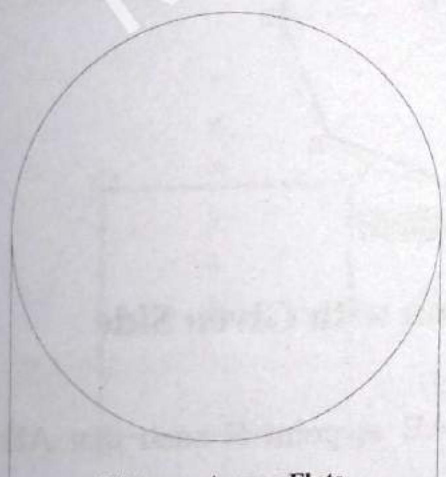


Figure 2.22(a)

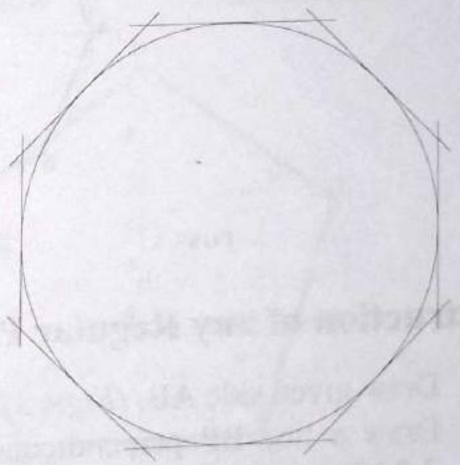


Figure 2.22(b)

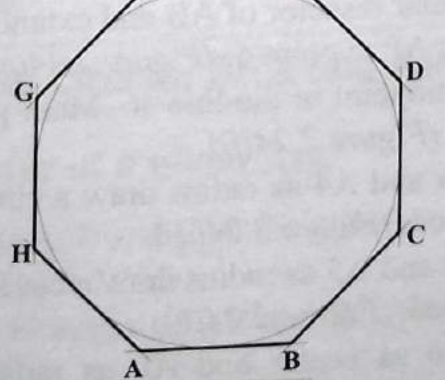


Figure 2.22(c)

(b) Given Distance across Corners or Diameter of the Circumscribing Circle

- Draw a circle with diameter equal to given distance across corners or given diameter of the circumscribing circle. *(Figure 2.23(a))*
- Divide the circle into eight equal parts to determine vertices of the octagon. *(Figure 2.23(b))*
- Join the vertices in proper sequence to get the required octagon. *(Figure 2.23(c))*

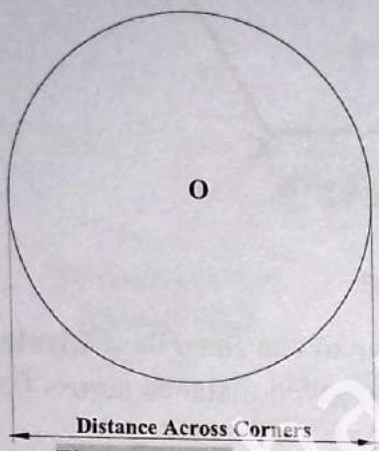


Figure 2.23(a)

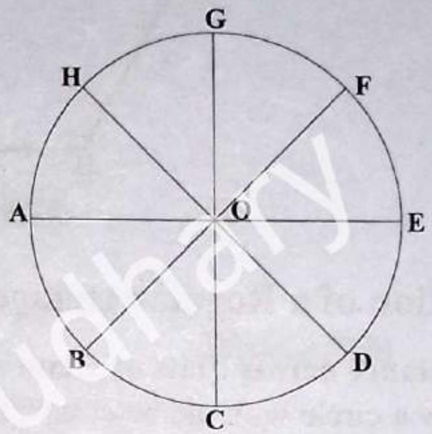


Figure 2.23(b)

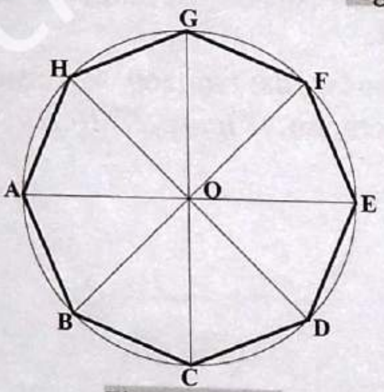


Figure 2.23(c)

2.2.8 Construction of any Regular Polygon with Given Side

- Draw given side AB. *(Figure 2.24(a))*
- Draw a line BP perpendicular to AB at point B such that AB = BP. *(Figure 2.24(b))*
- Join A and P with a straight line. *(Figure 2.24(c))*
- With B as center and BA as radius draw an arc AP. *(Figure 2.24(d))*
- Draw a perpendicular bisector of AB and extend it which intersects the line AP at point 4 and the arc AP at point 6. *(Figure 2.24(e))*
- Mark point 5 as midpoint of the line 46. Mark points 7, 8, 9, …… such that 56 = 67 = 78= 89 …… *(Figure 2.24(f))*
- With 4 as a center and A4 as radius draw a circle, in which a square with a side AB can be inscribed. *(Figure 2.24(g))*
- With 5 as a center and A5 as radius draw a circle, in which a pentagon with a side AB can be inscribed. *(Figure 2.24(h))*
- Similarly, using 6 as center and A6 as radius a circle inscribing a required hexagon can be drawn and so on.

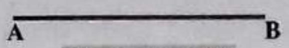
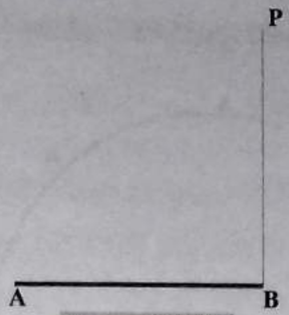


Figure 2.24(a)



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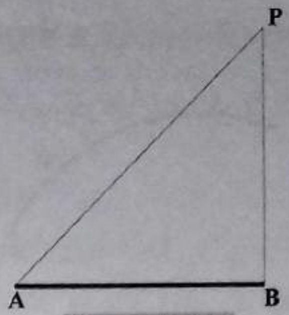

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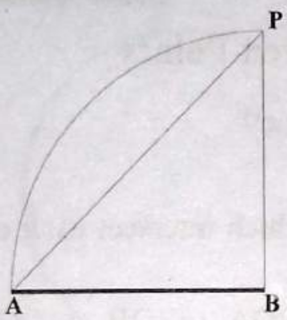

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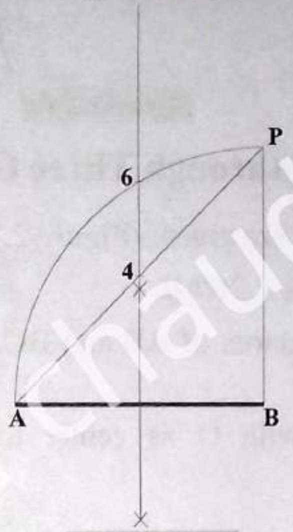

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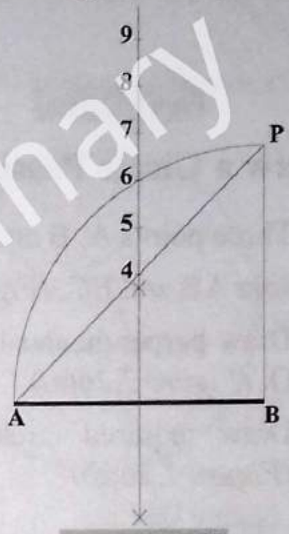


Figure 2.24(f)

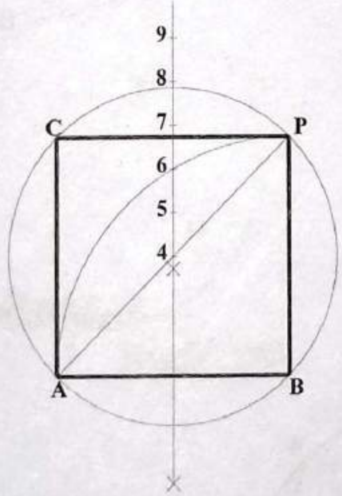


Figure 2.24(g)

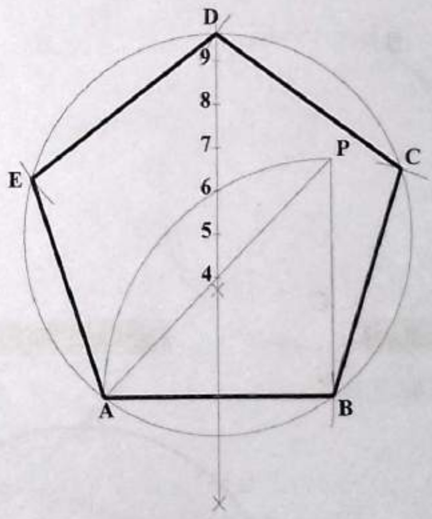


Figure 2.24(h)

2.3 Construction Involving Circular Arcs and Tangents

2.3.1 To Determine the Center of a given Arc

- Take any three points A, B and C on the given arc. *(Figure 2.25(a))*
- Join AB and BC. *(Figure 2.25(b))*
- Draw perpendicular bisectors of AB and BC, which intersect each other at point O. *(Figure 2.25(c))*
- Point O is the required center of the given arc.

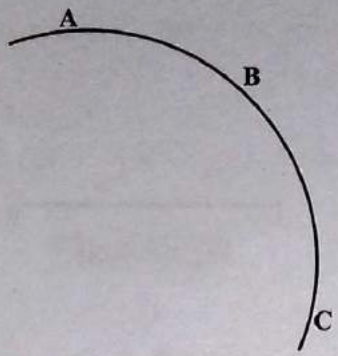


Figure 2.25(a)

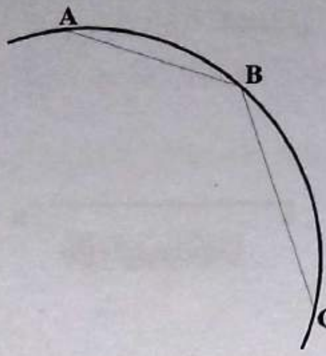


Figure 2.25(b)

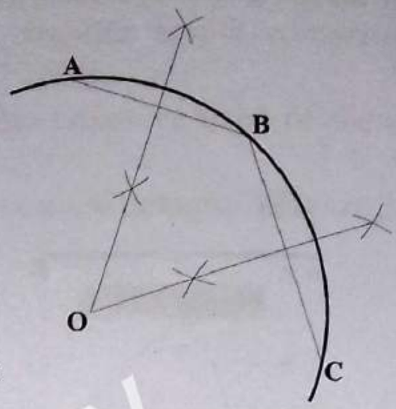


Figure 2.25(c)

2.3.2 To Draw a Circle Passing through Three Given Points

- Three points A, B and C are given. *(Figure 2.26(a))*
- Join AB and BC. *(Figure 2.26(b))*
- Draw perpendicular bisectors of AB and BC, which intersect each other at point O. *(Figure 2.26(c))*
- Draw required circle with O as center and OA (= OB = OC) as radius. *(Figure 2.26(d))*

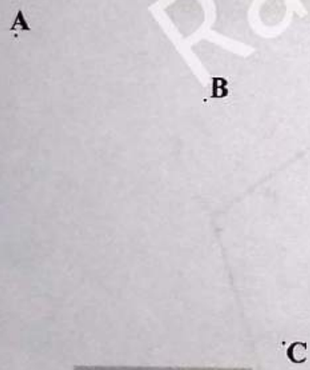


Figure 2.26(a)

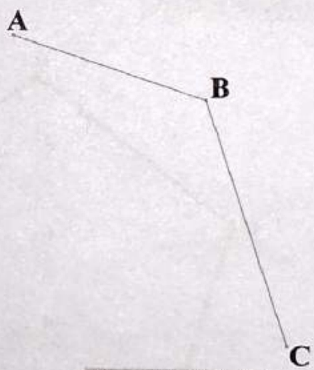


Figure 2.26(b)

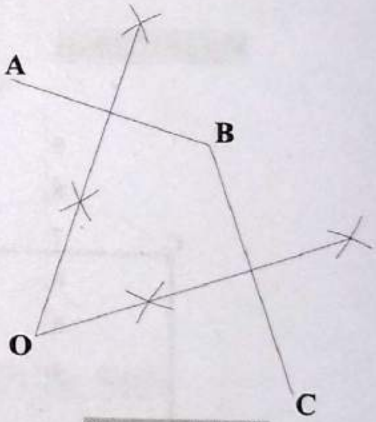


Figure 2.26(c)

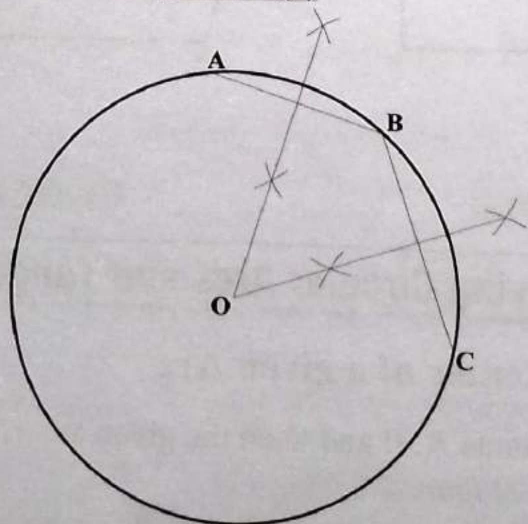


Figure 2.26(d)

2.3.3 To Draw a Tangent to a Circle Passing through a Given External Point

- A circle with its center at point O and a point P external to it are given. *(Figure 2.27(a))*
- Join O and P with a straight line. *(Figure 2.27(b))*
- Draw perpendicular bisector of OP and mark its midpoint M. *(Figure 2.27(c))*
- With M as center and MO (= OP) as radius, draw semicircle on the side where tangent is to be drawn. Mark intersection of the semicircle and the given circle as point T. *(Figure 2.27(d))*
- Join P and T by a straight line, which the required tangent. *(Figure 2.27(e))*

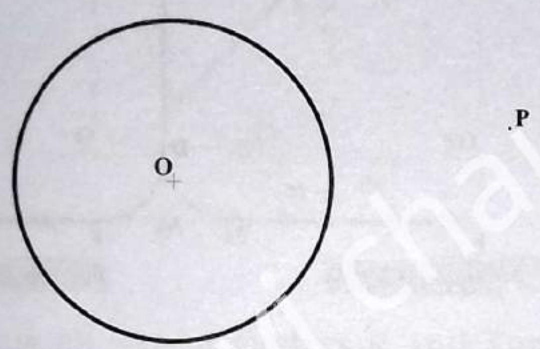


Figure 2.27(a)

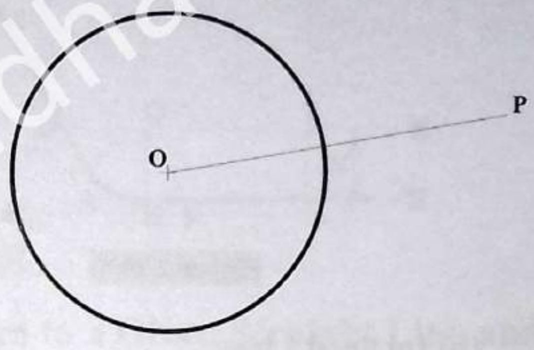


Figure 2.27(b)

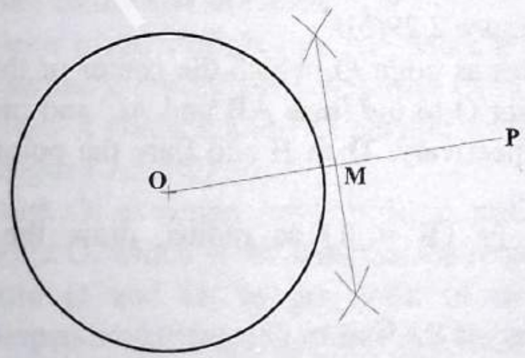
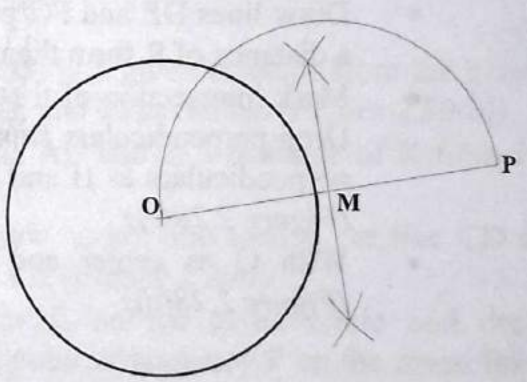


Figure 2.27(c)



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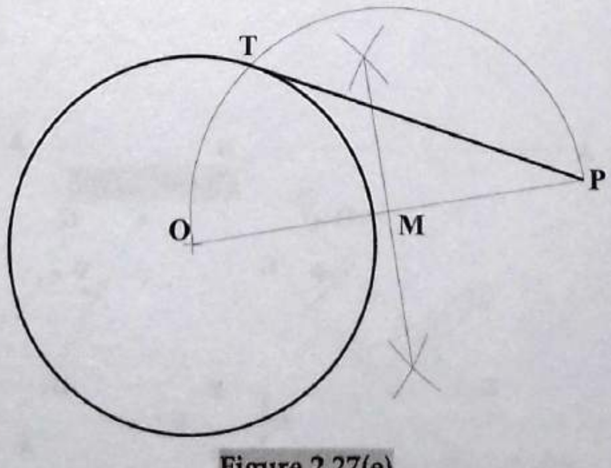


Figure 2.27(e)

2.3.4 To Draw an Arc of Radius R and Tangent to Two Given Lines

(a) Perpendicular Lines

- AB and AC are the two given lines perpendicular to each other. *(Figure 2.28(a))*
- Draw lines DE and FG parallel to the given lines AB and AC respectively and at a distance of R form them. *(Figure 2.28(b))*
- Mark intersection of these lines as point O, which the center of the required arc. With O as center and OD (= OF = R) as radius, draw the required arc. *(Figure 2.28(c))*

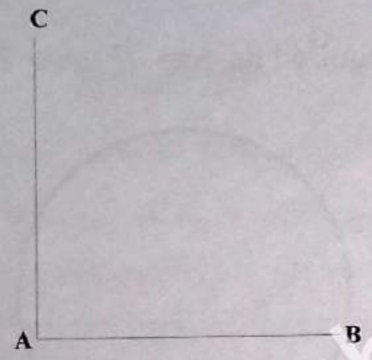


Figure 2.28(a)

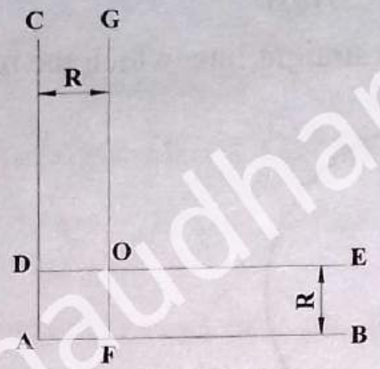


Figure 2.28(b)

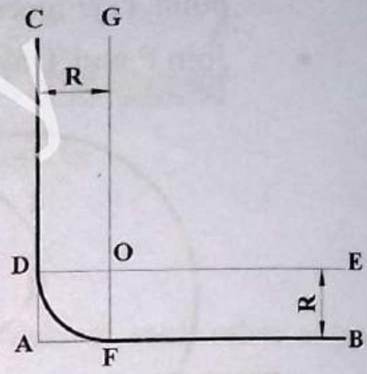


Figure 2.28(c)

(b) Inclined Lines

- AB and AC are the two given lines inclined to each other. *(Figure 2.29(a))*
- Draw lines DE and FG parallel to the given lines AB and AC respectively and at a distance of R form them. *(Figure 2.29(b))*
- Mark intersection of these lines as point O, which the center of the required arc. Drop perpendiculars from point O to the lines AB and AC and mark the foot of perpendiculars as H and I respectively. Then H and I are the points of tangency. *(Figure 2.28(c))*
- With O as center and OH (= OI = R) as radius, draw the required arc. *(Figure 2.28(d))*

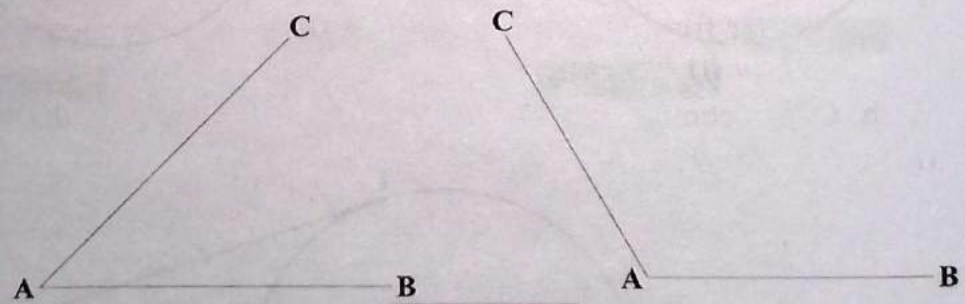


Figure 2.29(a)

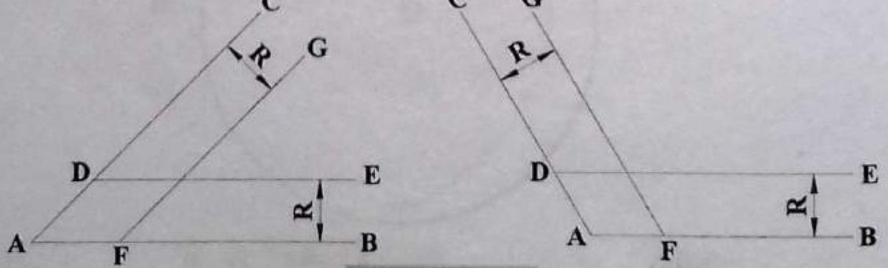


Figure 2.29(b)

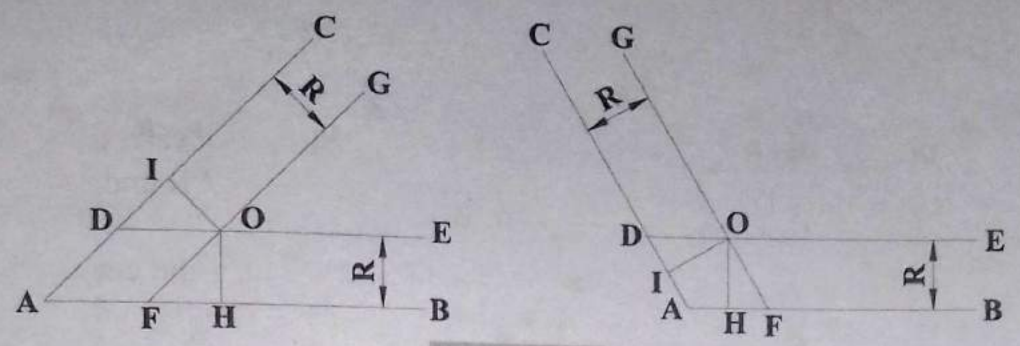


Figure 2.29(c)

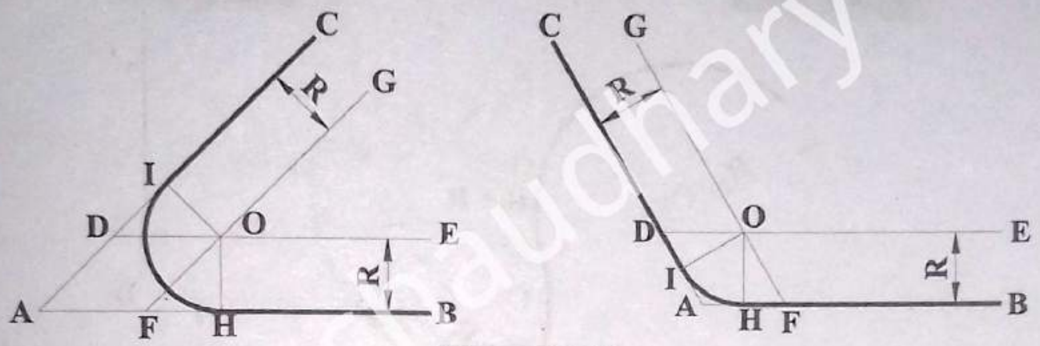


Figure 2.29(d)

2.3.5 To Draw an Arc of Radius R and Tangent to a Given Straight Line and a Given Circle (or a Circular Arc)

(a) Outside to the Given Circle

- Draw given straight line AB. Mark a point O_1 at a given distance from the given line and draw a given circle with O_1 as center and R_1 as radius. *(Figure 2.30(a))*
- Draw a straight line CD parallel to the line AB and at a distance of R from it. *(Figure 2.30(b))*
- With O_1 as center and $R + R_1$ as radius draw an arc intersecting the line CD at point O, which is the center of the required arc. *(Figure 2.30(c))*
- Join O and O_1 to get point of tangency E on the given circle and drop perpendicular from O to line AB to get the point of tangency F on the given line. *(Figure 2.30(d))*
- With O as center and OE (= OF = R) as radius, draw the required arc. *(Figure 2.30(e))*

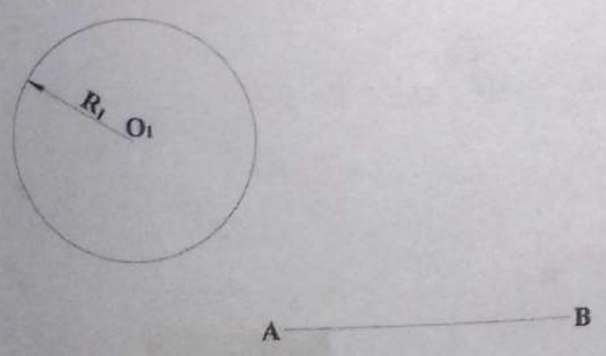


Figure 2.30(a)

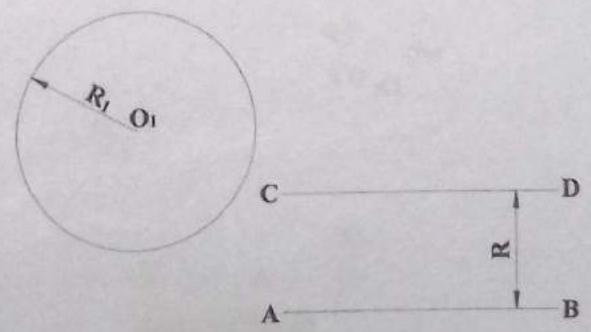


Figure 2.30(b)

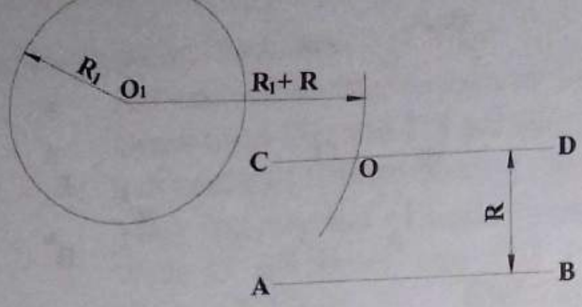


Figure 2.30(c)

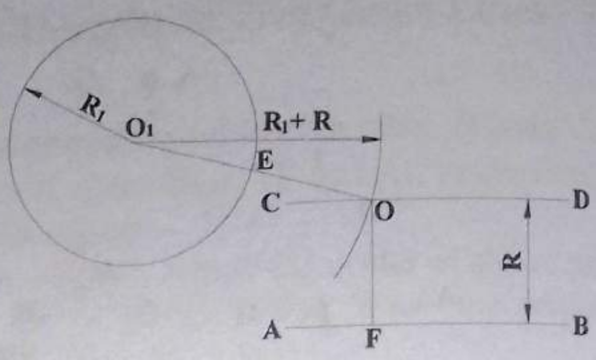


Figure 2.30(d)

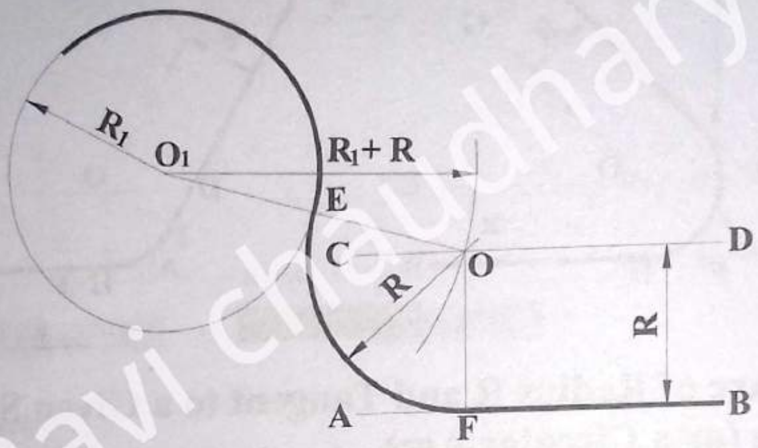


Figure 2.30(e)

(b) Including the Given Circle

- Draw given straight line AB. Mark a point O_1 at a given distance from the given line and draw a given circle with O_1 as center and R_1 as radius. *(Figure 2.31(a))*
- Draw a straight line CD parallel to the line AB and at a distance of R from it. *(Figure 2.31(b))*
- With O_1 as center and $R - R_1$ as radius draw an arc intersecting the line CD at point O, which is the center of the required arc. *(Figure 2.31(c))*
- Join O and O_1 and extend to get point of tangency E on the given circle and drop perpendicular from O to line AB to get the point of tangency F on the given line. *(Figure 2.31(d))*
- With O as center and OE (= OF = R) as radius, draw the required arc. *(Figure 2.31(e))*

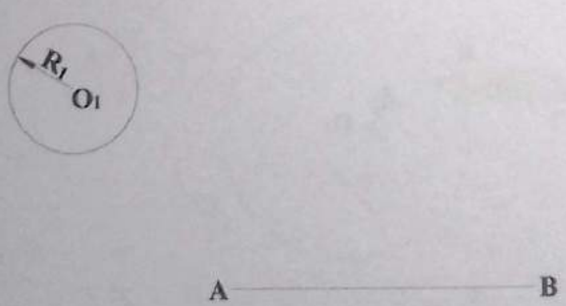
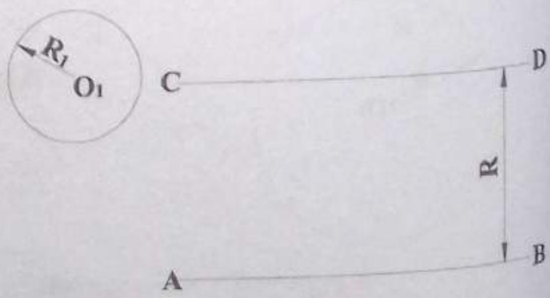


Figure 2.31(a)



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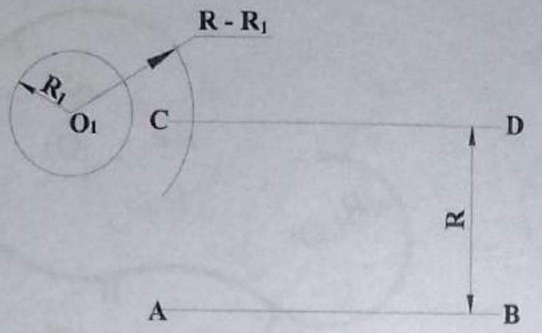

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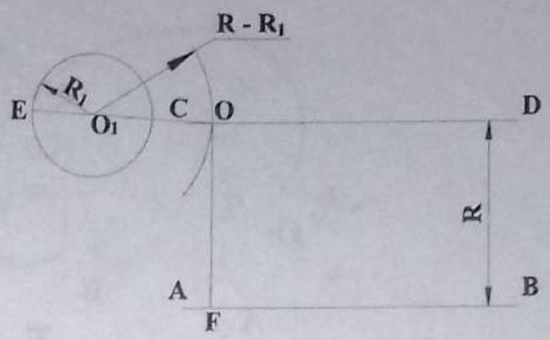

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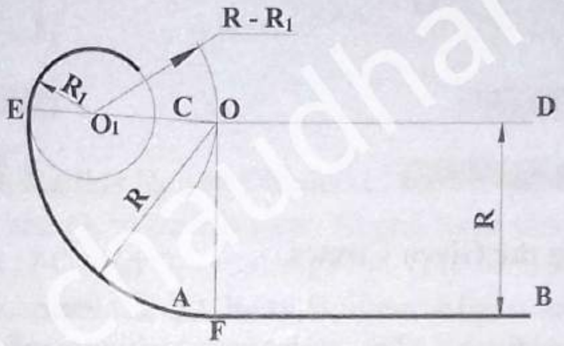


Figure 2.31(e)

2.3.6 To Draw an Arc of Radius R and Tangent to Given Two Circles (or Circular Arcs)

(a) Outside to the Given Circles

- Draw circles with O_1 and O_2 as their centers, R_1 and R_2 as their radii respectively. The relative positions of O_1 and O_2 are also given. *(Figure 2.32(a))*
- Draw arcs with O_1 as center and $R + R_1$ as radius and O_2 as center and $R + R_2$ as radius respectively. Intersection of these arcs gives the center O of the required arc. *(Figure 2.32(b))*
- Join O and O_1 and O and O_2 to get the point of tangencies A and B respectively. *(Figure 2.32(c))*
- Draw the required arc with O as center and OA (= OB = R) as radius. *(Figure 2.32(d))*

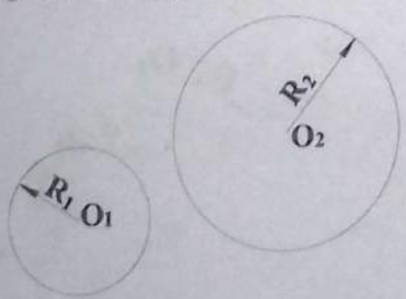


Figure 2.32(a)

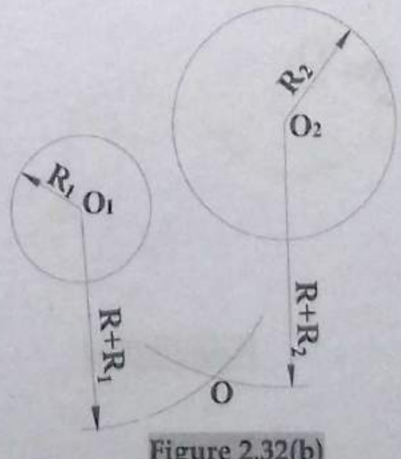
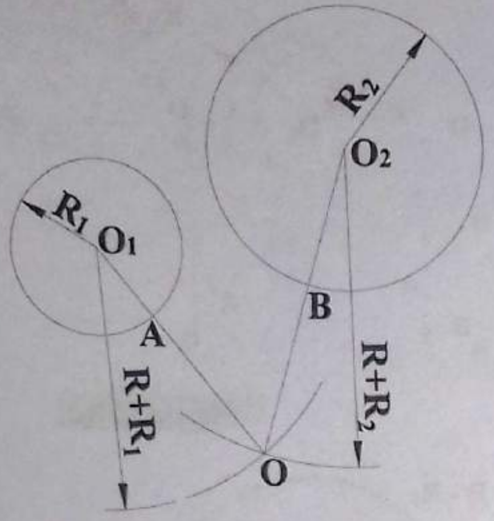


Figure 2.32(b)



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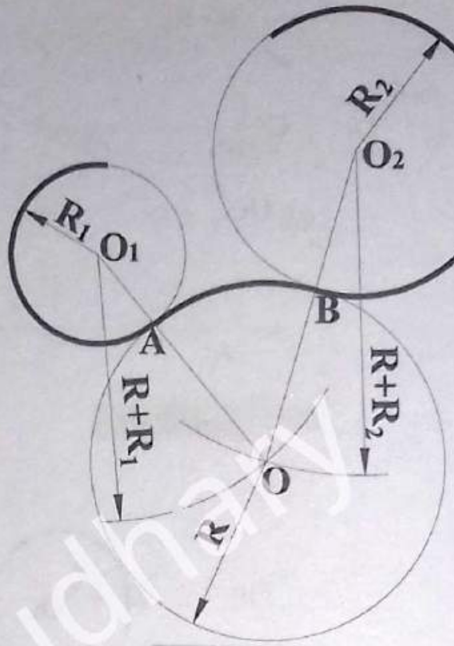


Figure 2.32(d)

(b) Including the Given Circles

- Draw circles with O_1 and O_2 as their centers, R_1 and R_2 as their radii respectively. The relative positions of O_1 and O_2 are also given. *(Figure 2.33(a))*
- Draw arcs with O_1 as center and $R - R_1$ as radius and O_2 as center and $R - R_2$ as radius respectively. Intersection of these arcs gives the center O of the required arc. *(Figure 2.33(b))*
- Join O and O_1 and O and O_2 and extend to get the point of tangencies A and B respectively. *(Figure 2.33(c))*
- Draw the required arc with O as center and OA (= OB = R) as radius. *(Figure 2.33(d))*

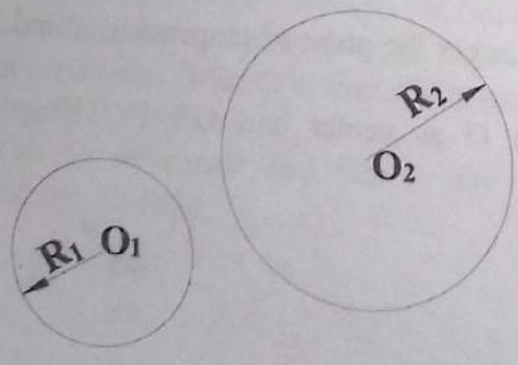
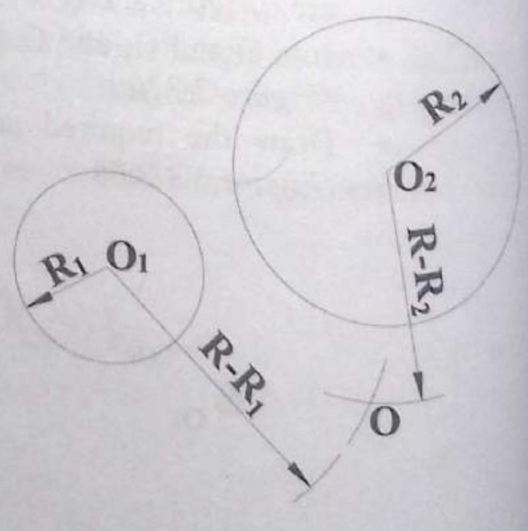


Figure 2.33(a)



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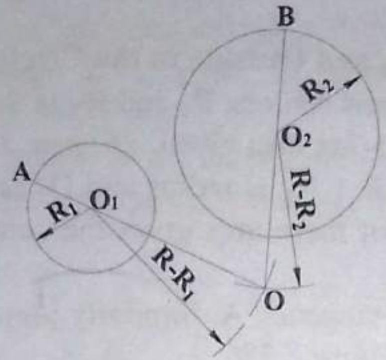


Figure 2.33(c)

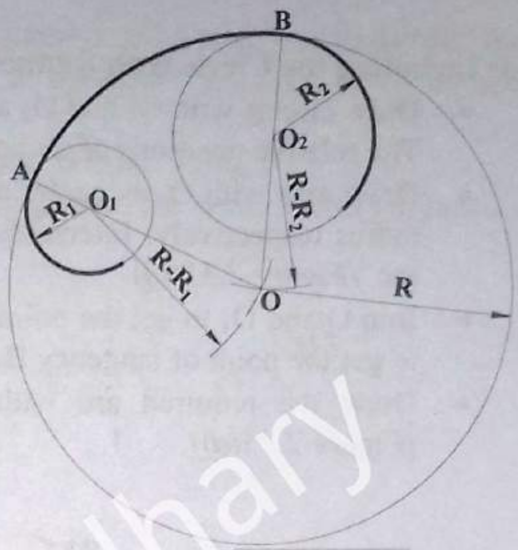


Figure 2.33(d)

(c) Including the Circle with Radius R_1 and Outside to the Circle with Radius R_2

- Draw circles with O_1 and O_2 as their centers, R_1 and R_2 as their radii respectively. The relative positions of O_1 and O_2 are also given. *(Figure 2.34(a))*
- Draw arcs with O_1 as center and R - R_1 as radius and O_2 as center and R + R_2 as radius respectively. Intersection of these arcs gives the center O of the required arc. *(Figure 2.34(b))*
- Join O and O_1 and extend to get the point of tangency A. Similarly join O and O_2 to get the point of tangency B. *(Figure 2.34(c))*
- Draw the required arc with O as center and OA (= OB = R) as radius. *(Figure 2.34(d))*

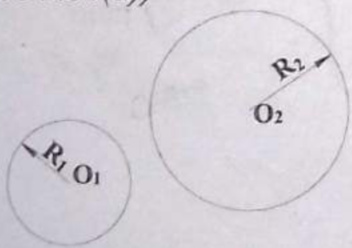
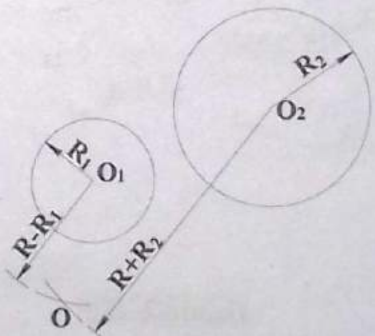


Figure 2.34(a)



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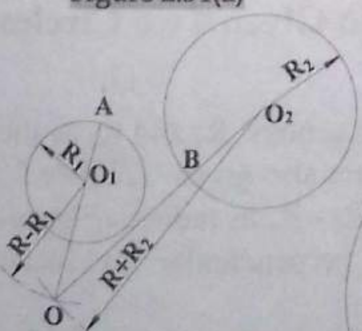

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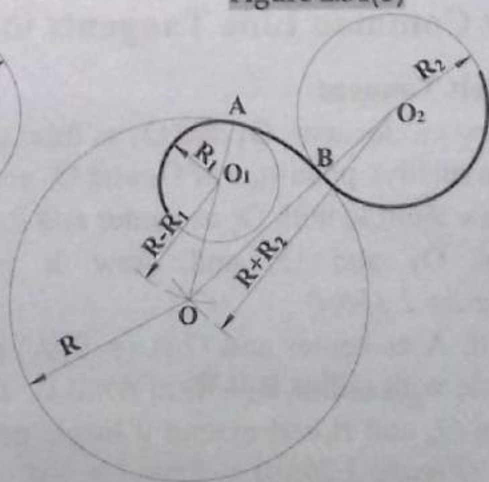


Figure 2.34(d)

(d) Including the Circle with Radius R_2 and Outside to the Circle with Radius R_1

- Draw circles with O_1 and O_2 as their centers, R_1 and R_2 as their radii respectively. The relative positions of O_1 and O_2 are also given. *(Figure 2.35(a))*
- Draw arcs with O_1 as center and $R + R_1$ as radius and O_2 as center and $R - R_2$ as radius respectively. Intersection of these arcs gives the center O of the required arc. *(Figure 2.35(b))*
- Join O and O_1 to get the point of tangency A. Similarly join O and O_2 and extend to get the point of tangency B. *(Figure 2.35(c))*
- Draw the required arc with O as center and OA (= OB = R) as radius. *(Figure 2.35(d))*

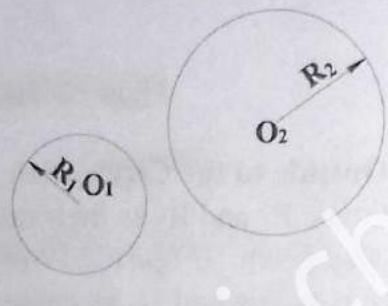


Figure 2.35(a)

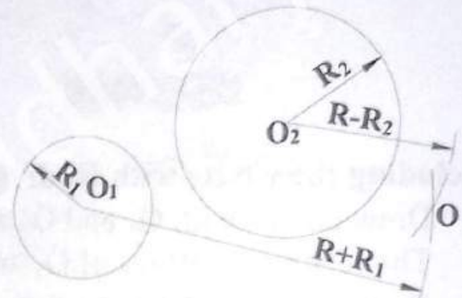


Figure 2.35(b)

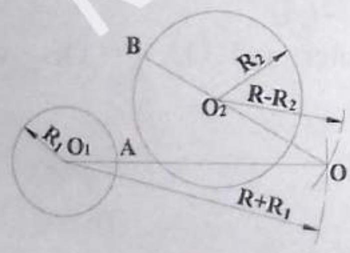


Figure 2.35(c)

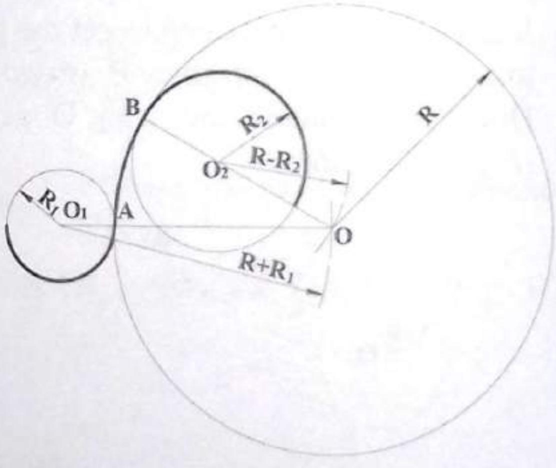


Figure 2.35(d)

2.3.7 To Draw Common Line Tangents to Given Two Circles

(a) Open Belt Tangent

- Draw circles with O_1 and O_2 as their centers, R_1 and R_2 as their radii respectively. The relative positions of O_1 and O_2 are also given. *(Figure 2.36(a))*
- Draw a circle with O_2 as center and $R_2 - R_1$ as radius. *(Figure 2.36(b))*
- Join O_1 and O_2 and draw it perpendicular to locate its midpoint A. *(Figure 2.36(c))*
- With A as center and O_1A (= O_2A) as radius draw a circle which intersects the circle with radius $R_2 - R_1$ at point B. *(Figure 2.36(d))*
- Join O_2 and B and extend it to get point of tangency T_2 on the circle with radius R_2. *(Figure 2.36(e))*

- Draw a straight line passing through O_1 and parallel to O_2T_2 intersecting the circle with radius R_1 at point T_1, which is the other required point of tangency. *(Figure 2.36(f))*
- Joint T_1 and T_2 to get the required tangent *(Figure 2.36(g))*
- Repeat the same procedure to get the tangent on the bottom side. *(Figure 2.36(h))*

Figure 2.36(a)

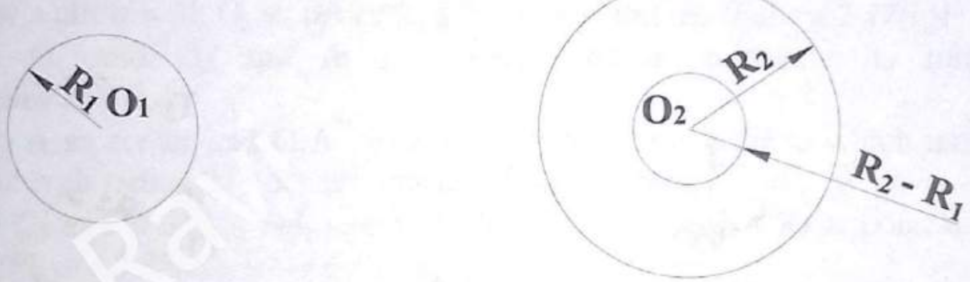


Figure 2.36(b)

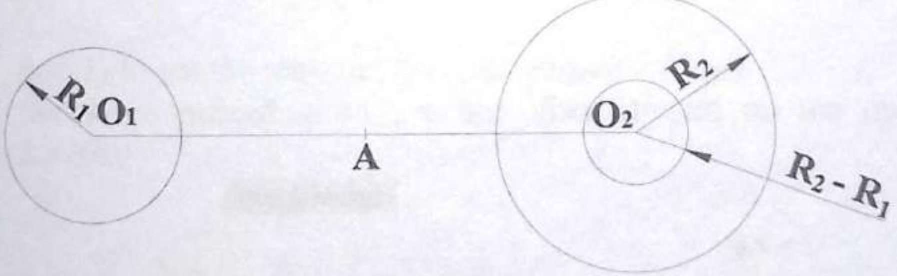


Figure 2.36(c)

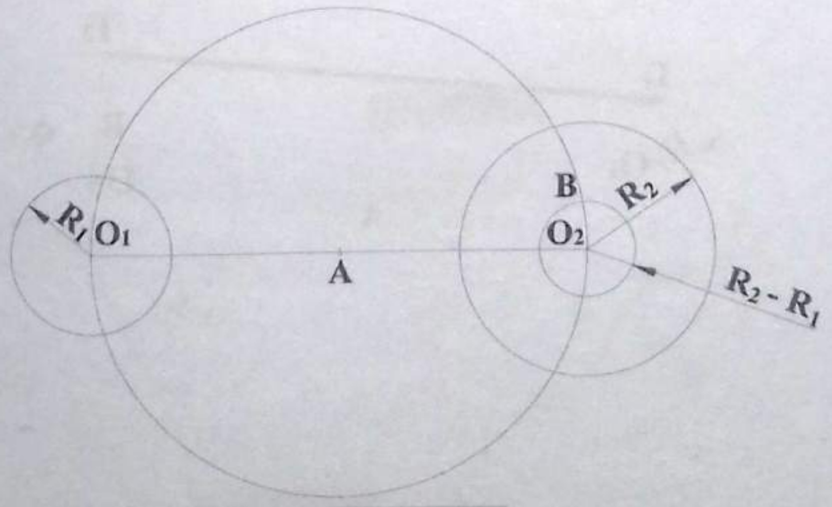


Figure 2.36(d)

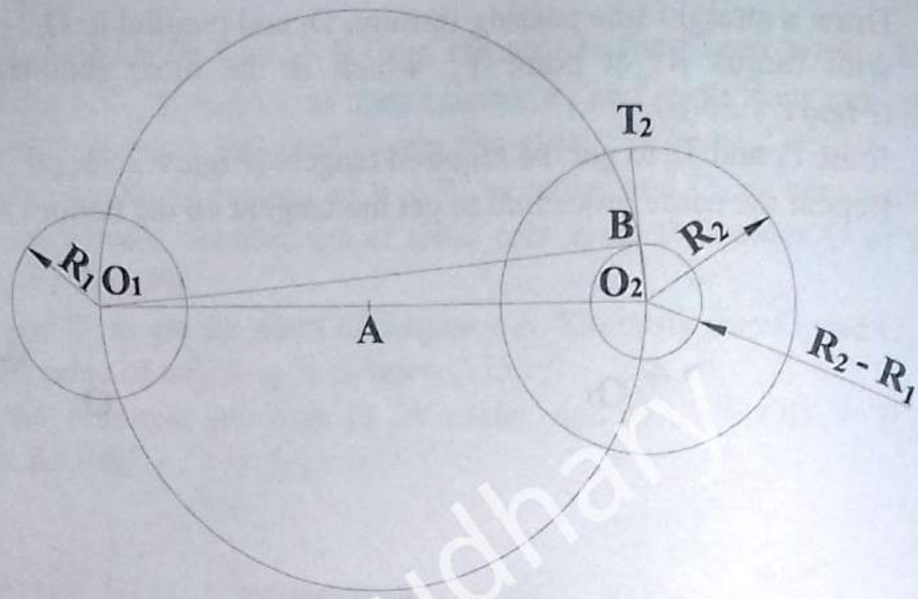


Figure 2.36(e)

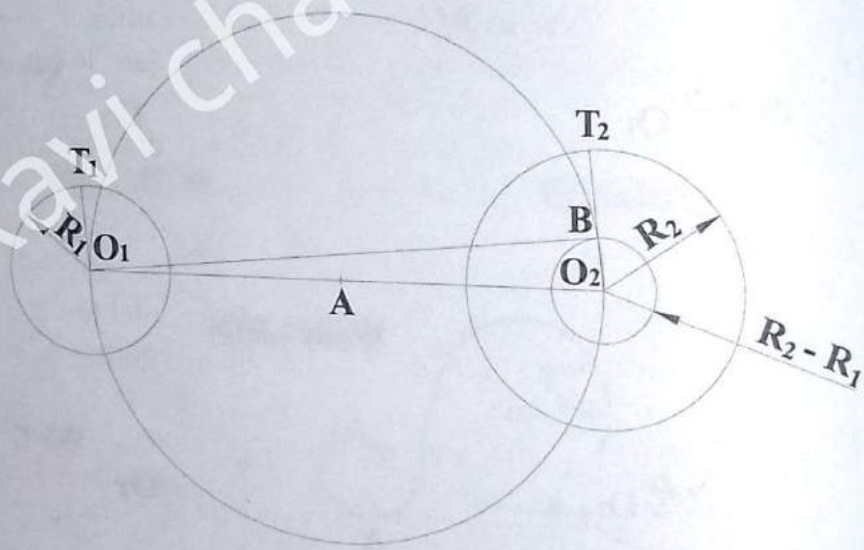


Figure 2.36(f)

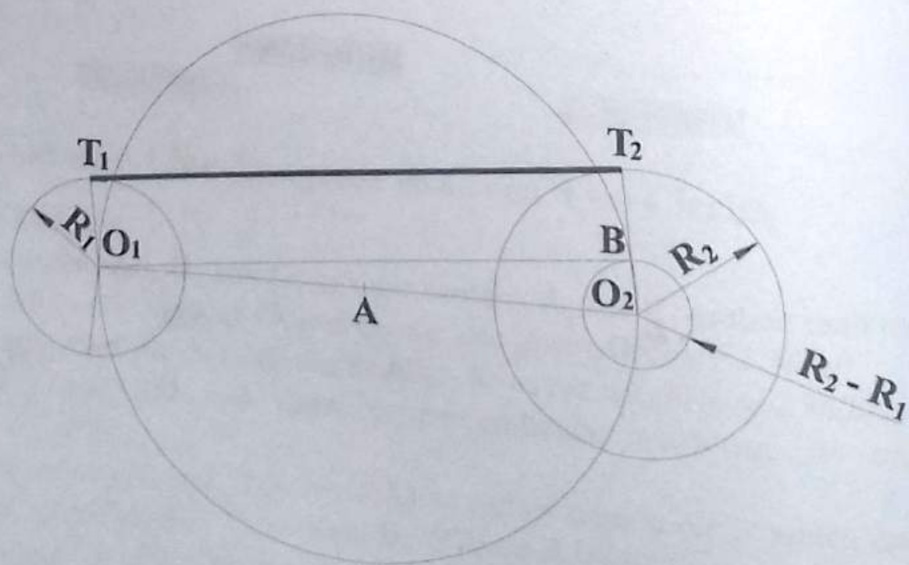


Figure 2.36(g)

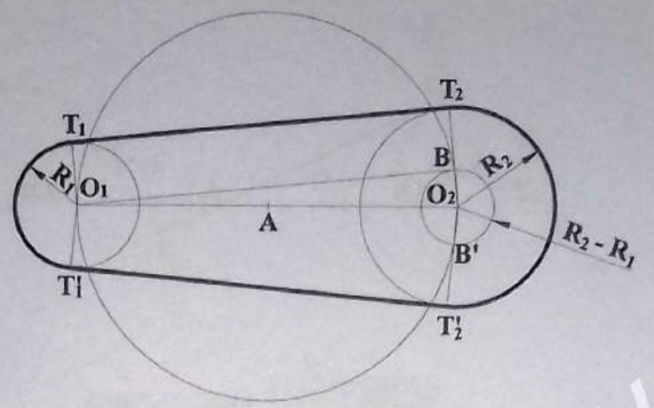


Figure 2.36(h)

(b) Cross Belt Tangent

- Draw circles with O_1 and O_2 as their centers, R_1 and R_2 as their radii respectively. The relative positions of O_1 and O_2 are also given. *(Figure 2.37(a))*
- Draw a circle with O_2 as center and $R_1 + R_2$ as radius. *(Figure 2.37(b))*
- Join O_1 and O_2 and draw it perpendicular to locate its midpoint A. *(Figure 2.37(c))*
- With A as center and O_1A (= O_2A) as radius draw a circle which intersects the circle with radius $R_1 + R_2$ at point B. *(Figure 2.36(d))*
- Join O_2 and B and which intersect the circle with radius R_2 at point of tangency T_2. *(Figure 2.37(e))*
- Draw a straight line passing through O_1 and parallel to O_2T_2 intersecting the circle with radius R_1 at point T_1, which is the other required point of tangency. *(Figure 2.37(f))*
- Joint T_1 and T_2 to get the required tangent. *(Figure 2.37(g))*
- Repeat the same procedure to get the other tangent on the opposite side. *(Figure 2.37(h))*

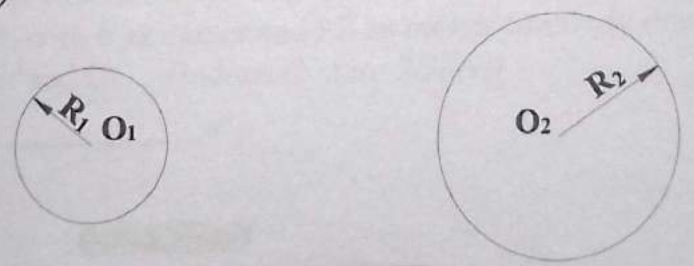


Figure 2.37(a)

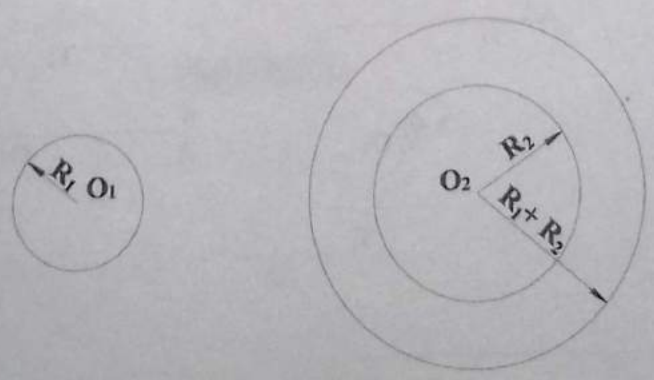


Figure 2.37(b)

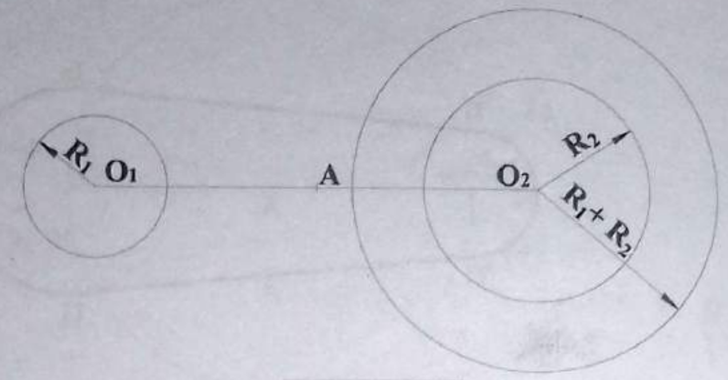


Figure 2.37(c)

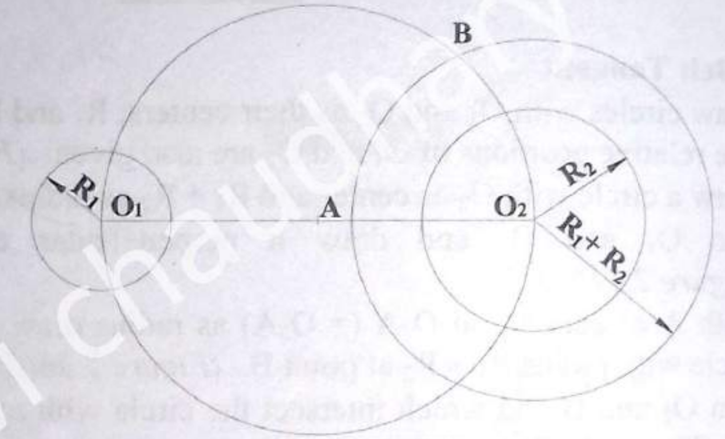


Figure 2.37(d)

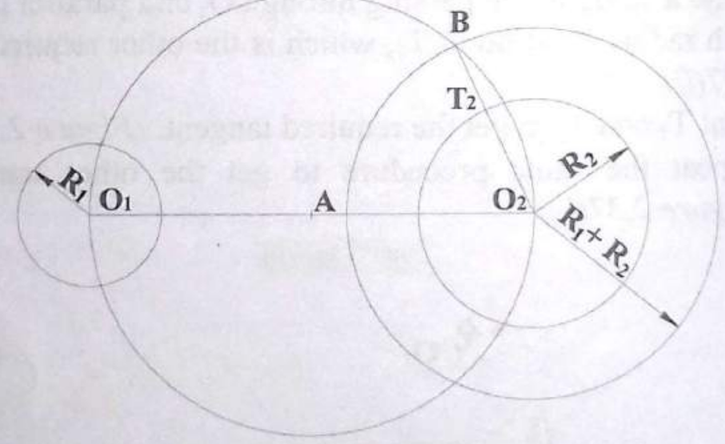


Figure 2.37(e)

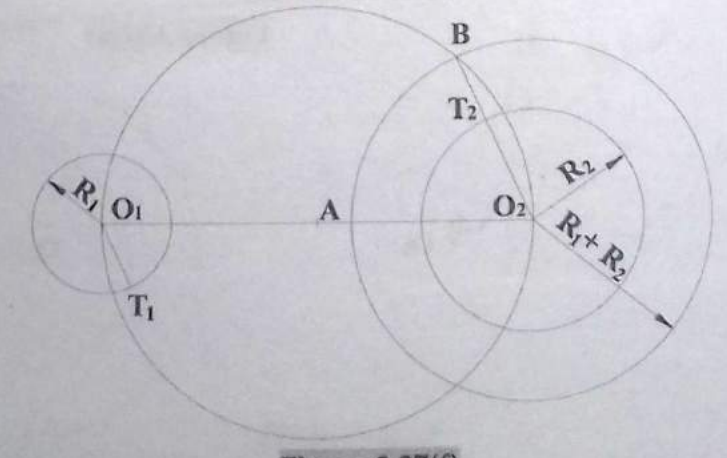


Figure 2.37(f)

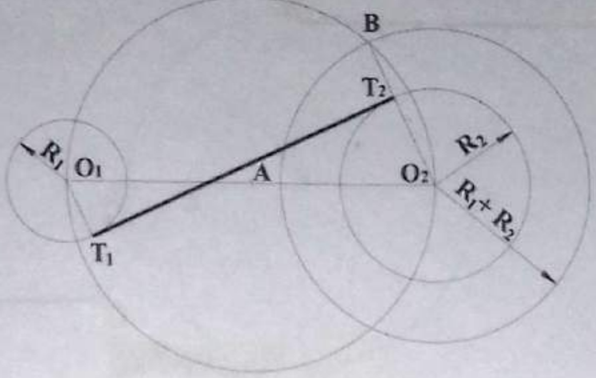


Figure 2.37(g)

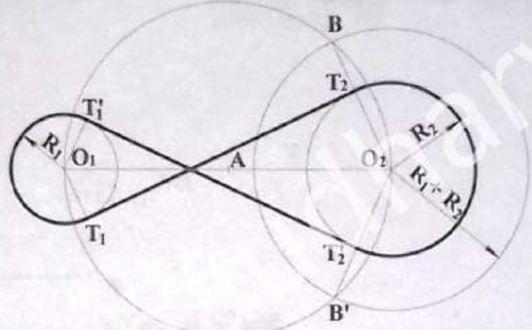


Figure 2.37(h)

2.3.8 To Draw a Reverse Curve (Ogee Curve)

- AB and CD are the given lines. *(Figure 2.38(a))*
- Join B and C and take any point E on the line BC. *(Figure 2.38(b))*
- Draw perpendicular bisectors of line segments BE and EC. *(Figure 2.38(c))*
- Draw perpendicular from point B such that it intersects the perpendicular bisector of BE at point F. Similarly draw perpendicular from point C such that it intersects the perpendicular bisector of EC at point G. *(Figure 2.38(d))*
- Draw an arc BE with F as center and FB as radius. Similarly draw another arc EC with G as center and GC as radius. *(Figure 2.38(e))*

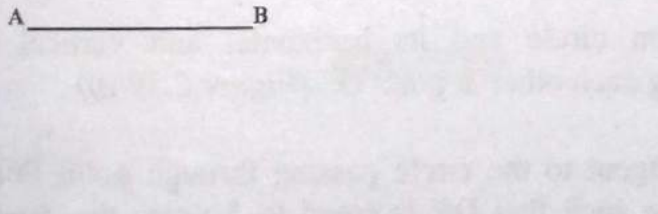


Figure 2.38(a)

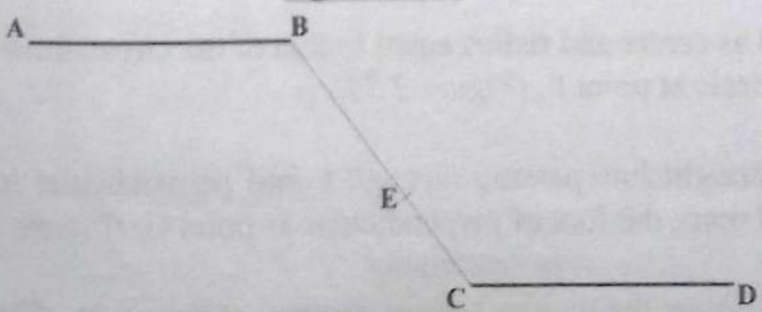


Figure 2.38(b)

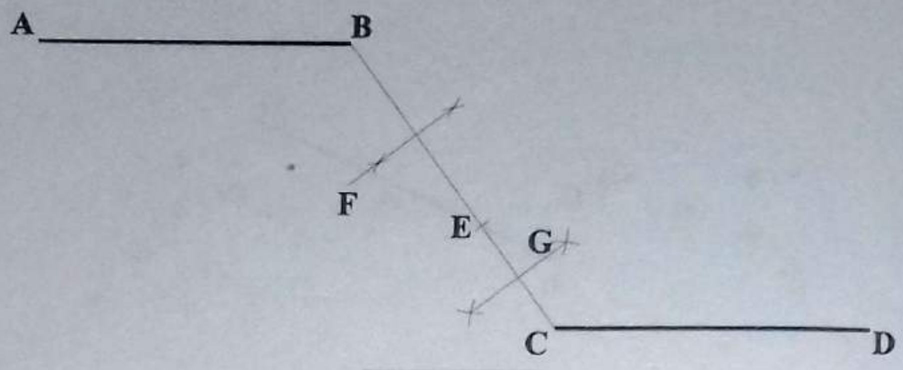


Figure 2.38(c)

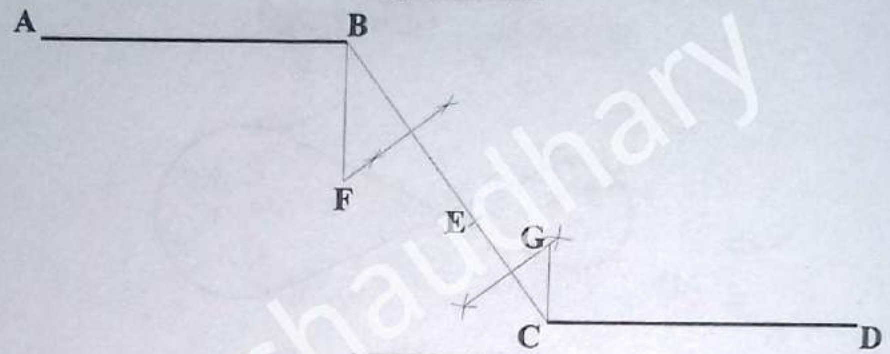


Figure 2.38(d)

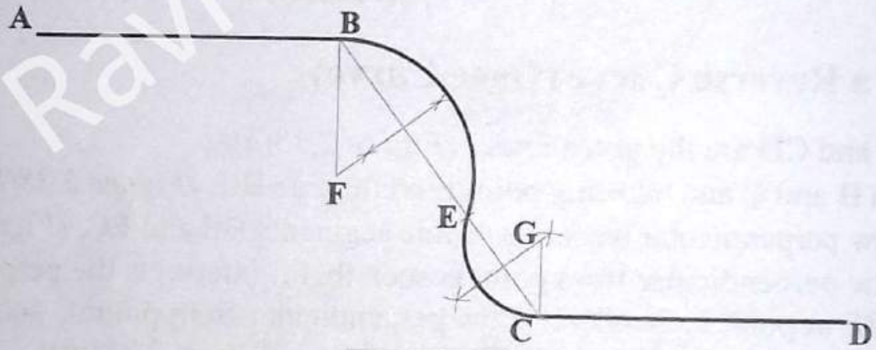


Figure 2.38(e)

2.3.9 To Determine the Circumference of a Circle

- Draw given circle and its horizontal and vertical diameters AB and CD intersecting each other at point O. *(Figure 2.39(a))*
- Draw a tangent to the circle passing through point D and mark point E on the tangent line such that DE is equal to 3 times the diameter of the given circle. *(Figure 2.39(b))*
- With B as center and radius equal to that of the circle, draw an arc intersecting the given circle at point F. *(Figure 2.39(c))*
- Draw straight line passing through F and perpendicular to the vertical diameter CD and mark the foot of perpendicular as point G. *(Figure 2.39(d))*
- Join GE to get the required circumference of the circle. *(Figure 2.39(e))*

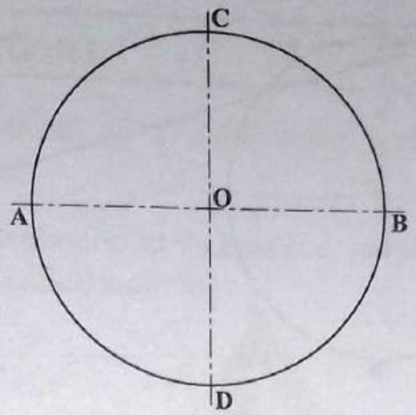


Figure 2.39(a)

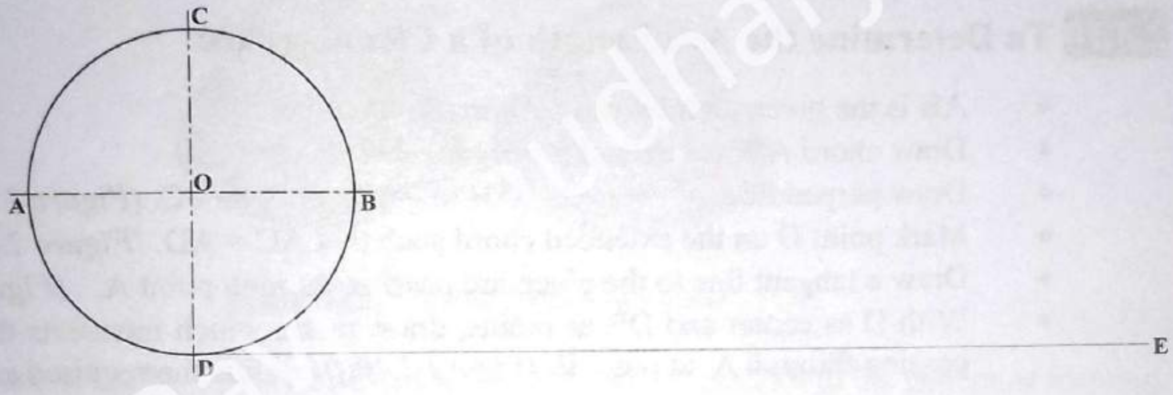


Figure 2.39(b)

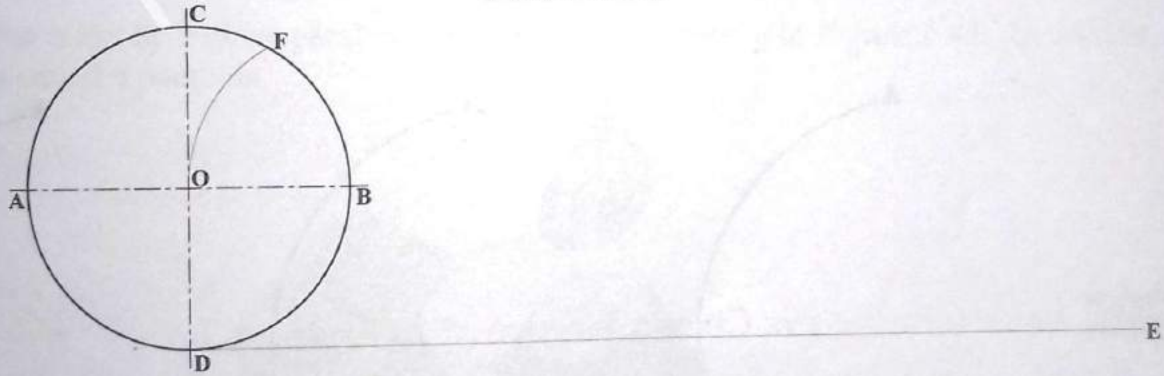


Figure 2.39(c)

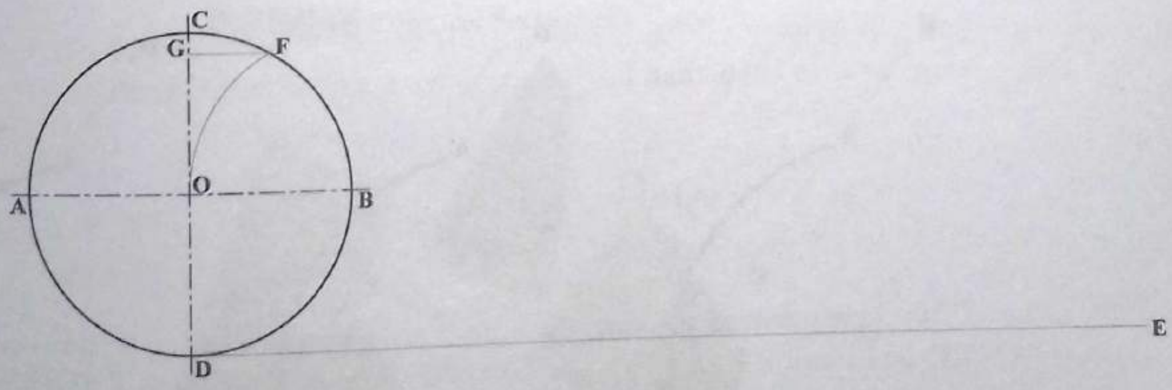


Figure 2.39(d)

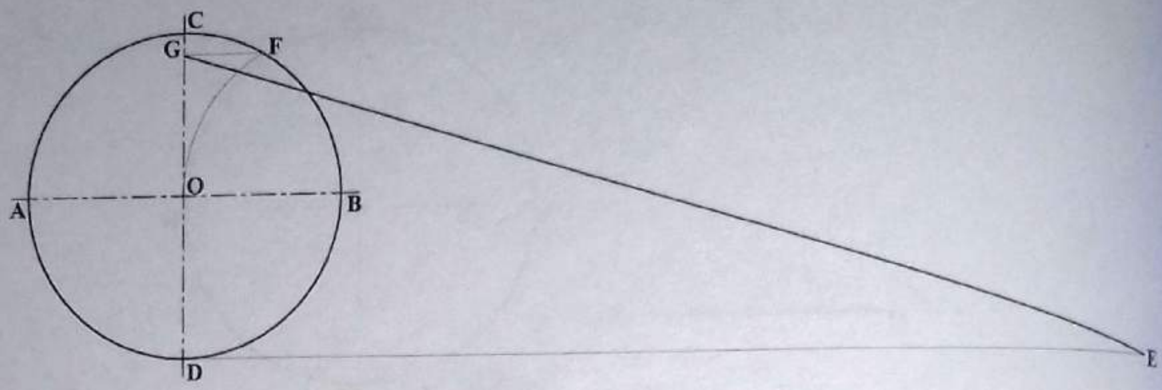


Figure 2.39(e)

2.3.10 To Determine the Arc Length of a Circular Arc

- AB is the given circular arc. *(Figure 2.40(a))*
- Draw chord AB and extend it. *(Figure 2.40(b))*
- Draw perpendicular bisector of AB to get its midpoint C. *(Figure 2.40(c))*
- Mark point D on the extended chord such that AC = AD. *(Figure 2.40(d))*
- Draw a tangent line to the given arc passing through point A. *(Figure 2.40(e))*
- With D as center and DB as radius, draw an arc which intersects the tangent line passing through A at point E. *(Figure 2.40(f))*. AE is the required arc length.

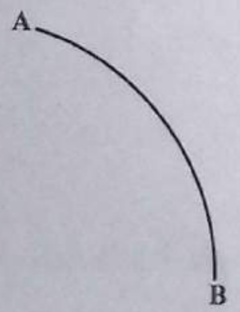


Figure 2.40(a)

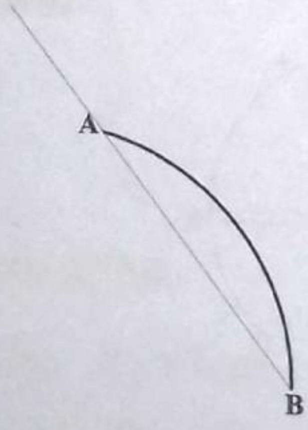


Figure 2.40(b)

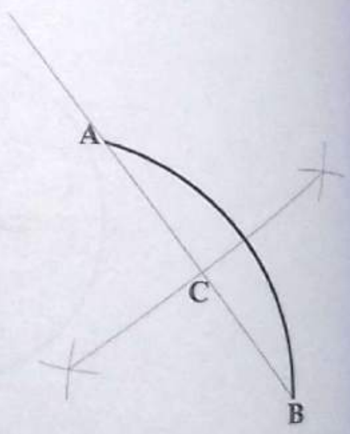


Figure 2.40(c)

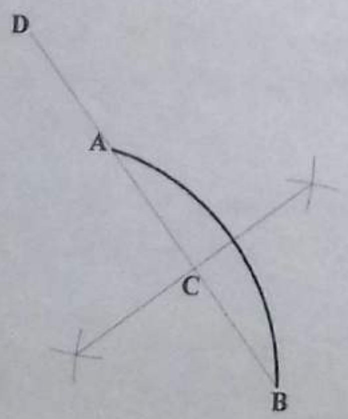


Figure 2.40(d)

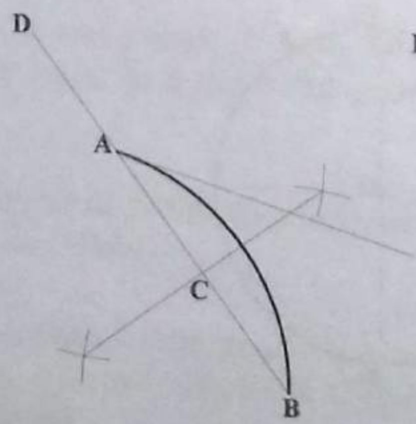


Figure 2.40(e)

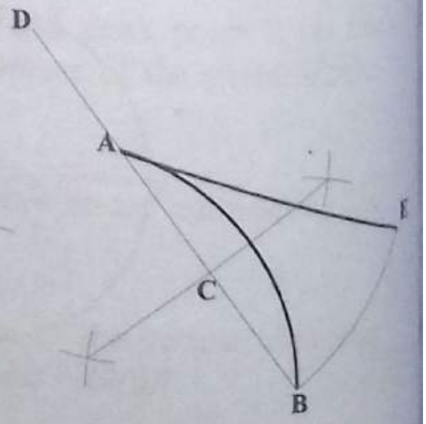


Figure 2.40(f)

2.4 Construction of Conic Sections

Sections obtained by the intersection of a solid cone and planes having different inclinations are called conic sections.

When a cone is cut by a plane perpendicular to its axis (i.e. parallel to its base) as shown in *Figure 2.41*, the section obtained is called a circle.

Figure 2.41: Circular Section of a Cone

When a cone is cut by a plane inclined to its base and not parallel to the generator forming a closed curve as shown in *Figure 2.42*, the section obtained is called an ellipse.

When a cone is cut by a plane parallel to the generator as shown in *Figure 2.43*, the section obtained is called a parabola.

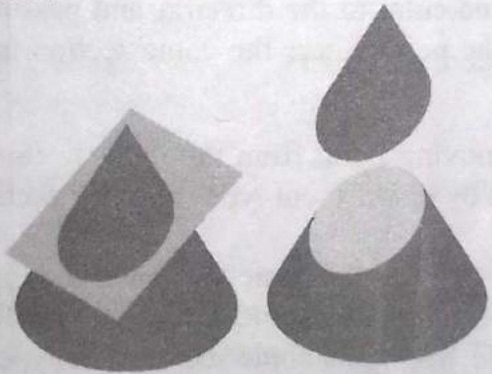


Figure 2.42: Elliptical Section of a Cone

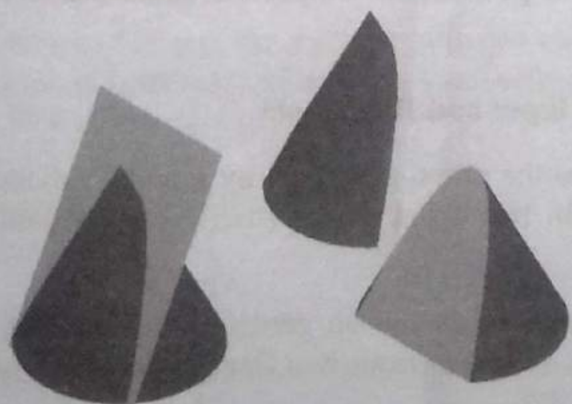


Figure 2.43: Parabolic Section of a Cone

When a cone is cut by a plane having inclination greater than that of the generator as shown in *Figure 2.44*, the section obtained is called a hyperbola. When the cutting plane is perpendicular to the base of the cone, the section obtained is called a rectangular hyperbola.

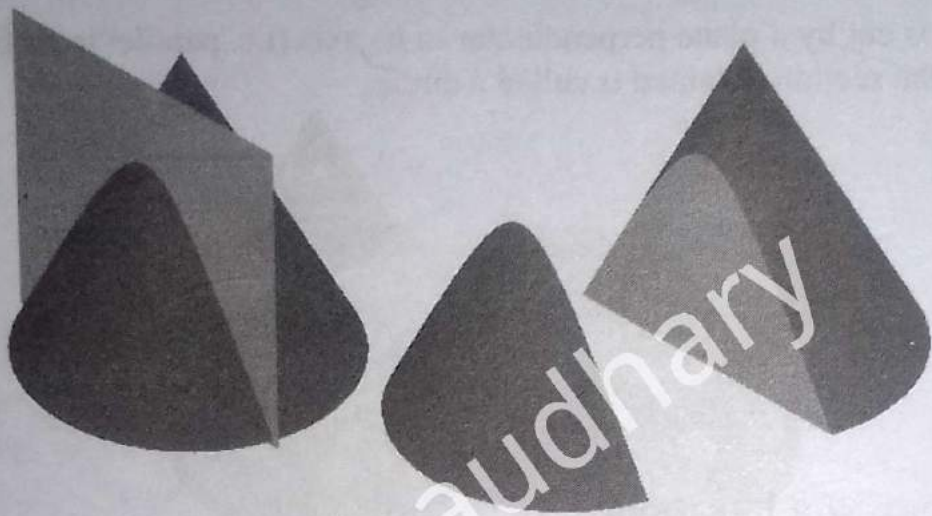


Figure 2.43: Hyperbolic Section of a Cone

2.4.1 Mathematical Definitions of Conic Sections

The locus of point moving in a plane in such a way that the ratio of its distances from a fixed point and a fixed straight line is always constant is called a conic section.

The fixed point is called the focus and the straight line is called the directrix of the conic section. The straight line perpendicular to the directrix and passing through the focus is called the axis of the conic section. The point where the conic section intersects the axis is called the vertex of the conic section.

The ratio of the distance of a moving point from the focus to its distance from the directrix is called eccentricity. It is denoted by e. Different types of conic section can be defined in terms of eccentricity as

If	$e < 1$	the conic section is an ellipse,
If	$e = 1$	the conic section is a parabola, and
If	$e > 1$	the conic section is a hyperbola.

Figure 2.44 shows the relative positions of ellipse, parabola and hyperbola with reference to a common directrix and focus.

Alternative Definitions of Ellipse and Hyperbola

Ellipse can also be defined as the curve generated by a point moving such that at any position the sum of its distances from two fixed points (foci) is a constant (equal to the major axis length).

Hyperbola can also be defined as the curve generated by a point moving such that at any position the difference of its distances from two fixed points (foci) is a constant (equal to the transverse axis distance).

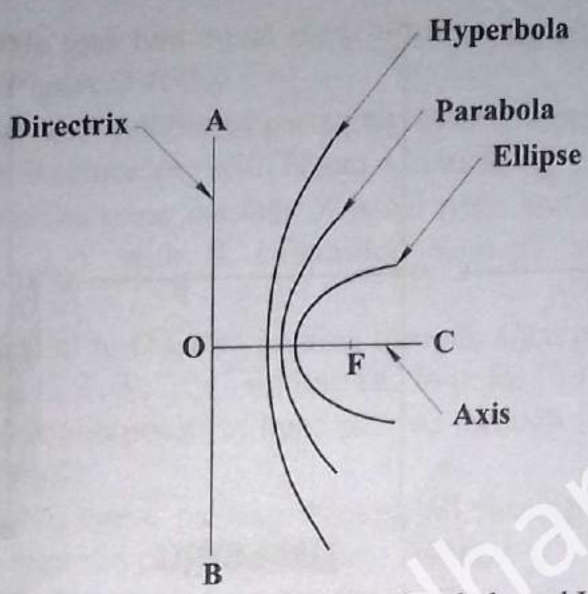


Figure 2.44: Relative Positions of Ellipse, Parabola and Hyperbola

2.4.2 Construction of a Parabola

(a) Definition Method (Given Directrix and Focus)

- Draw a straight line AB as the directrix of the parabola and draw another line OC perpendicular to AB and passing through the midpoint of the AB as an axis of the parabola. *(Figure 2.45(a))*

- Locate the given focus F on the axis OC according to the given distance. *(Figure 2.45(b))*

- Draw perpendicular bisector of OF to locate its midpoint V, which is the vertex of the required parabola. *(Figure 2.45(c))*

- Draw any number of lines 1, 2, 3, ….. parallel to the directrix AB and right to the vertex V. *(Figure 2.45(d))*

- With F as a center and the distance between the line 1 and the directrix as radius, draw arcs on both sides of the axis intersecting the line 1 at points P_1 and P_1'. *(Figure 2.45(e))*

- Again with F as a center and the distance between the line 2 and the directrix as radius, draw arcs on both sides of the axis intersecting the line 2 at points P_2 and P_2'. *(Figure 2.45(f))*

- In the similar way, determine points P_3, P_3', P_4, P_4', ….. and so on. *(Figure 2.45(g))*

- Join all the points V, P_1, P_2, P_3, P_4, ……… and P_1', P_2', P_3', P_4', ….. by a smooth curve to get the required parabola. *(Figure 2.45(h))*

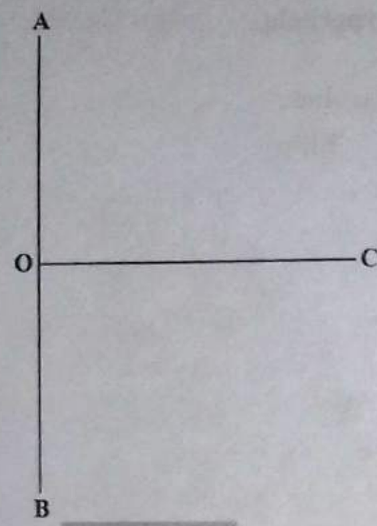


Figure 2.45(a)

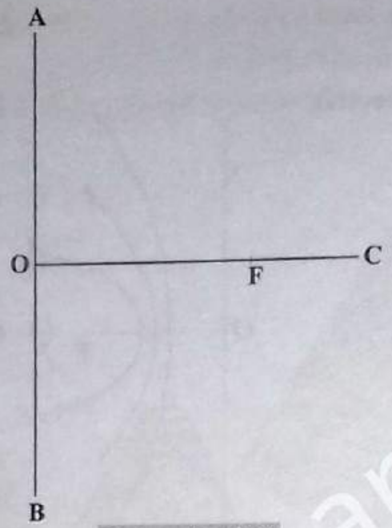


Figure 2.45(b)

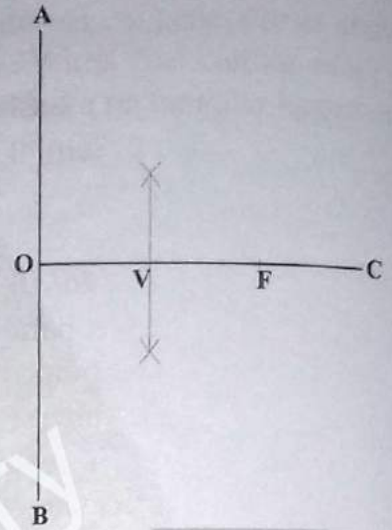


Figure 2.45(c)

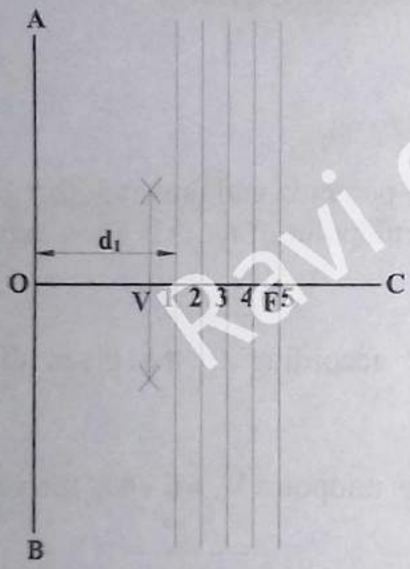


Figure 2.45(d)

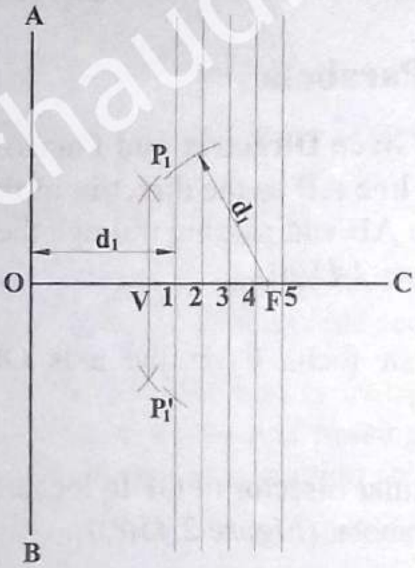
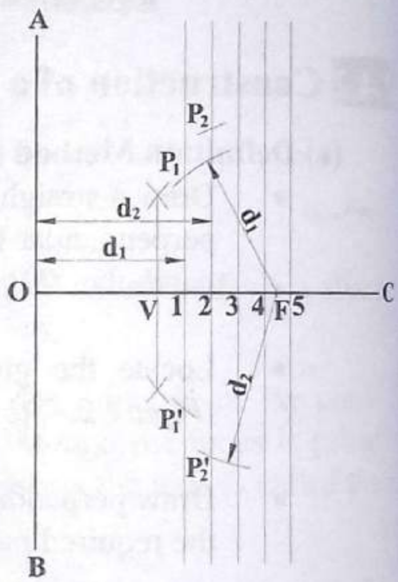


Figure 2.45(e)



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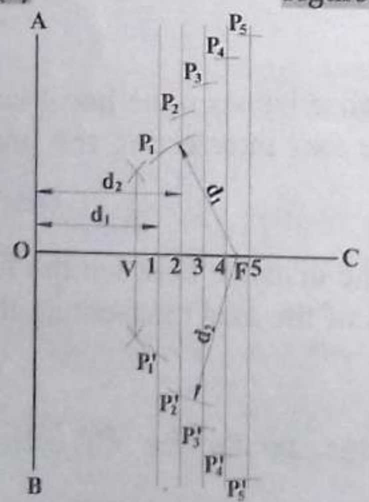


Figure 2.45(g)

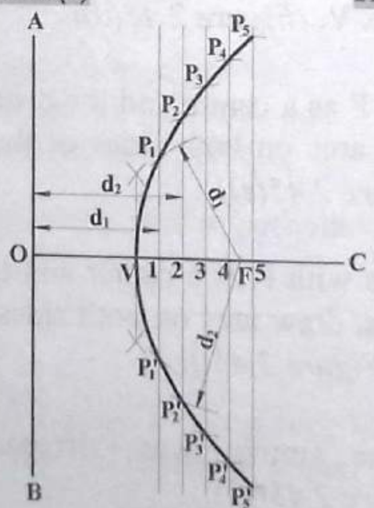


Figure 2.45(h)

(b) Rectangle Method (Given Double Ordinate and Axis Length)

- Draw a rectangle ABCD with its width as the given axis length (AB = DC) and height as the given double ordinate (AD = BC). *(Figure 2.46(a))*

- Divide rectangle into two equal parts by drawing perpendicular bisector OX of AD (or BC). *(Figure 2.46(b))*
- Divide OD into any number of parts (say 6) and name the dividing points as 0, 1, 2, 3, … 6 with 0 coinciding with O and 6 coinciding with D. *(Figure 2.46(c))*
- Divide DC into the same number of equal parts and name the dividing points as 0', 1', 2' ……. 6' with 0' coinciding with D and 6' coinciding with C. *(Figure 2.46(d))*
- Draw lines parallel to OX and passing through each point 1, 2, 3 …. on line OD. Join each point 1', 2', 3', ……. on line DC to point O. *(Figure 2.46(e))*
- Mark the intersection points of lines passing through 1 and 1', 2 and 2', …. and so on. *(Figure 2.46(f))*
- Draw the smooth curve passing through all these points to get the upper half portion of the required parabola. *(Figure 2.46(g))*
- Repeat the similar procedure to the bottom half portion of the parabola. *(Figure 2.46(h))*

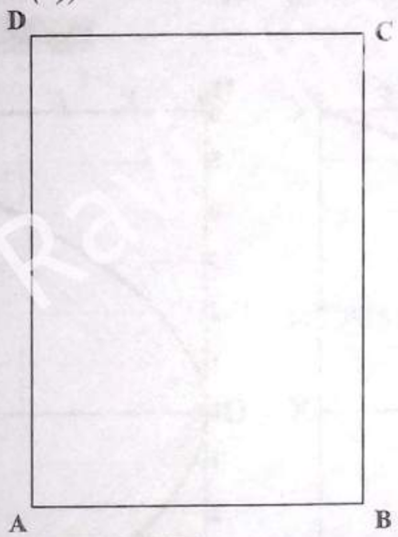
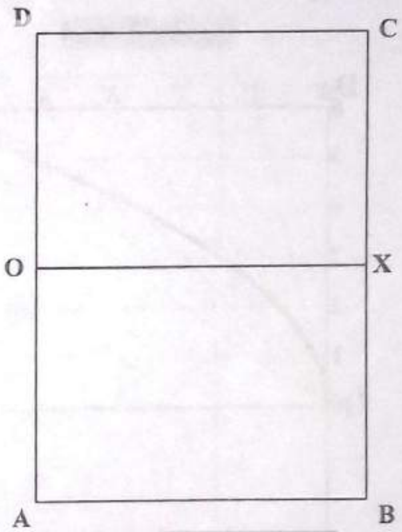


Figure 2.46(a)



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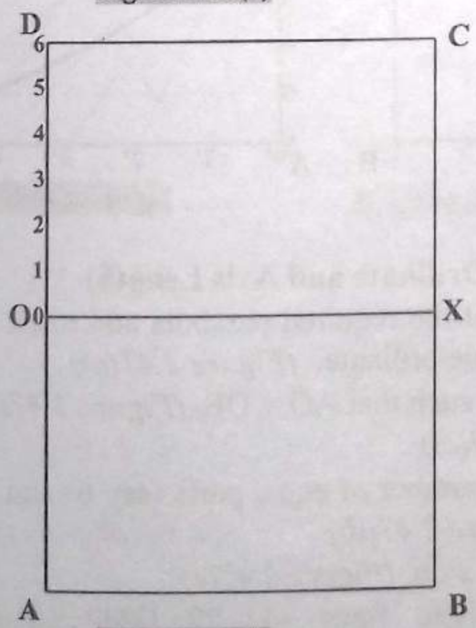

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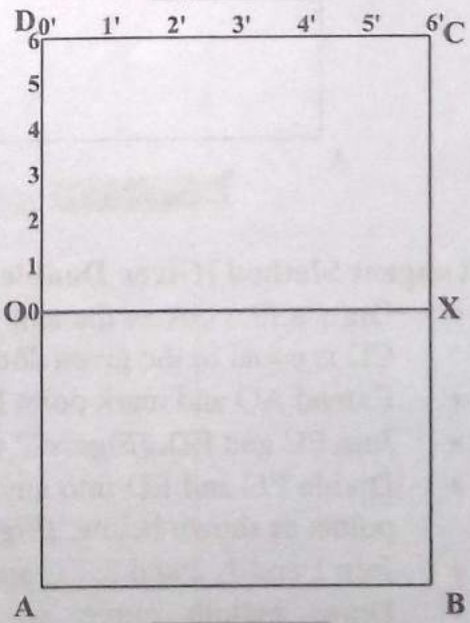

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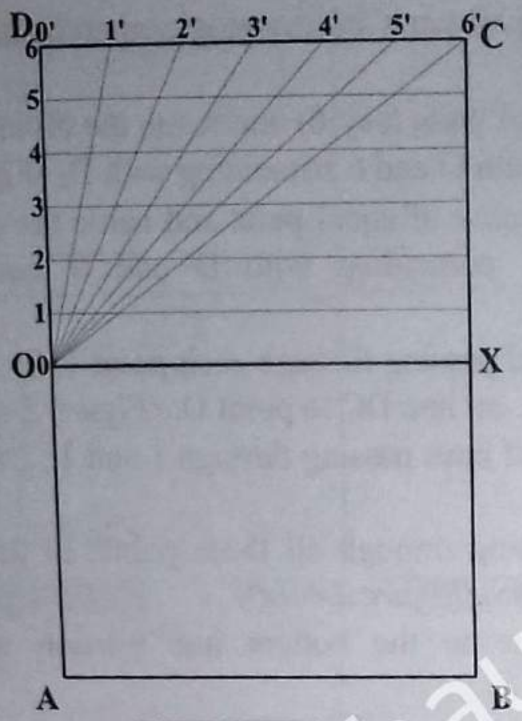

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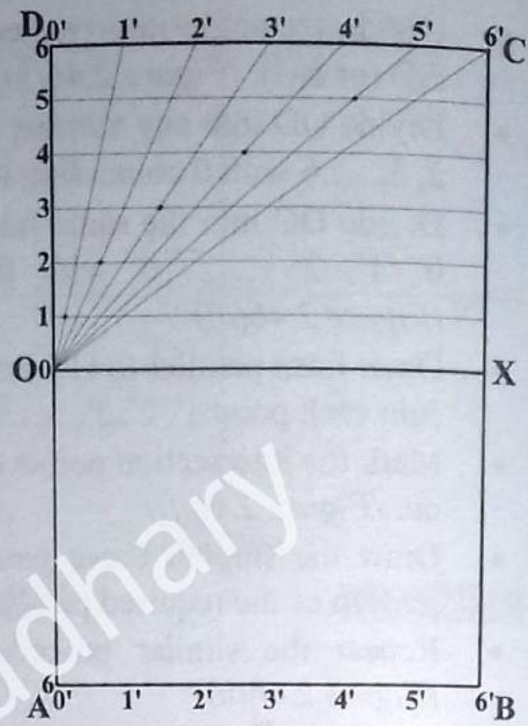

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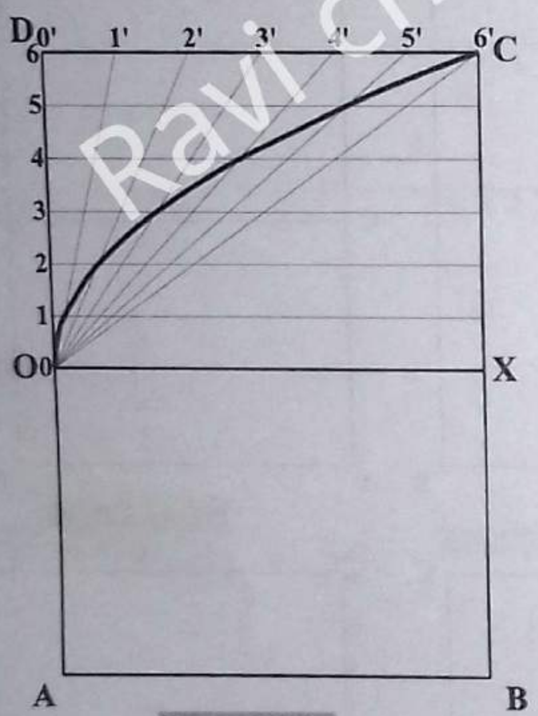

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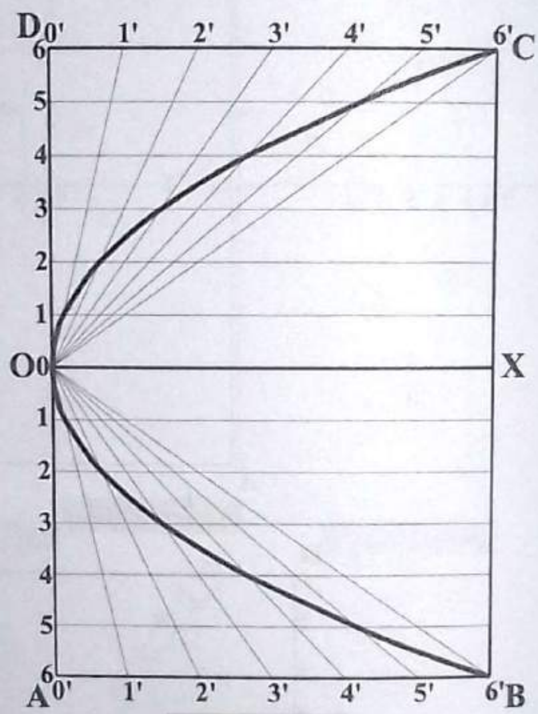


Figure 2.46(h)

(c) Tangent Method (Given Double Ordinate and Axis Length)

- Draw a line OA as the axis of the required parabola and mark C and D such that CD is equal to the given double ordinate. *(Figure 2.47(a))*
- Extend AO and mark point E such that AO = OE. *(Figure 2.47(b))*
- Join EC and ED. *(Figure 2.47(c))*
- Divide EC and ED into any number of equal parts (say 6) and name the dividing points as shown below. *(Figure 2.47(d))*
- Join 1 and 1, 2 and 2, …. and s on. *(Figure 2.47(e))*
- Draw smooth curve all these lines 11, 22, 33, ….. as tangent lines. *(Figure 2.47(f))*

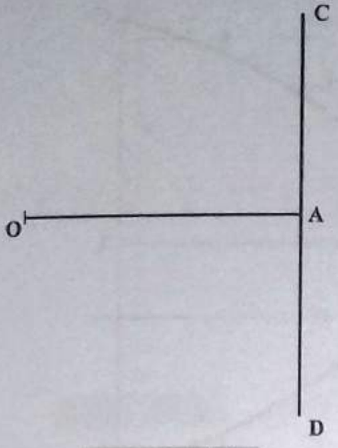
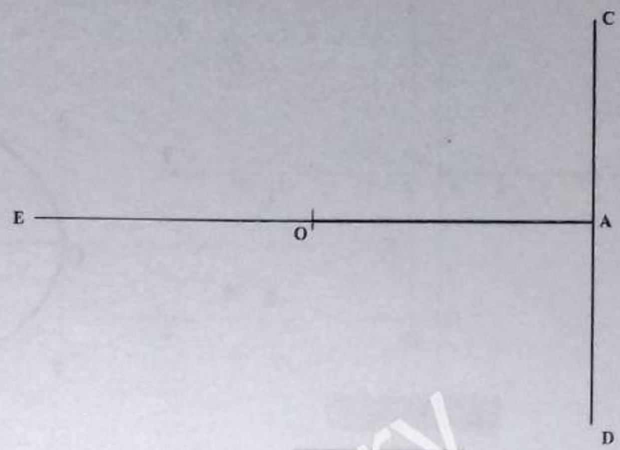


Figure 2.47(a)



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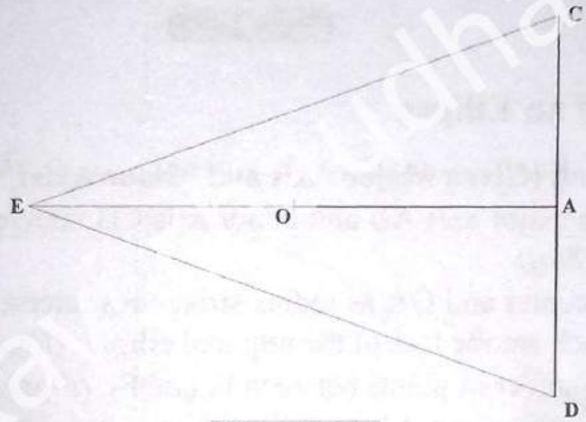

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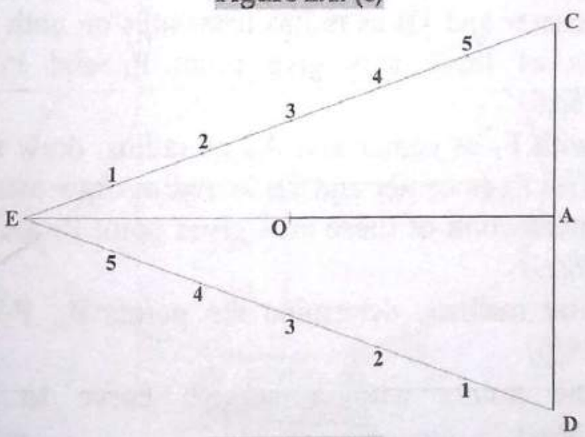


Figure 2.47(d)

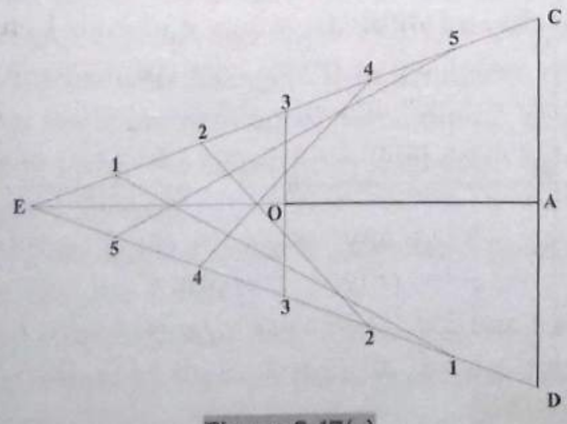


Figure 2.47(e)

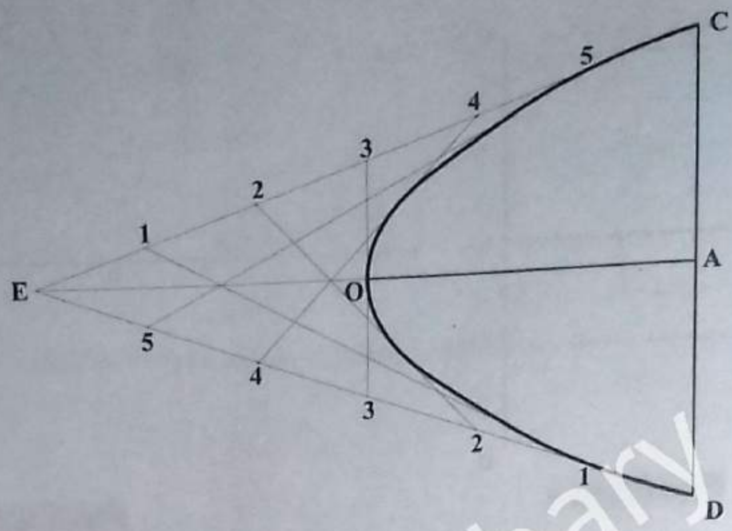


Figure 2.47(i)

2.4.3 Construction of an Ellipse

(a) Definition Method (Given Major Axis and Minor Axis)

- Draw given major axis AB and minor axis CD intersecting each other at point O. *(Figure 2.48(a))*
- With C as center and OA as radius strike arcs intersecting major axis AB and F_1 and F_2, which are the foci of the required ellipse. *(Figure 2.48(b))*
- Mark any number of points between F_1 and F_2. *(Figure 2.48(c))*
- With F_1 as center and A1 as radius, draw arcs on both sides of major axis AB. With F_2 as center and 1B as radius draw arcs on both sides of the major axis AB. Intersections of these arcs give point P_1 and P_1' of the required ellipse. *(Figure 2.48(d))*
- Similarly, with F_1 as center and A2 as radius, draw arcs on both sides of major axis AB. With F_2 as center and 2B as radius draw arcs on both sides of the major axis AB. Intersection of these arcs gives point P_2 and P_2' of the required ellipse. *(Figure 2.48(e))*
- In the similar manner, determine the points P_2, P_2', P_3, P_3', …… and so on. *(Figure 2.48(f))*
- Join all the points with a smooth curve to get the required ellipse. *(Figure 2.48(g))*

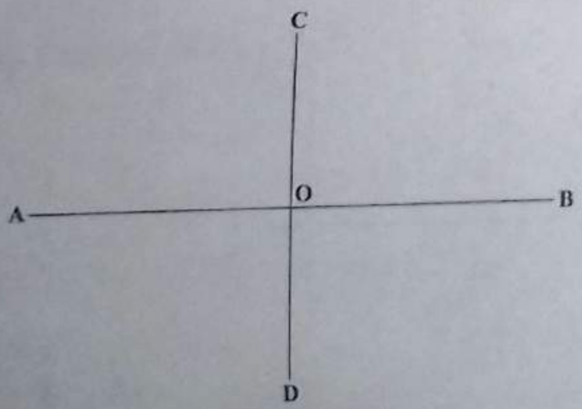


Figure 2.48(a)

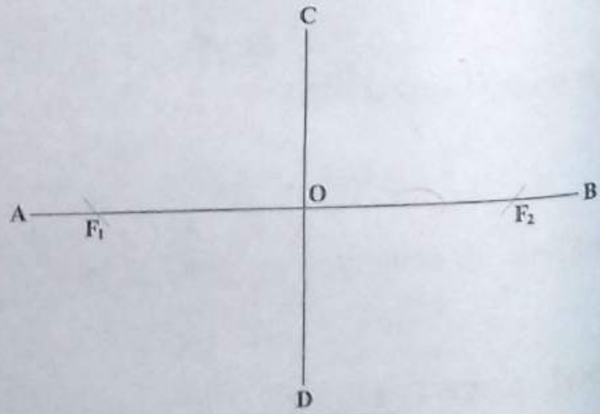


Figure 2.48(b)

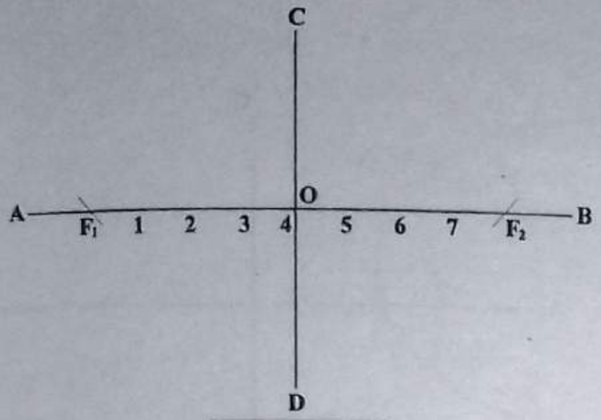


Figure 2.48(c)

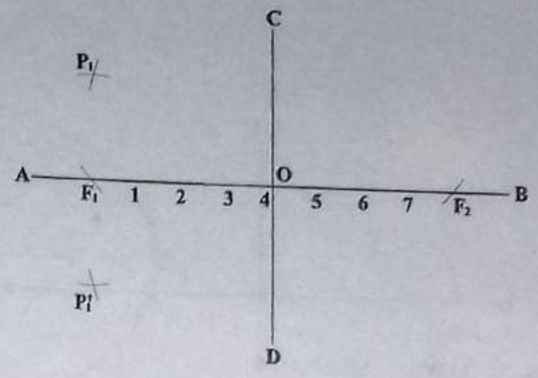


Figure 2.48(d)

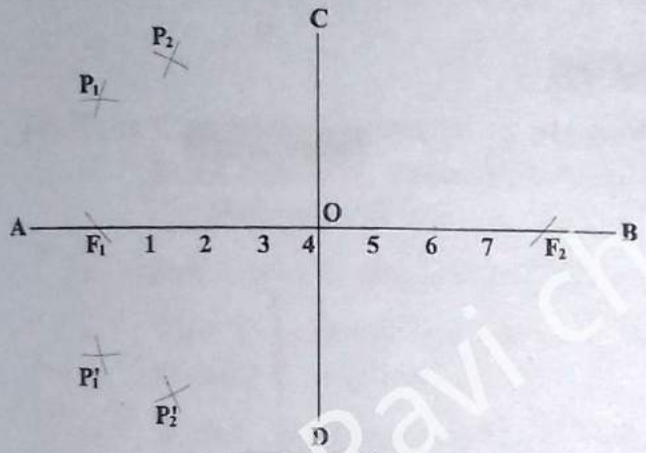


Figure 2.48(e)

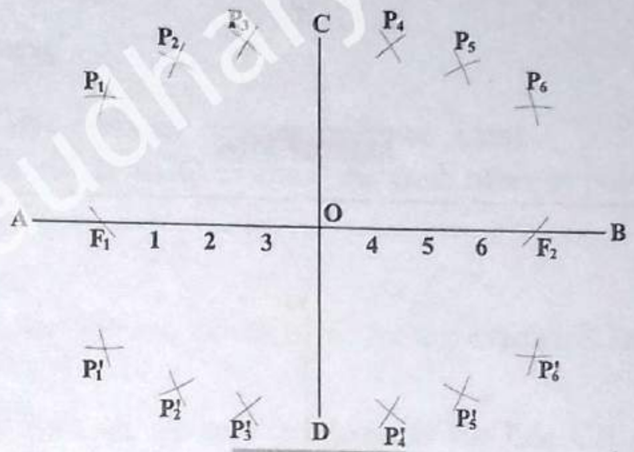


Figure 2.48(f)

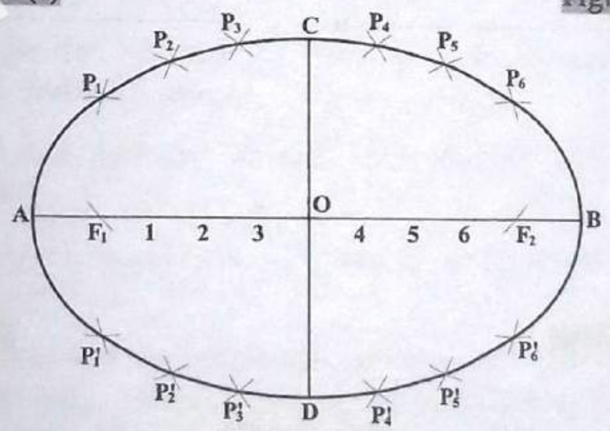


Figure 2.48(g)

(b) Concentric Circle Method (Given Major Axis and Minor Axis)

- Draw the given major axis AB and minor axis CD. *(Figure 2.49(a))*
- Draw concentric circles with AB and CD as diameters. *(Figure 2.49(b))*
- Divide circles into any number of equal parts, say 12. *(Figure 2.49(c))*
- Draw vertical lines inside the circle from each point on the circumference of the larger circle. *(Figure 2.49(d))*
- Draw horizontal lines inside the circle from each point on the circumference of the smaller circle. *(Figure 2.49(e))*
- Mark the points of intersection of these lines. *(Figure 2.49(f))*
- Draw smooth curve passing through these point including the points A, B, C and D to get the required ellipse. *(Figure 2.49(g))*

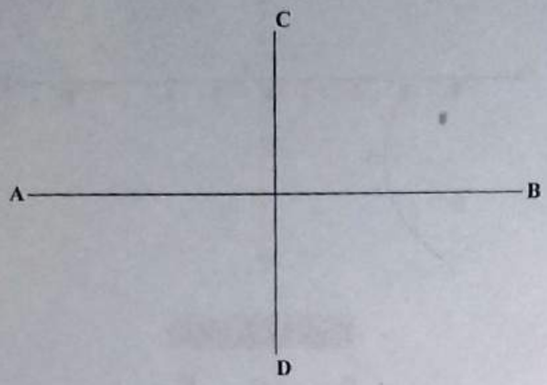


Figure 2.49(a)

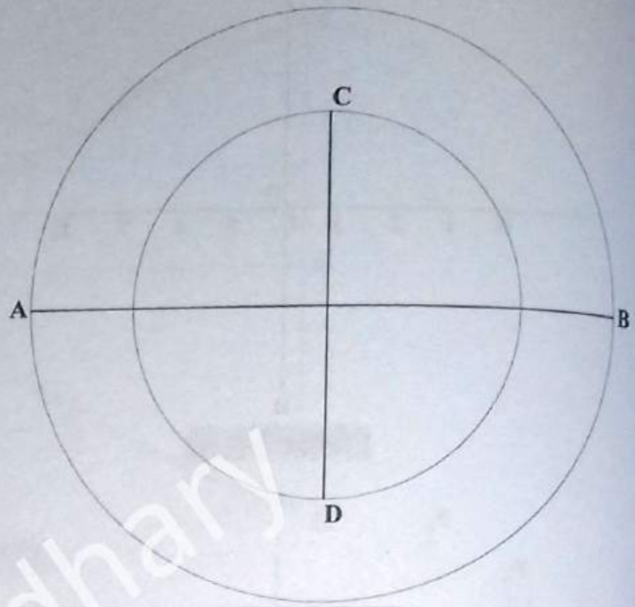


Figure 2.49(b)

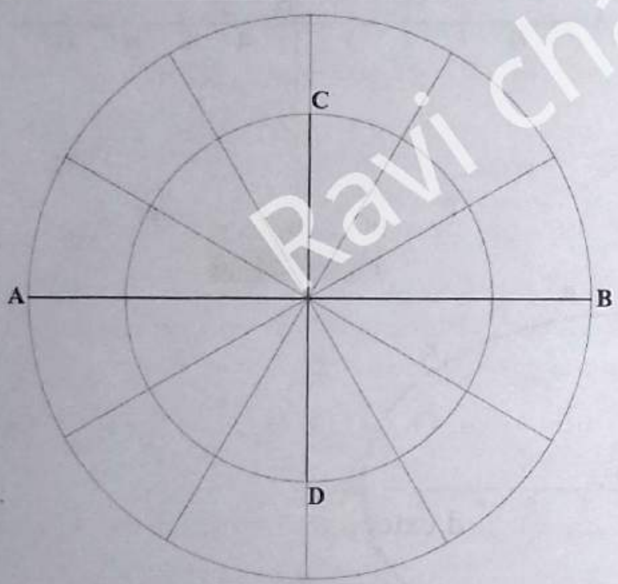
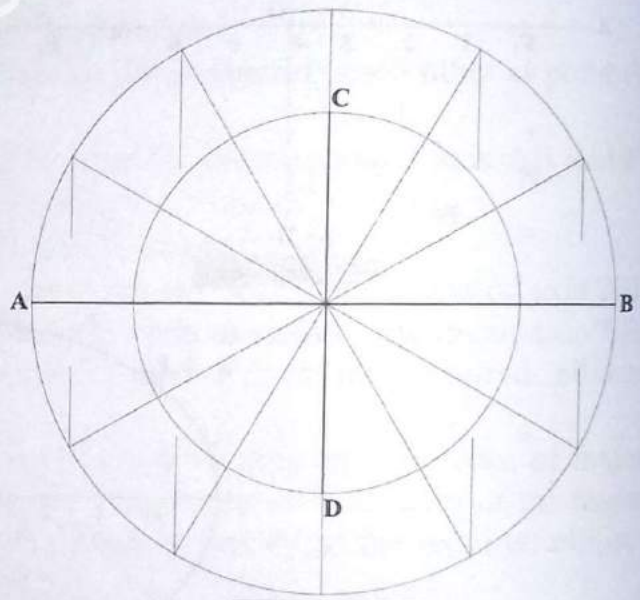


Figure 2.49(c)



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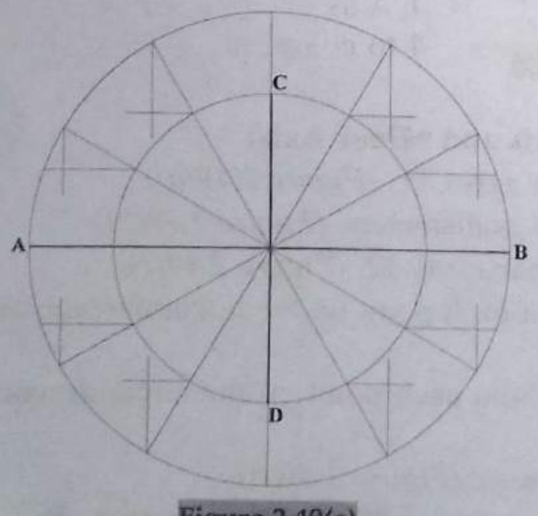

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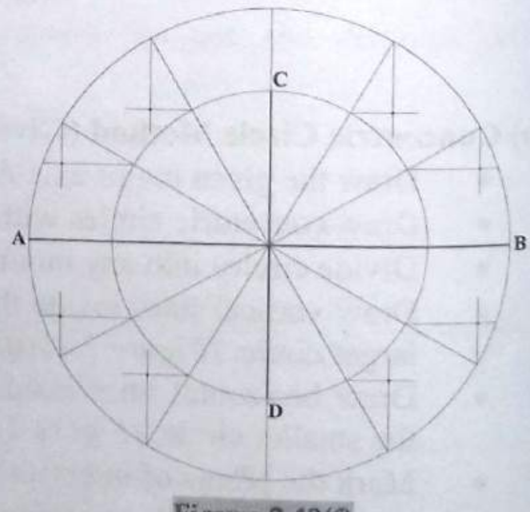

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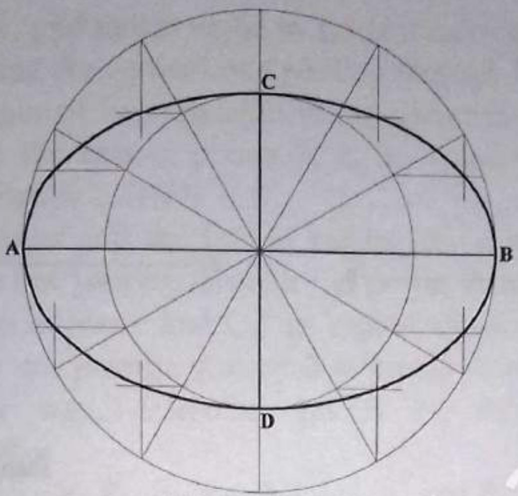


Figure 2.49(g)

(c) Four Center or Approximate Method (Given Major Axis and Minor Axis)

- Draw the given major axis AB and minor axis CD intersecting each other at point O. *(Figure 2.50(a))*
- Join C and B. *(Figure 2.50(b))*
- With O as center and OB as radius, draw an arc which interests the extended DC at point E. *(Figure 2.50(c))*
- With C as center and CE as radius, draw an arc which intersects the line CB at point F. *(Figure 2.50(d))*
- Draw perpendicular bisector of FB which intersects OB and (extended) OD respectively at points O_1 and O_2. *(Figure 2.50(e))*
- Mark O_1' on AO and O_2' on OC such that $OO_1 = OO_1'$ and $OO_2 = OO_2'$. *(Figure 2.50(f))*
- Join O_1' and O_2, O_1' and O_2' and O_2' and O_1 and extend in the direction as shown. *(Figure 2.50(g))*
- With O_1 as center and O_1B as radius, draw an arc 1. With O_2 as center and O_2C as radius, draw an arc 2. With O_1' as center and $O_1'A$ as radius, draw an arc 3. With O_2' as center and $O_2'D$ as radius, draw an arc 4 to complete the required ellipse. *(Figure 2.50(h))*

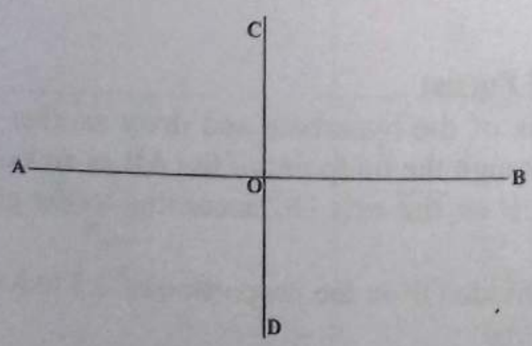
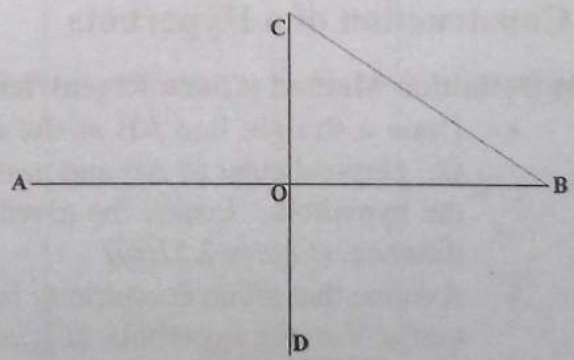


Figure 2.50(a)



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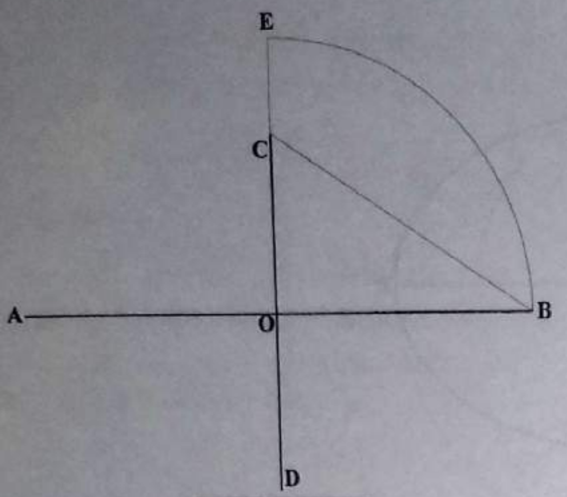

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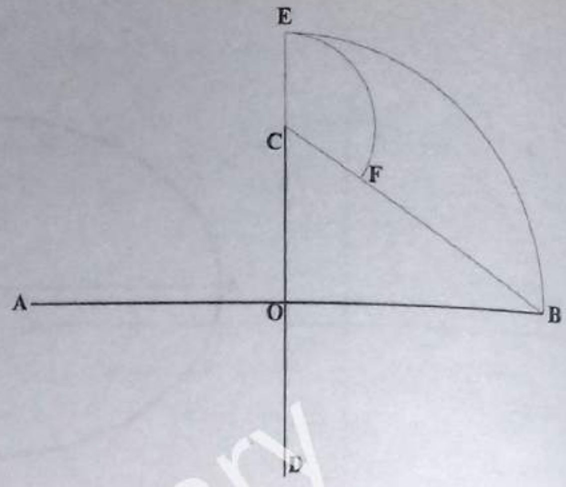

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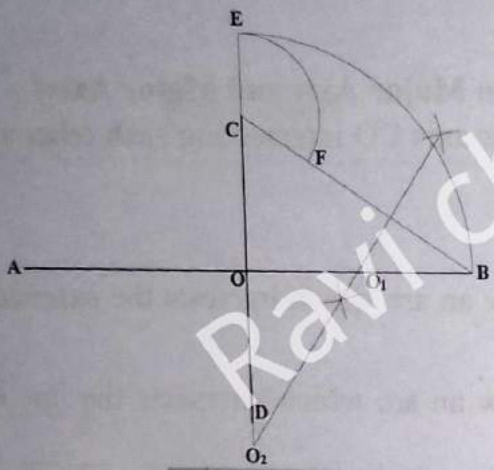

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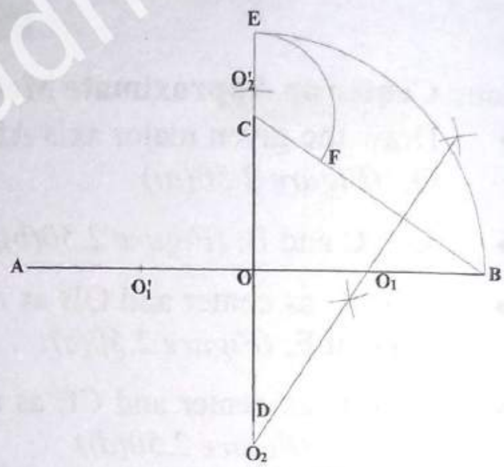

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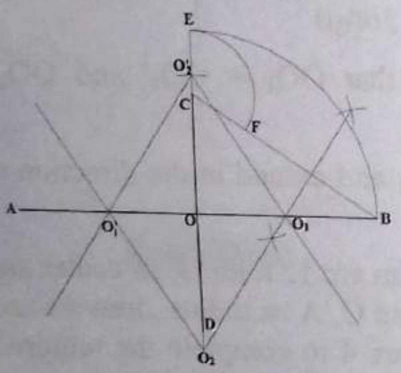

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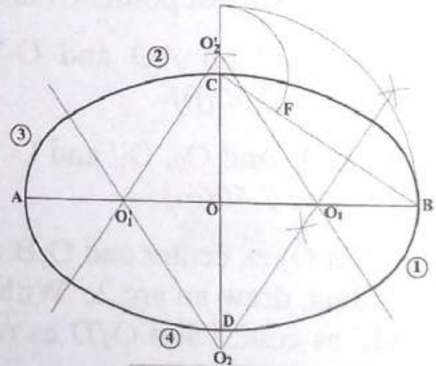


Figure 2.50(h)

2.4.4 Construction of a Hyperbola

(a) Definition Method (Given Eccentricity and Focus)

- Draw a straight line AB as the directrix of the hyperbola and draw another line OC perpendicular to AB and passing through the midpoint of the AB as an axis of the hyperbola. Locate the given focus F on the axis OC according to the given distance. *(Figure 2.51(a))*
- Assume that given eccentricity is 1.5. Divide OF in the proportion of 2:3 to locate vertex V of the hyperbola. *(Figure 2.51(b))*
- Mark any point M on the axis OC at any convenient distance, say 30. Draw a line perpendicular to the axis OC at point M. *(Figure 2.51(c))*

- With O as center and radius equal to 1.5 (eccentricity) times the OM (= 45), draw an arc intersecting the vertical line passing through M at point N. *(Figure 2.51(d))*
- Draw any number of lines parallel to the directrix AB and right to the vertex V which intersect the axis at points 1, 2, 3…. and the line ON at 1', 2', 3', ….. respectively. *(Figure 2.51(e))*
- With F as a center and the O1' as radius, draw arcs on both sides of the axis intersecting the line passing through 1 at points P_1 and P_1'. *(Figure 2.51(f))*
- Again with F as a center and O2' as radius, draw arcs on both sides of the axis intersecting the line passing through 2 at points P_2 and P_2'. *(Figure 2.51(g))*
- In the similar way, determine points P_3, P_3', P_4, P_4', ….. and so on. *(Figure 2.45(h))*
- Join all the points V, P_1, P_2, P_3, P_4, ……… and P_1', P_2', P_3', P_4', ….. by a smooth curve to get the required hyperbola. *(Figure 2.51(i))*

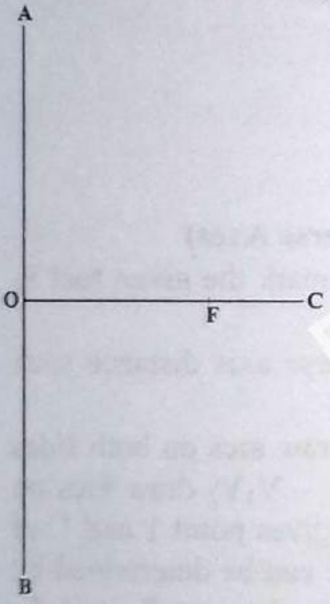
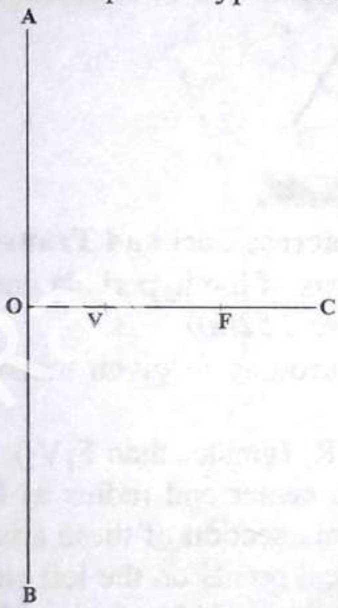


Figure 2.51(a)



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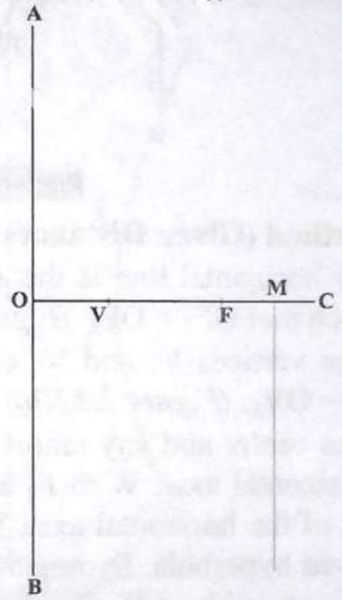

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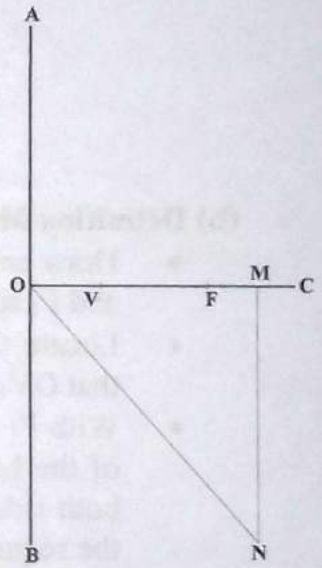

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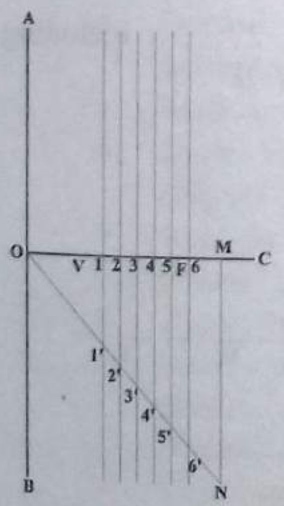

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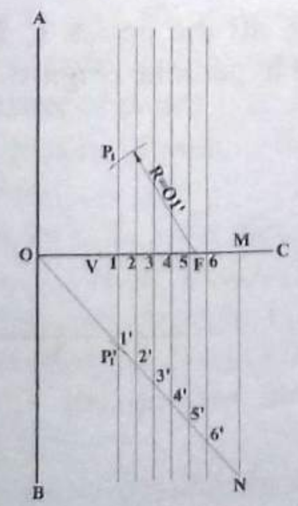

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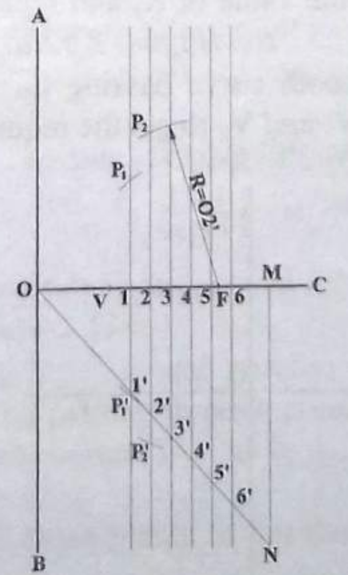

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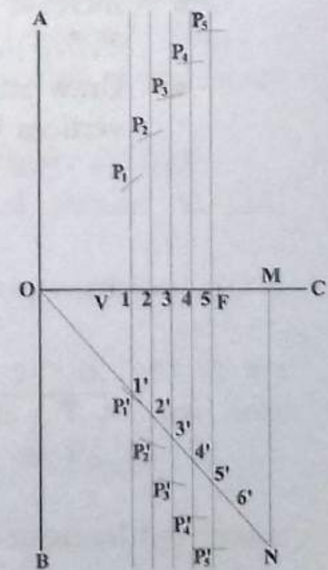


Figure 2.51(h)

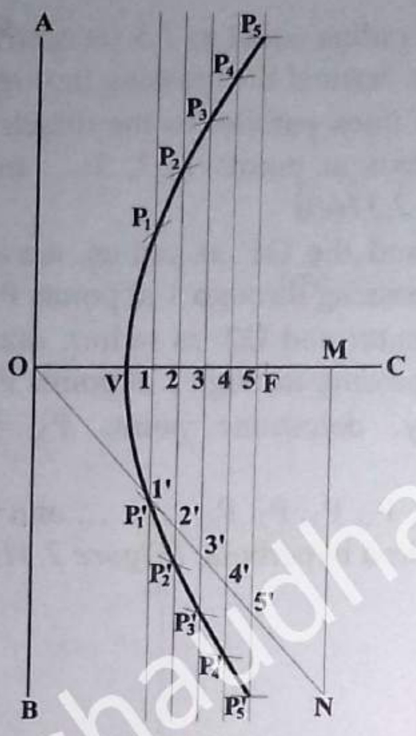


Figure 2.51(i)

(b) Definition Method (Given Distances across Foci and Transverse Axes)

- Draw any horizontal line as the axis of the hyperbola and mark the given foci F_1 and F_2 such that $OF_1 = OF_2$. *(Figure 2.52(a))*
- Locate the vertices V_1 and V_2 corroding to given transverse axis distance such that $OV_1 = OV_2$. *(Figure 2.52(b))*
- With F_1 as center and any radius R_1 (greater than F_1V_2), draw arcs on both sides of the horizontal axis. With F_2 as center and radius as $R_1 - V_1V_2$ draw arcs on both side of the horizontal axis. Intersection of these arcs gives point 1 and 1' of the required hyperbola. Symmetrical points on the left side can be determined by drawing arcs with radii R_1 and $R_1 - V_1V_2$ and with centers at F_2 and F_1 respectively. *(Figure 2.52(c))*
- Increase the value of R_1 and repeat the same procedure to get the other points 2, 2', 3, 3', …… etc. *(Figure 2.52(d))*
- Draw smooth curve passing through all the points 1, 1', 2, 2', …… including vertices V_1 and V_2 to get the required hyperbola. *(Figure 2.52(e))*

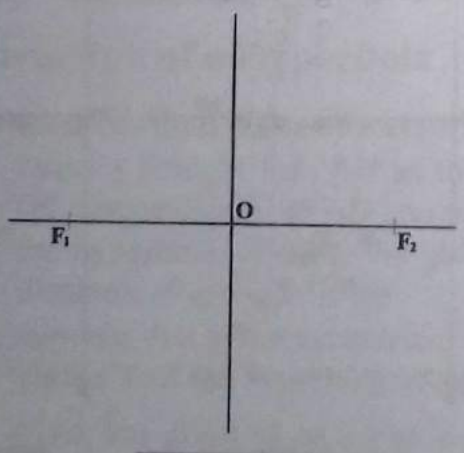
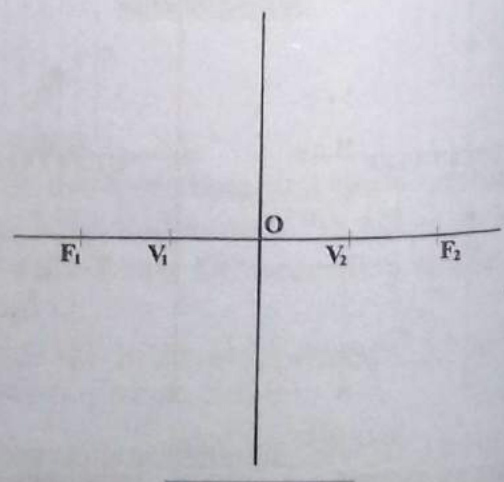


Figure 2.52(a)



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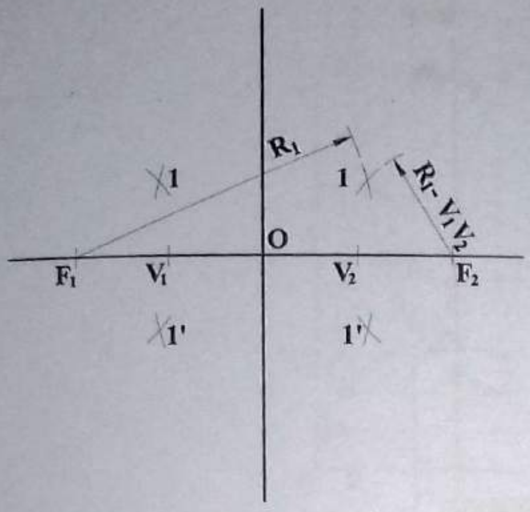


Figure 2.52(c)

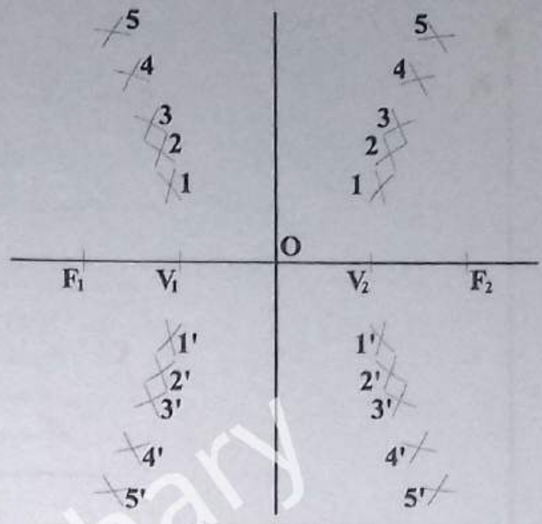


Figure 2.52(d)

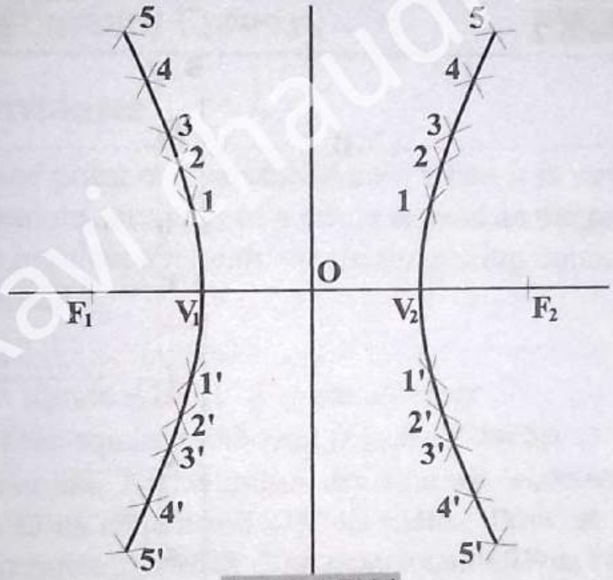


Figure 2.52(e)

(c) Rectangular Hyperbola (Given Asymptotes and Any Point of the Hyperbola)

- Draw given asymptotes OA and OB and mark given point P_0. *(Figure 2.53(a))*
- Draw lines CD and EF passing through point P_0 and parallel to the given asymptotes OA and OB. *(Figure 2.53(b))*
- Mark any number of points 1, 2, 3, … on line segment EP_0. *(Figure 2.53(c))*
- Draw lines passing through the points 1, 2, 3, …. and parallel to OA. *(Figure 2.53(d))*
- Joint each points 1, 2, 3, … with the point O and extend them to intersect CD at points 1', 2', 3', …. respectively. *(Figure 2.53(e))*
- Draw lines passing through 1', 2', 3', ….. and parallel to OB. Mark the intersection of lines parallel to OA and passing through points 1, 2, 3… and lines parallel to OB and passing through points 1', 2', 3', … as P_1, P_2, P_3, …. *(Figure 2.53(f))*
- Draw smooth curve passing through these points to get the required hyperbola. *(Figure 2.53(g))*

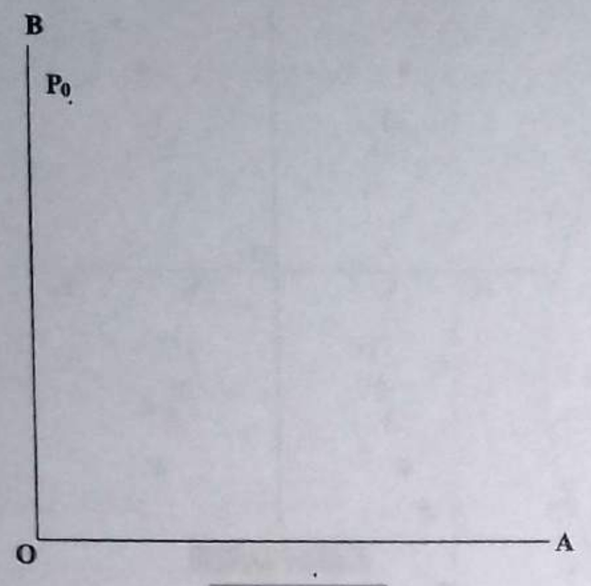


Figure 2.53(a)

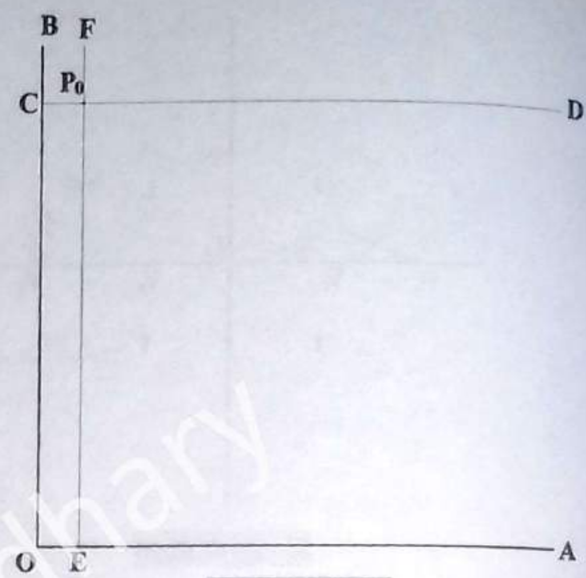


Figure 2.53(b)

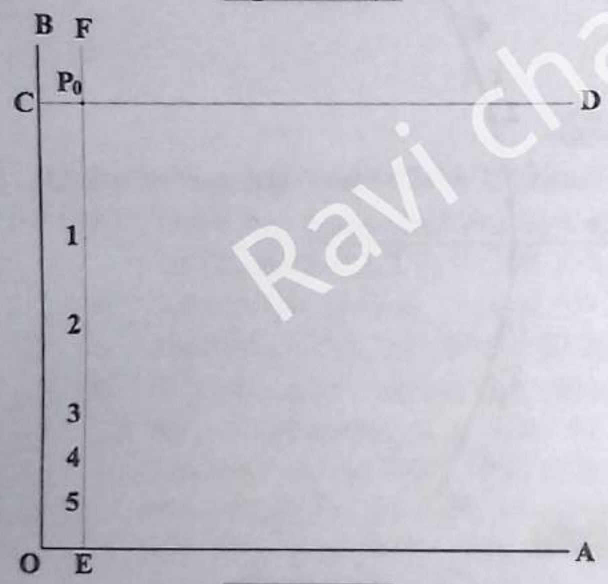


Figure 2.53(c)

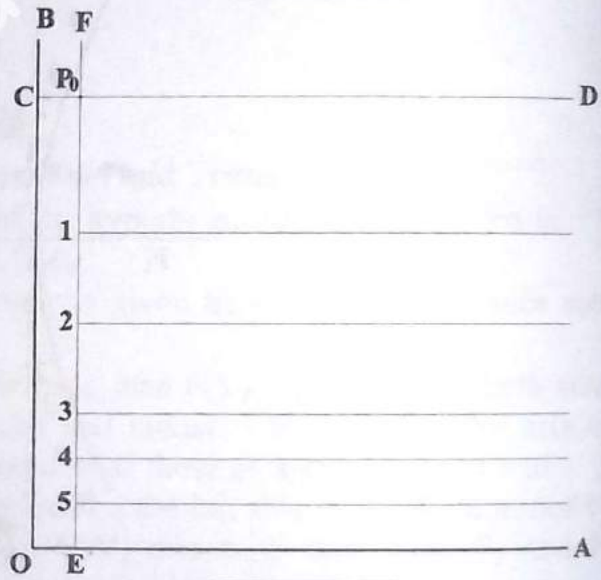


Figure 2.53(d)

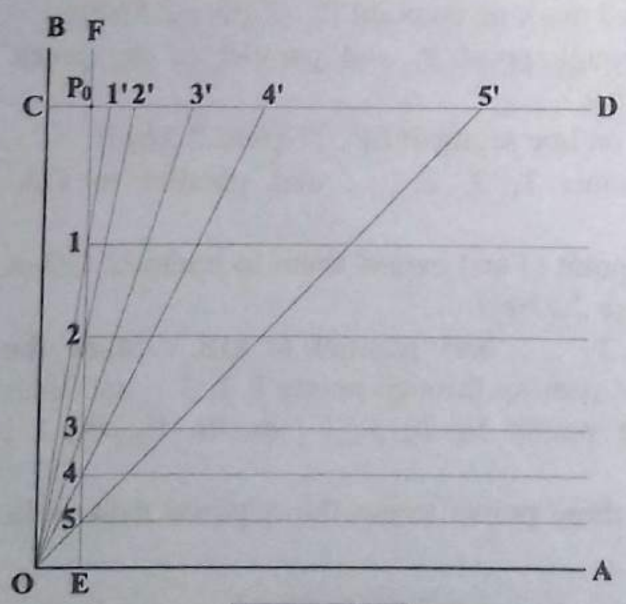


Figure 2.53(e)

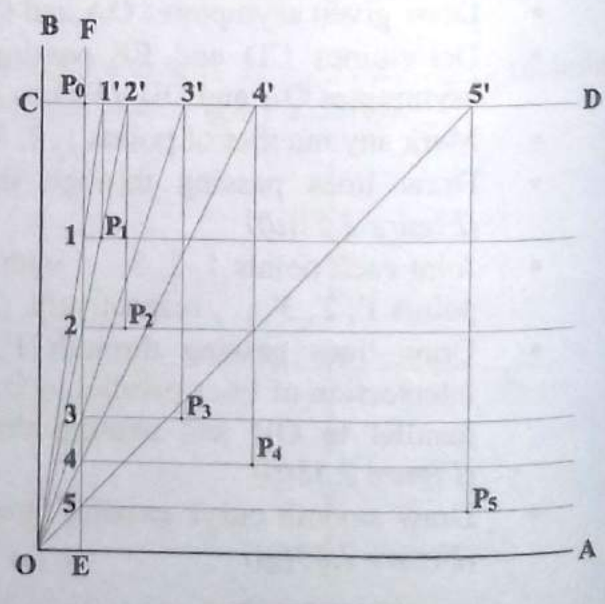


Figure 2.53(f)

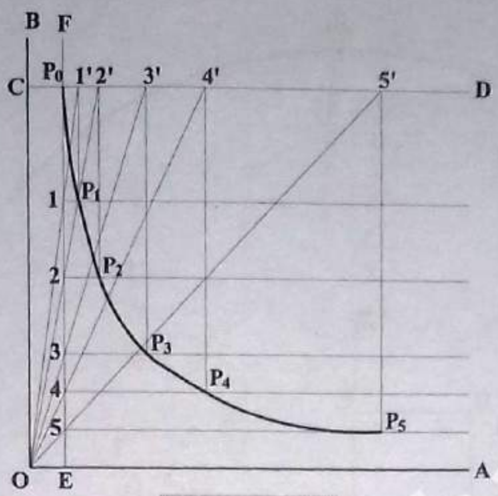


Figure 2.53(g)

2.5 Construction of Standard Curves

2.5.1 Construction of Involutes

The curve traced by a fixed point on a stretched cord when it is unwounded from a circle or a polygon is called an involute. Involute of a circle is used as the profile of gear teeth. Cams are often designed to the involute shape to ensure the rolling contact between the roller and the follower at constant speed.

(a) Involute of a Square

- Draw the given square ABCD. *(Figure 2.54(a))*
- Extend sides of the square as shown. *(Figure 2.54(b))*
- With A as center and AB as radius draw an arc intersecting the extended DA at point A'. With D as center and DA' as radius, draw an arc which intersects the extended CD at point D'. With C as center and CD' as radius, draw an arc which intersects the extended BC at point C'. With B as center and BC' as radius, draw an arc which intersects the extended AB at point B'. *(Figure 2.54(c))*

When one turn of the cord is taken out from the square, the length of the cord (BB') will be equal to perimeter of the square.

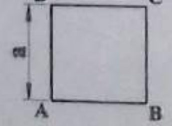


Figure 2.54(a)

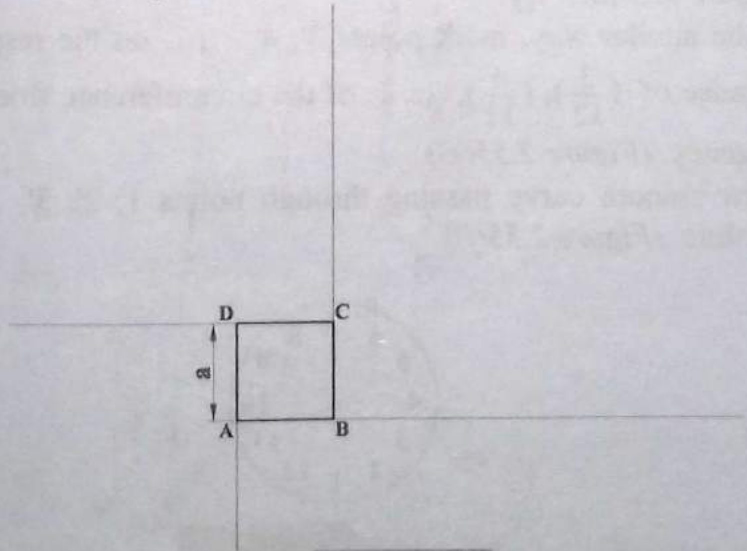


Figure 2.54(b)

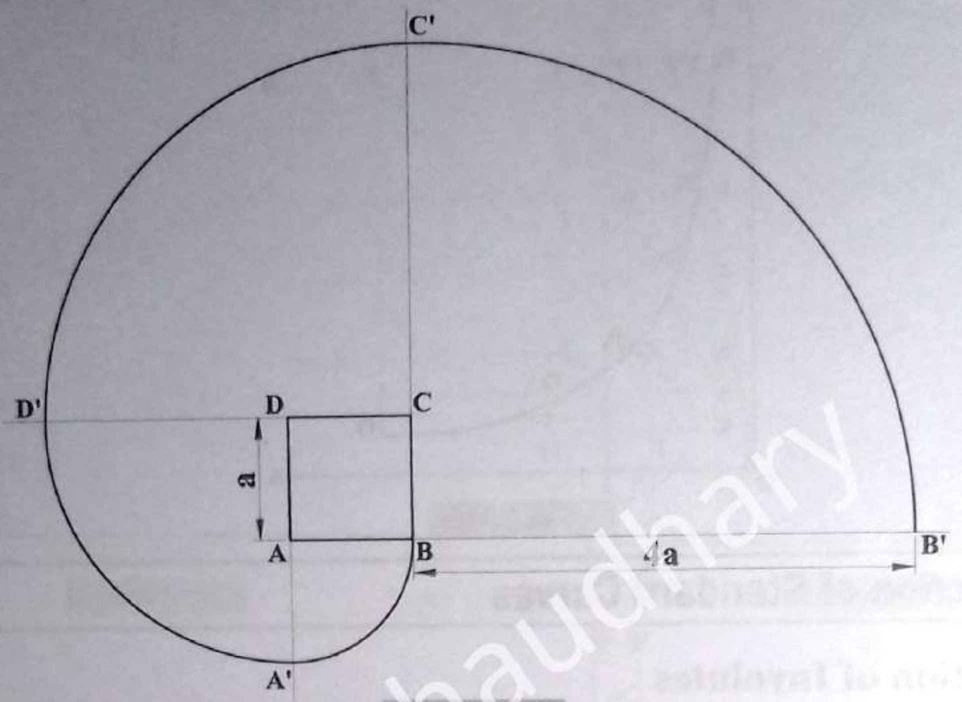


Figure 2.54(c)

(b) Involute of a Circle

- Draw the given circle and divide the circle into any number of equal parts, say 12. *(Figure 2.55(a))*
- Draw tangent lines from each point on the circumference of the circle in direction as shown. *(Figure 2.55(b))*
- Mark point 1' on the tangent line passing through point 1 such that the distance between 1 and 1' is equal to the circumference of the given circle. Divide 11' into the same number of equal parts as that of the circle. *(Figure 2.55(c))*
- Mark point 2' on the tangent line passing through point 2 such that 22' is equal to $(\frac{1}{12})$ of the circumference, i.e., 11'. Again, mark point 3' on the tangent line passing through point 3 such that 33' is equal to $(\frac{2}{12})$ of the circumference'. *(Figure 2.55(d))*
- In the similar way, mark points 3', 4', ... on the respective tangent lines at a distance of $(\frac{3}{12})$, $(\frac{4}{12})$, ... of the circumference from the respective points of tangency. *(Figure 2.55(e))*
- Draw smooth curve passing through points 1, 2', 3', ... to get the required involute. *(Figure 2.55(f))*

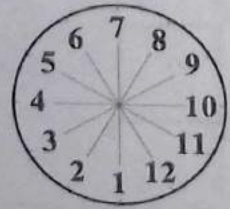


Figure 2.55(a)

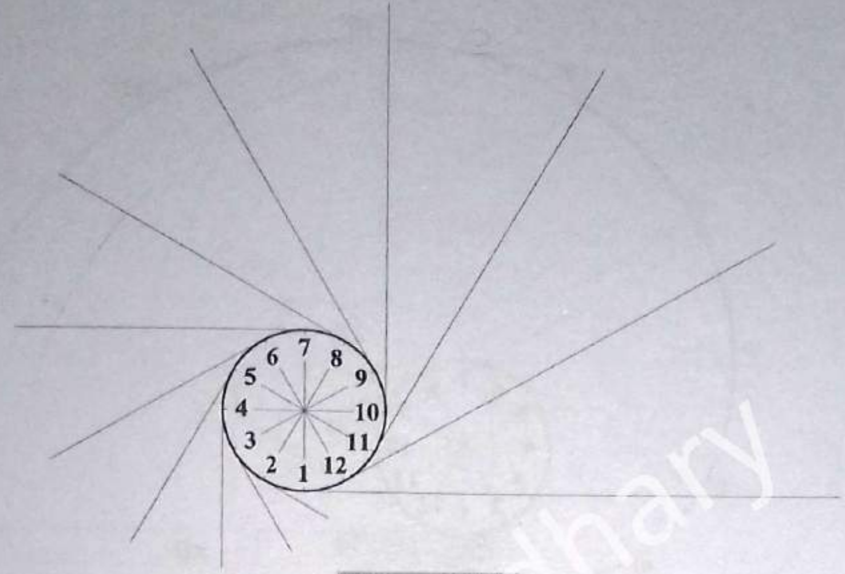


Figure 2.55(b)

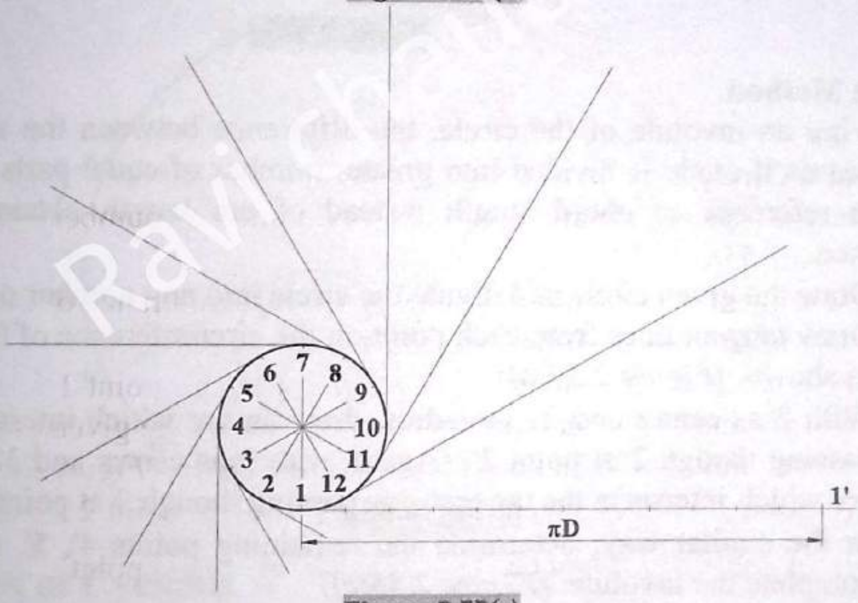


Figure 2.55(c)

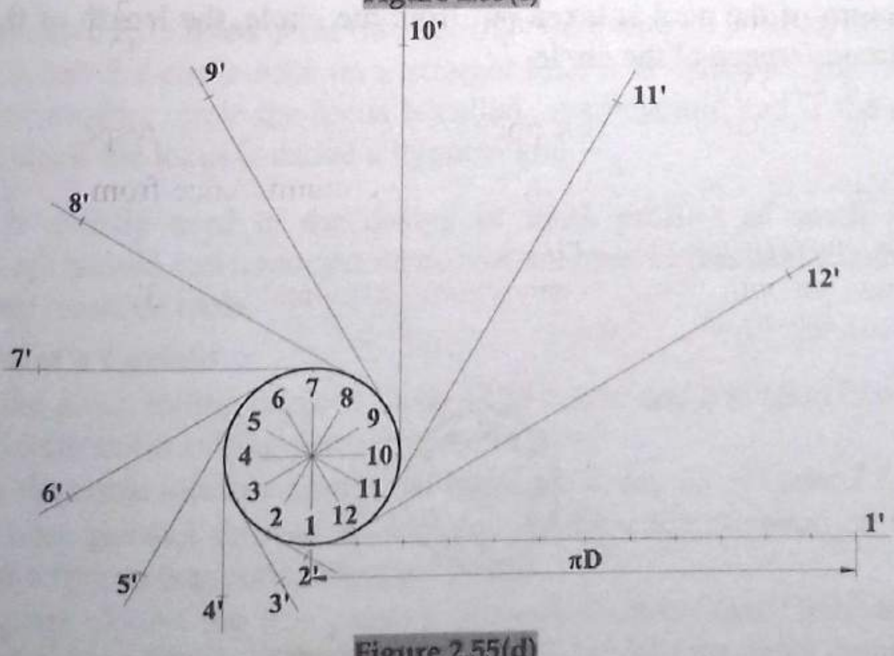


Figure 2.55(d)

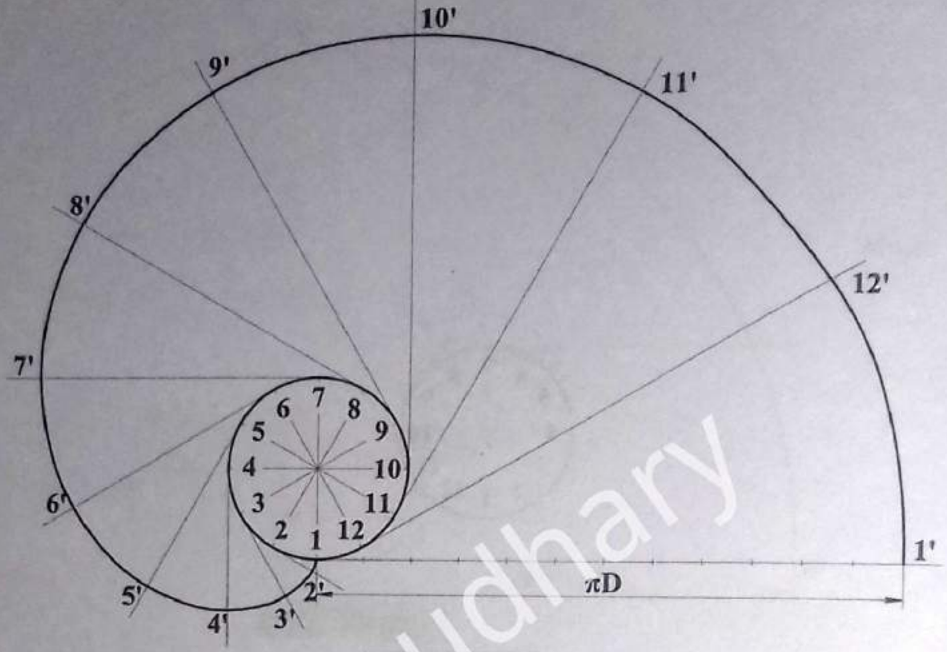


Figure 2.55(e)

Alternative Method

While drawing an involute of the circle, the difference between the arc length and chord length decreases if circle is divided into greater number of equal parts and involute can be drawn with reference to chord length instead of arc length obtained by dividing the circumference.

- Draw the given circle and divide the circle into any number of equal parts, say 12. Draw tangent lines from each point on the circumference of the circle in direction as shown. *(Figure 2.56(a))*
- With 2 as center and 21 as radius, draw an arc which intersects the tangent line passing though 2 at point 2'. (Again, with 3 as center and 32' as radius, draw an arc which intersects the tangent line passing though 3 at point 3'. *(Figure 2.56(b))*
- In the similar way, determine the remaining points 4', 5', 6', …………, 12' to complete the involute. *(Figure 2.56(c))*

When one turn of the cord is taken out from the circle, the length of the cord (11') will be equal to circumference of the circle.

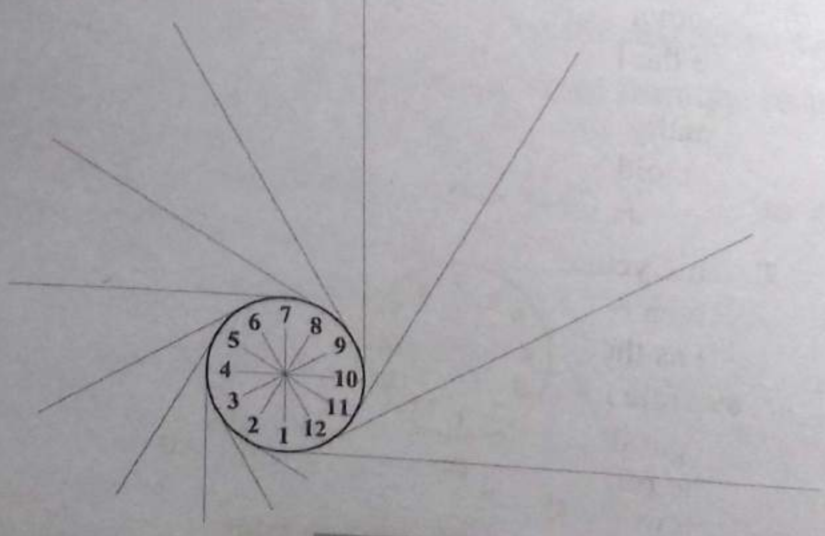


Figure 2.56(a)

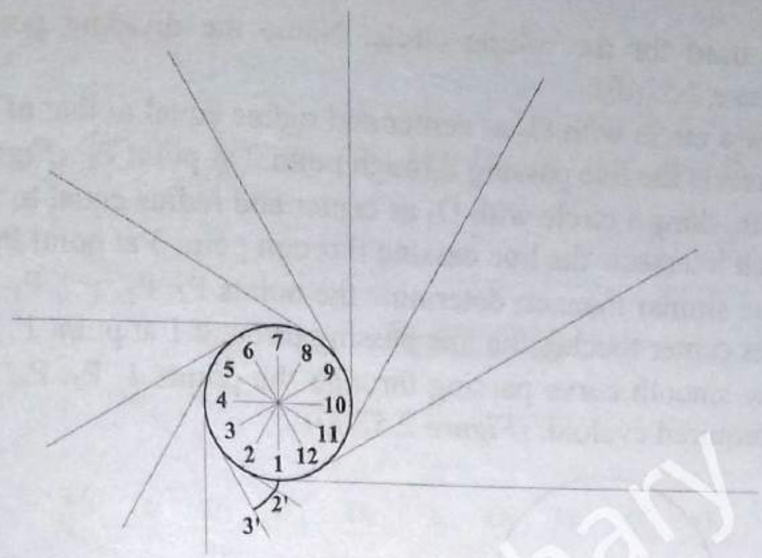


Figure 2.56(b)

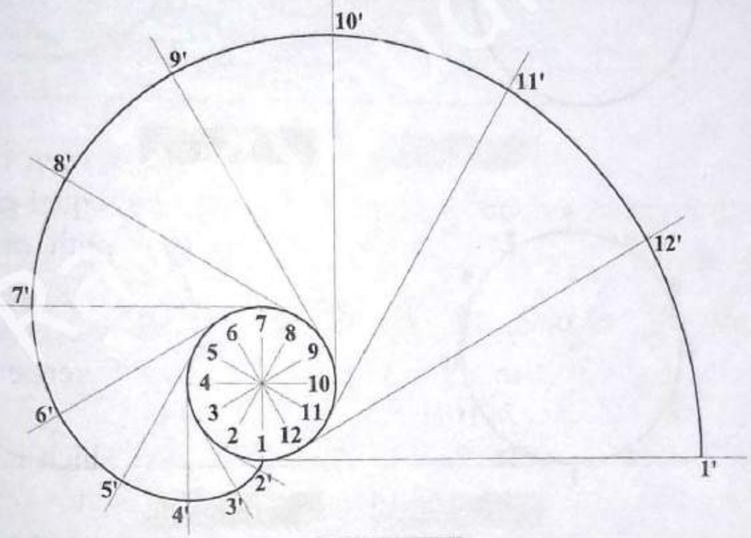


Figure 2.56(c)

2.5.2 Construction of Cycloids

The plane curves generated by a fixed point on a rolling circle when it rolls on different surfaces are called cycloids. When the circle rolls on a straight line, it is called a cycloid. If the circle rolls on the outside of another circle the locus is called an epicycloid and if the circle rolls on the inside of another circle the locus is called a hypocycloid.

The cycloid curve is usually used in the design of tooth profiles of small gears used in instruments whereas epicycloid and hypocycloid curves are used in mechanisms for cutting gear teeth and metal cutting machine tools.

(a) Construction of a Cycloid

- Draw the given rolling circle with O_1 as its center and a tangent line at the bottom of the circle as the rolling path. *(Figure 2.57(a))*
- Divide the circle into any number of equal parts, say 12. *(Figure 2.57(b))*
- Draw lines passing through each point on the circumference of the circle and parallel to the rolling path. *(Figure 2.57(c))*
- Mark point O_1' on the line passing through O_1 such that O_1O_1' is equal to the circumference of the rolling circle. Divide O_1O_1' into the same number of parts as

that used for the rolling circle. Name the dividing points as O_2, O_3, ... O_{12}. *(Figure 2.57(d))*

- Draw a circle with O_2 as center and radius equal to that of the rolling circle which intersects the line passing through point 2 at point P_2. *(Figure 2.57(e))*
- Again, draw a circle with O_3 as center and radius equal to that of the rolling circle which intersects the line passing through point 3 at point P_3. *(Figure 2.57(f))*
- In the similar manner, determine the points P_4, P_5, ..., P_{12}. The circle drawn with O_1' as center touches the line passing through 1 at point 1'. *(Figure 2.57(g))*
- Draw smooth curve passing through the points 1, P_2, P_3, ..., P_{12} and 1' to get the required cycloid. *(Figure 2.57(h))*

Figure 2.57(a)

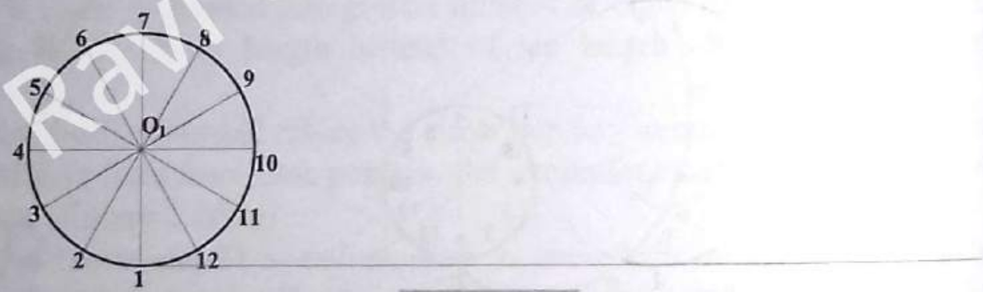


Figure 2.57(b)

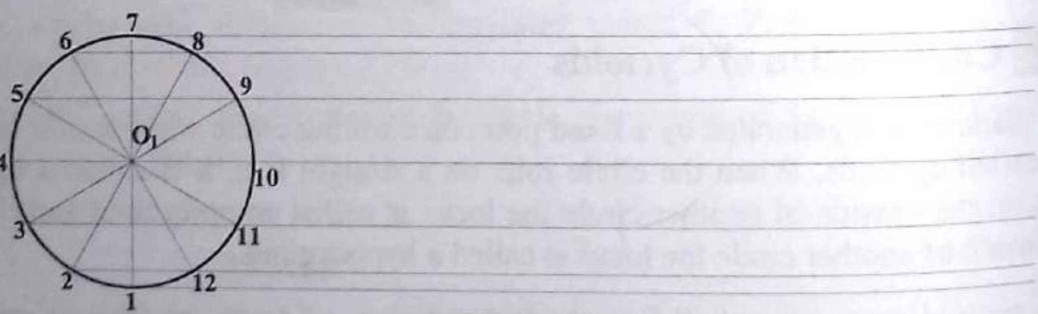


Figure 2.57(c)

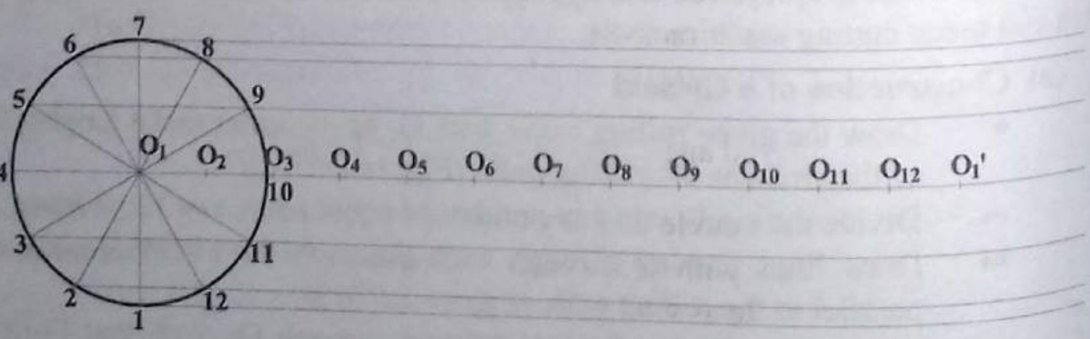


Figure 2.57(d)

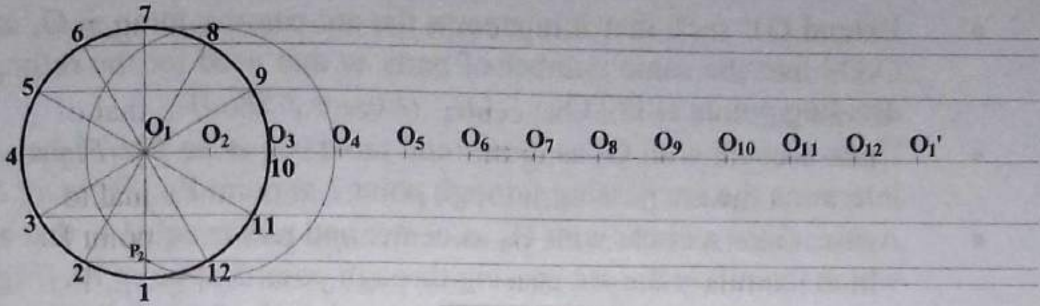


Figure 2.57(e)

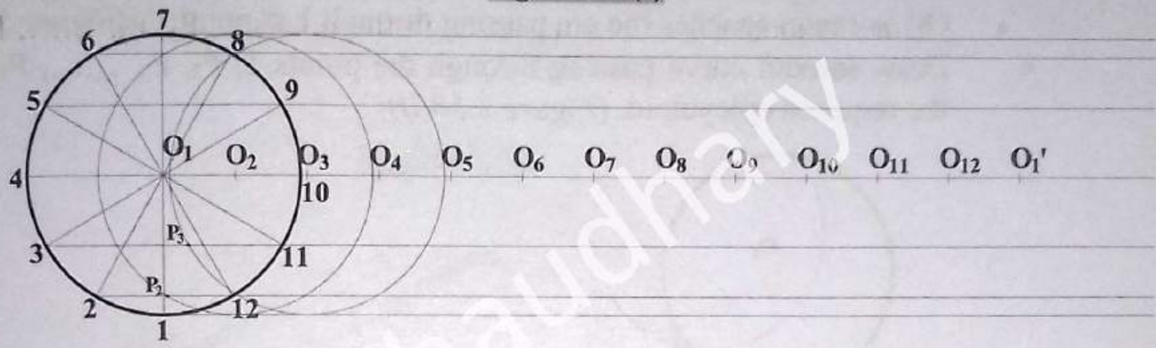


Figure 2.57(f)

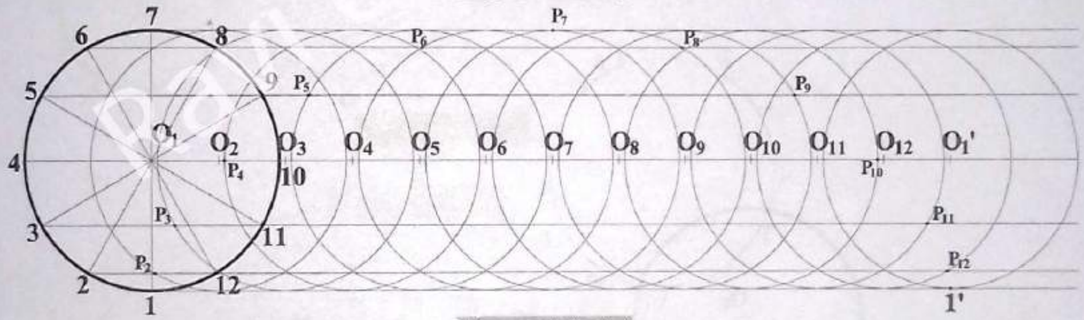


Figure 2.57(g)

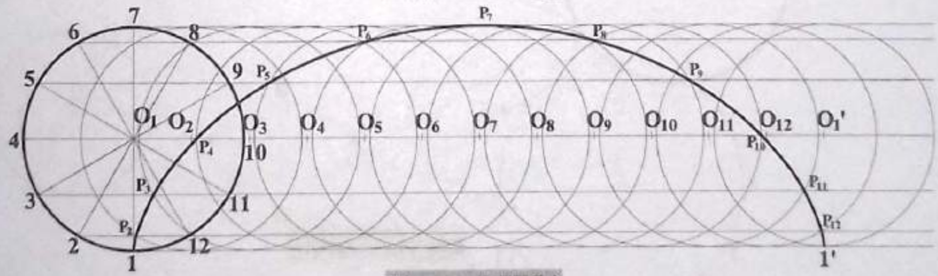


Figure 2.57(h)

(b) Construction of an Epicycloid

- Draw the given rolling circle with O_1 as its center and a guiding circle with O as its center and radius R tangent at point 1. *(Figure 2.58(a))*
- Mark arc length 11' equal to the circumference of the rolling circle (πD) such that the included angle between the point 1 and 1' is $\theta = \left(\frac{180D}{R}\right)^0$. *(Figure 2.58(b))*
- Divide the circle into any number of equal parts, say 12. *(Figure 2.58(c))*
- Draw arcs with O as center and passing through each point on the circumference of the circle as well as through point O_1. *(Figure 2.58(d))*

- Extend O1' such that it intersects the arc passing through O_1 at point O_1'. Divide O_1O_1' into the same number of parts as that used for the rolling circle. Name the dividing points as $O_2, O_3, \ldots O_{12}$. *(Figure 2.58(e))*
- Draw a circle with O_2 as center and radius equal to that of the rolling circle which intersects the arc passing through point 2 at point P_2. *(Figure 2.58(f))*
- Again, draw a circle with O_3 as center and radius equal to that of the rolling circle which intersects the arc passing through point 3 at point P_3. *(Figure 2.58(g))*
- In the similar manner, determine the points $P_4, P_5, \ldots, P_{12}$. The circle drawn with O_1' as center touches the arc passing through 1 at point 1'. *(Figure 2.58(h))*
- Draw smooth curve passing through the points 1, $P_2, P_3, \ldots\ldots, P_{12}$ and 1' to get the required epicycloid. *(Figure 2.58(i))*

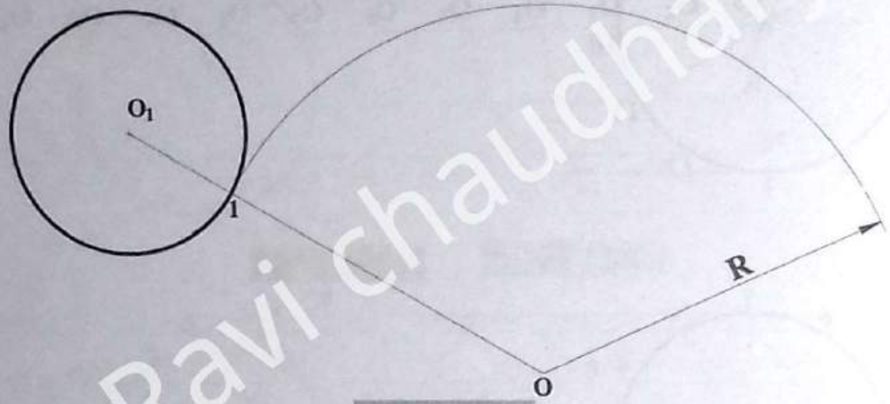


Figure 2.58(a)

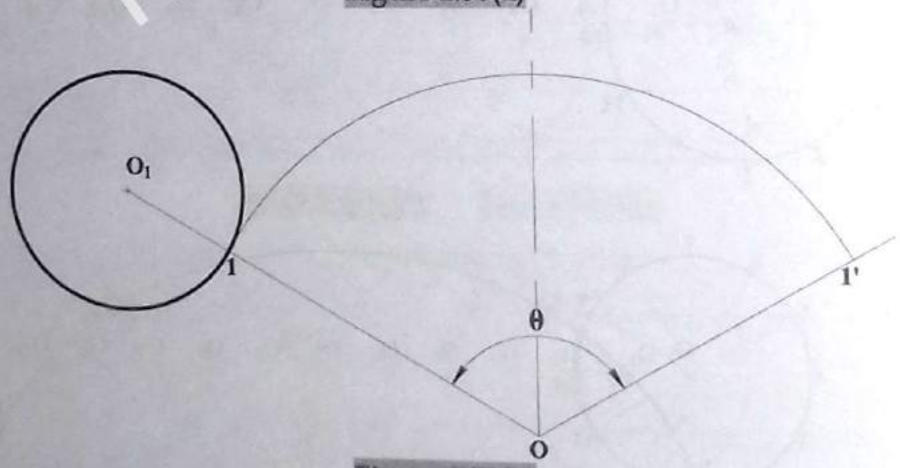


Figure 2.58(b)

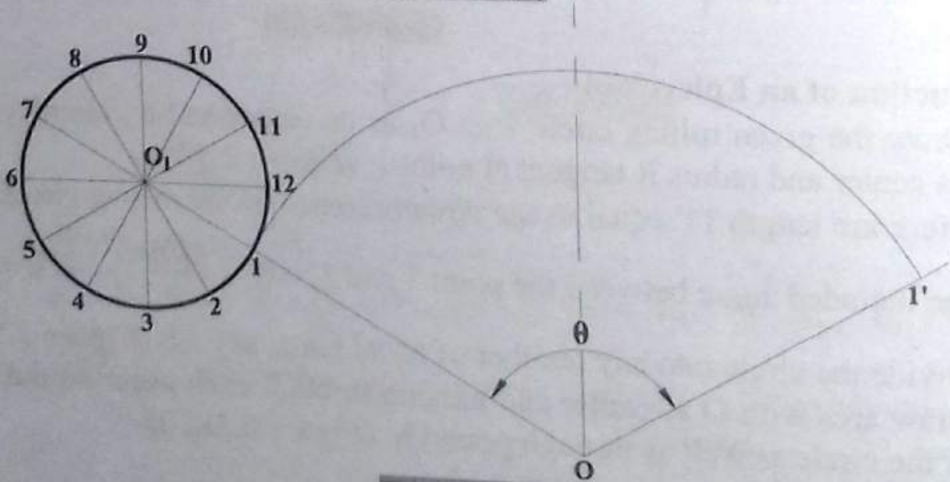


Figure 2.58(c)

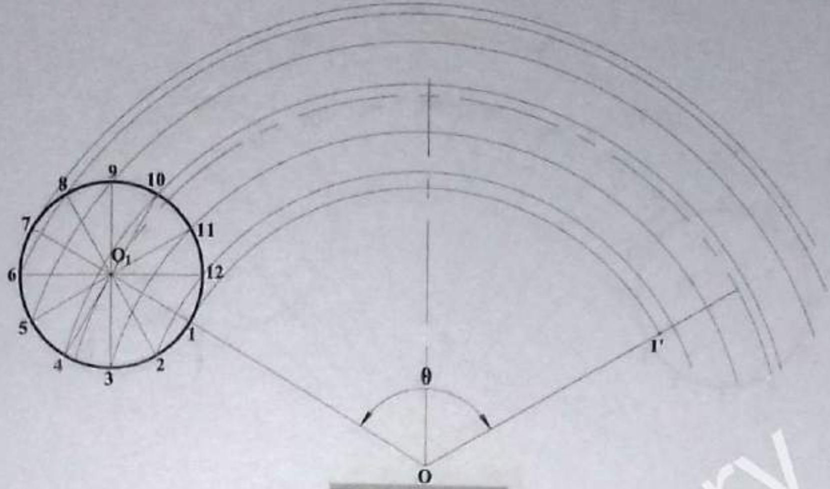


Figure 2.58(d)

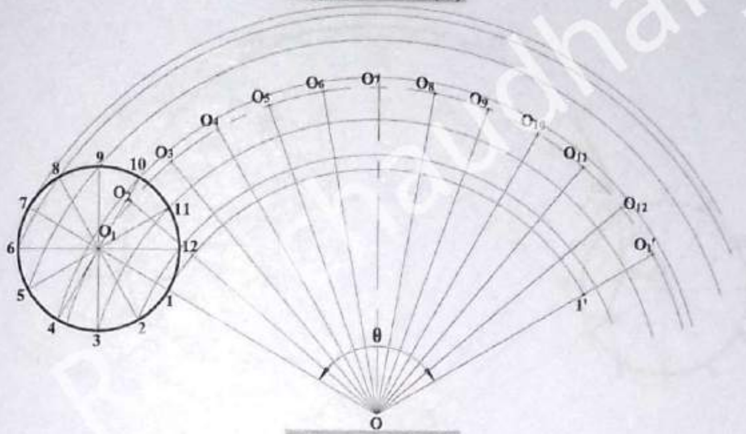


Figure 2.58(e)

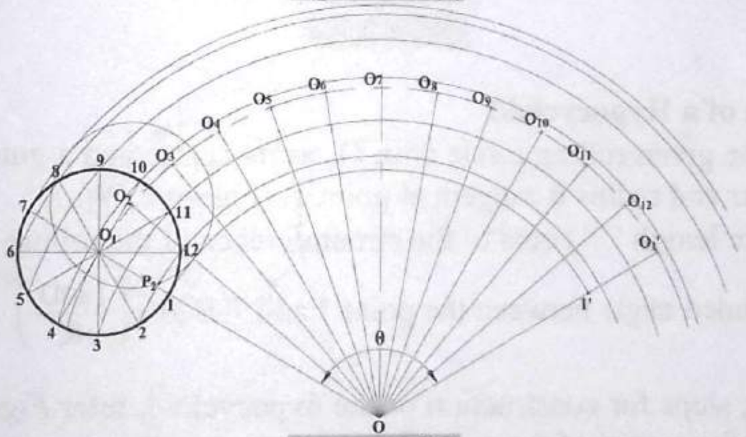


Figure 2.58(f)

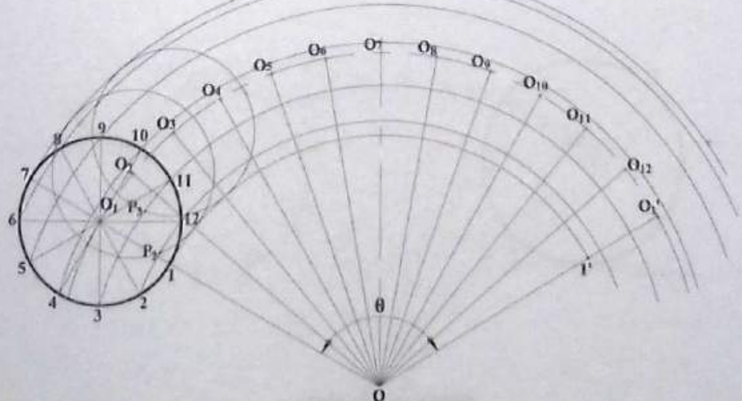


Figure 2.58(g)

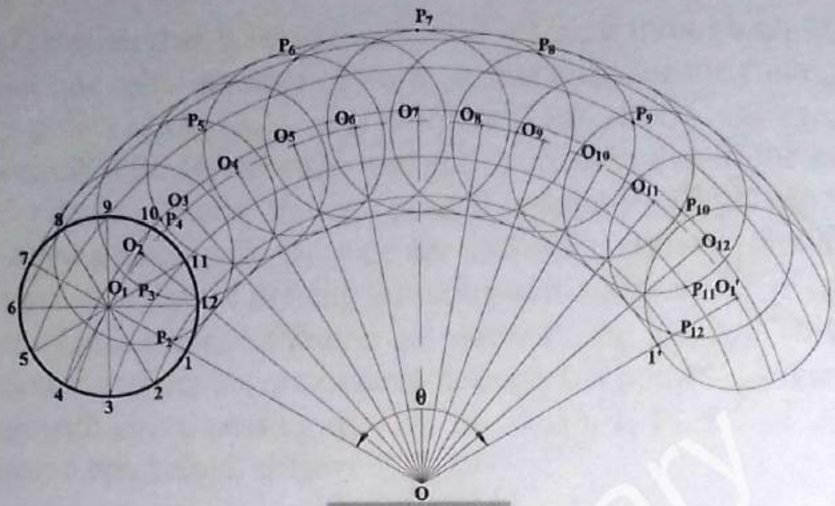


Figure 2.58(h)

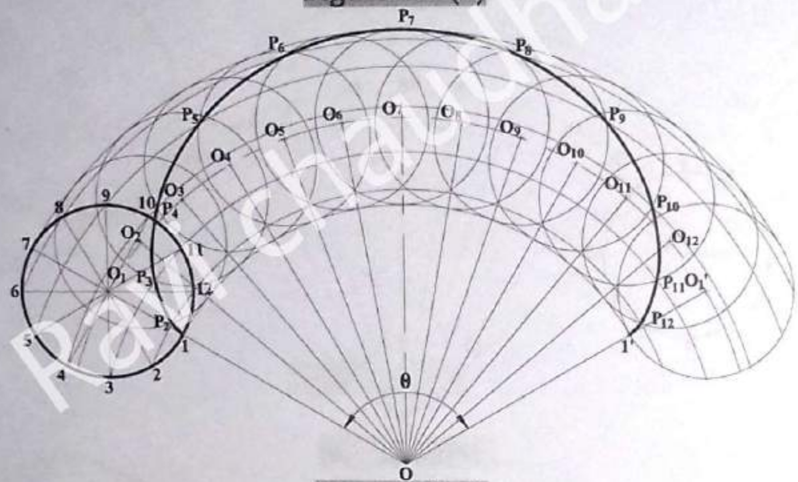


Figure 2.58(i)

(c) Construction of a Hypocycloid

- Draw the given rolling circle with O_1 as its center and a guiding circle with O as its center and radius R tangent at point 1. *(Figure 2.59(a))*
- Mark arc length 77' equal to the circumference of the rolling circle (πD) such that the included angle between the point 7 and 7' is $\theta = \left(\frac{180D}{R}\right)^0$. *(Figure 2.59(b))*

For the remaining steps for construction of the hypocycloid, refer *Figure 2.59(c)* to *2.59(i)* which is quite similar to that of the epicycloid.

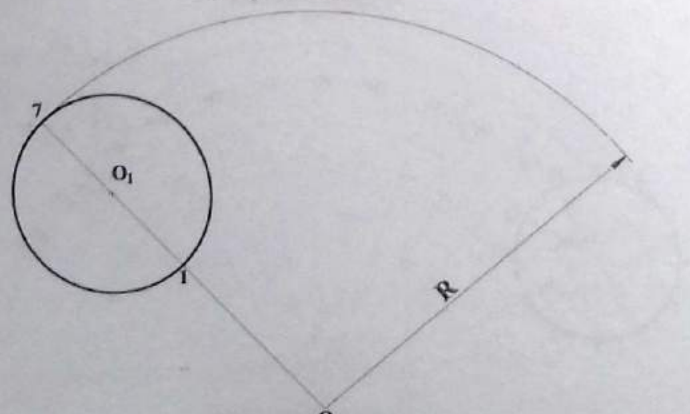


Figure 2.59(a)

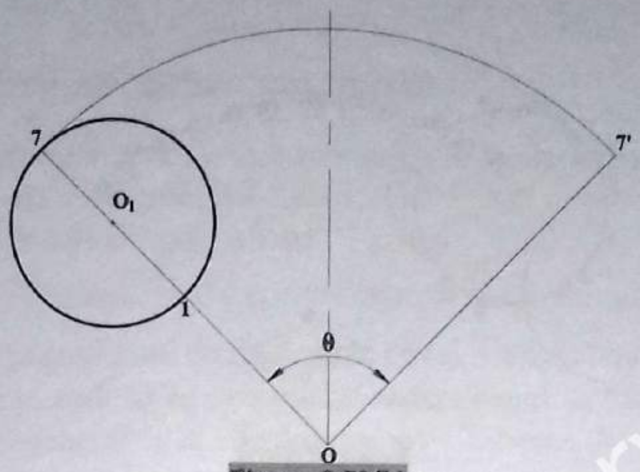


Figure 2.59(b)

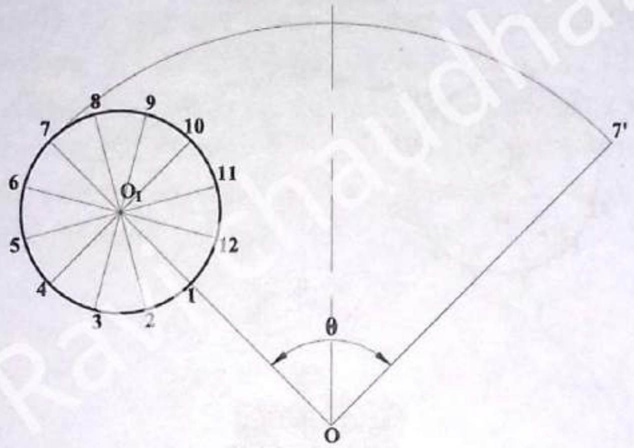


Figure 2.59(c)

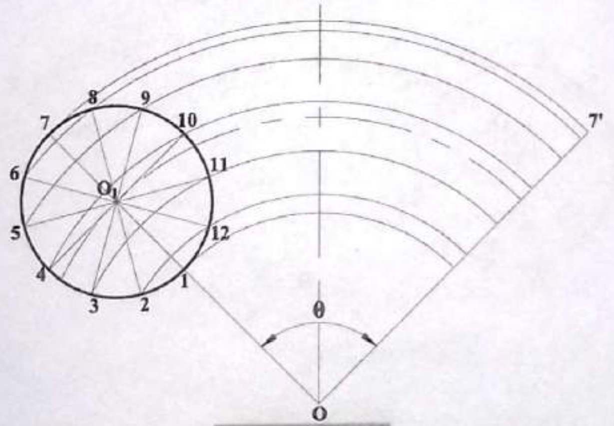


Figure 2.59(d)

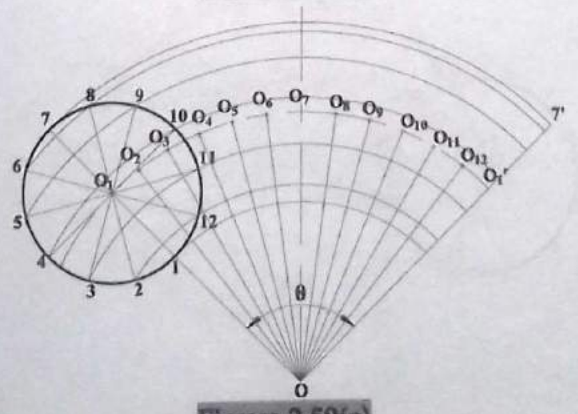


Figure 2.59(e)

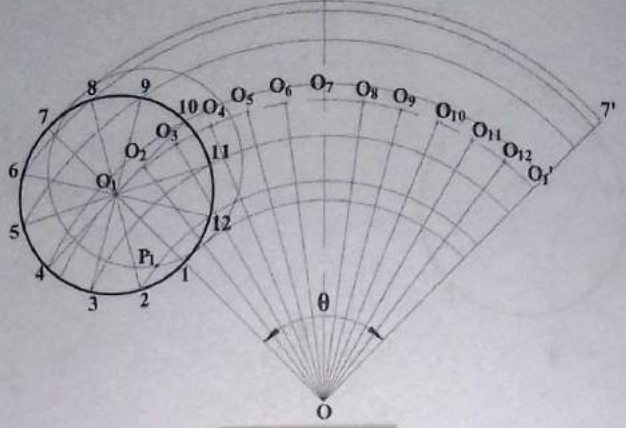


Figure 2.59(f)

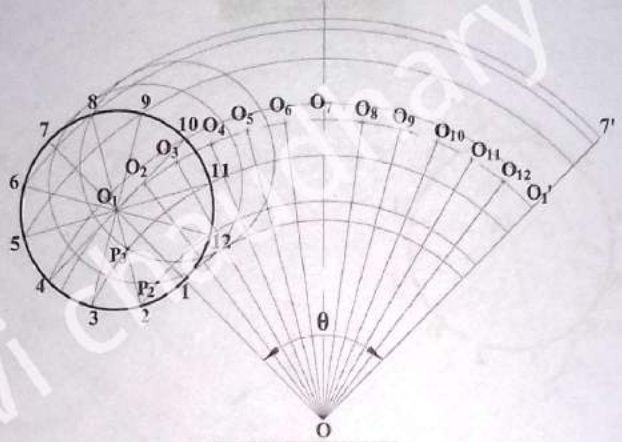


Figure 2.59(g)

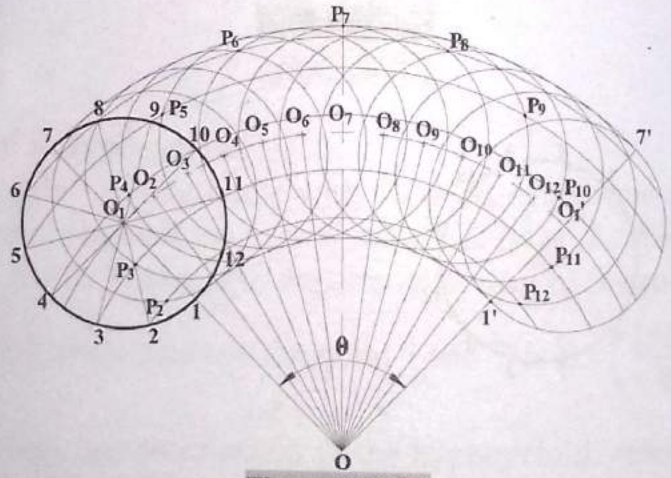


Figure 2.59(h)

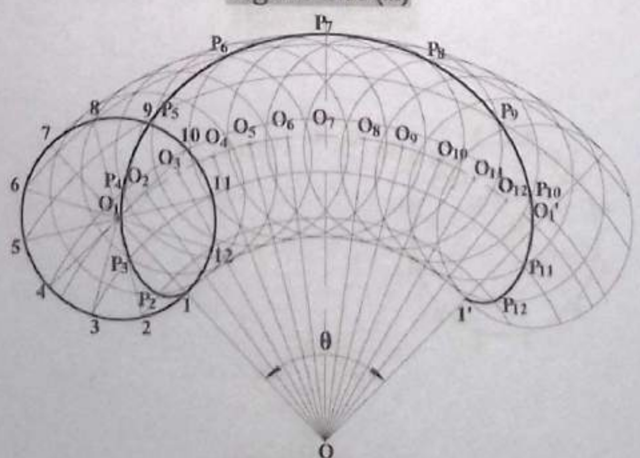


Figure 2.59(i)

2.5.3 Construction of an Archemedian Spiral

The plane curve traced by a point which moving uniformly away and uniformly around a fixed point is called an Archemedian Spiral. The linear distance travelled by the point during one revolution is called pitch or lead.

The Archemedian Spiral is used in the construction of cams, threads, scroll shocks, etc.

`The construction procedure for an Archemedian spiral is explained below.

- Draw a circle with O as center and radius equal to the given pitch. Divide the circle with any number of equal parts, say 12. Name the dividing points as 1, 2, 3, ….., 12.*(Figure 2.60(a))*
- Divide radius O-12 into the same number of parts as that of the circle and again name the dividing points on the radius as 1, 2, 3, ….., 12. *(Figure 2.60(b))*
- With O as center and O1, O2, O3 as radii, draw arcs which intersect the radial line O1 at point a, O2 at point b, O3 at point c, and so on. *(Figure 2.60(c))*
- Draw smooth curve passing through the points O, a, b, c, ….. and 12 to get the required spiral. *(Figure 2.60(d))*

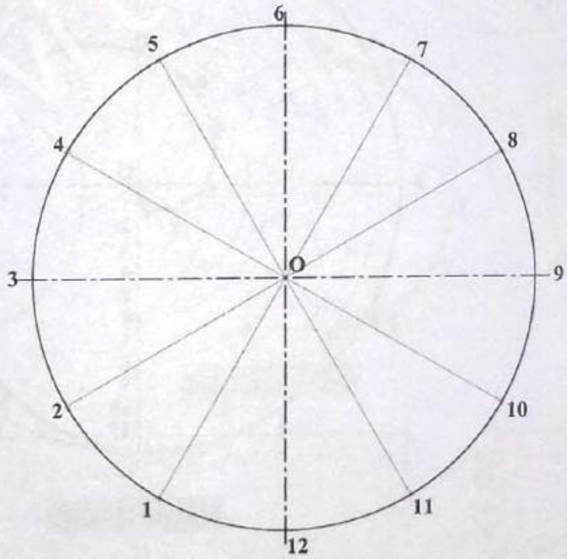


Figure 2.60(a)

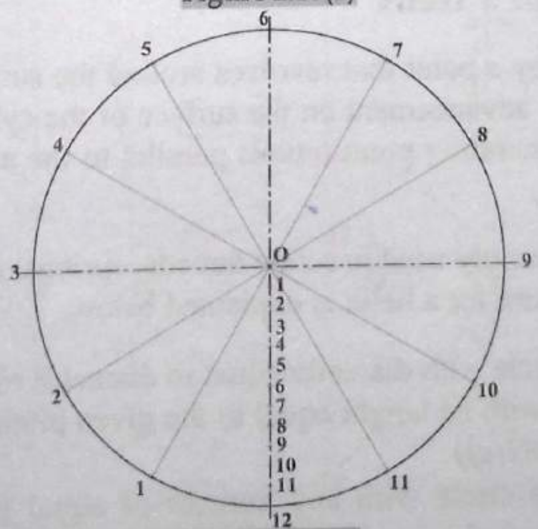


Figure 2.60(b)

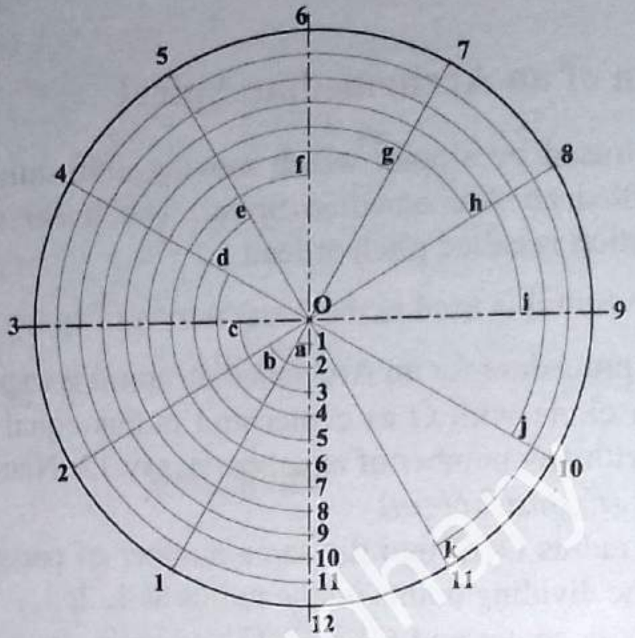


Figure 2.60(c)

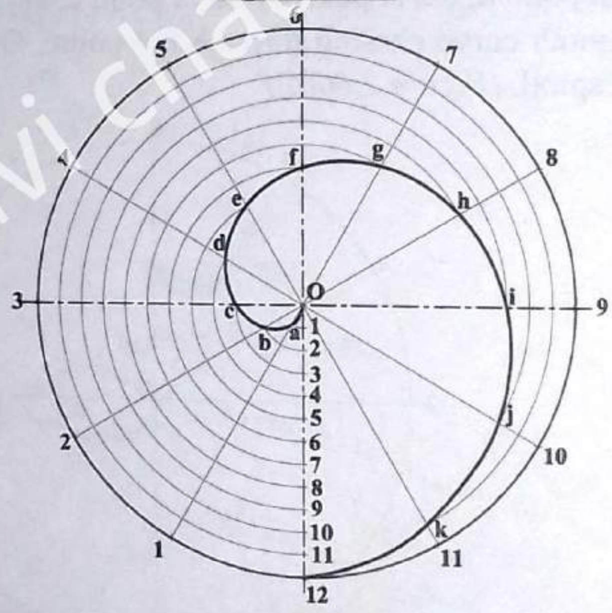


Figure 2.60(d)

2.5.4 Construction of a Helix

The space curve traced by a point that revolves around the surface of a right circular cylinder in such a way that its axial advancement on the surface of the cylinder is uniform is called a helix. The distance that the generating point travels parallel to the axis in one revolution is called the pitch or lead of the helix.

Helical profiles are commonly used in screw threads, springs, spiral staircase, conveyers, etc. The construction procedure for a helix is explained below.

- Draw a circle with diameter equal to diameter of the cylinder as its top view and a rectangle with its height equal to the given pitch as the front view of the cylinder. *(Figure 2.61(a))*
- Divide the circle with any number of equal parts, say 12. Name the dividing points as 1, 2, 3, ….., 12. *(Figure 2.61(b))*

- Divide the height of the cylinder (pitch) into the same number of parts as that of the circle and again name the dividing points on the radius as 1, 2, 3, ….., 12. *(Figure 2.61(c))*
- Draw vertical lines from each point on the circumference of the circle and horizontal lines from each point one the pitch. *(Figure 2.61(d))*
- Mark the intersection points of horizontal and vertical lines passing through 1 and 1, 2 and 2, 3 and 3, and so on. *(Figure 2.61(e))*
- Draw smooth curve passing through each points. Half portion of the curve will not be visible from the front and therefore drawn as hidden line. *(Figure 2.61(f))*

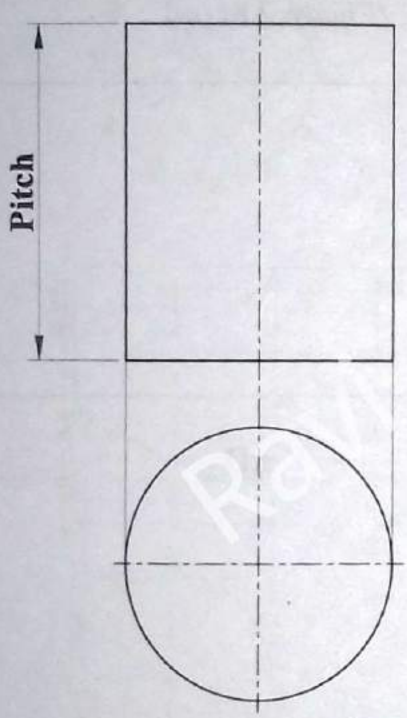
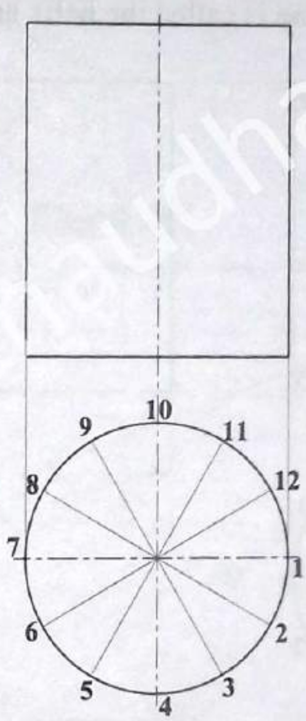


Figure 2.61(a)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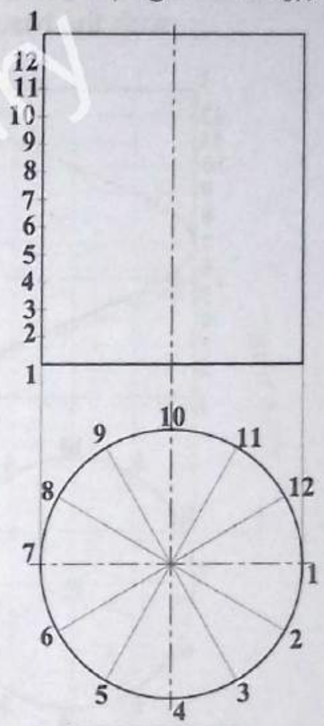


Figure 2.61(c)

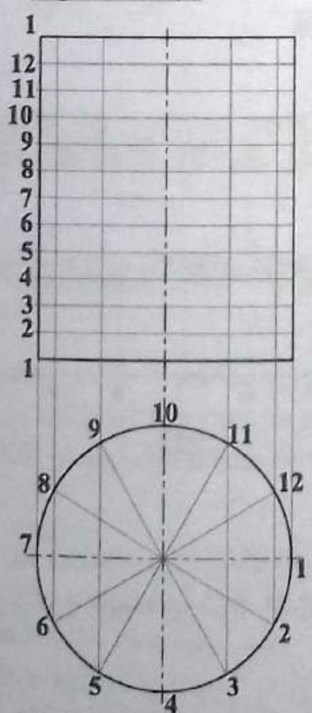


Figure 2.61(d)

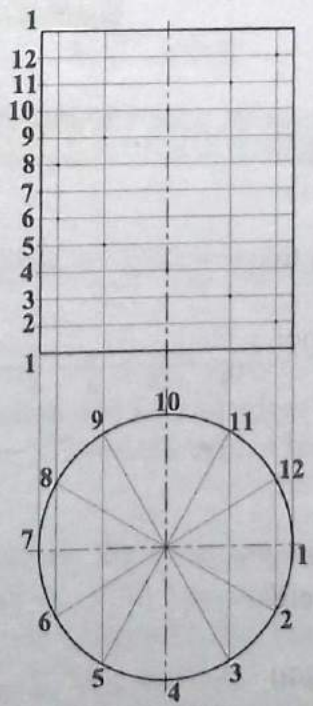


Figure 2.61(e)

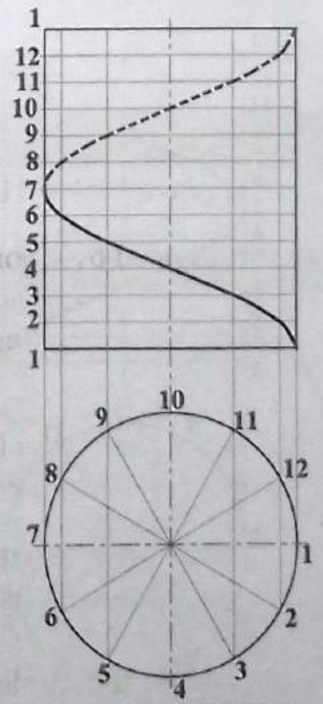


Figure 2.61(f)

Helix Development

- Draw a rectangle with its length equal to the circumference of the cylinder as the development of the surface of the cylinder. *(Figure 2.61(g))*
- Divide the circumference into the same number of equal parts as that used for the circle. Draw vertical lines from each dividing points. *(Figure 2.61(h))*
- Extend horizontal lines from the front view towards the development. Mark the intersection points of lines passing through 1 and 1, 2 and 2, 3 and 3, and so on. *(Figure 2.61(i))*
- A straight line is formed, when all these points are joined. Inclination of the line with the base line is called the helix angle. *(Figure 2.61(h))*

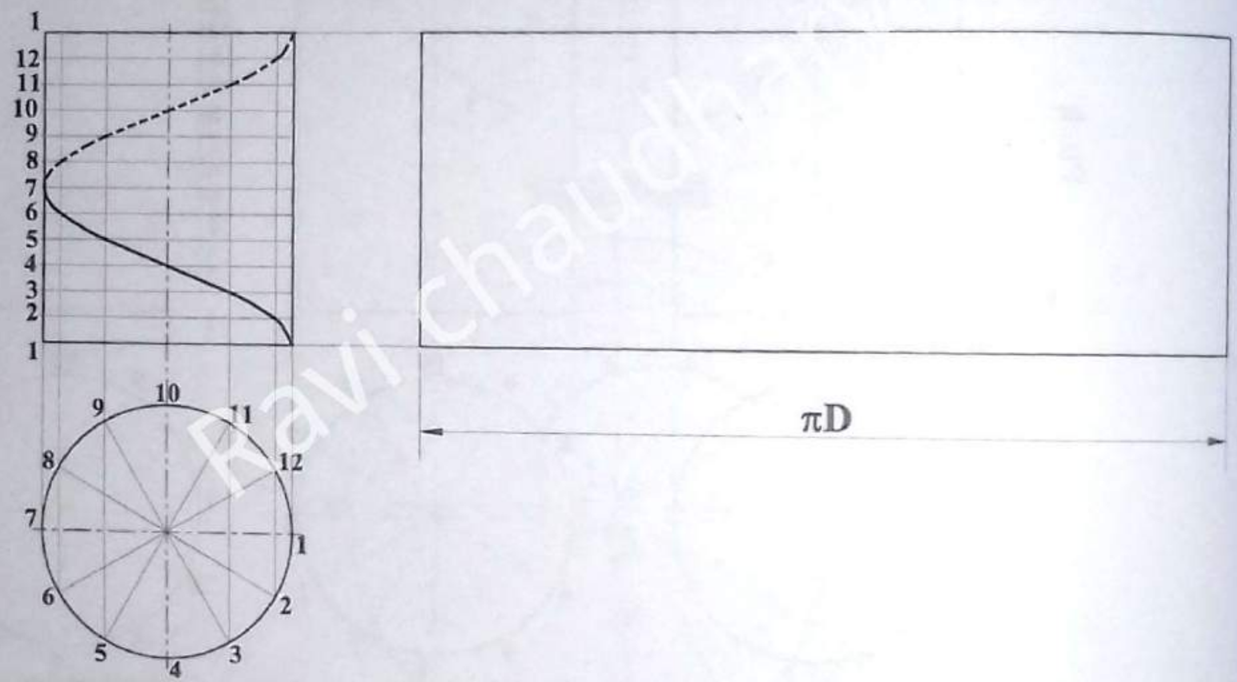


Figure 2.61(g)

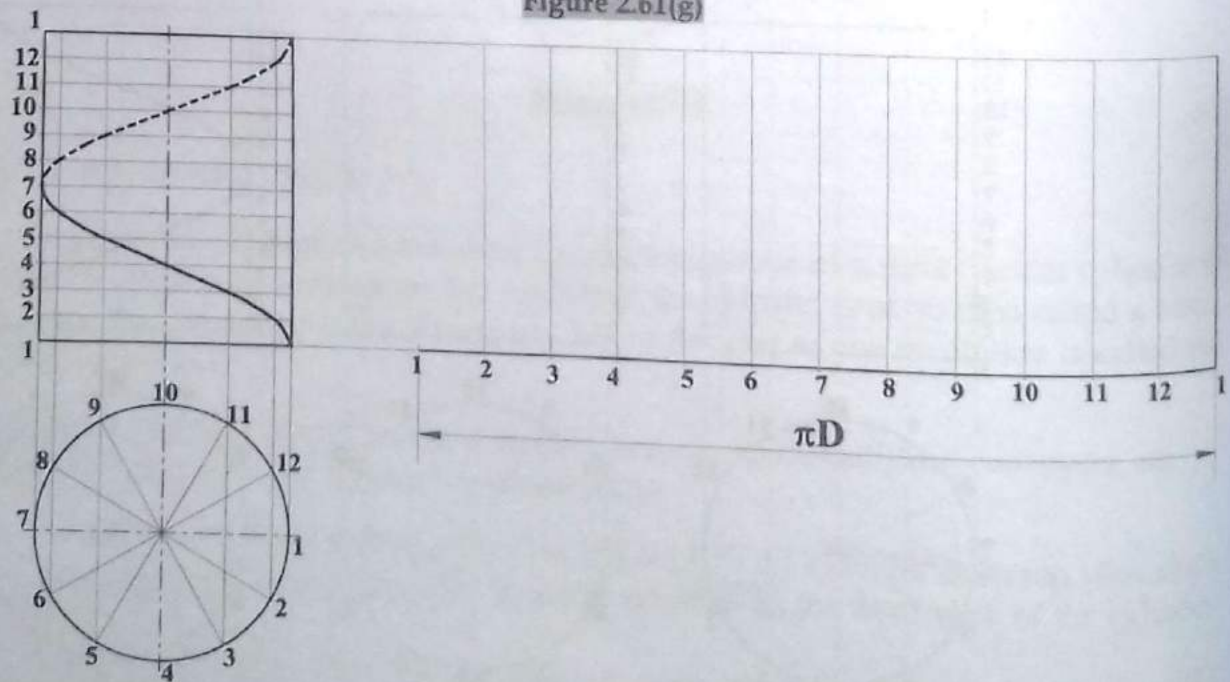


Figure 2.61(h)

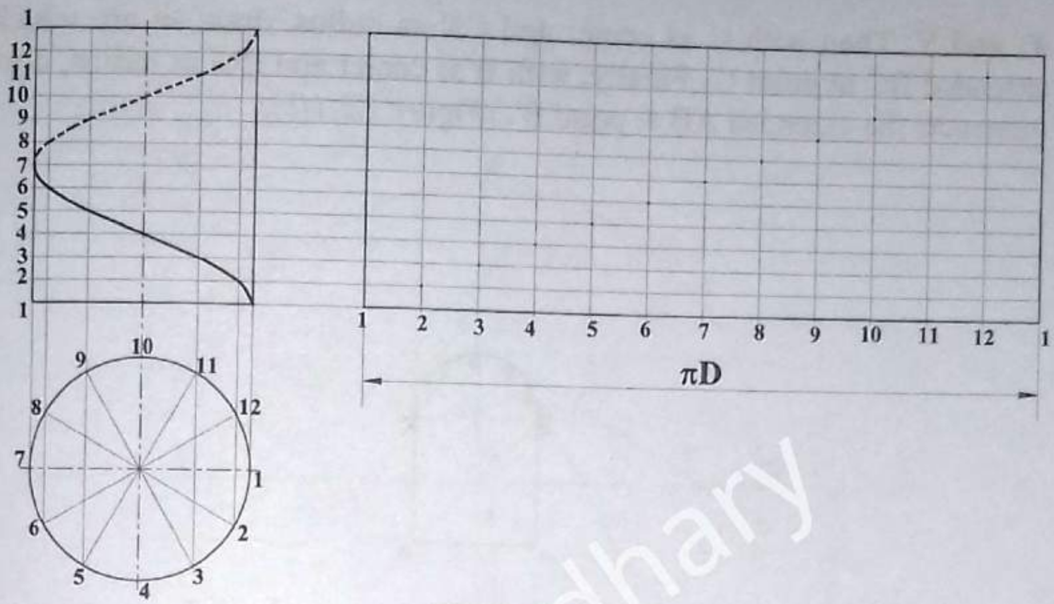


Figure 2.61(i)

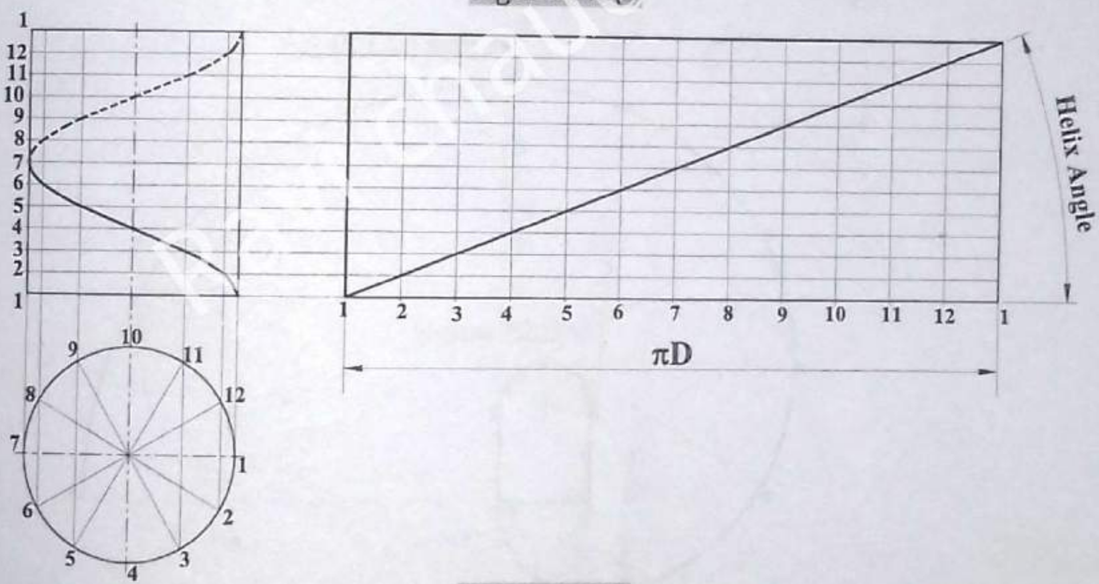


Figure 2.61(j)

Workout Examples

Example 2.1

Draw an involute of the plane figure shown in *Figure E2.1*.

Solution

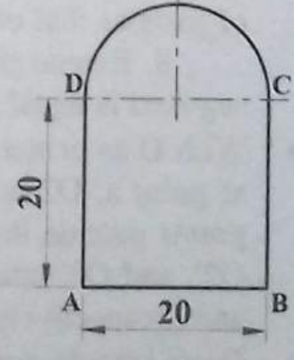


Figure E2.1

- Divide semicircle into 6 numbers of equal parts. Draw tangents at each of the dividing points and extend edges BC, AB and AD in the direction shown. *(Figure E2.1(a))*
- With A as center and AB as radius, draw an arc which intersects the extended DA at point A'. With 1 as center and 1A' as radius, draw an arc which intersects the tangent passing through point 1 at point 1'. With 2 as center and 21' as radius, draw an arc which intersects the tangent passing through 2 at point 2'. In the similar way, determine points 3',

4', and 5'. Then with C as center and C5' as radius, draw an arc which intersects the extended BC at point C'. Finally, with B as center and BC' as radius, draw an arc which intersects the extended AB at point B'. *(Figure E2.1(b))*

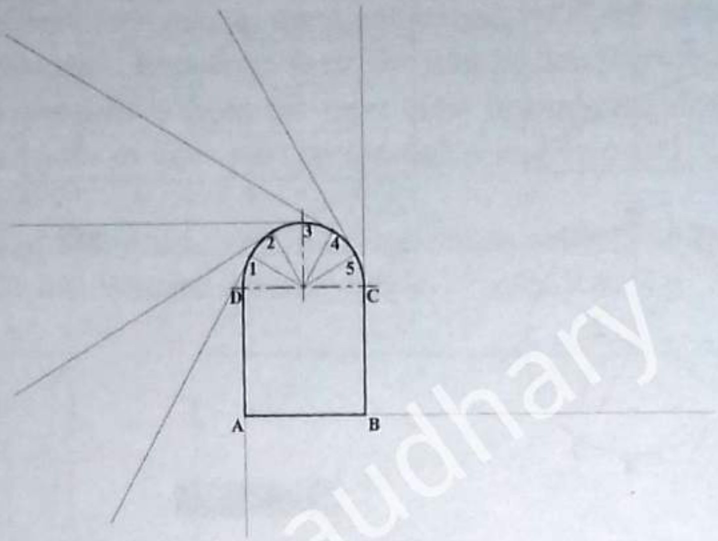


Figure E2.1(a)

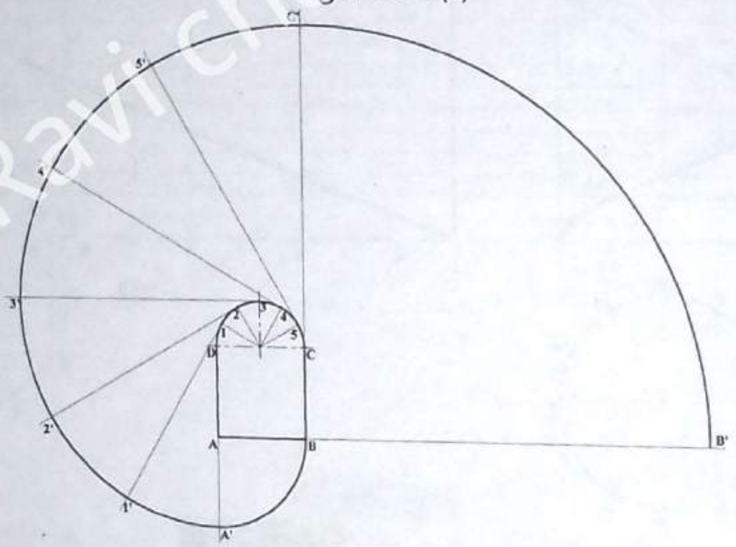


Figure E2.1(b)

Example 2.2

Draw an Archemedian Spiral for 1.5 convolutions with pitch equal to 40 mm.

Solution

- Draw a circle with O as center and radius of 40 mm. Divide the circle into 8 equal parts and name the dividing points as 1, 2, 3, ….. 8. Divide radius O-8 into the same number of parts as that of the circle and again name the dividing points on the radius as 1, 2, 3, ….., 8. Extend the radial line O-8 and mark the points 1', 2', 3' and 4' such that each segment is equal to the 1/8 th part of the radius. *(Figure E2.2(a))*
- With O as center and O1, O2, O3 as radii, draw arcs which intersect the radial line O1 at point a, O2 at point b, O3 at point c, and so on. In the similar way, determine the points outside the pitch radius by drawing arc with center at point O and radii as O1', O2', and O3' intersecting the extended radial lines O1, O2, O3 and O4 at points i, j, k and l respectively. *(Figure E2.2 (b))*
- Draw smooth curve passing through the points O, a, b, c, …..k and l to get the required spiral. *(Figure E2.2 (c))*

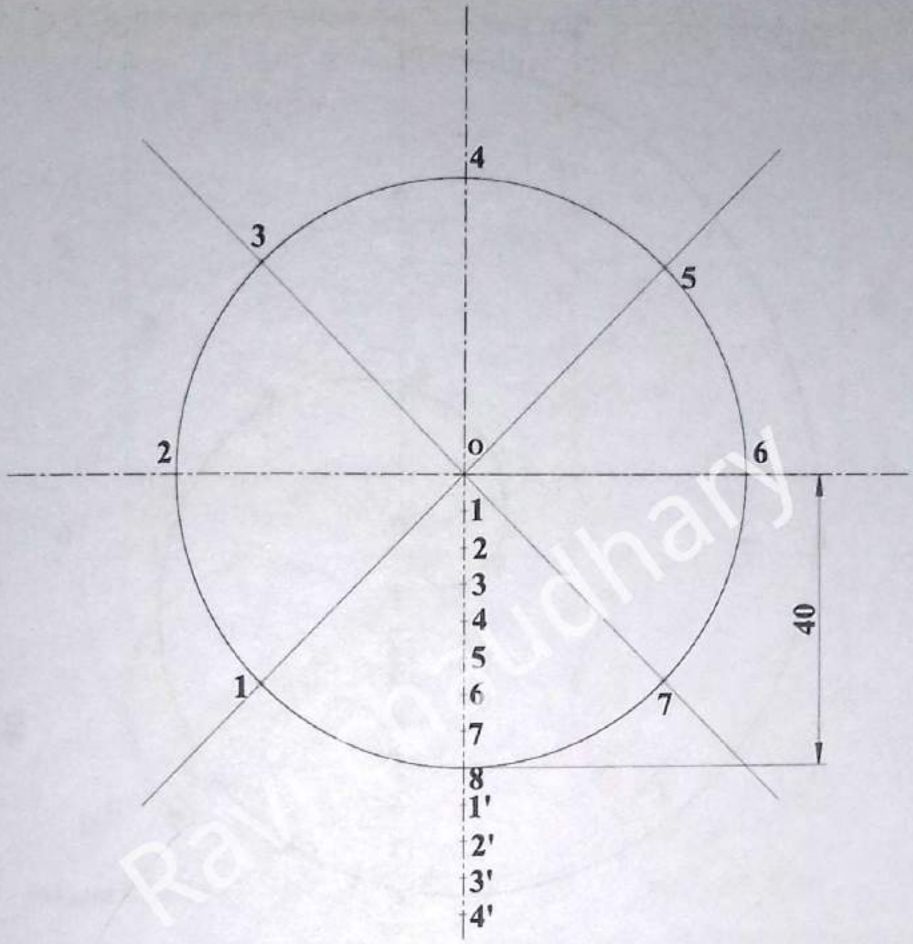


Figure E2.2(a)

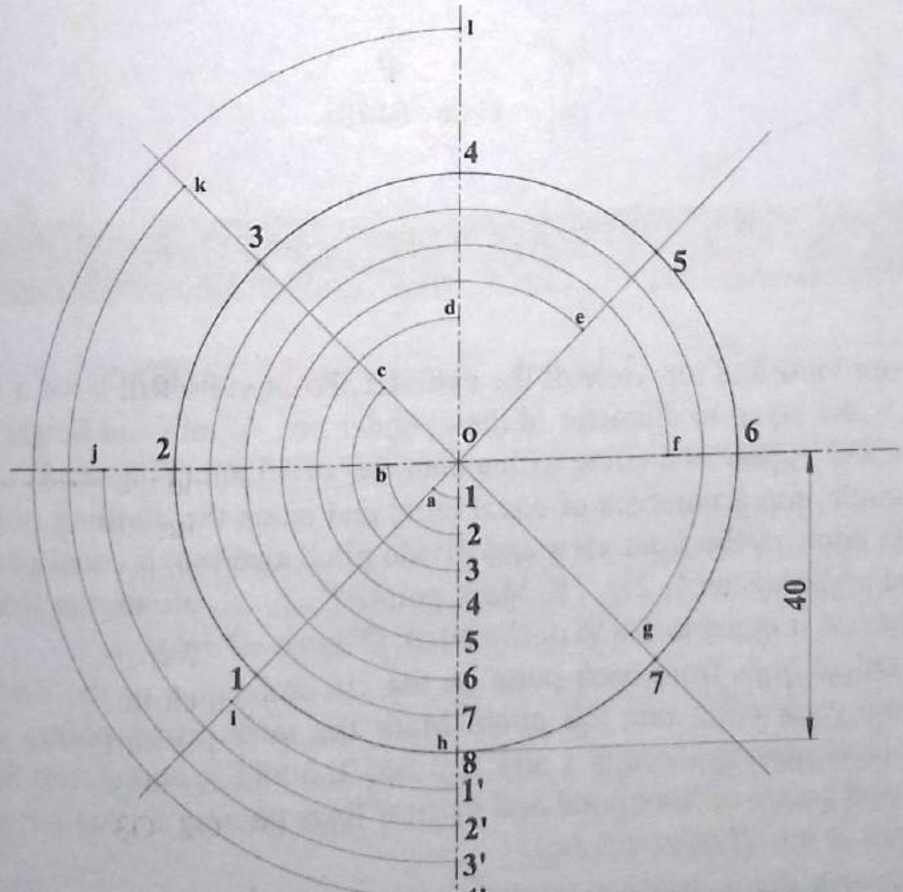


Figure E2.2(b)

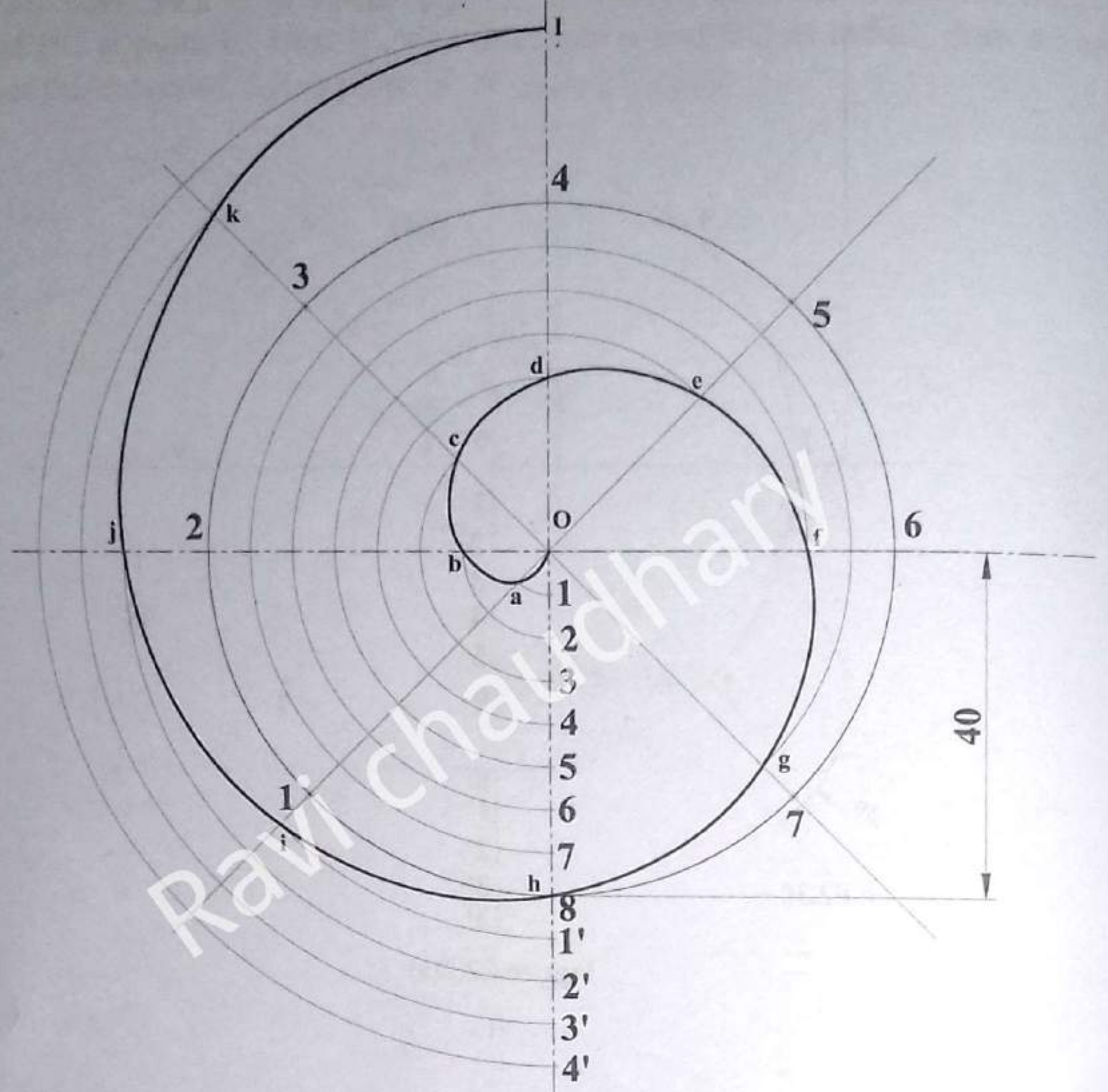


Figure E2.2(c)

Example 2.3

Draw helix for having a pitch of 50 mm on a cylinder with the diameter of 40 mm and height of 65 mm.

Solution

- Draw front view and top view of the cylinder. Front view will have a rectangular shape with its width equal to diameter of the cylinder i.e., 40 mm and height of 65 mm. while top view will appear as a circle with a diameter of 40 mm. *(Figure E2.3(a))*
- Divide circle into 8 numbers of equal parts and name the dividing points as 1, 2 …, 8. Mark the pitch on the front view and divide pitch also into 8 equal parts and also name the dividing points as 1, 2 …, 8. Mark points 1', 2', …. etc on the front view such that each segment is equal to 1/8 th of the pitch. *(Figure E2.3(b))*
- Draw vertical lines from each point on the circumference of the circle and horizontal lines from each point one the pitch. Mark the intersection points of horizontal and vertical lines passing through 1 and 1, 2 and 2, 3 and 3, and so on. Similarly mark the intersection points of horizontal and vertical lines passing through 1' and 1, 2' and 2, 3' and 3, and so on. *(Figure E2.3(c))*
- Draw smooth curve passing through each points to get the required helix. *(Figure E2.3(d))*

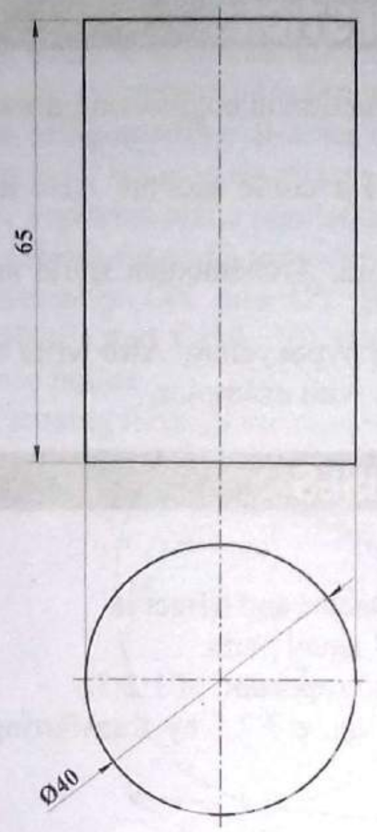


Figure E2.3(a)

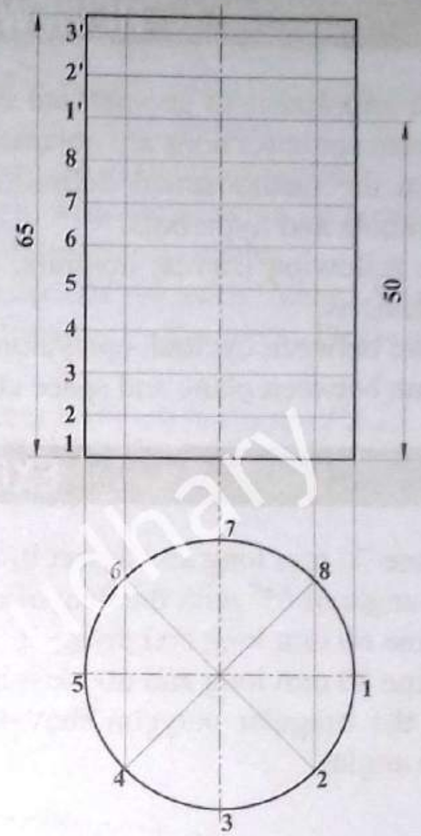


Figure E2.3(b)

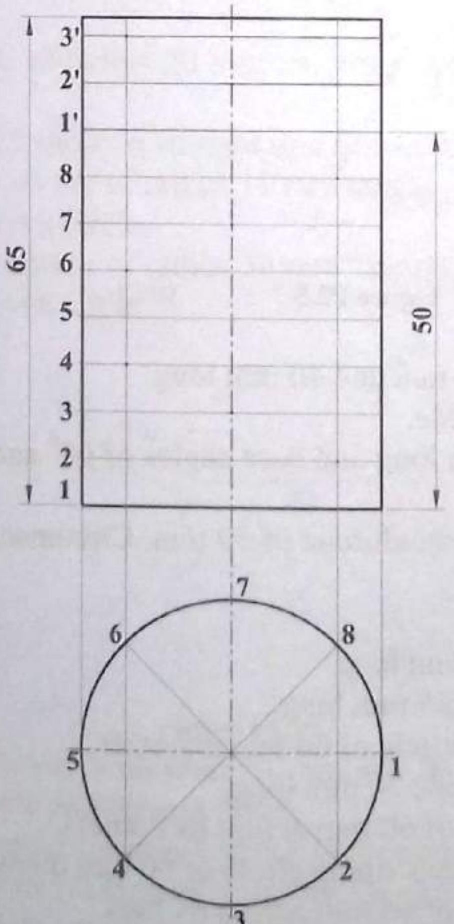


Figure E2.3(c)

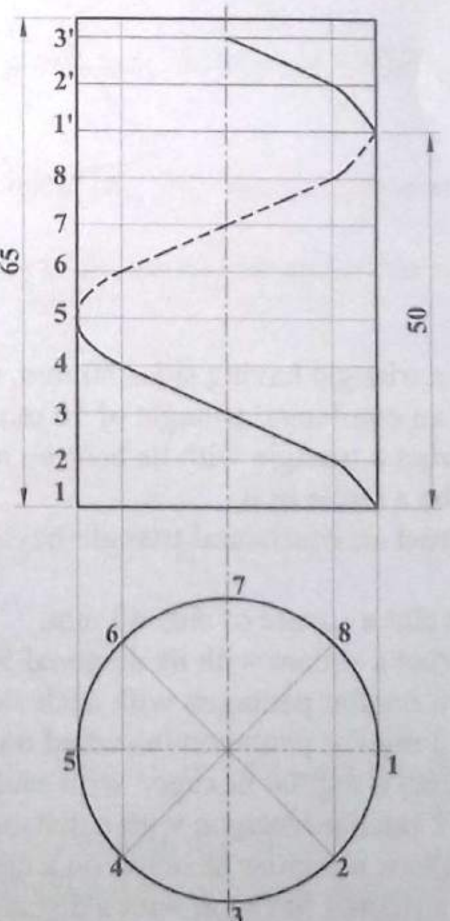


Figure E2.3(d)

REVIEW QUESTIONS

1. Explain the importance of geometrical construction in engineering drawing.
2. How different conic sections are obtained?
3. Write down the mathematical definitions of a conic section. Also differentiate between ellipse, parabola and hyperbola.
4. Define the following curves: involute, cycloid, Archemedian spiral and helix. Also write their applications.
5. Differentiate between cycloid, epicycloid and hypocycloid. Also write their applications.
6. Differentiate between plane and space curves with examples.

EXERCISES

1. Draw a line 70 mm long and trisect it.
2. Draw an angle of 65^0 with the help of a protector and trisect it.
3. Draw a line 60 mm long and divide it into 7 equal parts.
4. Draw a line 80 mm long and divide it in the proportion of 1:2:3.
5. Transfer the irregular polygon shown in *Figure P2.5* by transferring the corresponding sides and angles.

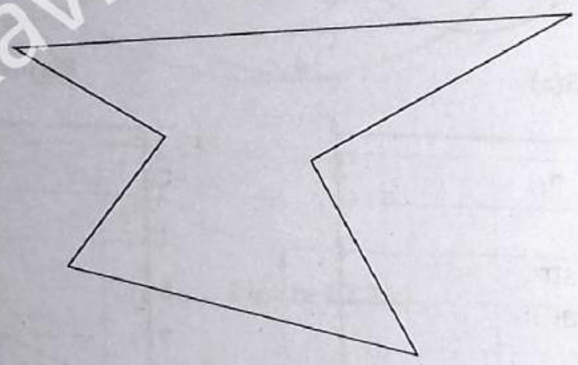


Figure P2.5

6. Draw a triangle having sides 50 mm, 60 mm and 70 mm long.
7. Draw an equilateral triangle of 50 mm side.
8. Construct a triangle with its base 60 mm long and base angles of 60^0 and 45^0 respectively. Inscribe a circle in it.
9. Construct an equilateral triangle having an altitude of 50 mm. Circumscribe a circle about it.
10. Constraint a square of side 40 mm.
11. Construct a square with its diagonal 50 mm long.
12. Draw a regular pentagon with each side 25 mm long.
13. Draw a regular pentagon inscribed on a circle of 60 mm diameter.
14. Construct a regular hexagon with each side 20 mm long.
15. Draw a regular hexagon with a distance of 60 mm across its corners.
    (OR) Draw a regular hexagon on a circumscribing circle of 60 mm diameter.
16. Draw a regular hexagon with a distance of 66 mm across its flats.
    (OR) Draw a regular hexagon on an inscribed circle of 66 mm diameter.

17. Construct a regular octagon with each side 20 mm long.
18. Draw a regular octagon with a distance of 60 mm across its corners.
(OR) Draw a regular octagon on a circumscribing circle of 60 mm diameter.
19. Draw a regular octagon with a distance of 66 mm across its flats.
(OR) Draw a regular octagon on an inscribed circle of 66 mm diameter.
20. Draw a regular heptagon and a regular nonagon with each having 20 mm side.
21. Draw an arc as the outline of a protector and determine its center.
22. Draw two references OX and OY perpendicular to each other. Mark three points A(20,20), B(50,40) and C(10, 50) with reference to the axes. Draw a circle passing through all these points.
23. Draw tangent passing through the external point P shown in *Figure P2.23*.

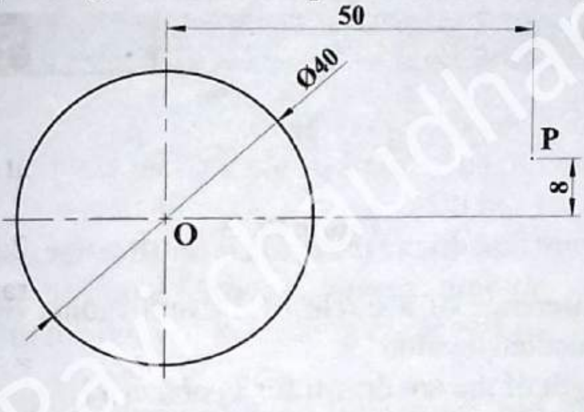


Figure P2.23

24. Draw an arc of radius 20 mm tangent to two given lines inclined at **(a)** 60^0, **(b)** 90^0, and **(c)** 120^0.
25. *Figure P2.25* shows a straight line and a circle.
    (a) Draw an arc of radius 18 mm tangent to both the given line and circle and outside to the given circle.
    (b) Draw an arc of radius 30 mm tangent to both the given line and circle and including the given circle.

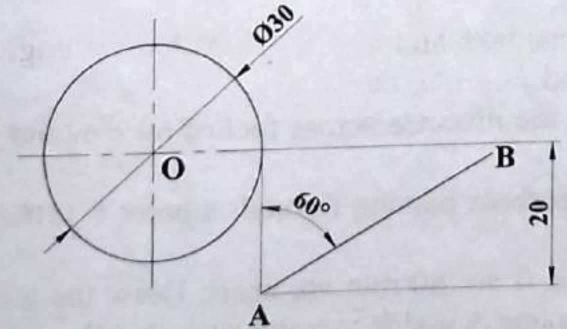


Figure P2.25

26. Draw two circles with radii 15 mm and 20 mm respectively with their centers lying on a horizontal line and 60 mm apart.
    (a) Draw an arc tangent of radius 40 mm outside to both the circles.
    (b) Draw an arc tangent of radius 50 mm including the circle with radius 15 mm.
    (c) Draw an arc tangent of radius 55 mm including the circle with radius 20 mm.
    (d) Draw an arc tangent of radius 60 mm including both the circles.

27. Draw two circles with radii 20 mm and 30 mm respectively with their centers lying on a horizontal line and 60 mm apart. Draw internal and external line tangents to the circles.

28. Draw a reverse curve (ogee curve) between the given lines AB and CD shown in *Figure P2.28*.

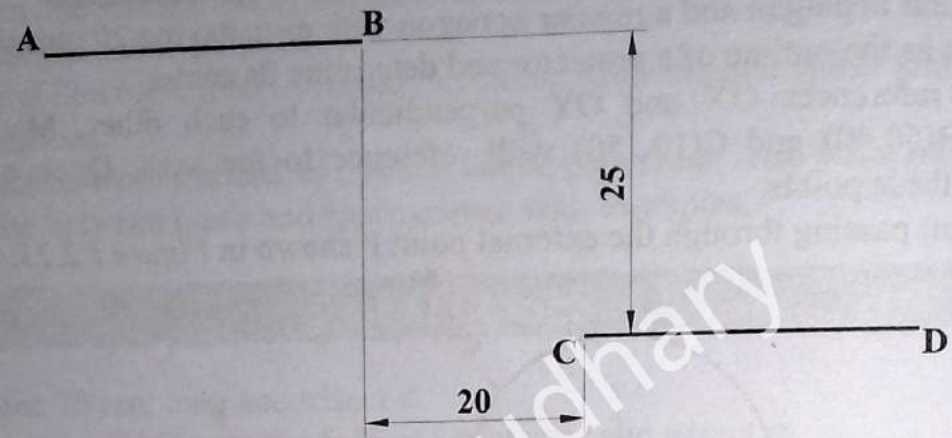


Figure P2.28

29. Determine the circumference of a circle of 14 mm radius by construction method and compare it with the calculated value.

30. Determine the arc length of the arc drawn for *Problem 21*.

31. Draw any line AB as the directrix of the conic section. Draw any line CD perpendicular to AB as the axis. Mark a point F on the line CD at a distance of 50 mm from the line AB as the focus of the conic section. Draw the locus of point moving such that the ratio of its distance from the point F to its distance from the line AB is **(a)** 2/3, **(b)** 1 and **(c)** 3/2. Take at least 5 points for each curve and name the curve.

32. Draw a parabola with axis length of 60 mm and double ordinate of 80 mm using
    (a) rectangle method, and
    (b) tangent method.

33. Draw an ellipse with major and minor axes of 80 mm and 60 mm respectively by using
    (a) definition method,
    (b) concentric circle method, and
    (c) four center method.

34. Draw a hyperbola with the distance across foci of 60 mm and transverse axis distance of 40 mm.

35. Draw a rectangular hyperbola passing through a point P (5,50) with reference to X- and Y-axes.

36. Two fixed points A and B are 80 mm are apart. Draw the locus of point P lying on the same plane as that of points A and B moving such that the sum of its distance from fixed points A and B is always constant and equal to 100 mm. Name the curve.

37. Two fixed points A and B are 60 mm are apart. Draw the locus of point P lying on the same plane as that of points A and B moving such that the difference of its distance from fixed points A and B is always constant and equal to 30 mm. Name the curve.

38. Draw the involutes of the plane figures shown in *Figure P2.38*.

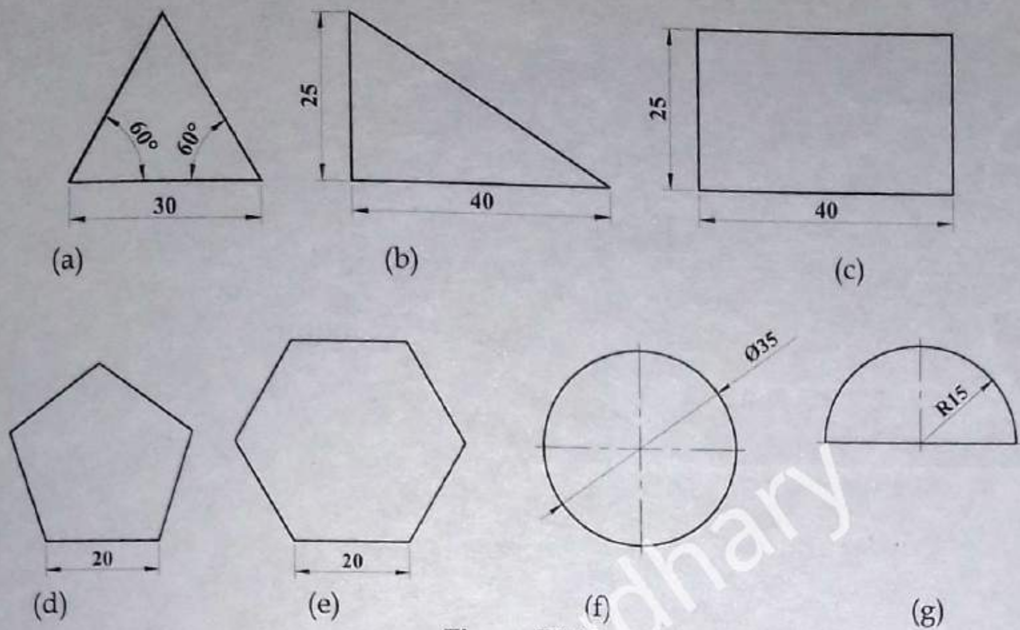
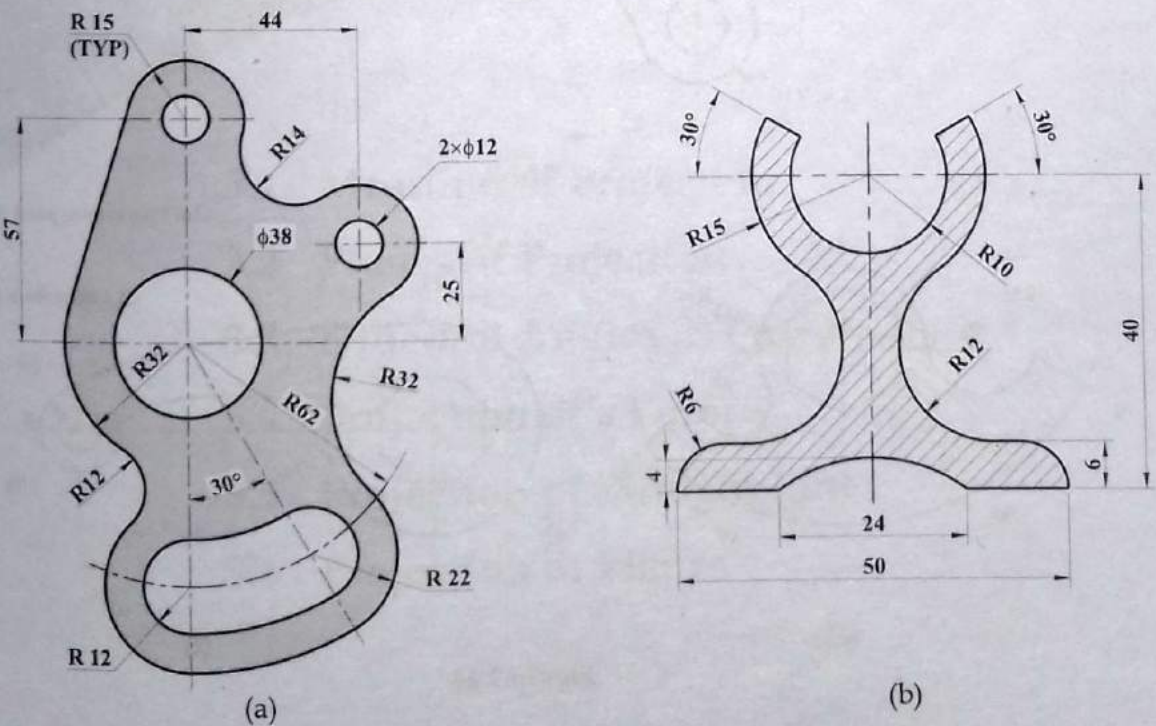
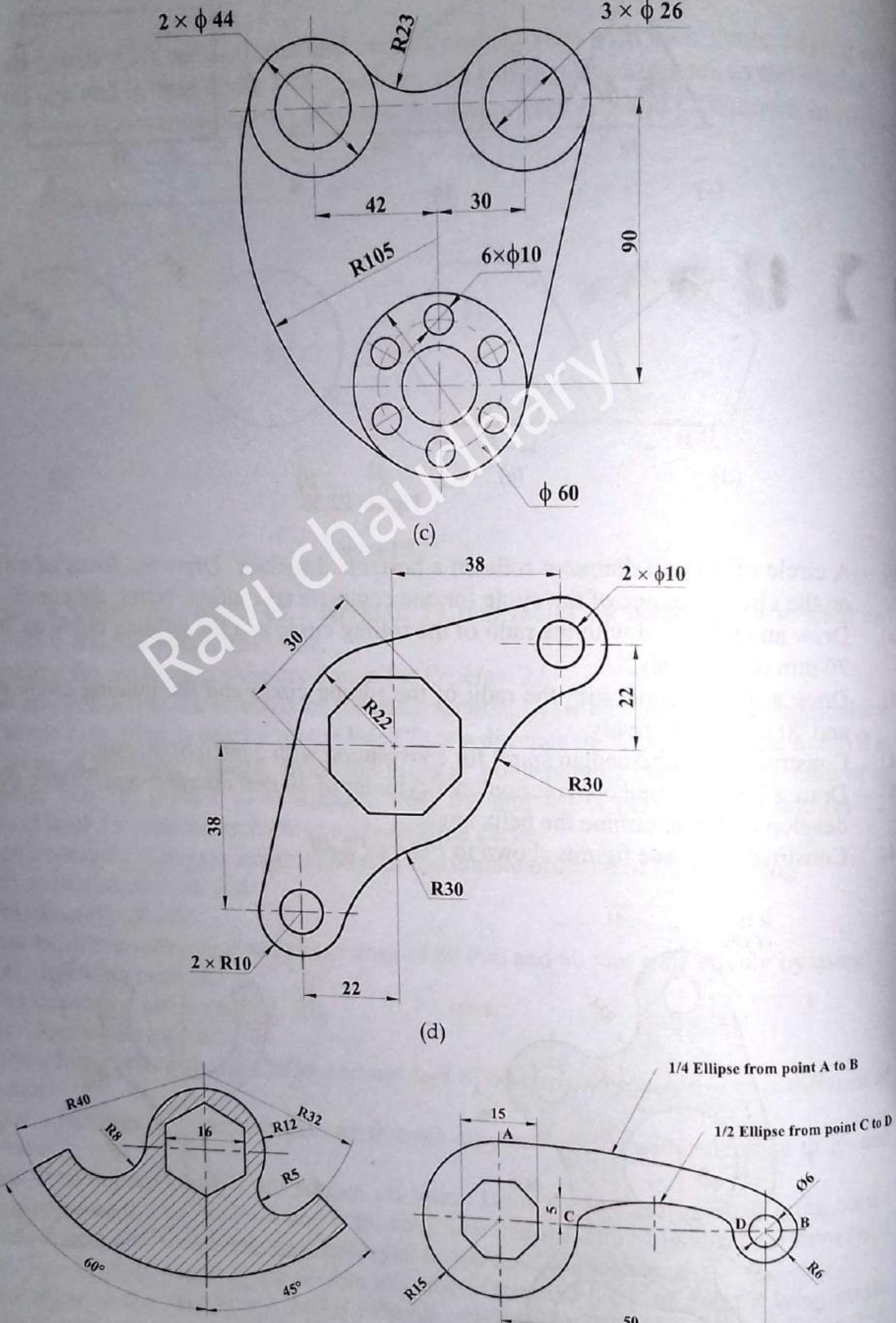


Figure P2.38

39. A circle of 50 mm diameter rolls on a horizontal surface. Draw the locus of a fixed point on the circumference of the circle for one complete revolution. Name the curve.
40. Draw an epicycloid with the radii of the rolling circle and the guiding circle as 20 mm and 70 mm respectively.
41. Draw a hypocycloid with the radii of the rolling circle and the guiding circle as 20 mm and 70 mm respectively.
42. Construct an Archemedian spiral for convolution with a pitch of 40 mm.
43. Draw a helix for one convolution on a cylinder of 40 mm diameter and 50 mm pitch. Also develop it and determine the helix angle.
44. Construct the plane figures shown in *Figure P2.44*.





2 × ϕ 44
R23
3 × ϕ 26
42
30
90
R105
6×ϕ10
ϕ 60
(c)
38
2 × ϕ10
30
R22
22
R30
38
R30
2 × R10
22
(d)
R40
R8
16
R12
R32
R5
60°
45°
(e)
1/4 Ellipse from point A to B
15
A
1/2 Ellipse from point C to D
Ø6
5
C
D
B
R15
R6
50
(f)

Figure 2.44

CHAPTER 3

DESCRIPTIVE GEOMETRY

The representation of any three dimensional objects on a two dimensional sheet is done by the projection. Projection drawing is based on descriptive geometry. Descriptive geometry is also called the theory of projection. Descriptive geometry provides practical methods of representation of the geometrical solids and their combinations, various objects, machine parts and instruments. It also provides the foundation for the three dimensional visualization and knowledge of logical reasoning. It can also be applied to solve different space geometrical problems.

3.1 Meaning of Projection

Consider an arrangement for a simple phenomenon of the formation of the shadow, as shown in **Figure 3.1**. When an object is placed between a light source and a screen, shadow is formed on the screen when the light rays coming from the light source strike the object. The size of the shadow formed in this case will be greater than the real size of the object.

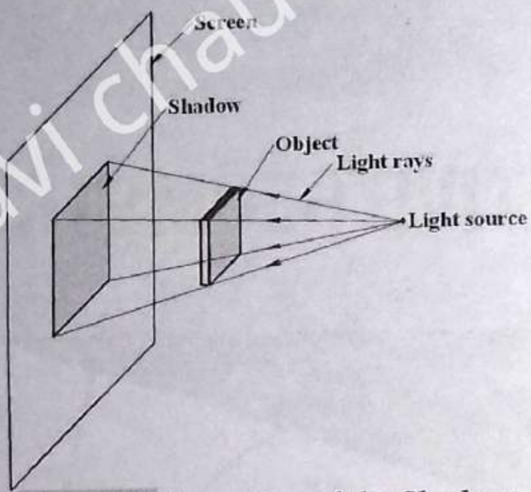


Figure 3.1: Formation of the Shadow

If the light source is moved far from the object, the size of the shadow decreases. When the light source is at infinite distance from the objects, the light rays become almost parallel to each other and the size of the shadow thus formed will be equal to that of the real size of the object, as shown in *Figure 3.2*.

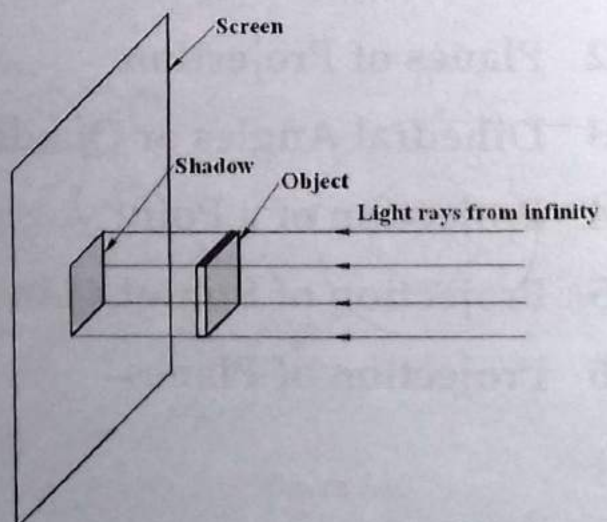


Figure 3.2: Formation of the Shadow due to Light Source at Infinity

This is the condition which is called projection in the language of drawing. The terms used in *Figure 3.2* such as screen, shadow and rays are replaced accordingly by the appropriate terms as shown in *Figure 3.3*. Light rays are called projection lines or projectors, screen is called a projection plane and the shadow is called a projection or view of the object.

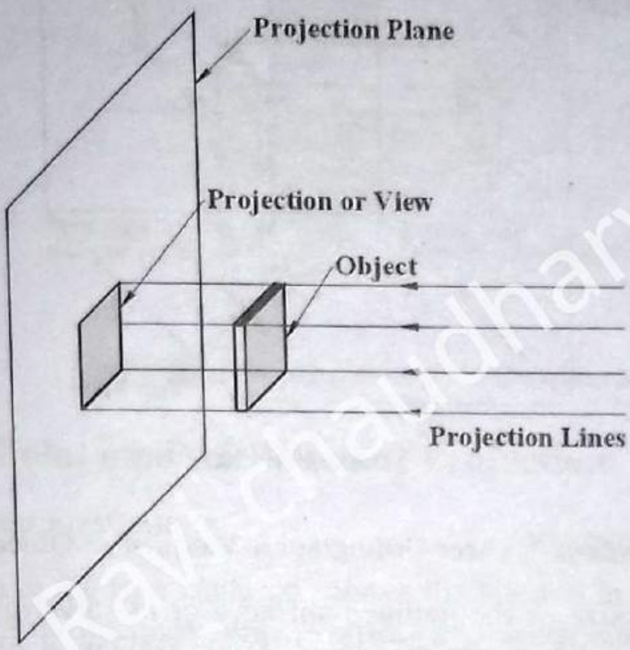


Figure 3.3: Orthographic Projection

Hence, the projection in which projection line are parallel to each other and perpendicular to the projection plane is called an orthographic projection.

3.2 Planes of Projection

The plane used to obtain the projection or view of an object is called a plane of projection. According to its orientation, the projection plane shown in *Figure 3.3* is called a vertical plane (VP). The view of the object obtained on the vertical plane is called a front view or frontal elevation of the object.

The front view of the object shown in *Figure 3.3* cannot describe the object completely because it shows only to dimensions of the object, i.e. length and height. If the top view of the object is to be drawn simultaneously, horizontal projection (HP) should be used, as shown in *Figure 3.4*. The intersection of vertical projection lines passing through the corners of the object with the horizontal plane gives the top view or plan of the object. Similarly, if the side view of the object is also to be drawn, the projection plane perpendicular to both the vertical and horizontal plane should also be used, as shown in *Figure 3.4*. The plane of projection used to draw side view is called an auxiliary vertical plane or profile plane (PP).

The three projection planes used for drawing front view, top view and side view are called principal planes of projection and the corresponding views are called the principal views of the object.

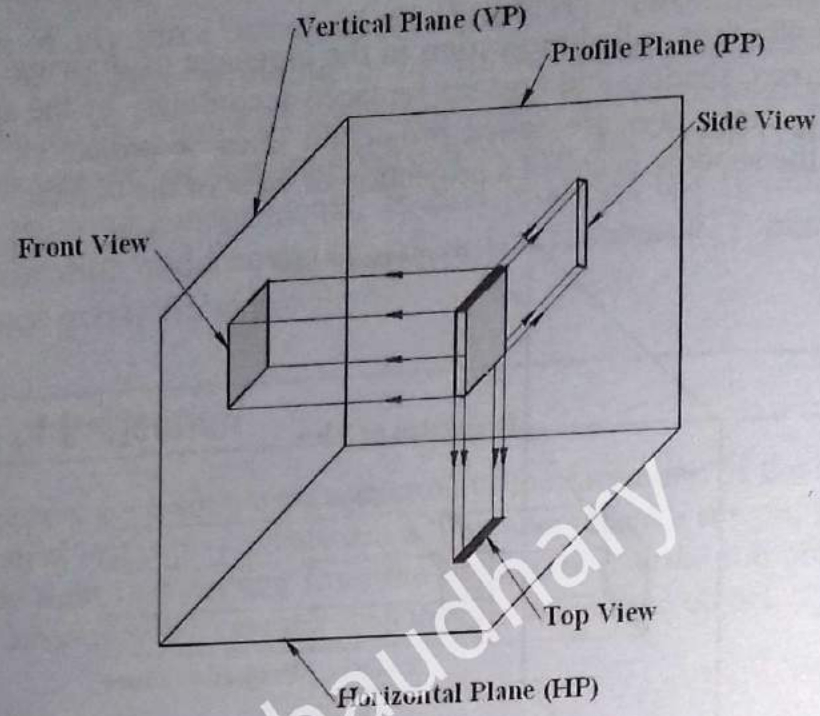


Figure 3.4: Three Orthographic Views of an Object

Sometimes to get the real size of the inclined surfaces or to solve different spatial descriptive problems incline projection planes are also used. Such projection planes are called auxiliary planes of projection and the views of the object obtained on these planes are called auxiliary views.

3.3 Dihedral Angles or Quadrants

The two principal planes of projection most commonly used are horizontal plane (HP) and vertical plane (VP). These planes intersecting each other at right angle, divide the space into four dihedral angles or quadrants, as shown in *Figure 3.5*. The line of intersection between these planes OX is called a reference line. Any position in space with reference to the principal planes can be defined as:

First angle:	above the HP and in front of the VP
Second angle:	above the HP and behind the VP
Third angle:	below the HP and behind the VP
Fourth angle:	below the HP and in front of the VP

3.4 Projection of a Point

The objective of projection is to represents any complex shape with the help of its views. Any complex three dimensional shape is formed by the combination of different surfaces and any surface is formed by the combination of different lines. Any line is formed by the locus of points. Hence, any object is described graphically by the projections of its constituent elements. The simplest geometrical element is a point and therefore to have clear understanding of the projection of any complex object, orthographic projection of the point should be studied.

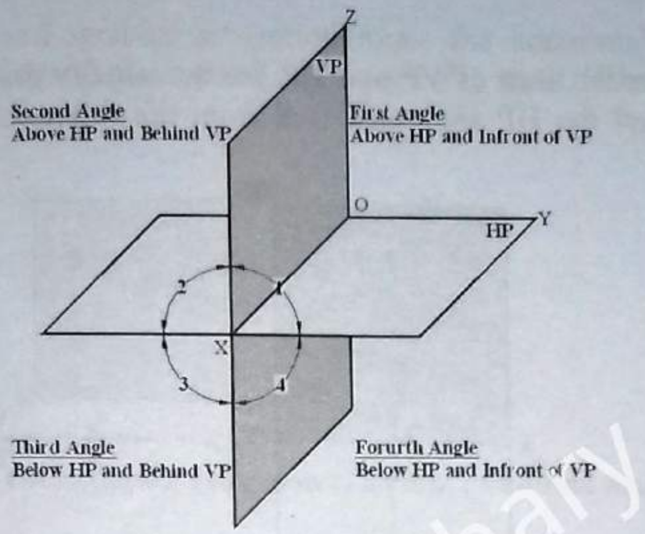


Figure 3.5: Four Angles or Four Quadrants

3.4.1 Projection of a Point on Two Planes of Projection

(a) Projection of a Point on First Angle

Consider a point **A** placed in the first angle i.e., above the HP and in front of the VP, as shown in *Figure 3.6*. To draw its projections, draw vertical and horizontal projection lines passing through point A. The intersection of the vertical projection line and the horizontal plane gives the top view **a** of the given point. Similarly, the intersection of the horizontal projection line and the vertical plane gives the front of the given **a'** of the given point. To illustrate the relative distance of the point from the principal planes of projections, draw a straight line passing through point **a** and parallel to **Aa'** which intersects the reference line OX at point $\mathbf{a_x}$. Similarly, draw another straight line passing through point **a'** and parallel to **Aa** which also passes through the point $\mathbf{a_x}$. Then $\mathbf{aa_x}$ is the distance of the given point **A** from the vertical plane (VP) and $\mathbf{a'a_x}$ is the distance of the point **A** from the horizontal plane (HP). Such type of projection in three dimensional form is called the pictorial projection. To convert it into an orthographic projection rotate the horizontal plane in clockwise direction, keeping the vertical plane fixed, such until ZO and OY become a straight line, as shown in *Figure 3.7(a)*.

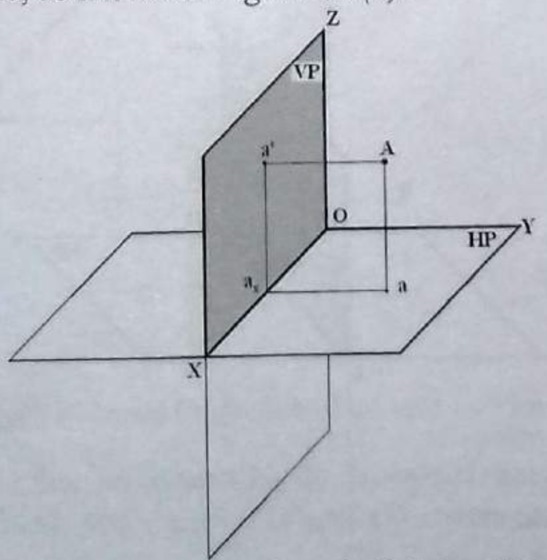


Figure 3.6: Pictorial Projection of a Point on First Angle

For the simplicity, border lines of VP and HP are not usually drawn and the distances of the point from the VP and the HP are transferred from the reference line OX as shown *Figure 3.7(b)*.

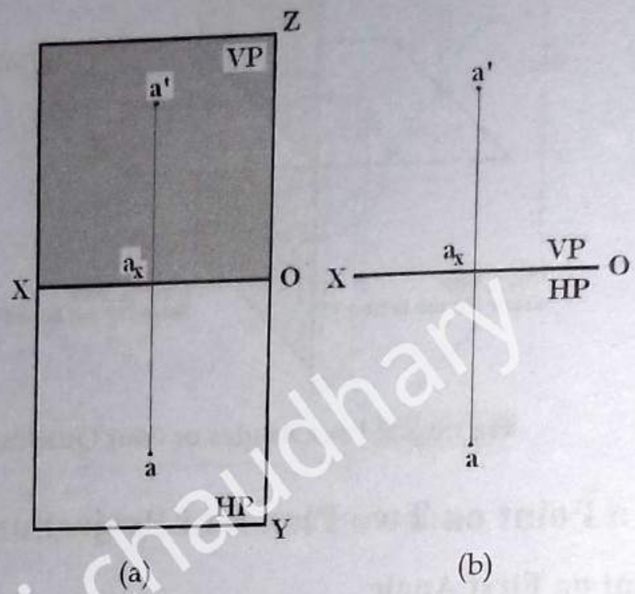


Figure 3.7: Orthographic Projection of a Point on First Angle

(b) Projection of a Point on Second Angle

Consider a point **B** placed in the second angle i.e., above the HP and behind the VP, as shown in *Figure 3.8*. To draw its projections, draw vertical and horizontal projection lines passing through point B. The intersection of the vertical projection line and the horizontal plane gives the top view **b** of the given point. Similarly, the intersection of the horizontal projection line and the transparent vertical plane gives the front of the given **b'** of the given point. To illustrate the relative distance of the point from the principal planes of projections, draw a straight line passing through point **b** and parallel to **Bb'** which intersects the reference line OX at point b_x. Similarly, draw another straight line passing through point **b'** and parallel to **Bb** which also passes through the point b_x. Then $\mathbf{bb_x}$ is the distance of the given point **B** from the vertical plane (VP) and $\mathbf{b'b_x}$ is the distance of the point **B** from the horizontal plane (HP).

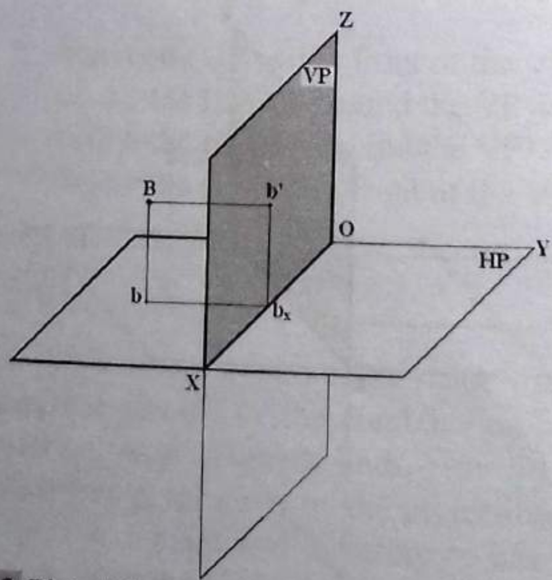


Figure 3.8: Pictorial Projection of a Point on Second Angle

To convert it into an orthographic projection rotate the horizontal plane in clockwise direction, keeping the vertical plane fixed, such until ZO and OY become a straight line, as shown in *Figure 3.9.*

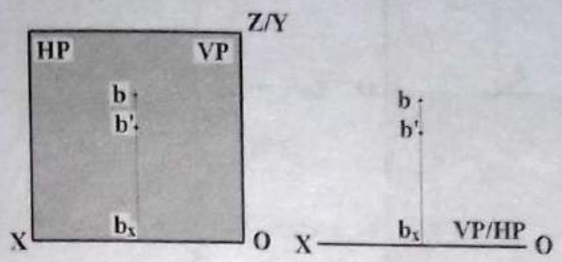


Figure 3.9: Orthographic Projection of a Point on Second Angle

(c) Projection of a Point on Third Angle

Consider a point **C** placed in the third angle i.e., below the HP and behind the VP, as shown in *Figure 3.10.* To draw its projections, draw vertical and horizontal projection lines passing through point C. The intersection of the vertical projection line and the transparent horizontal plane gives the top view **c** of the given point. Similarly, the intersection of the horizontal projection line and the transparent vertical plane gives the front of the given **c'** of the given point. To illustrate the relative distance of the point from the principal planes of projections, draw a straight line passing through point **c** and parallel to **Cc'** which intersects the reference line OX at point c_x. Similarly, draw another straight line passing through point **c'** and parallel to **Cc** which also passes through the point c_x. Then cc_x is the distance of the given point C from the vertical plane (VP) and $c'c_x$ is the distance of the point C from the horizontal plane (HP).

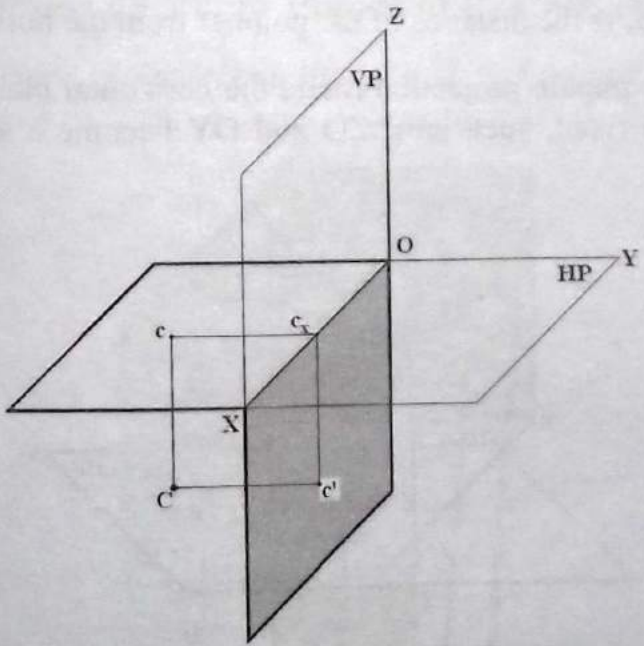


Figure 3.10: Pictorial Projection of a Point on Third Angle

To convert it into an orthographic projection rotate the horizontal plane in clockwise direction, keeping the vertical plane fixed, such until ZO and OY become a straight line, as shown in *Figure 3.11.*

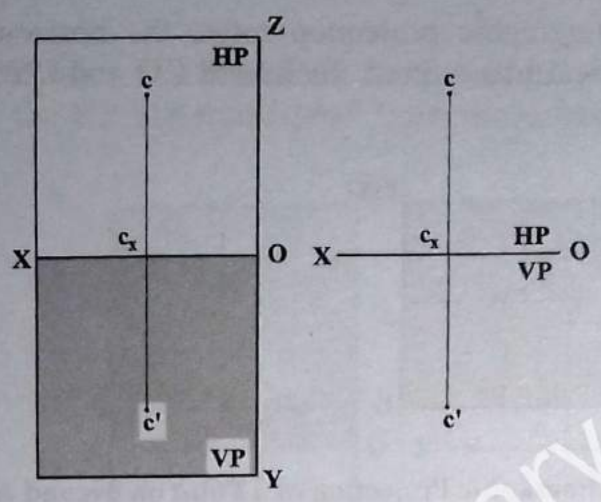


Figure 3.11: Orthographic Projection of a Point on Third Angle

(d) Projection of a Point on Fourth Angle

Consider a point **D** placed in the fourth angle i.e., below the HP and in front of the VP, as shown in *Figure 3.12*. To draw its projections, draw vertical and horizontal projection lines passing through point D. The intersection of the vertical projection line and the transparent horizontal plane gives the top view **d** of the given point. Similarly, the intersection of the horizontal projection line and the vertical plane gives the front of the given **d'** of the given point. To illustrate the relative distance of the point from the principal planes of projections, draw a straight line passing through point **c** and parallel to **Dd'** which intersects the reference line OX at point $\mathbf{d_x}$. Similarly, draw another straight line passing through point **d'** and parallel to Dd which also passes through the point $\mathbf{d_x}$. Then $\mathbf{dd_x}$ is the distance of the given point **D** from the vertical plane (VP) and $\mathbf{d'd_x}$ is the distance of the point **D** from the horizontal plane (HP).

To convert it into an orthographic projection rotate the horizontal plane in clockwise direction, keeping the vertical plane fixed, such until ZO and OY become a straight line, as shown in *Figure 3.13*.

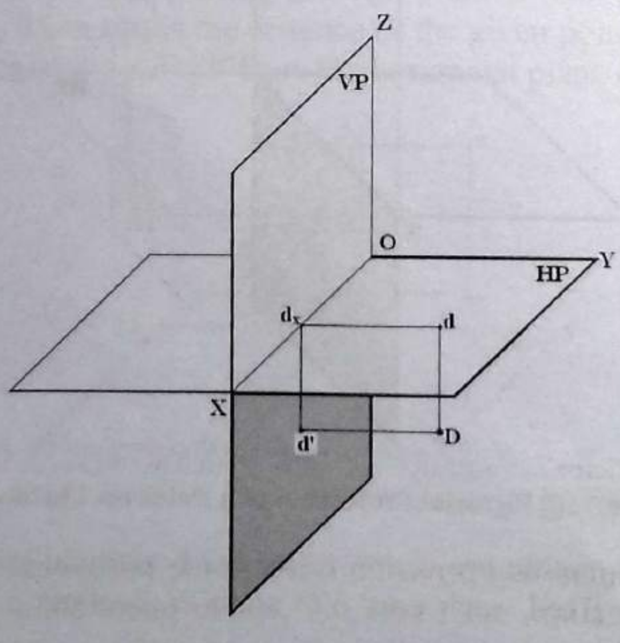


Figure 3.12: Pictorial Projection of a Point on Fourth Angle

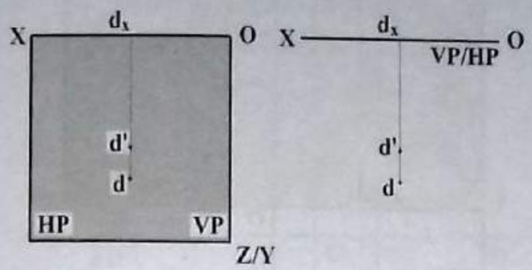


Figure 3.13: Orthographic Projection of a Point on Fourth Angle

Comparing *Figures 3.7, 3.9, 3.11, 3.13*, it can be concluded that

- Distance between he given point and the HP is used to locate the front views of the given points **a'**, **b'**, **c'** and **d'**.
- Distance between he given point and the VP is used to locate the front top views of the given points **a**, **b**, **c** and **d**.
- In case of first angle and third angle projections, two views appear on opposite side of the reference line whereas in case of second and fourth angle projection, both views lie on the same side of the reference line.

In the similar manner, if any object is placed on second or fourth angle, its views will overlap with each other. Such overlapped views cannot convey the information about the object clearly. Hence, in engineering drawing projections are made either on first angle or third angle.

3.4.2 Projection of a Point on Three Planes of Projection

If side view of the given point is also being drawn, it should be projected on the profile plane (PP), as shown in *Figure 3.14*.

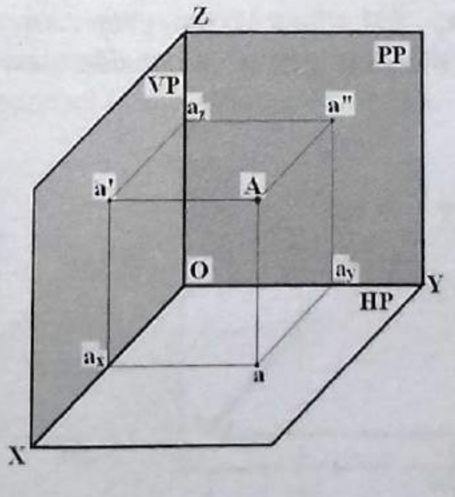


Figure 3.14: Pictorial Projection of a Point on Three Planes of Projection

To convert it into the orthographic projection, rotate the horizontal plane about the reference line OX in downward direction and rotate the profile plane about the reference line OZ in anticlockwise direction while keeping the vertical plane fixed until the entire construction becomes a single plane as shown in *Figure 3.15*.

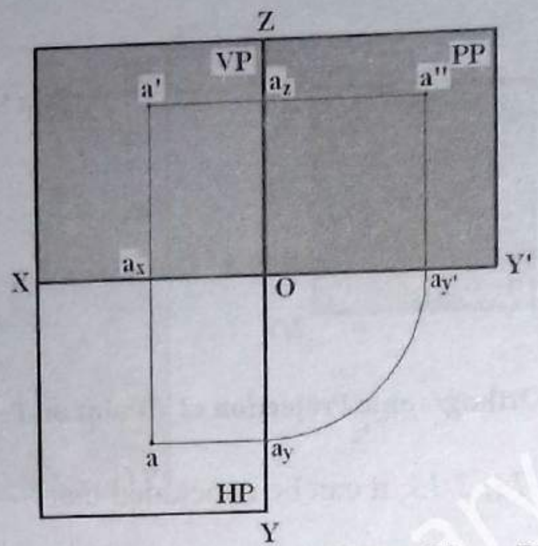


Figure 3.15: Orthographic Projection of a Point on Three Planes of Projection

Again for the simplified projection, only reference lines separating VP and HP, VP and PP and HP and PP can be drawn without drawing the complete planes.

To draw the orthographic projection of any point with given three coordinates

- Draw mutually perpendicular lines ZY and XY' intersecting at point O as the reference lines for the projection. Reference line OX separates VP and HP and the reference line OY separates VP and PP. *(Figure 3.16 (a))*
- Mark point $\mathbf{a_x}$ on the line OX using the given X-coordinate of the point. Draw a projection line passing through the point $\mathbf{a_x}$ and parallel to the reference line ZY. Mark the top view **a** and front view **a'** of the point using its Y- and Z-coordinates respectively. *(Figure 3.16 (b))*
- Draw a projection line passing through **a** and parallel to the reference line OX intersecting the reference line OY at point $\mathbf{a_y}$. Draw another line passing through a_y and inclined at 45^0 intersecting the reference line OY' at point $\mathbf{a_{y'}}$. Draw a vertical projection line passing through $\mathbf{a_{y'}}$ and a horizontal projection line passing through **a'**. The intersection of theses projection lines give the side view **a''** of the given point. *(Figure 3.16 (c))*

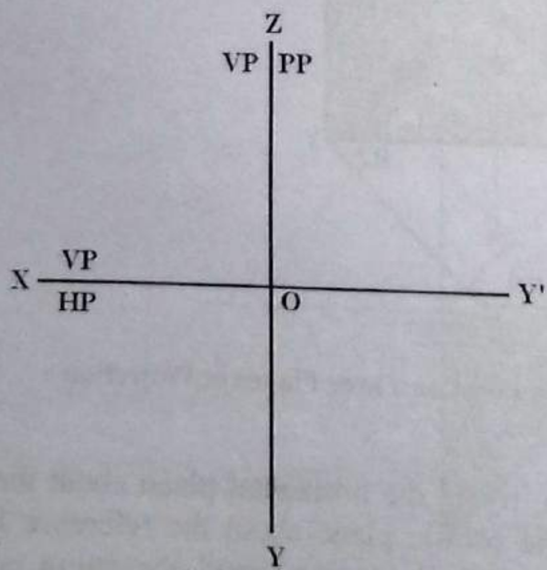


Figure 3.16 (a)

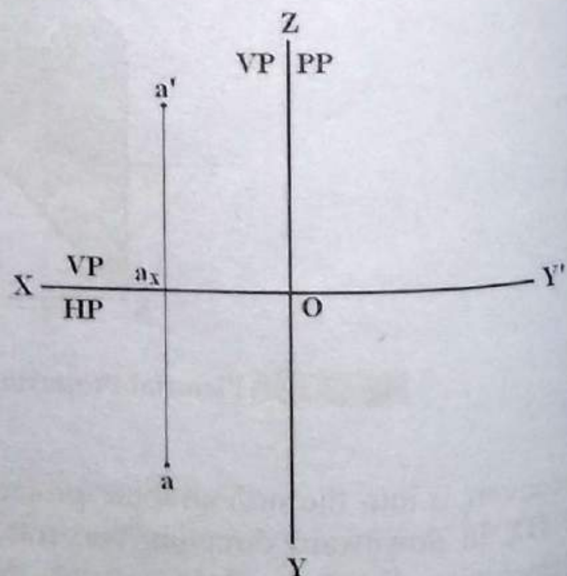


Figure 3.16 (b)

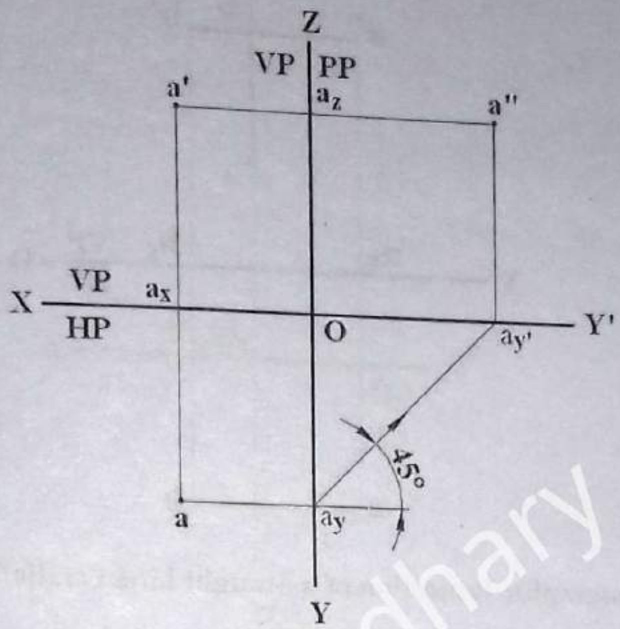


Figure 3.16 (c)

3.5 Projection of Straight Lines

Straight line can be defined as the shortest distance between the given two points. Hence, the projection a straight line can be drawn by projecting its end points and joining the views of the end points in each projection planes. Straight line can also be defined according to its position and orientation with reference to the principal planes of projection. Projections of straight lines with different orientations in space are discussed below.

3.5.1 Straight Line Parallel to both the VP and HP

A straight line **AB** parallel to both the VP and HP is shown in *Figure 3.17*. Project its end points **A** and **B** to get its top view **ab** on the HP and its front view **a'b'** on the VP respectively. The orthographic projections of the line with respect to the reference line OX is shown in *Figure 3.18*. In this case, both the top view **ab** and the view front **a'b'** have same length equal to true length of the line and both are parallel to the reference line OX.

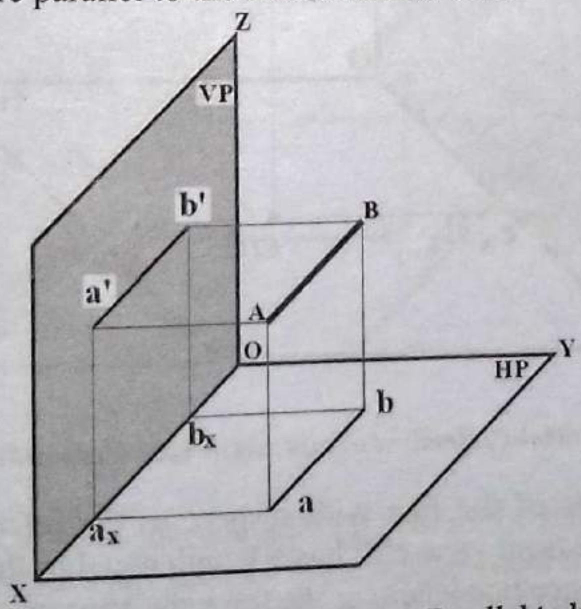


Figure 3.17: Pictorial Projection of a Straight Line Parallel to both the VP and HP

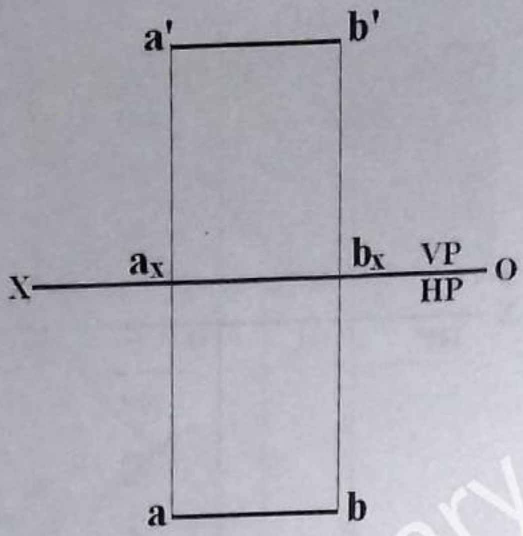


Figure 3.18: Orthographic Projection of a Straight Line Parallel to both the VP and HP

3.5.2 Straight Line Perpendicular to the HP

A straight line **CD** perpendicular to the HP and hence parallel to the VP is shown in *Figure 3.19*. The vertical projection line passing through the point **D** will also pass through the point **C** and the intersection of the common vertical projection line and the HP gives a point **c/d** as the top view of the given line. Intersections of the horizontal projection lines passing through the points **C** and **D** with the HP give the front view **c'd'** of the given line.

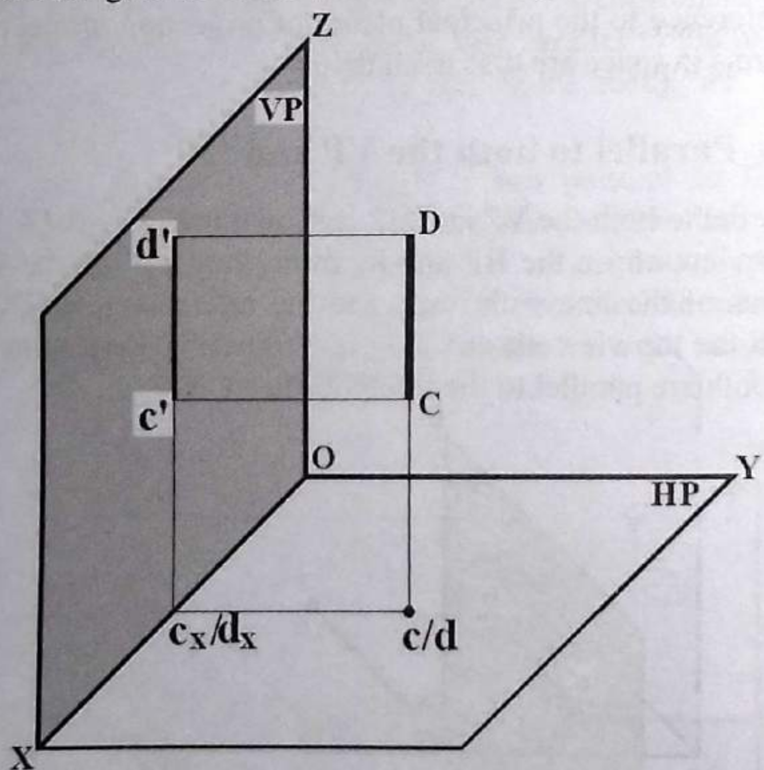


Figure 3.19: Pictorial Projection of a Straight Line Perpendicular to the HP

The orthographic projections of the line with respect to the reference line OX is shown in *Figure 3.20*. In this case, the front view **c'd'** has a length equal to the true length of the line and is perpendicular to the reference line where as the top view appears as a point.

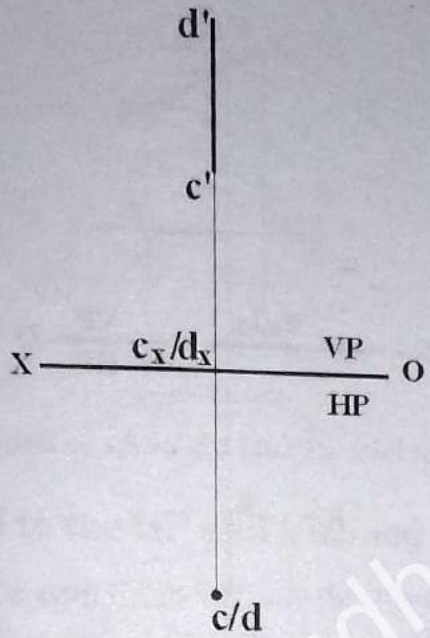


Figure 3.20: Orthographic Projection of a Straight Line Perpendicular to the HP

3.5.3 Straight Line Perpendicular to the VP

A straight line **EF** perpendicular to the VP and hence parallel to the HP is shown in *Figure 3.21*. The horizontal projection line passing through the point **F** will also pass through the point **E** and the intersection of the common vertical projection line and the VP gives a point **e'/f'** as the front view of the given line. Intersections of the vertical projection lines passing through the points **E** and **F** with the VP give the top view **ef** of the given line.

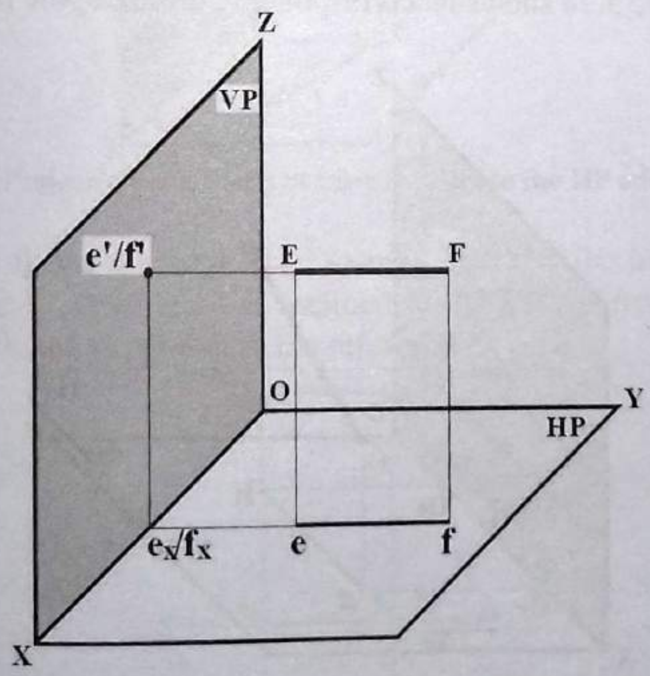


Figure 3.21: Pictorial Projection of a Straight Line Perpendicular to the VP

The orthographic projections of the line with respect to the reference line OX is shown in **Figure 3.22**. In this case, the top view **ef** has a length equal to the true length of the line and is perpendicular to the reference line where as the front view appears as a point.

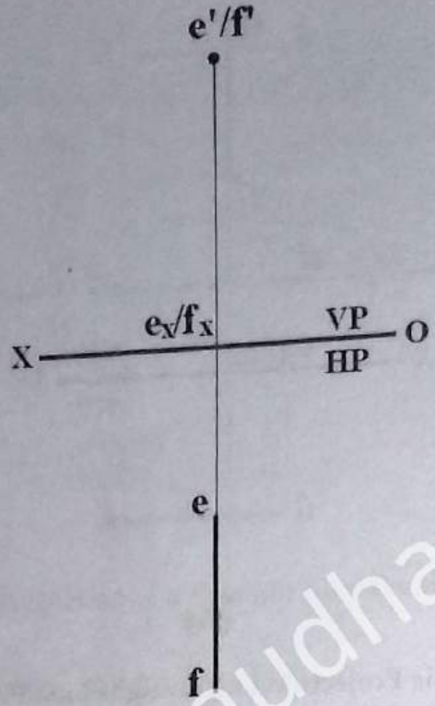


Figure 3.22: Orthographic Projection of a Straight Line Perpendicular to the VP

3.5.4 Straight Line Parallel to the VP and Inclined to the HP

When each points on any line are equidistance from the vertical plane and have different distances from the horizontal plane it is said to be parallel to the VP and inclined to the HP. *Figure 3.23* shows pictorial projection of the line **GH** parallel to the VP and inclined at an angle of θ_H to the HP and *Figure 3.24* shows its corresponding orthographic projections.

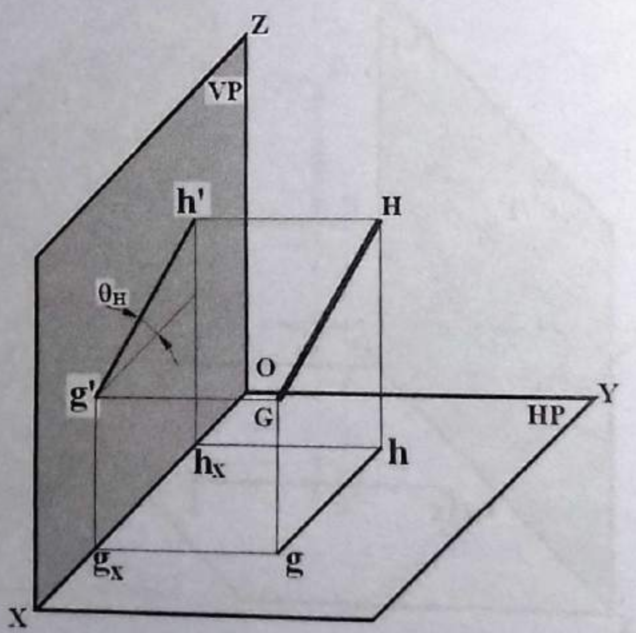


Figure 3.23: Pictorial Projection of a Straight Line Parallel to the VP and Inclined to the HP

In this case, its front view **g'h'** has a length equal to true length of the line and is inclined to the reference line at an angle θ_H at which it is inclined to the HP. Its top view **gh** has a length shorter than the true length and is parallel to the reference line OX.

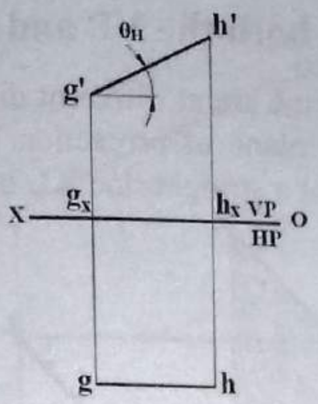


Figure 3.24: Orthographic Projection of a Straight Line Parallel to the VP and Inclined to the HP

3.5.5 Straight Line Parallel to the HP and Inclined to the VP

When each point on any line are equidistance from the horizontal plane and have different distances from the vertical plane it is said to be parallel to the HP and inclined to the VP. *Figure 3.25* shows pictorial projection of the line **IJ** parallel to the VP and inclined at an angle of θ_H to the VP and *Figure 3.26* shows its corresponding orthographic projections.

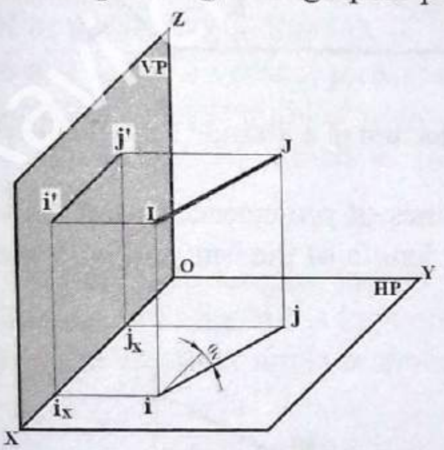


Figure 3.25: Pictorial Projection of a Straight Line Parallel to the HP and Inclined to the VP

In this case, its top view **ij** has a length equal to true length of the line and is inclined to the reference line at an angle θ_V at which it is inclined to the VP. Its front view **i'j'** has a length shorter than the true length and is parallel to the reference line OX.

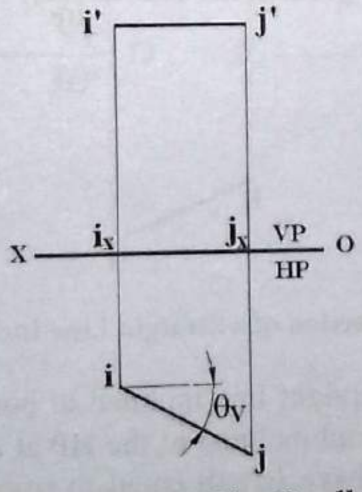


Figure 3.26: Orthographic Projection of a Straight Line Parallel to the HP and Inclined to the VP

3.5.6 Straight Line Inclined to both the VP and HP

When two end points of any straight line are at different distances from any plane of projection then it is said to be inclined to that plane of projection. *Figure 3.27* and *Figure 3.28* show pictorial and orthographic projection of a straight line **KL** inclined to both the VP and HP.

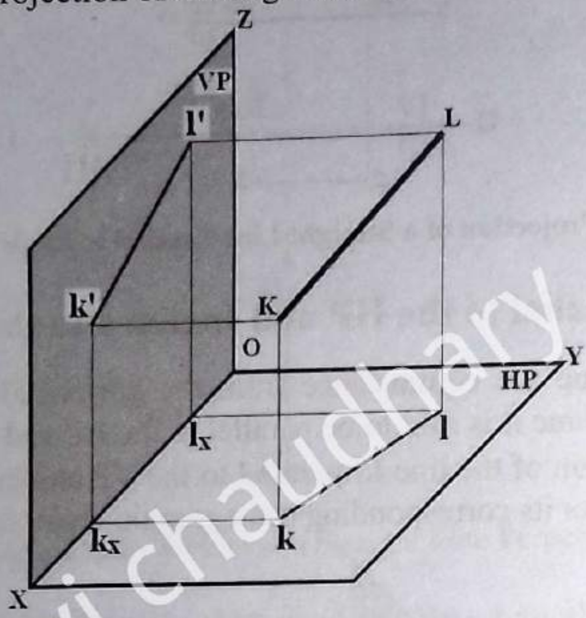


Figure 3.27: Pictorial Projection of a Straight Line Inclined to both the HP and VP

As the line is inclined to both planes of projection, its top view **kl** and the front view **k'l'** both have lengths shorter than the true length of the line and both views are inclined to the reference line OX.

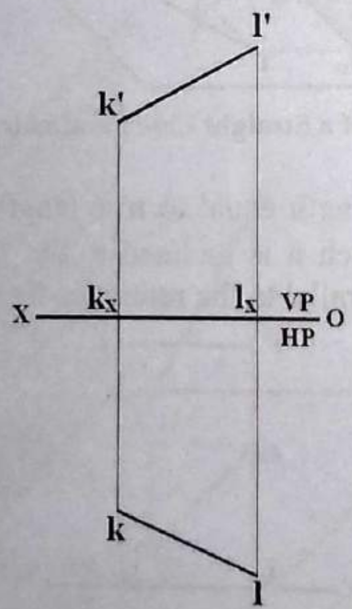


Figure 3.28: Orthographic Projection of a Straight Line Inclined to both the HP and VP

To have a clear visualization of a straight line inclined to both the principal planes, consider a straight line **MN** parallel to the VP and inclined to the HP at an angle of θ_H as shown in *Figure 3.29*. In this case, front view **m'n'** have length equal to true length of the line and gives true inclination of the line with the HP.

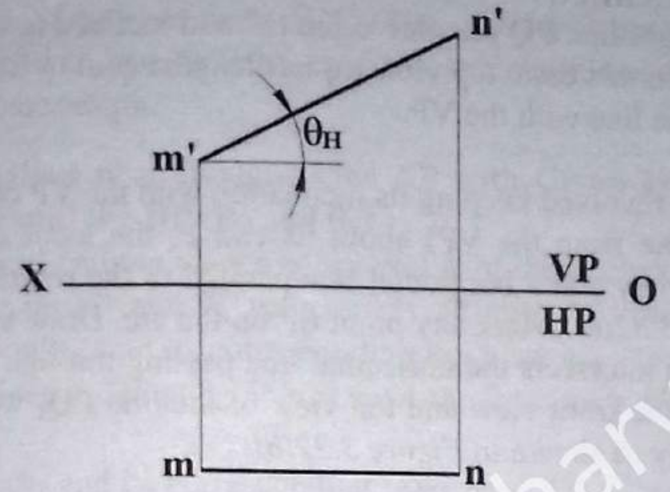


Figure 3.29: Orthographic Projections of a Straight Line MN Parallel to the VP and Inclined to the HP

When end **N** of the line is revolved keeping its inclination with the HP constant (i.e. keeping the height of point **N** same) about its end **M**, the locus of end point **N** will be circular arc on the top view and a horizontal line parallel to the reference line OX in the front view as shown in *Figure 3.30(a)*. Mark any point $\mathbf{n_1}$ on the arc. Draw a vertical projection line passing through $\mathbf{n_1}$ which intersects the horizontal line passing through the point **n'** at point $\mathbf{n_1'}$. Join $\mathbf{m'n_1'}$ and $\mathbf{mn_1}$ to get the front view and top view of the line $\mathbf{MN_1}$ which is inclined to both the VP and HP respectively as shown in *Figure 3.30(b)*.

In this position, both views of the line have lengths shorter than the true length of the line. While revolving the line, its inclination with the HP is kept constant, therefore the length of its top view remains unchanged while the apparent angle α made by the front view $\mathbf{m'n_1'}$ is not equal to the true inclination θ_H.

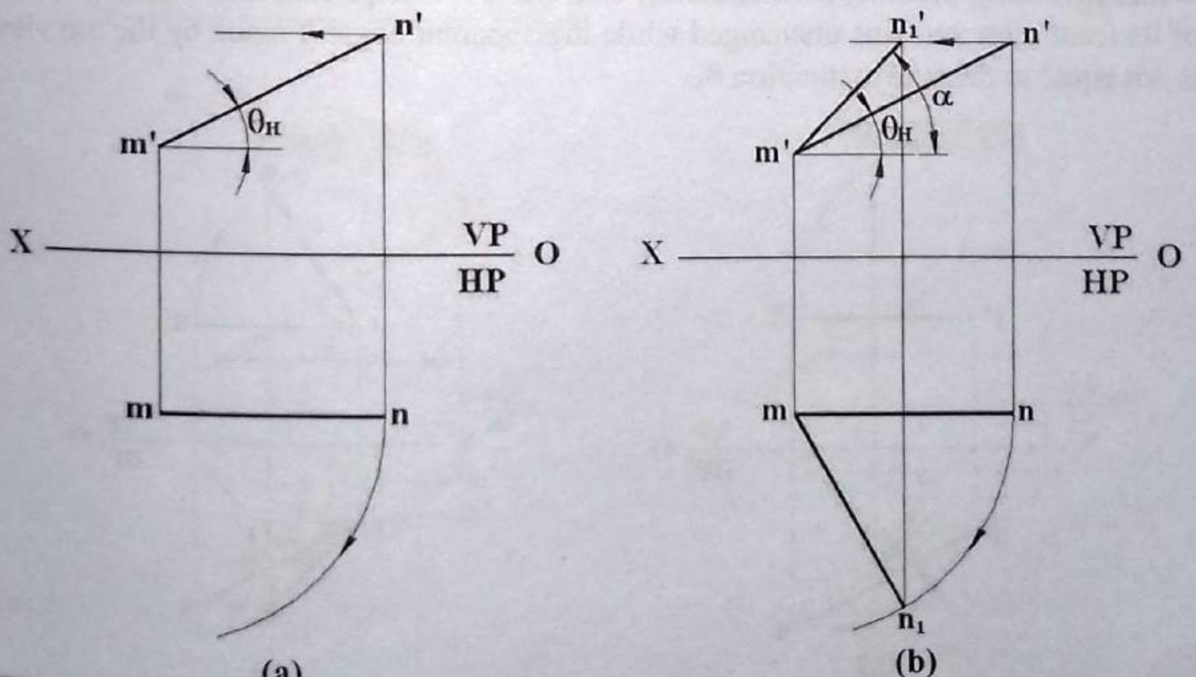


Figure 3.30: Orthographic Projections of a Straight Line MN_1 When it is Inclined to both the VP and the HP

Similarly, consider a straight line **PQ** parallel to the HP and inclined to the VP at an angle of θ_V as shown in *Figure 3.31*. In this case, top view **pq** has length equal to true length of the line and gives true inclination of the line with the VP.

When end **Q** of the line is revolved keeping its inclination with the VP constant (i.e. keeping the distance of its end **Q** same from the VP) about its end **P**, the locus of end point **Q** will be circular arc on the front view and a horizontal line parallel to the reference line OX in the top view as shown in *Figure 3.32(a)*. Mark any point **q_1'** on the arc. Draw a vertical projection line passing through **q_1'** which intersects the horizontal line passing through the point **q** at point q_1. Join **$p'q_1'$** and **pq_1** to get the front view and top view of the line **PQ_1** which is inclined to both the VP and HP respectively as shown in *Figure 3.32(b)*.

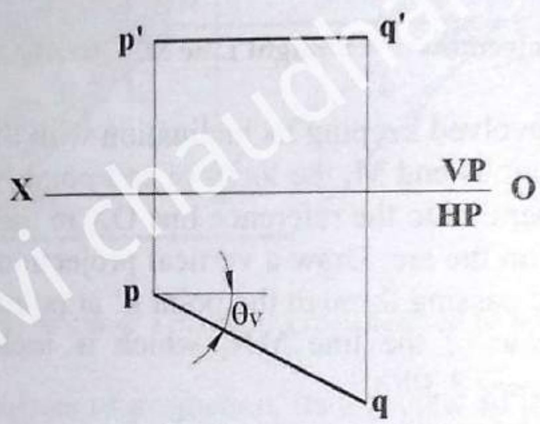


Figure 3.31: Orthographic Projections of a Straight Line PQ Parallel to the HP and Inclined to the VP

In this position, both views of the line have lengths shorter than the true length of the line. While revolving the line, its inclination with the VP is kept constant, therefore the length of its front view remains unchanged while the apparent angle β made by the top view **pq_1** is not equal to the true inclination θ_V.

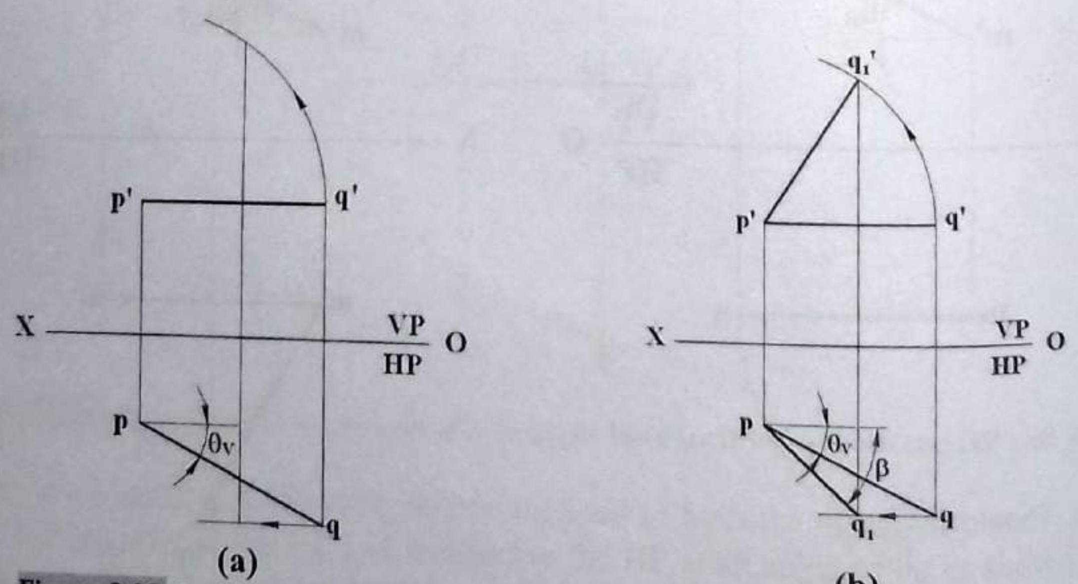


Figure 3.32: Orthographic Projections of a Straight Line PQ_1 When it is Inclined to both the VP and the HP

Hence, comparing the projections of lines MN_1 *(Figure 3.30(b))* and PQ_1 *(Figure 3.32(b))*, it can be clearly understood that, true inclination of any straight line can be measured in the view only when it appears in true length.

Construction of Projections of a Straight Line AB with Given True Length (TL) and its Inclination with the VP and the HP (θ_H and θ_V)

- Draw front view **a'** and top view **a** of the end **A** of the line according to given distances of the point from the HP and VP respectively. *(Figure 3.33(a))*
- Draw front view **a'b₁'** and its corresponding top view **ab₁** assuming that it is parallel to the VP and inclined to HP at θ_H. In this case, the front view **a'b₁'** appears in true length. *(Figure 3.33(b))*
- Draw top view **ab₂** and its corresponding front view **a'b₂'** assuming that it is parallel to the HP and inclined to VP at θ_V. In this case, the top view **ab₂** appears in true length. *(Figure 3.33(c))*
- Draw straight lines passing through the ends of true length line **b₁'** and **b₂** and parallel to the reference line OX. *(Figure 3.33(d))*
- With **a'** as center and **a'b₂'** as radius, draw an arc intersecting straight line passing through the point **b₁'** at point **b'**. Similarly, with **a** as center and **ab₁** as radius, draw an arc intersecting straight line passing through the point **b₂** at point **b**. *(Figure 3.33(e))*
- Draw vertical projection line passing through points **b'** and **b**. Join **a'b'** and **ab** to get the required front view and top view of the straight line **AB**. *(Figure 3.33(f))*

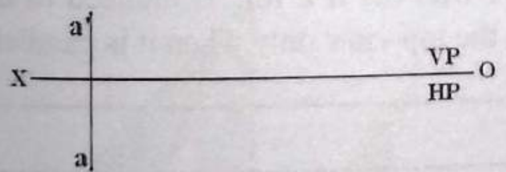


Figure 3.33(a)

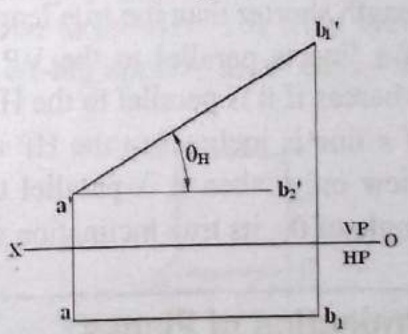


Figure 3.33(b)

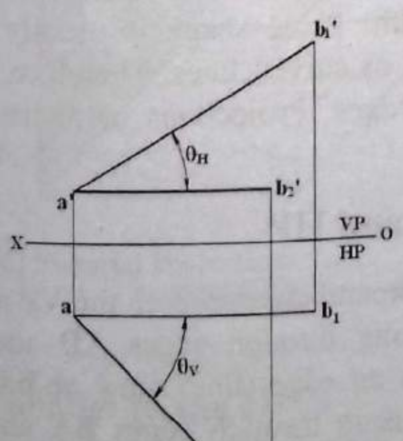


Figure 3.33(c)

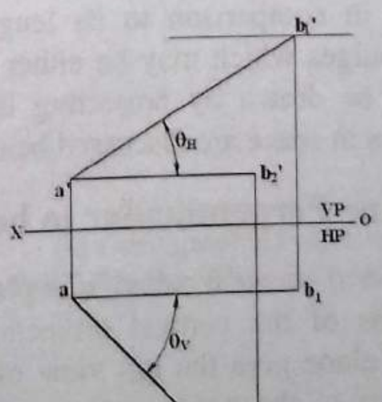


Figure 3.33(d)

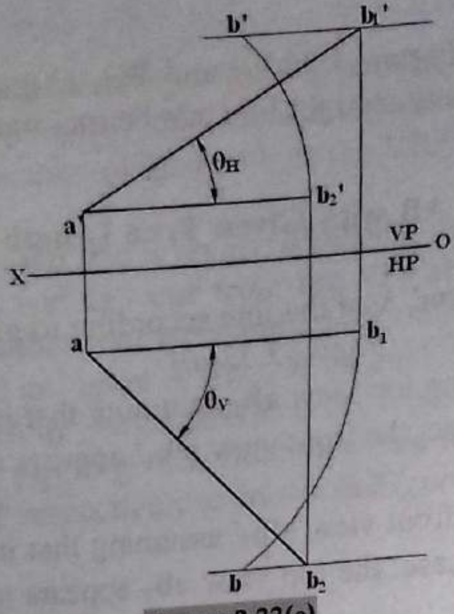


Figure 3.33(e)

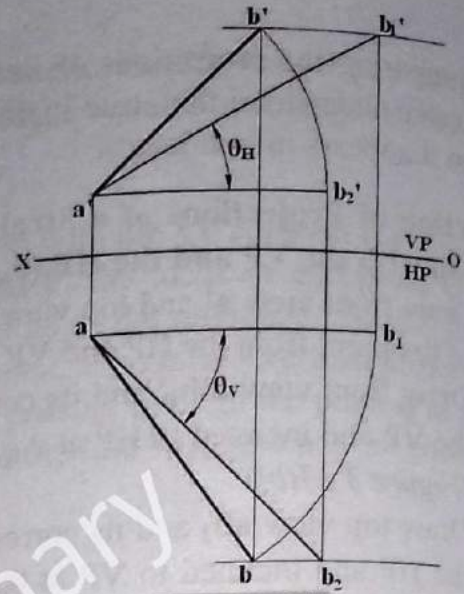


Figure 3.33(f)

Comparing the projections of the lines having different orientations, it can be concluded that

- When a line is perpendicular to any plane of projection, it appears as point view in that plane.
- When a line is parallel to any plane of projection, it appears as a true length in that plane.
- When a line is inclined to any plane of projection, its projection in that plane will have length shorter than the true length.
- If a line is parallel to the VP and its top view will be parallel to the reference line whereas if it is parallel to the HP its front view will be parallel to the reference line.
- If a line is inclined to the HP at an angle of θ_H, its true inclination is seen in the front view only when it is parallel to the VP; whereas if a line is inclined to the VP at an angle of θ_V, its true inclination is seen in the top view only when it is parallel to the HP.

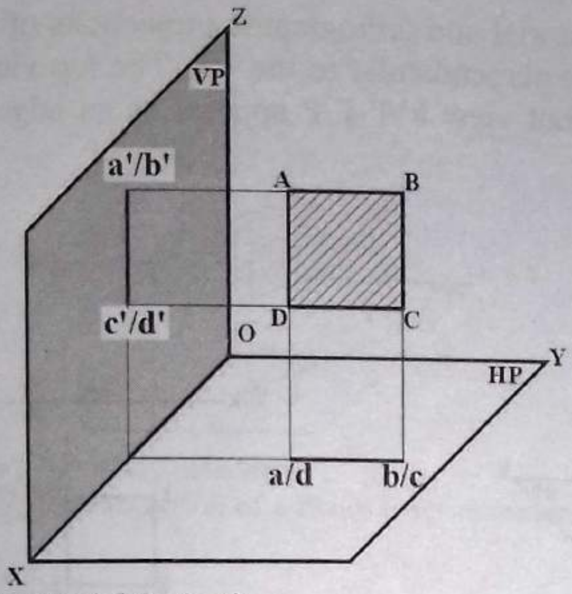
3.6 Projection of Planes

Any object having only two dimensions is called a plane. The thickness of the plane is negligible in comparison to its length and breadth. Plane shape is usually defined by its boundary edges which may be either straight lines or curved lines. Therefore, projection of a plane can be drawn by projecting its boundary edges. Projections of planes with different orientations in space are discussed below.

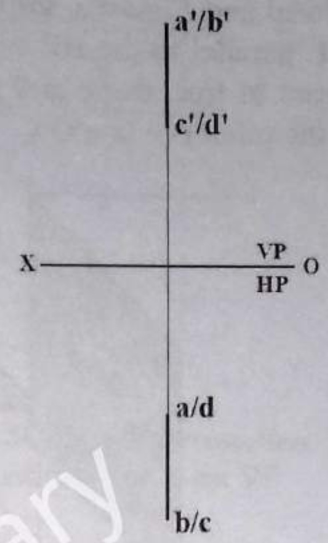
3.6.1 Plane Perpendicular to both the VP and HP

Figure 3.34(a) shows a rectangular plane **ABCD** perpendicular to both the VP and the HP. The intersections of the vertical projection lines passing through edges **AD** and **BC** with the horizontal plane give the top view of the plane as an edge (line) view **ac-bd**. Similarly, the intersections of the horizontal projection lines passing through edges **BA** and **CD** with the vertical plane give the front view of the plane as an edge (line) view **a'b'-c'd'**.

Figure 3.34(b) shows the corresponding orthographic views, where both views are perpendicular to the reference line OX.



(a) Pictorial Projection



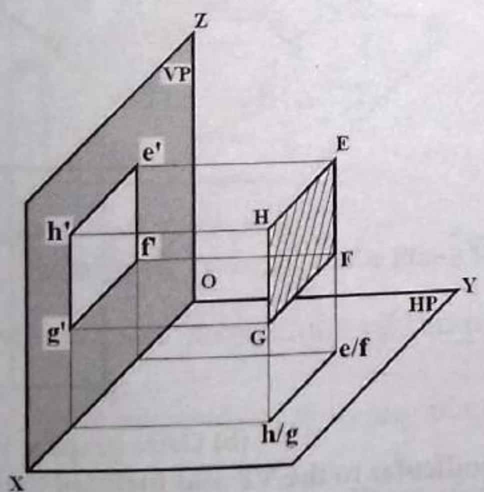
(b) Orthographic Projection

Figure 3.34: Plane Perpendicular to both the HP and VP

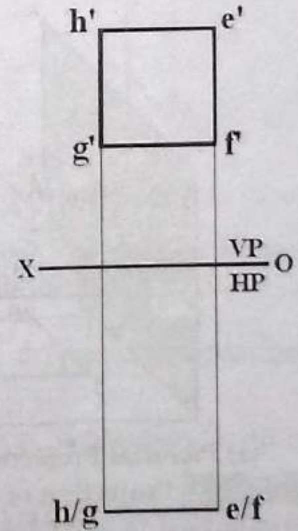
3.6.2 Plane Parallel to the VP

When each points of a given plane are equidistance from the vertical plane, it is said to be parallel to the VP. Again, when any plane is parallel to the VP, it will be perpendicular to the HP.

Figure 3.35(a) and *Figure 3.35(b)* show pictorial and orthographic projections of a rectangular plane **EFGH** parallel to the VP and therefore perpendicular to the HP. The front view **e'f'g'h'** of the plane appears in true shape and size. Its top view **e/f-h/g** appears as an edge view and is parallel to the reference line OX.



(a) Pictorial Projection



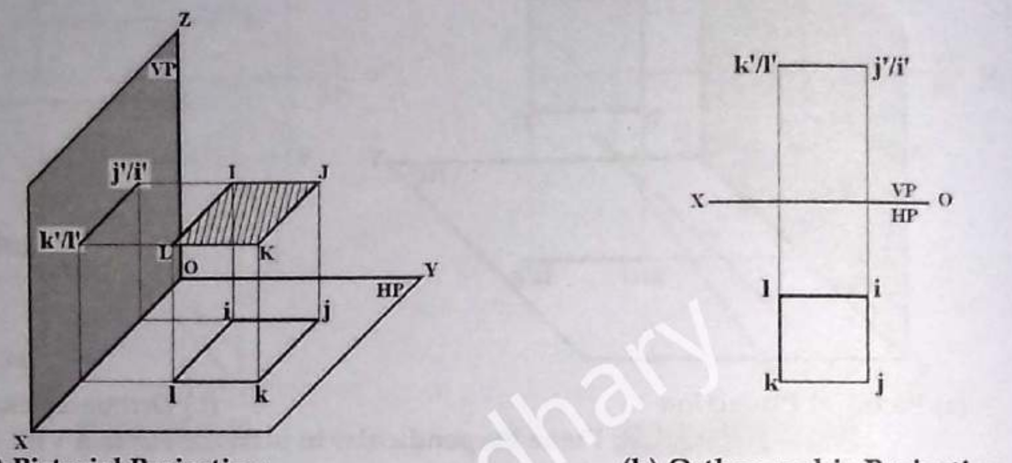
(b) Orthographic Projection

Figure 3.35: Projection of a Plane Parallel to the VP

3.6.3 Plane Parallel to the HP

When each points of a given plane are equidistance from the horizontal plane, it is said to be parallel to the HP. Again, when any plane is parallel to the HP, it will be perpendicular to the VP.

Figure 3.36(a) and *Figure 3.36(b)* show pictorial and orthographic projections of a rectangular plane **IJKL** parallel to the HP and therefore perpendicular to the VP. The top view **ijkl** of the plane appears in true shape and size. Its front view **k'/l'-j'/i'** appears as an edge view and is parallel to the reference line OX.

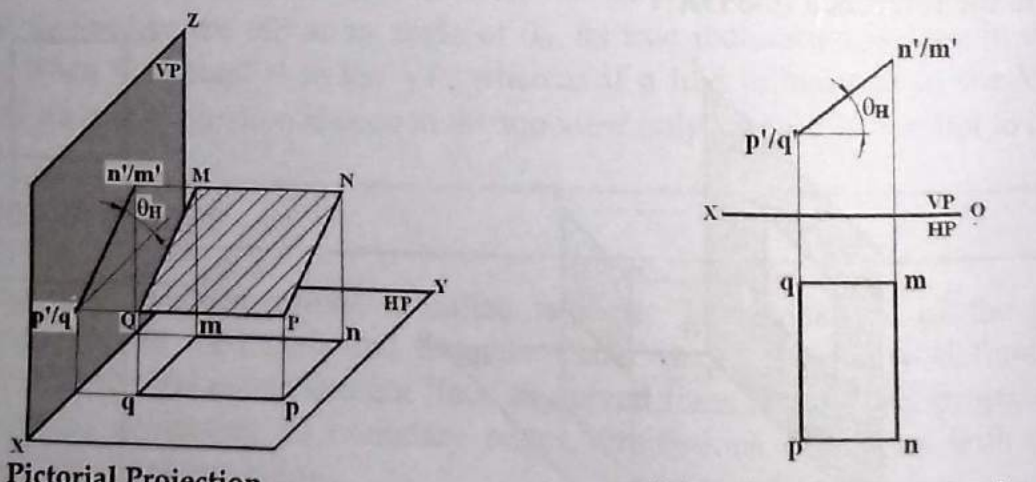


(a) Pictorial Projection (b) Orthographic Projection

Figure 3.36: Projection of a Plane Parallel to the HP

3.6.4 Plane Perpendicular to the VP and inclined to the HP

Figure 3.37(a) and *Figure 3.37(b)* show pictorial and orthographic projections of a rectangular plane **MNPQ** perpendicular to the VP and inclined to the HP at an angle of θ_H. In this case, top view of the plane **mnpq** appears in distorted form (reduced shape and size). Its front view **m'n'-p'q'** appears as an edge view and is inclined to the reference line at an angle θ_H at which the plane is inclined to the HP.

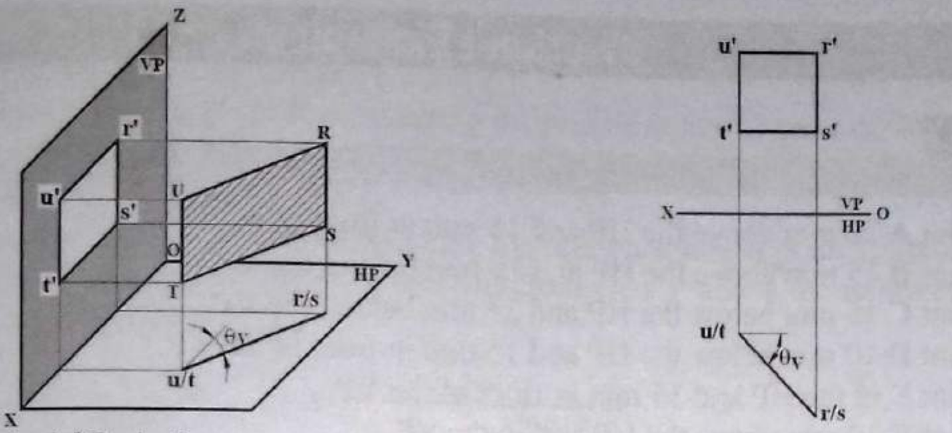


(a) Pictorial Projection (b) Orthographic Projection

Figure 3.37: Projection of a Plane Perpendicular to the VP and Inclined to the HP

3.6.5 Plane Perpendicular to the HP and inclined to the VP

Figure 3.38(a) and *Figure 3.38(b)* show pictorial and orthographic projections of a rectangular plane **RSTU** perpendicular to the HP and inclined to the VP at an angle of θ_V. In this case, front view of the plane **r's't'u'** appears in distorted form (reduced shape and size). Its top view **r/s-u/t** appears as an edge view and is inclined to the reference line at an angle θ_V at which the plane is inclined to the VP.

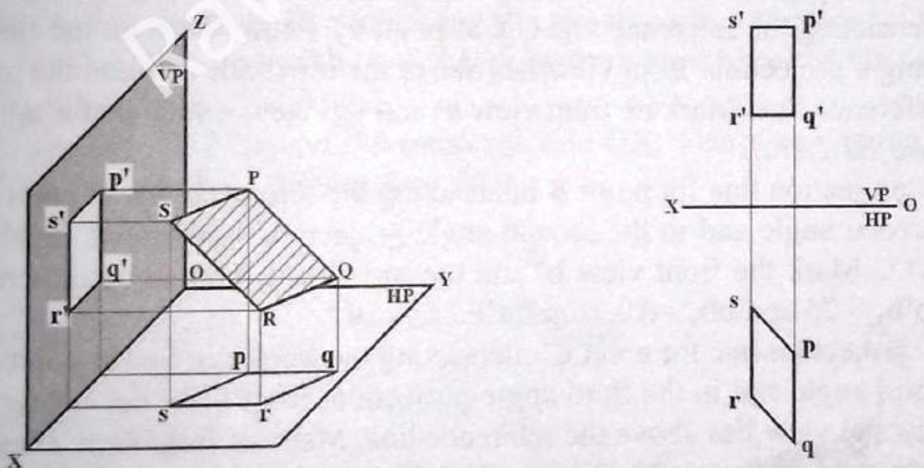


(a) Pictorial Projection (b) Orthographic Projection

Figure 3.38: Projection of a Plane Perpendicular to the HP and Inclined to the VP

3.6.6 Plane Inclined to both the VP and HP

Figure 3.39(a) and *Figure 3.39(b)* show pictorial and orthographic projections of a rectangular plane **PQRS** inclined to both the HP and VP. In this case, both the front and top views of the planes appears in the form of plane figures but both have size smaller than the rue size of the plane.



(a) Pictorial Projection (b) Orthographic Projection

Figure 3.39: Projection of a Plane Inclined to both the VP and HP

Comparing the orthographic projections of the planes having different orientations, it can be concluded that

- When a plane is perpendicular to any plane of projection, it appears as an edge view in that plane.
- When a plane is parallel to any plane of projection, it appears in true shape and size in that plane.
- When a plane is inclined to any plane of projection, its projection in that plane will be smaller in size than the true size of the plane.
- If a plane is perpendicular to the VP and inclined to the HP at an angle of θ_H, its true inclination is seen in the front view which appears as an edge view. Similarly, if a plane is perpendicular to the HP and inclined to the VP at an angle of θ_V, its true inclination is seen in the top view which appears as an edge view

Example 3.1

Draw the projections of the following points.

(a) Point **A** 20 mm above the HP and 15 mm in front of the VP.
(b) Point **B** 25 mm above the HP and 10 mm behind the VP.
(c) Point **C** 15 mm below the HP and 25 mm behind the VP.
(d) Point **D** 10 mm below the HP and 15 mm in front of the VP.
(e) Point **E** in the HP and 15 mm in front of the VP.
(f) Point **F** 20 mm above the HP and in the VP.
(g) Point **G** in the HP and 25 mm behind the VP.
(h) Point **H** 10 mm below the HP and in the VP.
(i) Point **I** in the HP and in the VP.

Solution

Hints: To draw orthographic projections of a point, use its distance from the HP to locate its front view and its distance from the VP to locate its top view respectively.

(a) Draw the reference line OX and any line perpendicular to it as a projection line for point **A** intersecting the reference line OX at point $\mathbf{a_x}$. Point **A** lies on the first angle and in the first angle projection, front view lies above the reference line and the top view lies below the reference line. Mark its front view **a'** and top view **a** such that $\mathbf{a'a_x} = 20$ and $\mathbf{aa_x} = 15$. *(Figure E3.1.1(a))*

(b) Draw projection line for point **B** intersecting the reference line at point $\mathbf{b_x}$. Point B lies in the second angle and in the second angle projection, both views lie above the reference line OX. Mark the front view **b'** and the top view **b** both above the reference line such that $\mathbf{b'b_x} = 25$ and $\mathbf{bb_x} = 10$. *(Figure E3.1.1(b))*

(c) Draw projection line for point **C** intersecting the reference line at point $\mathbf{c_x}$. Point **C** lies on the third angle and in the third angle projection, front view lies below the reference line and the top view lies above the reference line. Mark its front view **c'** and top view **c** such that $\mathbf{c'c_x} = 15$ and $\mathbf{cc_x} = 25$. *(Figure E3.1.1(c))*

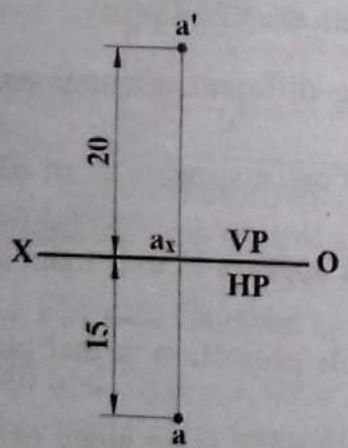


Figure E3.1.1(a)

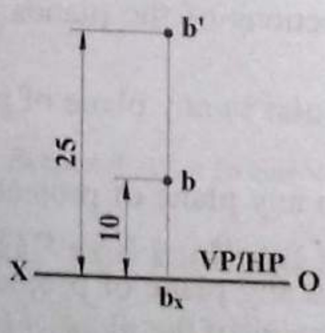


Figure E3.1.1(b)

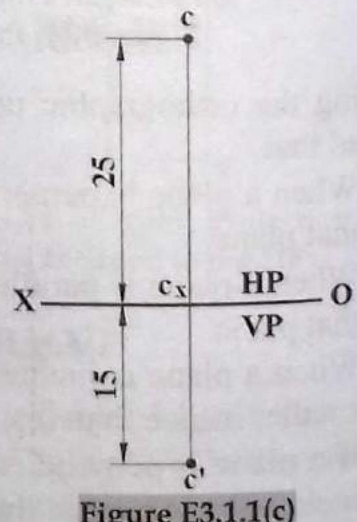


Figure E3.1.1(c)

(d) Draw projection line for point **D** intersecting the reference line at point $\mathbf{d_x}$. Point D lies in the fourth angle and in the fourth angle projection, both views lie below the reference line OX. Mark the front view **d'** and the top view **d** both below the reference line such that **d'd**$_x$ = 10 and **dd**$_x$ = 15. *(Figure E3.1.1(d))*

(e) Draw projection line for point **E** intersecting the reference line at point **e**$_x$. In this case, its front view **e'** coincides with **e**$_x$ (**e'e**$_x$ = 0). Mark its top view **e** below the reference line such that **ee**$_x$ = 15. *(Figure E3.1.1(e))*

(f) Draw projection line for point **F** intersecting the reference line at point **f**$_x$. In this case, its top view **f** coincides with **f**$_x$ (**ff**$_x$ = 0). Mark its front view **f'** above the reference line such that **f'f**$_x$ = 20. *(Figure E3.1.1(f))*

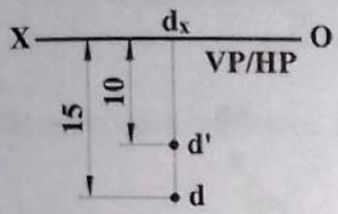


Figure E3.1.1(d)

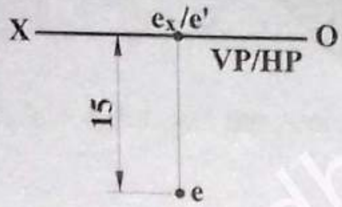


Figure E3.1.1(e)

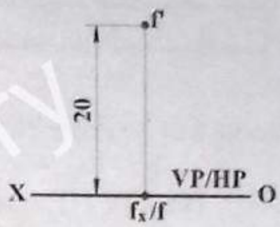


Figure E3.1.1(f)

(g) Draw projection line for point **G** intersecting the reference line at point g$_x$. In this case, its front view **g'** coincides with g$_x$ (g'g$_x$ = 0). Mark its top view g above the reference line such that **gg**$_x$ = 25. *(Figure E3.1.1(g))*

(h) Draw projection line for point **H** intersecting the reference line at point **h**$_x$. In this case, its top view **h** coincides with **h**$_x$ (**hh**$_x$ = 0). Mark its front view **h'** below the reference line such that **h'h**$_x$ = 10. *(Figure E3.1.1(h))*

(i) In this case, given point **I** lies on the reference line OX. Therefore, its top view **i** and front view **i'** coincide with **i**$_x$. *(Figure E3.1.1(i))*

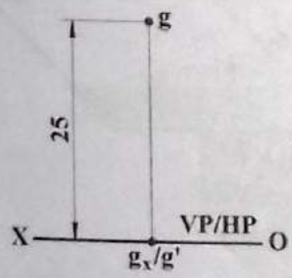


Figure E3.1.1(g)

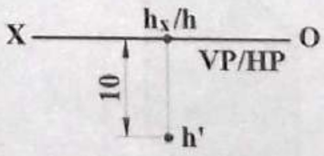


Figure E3.1.1(h)

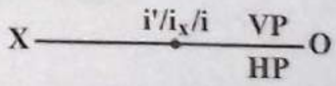
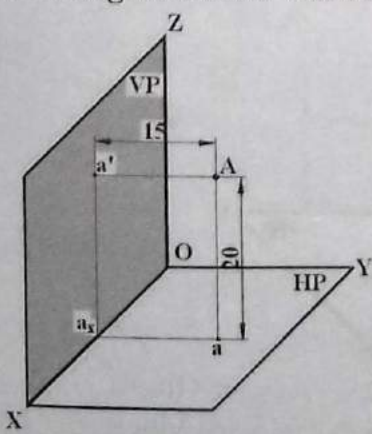
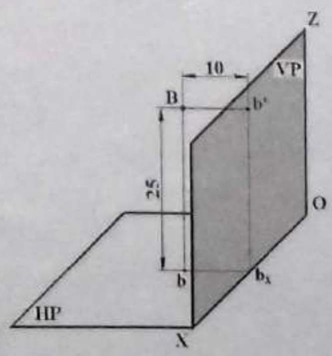


Figure E3.1.1(i)

If the direct construction of orthographic projections is troublesome, pictorial projection of each point shown in *Figure E3.1.2* can be referred.



(a)



(b)

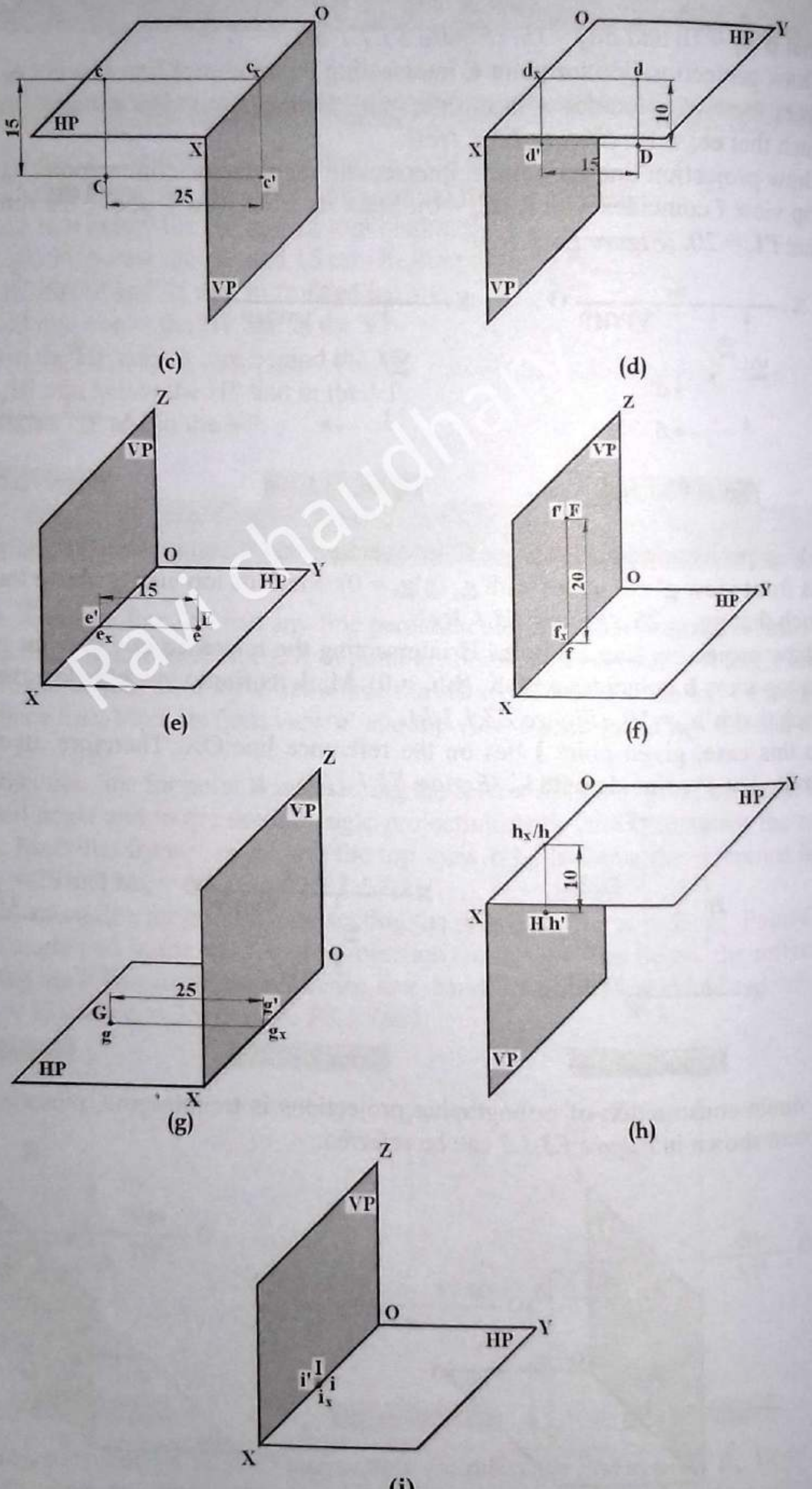


Figure E3.1.2: Pictorial Projection of Points Given in Example 1

Example 3.2

Draw three projections of the following points with given three coordinates (i.e. distances from three principal planes).

Point	Coordinates		
	X (Distance form PP)	Y (Distance from VP)	Z (Distance from HP)
A	40	15	25
B	30	20	35
C	20	35	15

Solution

Follow the procedure explained in *Article 3.4.2* to get the projections of each of the given point.

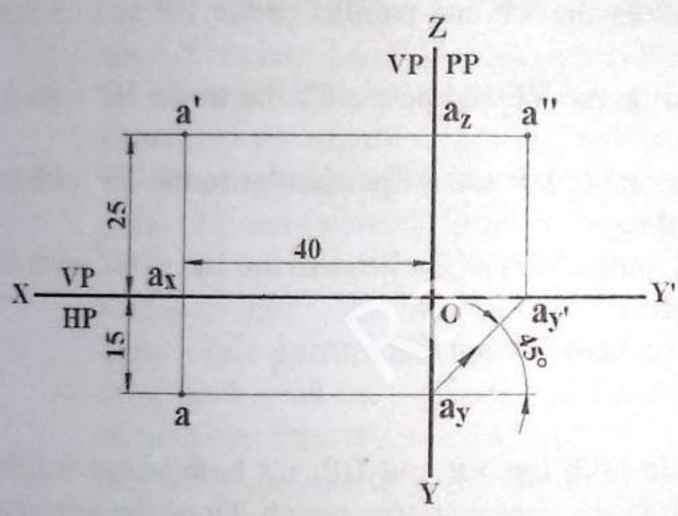


Figure E3.2(a)

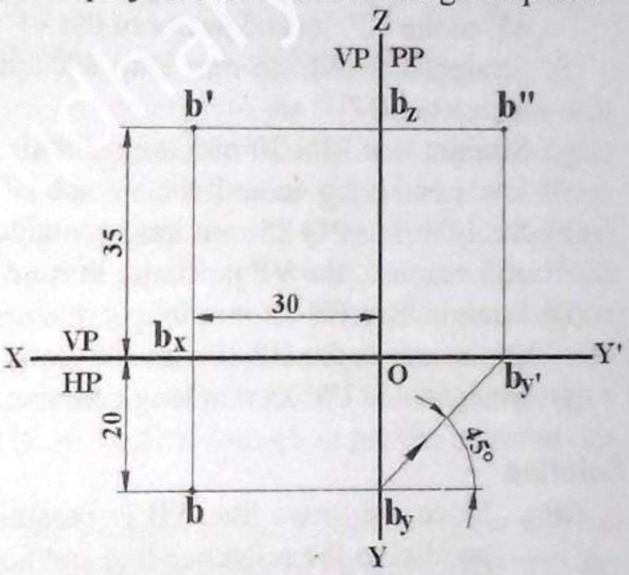


Figure E3.2(b)

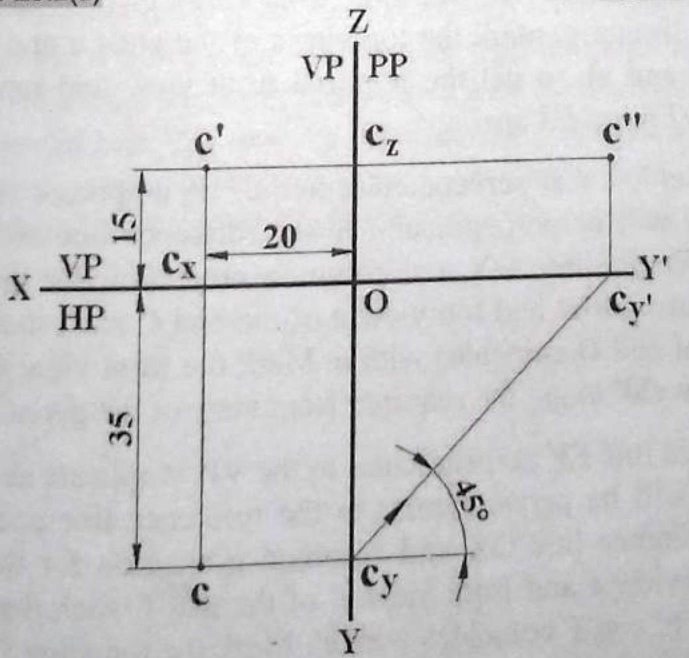


Figure E3.2(c)

Example 3.3

Draw the projections of the following straight lines.

(a) Straight line **AB** 20 mm long parallel to both the VP and HP. and lying 10 mm above the HP and 15 mm in front of the VP.

(b) Straight line **CD** 25 mm long perpendicular to the HP and 20 mm in front of the VP with its lower end 10 mm above the HP.

(c) Straight line **EF** 30 mm long perpendicular to the VP and 15 mm above the HP and its end nearer to the VP is 10 mm in front of it.

(d) Straight line **GH** 25 mm long parallel to the VP and inclined to the HP at 30^0. One of its ends is 12 mm above the HP and 18 mm in front of the VP.

(e) Straight line **IJ** 30 mm long parallel to the HP and 15 mm above the HP and inclined at 45^0 to the VP. Its end nearer to the VP is 10 mm in front of it.

(f) Straight line **KL** 25 mm long contained on the VP and parallel to the HP and 20 mm above the HP.

(g) Straight line **MN** 30 mm long contained on the VP and perpendicular to the HP with its lower end lying on both the VP and HP.

(h) Straight line **PQ** 25 mm long contained on the HP and perpendicular to the VP with its end nearer to the VP is 15 mm in front of it.

(i) Straight line **RS** 30 mm long contained on the VP and inclined to the HP at 60^0 with its end nearer to the HP 10 mm above the HP.

(j) Straight line **UV** 25 mm long contained on both VP and HP.

Solution

(a) Since the given line **AB** is parallel to both the VP and HP, it's both views will be parallel to the reference line and both views appear in true length. Draw the reference line OX and end projectors for the ends of the line which are 20 mm apart to locate the points $\mathbf{a_x}$ and $\mathbf{b_x}$ on line OX. Mark the front views of the ends **a'** and **b'** such that $\mathbf{a'a_x} = \mathbf{b'b_x} = 10$. Similarly, mark the top views of the ends **a** and **b** such that $\mathbf{aa_x} = \mathbf{bb_x} = 15$. Join **a'b'** and **ab** to get the required front view and top view of the given line respectively. *(Figure E3.3(a))*

(b) Since the given line **CD** perpendicular to the HP, it appears as a point in top view and its front view will be perpendicular to the reference line and appears in true length. Draw the reference line OX and common projection for the both ends of the line. Mark the front view **c'** and top view **c** of the end **C** such that $\mathbf{c'c_x} = 10$ and $\mathbf{cc_x} = 20$. Top view **d** of end **D** coincides with **c**. Mark the front view **d'** of the end **D** such that **c'd'** = 25. Join **c'd'** to get the required front view of the given line. *(Figure E3.3(b))*

(c) Since the given line **EF** perpendicular to the VP, it appears as a point in front view and its top view will be perpendicular to the reference line and appears in true length. Draw the reference line OX and common projection for the both ends of the line. Mark the top view **e** and front view **e'** of the end **E** such that $\mathbf{ee_x} = 10$ and $\mathbf{e'e_x} = 15$. Front view **f'** of end **F** coincides with **e'**. Mark the top view **f** of the end **F** such that **ef** = 30. Join **ef** to get the required top view of the given line. *(Figure E3.3(c))*

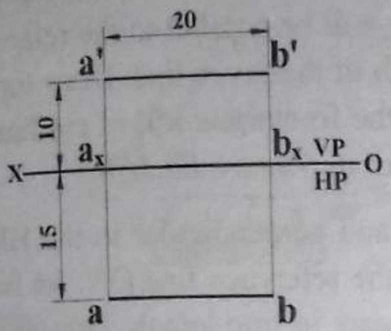


Figure E3.3(a)

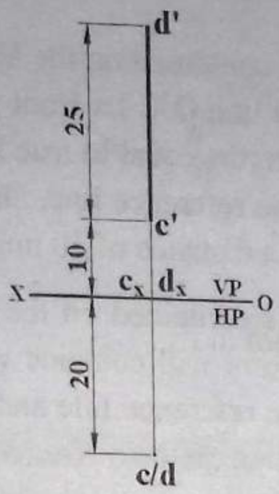


Figure E3.3(b)

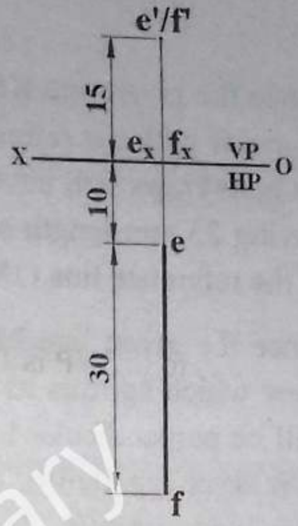


Figure E3.3(c)

(d) Since the given line **GH** is parallel to the VP and inclined to the HP, its front view will be inclined to the reference line and appears in true length whereas its top view will be parallel to the reference line and will have a length shorter than the true length. Draw the reference line OX and draw front view **g'** and top view **g** of the end **G** such that **g'g**$_x$ = 12 and **gg**$_x$ = 18. Draw a straight line passing through the point **g'** and inclined at 30^0 to the reference line. Mark the front view **h'** of the end **H** on the inclined line such that **g'h'** = 25. Draw a vertical projection line passing through the point **h'** and draw a line passing through point **g** and parallel to the reference line. Intersection of these lines gives the top view **h** of the end **H** of the line. Join **gh** to get the required top of the given line. *(Figure E3.3(d))*

(e) Since the given line **IJ** is parallel to the HP and inclined to the VP, its top view will be inclined to the reference line and appears in true length whereas its front view will be parallel to the reference line and will have a length shorter than the true length. Draw the reference line OX and draw front view **i'** and top view **i** of the end **I** such that **i'i**$_x$ = 15 and **ii**$_x$ = 10. Draw a straight line passing through the point **i** and inclined at 45^0 to the reference line. Mark the top view **j** of the end **J** on the inclined line such that **ij** = 30. Draw a vertical projection line passing through the point **j** and draw a line passing through point **i'** and parallel to the reference line. Intersection of these lines give the front view **j'** of the end J of the line. Join **i'j'** to get the required front of the given line. *(Figure E3.3(e))*

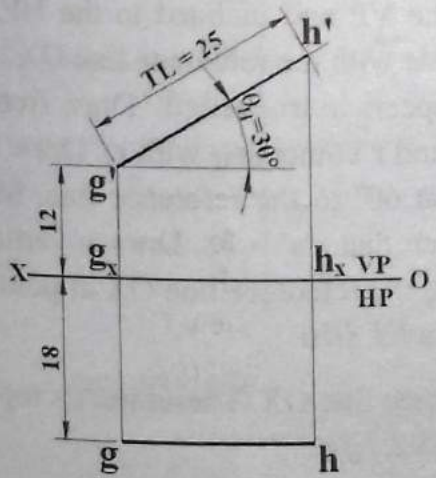


Figure E3.3(d)

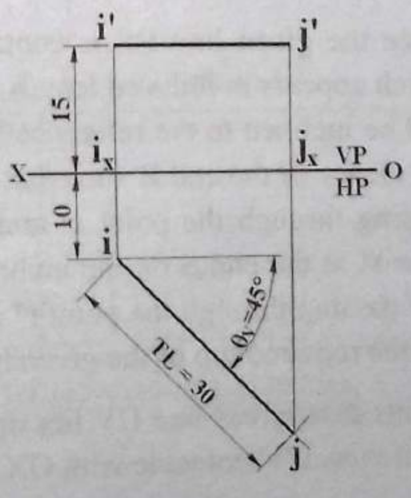


Figure E3.3(e)

(f) Since the given line **KL** is contained on the VP and parallel to the HP, its top view will coincide with the reference line OX. Its front view will be parallel to the reference line. Its both views will have lengths equal to true length of the given line. Draw top view **kl** having 25 mm length on the reference line. Draw the front view **k'l'** of the line parallel to the reference line OX at a distance of 20 mm from it. *(Figure E3.3(f))*.

(g) Since the given line **MN** is contained on the VP and perpendicular to the HP, its top view which appears as a point will coincide with the reference line OX. Its front view will be perpendicular to the reference line and appears in true length. Draw point view **m/n** as the top view of the line on the reference line OX. Front view **n'** of the end **N** also coincides with **m/n**. Draw front view **m'n'** of the line perpendicular to the reference line OX at point **n'** such that **m'n'** = 30. *(Figure E3.3(g))*

(h) Since the given line **PQ** is contained on the HP and perpendicular to the VP, its front view which appears as a point will coincide with the reference line OX. Its top view will be perpendicular to the reference line and appears in true length. Draw point view **p'/q'** as the front view of the line on the reference line OX. Draw a projection passing through **p'/q'** and perpendicular to OX and draw top views **p** and **q** of ends **P** and **Q** respectively on the projection line such that **p'p** = 15 and **pq** = 25. *(Figure E3.3(h))*

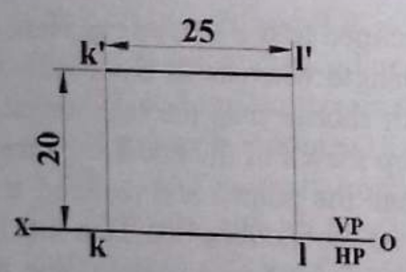


Figure E3.3(f)

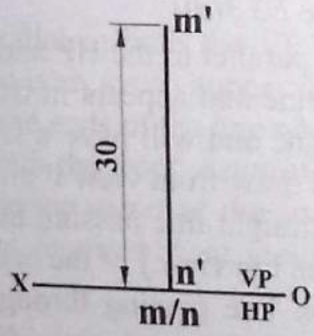


Figure E3.3(g)

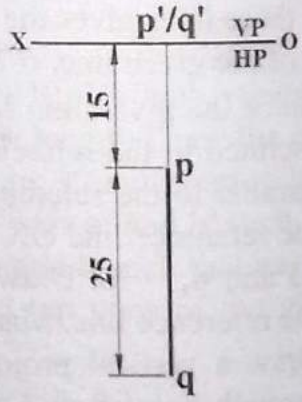


Figure E3.3(h)

(i) Since the given line **RS** is contained on the VP and inclined to the HP, its top view which appears in reduced length will coincide with the reference line OX. Its front view will be inclined to the reference line and appears in true length. Draw front view **r'** and top view **r** of the end **R** such that $\mathbf{r'r_x}$ = 10 and **r** coinciding with $\mathbf{r_x}$. Draw a straight line passing through the point **r'** and inclined at 60^0 to the reference line. Mark the front view **s'** of the end **S** on the inclined line such that **r's'** = 30. Draw a vertical projection line passing through the point **r'** intersecting the reference line OX at point **s**. Join **rs** to get the required top of the given line. *(Figure E3.3(i))*

(j) In this case, given line **UV** lies on the reference line OX. Therefore, its top view **uv** and front view **u'v'** coincide with OX. *(Figure E3.1.1(j))*

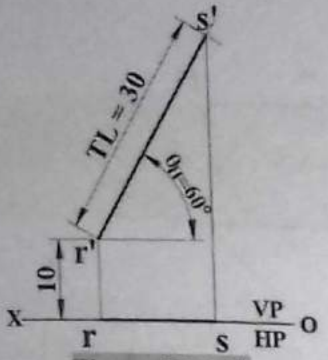


Figure E3.3(i)

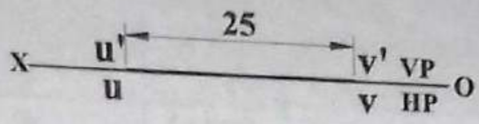


Figure E3.3(j)

Example 3.4

Draw three projections of the straight lines with given coordinates of the end points. Also determine their positions with respect to the principal planes of projections.

Line	AB		CD		EF		GH		IJ	
Coordinates	A	B	C	D	E	F	G	H	I	J
X	50	10	40	40	40	10	40	5	35	15
Y	20	20	12	46	10	30	20	20	30	10
Z	15	15	25	25	20	20	15	40	5	30

Solution

Follow the procedure explained in *Example 3.2* to get the projections of each end point of the given lines. Join the projections of end points in each view to get the three principal views of the given lines.

(a) As shown in *Figure E3.4(a)*, both top and front views of the line **AB** are parallel to the reference line OX, it is parallel to both the VP and HP and therefore perpendicular to the PP.

(b) As shown in *Figure E3.4(b)*, front view of the line **CD** is a point view; it is perpendicular to the VP and therefore parallel to the HP and VP.

(c) As shown in *Figure E3.4(c),* front view and side view of the line **EF** are parallel to the reference line OX and OY' respectively, it is parallel to the HP and inclined to the VP and PP.

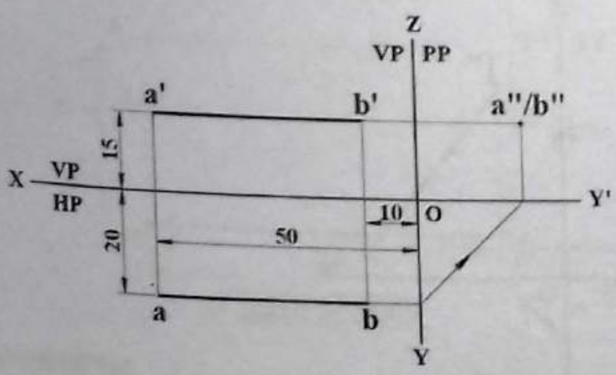


Figure E3.4(a)

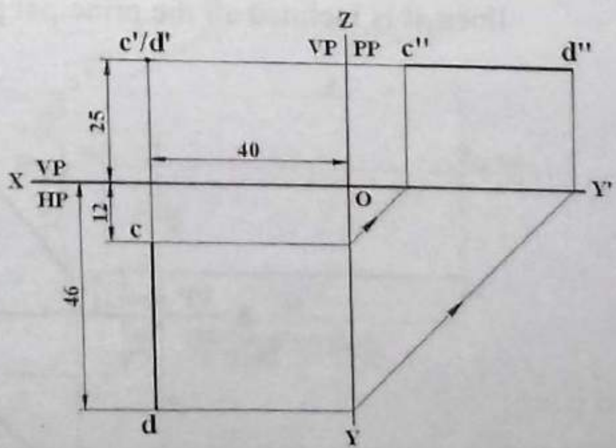


Figure E3.4(b)

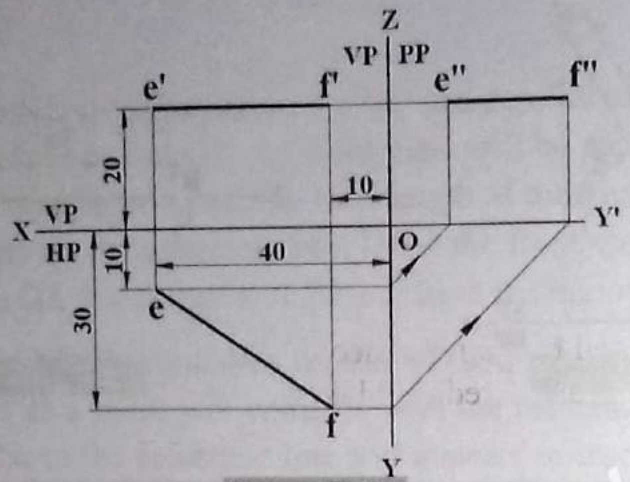


Figure E3.4(c)

(d) As shown in *Figure E3.4(d)*, top view and side view of the line **GH** are parallel to the reference line OX and OY' respectively, it is parallel to the VP and inclined to the HP and PP.

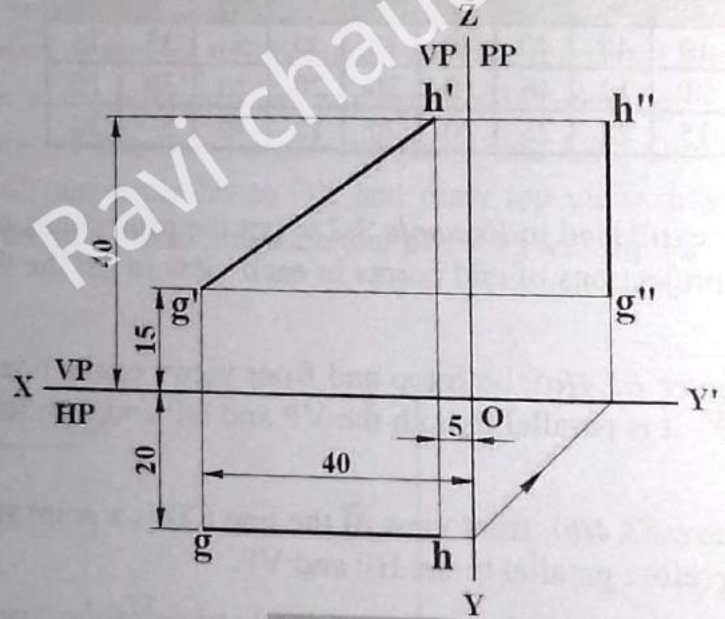


Figure E3.4(d)

(e) As shown in *Figure E3.4(e)*, all three views of the line **IJ** are inclined to the reference lines, it is inclined all the principal planes.

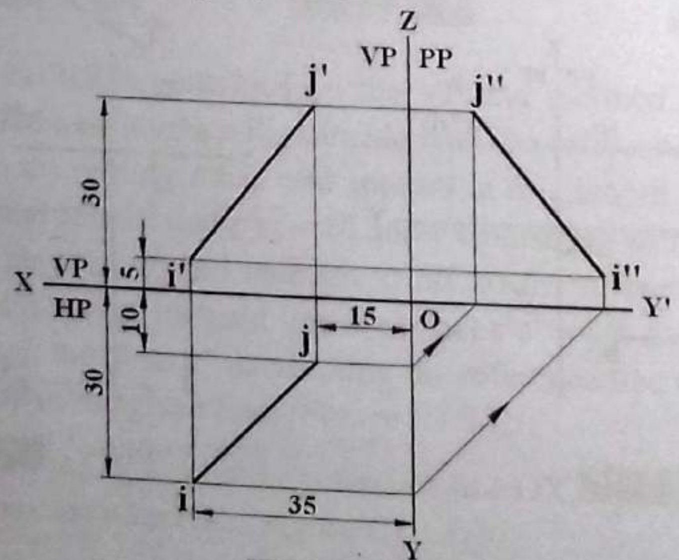


Figure E3.4(e)

Example 3.5

Top view of a straight line **AB** measures 50 mm. The line is parallel to the VP and inclined to the HP at 30°. Its end **A** is 10 mm above the HP and 15 mm in front of VP. Draw its projections and determine its true length.

Solution

Since the given line is parallel to the VP and inclined to the HP, its front view appears in true length and is inclined to the reference line at an angle at which it is inclined to the HP whereas its top view appears in reduced length and will be parallel to the reference line.

- Draw front view **a'** and top view **a** of the end **A** of the line such that **a'a**$_x$ = 10 and **aa**$_x$ = 15. *(Figure E3.5(a))*
- Draw a straight line passing through the point **a** and parallel to the reference line OX. Mark top view of **b** of the end **B** along the line such that **ab** = 50. *(Figure E3.5(b))*
- Draw vertical projection line passing through the point **b** and a line passing through the point **a'**and inclined at 30° to the reference line. Intersection of these lines give the front view **b'** of the end **B** of the line. *(Figure E3.5(c))*
- Then **ab** and **a'b'** are the required views of the line where **a'b'** (= 57.7 mm) is the true length of the line. *(Figure E3.5(d))*

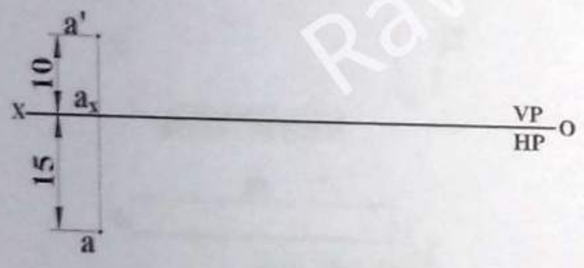


Figure E3.5(a)

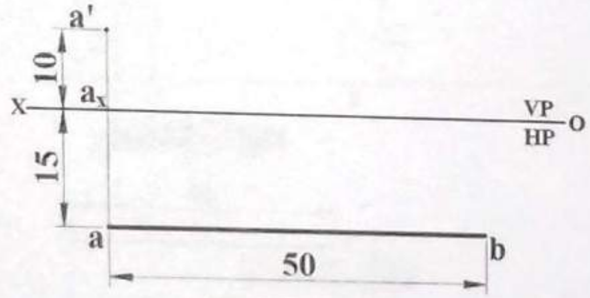


Figure E3.5(b)

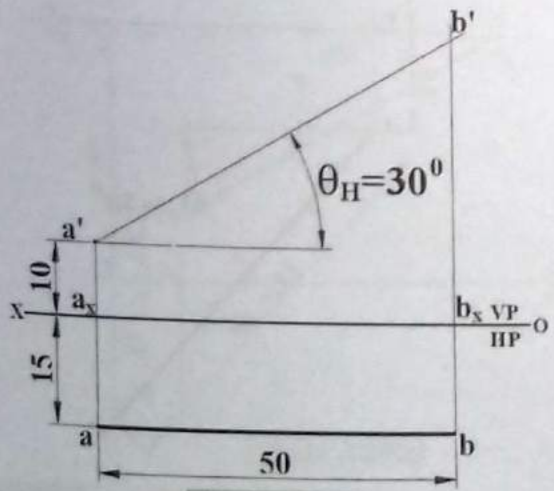


Figure E3.5(c)

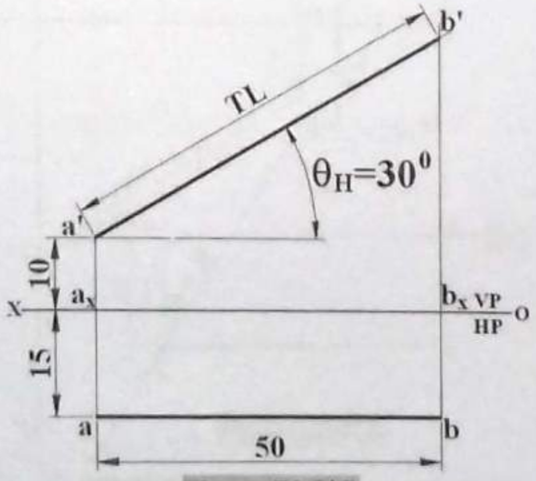


Figure E3.5(d)

Example 3.6

Front view of a straight line **CD** 60 mm long measures 40 mm. The line is parallel to the HP with its end **C** 20 mm above the HP and 15 mm in front of VP. Draw its projections and determine its true inclination with the VP.

Solution

Since the given line is parallel to the HP and inclined to the VP, its top view appears in true length and is inclined to the reference line at an angle at which it is inclined to the VP whereas its front view appears in reduced length and will be parallel to the reference line.

- Draw front view **c'** and top view **c** of the end **C** of the line such that $c'c_x = 20$ and $cc_x = 15$. *(Figure E3.6(a))*
- Draw a straight line passing through the point **c'** and parallel to the reference line OX. Mark front view of **d'** of the end **D** along the line such that **c'd'** = 40. *(Figure E3.6(b))*
- Draw vertical projection line passing through the point **d'** and an arc of radius 65 mm with **c** as center. Intersection of the line and the arc give the top view **d** of the end **D** of the line. *(Figure E3.6(c))*
- Then **cd** and **c'd'** are the required views of the line where inclination of the line **cd** gives the required inclination of the line with the VP, i.e. $\theta_V = 48.2^0$. *(Figure E3.5(d))*

Figure E3.6(a)

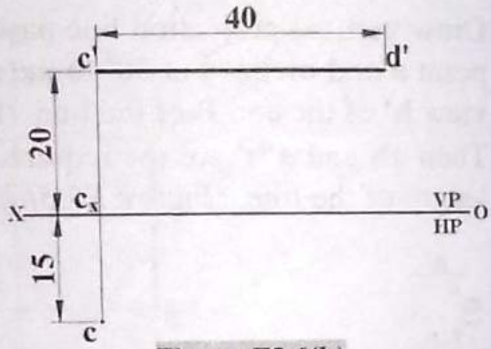


Figure E3.6(b)

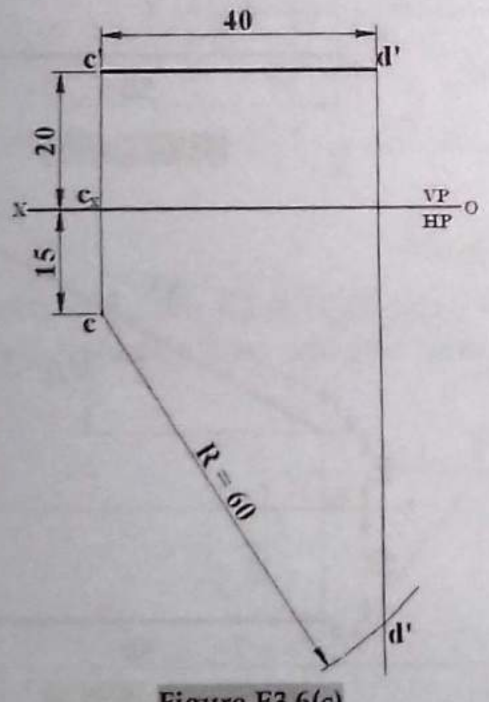


Figure E3.6(c)

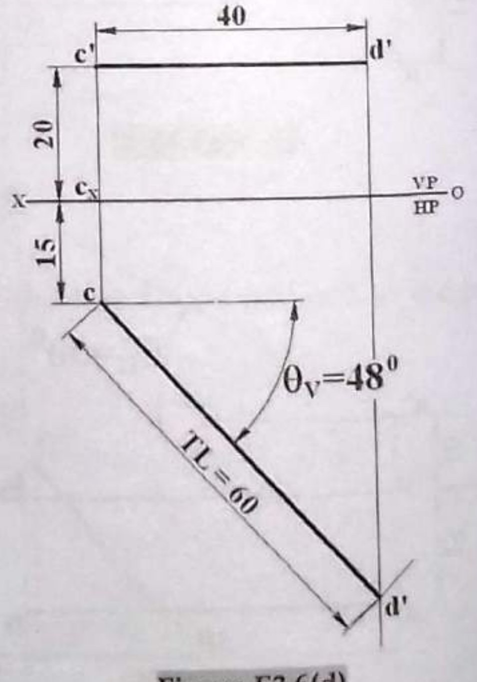


Figure E3.6(d)

Example 3.7

A straight line **EF** 60 mm long is parallel to the VP and 20 mm in front of it. Its end **E** is 10 mm above the HP while the other end is 40 mm above the HP. Draw its projections and determine its inclination with the HP.

Solution

Since the given line is parallel to the VP and inclined to the HP, its front view appears in true length and is inclined to the reference line at an angle at which it is inclined to the HP whereas its top view appears in reduced length and will be parallel to the reference line.

- Draw front view **e'** and top view **e** of the end **E** of the line such that **e'e$_x$** = 10 and **ee$_x$** = 20. *(Figure E3.7(a))*
- Draw a straight line parallel to the reference line OX 40 mm above it. *(Figure E3.7(b))*
- With **e'** as center and radius equal to 60, draw an arc intersecting the line at point **f'**. Join **e'** and **f'** to get the required front view of the given line. *(Figure E3.7(c))*
- Draw vertical projection line passing through the point **f'** and a line passing through the point **e** and parallel to the reference line. Intersection of these lines gives the top view **f** of the end **F** of the line. Then **ef** is the required top view of the line. The inclination of the line **e'f'** gives the required inclination of the line with the HP, i.e. $\theta_H = 29.7^0$. *(Figure E3.7(d))*

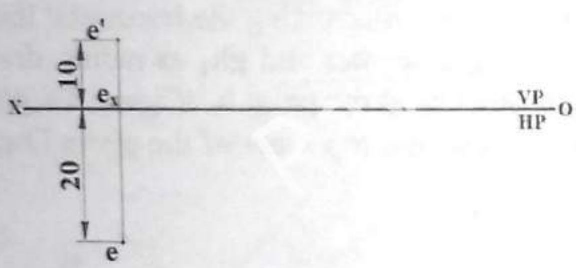


Figure E3.7(a)

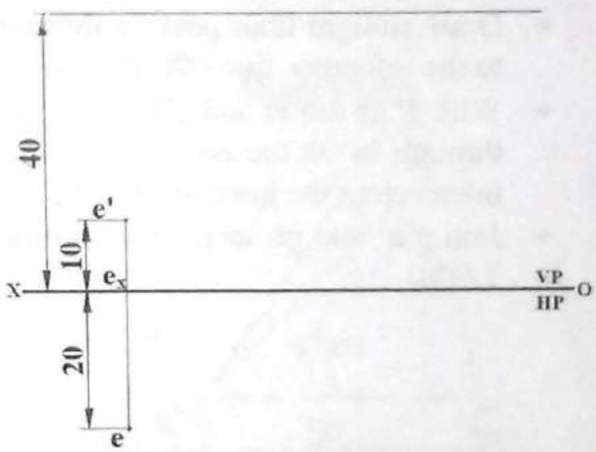


Figure E3.7(b)

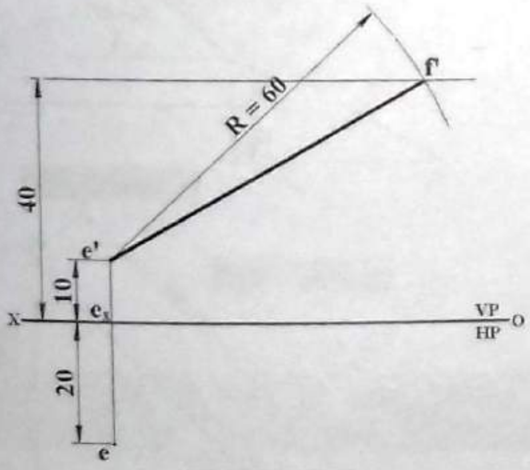


Figure E3.7(c)

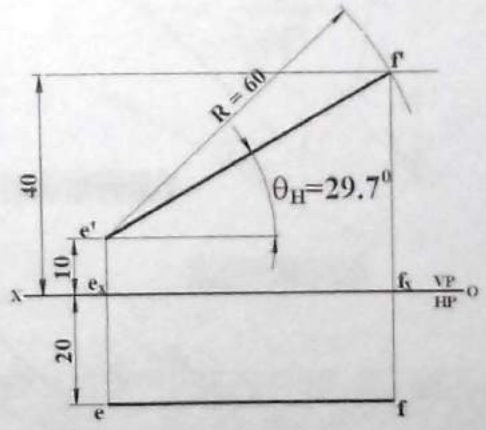


Figure E3.7(d)

Example 3.8

A straight line **GH** 60 mm long has its end **G** 10 mm above the HP and 12 mm in front of the VP. Draw its projections when it is inclined to the HP and VP at 45^0 and 30^0 respectively.

Solution

- Draw front view **g'** and top view **g** of the end **G** of the line such that **g'g$_x$** = 10 and **gg$_x$** = 12. *(Figure E3.8(a))*

- Draw the projections of the line assuming that it is parallel to the VP and inclined to the HP at $\theta_H = 45^0$.
  - Draw a line passing through **g'** and at an angle of 45^0 to the reference line. Mark point **h_1'** along the inclined line such that **g'h_1'** = 60. *(Figure E3.8(b))*
  - Draw a vertical projection passing through the point **h_1'** and a horizontal line passing through the point **g**. Intersection of these lines gives the point **h_1** on the top view. *(Figure E3.8(c))*
- Draw the projections of the line assuming that it is parallel to the HP and inclined to the VP at $\theta_V = 30^0$.
  - Draw a line passing through **g** and at an angle of 30^0 to the reference line. Mark point **h_2** along the inclined line such that **gh_2** = 60. *(Figure E3.8(d))*
  - Draw a vertical projection passing through the point **h_2** and a horizontal line passing through the point **g'**. Intersection of these lines gives the point **h_2'** on the front view. *(Figure E3.8(e))*
- Draw straight lines passing through the ends of true length lines **h_1'** and **h_2** and parallel to the reference line OX. *(Figure 3.8(f))*
- With **g'** as center and **g'h_2'** as radius, draw an arc intersecting the horizontal line passing through **h_1'** at the point **h'**. Similarly, with **g** as center and **gh_1** as radius, draw an arc intersecting the horizontal line passing through **h_2** at the point **h**. *(Figure 3.8(g))*
- Join **g'h'** and **gh** to get the required front view and top view of the given line. *(Figure 3.8(h))*

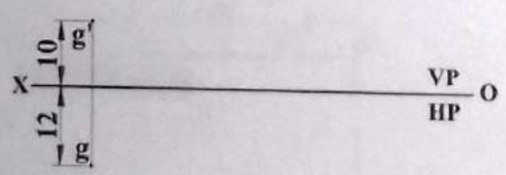


Figure E3.8(a)

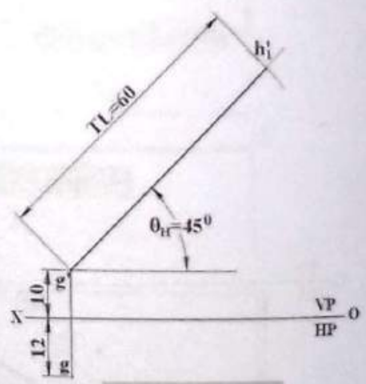


Figure E3.8(b)

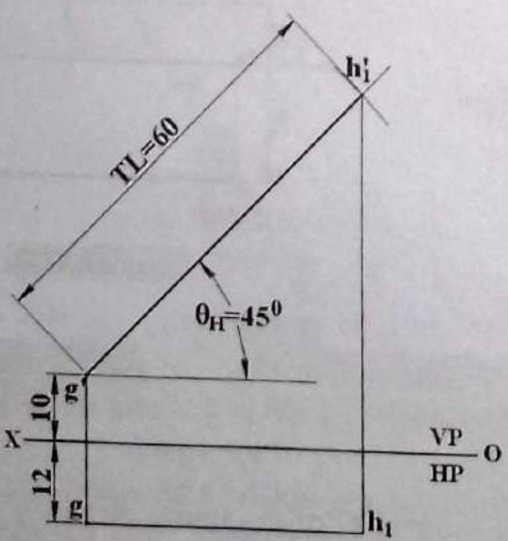


Figure E3.8(c)

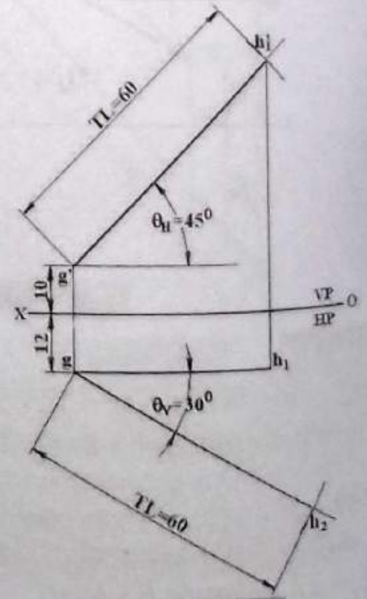


Figure E3.8(d)

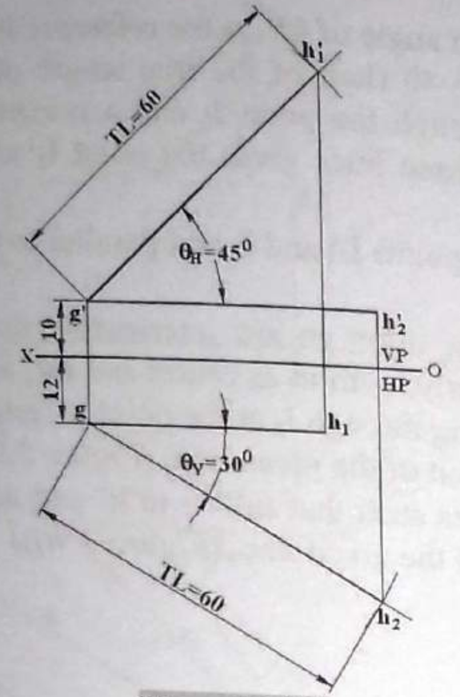


Figure E3.8(e)

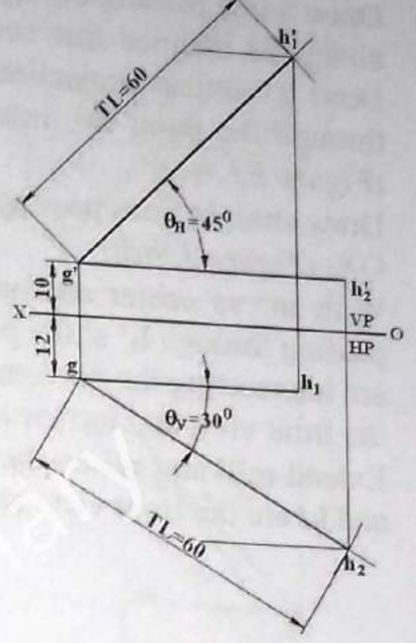


Figure E3.8(f)

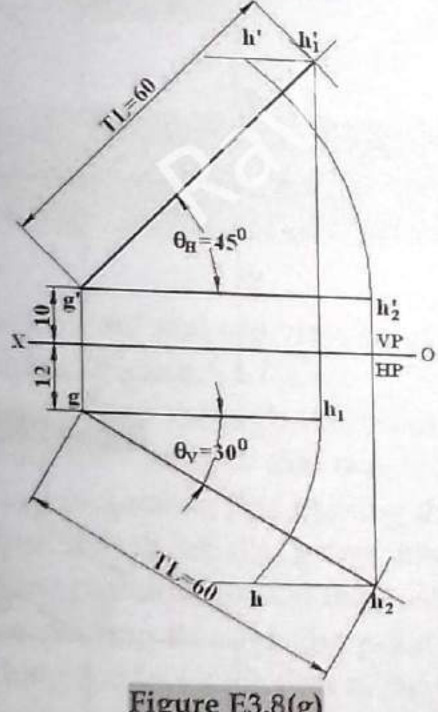


Figure E3.8(g)

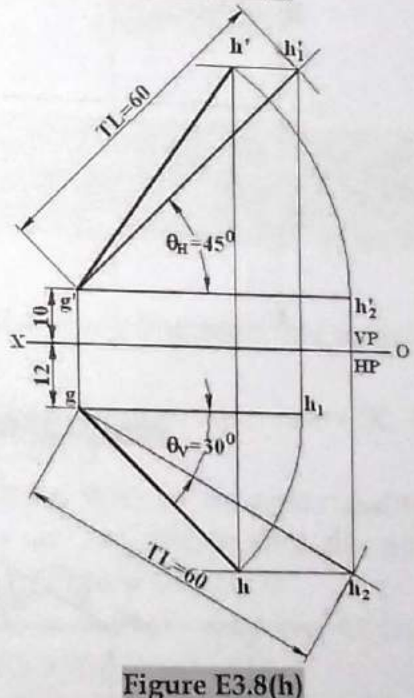


Figure E3.8(h)

Example 3.9

A straight line **KL** 80 mm long is inclined at 30^0 to the HP and 45^0 to the VP. Its midpoint is 30 mm above the HP and 35 mm in front of the VP. Draw its projections.

Solution

- Draw front view **m'** and top view **m** of the midpoint **M** of the given line such that $\mathbf{m'm_x}$ = 30 and $\mathbf{mm_x}$ = 35. *(Figure E3.9(a))*
- Draw a line passing through **m'** and at an angle of 30^0 to the reference line. Mark point $\mathbf{l_1'}$ along the inclined line such that $\mathbf{m'l_1'}$ = 40 (half of the true length of the given line). Draw a vertical projection passing through the point $\mathbf{l_1'}$ and a horizontal line passing through the point **m**. Intersection of these lines gives the point $\mathbf{l_1}$ on the top view. *(Figure E3.9(b))*

- Draw a line passing through **m** and at an angle of 45^0 to the reference line. Mark point $\mathbf{l_2}$ along the inclined line such that $\mathbf{ml_2}$ = 40 (half of the true length of the given line). Draw a vertical projection passing through the point $\mathbf{l_2}$ and a horizontal line passing through the point **m'**. Intersection of these lines gives the point $\mathbf{l_2}'$ on the front view *(Figure E3.9(c))*
- Draw straight lines passing through the points $\mathbf{l_1}'$ and $\mathbf{l_2}$ and parallel to the reference line OX. *(Figure 3.9(d))*
- With **m'** as center and $\mathbf{m'l_2}'$ as radius, draw an arc intersecting the horizontal line passing through $\mathbf{l_1}'$ at the point **l'**. Similarly, with **m** as center and $\mathbf{m}l_1$ as radius, draw an arc intersecting the horizontal line passing through $\mathbf{l_2}$ at the point **l**. Then **m'l'** and **ml** are the front view and top view of half portion of the given line. *(Figure 3.9(e))*
- Extend **m'l'** and **ml** on the opposite sides such that **m'l'** = **m'k'** and **ml** = **kl**. Then k'l' and **k*l*** are the front view and top view of the given line. *(Figure 3.9(f))*

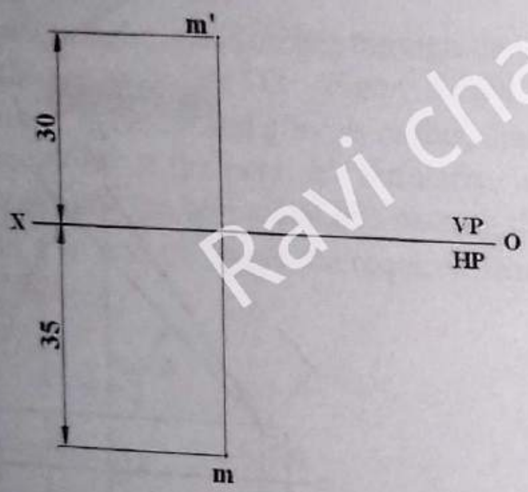


Figure E3.9(a)

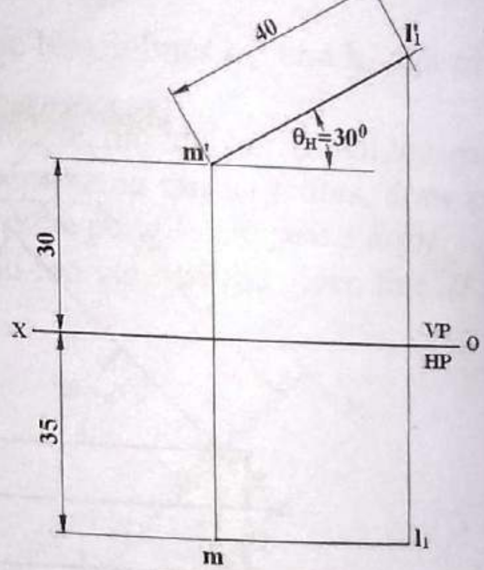


Figure E3.9(b)

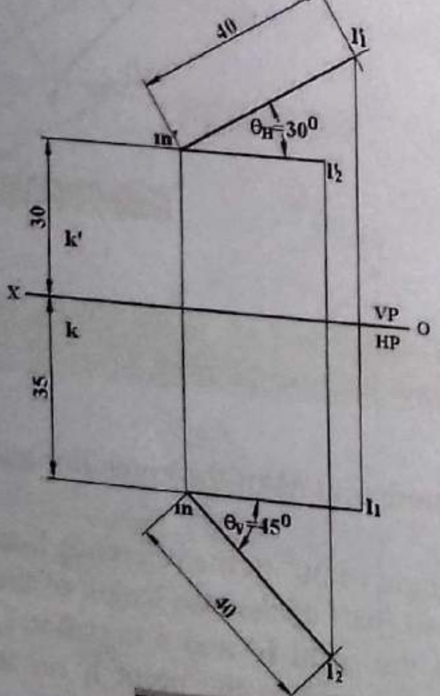


Figure E3.9(c)

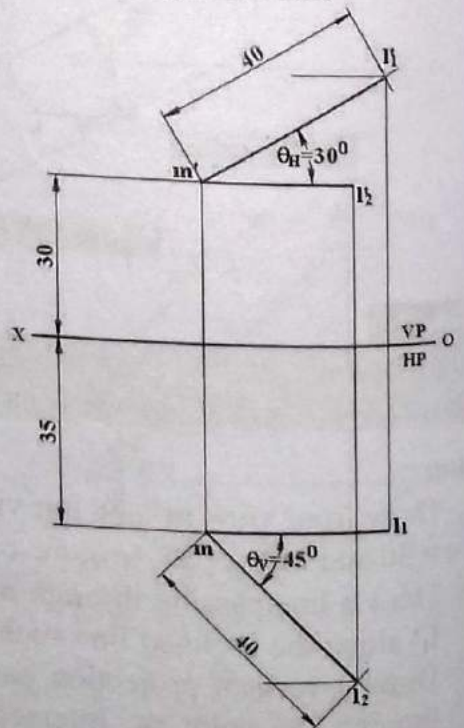


Figure E3.9(d)

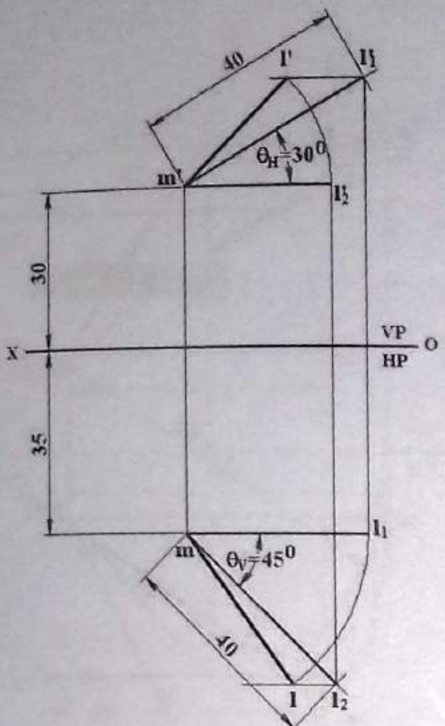


Figure E3.9(e)

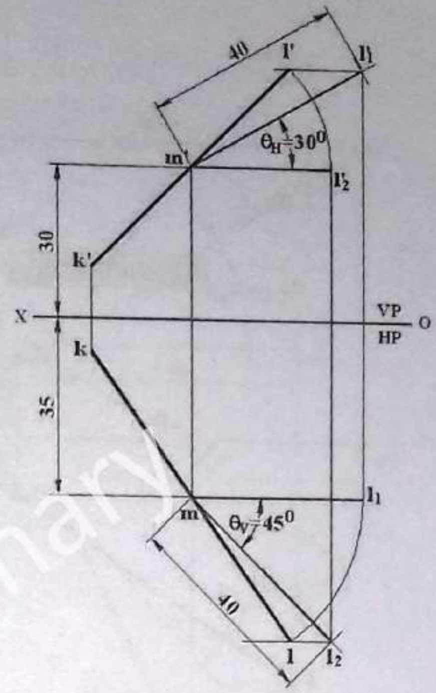


Figure E3.9(f)

Example 3.10

Top view and front view of a straight line MN 60 mm long measure 50 mm and 40 mm respectively. One of its ends is 15 mm above the HP and 10 mm in front of the VP. Draw its projections and determine its true inclination with the VP and HP.

Solution

- Draw front view **m'** and top view **m** of the end **M** of the given line such that $m'm_x = 15$ and $mm_x = 10$. *(Figure E3.10(a))*
- Draw a line passing through the point **m** and parallel to the reference line OX. Mark point n_1 along the line such that $mn_1 = 50$. *(Figure E3.10(b))*
- Draw vertical projection line passing through the point n_1. With **m'** as center and radius equal to true length of the given line (= 60), draw an arc intersecting the vertical projection line passing through the point n_1 at point n_1'. *(Figure E3.10(c))*
- Draw a line passing through the point **m'** and parallel to the reference line OX. Mark point n_2' along the line such that $m'n_2' = 40$. *(Figure E3.10(d))*
- Draw vertical projection line passing through the point n_2'. With **m** as center and radius equal to true length of the given line (= 60), draw an arc intersecting the vertical projection line passing through the point n_2' at point n_2. *(Figure E3.10(e))*
- Draw straight lines passing through the ends of true length lines n_1' and n_2 and parallel to the reference line OX. *(Figure E3.10(f))*
- With **m'** as center and $m'n_2'$ as radius, draw an arc intersecting the horizontal line passing through n_1' at the point **n'**. Similarly, with **m** as center and mn_1 as radius, draw an arc intersecting the horizontal line passing through n_2 at the point **n**. Join **m'n'** and **mn** to get the required front view and top view of the given line. *(Figure 3.10(g))*
- True inclination of the line with the HP is given by the angle made by the true length line in the front view $m'n_1'$ with the reference line ($\theta_H = 33.6^0$) and true inclination of the line with the VP is given by the angle made by the true length line in the top view mn_2 with the reference line ($\theta_V = 48.2^0$) *(Figure 3.10(h))*.

Figure E3.10(a)

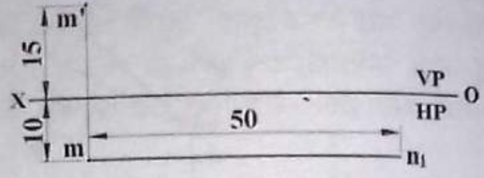


Figure E3.10(b)

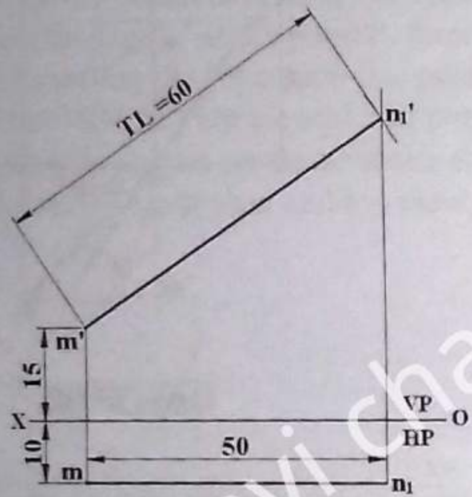
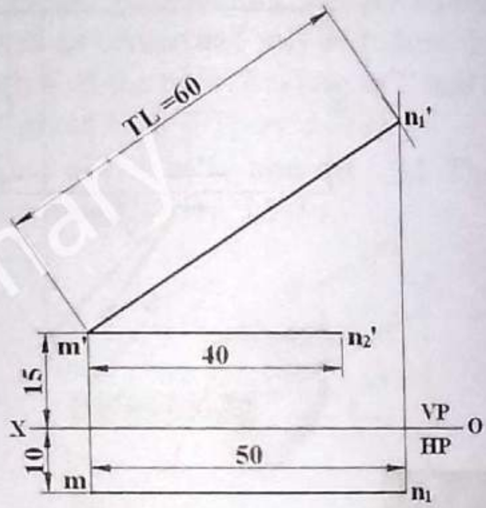


Figure E3.10(c)



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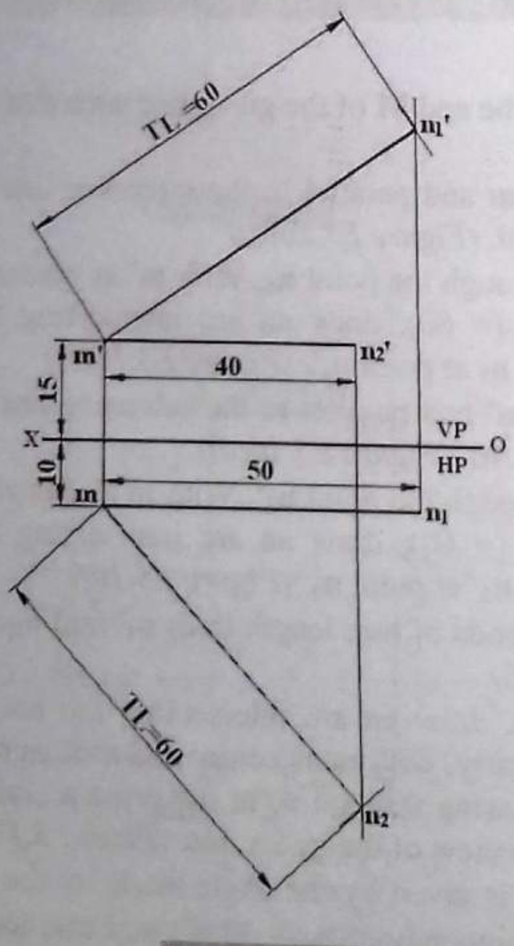


Figure E3.10(e)

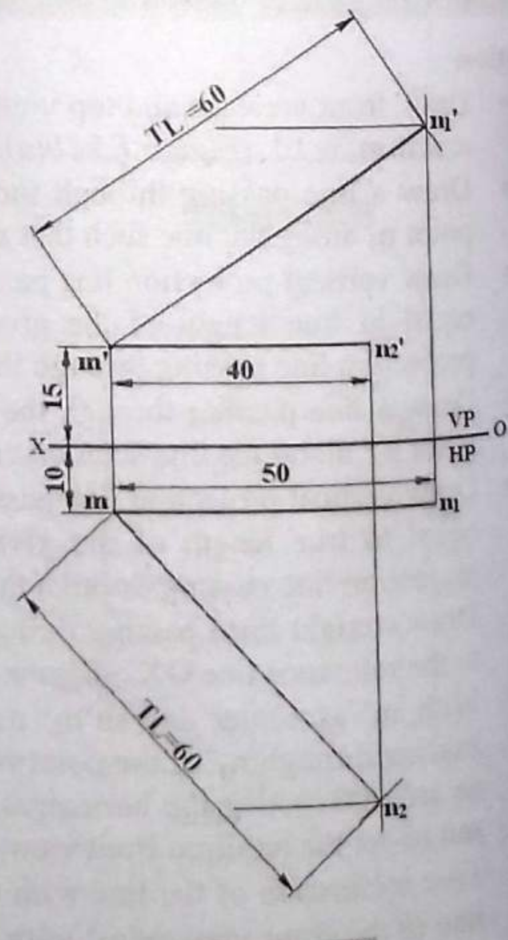


Figure E3.10(f)

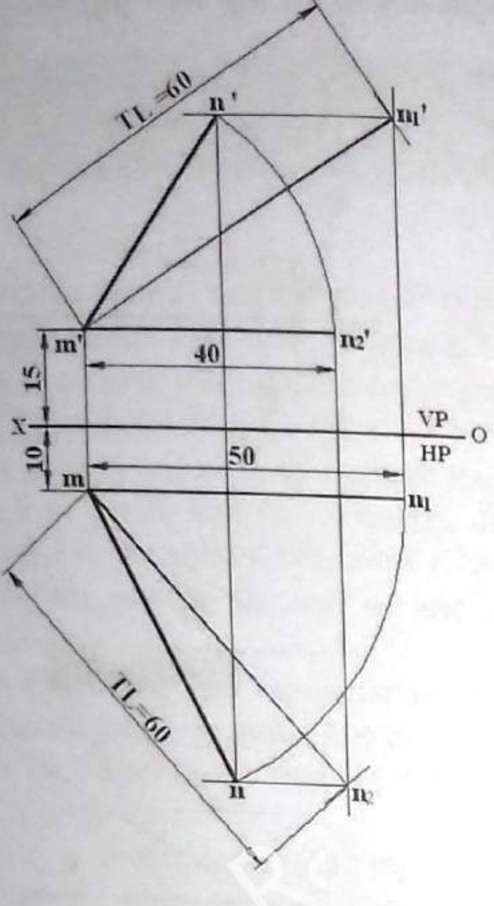


Figure E3.10(g)

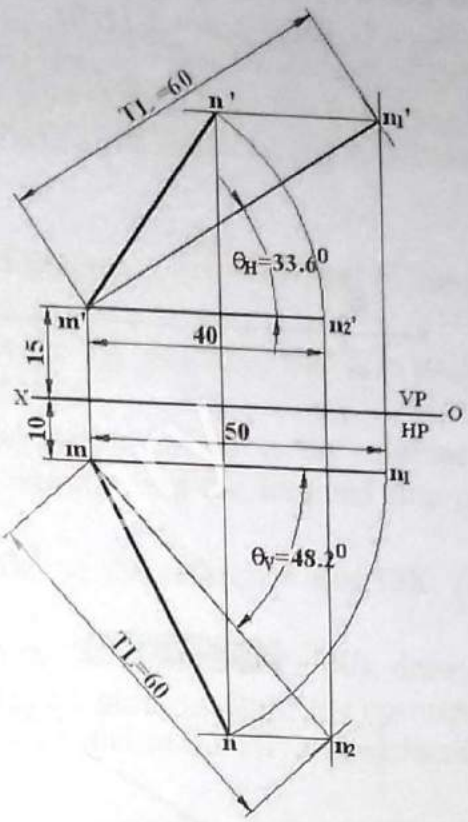


Figure E3.10(h)

Example 3.11

A straight line **PQ** 60 mm long has its end **P** 10 mm above the HP and 15 mm in front of the VP while its end **Q** 40 mm above the HP and 55 mm in front of the VP. Draw its projections and determine its true inclinations with the VP and HP.

Solution

- Draw front view **p'** and top view **p** of the end **P** of the given line such that $\mathbf{p'p_x} = 10$ and $\mathbf{pp_x} = 15$. *(Figure E3.11(a))*
- Draw straight lines **1** and **2** parallel to the reference line OX at distances of 40 mm above the reference line and 55 mm below the reference line respectively. *(Figure E3.11(b)).*
- With **p'** as center and radius equal to true length of the given line (= 60), draw an arc intersecting the line **1** at the point $\mathbf{q_1'}$. Draw its corresponding top view $\mathbf{pq_1}$ assuming that it is parallel to the VP. *(Figure E3.11(c))*
- With **p** as center and radius equal to true length of the given line (= 60), draw an arc intersecting the line **2** at the point $\mathbf{q_2}$. Draw its corresponding front view $\mathbf{pq_2'}$ assuming that it is parallel to the HP. *(Figure E3.11(d))*
- With **p'** as center and $\mathbf{p'q_2'}$ as radius, draw an arc intersecting the horizontal line **1** at the point **q'**. Similarly, with **p** as center and $\mathbf{pq_1}$ as radius, draw an arc intersecting the horizontal line **2** at the point **q**. Join **p'q'** and **pq** to get the required front view and top view of the given line. *(Figure 3.11(e))*
- True inclination of the line with the HP is given by the angle made by the true length line in the front view $\mathbf{p'q_1'}$ with the reference line ($\theta_H = 30^0$) and true inclination of the line with the

VP is given by the angle made by the true length line in the top view **pq₂** with the reference line ($\theta_V = 41.8^0$) *(Figure 3.11(f))*.

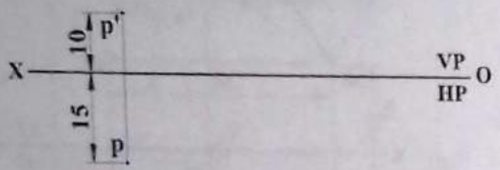
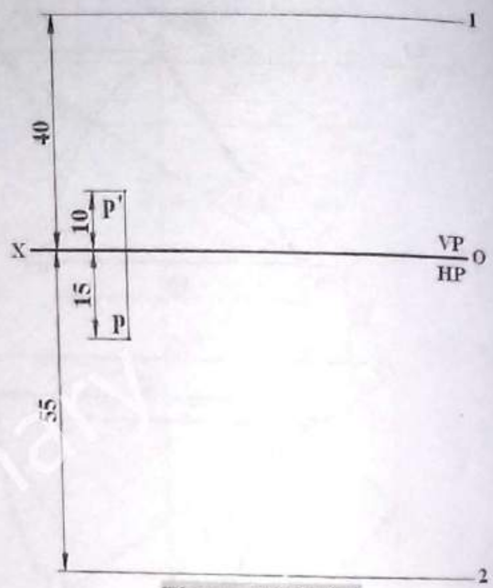


Figure E3.11(a)



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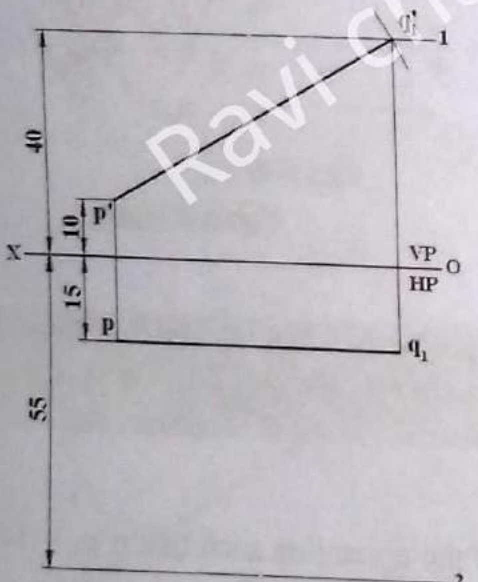

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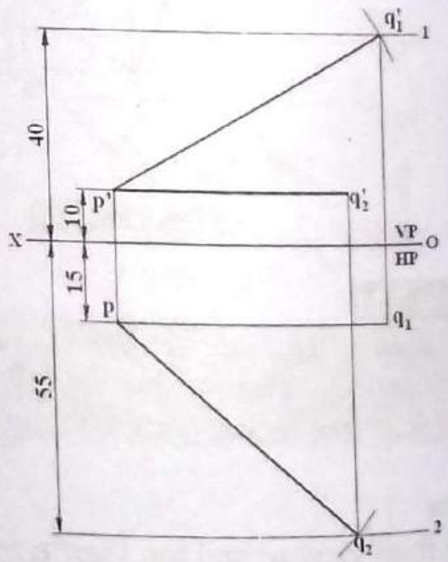


Figure E3.11(d)

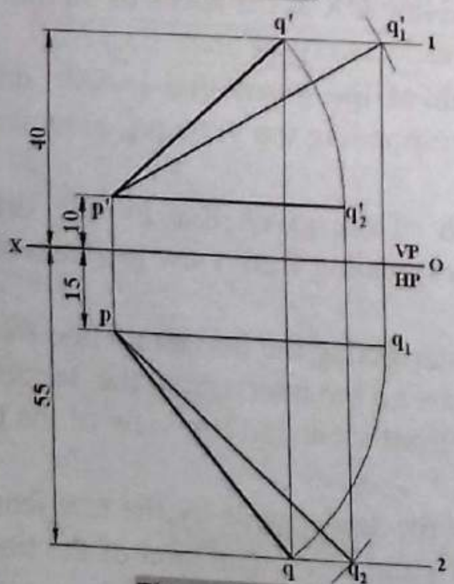


Figure E3.11(e)

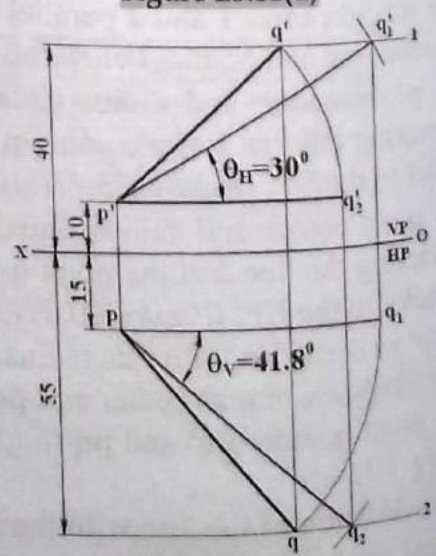


Figure E3.11(f)

Example 3.12

A straight line **RS** 60 mm long is inclined to the HP at 45^0 and its top view makes an angle of 60^0 with the reference line. Its end **R** is in the HP and 10 mm in front of the VP. Draw its projections and determine its inclination with the VP.

Solution

- Draw front view **r'** and top view **r** of the end **R** of the given line such that **r'** lies on the reference line and $\mathbf{rr_x} = 10$. *(Figure E3.12(a))*
- Draw front view $\mathbf{r's_1'}$ and the corresponding top view $\mathbf{rs_1}$ assuming that it is parallel to the VP and inclined to the HP at 45^0. *(Figure E3.12(b))*
- Draw straight line passing through the point **r** and inclined at 60^0 to the reference line. With **r** as center and $\mathbf{rs_1}$ as radius, draw an arc intersecting the inclined line passing through **r** at the point **s**. *(Figure E3.12(c))*
- Draw lines passing through $\mathbf{s_1'}$ and **s** and parallel to the reference line OX. *(Figure E3.12(d))*
- With **r** as center and radius equal to true length of the given line (= 60), draw an arc intersecting the horizontal line passing through **s** at the point $\mathbf{s_2}$. Draw the corresponding front view $\mathbf{r's_2'}$ of the line $\mathbf{rs_2}$ assuming that it is parallel to the HP and inclined to the VP. *(Figure E3.12(e))*
- With **r'** as center and $\mathbf{r's_2'}$ as radius, draw an arc intersecting the horizontal line passing through the point $\mathbf{s_1'}$ at **s'**. Then **r's'** and **rs** are the required front and top view of the line. True inclination of the line with the VP is given by the angle made by the true length line in the top view $\mathbf{rs_2}$ with the reference line ($\theta_V = 41.8^0$) *(Figure E3.12(f))*.

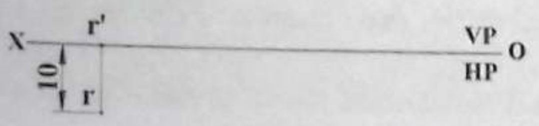
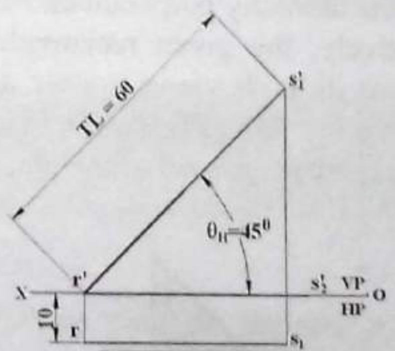


Figure E3.12(a)



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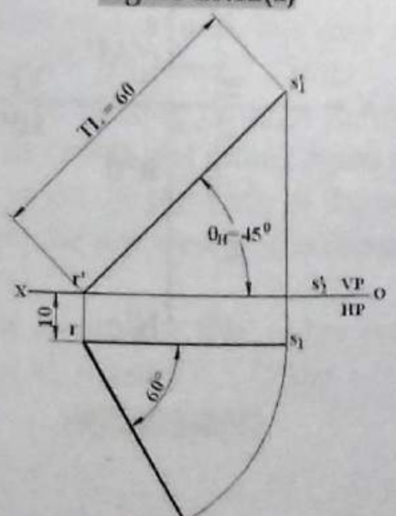


Figure E3.12(c)

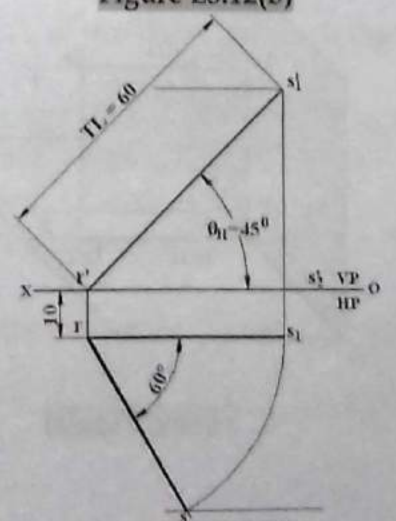


Figure E3.12(d)

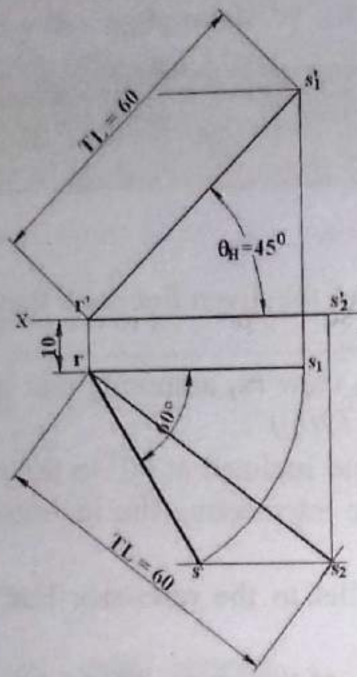


Figure E3.12(e)

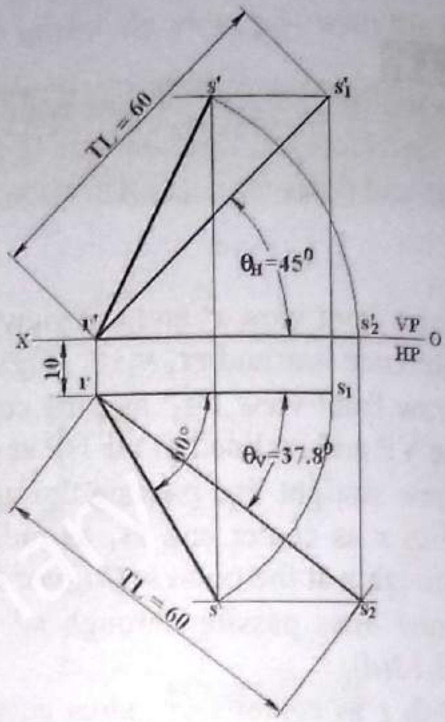


Figure E3.12(f)

Example 3.13

A rectangle **ABCD** 50 mm × 40 mm has its longer edges perpendicular to the HP and shorter edges perpendicular to the VP. Draw its projections when its edges nearer to the HP and VP are 10 mm and 20 mm from them respectively.

Solution

Since the mutually perpendicular edges of the rectangle are perpendicular to the VP and HP respectively, the given rectangular plane is perpendicular to both the VP and HP and therefore its both views appear as the edge view and both will be perpendicular to the reference line OX. *Figure E3.13(a)* shows pictorial projection and *Figure E3.13(b)* shows orthographic projection of the plane.

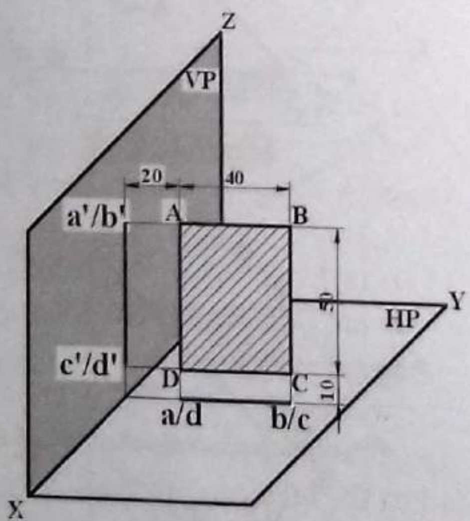


Figure E3.13(a)

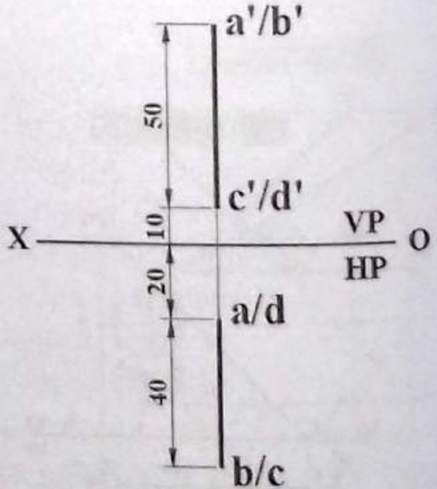


Figure E3.13(b)

Example 3.14

A square **ABCD** of 20 mm side is parallel to the VP and 15 mm in front of it. Draw its projections when its edge nearer to the HP is parallel to it and 10 mm above it.

Solution

Since the given plane is parallel to the VP, its front view appears in true shape and size and the top view appears as an edge view.

- Draw front view **a'b'** 20 mm long as base of the square parallel to the reference line and 10 mm above it. *(Figure E3.14(a))*
- Complete the square **a'b'c'd'** as the required front view of the given plane. *(Figure E3.14(b))*
- Draw its edge view as the top view parallel to the reference line and 15 mm below it. *(Figure E3.14(c))*

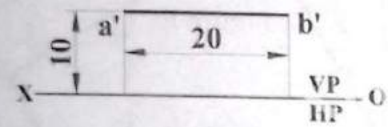
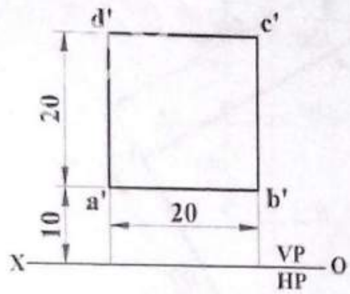


Figure E3.14(a)



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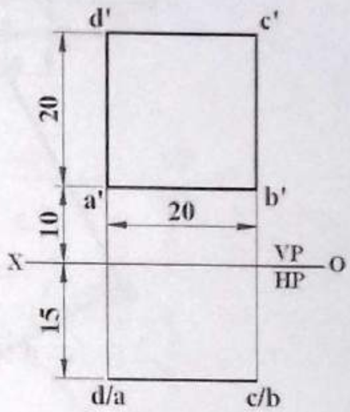


Figure E3.14(c)

Example 3.15

A rectangle **ABCD** 60 mm × 50 mm is parallel to the HP and 20 mm above it. Its longer edge is inclined at 30^0 to the VP. Draw its projections when its corner nearer to the VP is 15 mm in front of it.

Solution

Since the given rectangular plane is parallel to the HP, its top view appears in true shape and size and the front view appears as an edge view.

- Draw front view **a'** and top view **a** of the corner **A** of the rectangle such that $a'a_x = 20$ and $aa_x = 15$. *(Figure E3.15(a))*
- Draw a line passing through the point **a** and inclined at 30^0 to the reference line OX. With **a** as center and radius equal to 60, draw an arc intersecting the inclined line at the point **b** to get the top view of the corner **B**. *(Figure E3.15(b))*
- Complete the top view of the rectangular plane with **ab** as its base and **bc** = 50. *(Figure E3.15(c))*
- Draw the front view **d'b'** (edge view) parallel to the reference line and passing through the point **a'**. *(Figure E3.15(d))*

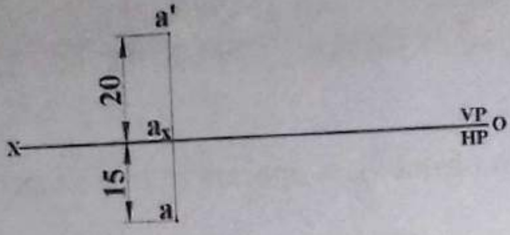


Figure E3.15(a)

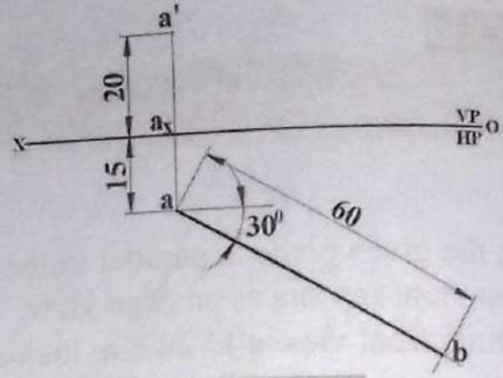


Figure E3.15(b)

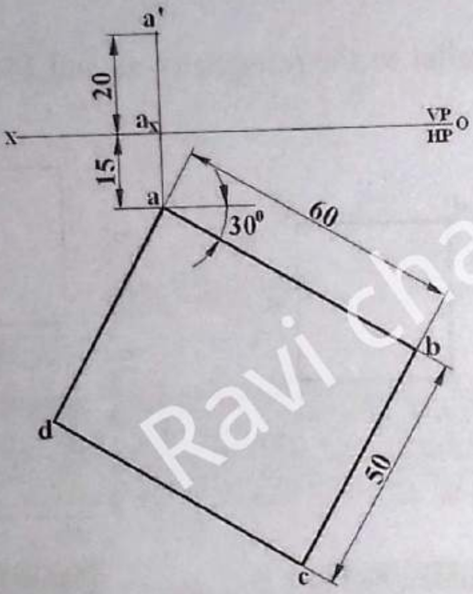


Figure E3.15(c)

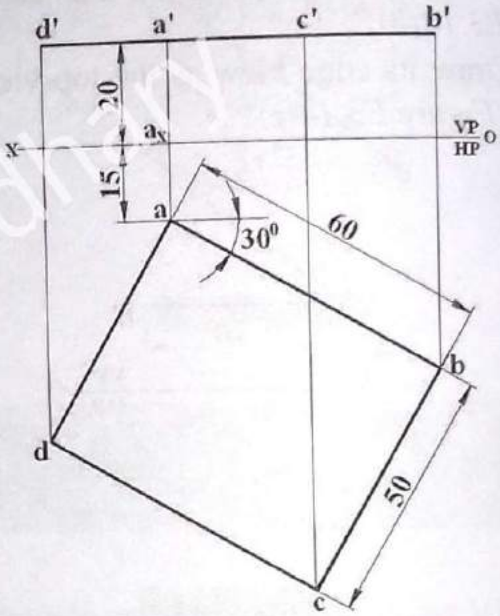


Figure E3.15(d)

Example 3.16

An equilateral triangle of 25 mm side is parallel to the HP and 15 mm above it. It is placed in such a way that one of the corners of the triangle is 10 mm in front of the VP and the edge containing that corner is inclined at 45^0 to the VP. Draw its projections.

Solution

Since the given triangular plane is parallel to the HP, its top view appears in true shape and size and the front view appears as an edge view.

- Draw front view **a'** and top view **a** of the corner **A** of the triangle such that **a'a**$_x$ = 10 and **aa**$_x$ = 10. *(Figure E3.16(a))*
- Draw a line passing through the point **a** and inclined at 45^0 to the reference line OX. With **a** as center and radius equal to 25, draw an arc intersecting the inclined line at the point **b** to get the top view of the corner **B**. *(Figure E3.16(b))*
- Complete the top view of the triangular plane with **ab** as its base. *(Figure E3.15(c))*
- Draw the front view **c'b'** (edge view) parallel to the reference line and passing through the point **a'**. *(Figure E3.16(d))*

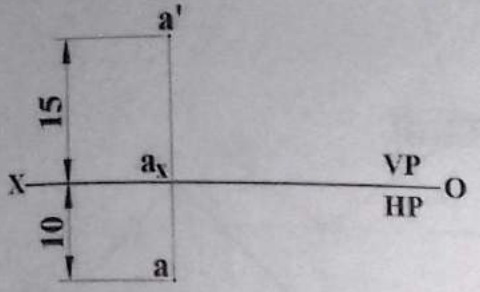


Figure E3.16(a)

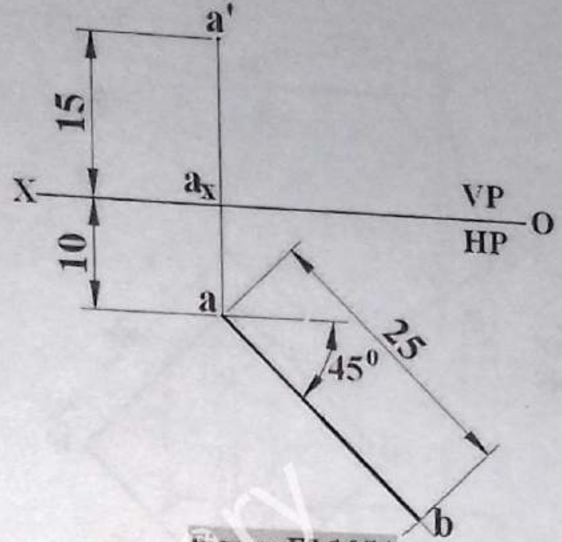


Figure E3.16(b)

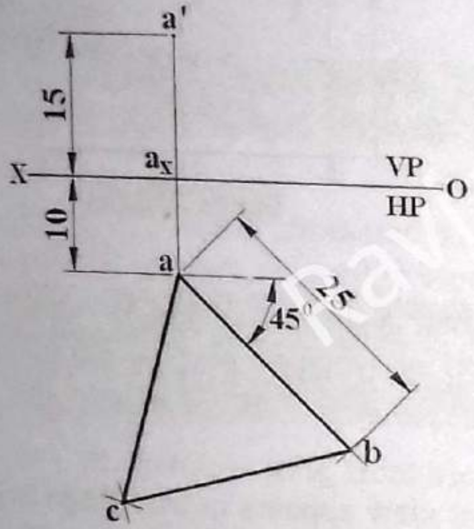


Figure E3.16(c)

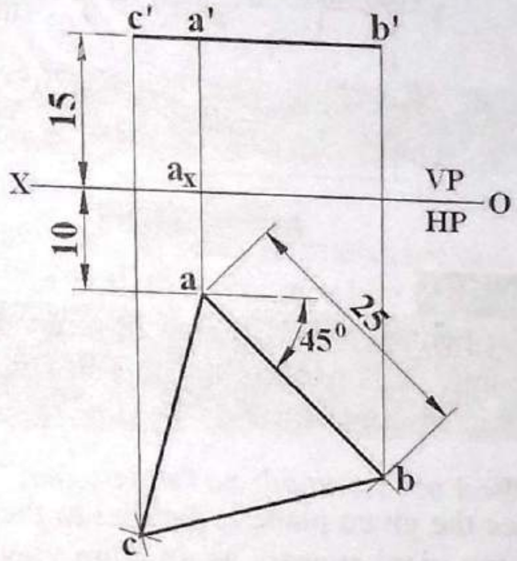


Figure E3.16(d)

Example 3.17

A square **ABCD** of 25 mm side has a corner resting on the HP and 20 mm in fornt of the VP. Its plane is parallel to the VP with all its edges equally inclined to the HP. Draw its projections.

Solution

Since the given plane is parallel to the VP, its front view appears in true shape and size and the top view appears as an edge view. It all will have equal inclination of 45^0 with the HP when one of its diagonal is perpendicular to the HP.

- Draw front view **a'** and top view **a** of the corner **A** of the square on which it rests on the HP such that **a'** coincides on the reference line OX and $\mathbf{aa_x} = 20$. *(Figure E3.17(a))*
- Draw straight lines passing through the point **a'** on both side of the point and inclined at 45^0 to the reference line OX. With **a'** as center and radius equal to 25, draw arcs intersecting the inclined lines at the points **b'**and **d'**. Draw arcs with **b'** and **d'** as centers and radii equal to 25 intersecting each other at the point **c'**. Complete the front view of the square plane **a'b'c'd'**. *(Figure E3.17(b))*
- Draw the top view **db** (edge view) parallel to the reference line and passing through the point **a**. *(Figure E3.17(c))*

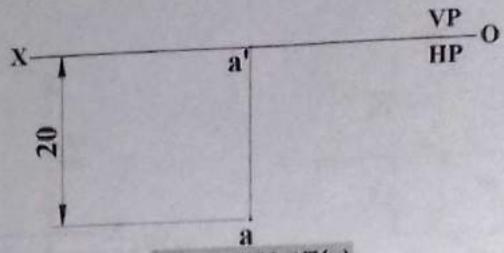
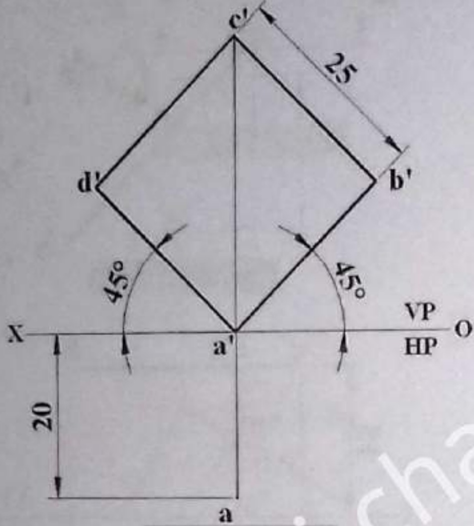


Figure E3.17(a)



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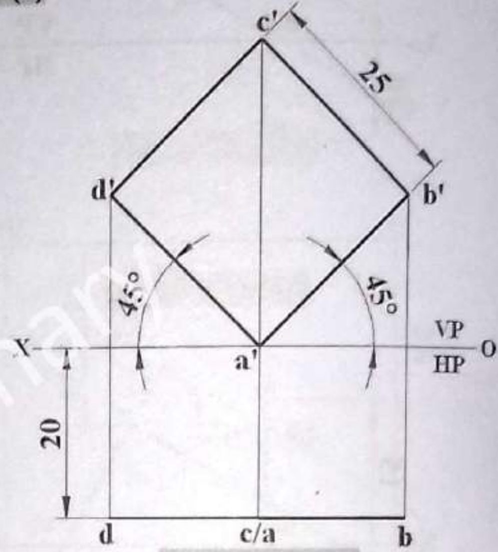


Figure E3.17(c)

Example 3.18

A regular pentagon **ABCDE** of 20 mm side has its corner **A** on the HP and its side **CD** opposite to the corner A is parallel to the HP. Draw its projections when its plane is parallel to the VP and 18 mm in front of it.

Solution

Since the given plane is parallel to the VP, its front view appears in true shape and size and the top view appears as an edge view. Its edge **CD** will be parallel to the HP when it is placed vertically inverted and with corner **A** on the HP.

- Draw front view **a'** and top view **a** of the corner **A** of the pentagon on which it rests on the HP such that **a'** coincides on the reference line OX and $aa_x = 18$. *(Figure E3.18(a))*
- Draw straight lines passing through the point **a'** on both side of the point and inclined at 36^0 to the reference line OX. With **a'** as center and radius equal to 20, draw arcs intersecting the inclined lines at the points **b'**and **e'**. Complete the front view of the pentagon **a'b'c'd'e'** such that internal angle between the adjacent sides is 108^0 and each side have e a length of 20 mm. *(Figure E3.18(b))*
- Draw the top view **eb** (edge view) parallel to the reference line and passing through the point **a**. *(Figure E3.18(c))*

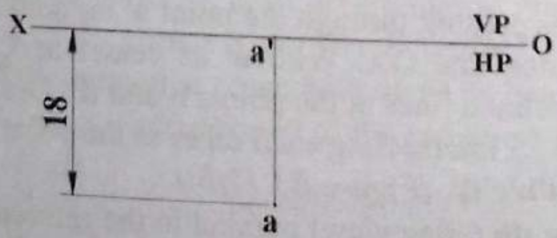


Figure E3.18(a)

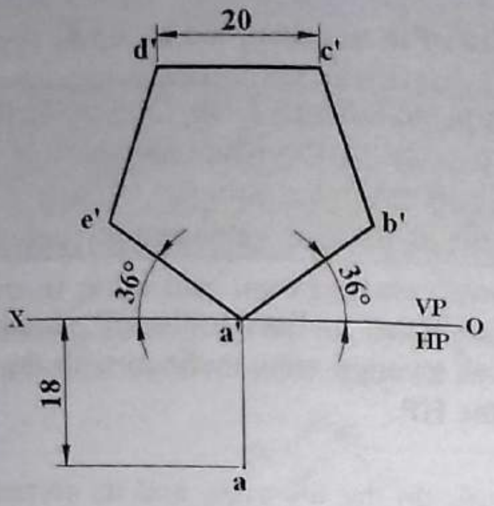


Figure E3.18(b)

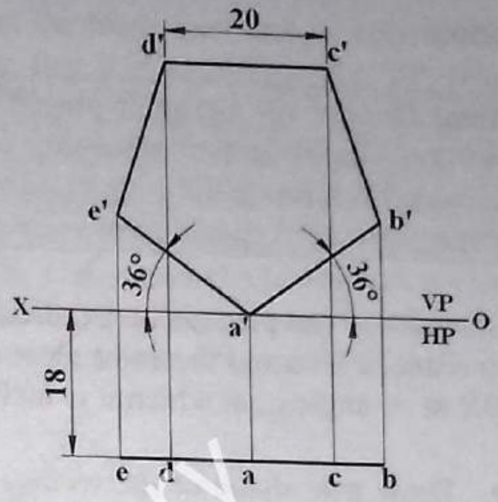


Figure E3.18(c)

EXAMPLE 3.19

A hexagonal plane of 20 mm side is lying on the HP such that one of its edge is perpendicular to the VP. Draw its projections when its corner nearer to the VP is 12 mm in front of it.

Solution

Since the given plane is contained on the HP, its top view appears in true shape and size and the front view appears as an edge view and coincides with the reference line OX.

- Draw regular hexagon of side 20 mm such that its corner **a** is 12 mm below the reference line OX and its edges **bc** and **ef** are perpendicular to the reference line. *(Figure E3.19(a))*
- Draw the corresponding front view **b'/c'-e'/f'** (edge view) on the reference line. *(Figure E3.19(b))*

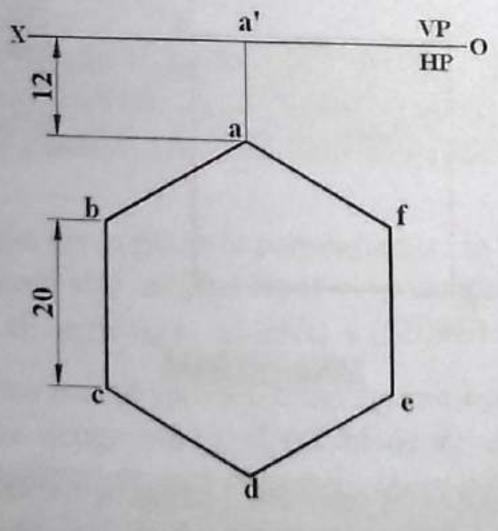


Figure E3.19(a)

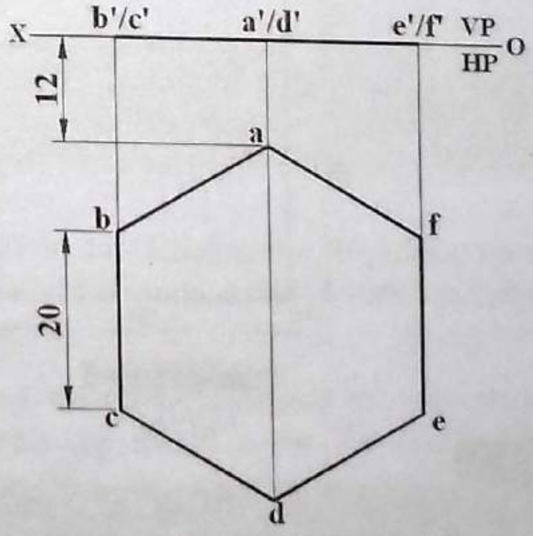


Figure E3.19(b)

Example 3.20

A rectangular plane **ABCD** of dimensions 60 mm × 40 mm is perpendicular to the VP and inclined to the HP at 30^0. Its edge **AB** is resting on the HP and perpendicular to the VP with its corner **A** 15 mm in front of it. Draw its projections.

Solution

> **Hints:** To draw orthographic projections of a plane inclined to any one of the principal plane, first draw its projection assuming it to be parallel to the plane to which it is given to be inclined. Then rotate the edge view thus obtained through the angle at which it is inclined and complete the corresponding reduced form of the plane on the adjacent projection plane.

Since the given plane is perpendicular to the VP and inclined to the HP, its top view appears in reduced size and the front view appears as an edge view inclined with the reference line OX at an angle q_H at which it is inclined to the HP.

- Draw true shape of the rectangle $\mathbf{a_1b_1c_1d_1}$ on the top view and its corresponding front view (edge view) of the plane $\mathbf{a_1'/b_1'\text{-}c_1'/d_1'}$ assuming that it is parallel to the HP. *(Figure E3.20(a))*
- Rotate the edge view $\mathbf{a_1'/b_1'\text{-}c_1'/d_1'}$ through an angle of 30^0 to get the front view **a'/b'-c'/d'** in new position. Draw vertical projection lines from each points **a'/b'** and **c'/d'** of the edge view. Draw horizontal projection lines form the true shape plane $\mathbf{a_1b_1c_1d_1}$. The intersection of the vertical projection lines passing through **a'**, **b'**, **c'**, **d'** and the horizontal projection lines passing through $\mathbf{a_1}$, $\mathbf{b_1}$, $\mathbf{c_1}$, $\mathbf{d_1}$ give the top view of each corners **a**, **b**, **c**, **d** of the plane. **abcd** is the required top view of the plane. *(Figure E3.20(b))*

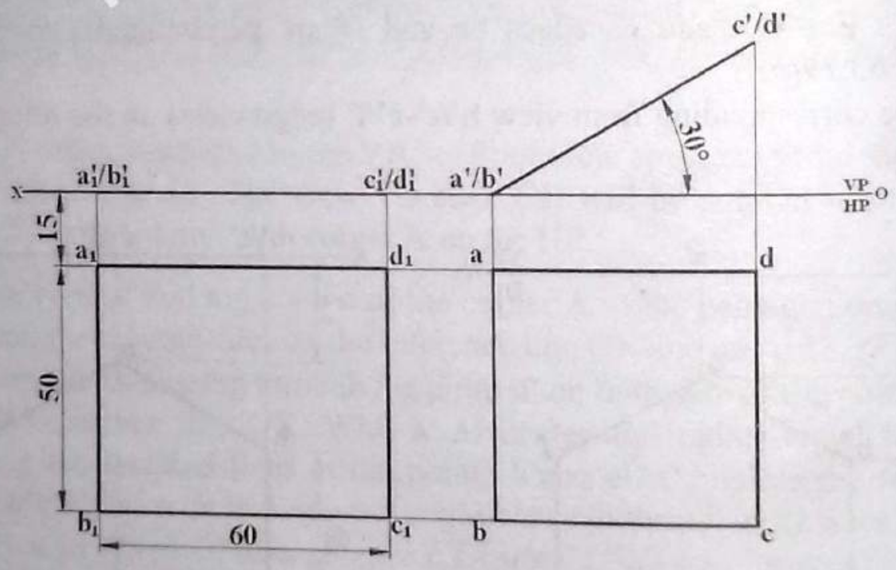


Figure E3.20(a) **Figure E3.20(b)**

Example 3.21

A regular pentagonal plane **ABCDE** of 20 mm side has its edge **BC** resting on the HP. Its plane is perpendicular to the HP and inclined to the VP at 45^0. Draw its projections when its corner nearer to the VP is 18 mm in front of the VP.

Solution

Since the given plane is perpendicular to the HP and inclined to the VP, its front view appears in reduced size and the top view appears as an edge view inclined with the reference line OX at an angle θ_V at which it is inclined to the VP.

- Draw true shape of the pentagon $a_1'b_1'c_1'd_1'e_1'$ on the front view and its corresponding top view (edge view) of the plane a_1d_1 assuming that it is parallel to the VP. *(Figure E3.21(a))*
- Rotate the edge view a_1d_1 through an angle of 45^0 to get the top view **ad** in new position. Draw vertical projection lines from each points of the edge view. Draw horizontal projection lines form each corners the true shape plane. The intersection of the vertical projection lines passing through **a, b, c, d, e** and the horizontal projection lines passing through a_1, b_1, c_1, d_1, e_1 give the front view of each corners **a', b', c', d' e'** of the plane. **a'b'c'd'e'** is the required front view of the plane. *(Figure E3.21(b))*

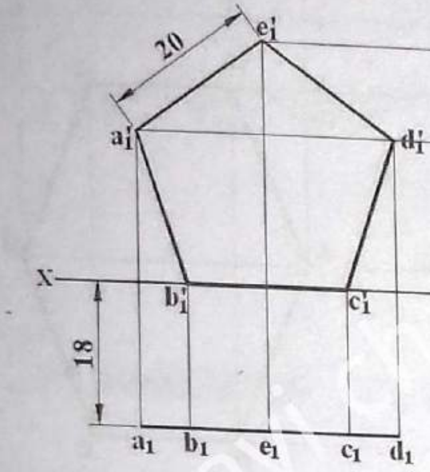


Figure E3.21(a)

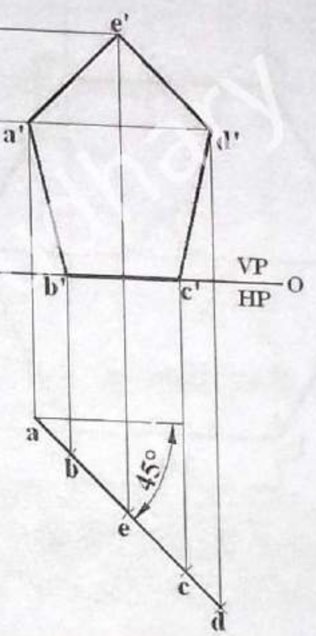


Figure E3.21(b)

Example 3.22

A regular hexagonal plane of 20 mm side has its one corner resting on the HP. Its plane is perpendicular to the VP and inclined to the HP at 45^0. Draw its projections when its edge nearer to the VP is 15 mm in front of the VP.

Solution

Since the given plane is perpendicular to the VP and inclined to the HP, its top view appears in reduced size and the front view appears as an edge view inclined with the reference line OX at an angle θ_H at which it is inclined to the VP.

- Draw true shape of the hexagon $a_1b_1c_1d_1e_1f_1$ on the top view and its corresponding front view (edge view) of the plane a_1'-d_1' assuming that it is parallel to the HP. *(Figure E3.22(a))*
- Rotate the edge view a_1'-d_1' through an angle of 45^0 to get the front view **a'-d'** in new position. Draw vertical projection lines from each point on the edge view. Draw horizontal projection lines form each corner of the true shape plane $a_1b_1c_1d_1e_1f_1$. The intersection of the vertical projection lines passing through **a', b', c', d', e', f'** and the horizontal projection lines passing through a_1, b_1, c_1, d_1, e_1, f_1 give the top view of each corners **a, b, c, d, e, f** of the plane. **abcdef** is the required top view of the plane. *(Figure E3.22(b))*

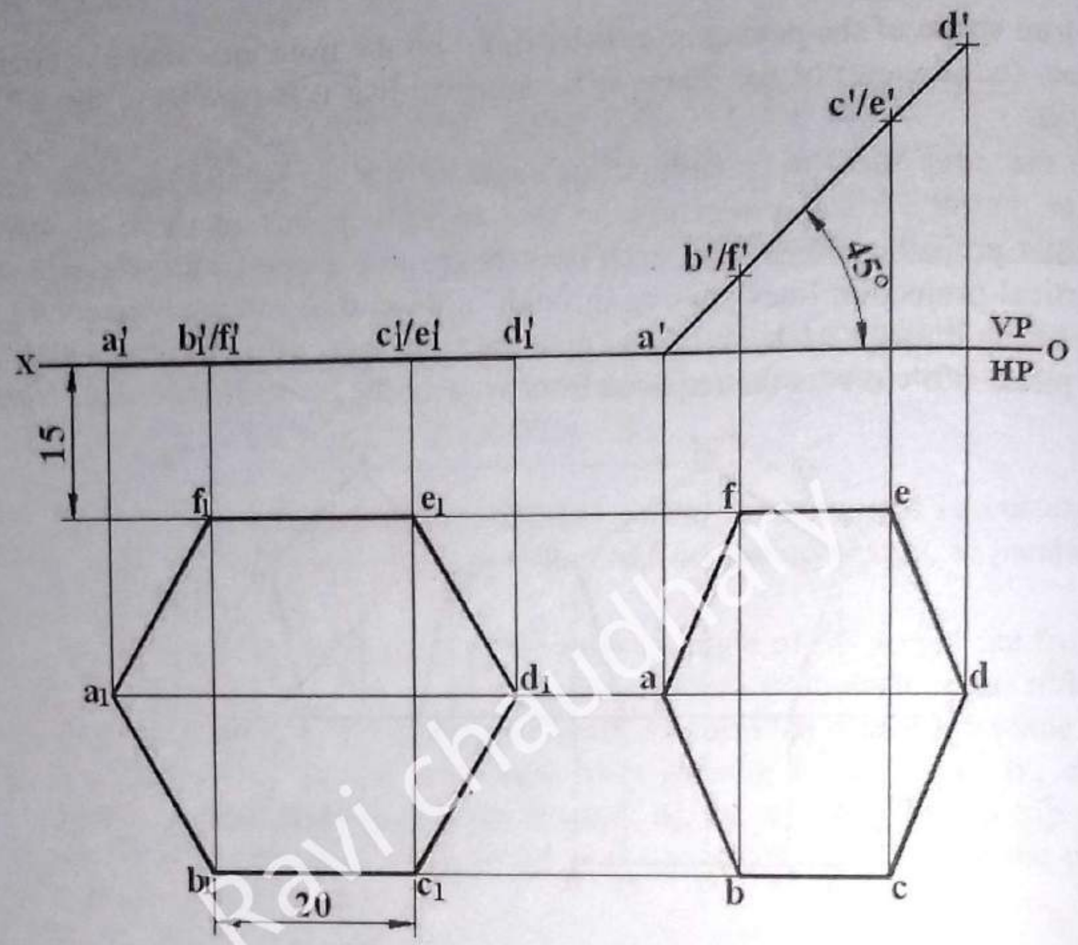


Figure E3.22(a) Figure E3.22(b)

Example 3.23

A circle of 40 mm diameter is held in such a way that it is perpendicular to the HP and inclined to the VP at 45^0. Draw its projections when a point on its circumference is nearer to the VP is 25 mm above the HP and 10 mm in front of the VP.

Solution

Since the given plane is perpendicular to the HP and inclined to the VP, its front view appears in reduced size and the top view appears as an edge view inclined with the reference line OX at an angle θ_V at which it is inclined to the VP.

- Draw true shape of the circle on the front view and its corresponding top view (edge view) of the plane assuming that it is parallel to the VP. Divide the true shape circle on the top view into any number of equal parts (say 8). Name the dividing points as a_1', b_1', ……..,h_1'. Transfer each point on the front view into the top view (edge view). *(Figure E3.23(a))*
- Rotate the edge view a_1e_1 through an angle of 45^0 to get the top view **ae** in new position. Draw vertical projection lines from each points of the edge view. Draw horizontal projection lines form each point on the true shape circle. The intersection of the vertical projection lines passing through **a**, **b**, ……. **h** and the horizontal projection lines passing through a_1', b_1', ….. h_1' give the front view of each point **a'**, **b'**…… **h'** of the circle. Join all the points **a'**, **b'**…… **h'** by a smooth curve to get the required front view of the circle. *(Figure E3.23(b))*

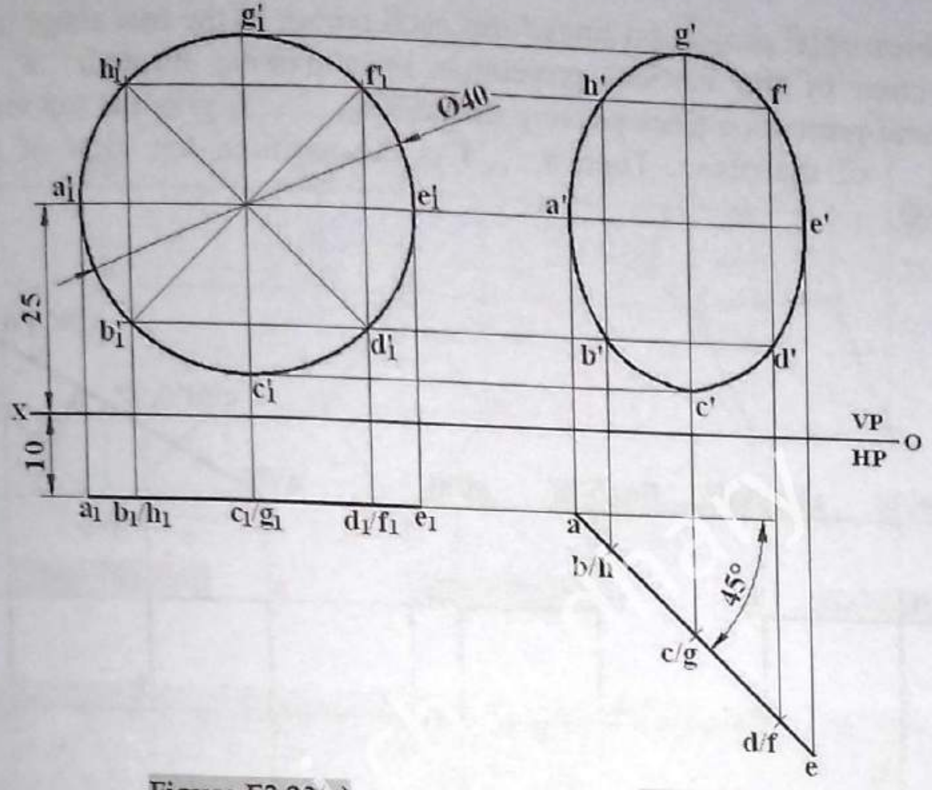


Figure E3.23(a) Figure E3.23(b)

Example 3.24

A plane shown in **Figure E3.24** is held in such a way that its edge **AB** is resting on the HP and perpendicular to the VP. Its plane is perpendicular to the VP and inclined to the HP at 30^0. Draw its projections when its corner **A** is 18 mm in front of the VP.

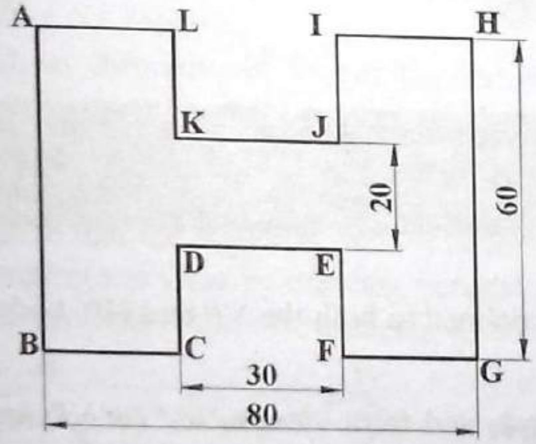


Figure E3.24

Solution

Since the given plane is perpendicular to the VP and inclined to the HP, its top view appears in reduced size and the front view appears as an edge view inclined with the reference line OX at an angle θ_H at which it is inclined to the VP.

- Draw true shape of plane $\mathbf{a_1} \ldots \mathbf{l_1}$ on the top view and its corresponding front view (edge view) of the plane $\mathbf{a_1'/b_1'\text{-}g_1'/h_1'}$ assuming that it is contained on the HP. *(Figure E3.24(a))*
- Rotate the edge view $\mathbf{a_1'/b_1'\text{-}g_1'/h_1'}$ through an angle of 30^0 to get the front view $\mathbf{a'/b'\text{-}g'/h'}$ in new position. Draw vertical projection lines from each point on the edge view.

Draw horizontal projection lines form each corner of the true shape plane a_1 .... l_1. The intersection of the vertical projection lines passing through **a'**, ..., **l'** and the horizontal projection lines passing through a_1, ..., l_1 give the top view of each corners **a**, ..., **l** of the plane. Then **a** ... **l** is the required top view of the plane. *(Figure E3.24(b))*

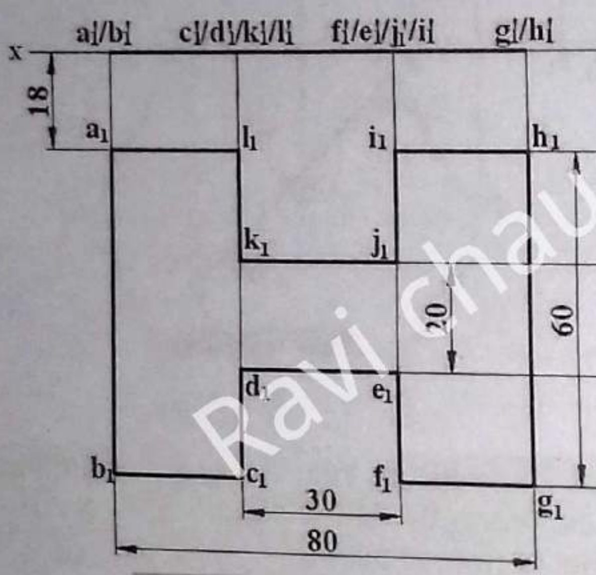


Figure E3.24(a)

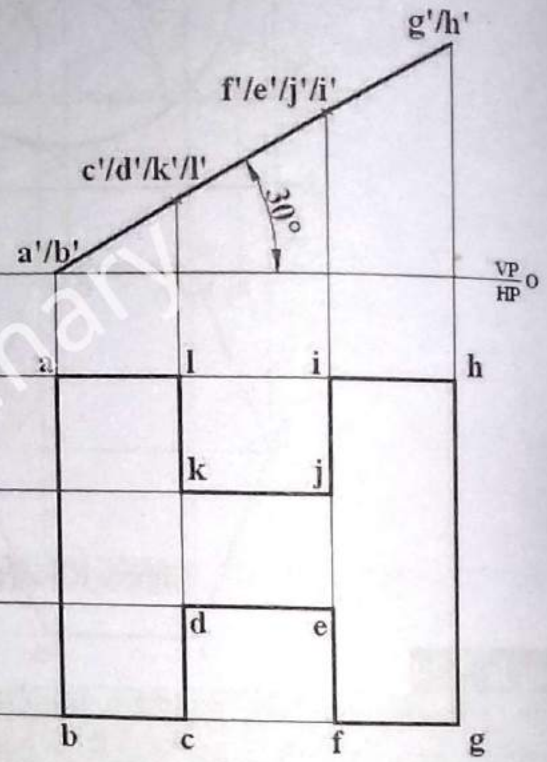


Figure E3.24(b)

Example 3.25

A square plane **ABCD** of 25 mm side rests on its corner **A** on the HP. Its plane is inclined at 45^0 to the HP and the diagonal DB inclined at 30^0 to the VP. Draw its projections when its corner nearer to the VP is 15 mm in front of it.

Solution

Since the given plane is inclined to both the VP and HP, both the front and top views appear in reduced size.

- Draw top view $\mathbf{a_1b_1c_1d_1}$ and front view $\mathbf{a_1'}$-$\mathbf{c_1'}$ (edge view) assuming that it is lying on the HP. *(Figure E3.25(a))*

- Rotate edge view $\mathbf{a_1'}$-$\mathbf{c_1'}$ through 45^0 to get the inclined front view $\mathbf{a_2'}$-$\mathbf{c_2'}$. Draw vertical projection lines from each point on the edge view and horizontal projection lines from each corners of the true shape square $\mathbf{a_1b_1c_1d_1}$. Intersection of thee projection lines give the corresponding top view $\mathbf{a_2b_2c_2d_2}$. *(Figure E3.25(b))*

- Rotate the top view about the corner **d** such that the diagonal **db** makes an angle of 30^0 to the reference line and the corner **d** is 15 mm from the reference line. Complete the corresponding front view by drawing vertical projection line from **a**, **b**, **c** and **d** and horizontal projection lines from $\mathbf{a_2'}$, $\mathbf{b_2'/d_2'}$ and $\mathbf{c_2'}$. *(Figure E3.25(c))*

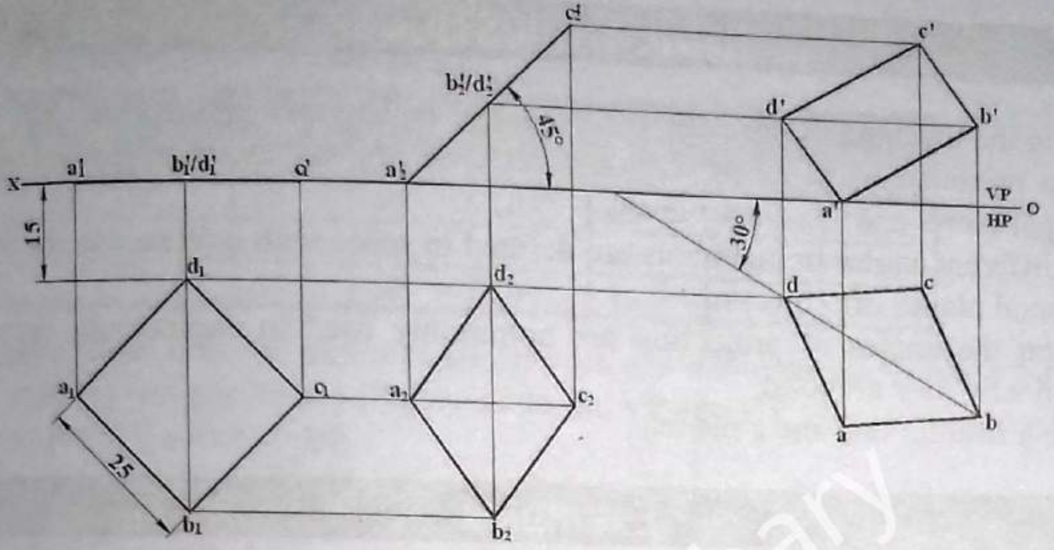


Figure E3.25(a) Figure E3.25(b) Figure E3.25(c)

Example 3.26

A regular pentagon **ABCDE** of 20 mm side has its edge **CD** parallel to the VP and inclined to the HP at 45^0. Its plane is inclined at 30^0 to the VP. Draw its projections when its corner **D** is 15 mm above the HP and 20 mm in front of the VP.

Solution

Since the given plane is inclined to both the VP and HP, both the front and top views appear in reduced size.

- Draw front view $\mathbf{a_1'b_1'c_1'd_1'e_1'}$ and top view $\mathbf{c_1/d_1\text{-}a_1}$ (edge view) assuming that it is parallel to the HP. *(Figure E3.26(a))*
- Rotate edge view $\mathbf{c_1/d_1\text{-}a_1}$ through 30^0 to get the inclined top view $\mathbf{c_2/d_2\text{-}a_2}$. Draw vertical projection lines from each point on the edge view and horizontal projection lines from each corners of the true shape $\mathbf{a_1'b_1'c_1'd_1'e_1'}$. Intersection of thee projection lines give the corresponding top view $\mathbf{a_2b_2c_2d_2e_2}$. *(Figure E3.26(b))*
- Rotate the front view such that the edge **d'b'** makes an angle of 45^0 to the reference line Complete the corresponding top view by drawing vertical projection line from **a'**, **b'**, **c'**, **d'** and **e'** and horizontal projection lines from $\mathbf{a_2}$, $\mathbf{b_2}$, $\mathbf{c_2}$, $\mathbf{d_2}$ and $\mathbf{e_2}$. *(Figure E3.26(c))*

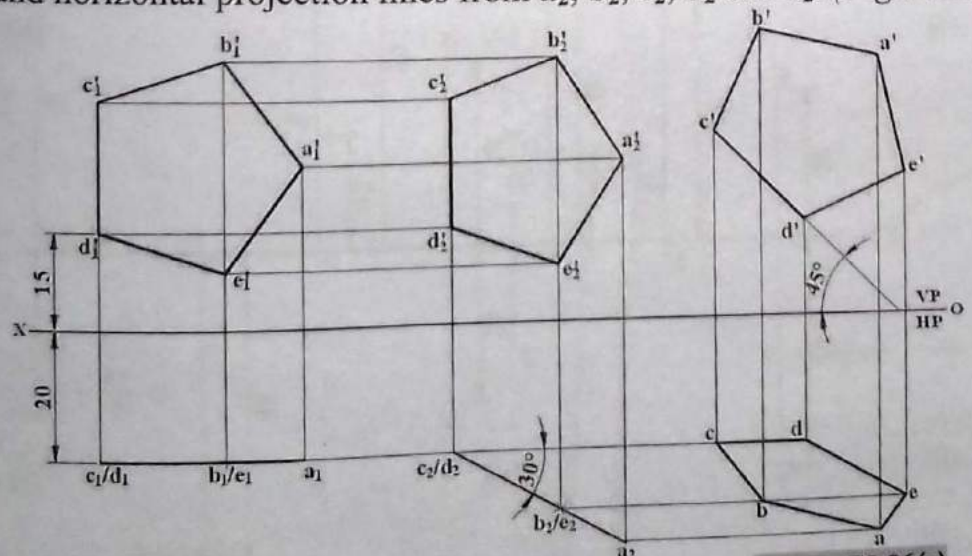


Figure E3.26(a) Figure E3.26(b) Figure E3.26(c)

CHAPTER 4

APPLICATION OF DESCRIPTIVE GEOMETRY

4.1 Introduction

Theory of projection drawing or descriptive geometry can be used to solve different spatial problems graphically. Few examples of such problems are determining the true length of an oblique line, true shape of an oblique plane, shortest distance between a point and a line, shortest distance between skew lines, dihedral angle between planes, etc. To solve such problems, projections only on the principal planes of projection may not be sufficient. Therefore, given element should also be projected on the auxiliary plane having different orientations. This chapter presents mainly the auxiliary view method to solve the engineering problems graphically.

4.2 Projection of Parallel Lines and Perpendicular Lines

While solving problems of descriptive geometry, important features of the projections of parallel lines and perpendicular lines should be understood. These are also called rules of projection.

Rule of Parallel Lines

When two lines are parallel with each other, their projections will be parallel in each view. In some case, these parallel views may coincide with each other or both may appear as point views on the same projection plane.

Rule of Perpendicular Lines

Lines which are perpendicular to each other will have their projections perpendicular to each other only when any one or both of the lines appear in true length in the same plane of projection.

When a line is perpendicular to a plane, it will be perpendicular to every line contained on that plane.

4.3 True Length of an Oblique Line

Any line which is inclined to all the principal planes of projection is called an oblique line. Therefore an oblique line appears in shorter length than the true length in its principal views.

As explained earlier any line appears as a true length when the plane of projection is parallel to the given line. So to determine the true length of any oblique line either the line should be revolved until it becomes parallel to any one of the principal plane or line should be projected on the auxiliary plane which is parallel to the given line. The first method is called the revolution method while the later is called the auxiliary view method.

4.3.1 Revolution Method

Figure 4.1 shows the orthographic projections of a line inclined to both the VP and HP. To determine its true length revolve it to make it parallel with either the HP or the VP. When the line appears in true length in its front view, its gives true inclination of the line with the HP and when it appears in true length in its top view, it gives true inclination of the line with the VP.

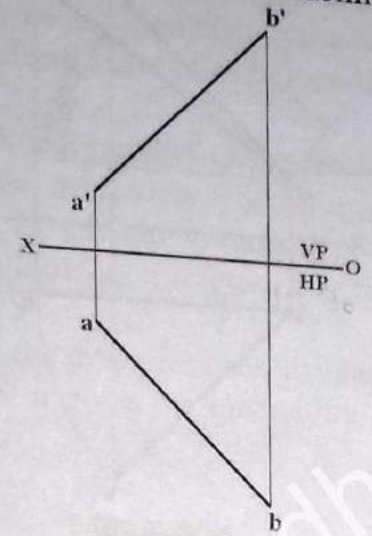


Figure 4.1: Orthographic Projections of a Line Inclined to both the VP and the HP

To Determine the True Length of a Given Line and Its Inclination of with HP

- Draw line passing through the point **a** and parallel to the reference line OX. With **a** as center and **ab** as radius draw an arc intersecting the horizontal line passing through **a** at point $\mathbf{b_1}$. Then $\mathbf{ab_1}$ is the top view of the given line when it is parallel to the VP. *(Figure 4.2(a))*
- Draw a line passing through **b'** and parallel to the reference line OX, which is the locus of end point **B** of the line in the front view when it is revolved about its end **A** keeping its inclination with the HP constant. Draw vertical projection line passing through the point $\mathbf{b_1}$ which intersects the horizontal line passing through **b'** at the point $\mathbf{b_1'}$. *(Figure 4.2(b))*
- Then $\mathbf{a'b_1'}$ is the true length of the given line and the angle made by it with the reference line passing through the point **a'** gives the inclination of the line with the HP. *(Figure 4.2(c))*

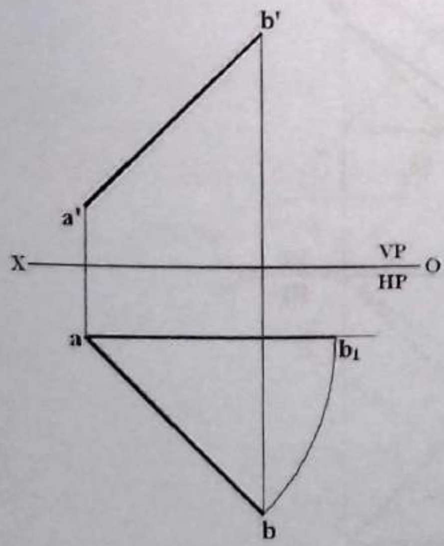


Figure 4.2(a)

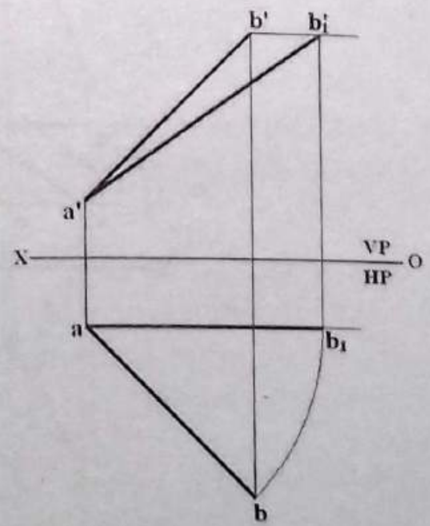


Figure 4.2(b)

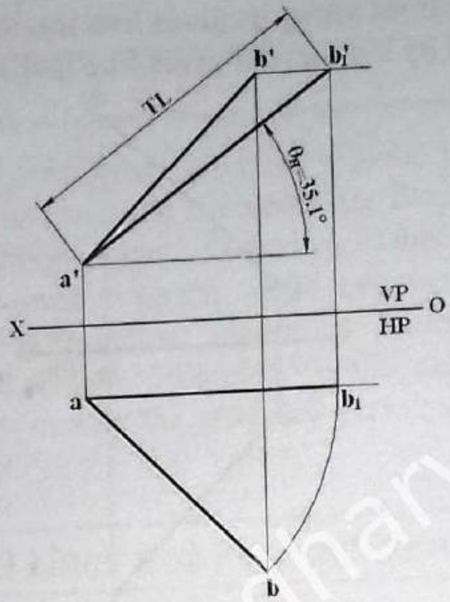


Figure 4.2(c)

Alternatively, end **A** of the given line can also be revolved about its end **B**, as shown in *Figure 4.3*.

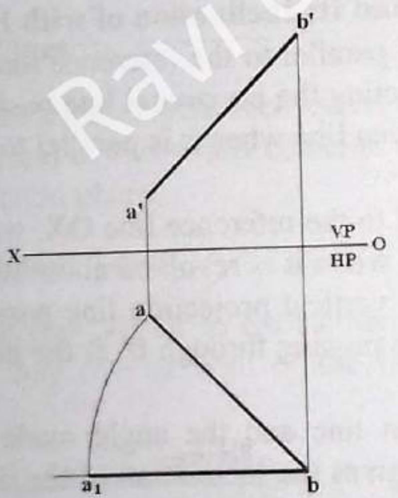


Figure 4.3(a)

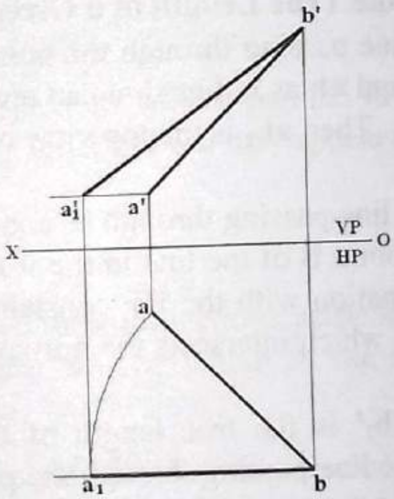


Figure 4.3(b)

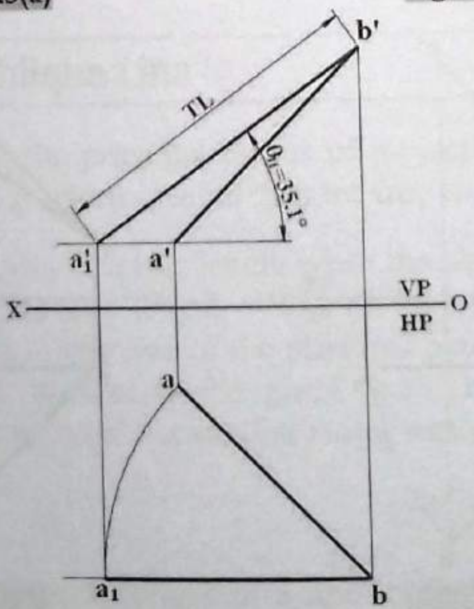


Figure 4.3(c)

To Determine the True Length of a Given Line and Its Inclination of with HP

- Draw line passing through the point **a'** and parallel to the reference line OX. With **a'** as center and **a'b'** as radius draw an arc intersecting the horizontal line passing through **a'** at point **b$_1$'**. Then **a'b$_1$'** is the front view of the given line when it is parallel to the HP. *(Figure 4.4(a))*
- Draw a line passing through **b** and parallel to the reference line OX, which is the locus of end point **B** of the line in the front view when it is revolved about its end **A** keeping its inclination with the VP constant. Draw vertical projection line passing through the point **b$_1$'** which intersects the horizontal line passing through **b** at the point **b$_1$**. *(Figure 4.4(b))*
- Then **ab$_1$** is the true length of the given line and the angle made by it with the reference line passing through the point **a** gives the inclination of the line with the VP. *(Figure 4.4(c))*

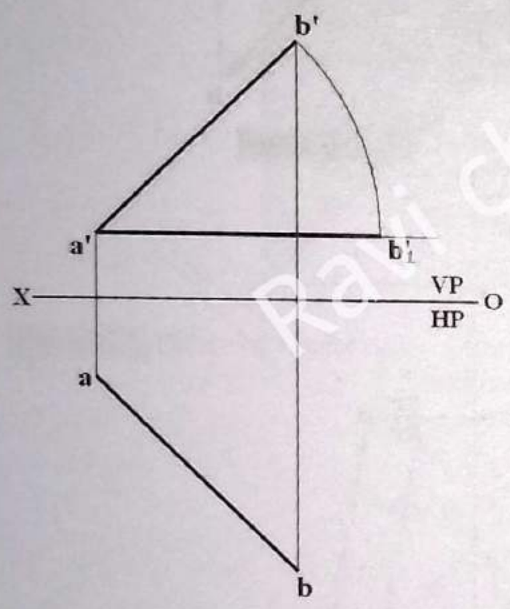


Figure 4.4(a)

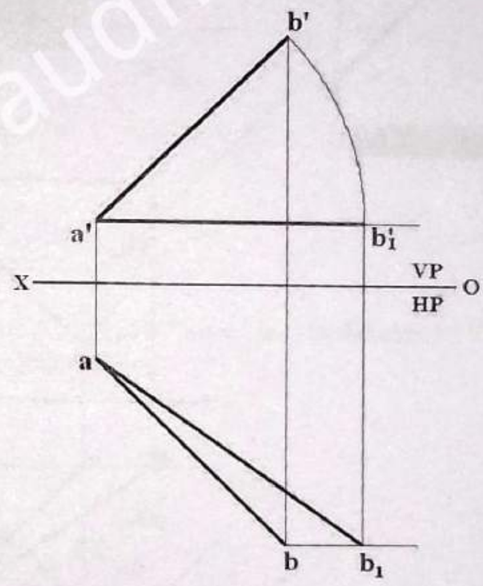


Figure 4.4(b)

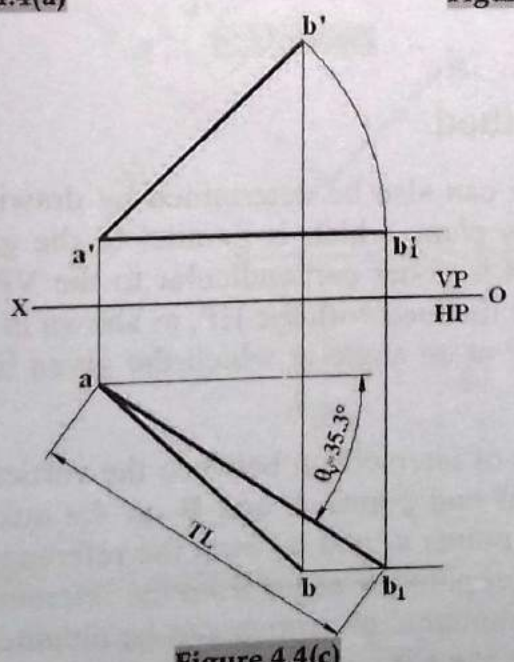


Figure 4.4(c)

Alternatively, end **A** of the given line can also be revolved about its end **B**, as shown in *Figure 4.5*.

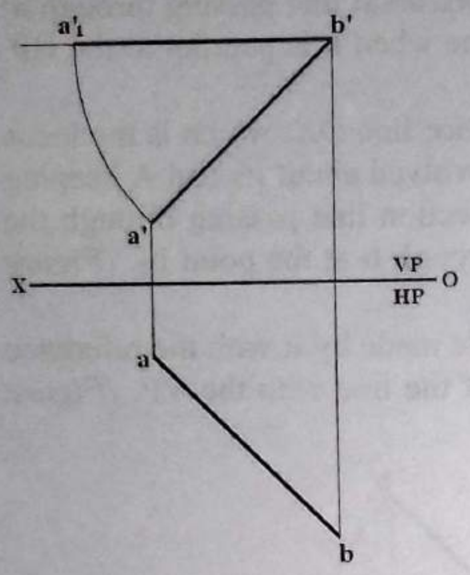


Figure 4.5(a)

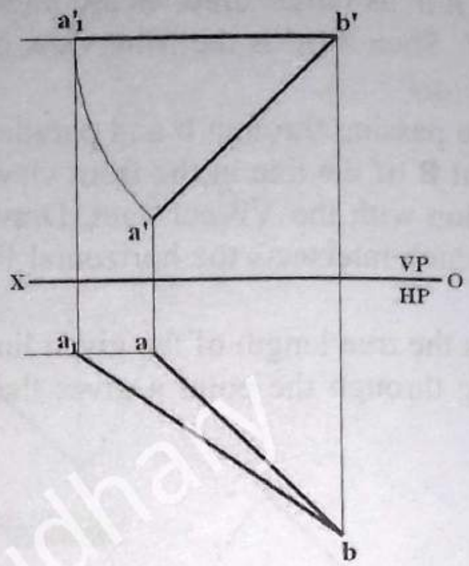


Figure 4.5(b)

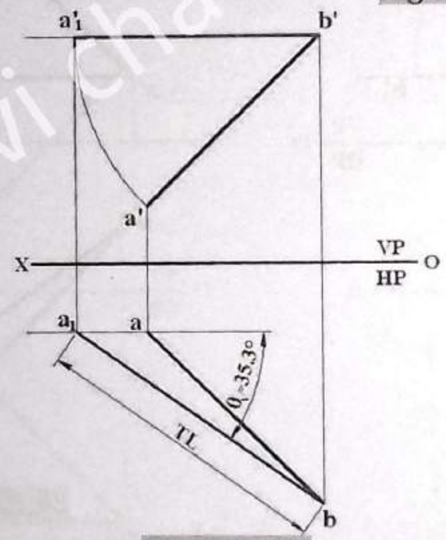


Figure 4.5(c)

4.3.2 Auxiliary View Method

True length of an oblique line can also be determined by drawing the projection of the given line on an auxiliary projection plane which is parallel to the given line. The auxiliary plane should be selected such that it is either perpendicular to the VP and inclined to the HP at an angle at which the given line is inclined with the HP, as shown in **Figure 4.6** or perpendicular to the HP and inclined to the VP at an angle at which the given line is inclined with the VP, as shown in *Figure 4.8*.

In *Figure 4.6*, O_1X_1 is the line of intersection between the vertical plane (VP) and the auxiliary plane (AP). The projections of end points **A** and **B** on the auxiliary plane AP are $\mathbf{a_1}$ and $\mathbf{b_1}$ respectively. The distances of points $\mathbf{a_1}$ and $\mathbf{b_1}$ from the reference line O_1X_1 along the auxiliary plane is same as the distances of points **a** and **b** from the reference line OX along the horizontal plane. The corresponding orthographic projection can be obtained by revolving the HP and AP downward about the reference line OX and upward about the reference line O_1X_1 respectively, as shown in *Figure 4.7*.

By comparing the views of the line on AP and VP, it can be understood that the line is parallel to the AP and inclined to the VP. Therefore, it appears as true length on the AP and the angle at which the true length line is inclined to the line parallel to reference line O_1X_1 gives the true inclination of the line with the VP, θ_V.

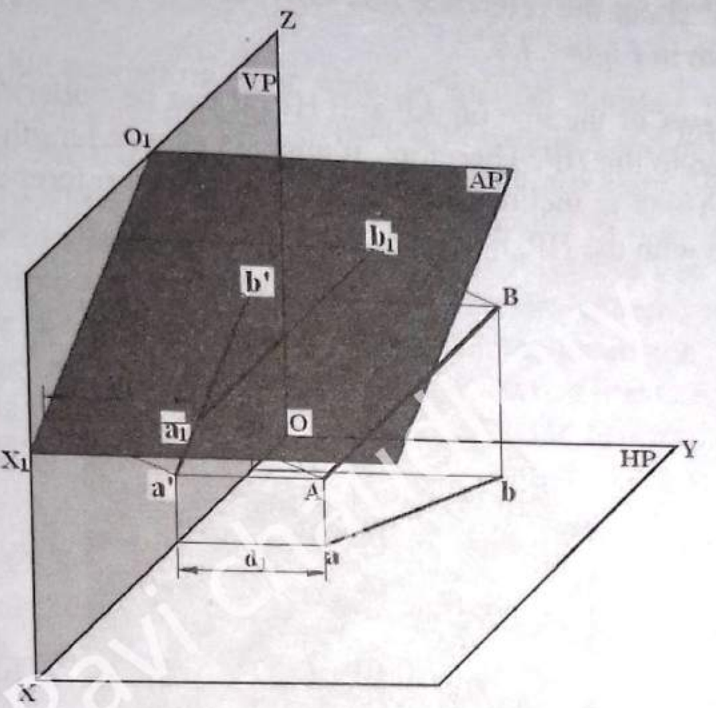


Figure 4.6: Pictorial Projection a Line on an Auxiliary Plane Perpendicular to the VP and Inclined to the HP

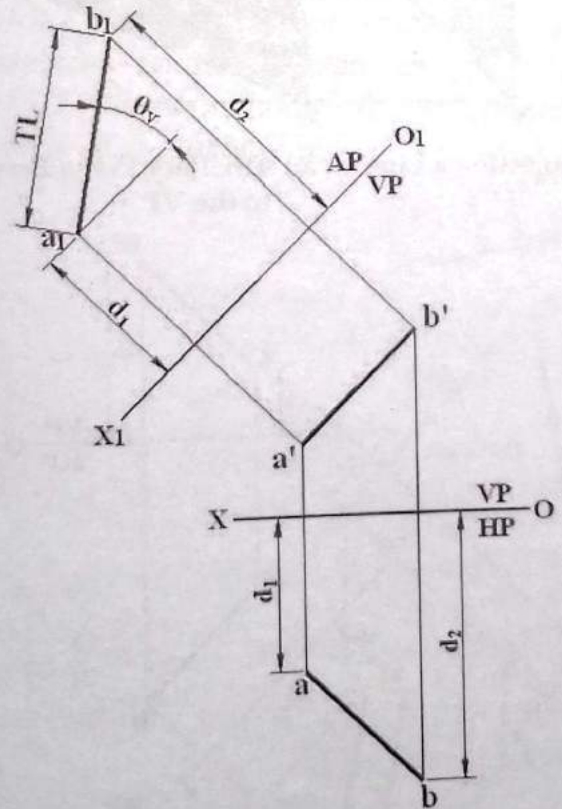


Figure 4.7: True Length and Inclination of a Line with the VP

In *Figure 4.8*, O_2X_2 is the line of intersection between the horizontal plane (HP) and the auxiliary plane (AP). The projections of end points **A** and **B** on the auxiliary plane AP are $\mathbf{a_2}$

and b_2 respectively. The heights of points a_2 and b_2 from the reference line O_2X_2 along the auxiliary plane is same as the distances of points **a'** and **b'** from the reference line OX along the vertical plane. The corresponding orthographic projection can be obtained by revolving the AP and HP towards right about the reference line O_2X_2 and downward about the reference line OX respectively, as shown in *Figure 4.9*.

By comparing the views of the line on AP and HP, it can be understood that the line is parallel to the AP and inclined to the HP. Therefore, it appears as true length on the AP and the angle at which the true length line is inclined to the line parallel to reference line O_2X_2 gives the true inclination of the line with the HP, θ_H.

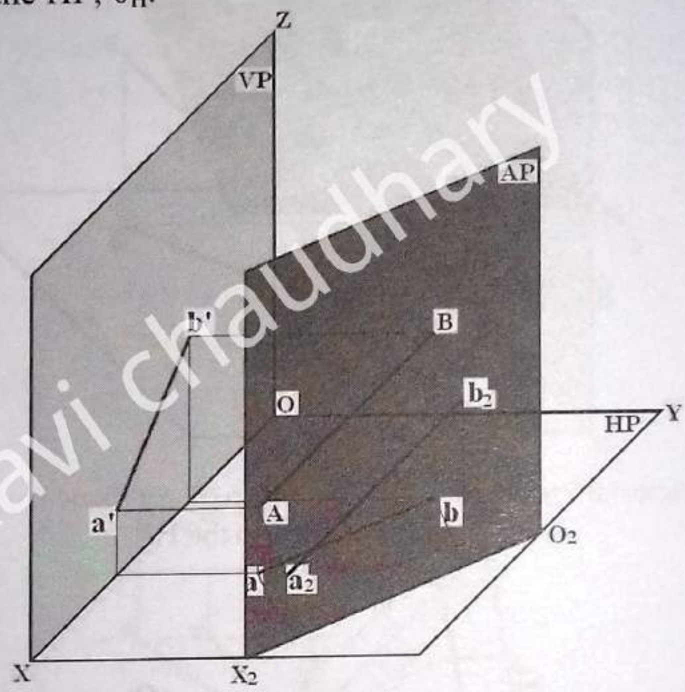


Figure 4.8: Pictorial Projection a Line on an Auxiliary Plane Perpendicular to the HP and Inclined to the VP

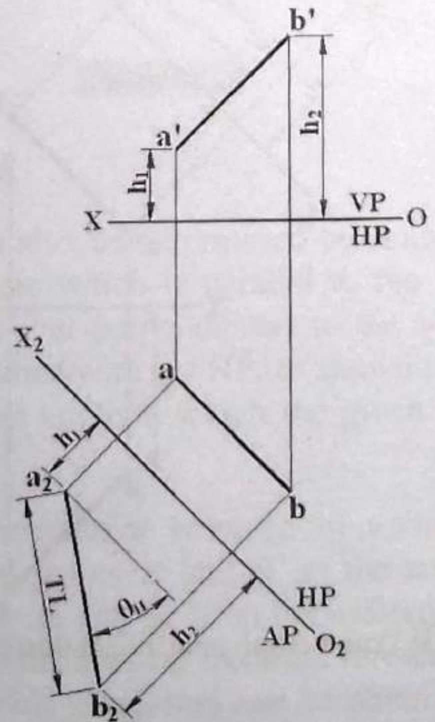


Figure 4.9: True Length and Inclination of a Line with the HP

4.4 Point View of a Line

To draw the point view of a given line an auxiliary plane should be placed such that it is perpendicular to the given line.

Figure 4.10 shows orthographic projection of an oblique line. An auxiliary plane perpendicular to the given line cannot be drawn directly with respected to its given views because both views are not in true length. Hence for the proper positioning of the auxiliary plane, its true length should be drawn first. For this draw a reference line O_1X_1 parallel to any view of the line, say the front view **a'b'** as shown in *Figure 4.11*. In this case, the reference line O_1X_1 separates the vertical plane (VP) and the first auxiliary plane (AP_1). Draw projection lines passing through each points on the front view **a'** and **b'** and perpendicular to the reference line O_1X_1. Measure distances of each point on the top view **a** and **b** from the reference line OX and transfer it into the respective projection lines from the reference line O_1X_1 into the first auxiliary plane (AP_1) to get the auxiliary view $\mathbf{a_1}$ and $\mathbf{b_1}$ of end points of the line. Then auxiliary view $\mathbf{a_1b_1}$ appears in true length.

Draw another reference line O_2X_2 perpendicular to the true length line $\mathbf{a_1b_1}$ which separates the first auxiliary plane (AP_1) and the second auxiliary plane (AP_2). Draw a common projection line passing through each point $\mathbf{a_1}$ and $\mathbf{b_1}$ on the first auxiliary plane (AP_1) and perpendicular to the reference line O_2X_2. To determine the auxiliary views of the end points on the second auxiliary plane (AP_2), skip its adjacent plane (AP_1) and measure the distance of points **a'** and **b'** on the vertical plane (VP) from the reference line O_1X_1 and transfer into the respective projection lines into the second auxiliary plane (AP_2) from the reference line O_2X_2. Since the projection line passing through points $\mathbf{a_1}$ and $\mathbf{b_1}$ are same on the AP_2 plane and the distances of both end points **a'** and **b'** from the reference line O_1X_1 is same, both end points coincide on the AP_2 plane as $\mathbf{a_2/b_2}$, which is the required point view of the given line.

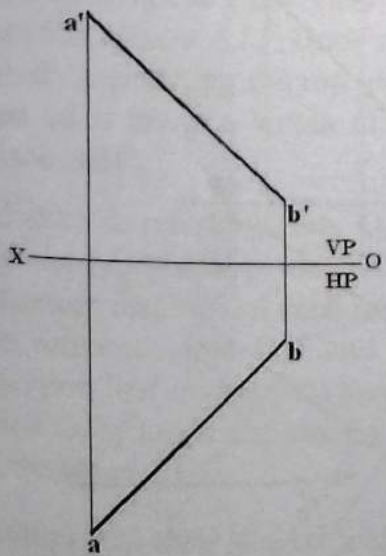


Figure 4.10

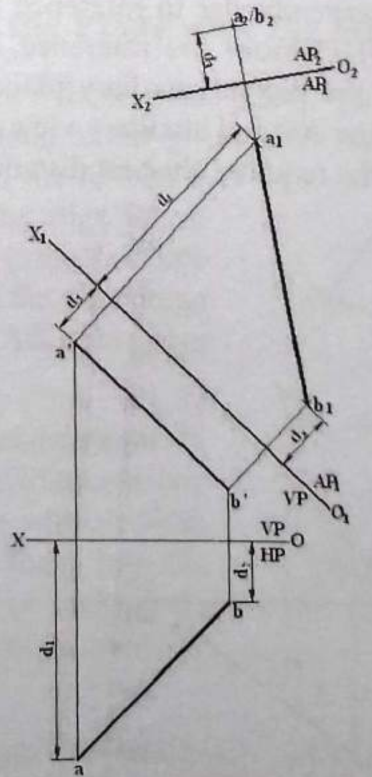


Figure 4.11

4.5 Shortest Distance between a Point and a Line

Shortest distance between a point and a line can be measured on a view in which the given line appears as a point view. The distance between the point view of the given line and the given point on a same auxiliary plane gives the required shortest distance.

Figure 4.12 shows orthographic projections of a line **AB** and a point **C**. Draw a reference line O_1X_1 parallel to the view **ab** of the line. In this case, the reference line O_1X_1 separates the vertical plane (HP) and the first auxiliary plane (AP_1), as shown in *Figure 4.13(a)*. Draw projection lines passing through each points on the top view **a**, **b** and **c** and perpendicular to the reference line O_1X_1. Measure distances each point **a'**, **b'** and **c'** on the front view from the reference line OX and transfer it into the respective projection lines from the reference line O_1X_1 into the first auxiliary plane (AP_1). Then auxiliary view $\mathbf{a_1b_1}$ appears in true length.

Draw a line passing through $\mathbf{c_1}$ and perpendicular to $\mathbf{a_1b_1}$ and mark the foot of the perpendicular as $\mathbf{d_1}$, as shown in *Figure 4.13(b)*. To draw top view and front view of the point **D**, draw projection lines passing through the point $\mathbf{d_1}$ and perpendicular to the reference line O_1X_1 towards the horizontal plane (HP). The intersection of the projection line and the top view **ab** of the given line gives the top view **d**. Again draw a projection line passing through the point **d** and perpendicular to the reference line OX towards the vertical plane (VP). The intersection of the vertical projection line and the front view **a'b'** of the given line gives the front view **d'**. Join **c** and **d** and **c'** and **d'** to get top view and front view of the line **CD** perpendicular to the given line **AB**.

To determine the true length of the line **CD** draw a reference line O_2X_2 parallel to the $\mathbf{c_1d_1}$ (i.e., parallel to $\mathbf{a_1b_1}$) which separates the first auxiliary plane (AP_1) and the second auxiliary plane (AP_2) , as shown in *Figure 4.13(c)*. Draw projection lines from each point on the first auxiliary plane (AP_1) and perpendicular to reference line O_2X_2. Measure distance of each point on the horizontal plane (HP) from the reference line O_1X_1 and transfer them into the respective projection line on the second auxiliary plane (AP_2) from the reference line O_2X_2 to get point view $\mathbf{a_2/b_2}$ of the line **AB** and auxiliary view $\mathbf{c_2}$ of the given point **C**. Then the distance between $\mathbf{a_2/b_2}$ and $\mathbf{c_2}$ gives the required shortest distance.

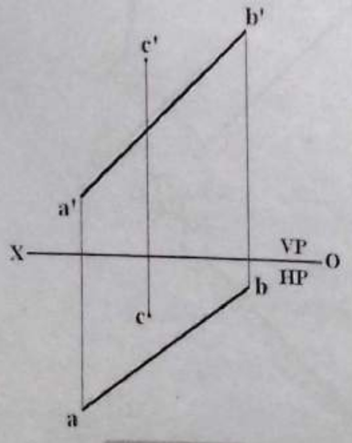


Figure 4.12

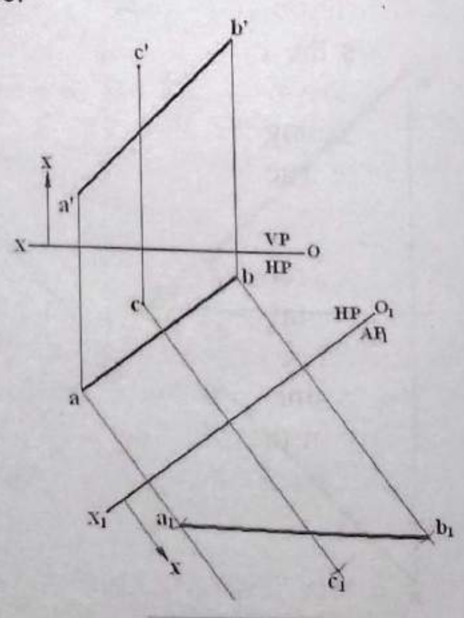


Figure 4.13(a)

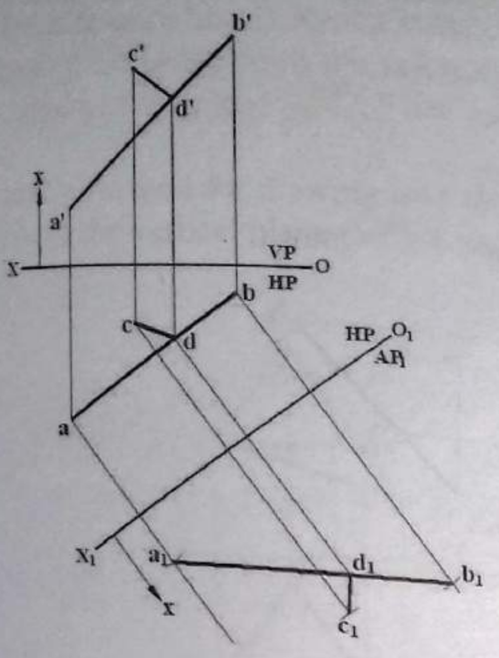


Figure 4.13(b)

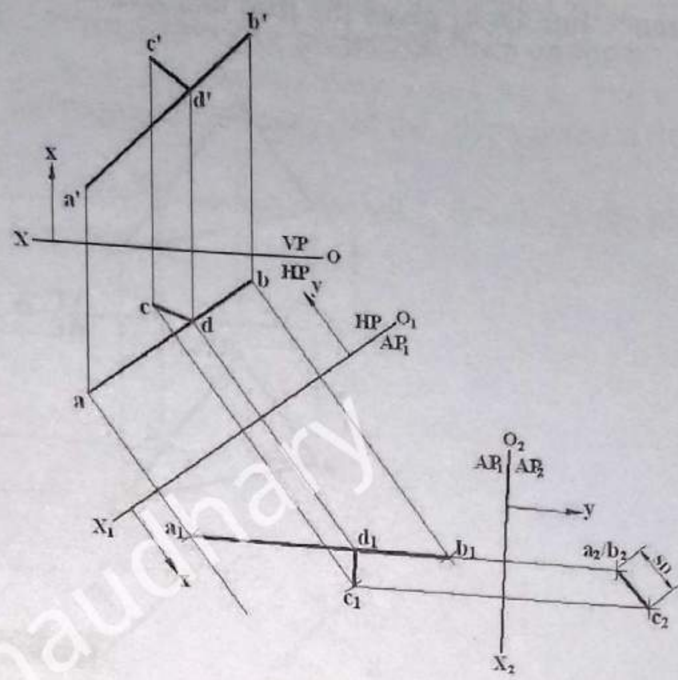


Figure 4.13(c)

4.6 Edge View of an Oblique Plane

Any plane inclined to all the principal planes is called an oblique plane. All principal views of an oblique plane will be plane figures with their sizes smaller that the true size of the plane. To draw an edge view of an oblique plane, an auxiliary plane should be placed such that it is perpendicular to the given plane.

4.6.1 Edge View and True Inclination of the Plane with the HP

Figure 4.14 shows orthographic projections of an oblique plane **ABC**. To draw an auxiliary plane perpendicular to it, a true length line containing on the given plane should be drawn. For this draw a line passing through **a'** and parallel to the reference line OX which intersects the front view **b'c'** of the edge **BC** at point **d'**, as shown in *Figure 4.15*. Draw a vertical projection line passing through **d'** intersecting the top view **bc** of the same edge at point **d**. Then **ad** is the true length of the line **AD** containing on the given plane **ABC**.

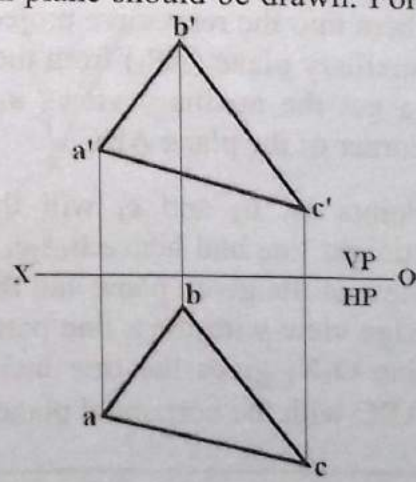


Figure 4.14

Extend **ad** and draw a reference line O_1X_1 perpendicular to it, which separates the horizontal plane (HP) and the first auxiliary plane (AP_1). Measure distance of each point on the vertical plane (VP) from the reference line OX and transfer them into the respective projection line on the first auxiliary plane (AP_1) from the reference line O_1X_1 to get the auxiliary views $\mathbf{a_1}$, $\mathbf{b_1}$ and $\mathbf{c_1}$ of each corner of the plane **ABC**.

Points $\mathbf{a_1}$, $\mathbf{b_1}$ and $\mathbf{c_1}$ will then line on a same straight line and hence $\mathbf{c_1a_1b_1}$ is the required edge view of the given plane and the angle made by the edge view with the a line parallel to the reference line O_1X_1 gives the true inclination of the plane **ABC** with the horizontal plane (HP), θ_H.

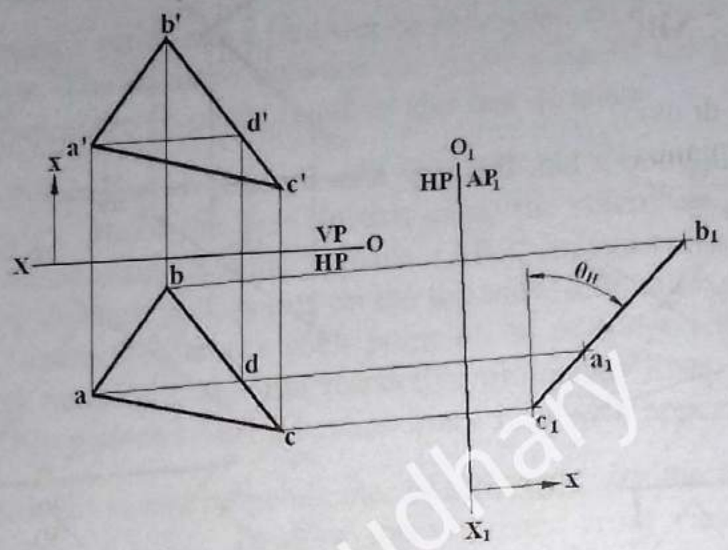


Figure 4.15

4.6.2 Edge View and True Inclination of the Plane with the VP

Consider the same plane shown in *Figure 4.14*. To determine its true inclination with the VP, a true length line containing on the given plane should be drawn on the front view. For this draw a line passing through **a** and parallel to the reference line OX which intersects the top view **bc** of the edge **BC** at point **d**, as shown in *Figure 4.16*. Draw a vertical projection line passing through **d** intersecting the front view **b'c'** of the same edge at point **d'**. Then **a'd'** is the true length of the line **AD** containing on the given plane **ABC**.

Extend **a'd'** and draw a reference line O_1X_1 perpendicular to it, which separates the vertical plane (VP) and the first auxiliary plane (AP_1). Measure distance of each point on the horizontal plane (HP) from the reference line OX and transfer them into the respective projection line on the first auxiliary plane (AP_1) from the reference line O_1X_1 to get the auxiliary views a_1, b_1 and c_1 of each corner of the plane **ABC**.

Points a_1, b_1 and c_1 will then line on a same straight line and hence $b_1a_1c_1$ is the required edge view of the given plane and the angle made by the edge view with the a line parallel to the reference line O_1X_1 gives the true inclination of the plane **ABC** with the horizontal plane (VP), θ_V.

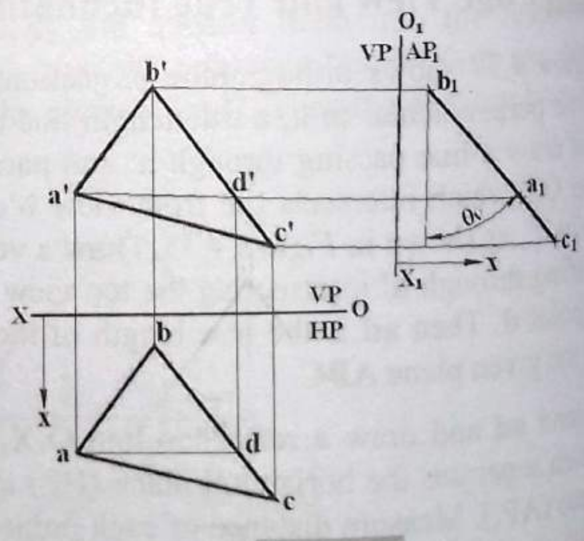


Figure 4.16

4.7 True Shape of an Oblique Plane

To draw the true shape of any plane, an auxiliary plane should be placed such that it is parallel to the given plane. For this, edge view of the given plane should be drawn first by following the procedure explained earlier. Then draw another reference line O_2X_2 parallel to the edge view

$c_1a_1b_1$, as shown in *Figure 4.17*. Measure distance of each point on the horizontal plane (HP) from the reference line O_1X_1 and transfer them into the respective projection lines on the second auxiliary plane (AP_2) from the reference line O_2X_2 to get the auxiliary views $\mathbf{a_2}$, $\mathbf{b_2}$ and $\mathbf{c_2}$ of each corners of the plane **ABC**. Then $\mathbf{a_2b_2c_2}$ is the required true shape of the given plane **ABC**.

Alternative method for drawing true shape of the plane by using edge view drawn on the plane adjacent to the vertical plane (VP) is shown in *Figure 4.18*.

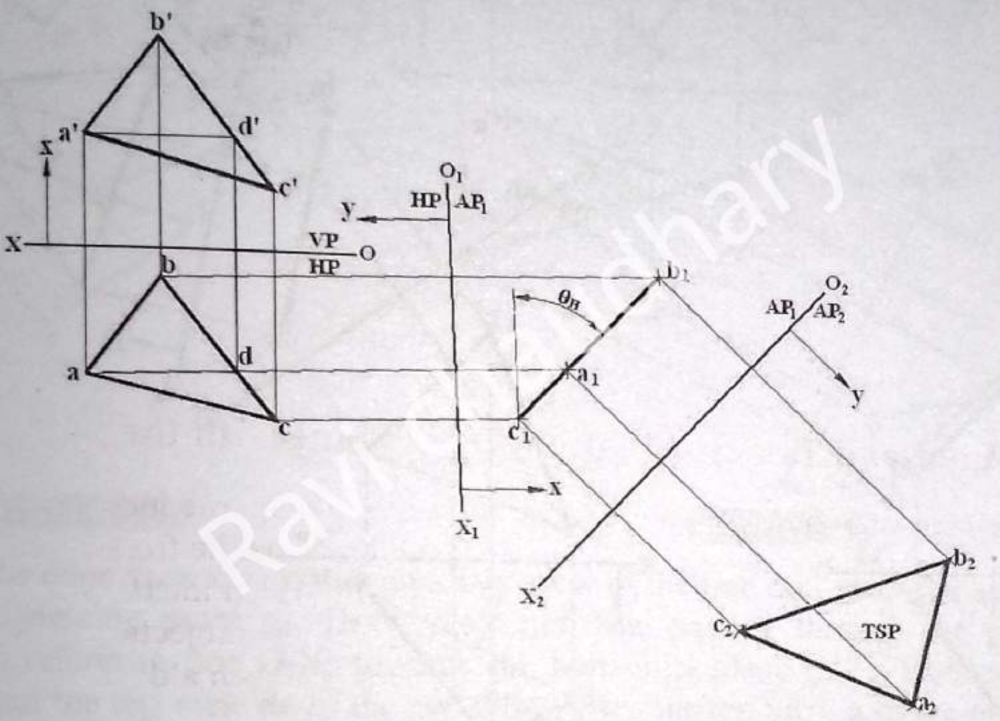


Figure 4.17

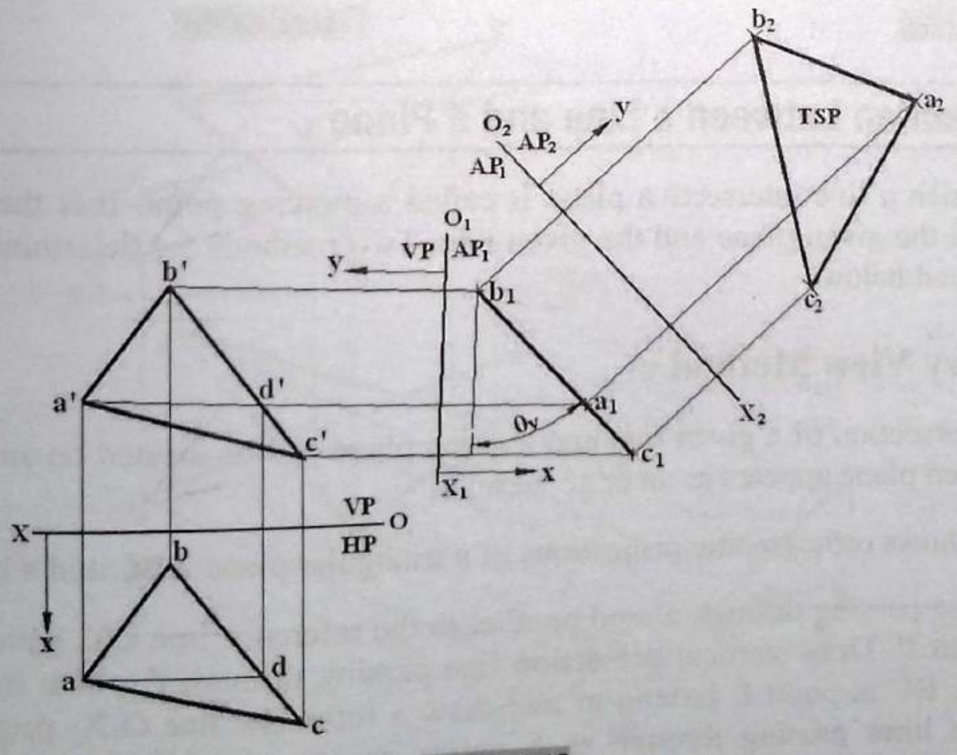


Figure 4.18

4.8 True Angle between Two Intersecting Lines

Figure 4.19(a) shows the orthographic projections of two intersecting line **AB** and **BC** intersecting each other at point **B**. Join free ends **A** and **C** in each views to get the front view **a'b'c'** and the top view **abc** of the triangular plane **ABC**. Draw true shape of the triangle by following the procedure explained earlier. Then $\measuredangle a_2b_2c_2$ on the true shape plane shown in *Figure 4.19(b)* gives the required true angle between the lines **AB** and **BC**.

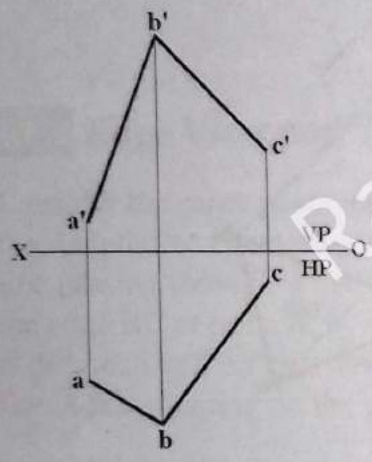


Figure 4.19(a)

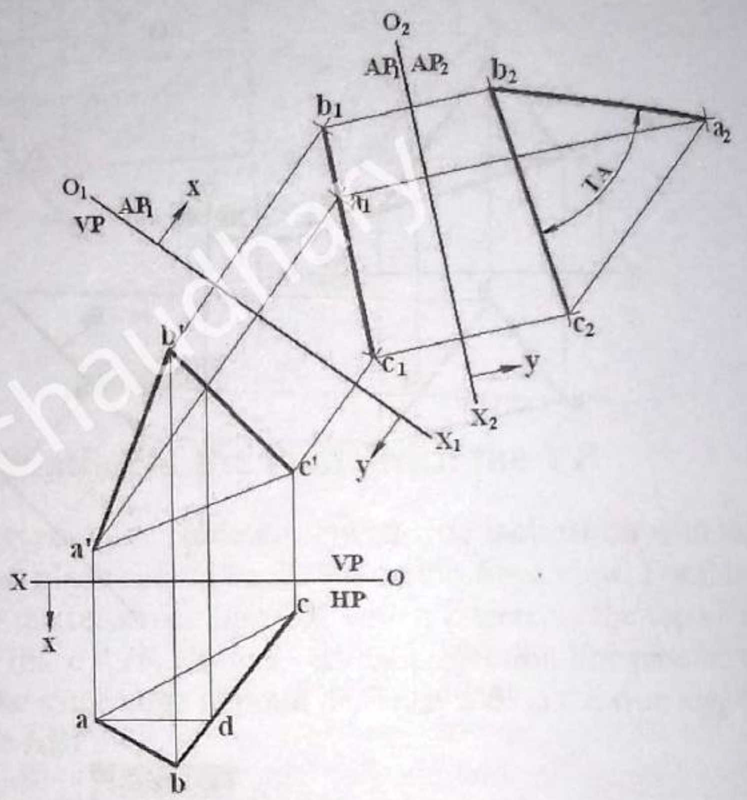


Figure 4.19(b)

4.9 Intersection between a Line and a Plane

The point at which a line intersects a plane is called a piercing point. It is the point which is common to both the given plane and the given line. Two methods for determining the piercing point are explained below.

4.9.1 Auxiliary View Method

The point of intersection of a given line and a given plane can be located on an auxiliary plane in which the given plane appears as an edge view.

Figure 4.20(a) shows orthographic projections of a triangular plane **ABC** and a line **DE**.

Draw straight line passing through **a'** and parallel to the reference line OX, which intersects the edge **b'c'** at point **f'**. Draw vertical projection line passing through **f'** which intersects the top view **bc** of edge **BC** at point **f**. Extend **af** and draw a reference line O_1X_1 perpendicular to it. Draw projection lines passing through each points on top view (**a**, **b**, **c**, **d**, **e**, and **f**) and perpendicular to the reference line O_1X_1. Measure distances of each point on the vertical plane

(VP) from the reference line OX and transfer them into the respective projection lines on the first auxiliary plane (AP_1) to get the auxiliary views of each corners of the triangle and end points of the line, as shown in *Figure 4.20(b)*.

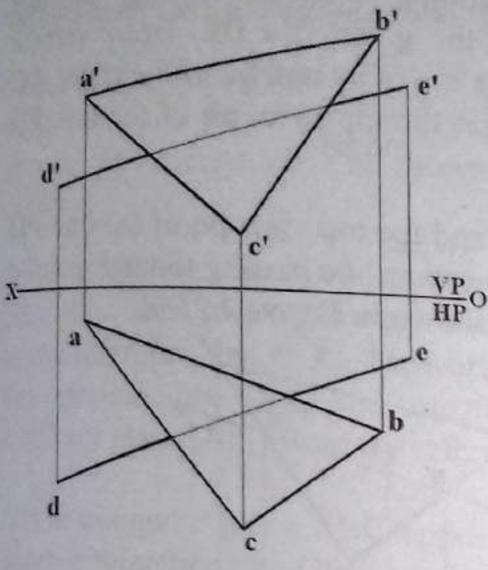


Figure 4.20(a)

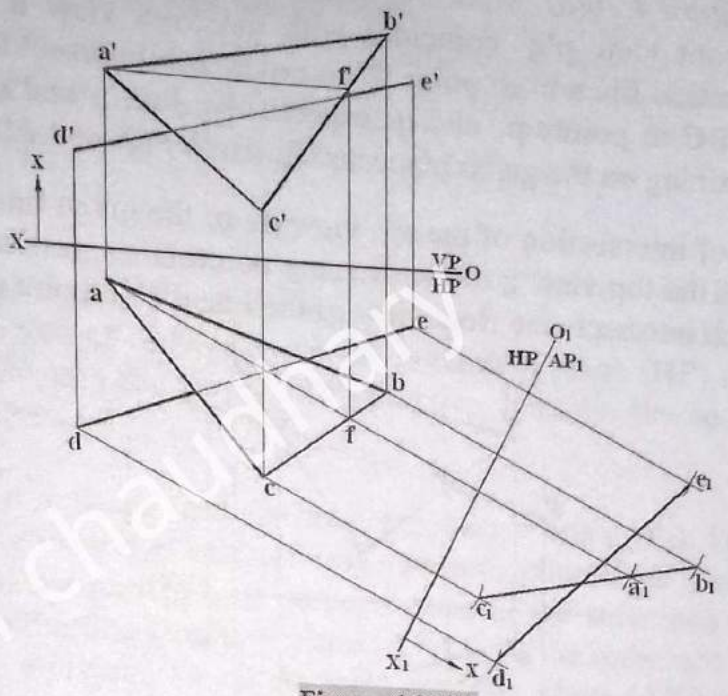


Figure 4.20(b)

Intersection of the edge view $\mathbf{c_1a_1b_1}$ and auxiliary view of the line $\mathbf{d_1e_1}$ gives the auxiliary view of the required piercing point $\mathbf{g_1}$. Draw projection line passing through the point $\mathbf{g_1}$ and perpendicular to reference line O_1X_1 towards the horizontal plane (HP). Intersection of this projection line and the top view **de** of the given line gives the top view **g** of the piercing point. Draw vertical projection line passing through point **g**. Intersection of this projection line with the front view **d'e'** gives front view g' of the piercing point, as shown *Figure 4.20(c)*.

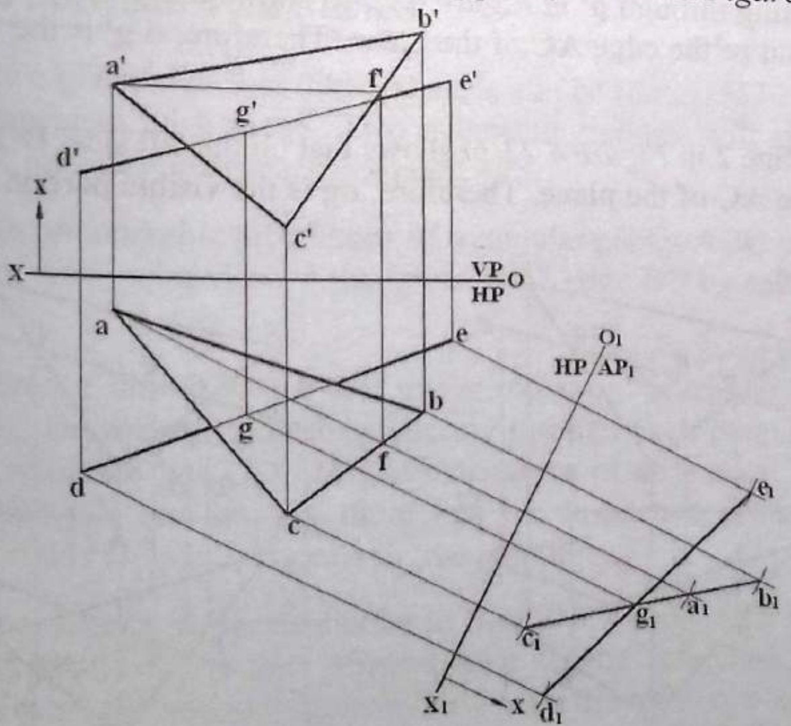


Figure 4.20(c)

4.9.2 Cutting Plane Method

Consider again the orthographic projections of the triangular plane **ABC** and the line **DE** shown in *Figure 4.20(a)*. Mark the front view of any line **PQ** containing on the given plane such that its front view **p'q'** coincides with the front view **d'e'** of the given line **DE**. Draw vertical projection lines from point **p'** and **q'** which intersect the top views **ac** and **bc** of the edges **AC** and **BC** at points **p** and **q** respectively. Join **p** and **q** to get the top view **pq** of the line **PQ** containing on the given plane, as shown in *Figure 4.21(a)*.

The of intersection of the top view **de** of the given line **DE** and the top view **pq** of the line **PQ** gives the top view **g** of the piercing point. Draw vertical projection line passing through point **g** which intersects the front view of the line **d'e'** at point **g'**, as shown in *Figure 4.21(b)*.

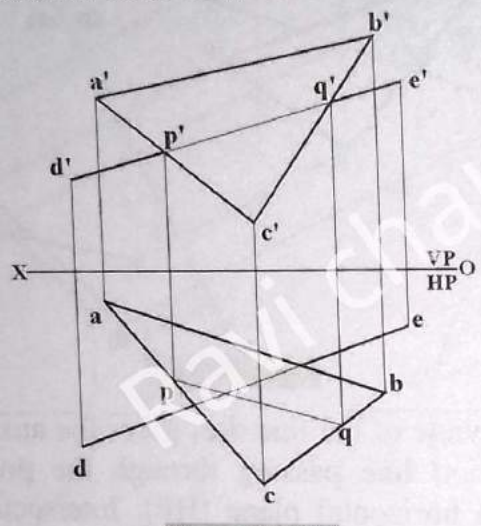


Figure 4.21(a)

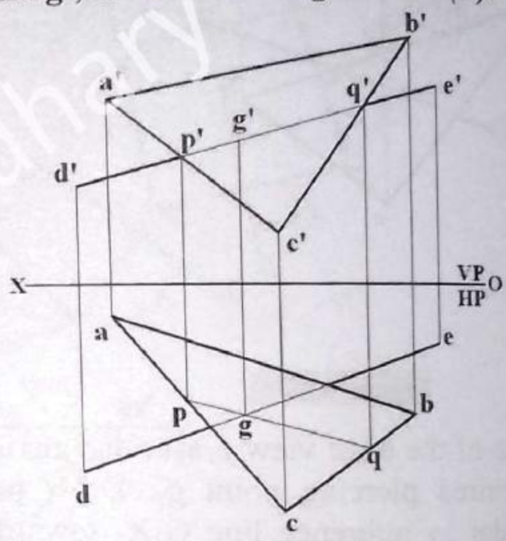


Figure 4.21(b)

4.9.3 Visible Portion of the Line

Projection line **1** passing through **p'** in *Figure 4.22(a)* shows that on the left side, **DG** portion of the give line is in front of the edge **AC** of the plane. Therefore, **d'g'** is the visible portion of the line in the front view.

Similarly projection line **2** in *Figure 4.22(b)* shows that on the left side, **DG** portion of the given line is above the edge **AC** of the plane. Therefore, **dg** is the visible portion of the line in the top view.

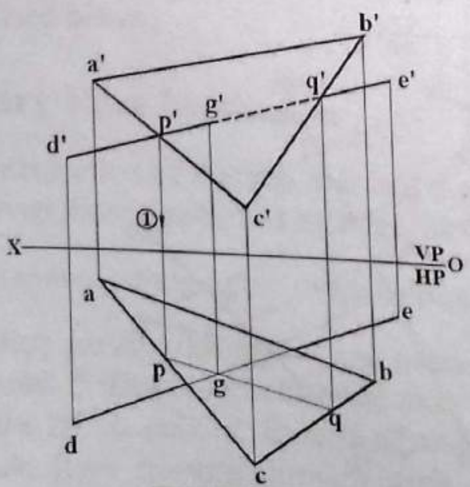


Figure 4.22(a)

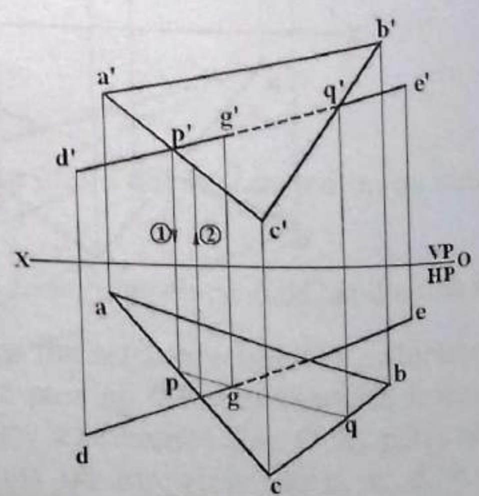


Figure 4.22(b)

4.10 True Angle between a Line and a Plane

True angle between a line and a plane can be measured in an auxiliary plane in which the given plane appears as an edge view and the given line appears in true length.

Consider the orthographic projections of the triangular plane **ABC** and the line **DE** shown in *Figure 4.20(a)*. Draw auxiliary views of both the line and plane such that the given plane appears as an edge view as shown in *Figure 4.20 (b)* by following the procedure explained earlier.

Draw a reference line O_2X_2 parallel to the edge view $\mathbf{c_1a_1b_1}$, as shown in *Figure 4.23*. Draw projection lines passing through each point on the first auxiliary plane (AP_1) and perpendicular to the reference line O_2X_2. Measure distances of each point on the horizontal plane (HP) from the reference line O_1X_1 and transfer them into the respective projection lines on the second auxiliary plane (AP_2) from the reference line O_2X_2.

Given triangular plane **ABC** appears as true shape in the second auxiliary plane (AP_2). Draw another reference line O_3X_3 parallel to the view $\mathbf{d_2e_2}$ of the line. Draw projection lines passing through each points on the second auxiliary plane (AP_2) perpendicular to the reference line O_3X_3. Measure distances of each point on the first auxiliary plane (AP_1) from the reference line O_2X_2 and transfer them into the respective projections on the third auxiliary plane (AP_3) from the reference line O_3X_3.

In the third auxiliary plane, given planes appears as an edge view $\mathbf{b_3c_3a_3}$ and the given line appears in true length $\mathbf{d_3e_3}$ and the required true angle between the given line and the plane can be measured in this view.

4.11 True Angle between Two Planes

True angle between two given planes or dihedral angle can be measured in an auxiliary plane in which both planes appear as edge views. Two intersecting planes will appear as edge views when common line of intersection appears as a point view.

Figure 4.24 shows the orthographic projections of triangular planes **ABC** and **BCD** with line of intersection **BC**. Then draw point view of the intersecting edge **BC** by following the procedure explained earlier.

For this, draw a reference line O_1X_1 parallel to the top view **bc** of the intersecting edge, as shown in *Figure 4.25*. Draw projection lines passing through each point on the top view and perpendicular to the reference line O_1X_1.Measure distances of each point on the vertical plane from the reference line OX and transfer them into the respective projection lines from the reference line O_1X_1. In this view $\mathbf{b_1c_1}$ appears in true length.

Draw another reference line O_2X_2 perpendicular to $\mathbf{b_1c_1}$. Draw projection lines from each point on the first auxiliary plane (AP_1) and perpendicular to the reference line O_2X_2. Measure distances of each point on the horizontal plane (HP) from the reference line O_1X_1 and transfer them into the respective projection lines from the reference line O_2X_2. In this plane, common edge **BC** appears as point view b_2/c_2 and both the planes appear as edge views. Then the angle between these edge views gives the required true angle between the planes.

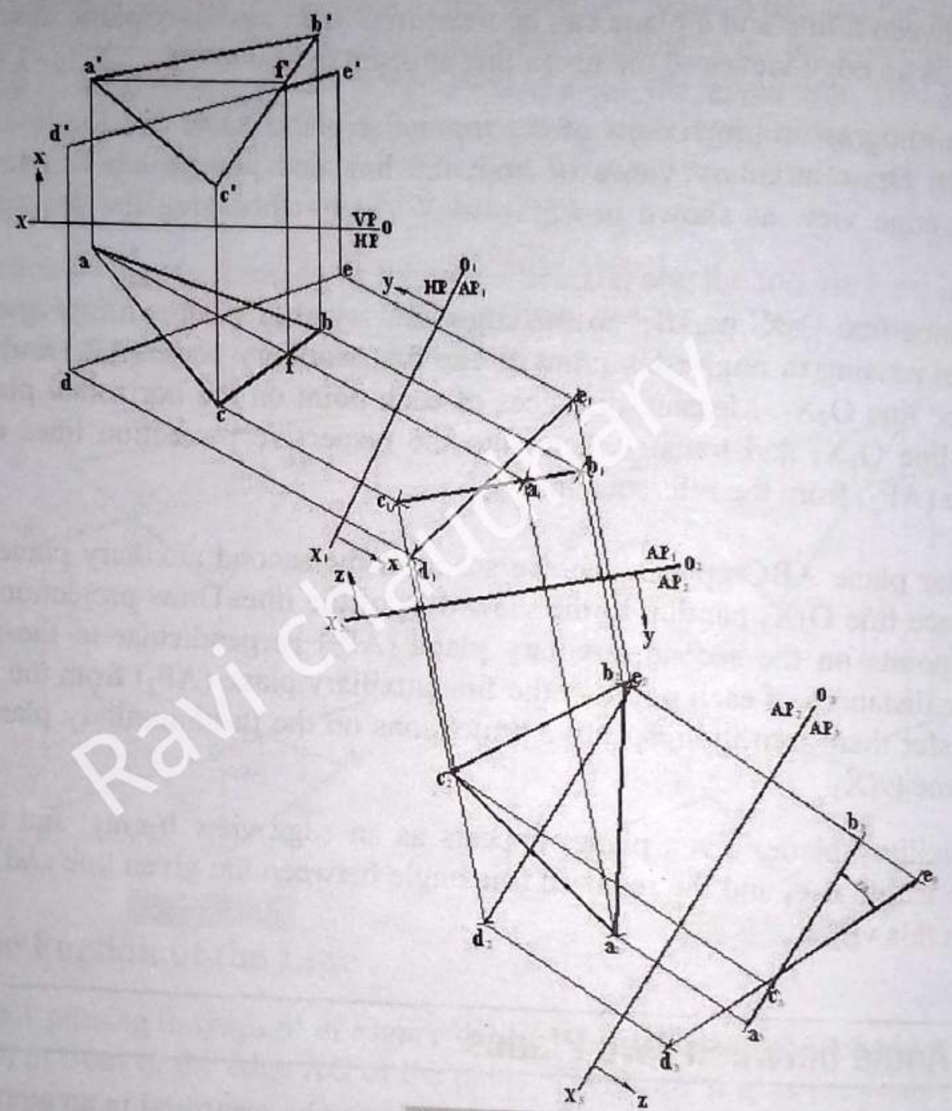


Figure 4.23

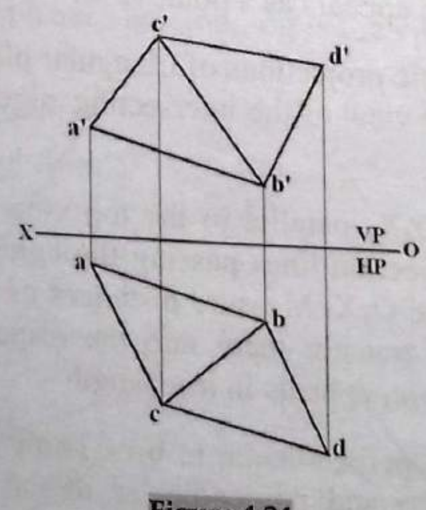


Figure 4.24

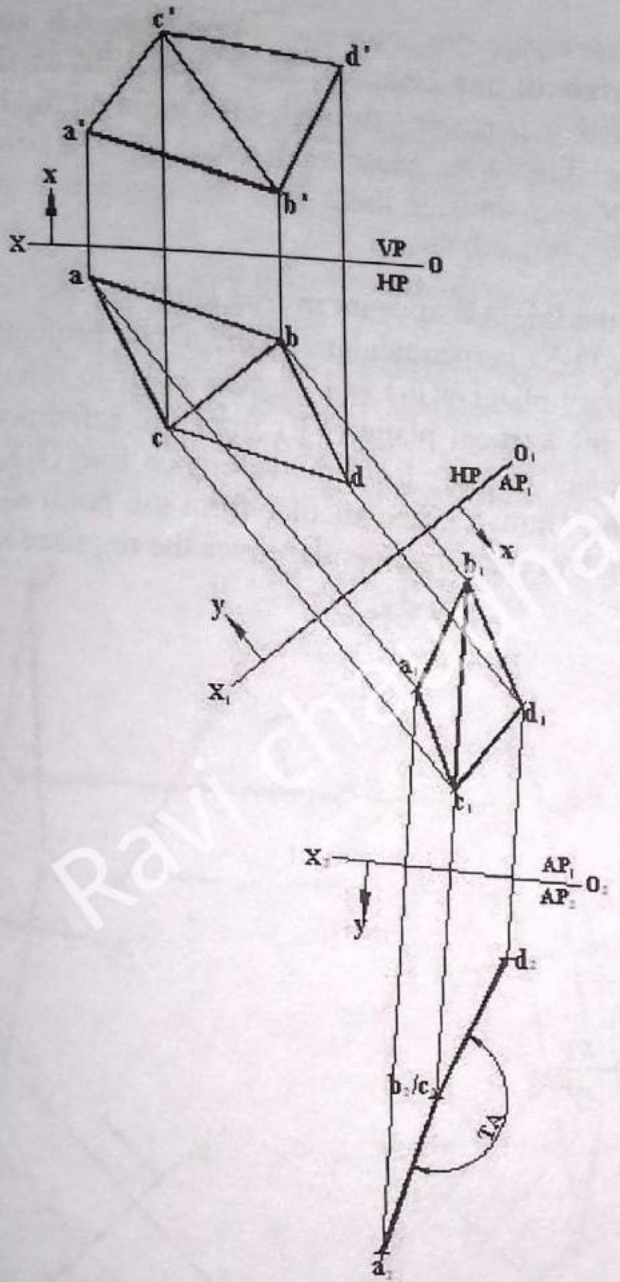
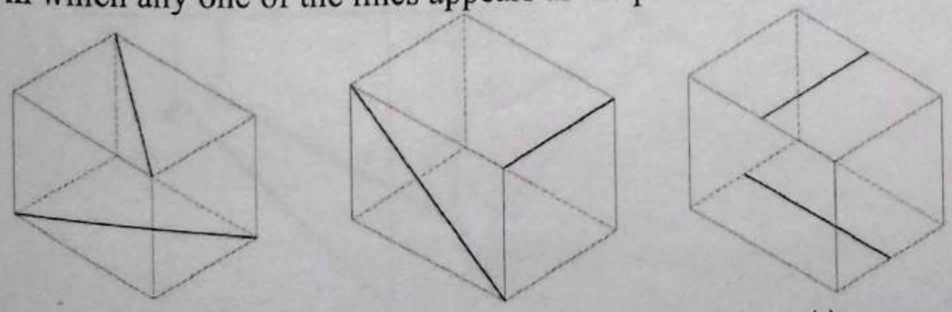


Figure 4.25

4.12 Shortest Distance between Skew-Lines

Any lines which are non-intersecting and non-parallel are called skew lines. *Figure 4.26* shows different sets of skew lines. Shortest distance between skew-lines can be measured in an auxiliary view in which any one of the lines appears as the point view.



(a) (b) (c)

Figure 4.26

Figure 4.27 shows the orthographic projections of skew lines **AB** and **CD**. Draw a reference line O_1X_1 parallel to any view of any line, say front view **a'b'** of the line **AB**, as shown in **Figure 4.28**. Draw projection lines passing through each point on the horizontal plane (HP) and perpendicular to the reference line O_1X_1. Measure distance of each point on the horizontal plane from the reference line OX and transfer them into the respective projection lines from the reference line O_1X_1.

The auxiliary view $\mathbf{a_1b_1}$ of the line **AB** appears in true length in the first auxiliary plane (AP_1). Draw another reference line O_2X_2 perpendicular to $\mathbf{a_1b_1}$. Draw projection lines passing through each point on the first auxiliary plane (AP_1) and perpendicular to reference line O_2X_2. Measure distances of each point on the vertical plane (VP) from the reference line O_1X_1 and transfer them into the respective projection lines from the reference line O_2X_2. In this plane, line AB appears as a point view $\mathbf{a_2/b_2}$. Draw a perpendicular from the point $\mathbf{a_2/b_2}$ to the auxiliary view $\mathbf{c_2d_2}$ of the line **CD**. The length of the perpendicular gives the required shortest distance.

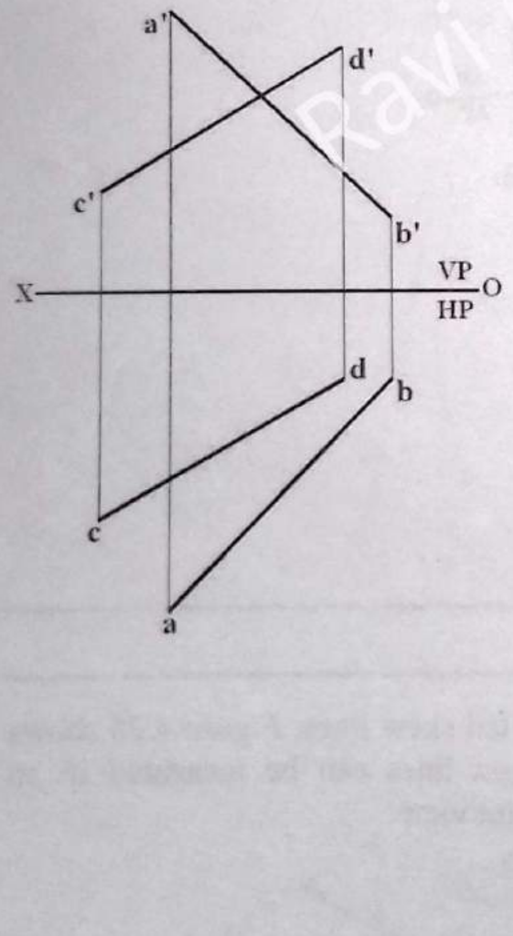


Figure 4.27

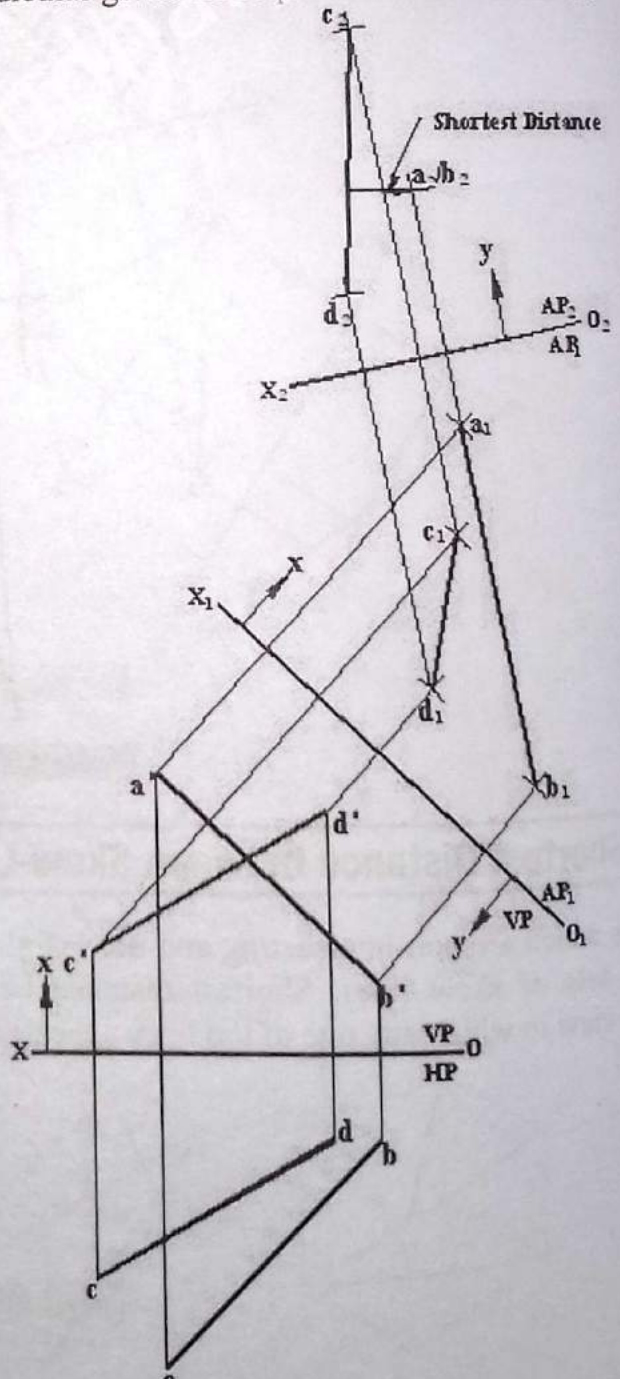


Figure 4.28

4.13 True Angle between Skew-Lines

Apparent true angle between two skew lines can be measured in a view in which both lines appear in true length.

For this, draw point view of any one line, say line **AB** as explained earlier. Then draw a reference line O_3X_3 such that it is parallel to the auxiliary view $\mathbf{c_2d_2}$ of the line **CD**. Draw projection lines passing through each point on the second auxiliary plane (AP_2) and perpendicular to the reference line O_3X_3. Measure distance of each point on the first auxiliary plane (AP_1) from the reference line O_2X_2 and transfer them into the respective projection lines from the reference line O_3X_3. In the third auxiliary plane (AP_3), both lines appear in true lengths. The angle made by the auxiliary views $\mathbf{a_3b_3}$ and $\mathbf{c_3d_3}$ gives the required true angle.

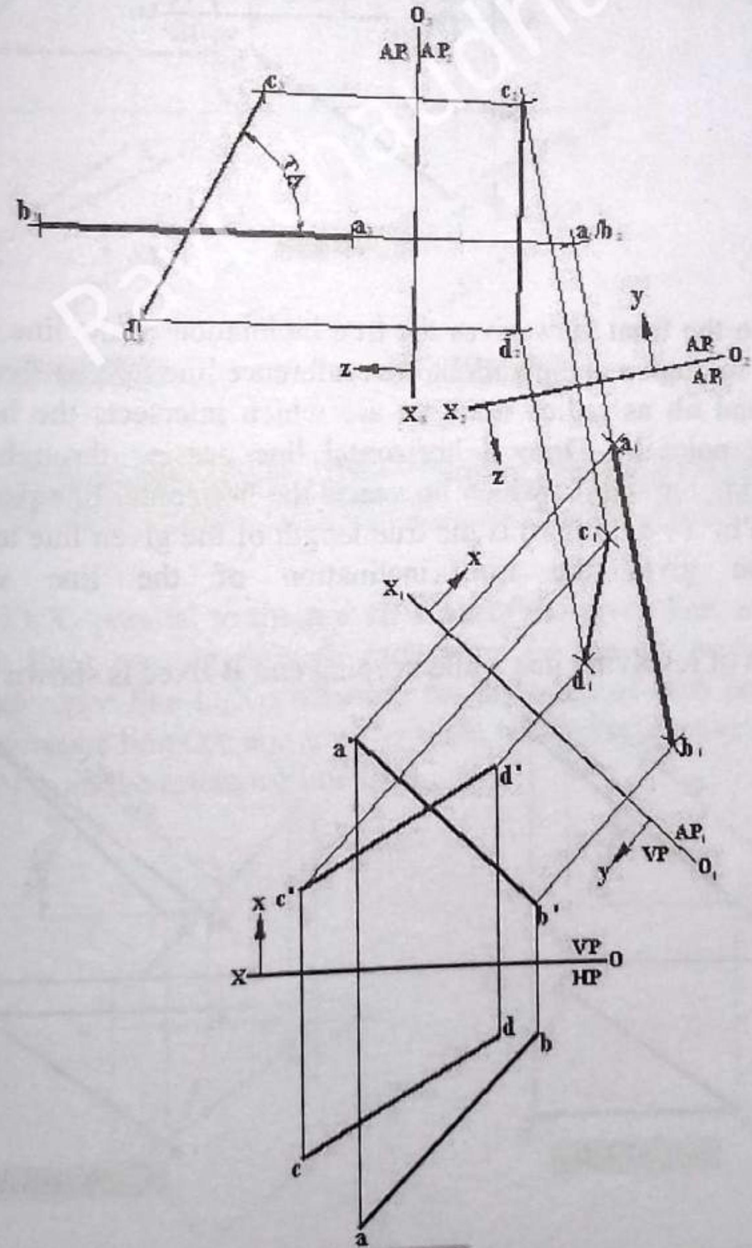


Figure 4.29

Example 4.1

Orthographic projection of a line **AB** is given in *Figure E4.1*. Determine its true length and inclination with the HP using revolution method.

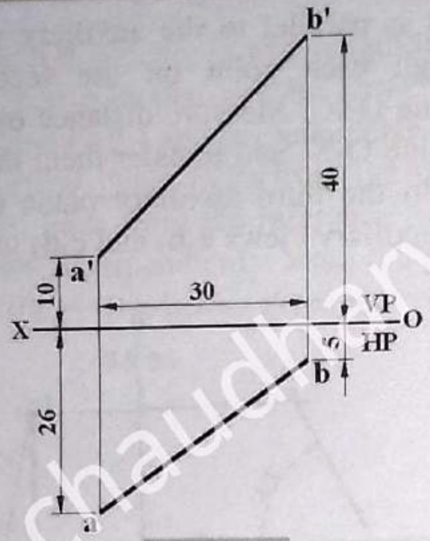


Figure E4.1

Solution

A true length line in the front view gives the true inclination of the line with the HP. For this, draw a line passing through **a** and parallel to the reference line OX, as shown in *Figure E4.1(a)*. With **a** as center and **ab** as radius draw an arc which intersects the horizontal line passing through point **a** at point $\mathbf{b_1}$. Draw a horizontal line passing through **b'**. Draw a vertical projection line passing through $\mathbf{b_1}$ which intersects the horizontal line passing through point b' at point $\mathbf{b_1'}$. Then $\mathbf{a'b_1'}$ (= 47.3 mm) is the true length of the given line and its inclination with the reference line gives the true inclination of the line with the HP, i.e., $\theta_H = 39.3^0$.

An alternate method of revolving line while keeping end **B** fixed is shown in **Figure E4.1(b)**.

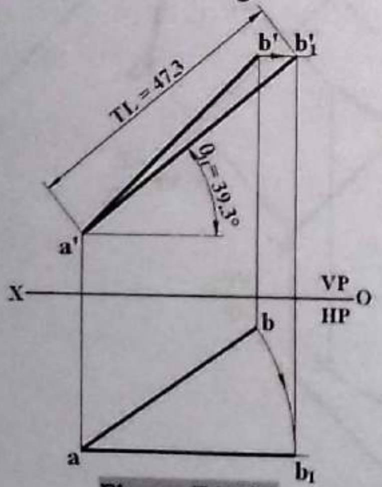


Figure E4.1(a)

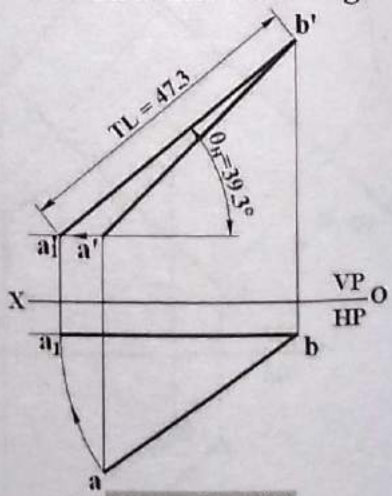


Figure E4.1(b)

Example 4.2

Determine the true length and inclination of the line shown in *Figure E4.1* with the VP using revolution method.

Solution

A true length line in the top view gives the true inclination of the line with the VP. For this, draw a line passing through **a'** and parallel to the reference line OX, as shown in *Figure E4.2(a)*. With **a'** as center and **a'b'** as radius draw an arc which intersects the horizontal line passing through point **a'** at point $\mathbf{b_1'}$. Draw a horizontal line passing through **b**. Draw a vertical projection line passing through $\mathbf{b_1'}$ which intersects the horizontal line passing through point **b** at point $\mathbf{b_1}$. Then $\mathbf{a_1b_1}$ (= 47.3 mm) is the true length of the given line and its inclination with the reference line gives the true inclination of the line with the VP, i.e., $\theta_V = 26.3^0$.

An alternate method of revolving line while keeping end B fixed is shown in Figure E4.2(b).

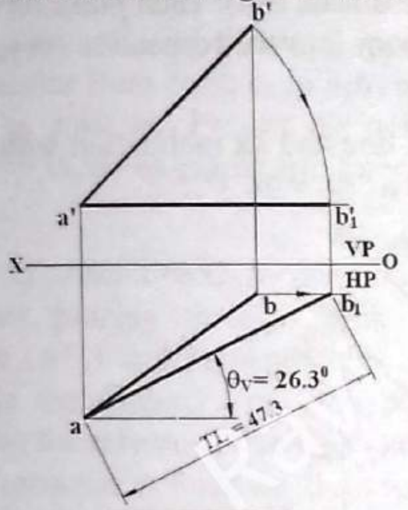


Figure E4.2(a)

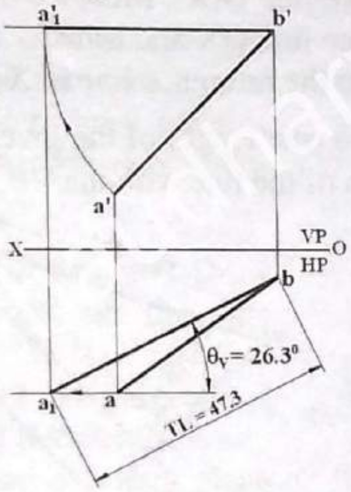


Figure E4.2(b)

Example 4.3

Determine the true length and inclination of the line shown in *Figure E4.1* with the HP using auxiliary view method.

Solution

Draw a reference line O_1X_1 parallel to the top view **ab** of the given line, as shown in *Figure E4.3*. Draw projection lines passing through each point on the horizontal plane (HP) and perpendicular to the reference line O_1X_1. Measure the distances of each point on the vertical plane (VP) from the reference line OX and transfer them into the respective projection lines on the auxiliary plane (AP) from the reference line O_1X_1.

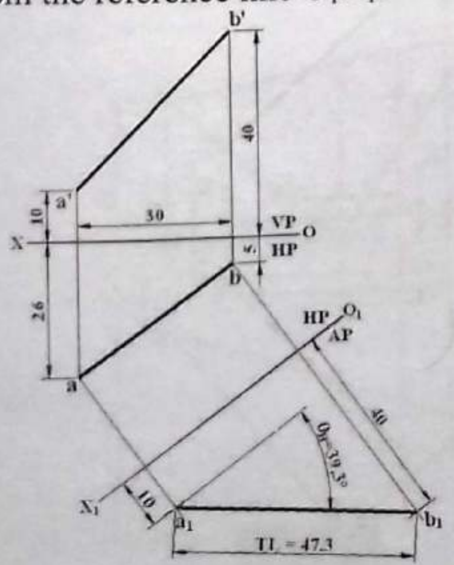


Figure E4.3

Then a_1b_1 (= 47.3 mm) is the true length of the given line and its inclination with the reference line gives the true inclination of the line with the HP, i.e., $\theta_H = 39.3^0$.

Example 4.4

Determine the true length and inclination of the line shown in *Figure E4.1* with the VP using auxiliary view method.

Solution

Draw a reference line O_1X_1 parallel to the front view **a'b'** of the given line, as shown in *Figure E4.4*. Draw projection lines passing through each point on the vertical plane (VP) and perpendicular to the reference line O_1X_1. Measure the distances of each point on the horizontal plane (HP) from the reference line OX and transfer them into the respective projection lines on the auxiliary plane (AP) from the reference line O_1X_1.

Then a_1b_1 (= 47.3 mm) is the true length of the given line and its inclination with the reference line gives the true inclination of the line with the VP, i.e., $\theta_V = 26.3^0$.

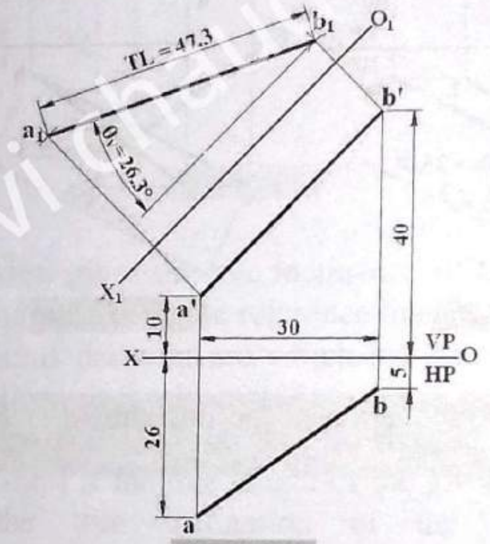


Figure E4.4

Example 4.5

Orthographic projections of a line **AB** and a point **C** are given in *Figure E4.5*. Determine shortest distance between them.

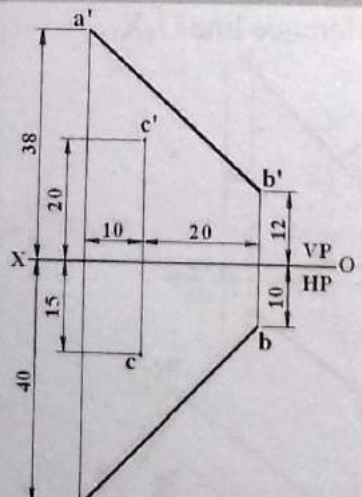


Figure E4.5

Solution

Draw a reference line O_1X_1 parallel to the front view **a'b'** of the given line, as shown in *Figure E4.5(a)*. Draw projection lines passing through each point on the vertical plane (VP) and perpendicular to the reference line O_1X_1. Measure the distances of each point on the horizontal plane (HP) from the reference line OX and transfer them into the respective projection lines on the first auxiliary plane (AP_1) from the reference line O_1X_1.

The auxiliary view $\mathbf{a_1b_1}$ of the line appears in true length. Draw perpendicular from point $\mathbf{c_1}$ to $\mathbf{a_1b_1}$ and name the foot of perpendicular as point $\mathbf{p_1}$. Project the point $\mathbf{p_1}$ back towards the front and top views to get its front and top views **p'** and **p** respectively.

Draw another reference line O_2X_2 perpendicular to $\mathbf{a_1b_1}$. Draw projection lines passing through each point on the first auxiliary plane (AP_1) and perpendicular to the reference line O_2X_2. Measure the distances of each point on the vertical plane (VP) from the reference line O_1X_1 and transfer them into the respective projection lines on the second auxiliary plane (AP_2) from the reference line O_2X_2 to get point view $\mathbf{a_2/b_2}$ of the line **AB** and auxiliary view $\mathbf{c_2}$ of the given point **C**. Then the distance between $\mathbf{a_2/b_2}$ and $\mathbf{c_2}$ gives the required shortest distance, i.e. SD = 10.9 mm.

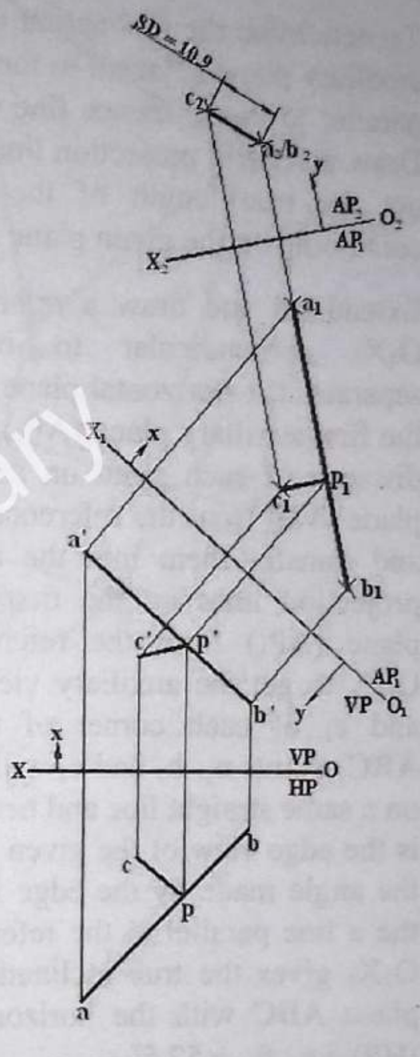


Figure E4.5(a)

Example 4.6

Orthographic projection of a triangular plane **ABC** is given in *Figure E4.6*. Determine its true shape. Draw auxiliary views in the direction from which its inclination with the HP can be determined.

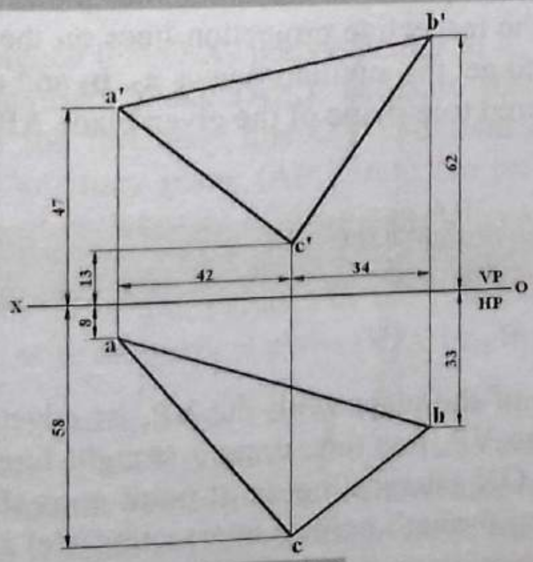


Figure E4.6

Solution

To determine the inclination of the plane with the HP, its edge view should be drawn on an auxiliary plane adjacent to the HP. For this, draw a straight line passing through point **a'** and parallel to the reference line OX intersecting **b'c'** at point **d'**, as shown in *Figure E4.6(a)*. Draw a vertical projection line passing through point **d'** intersecting **bc** at point **d**. Join **ad** to get the true length of the line **AD** containing on the given plane **ABC**.

Extend **ad** and draw a reference line O_1X_1 perpendicular to it, which separates the horizontal plane (HP) and the first auxiliary plane (AP_1). Measure distance of each point on the vertical plane (VP) from the reference line OX and transfer them into the respective projection lines on the first auxiliary plane (AP_1) from the reference line O_1X_1 to get the auxiliary views $\mathbf{a_1}$, $\mathbf{b_1}$ and $\mathbf{c_1}$ of each corner of the plane **ABC**. Points $\mathbf{a_1}$, $\mathbf{b_1}$ and $\mathbf{c_1}$ will then line on a same straight line and hence $\mathbf{c_1a_1b_1}$ is the edge view of the given plane and the angle made by the edge view with the a line parallel to the reference line O_1X_1 gives the true inclination of the plane **ABC** with the horizontal plane (HP), i.e., $\theta_H = 52.5^0$.

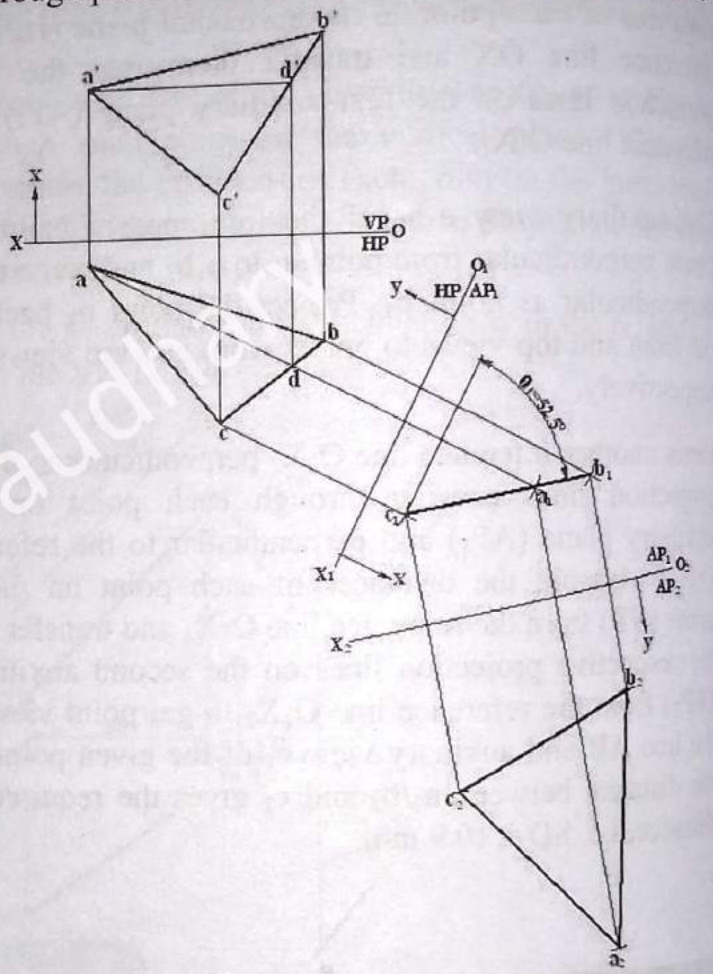


Figure E4.6(a)

To draw the true shape of the plane, draw another reference line O_2X_2 parallel to the edge view $\mathbf{c_1a_1b_1}$. Measure distance of each point on the horizontal plane (HP) from the reference line O_1X_1 and transfer them into the respective projection lines on the second auxiliary plane (AP_2) from the reference line O_2X_2 to get the auxiliary views $\mathbf{a_2}$, $\mathbf{b_2}$ and $\mathbf{c_2}$ of each corners of the plane **ABC**. Then $\mathbf{a_2b_2c_2}$ is the required true shape of the given plane **ABC**.

Example 4.7

Determine the true shape a triangular plane **ABC** given in *Figure E4.6*. Draw auxiliary views in the direction from which its inclination with the VP can be determined.

Solution

To determine the inclination of the plane with the VP, its edge view should be drawn on an auxiliary plane adjacent to the VP. For this, draw a straight line passing through point **b** and parallel to the reference line OX intersecting **ac** at point **e**, as shown in *Figure E4.7*. Draw a vertical projection line passing through point **e** intersecting **a'c'** at point **e'**. Join **b'e'** to get the true length of the line **BE** containing on the given plane **ABC**.

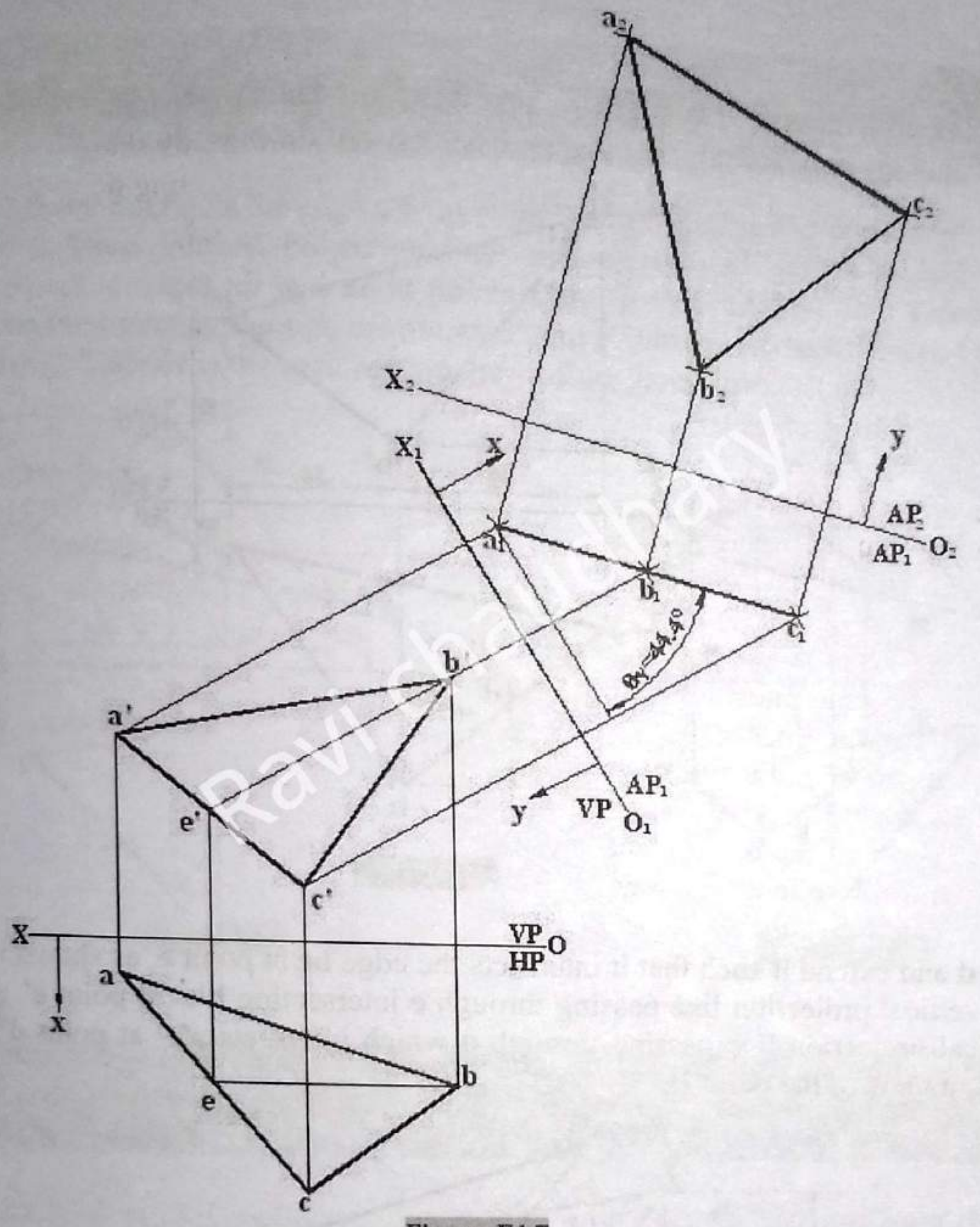


Figure E4.7

Extend **b'e'** and draw a reference line O_1X_1 perpendicular to it, which separates the vertical plane (VP) and the first auxiliary plane (AP_1). Measure distance of each point on the horizontal plane (HP) from the reference line OX and transfer them into the respective projection lines on the first auxiliary plane (AP_1) from the reference line O_1X_1 to get the auxiliary views $\mathbf{a_1}$, $\mathbf{b_1}$ and $\mathbf{c_1}$ of each corner of the plane **ABC**. Points $\mathbf{a_1}$, $\mathbf{b_1}$ and $\mathbf{c_1}$ will then line on a same straight line and hence $\mathbf{a_1b_1c_1}$ is the edge view of the given plane and the angle made by the edge view with the a line parallel to the reference line O_1X_1 gives the true inclination of the plane **ABC** with the vertical plane (VP), i.e., $\theta_V = 44.4^0$.

To draw the true shape of the plane, draw another reference line O_2X_2 parallel to the edge view $\mathbf{a_1b_1c_1}$. Measure distance of each point on the vertical plane (VP) from the reference line O_1X_1 and transfer them into the respective projection lines on the second auxiliary plane (AP_2) from the reference line O_2X_2 to get the auxiliary views $\mathbf{a_2}$, $\mathbf{b_2}$ and $\mathbf{c_2}$ of each corners of the plane **ABC**. Then $\mathbf{a_2b_2c_2}$ is the required true shape of the given plane **ABC**.

Example 4.8

Orthographic projection of a triangular plane **ABC** and the top view of a point **D** lying on the plane **ABC** are given in *Figure E4.8*. Draw front view of the point **D**.

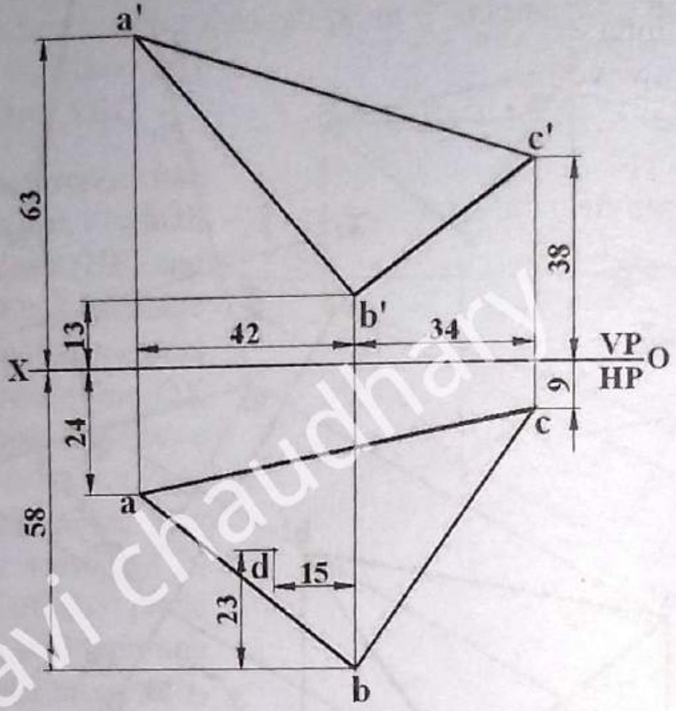


Figure E4.8

Solution

Join **a** and **d** and extend it such that it intersects the edge **bc** at point **e**, as shown in *Figure E4.8 (a)*. Draw vertical projection line passing through **e** intersecting **b'c'** at point **e'**. Join **a'** and **e'**. Draw vertical projection line passing through **d** which intersects **a'e'** at point **d'**, which is the required front view of the point **D**.

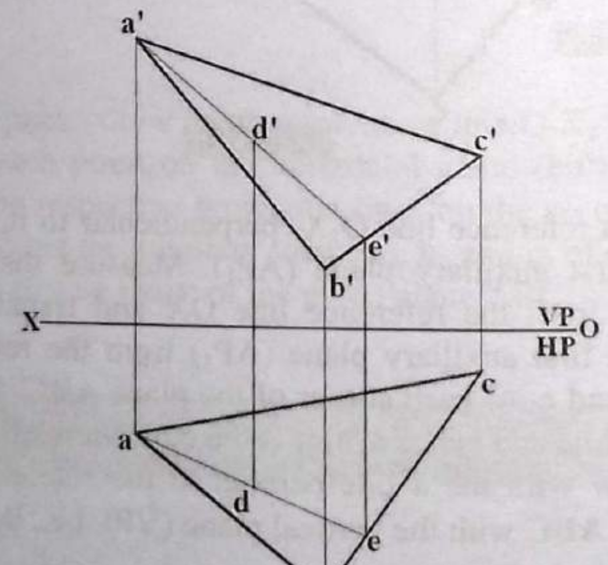


Figure E4.8(a)

Example 4.9

Orthographic projection of a triangular plane **ABC** and the front view of a line **DE** lying on the plane **ABC** are given in *Figure E4.9*. Draw the top view of the line **DE**.

Solution

Extend **d'e'** such that it intersects the edge **a'b'** at point **f'** and the edge **a'c'** at point **g'**, as shown in *Figure E4.9(a)*. Draw vertical projection lines passing through points **f'** and **g'** which intersects the respective edges **ab** and **ac** at points **f** and **g** respectively. Join **f** and **g**. Draw vertical projection lines passing through the points **d'** and **e'** which intersect the line **fg** at points **d** and **e** respectively. Then **de** is the required top view of the given line.

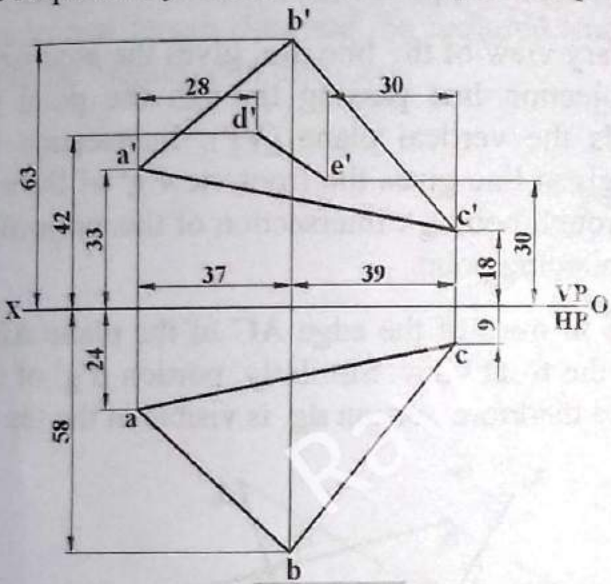


Figure E4.9

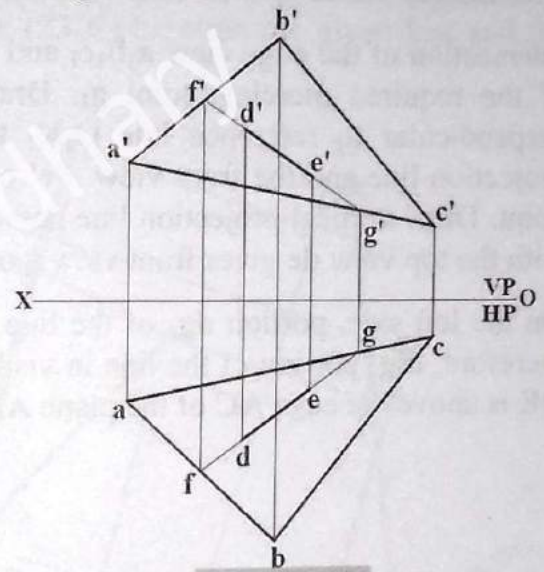


Figure E4.9(a)

Example 4.10

Orthographic projections of a plane **ABC** and a straight line **DE** are given in *Figure E4.10*. Draw the front and top views of the piercing point and determine the visible portion of the line. Also determine the true angle between them.

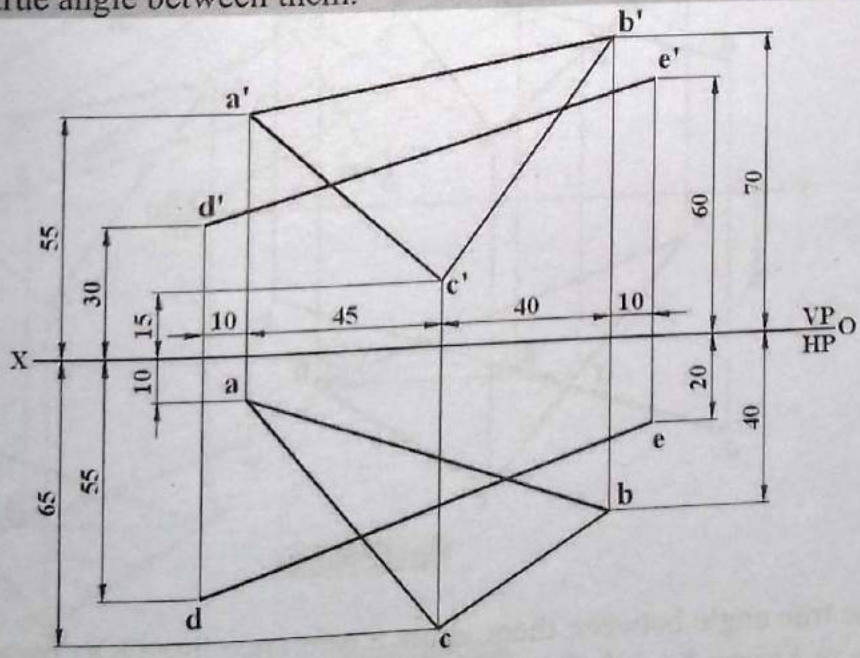


Figure E4.10

Solution

Draw a straight line passing through point **b** and parallel to the reference line OX intersecting ac at point **f**, as shown in *Figure E4.10a*. Draw a vertical projection line passing through the point f intersecting **a'c'** at point **f'**. Join **f'b'** to get a true length of the line **FB** lying on the plane **ABC**.

Extend **f'b'** and draw a reference line O_1X_1 perpendicular to it. Draw projection lines passing through each points on the front view (**a'**, **b'**, **c'**, **d'**, **e'**, and **f'**) and perpendicular to the reference line O_1X_1. Measure distances of each point on the horizontal plane (HP) from the reference line OX and transfer them into the respective projection lines on the first auxiliary plane (AP_1) to get the auxiliary views of each corners of the triangle and end points of the line.

Intersection of the edge view $\mathbf{a_1b_1c_1}$ and auxiliary view of the line $\mathbf{d_1e_1}$ gives the auxiliary view of the required piercing point $\mathbf{g_1}$. Draw projection line passing through the point g_1 and perpendicular to reference line O_1X_1 towards the vertical plane (VP). Intersection of this projection line and the front view **d'e'** of the given line gives the front view **g'** of the piercing point. Draw vertical projection line passing through point **g'**. Intersection of this projection line with the top view **de** gives front view **g** of the piercing point.

On the left side, portion **dg** of the line **DE** is in front of the edge **AC** of the plane **ABC** and therefore, **d'g'** portion of the line in visible in the front view. Similarly, portion **d'g'** of the line **DE** is above the edge **AC** of the plane **ABC** and therefore portion **dg** is visible in the top view.

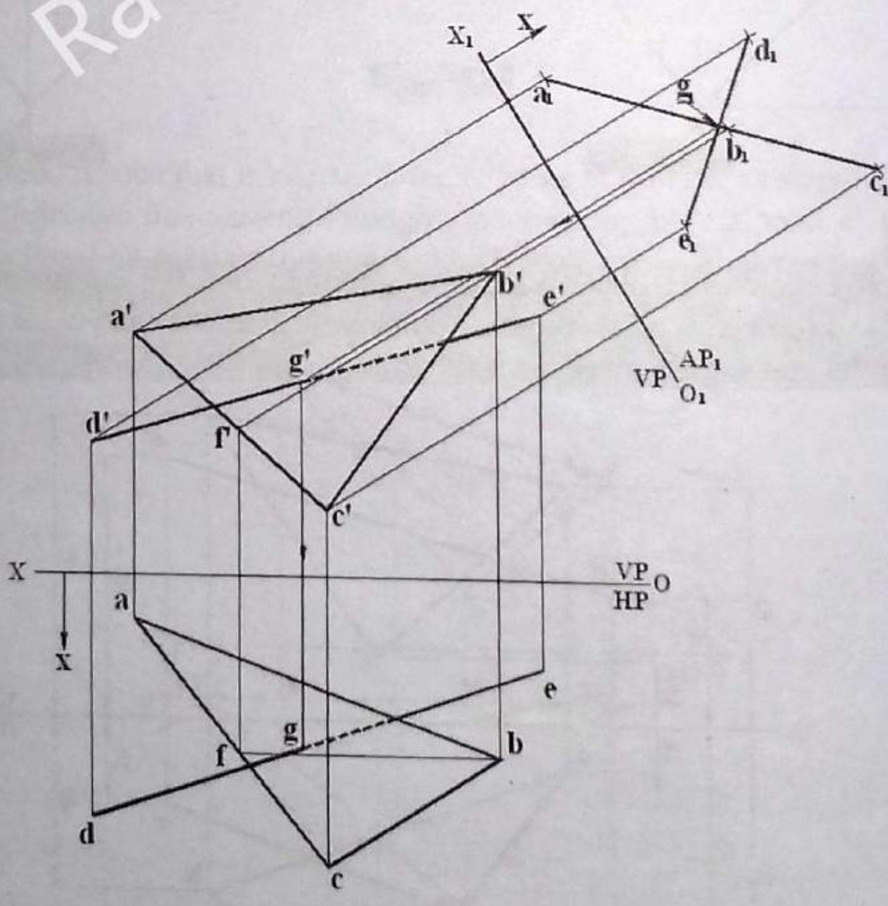


Figure E4.10a

To determine the true angle between them, draw a reference line O_2X_2 parallel to the edge view $\mathbf{a_1b_1c_1}$, as shown in *Figure E4.10b*. Draw projection lines passing through each point on the first auxiliary plane (AP_1) and perpendicular to the reference line O_2X_2. Measure distances of each point on the vertical plane (VP) from the reference line O_1X_1 and transfer them into the respective projection lines on the second auxiliary plane (AP_2) from the reference line O_2X_2.

Given triangular plane **ABC** appears as true shape in the second auxiliary plane (AP_2). Draw another reference line O_3X_3 parallel to the view $\mathbf{d_2e_2}$ of the line. Draw projection lines passing through each point on the second auxiliary plane (AP_2) perpendicular to the reference line O_3X_3. Measure distances of each point on the first auxiliary plane (AP_1) from the reference line O_2X_2 and transfer them into the respective projections on the third auxiliary plane (AP_3) from the reference line O_3X_3.

In the third auxiliary plane, given planes appears as an edge view $\mathbf{a_3c_3b_3}$ and the given line appears in true length $\mathbf{d_3e_3}$ and the required true angle (23.3^0) between the given line and the plane can be measured in this view.

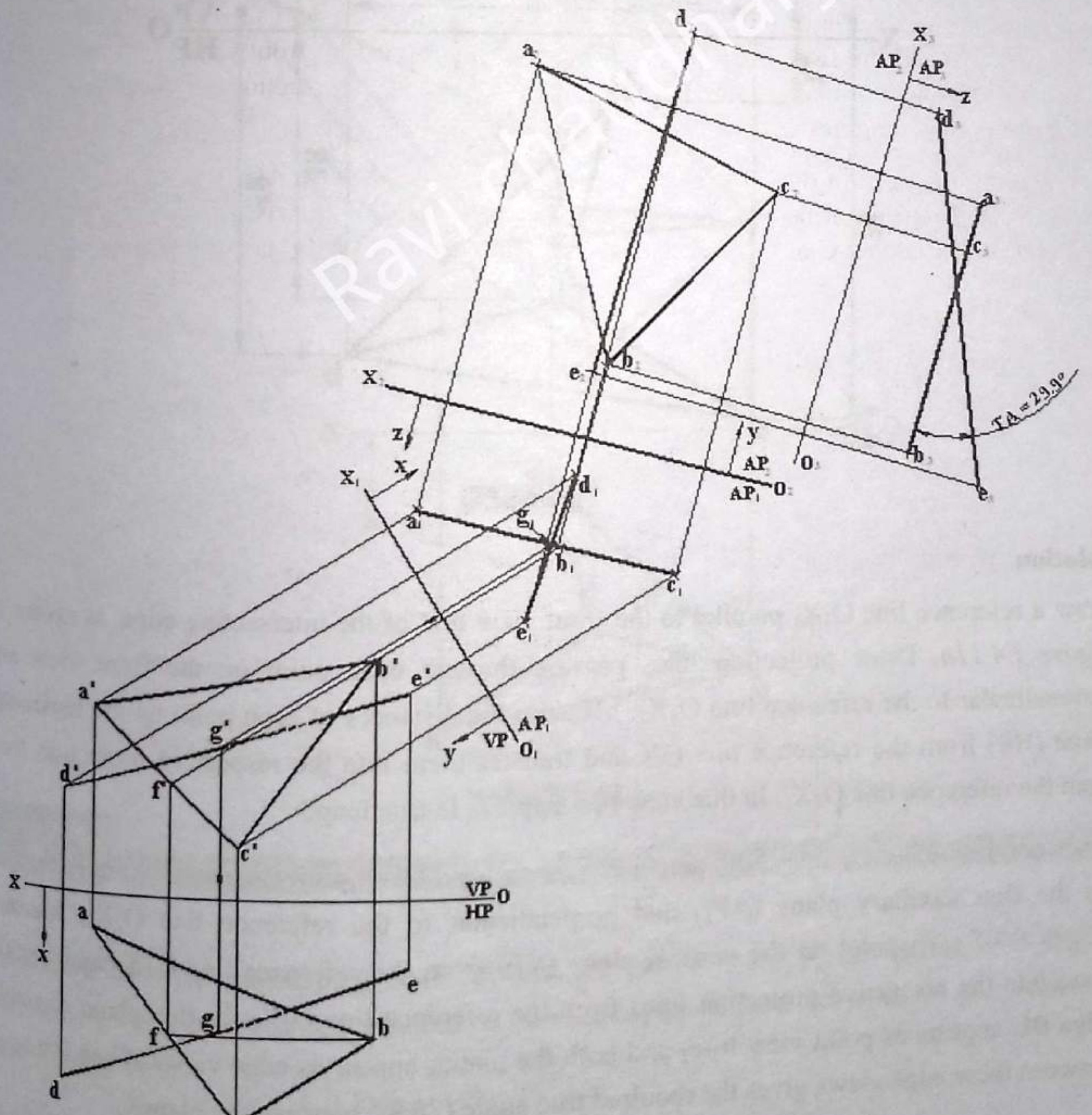


Figure E4.10b

Example 4.11

Determine the true angle between the given planes **ABC** and **BCD** shown in *Figure E4.11.*

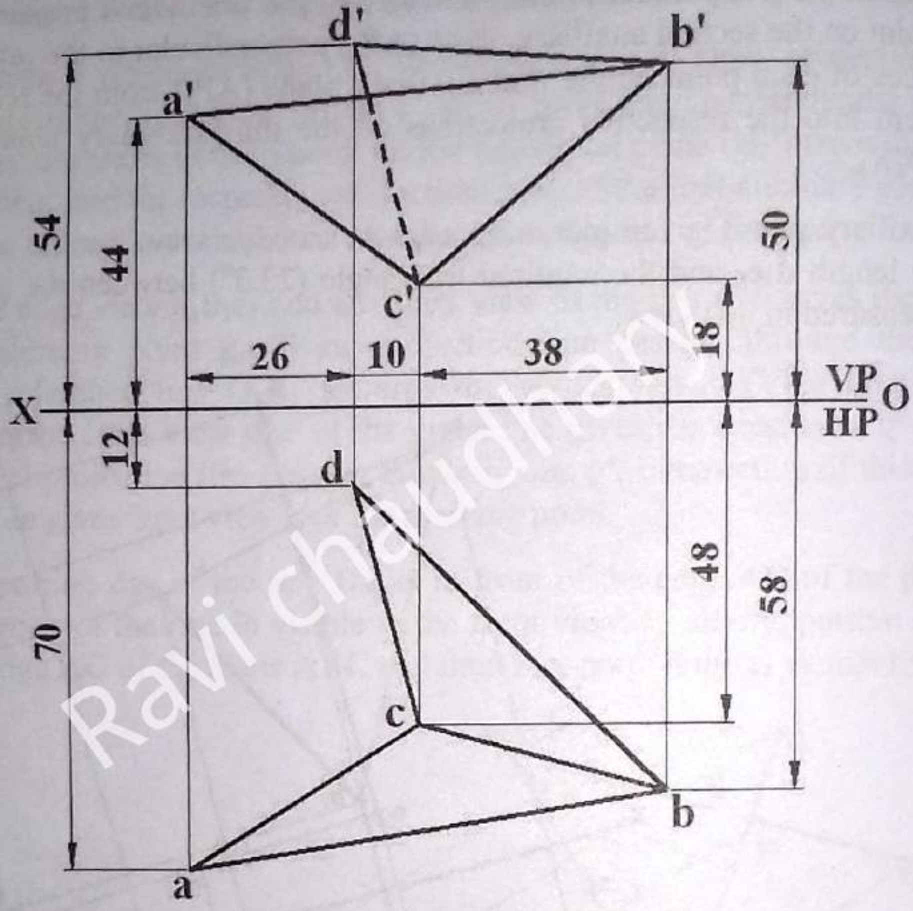


Figure E4.11

Solution

Draw a reference line O_1X_1 parallel to the front view **b'c'** of the intersecting edge, as shown in *Figure E4.11a.* Draw projection lines passing through each point on the front view and perpendicular to the reference line O_1X_1. Measure the distances of each point on the horizontal plane (HP) from the reference line OX and transfer them into the respective projection lines from the reference line O_1X_1. In this view $\mathbf{b_1c_1}$ appears in true length.

Draw another reference line O_2X_2 perpendicular to $\mathbf{b_1c_1}$. Draw projection lines from each point on the first auxiliary plane (AP_1) and perpendicular to the reference line O_2X_2. Measure distances of each point on the vertical plane (VP) from the reference line O_1X_1 and transfer them into the respective projection lines from the reference line O_2X_2. In this plane, common edge **BC** appears as point view $\mathbf{b_2/c_2}$ and both the planes appear as edge views. Then the angle between these edge views gives the required true angle (76.8^0) between the planes.

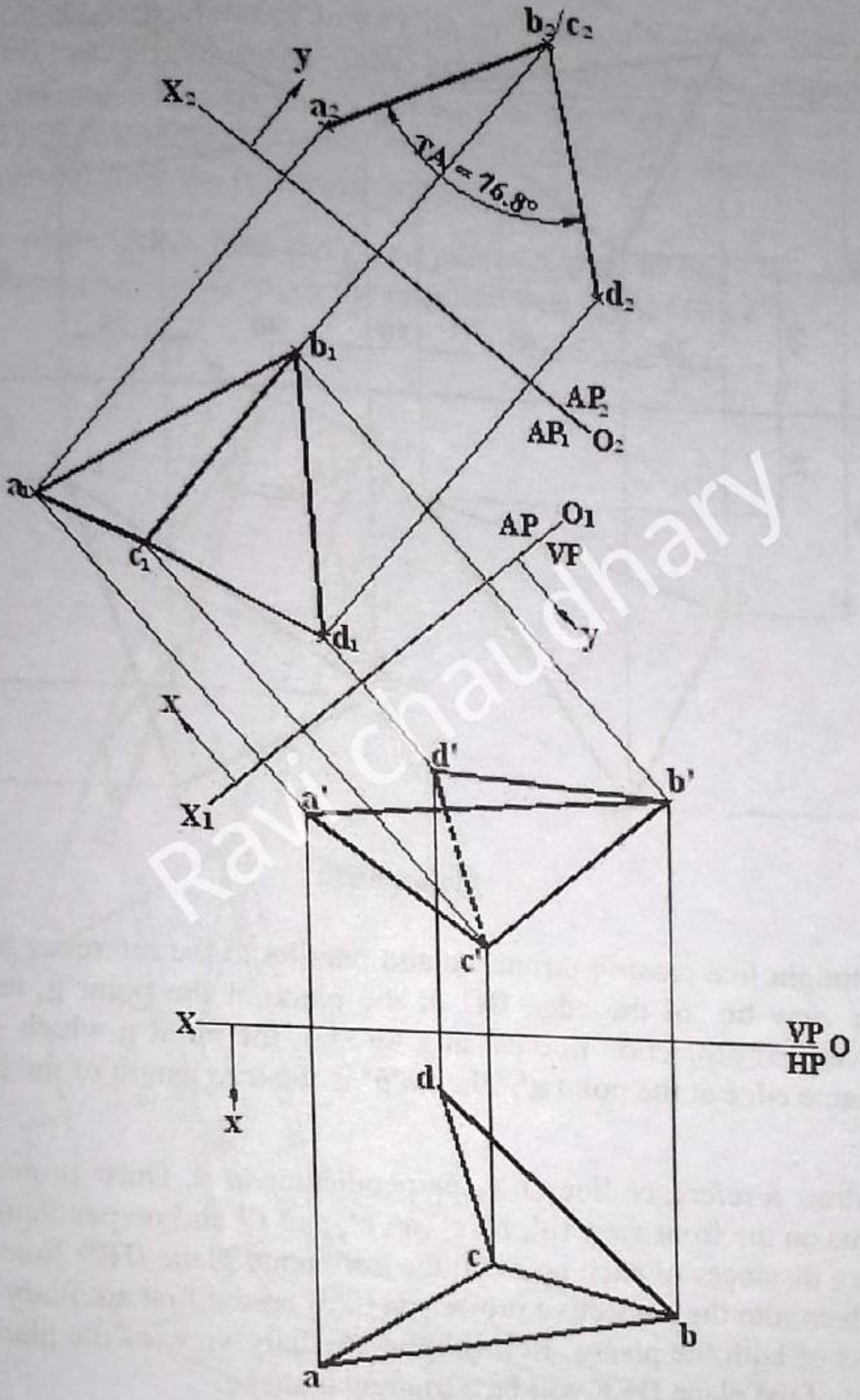


Figure E4.11a

Example 4.12

Determine the true angle between the given planes **ABC** and **DEF** shown in *Figure E4.11.*

Solution

In this case, common intersecting edge between the planes is not given. Hence, true angle between the given planes can be measured on an auxiliary plane on which both the given planes appear in edge views.

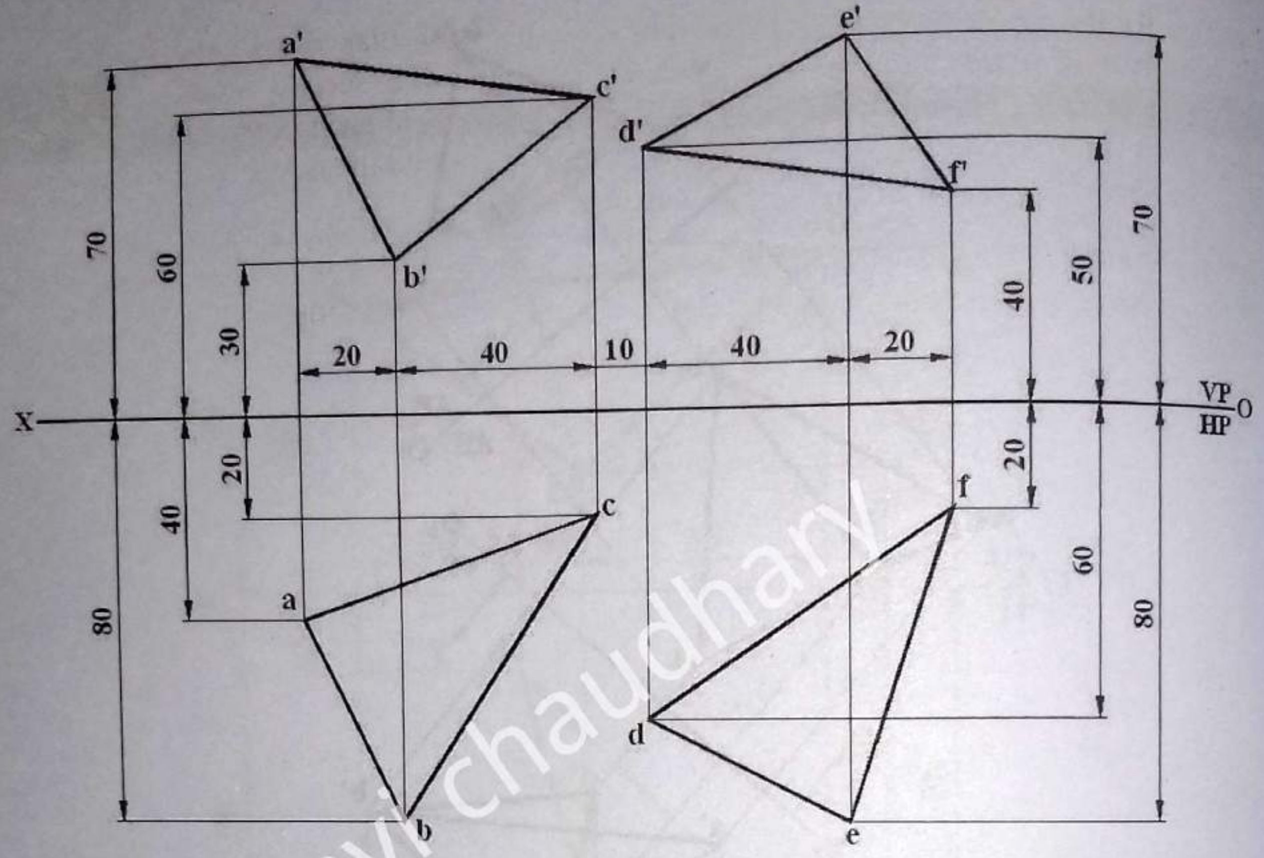


Figure E4.12

For this draw a straight line passing through **a** and parallel to the reference line OX such that it intersects the top view **bc** of the edge **BC** of the plane at the point g, as shown in *Figure E4.12a*. Draw a vertical projection line passing through the point g which intersects the front view **b'c'** of the same edge at the point **g'**. Then **a'g'** is the true length of the line **AG** containing on the plane **ABC**.

Extend **a'g'** and draw a reference line O_1X_1 perpendicular to it. Draw projection lines passing through each points on the front view (**a'**, **b'**, **c'**, **d'**, **e'**, and **f'**) and perpendicular to the reference line O_1X_1. Measure distances of each point on the horizontal plane (HP) from the reference line OX and transfer them into the respective projection lines on the first auxiliary plane (AP_1) to get the auxiliary views of both the planes. In this view auxiliary view of the plane **ABC** will be an edge view and that of the plane **DEF** will be a triangular shape.

Draw another reference line O_2X_2 parallel to the edge view $\mathbf{b_1a_1c_1}$. Draw projection lines passing through each point on the first auxiliary plane (AP_1) and perpendicular to the reference line O_2X_2. Measure distances of each point on the vertical plane (VP) from the reference line O_1X_1 and transfer them into the respective projection lines on the second auxiliary plane (AP_2) from the reference line O_2X_2. Given triangular plane **ABC** appears as true shape in the second auxiliary plane (AP_2) whereas plane **DEF** appears in distorted triangular form.

Draw a straight line passing though the point $\mathbf{d_1}$ in the first auxiliary plane (AP_1) and parallel to the reference line O_2X_2 intersecting the line $\mathbf{e_1f_1}$ at the point $\mathbf{h_1}$. Draw a projection line passing thorough $\mathbf{h_1}$ and perpendicular to the reference line O_2X_2 which intersects the line $\mathbf{e_2f_2}$ at the point $\mathbf{h_2}$.

Extend $\mathbf{d_2h_2}$ and draw another reference line O_3X_3 perpendicular to the view $\mathbf{d_2h_2}$ of the line. Draw projection lines passing through each point on the second auxiliary plane (AP_2) perpendicular to the reference line O_3X_3. Measure distances of each point on the first auxiliary plane (AP_1) from the reference line O_2X_2 and transfer them into the respective projections on the third auxiliary plane (AP_3) from the reference line O_3X_3.

In the third auxiliary plane (AP_3), both the given planes appear as edge views $\mathbf{b_3c_3a_3}$ and $\mathbf{e_3d_3f_3}$. The angle between these edge views gives the required true angle (106.8^0).

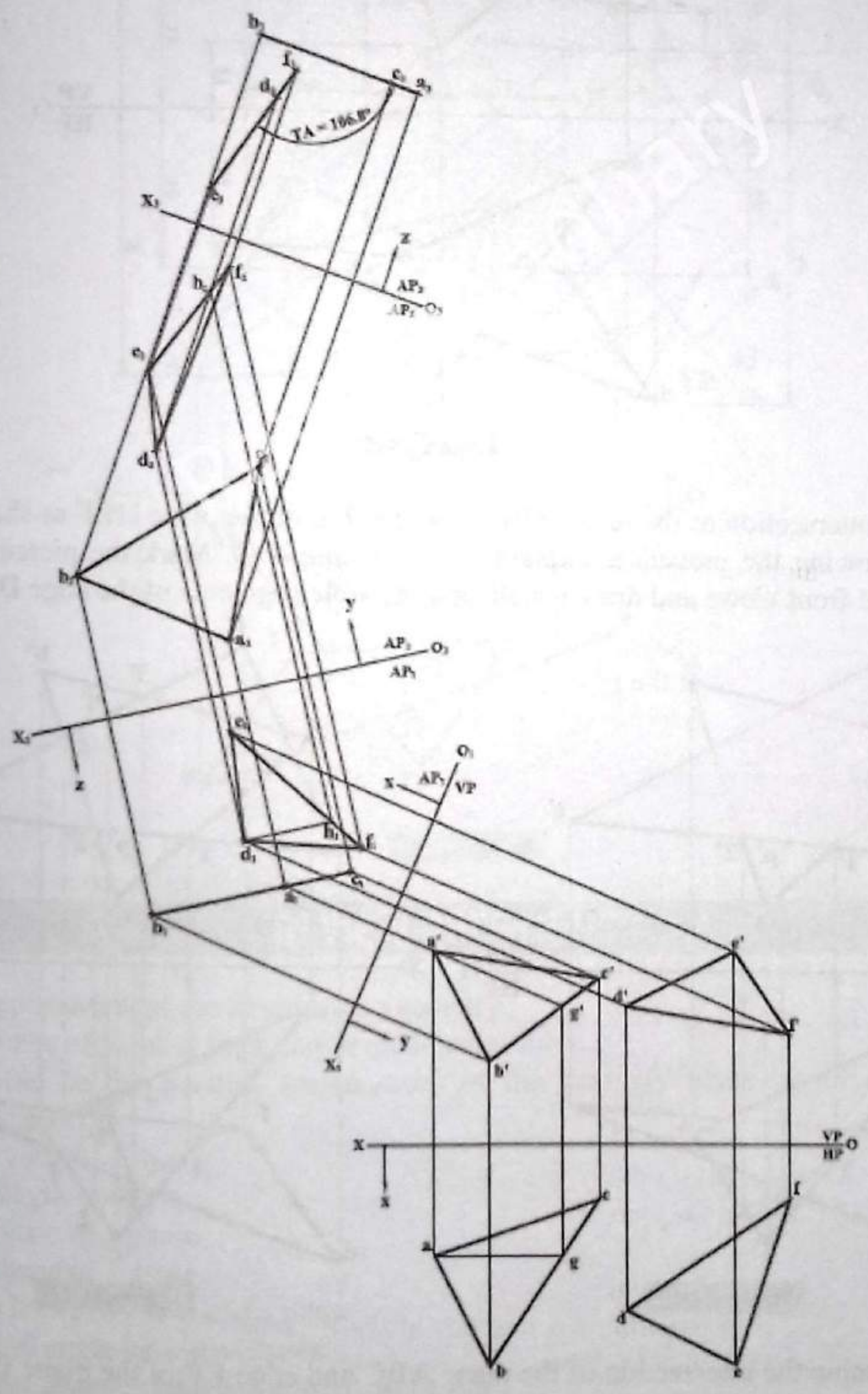


Figure E4.12a

Example 4.13

Complete the intersection between the planes **ABC** and **DEF** shown in *Figure E4.13.*

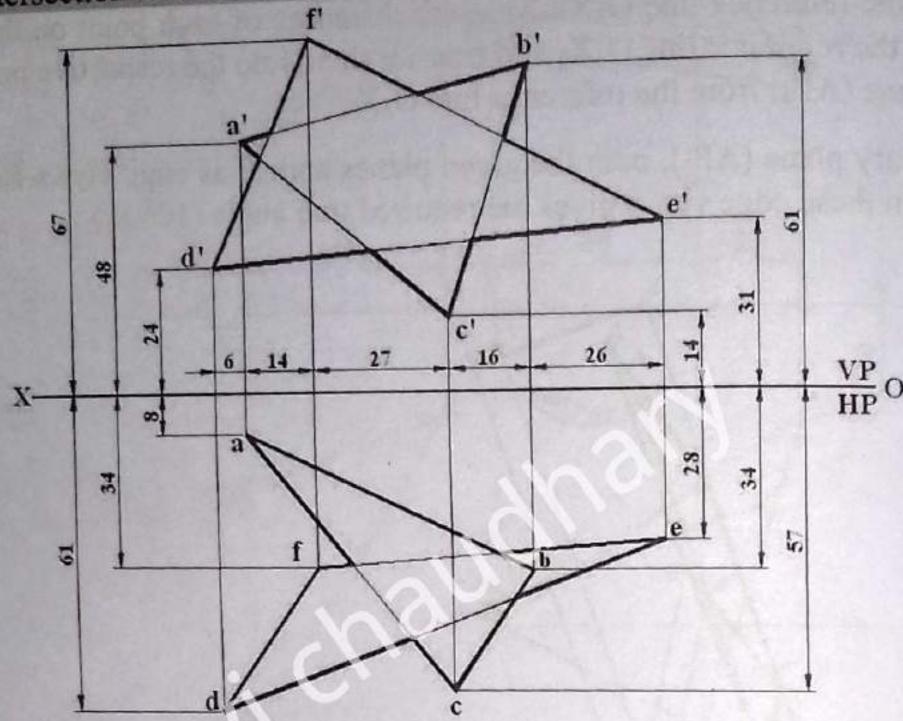


Figure E4.13

Solution

Determine the intersection of the plane **ABC** and edge **DE** of the plane **DEF** as shown in *Figure E4.13a* by following the procedure explained in *Example 4.10*. Mark the piercing point P on both the top and front views and draw visible and invisible segments of the edge **DE**.

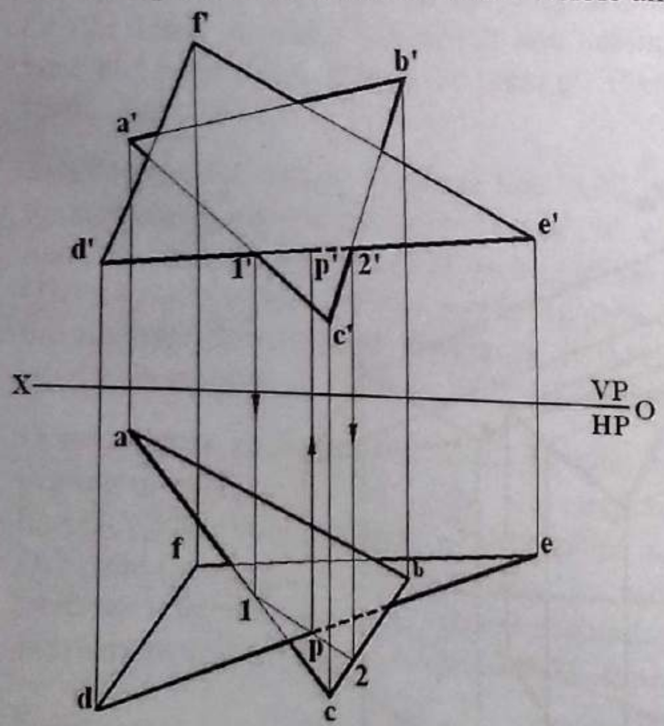


Figure E4.13a

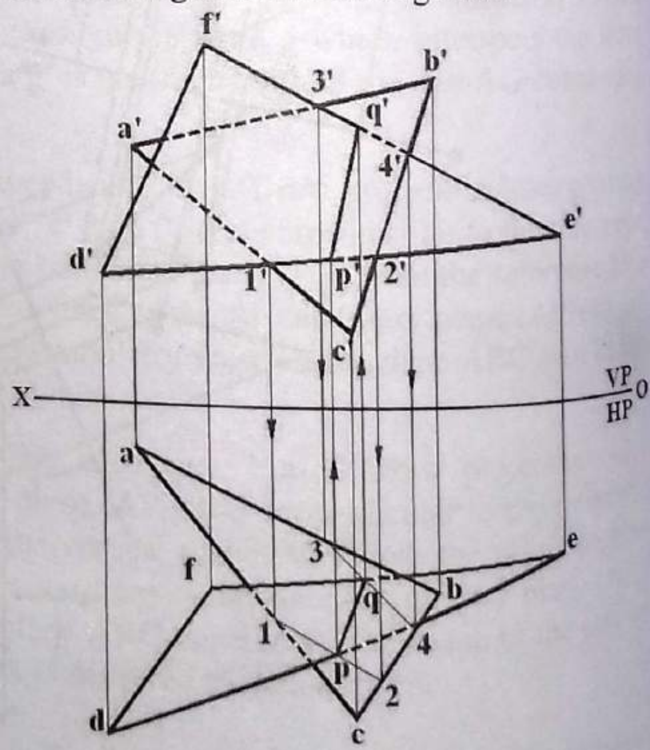


Figure E4.13b

Similarly determine the intersection of the plane **ABC** and edge **EF** of the plane **DEF** as shown in *Figure E4.13b*. Mark the piercing point **Q** on both the top and front views and draw visible and invisible segments of the edge **EF**.

Alternative Method (Auxiliary View Method)

Draw edge view $a_1b_1c_1$ of the plane **ABC** as shown in *Figure E4.13c*. Mark the intersection of the edge d_1e_1 and the edge view $a_1b_1c_1$ as p_1 and that of the edge e_1f_1 and the edge view $a_1b_1c_1$ as q_1, respectively. Project back the points p_1 and q_1 towards front view and then top view to complete the intersection.

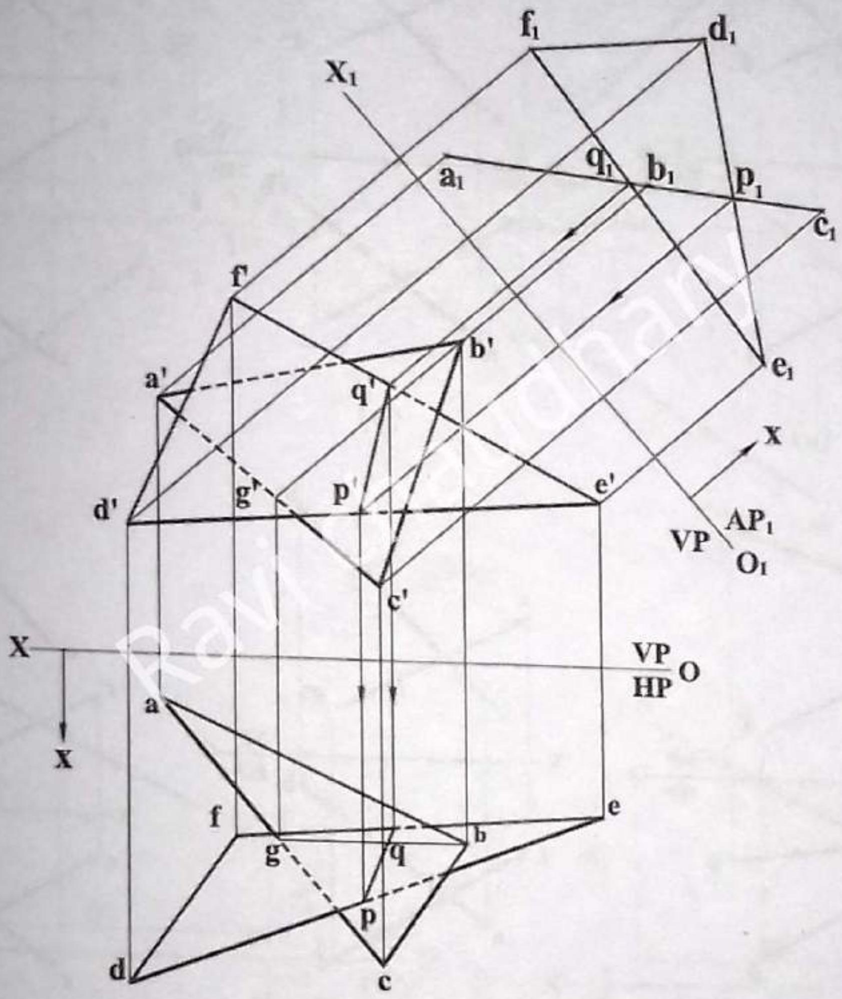


Figure E4.13c

REVIEW QUESTIONS

1. List the applications of the descriptive geometry.
2. State the rules of parallel lines and perpendicular lines.
3. What should be the position (orientation) of the auxiliary plane to solve the following spatial problems?
   (a) point view of a line
   (b) true length of a line
   (c) edge view of a plane
   (d) true shape of a plane
   (e) angle between a line and a plane
   (f) dihedral angle between planes
   (g) shortest distance between a point and a line
   (h) shortest distance between two skew lines
   (i) true angle between two skew lines.

CHAPTER 5

MULTIVIEW DRAWINGS

5.1 Introduction

The shape and size of any object is described by a set of different views of the object observed by the observer from different positions and arranged in a systematic way by means of projections.

When projection lines are drawn from the various points on the contour of the object to meet a plane, the object is said to be projected on that plane. The figure thus obtained by joining, in correct sequence, the points at which the projection lines meet the plane is called the projection or view of the object.

5.2 Classification of Projections

The projection or view of any object on any plane is produced by the piercing points of the projection lines on the plane of projection. The projection methods are classified according to the orientation of the object with reference to the plane of projection and the direction of sight (or relation of the projection lines with the plane of projection). The different systems of projections are shown in *Figure 5.1*.

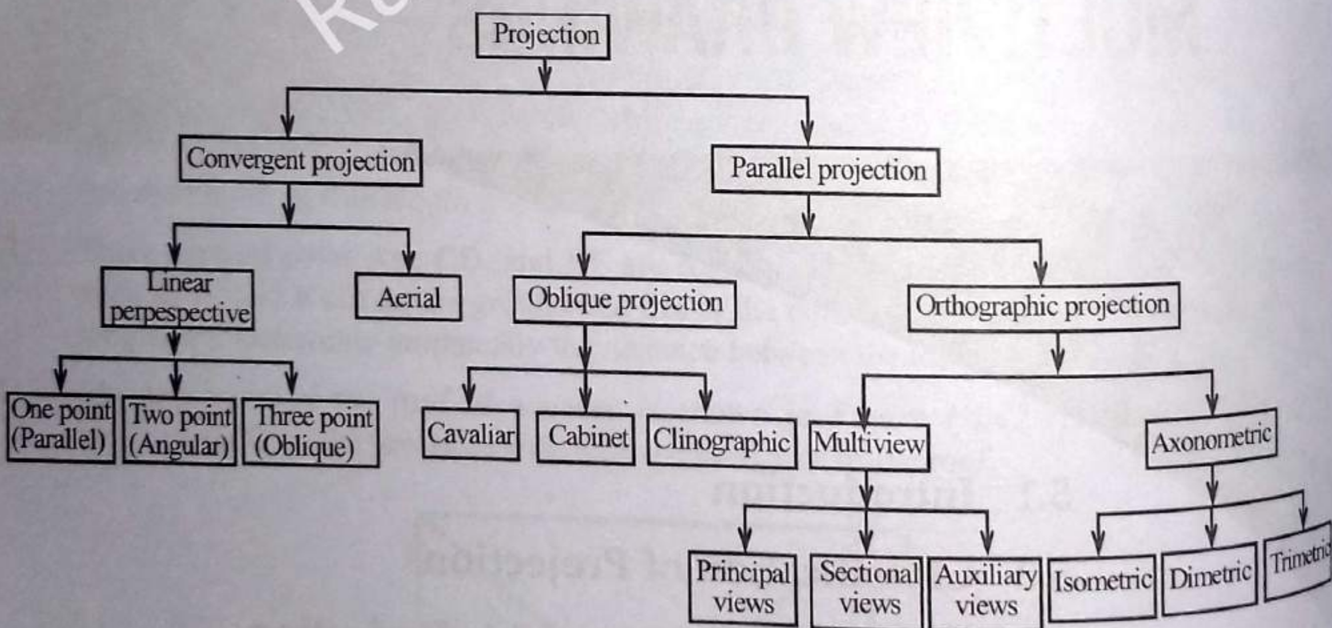


Figure 5.1: Classification of Projections

5.2.1 Convergent (Perspective) Projection

The projection of an object obtained on a plane when the projection lines converge to a point is called a convergent or perspective projection.

Perspective projections are not used for the preparation of working drawings of any part because it does not reveal the exact size and shape. However, it can be used to describe the natural view of the object to the non-technical persons.

5.2.2 Oblique Projection

The projection of an object produced by using parallel projection lines that make some angle other than 90^0, with the plane of projection is called an oblique projection. In this type of projection, one face of the object is kept parallel to the projection plane and the projection lines are taken at an angle of 30^0, 45^0 or 60^0 to the projection plane.

As the projection plane is parallel to the front face of the object, the projection of the front face will be in true shape and size. The shape and size of other faces depends upon the inclination of the projection lines with the projection plane.

5.2.3 Orthographic Projection

The projection of an object obtained on a projection plane when the projection lines are parallel to each other and perpendicular to the projection plane. Normally object is placed such that its face is parallel to the plane of projection.

5.2.4 Axonometric Projection

A special type of orthographic projection when the object is turned and tilted such that all three faces of the object are inclined to the plane of projection is called an axonometric projection. A single axonometric view shows three dimensions of the object in one projection therefore it is also called a one-plane pictorial projection.

5.2.5 Multiview Drawings

Multiview projection is the method used by the engineers to describe the exact shape of an object with the help of two or more orthographic views. Combination of such views when systematically arranged in accordance with the universally recognized standard provides information about all three dimensions: length, breadth and height of any object.

By projecting any object in three principal planes of projection (VP, HP and PP), three principal views (respectively front view, top view and side view) can be obtained. But the number of views required to describe the object completely depends upon the complexity of the object. Simple cylindrical objects can be described with a single view, while some object may need two views and complex object may need all three views.

If the object has oblique surfaces, auxiliary views should also be used with the principal views. Similarly, if the object has complex internal features, sectional views should also be used.

5.3 Systems of Orthographic Projection

5.3.1 First Angle Projection

In the first angle system of projection, the object is assumed to be placed in the first angle (or quadrant) i.e. above the HP and in front of the VP. Profile plane is considered at the right side if the projection of the object from the left side is to be made while it is considered at the left side if the projection of the object from the right side is to be made. In this projection, the object lies in between the observer and the projection planes.

Relative positions of the principal planes for the top, front and left side view of the object for the first angle projection is shown in *Figure 5.2*. To convert it into two-dimensional form, horizontal plane (HP) is rotated about the reference line OX and the profile plane (PP) is revolved about the reference line OZ keeping the vertical plane fixed until both of these planes align with the VP to form a two dimensional layout as shown in *Figure 5.3*.

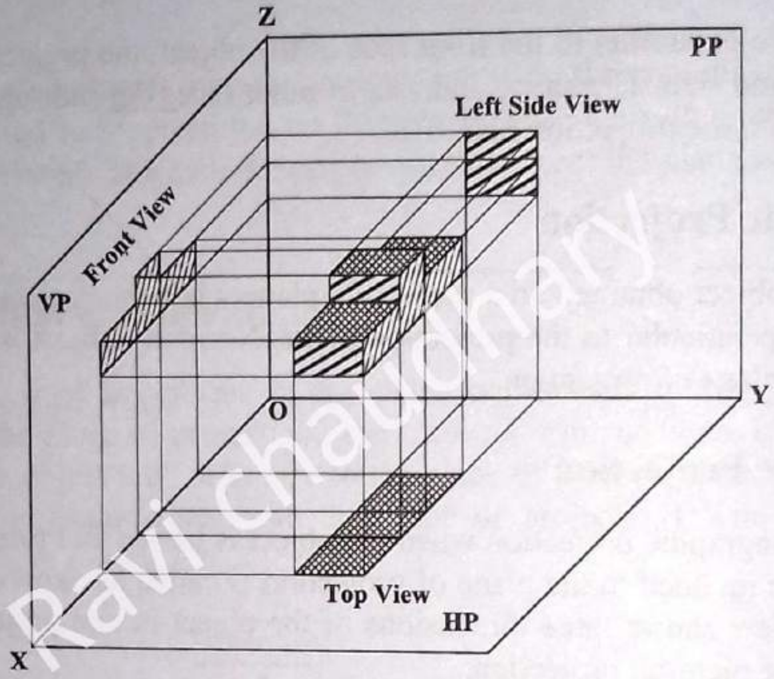


Figure 5.2: Relative Positions of Principal Planes for First Angle Projection with the Left Side View

Similarly, relative positions of the principal planes for the top, front and right side view of the object is shown in *Figure 5.4*. When it is converted into two dimensional form, the relative positions of the views will be as shown in *Figure 5.5*.

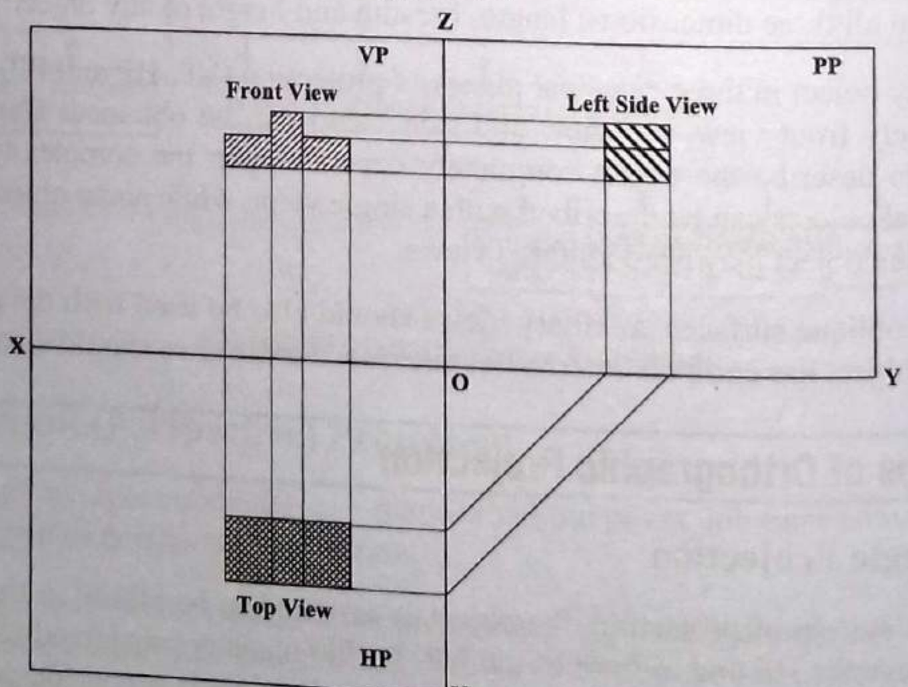


Figure 5.3: First Angle Projection with the Left Side View

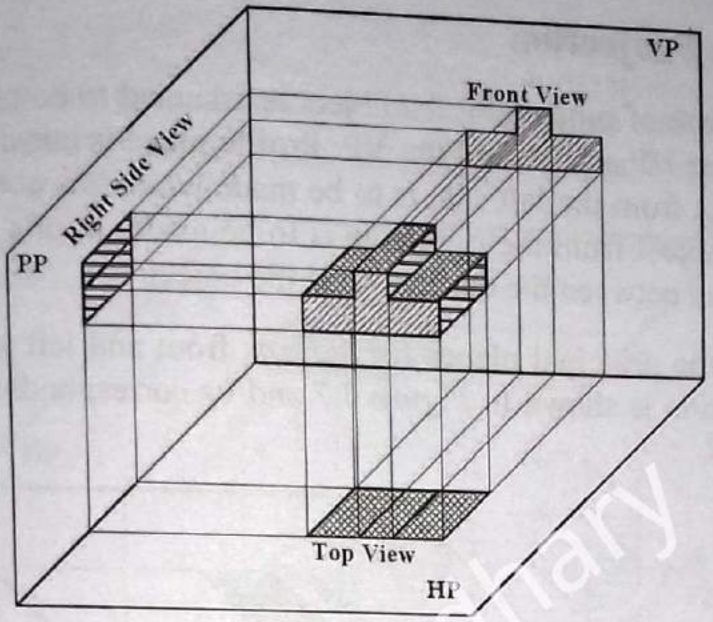


Figure 5.4: Relative Positions of Principal Planes for First Angle Projection with the Right Side View

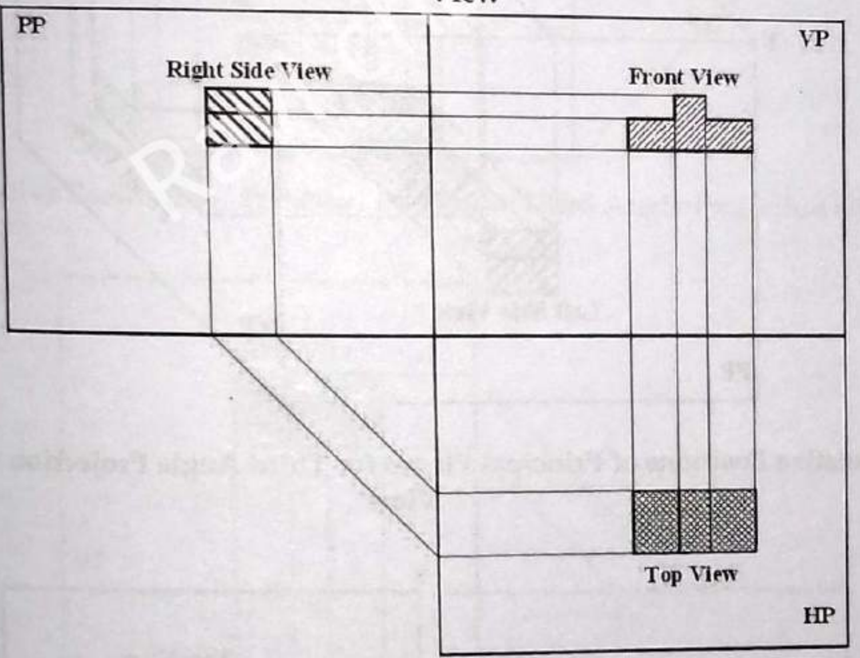


Figure 5.5: First Angle Projection with the Right Side View

When a drawing is made by following the first angle system of projection, it is specified by the standard symbol shown in *Figure 5.6.*

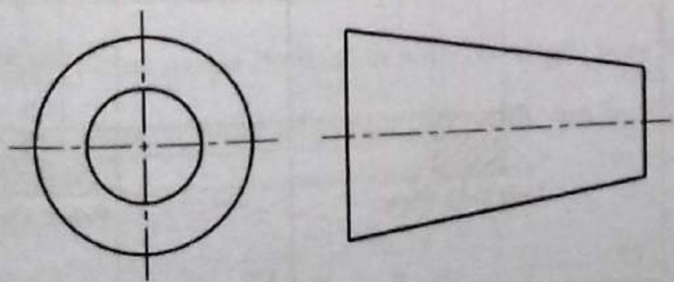


Figure 5.6: Symbol for First Angle Projection

5.3.2 Third Angle Projection

In the third angle system of projection, the object is assumed to be placed in the third angle (or quadrant) i.e. below the HP and behind the VP. Profile plane is considered at the left side if the projection of the object from the left side is to be made while it is considered at the right side if the projection of the object from the right side is to be made. In this projection, the transparent projection planes lies in between the observer and the object.

Relative positions of the principal planes for the top, front and left side view of the object for the third angle projection is shown in *Figure 5.7* and its corresponding two dimensional layout is shown in *Figure 5.8.*

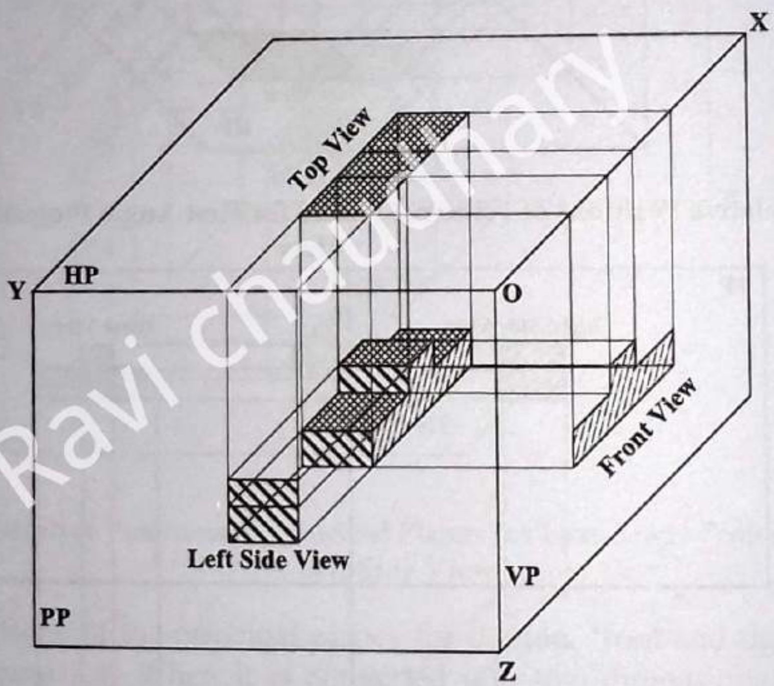


Figure 5.7: Relative Positions of Principal Planes for Third Angle Projection with the Left Side View

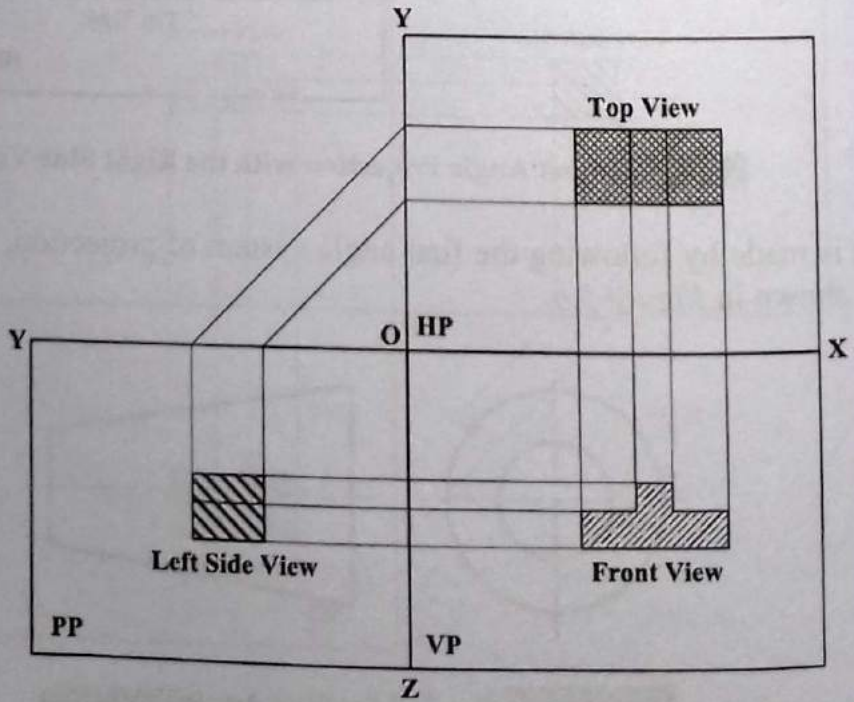


Figure 5.8: Third Angle Projection with the Left Side View

Similarly, relative positions of the principal planes for the top, front and right side view of the object for the third angle projection is shown in *Figure 5.9* and its corresponding two dimensional layout is shown in *Figure 5.10*.

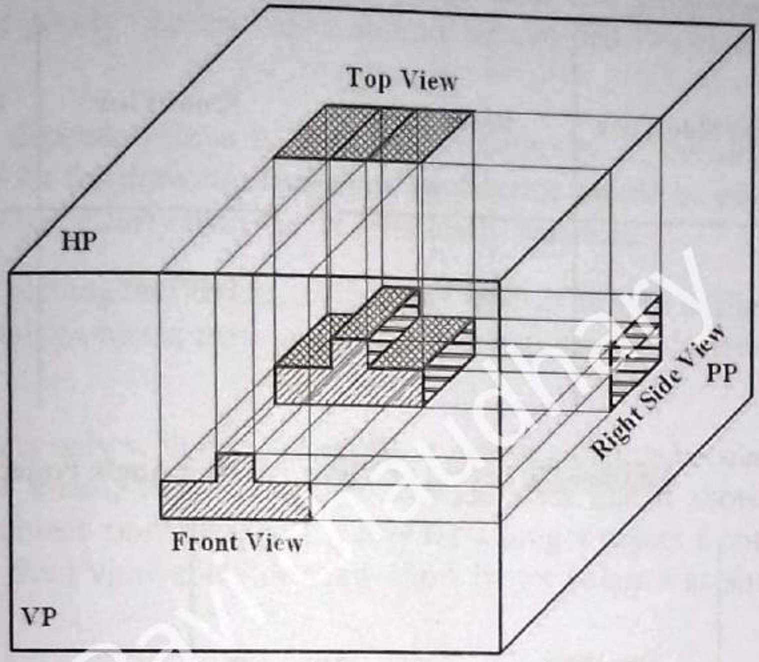


Figure 5.9: Relative Positions of Principal Planes for Third Angle Projection with the Right Side View

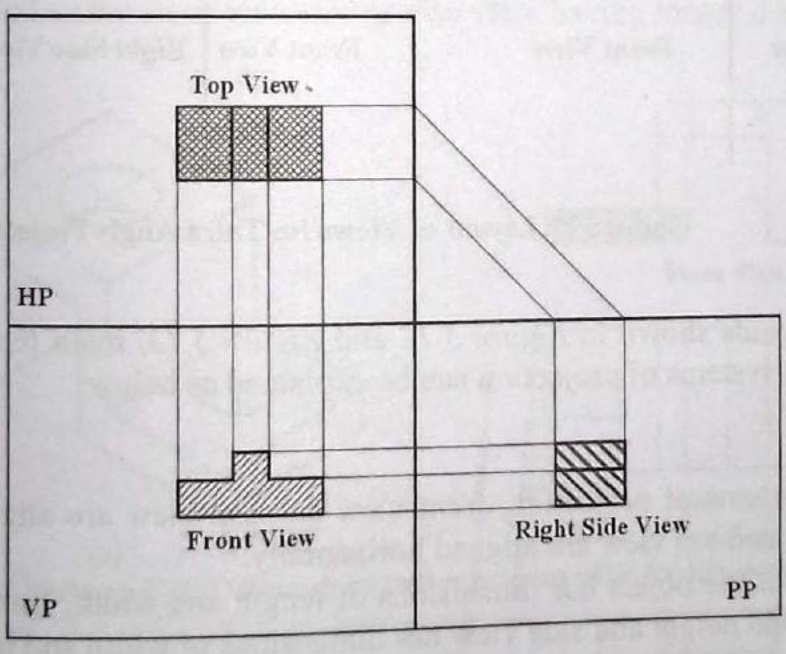


Figure 5.10: Third Angle Projection with the Right Side View

The standard symbol for the third angle system of projection is shown in *Figure 5.11*.

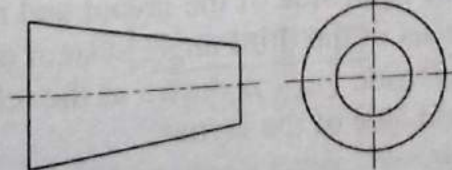


Figure 5.11: Symbol for Third Angle Projection

5.4 Comparison between First Angle and Third Angle Projections

Layouts of the views for the first angle and third angle systems of projection are shown in *Figure 5.12* and *Figure 5.13.*

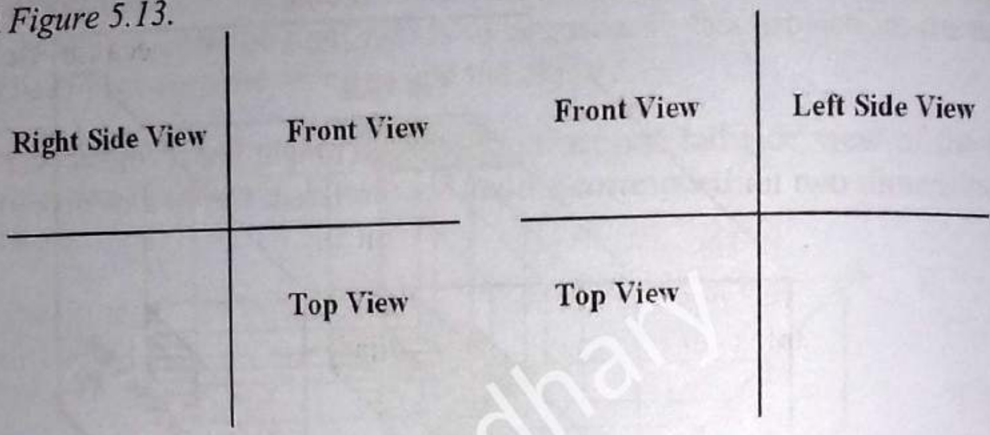


Figure 5.12: Layout of Views for First Angle Projection

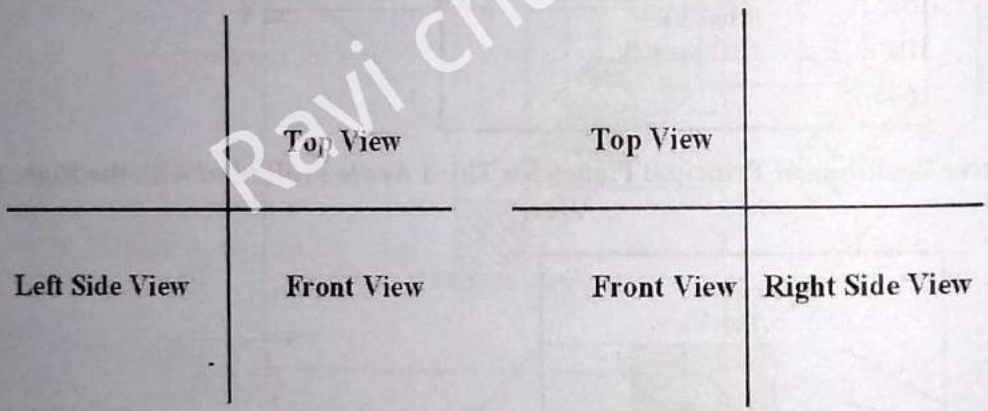


Figure 5.13: Layout of Views for Third Angle Projection

Comparing the layouts shown in *Figure 5.12* and *Figure 5.13,* main features of the first angle and the third angle systems of projection can be explained as below.

Similarities

- In both systems of projection, front view and top view are aligned vertically whereas front view and top view are aligned horizontally.
- Top view of the object has dimensions of length and width, front view has dimensions of length and height and side view has dimensions of width and height.

Differences

- In the first angle system of projection, top view is drawn at the bottom of the layout, left side view is drawn at the right side of the layout and right side view is drawn at the left side of the layout whereas in the third angle system of projection, top view is drawn at the top of the layout, left side view is drawn at the left side of the layout and right side view is drawn at the right side of the layout.

5.5 Selection of Views

The proper selection of views is very important in engineering drawing. Only necessary and least number of views should be selected which gives clear and complete description of the object. Unnecessary and poorly chosen views should be avoided because it may confuse the reader.

The selection of views depends primarily upon the complexity of the object and sometimes upon the space available for the drawing. However, preference should be given to a set of views that describes an object most clearly than that is artistically balanced.

Simpler objects such as rectangular blocks, cylindrical objects can be sufficiently described by two views whereas complex objects may need auxiliary and sectional views in addition to the three principal views.

While describing an object only with two views, there is also a choice between the top view and side view. Among these views, view should be selected such that it shows the characteristic shape or contour of the object more clearly. Usually for a longer object front view and top view and for a shorter object front view and side view show better balance as shown in *Figure 5.14* and *Figure 5.15*.

Another important decision that should be taken while making multiview drawing is the selection of the front view. The view showing the characteristic contour of the object more clearly should be selected as the front view. Otherwise view having longer dimension should be taken as the front view.

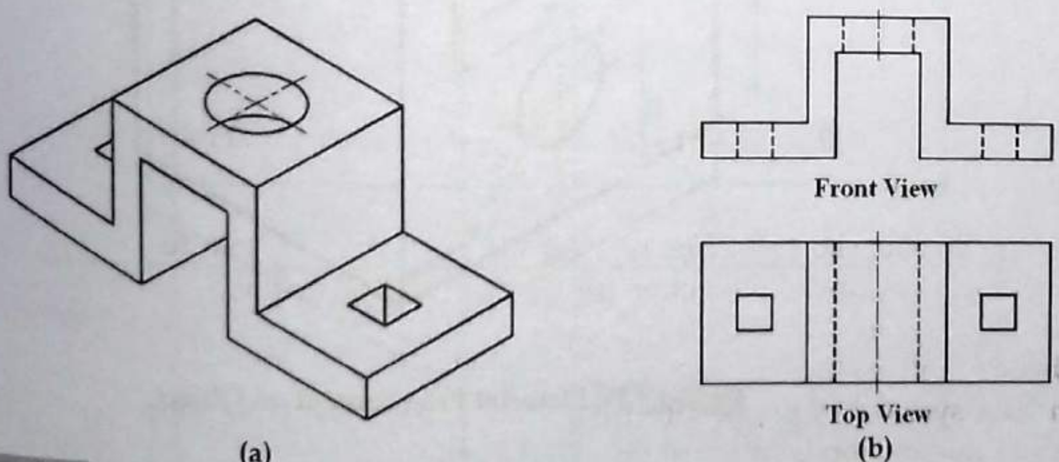


Figure 5.14: Front View and Top View show better presentation for the longer objects

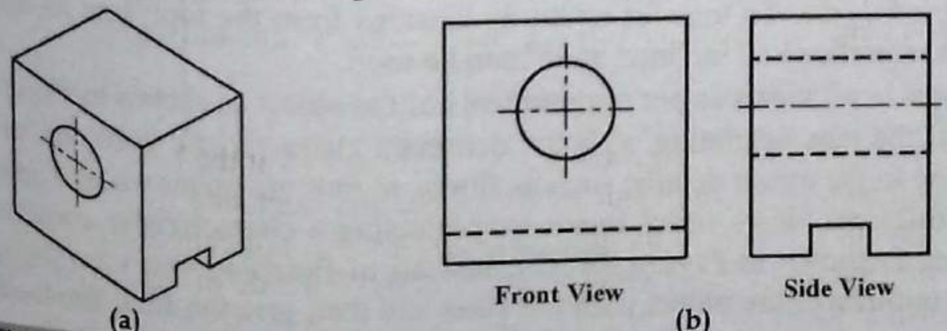


Figure 5.15: Front View and Side View show better presentation for the shorter objects

5.6 Procedure for Making a Multiview Drawing

The order of constructing different features of the object is important because speed and accuracy of a drawing depend upon the sequence used in laying down the lines. In general, following basic steps should be noted while making the multiview drawing of an object.

- Determine the overall dimensions of the required views. Select a suitable scale so that the required views can be accommodated on the drawing sheet.
- Decide the system of projection to be used and decide the combination of views that will describe the object in better way. A free hand sketch may help to plan the arrangement views.

For example, consider an object shown in *Figure 5.16.* It has symmetrical left and right side sections. Hence the object can be described by the combination of top view, front view and left side or right side view. If the system of projection is chosen as the first angle, the layout for the top view, front view and left side view will be as shown in *Figure 5.17(a).*

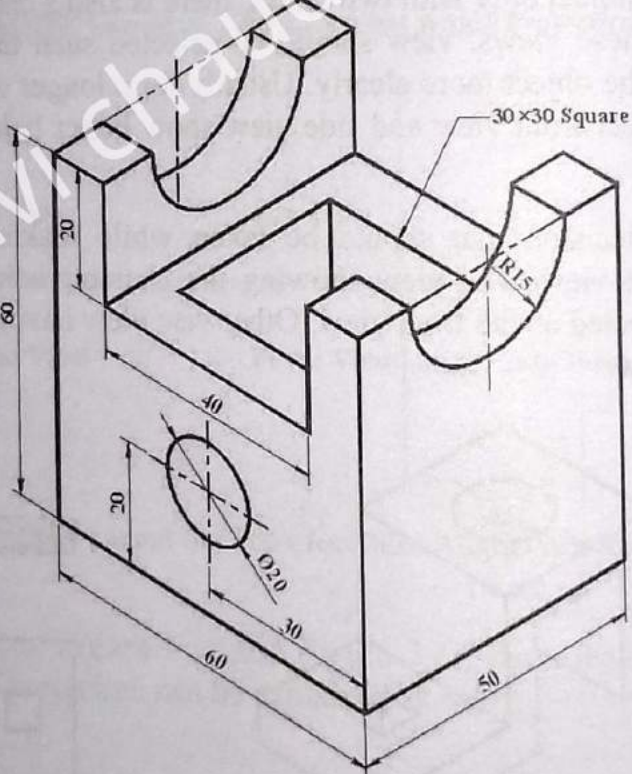


Figure 5.16: Pictorial Projection of an Object

- Make rectangular blocks for each view according to the overall dimensions of the object as shown in *Figure 5.17(b).* To transfer width dimensions from the top view to the side view or vice versa, a reference line inclined at 45^0 can be used.
- Draw center lines in all views as per requirements of the object as shown in *Figure 5.17(c).*
- Draw details of the part beginning with the dominant characteristic contours of the object and then precede to the minor details, such as fillets, rounds etc as shown in *Figure 5.17 (d).* Construct the different views simultaneously, projecting a characteristic contour from one view to another as shown in *Figure 5.17(e),* instead of finishing one view before starting another. Draw outlines of the object with fair lines and then give the final finished weight.
- Place the dimensions, specify the scale, provide titles and required explanatory notes by lettering as shown in *Figure 5.17(f).*

- Check the drawing carefully for its completeness in all aspects.

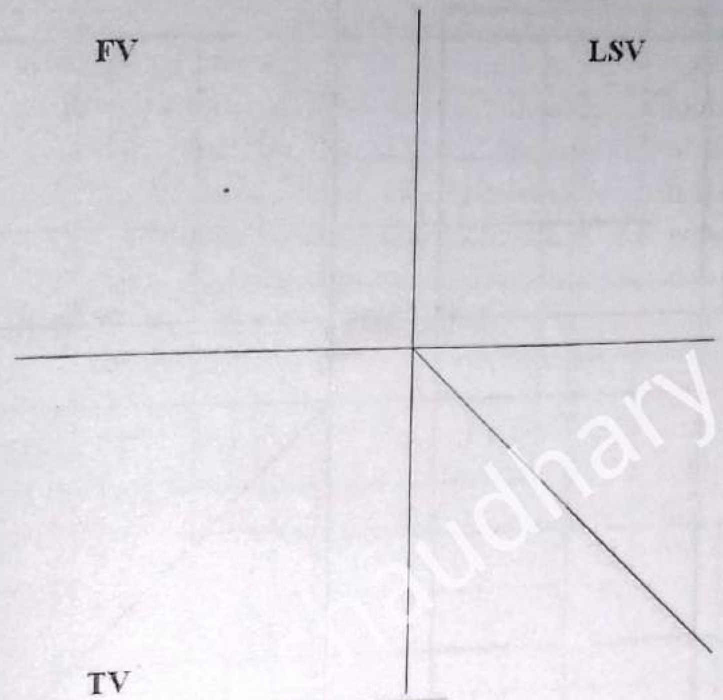


Figure 5.17(a)

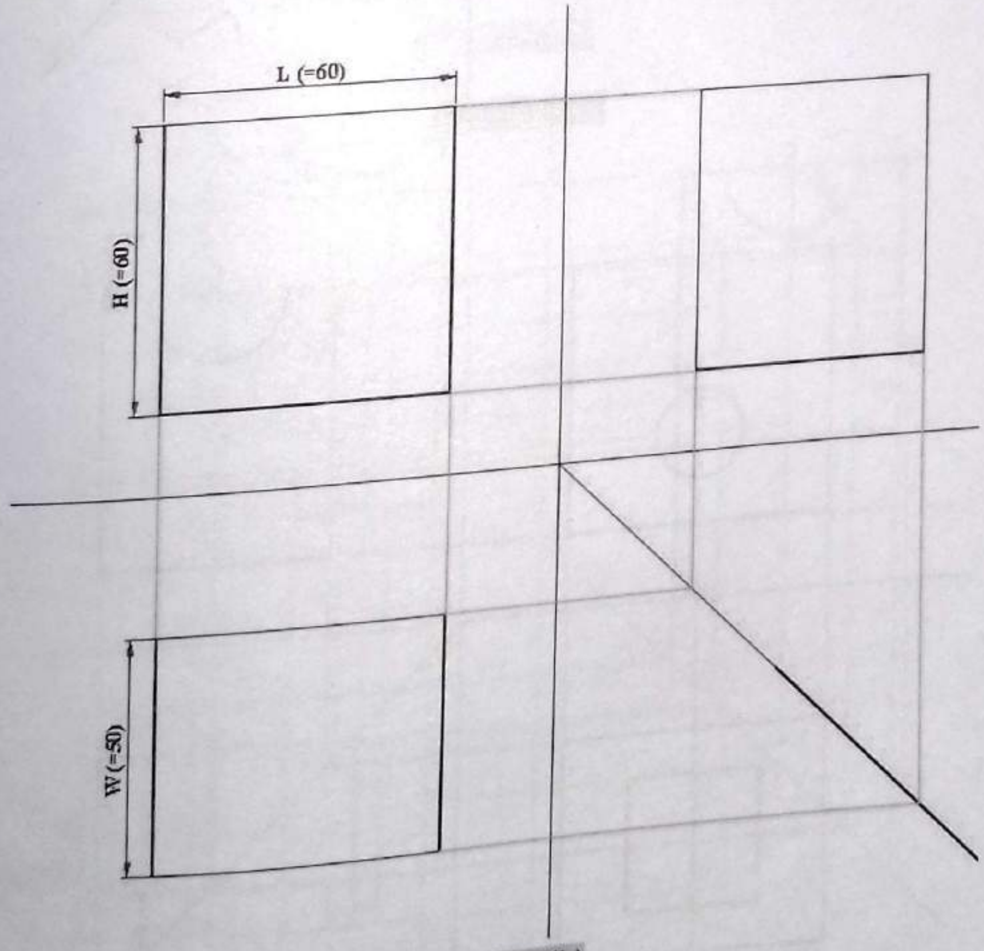


Figure 5.17(b)

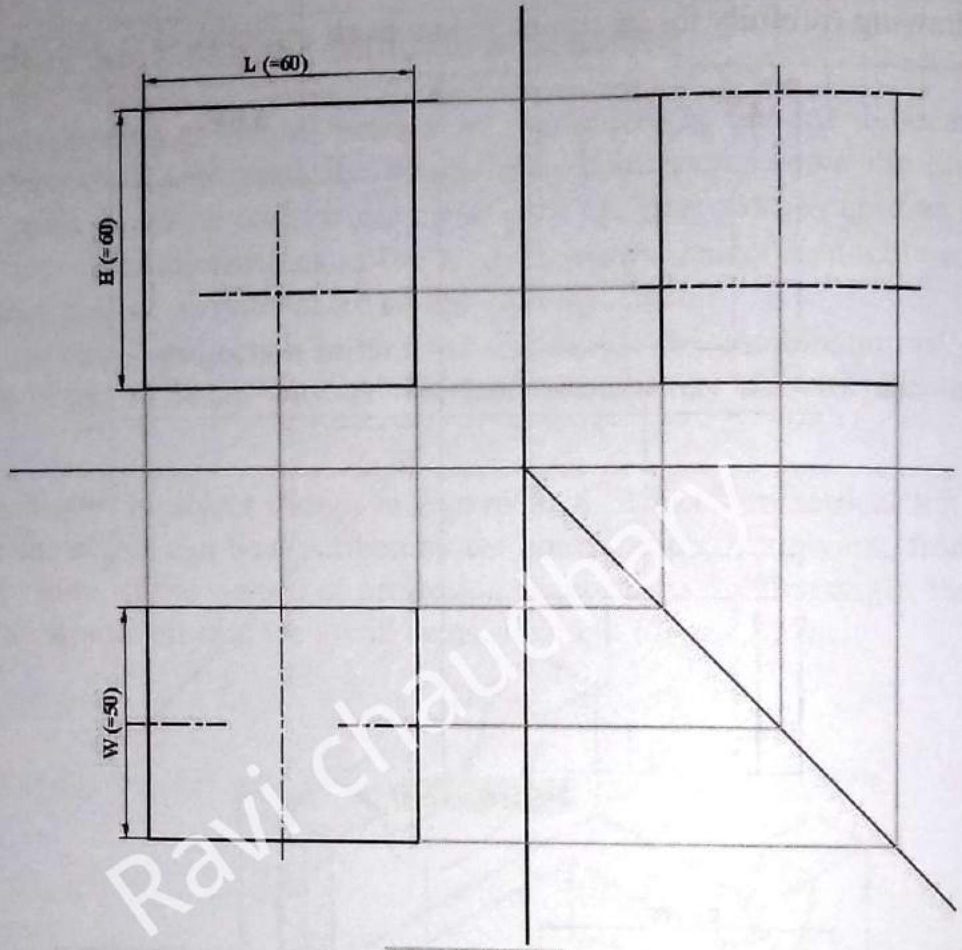


Figure 5.17(c)

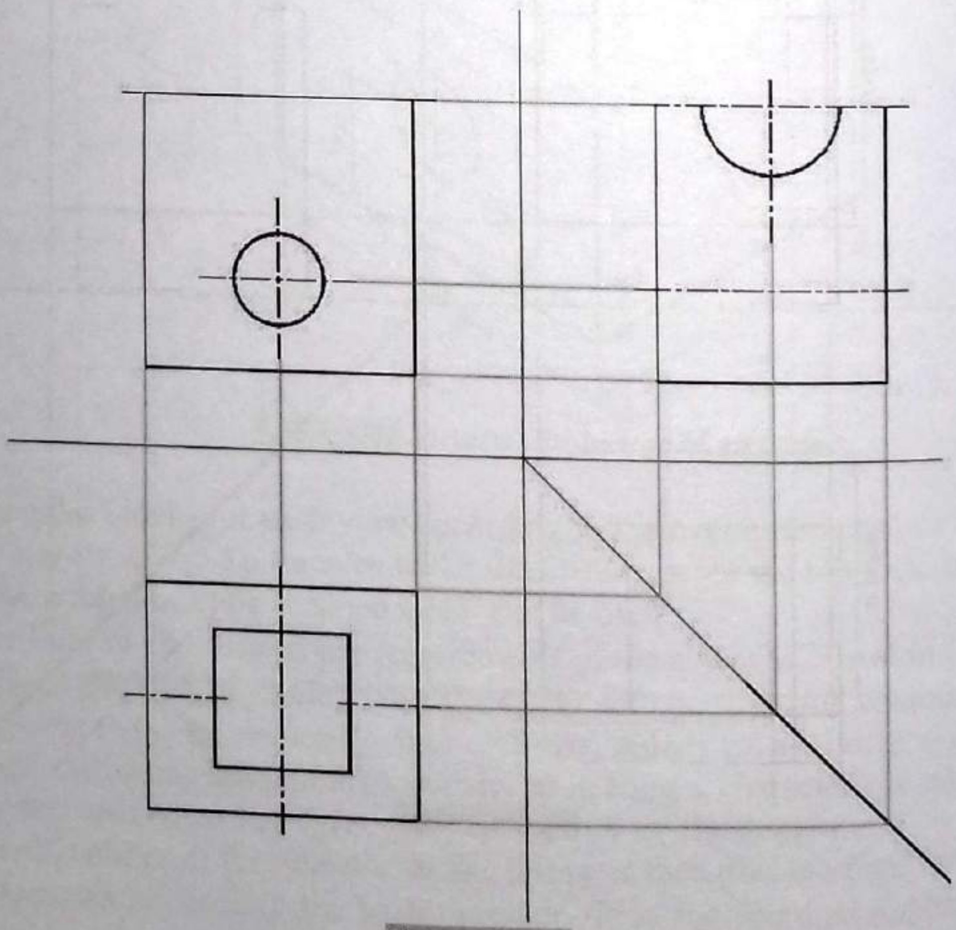


Figure 5.17(d)

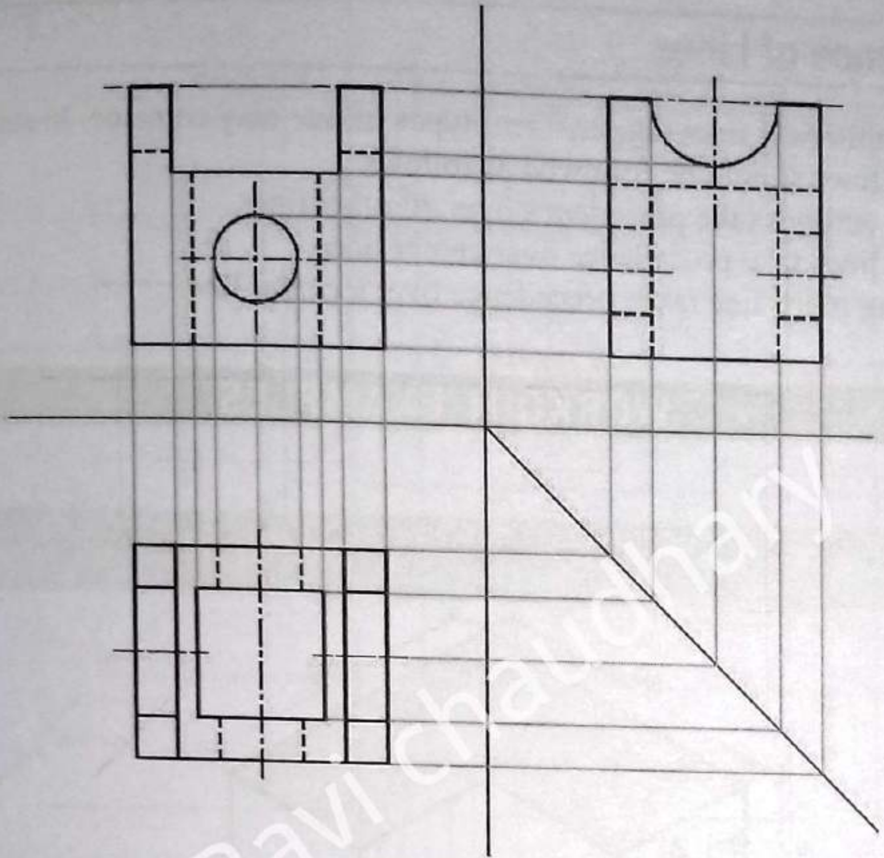


Figure 5.17(e)

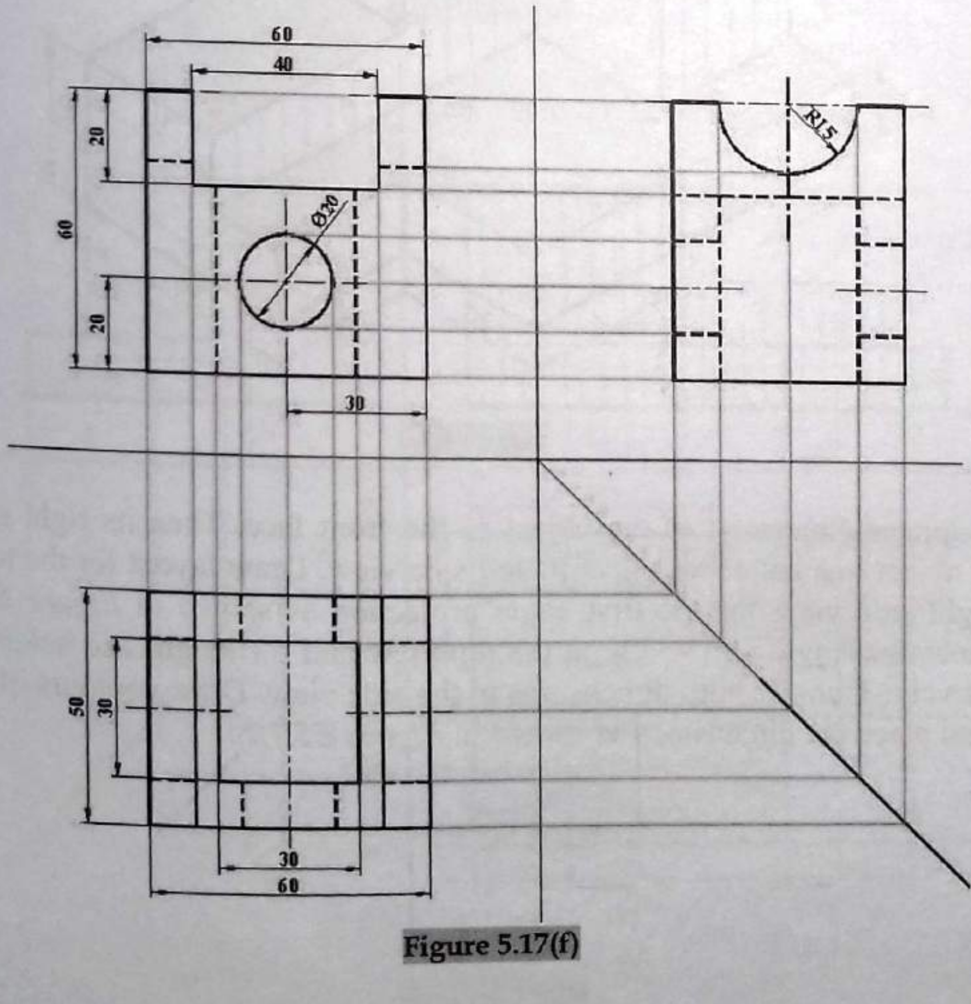


Figure 5.17(f)

5.7 Precedence of Lines

While making a multiview drawing, different types of line may coincide. In such circumstances, the precedence of lines should be followed as follows:

(a) Visible outlines take precedence over all other lines.

(b) Hidden lines take precedence over center lines.

(c) A cutting plane line takes precedence over a center line.

WORKOUT EXAMPLES

Example 5.1

Draw orthographic views of the object shown in *Figure E5.1*. Use the first angle system of projection.

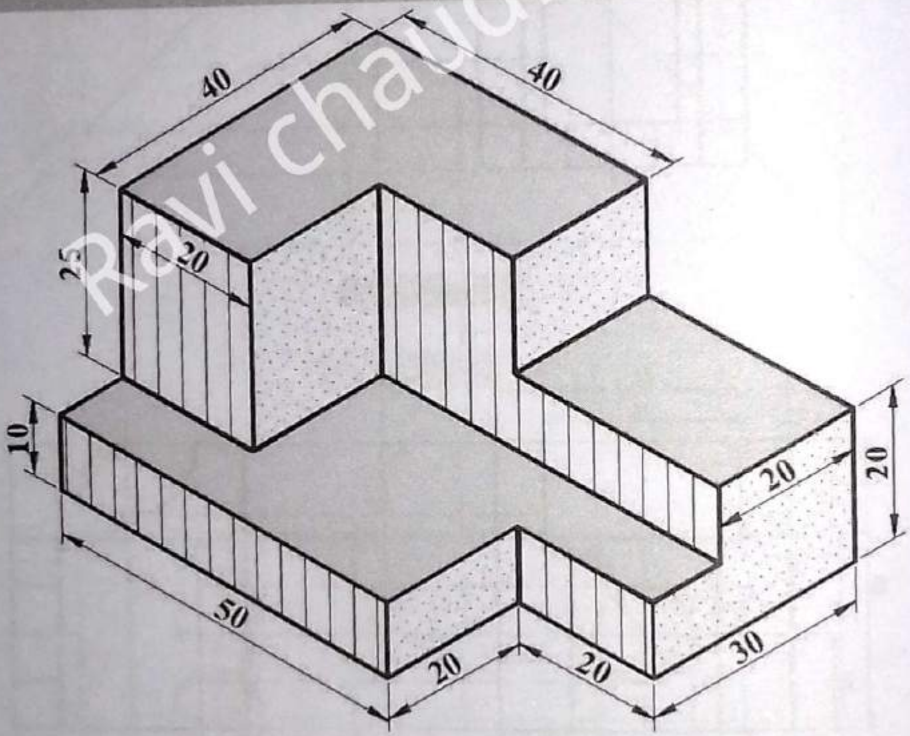


Figure E5.1

Solution

Assume the longer dimension of the object as the front face. Then its right side view will describe the object in a better way than its left side view. Draw layout for the top view, front view and right side view for the first angle projection as shown in *Figure E5.1(a)*. Draw projection lines for length and width on the top view and for length and height on the front view respectively. Transfer both dimensions to the side view. Draw contours of the object on each view and place the dimensions as shown in *Figure E5.1(b)*.

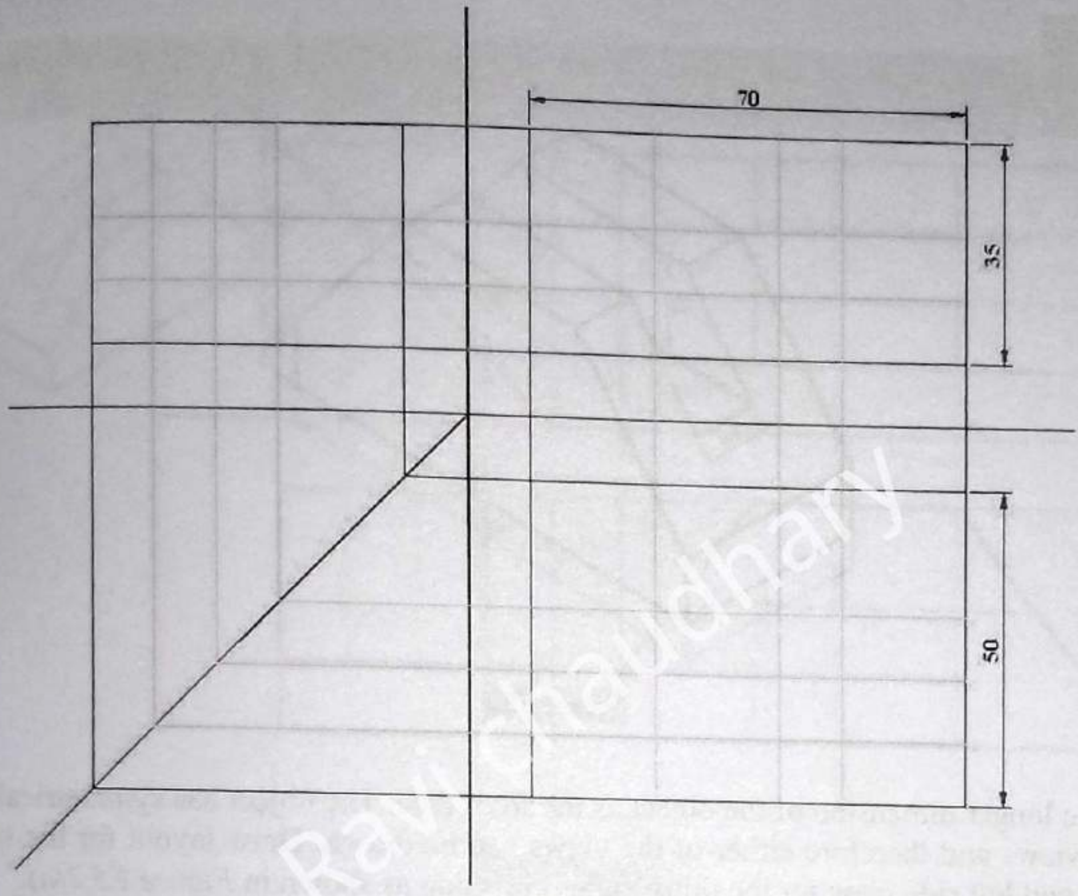


Figure E5.1(a)

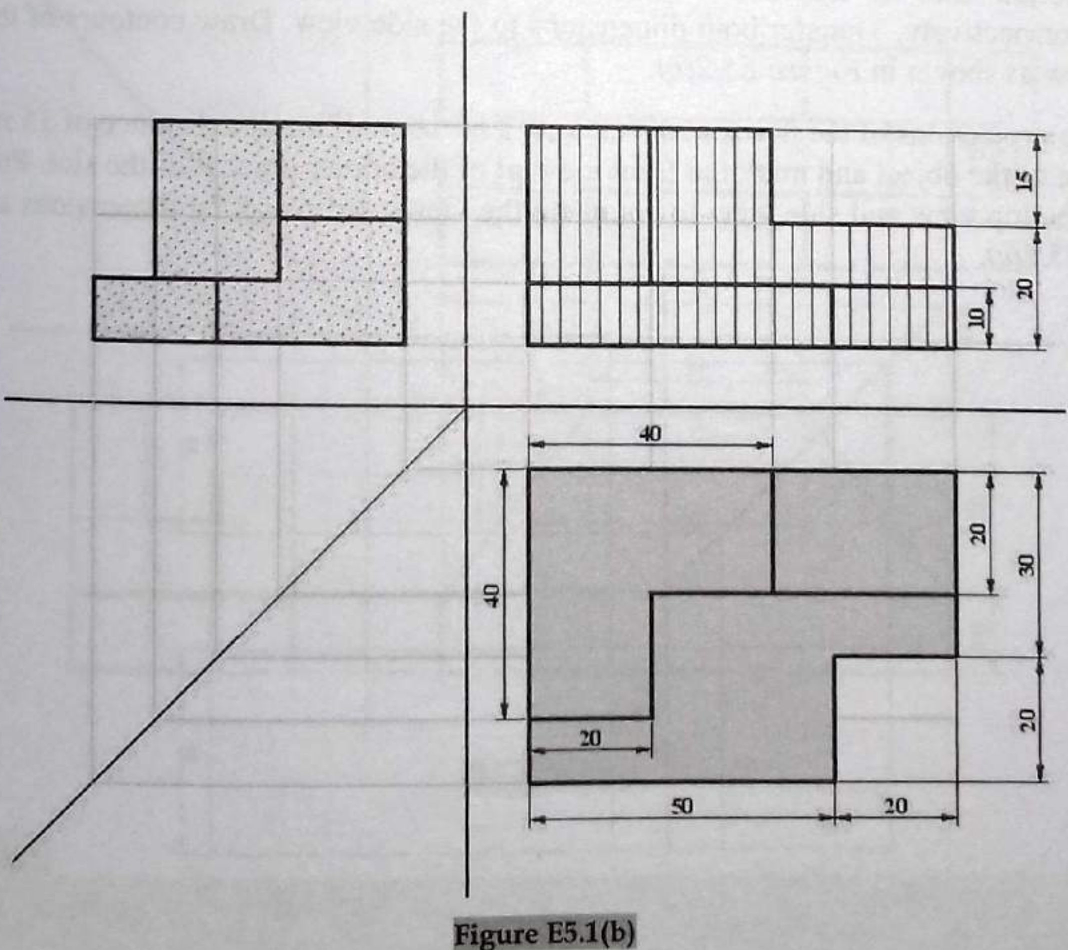


Figure E5.1(b)

Example 5.2

Draw orthographic views of the object shown in *Figure E5.2*. Use the third angle system of projection.

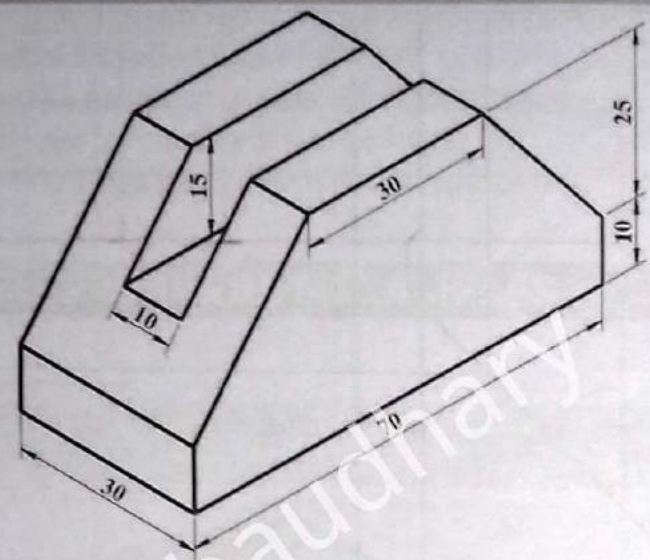


Figure E5.2

Solution

Assume the longer dimension of the object as the front face. The object has symmetrical left and right side views and therefore either of the views can be drawn. Draw layout for the top view, front view and left side view for the third angle projection as shown in *Figure E5.2(a)*.

Draw projection lines for length and width on the top view and for length and height on the front view respectively. Transfer both dimensions to the side view. Draw contours of the object on each view as shown in *Figure E5.2(b)*.

To draw the projections of the horizontal slot draw a horizontal line at a distance of 15 mm from the top edge of the object and mark the front view **p'** of the corner point **P** of the slot. Project the feature to the top view and side view to complete the views and place the dimensions as shown in *Figure E5.2(c)*.

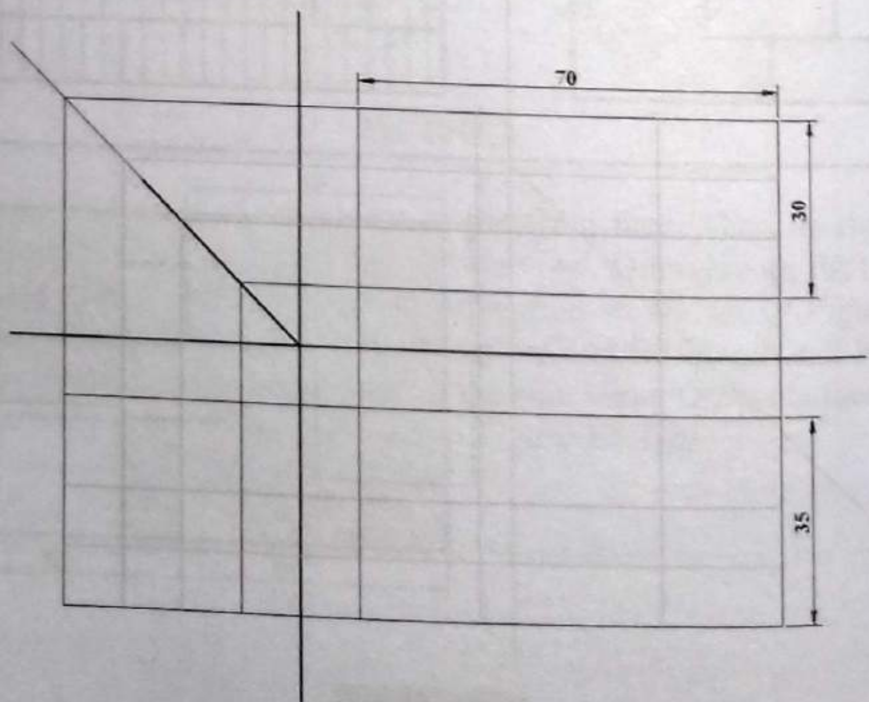


Figure E5.2(a)

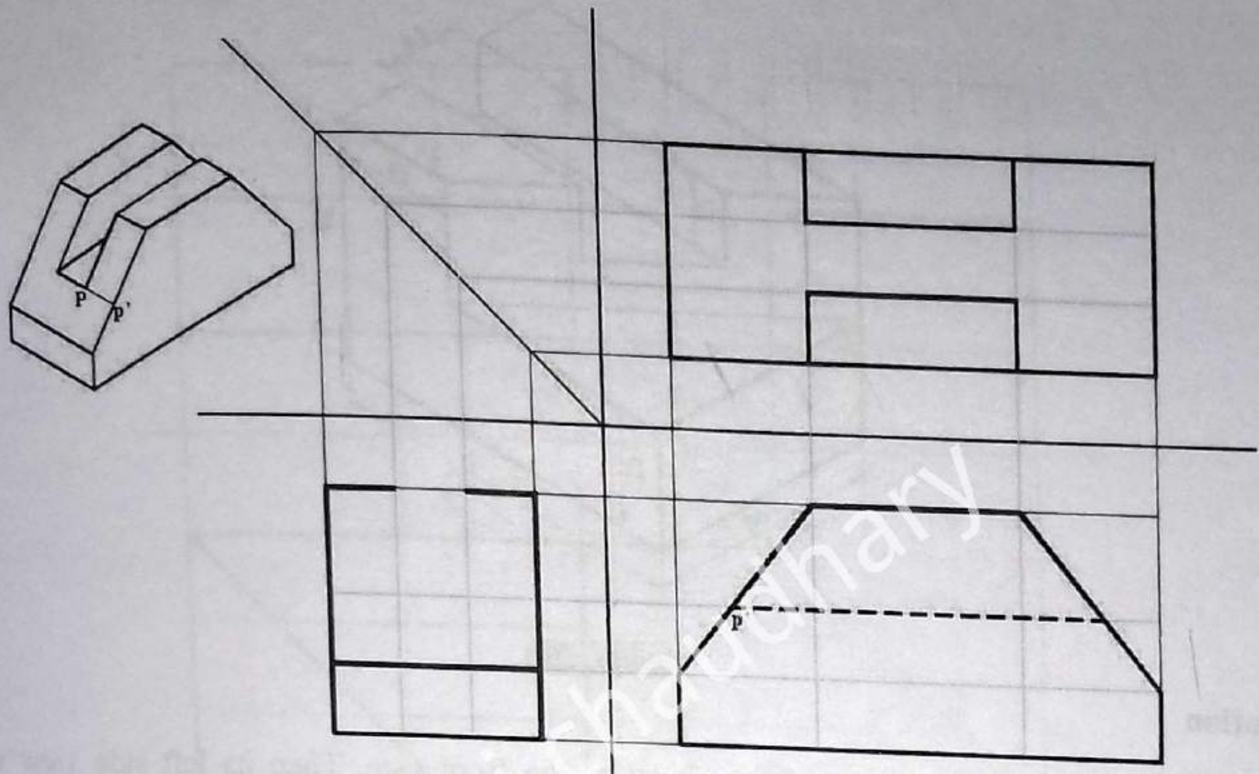


Figure E5.2(b)

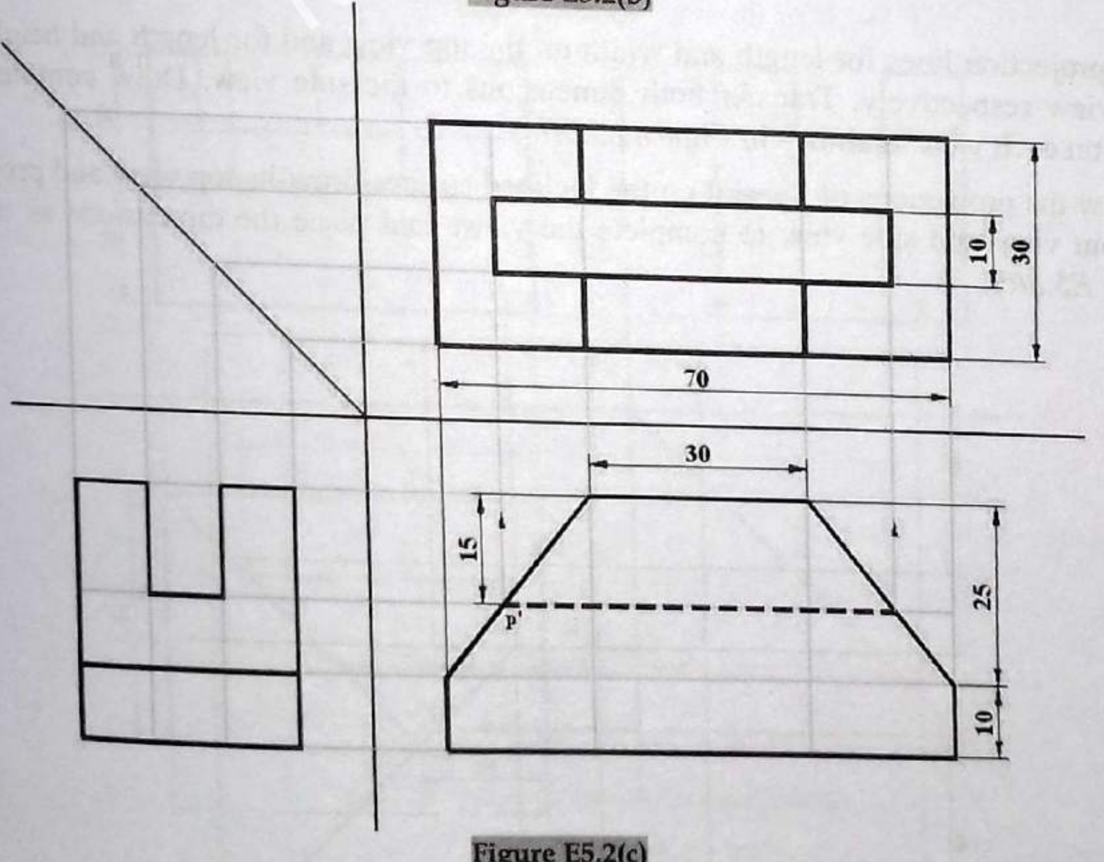


Figure E5.2(c)

Example 5.3

Draw orthographic views of the object shown in *Figure E5.3*. Use the first angle system of projection.

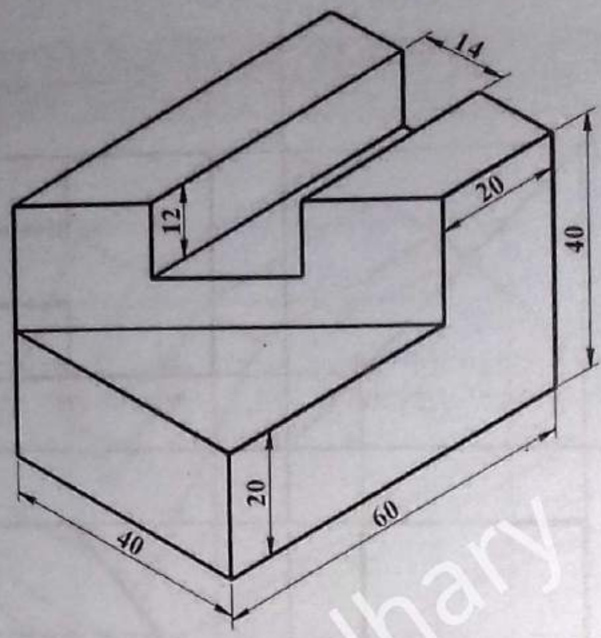


Figure E5.3

Solution

Assume the longer dimension of the object as the front face. Then its left side view will describe the object in a better way than its right side view. Draw layout for the top view, front view and left side view for the first angle projection as shown in *Figure E5.3(a).*

Draw projection lines for length and width on the top view and for length and height on the front view respectively. Transfer both dimensions to the side view. Draw contours of the object on each view as shown in *Figure E5.3(b).*

To draw the projections of the slot on the inclined surface draw its top view and project it to the front view and side view to complete the views and place the dimensions as shown in *Figure E5.3(c).*

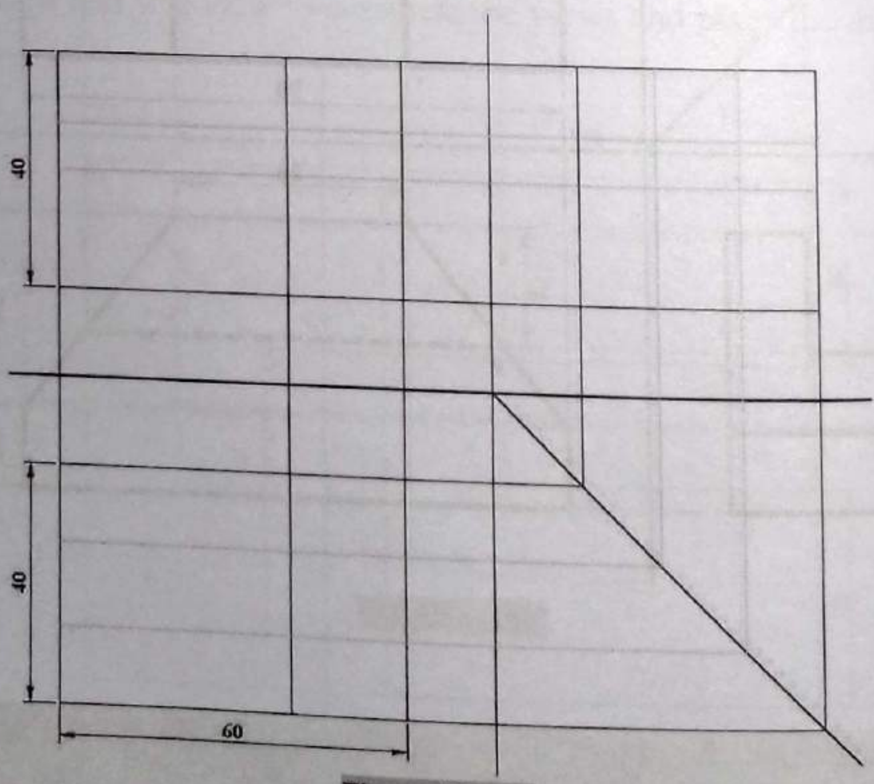


Figure E5.3(a)

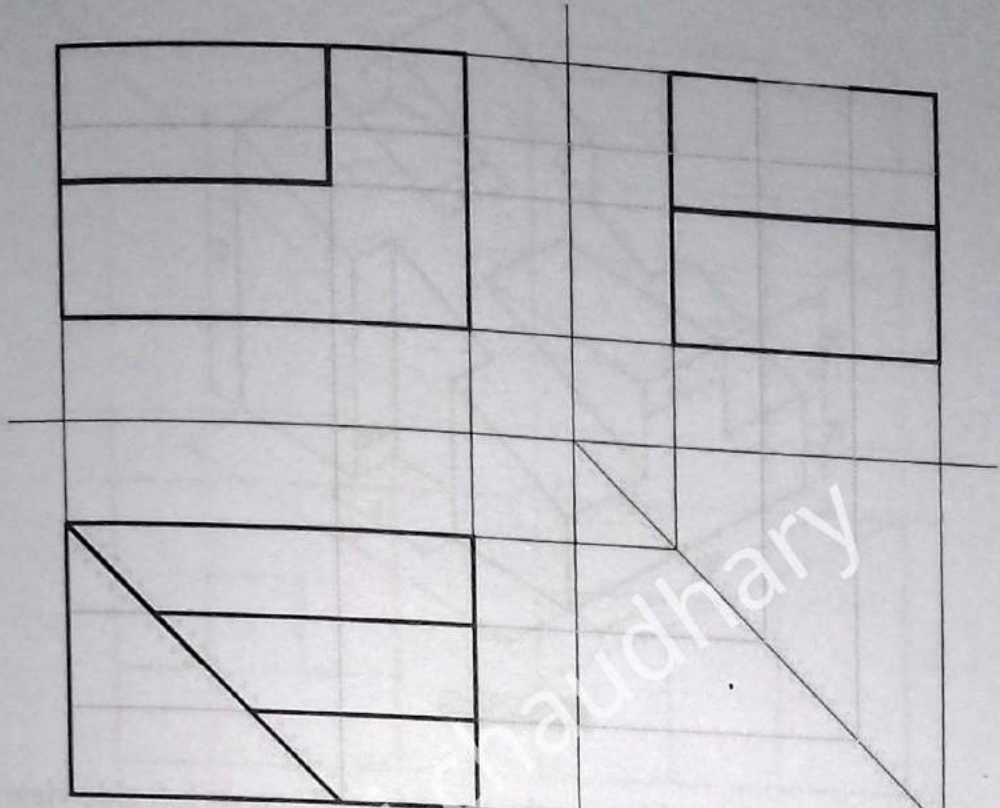


Figure E5.3(b)

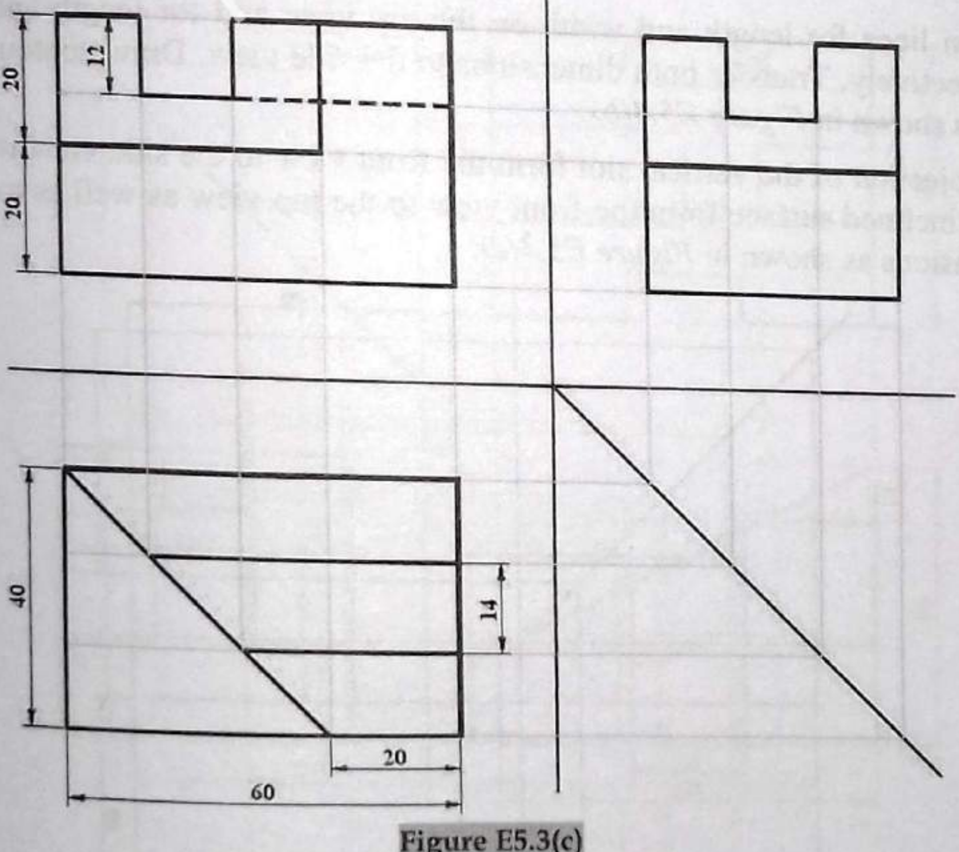


Figure E5.3(c)

Example 5.4

Draw orthographic views of the object shown in *Figure E5.4.* Use the third angle system of projection.

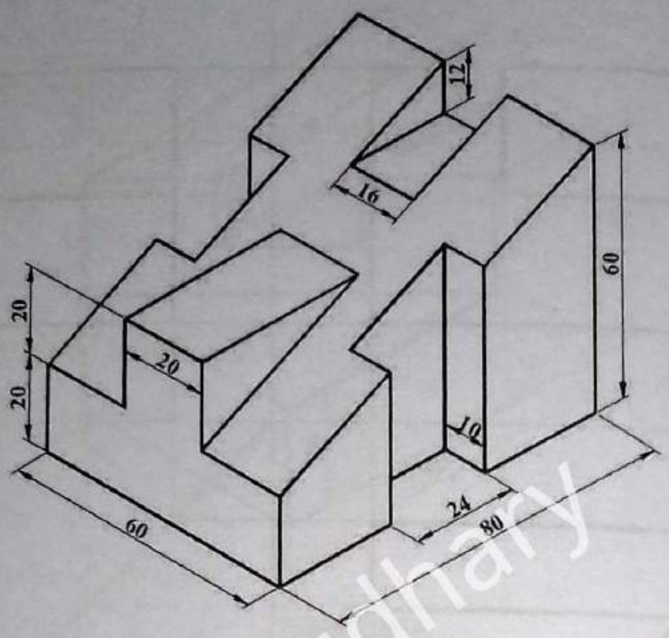


Figure E5.4

Solution

Assume the longer dimension of the object as the front face. Then its left side view will describe the object in a better way than its right side view. Draw layout for the top view, front view and left side view for the third angle projection as shown in *Figure E5.4(a)*.

Draw projection lines for length and width on the top view and for length and height on the front view respectively. Transfer both dimensions to the side view. Draw contours of the object on each view as shown in *Figure E5.4(b)*.

Transfer the projection of the vertical slot form the front view to the side view whereas transfer the features on inclined surface from the front view to the top view as well as to the side view. Place the dimensions as shown in *Figure E5.4(c)*.

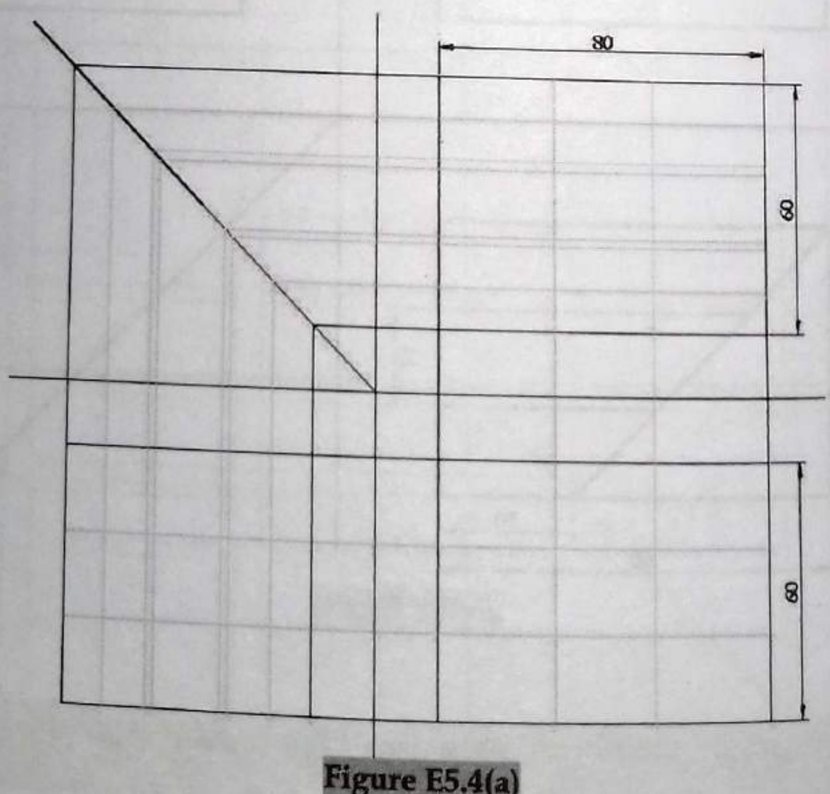


Figure E5.4(a)

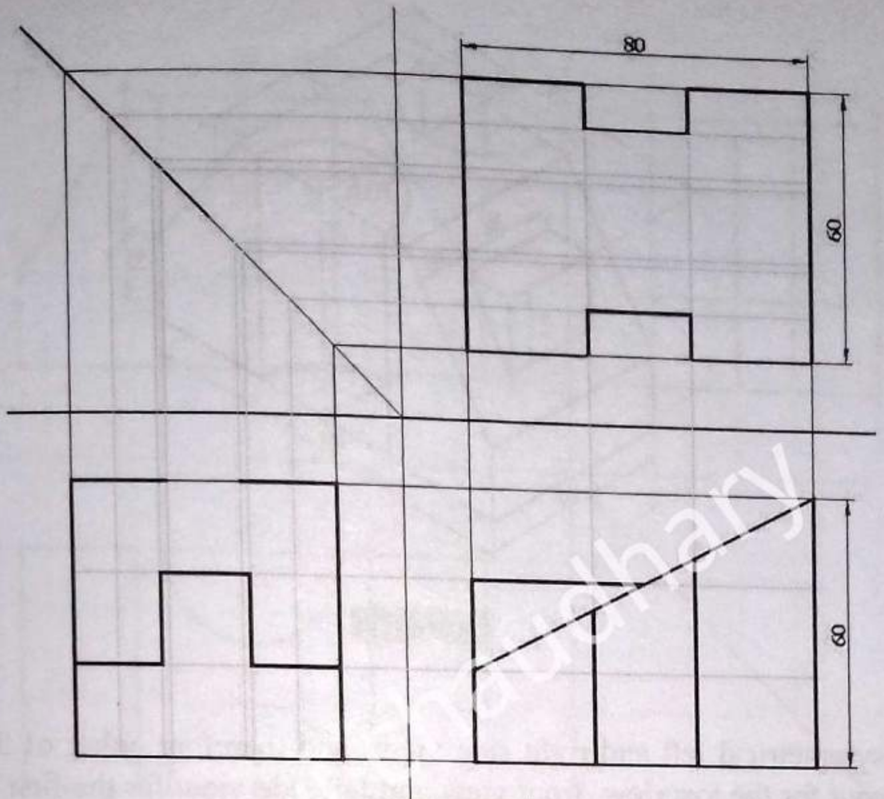


Figure E5.4(b)

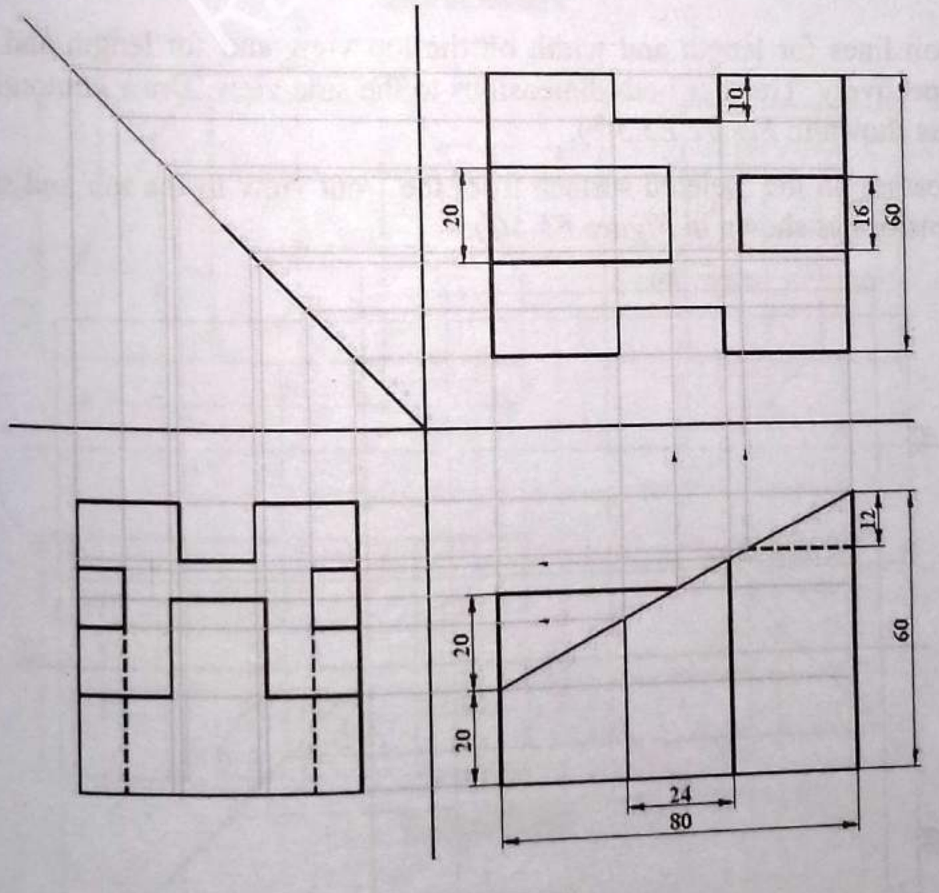


Figure E5.4(c)

Example 5.5

Draw orthographic views of the object shown in *Figure E5.5*. Use the first angle system of projection.

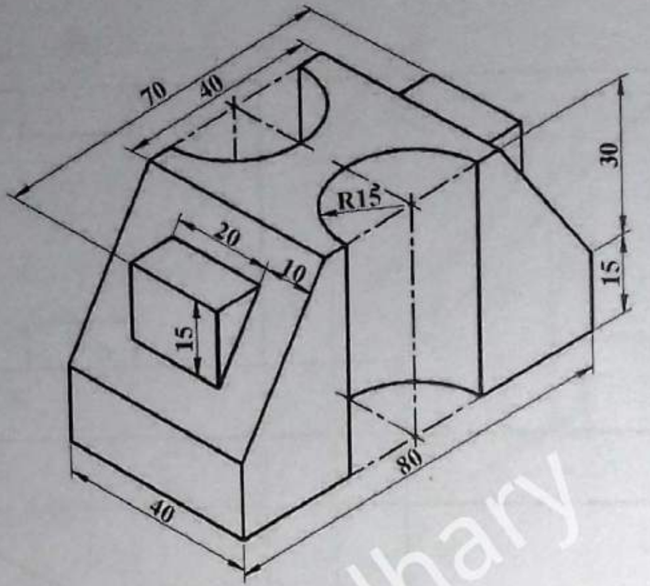


Figure E5.5

Solution

The object has symmetrical left and right side views and therefore either of the views can be drawn. Draw layout for the top view, front view and left side view for the first angle projection as shown in *Figure E5.5(a)*.

Draw projection lines for length and width on the top view and for length and height on the front view respectively. Transfer both dimensions to the side view. Draw contours of the object on each view as shown in *Figure E5.5(b)*.

Transfer the feature on the inclined surface from the front view to the top and side views and place the dimensions as shown in *Figure E5.5(c)*.

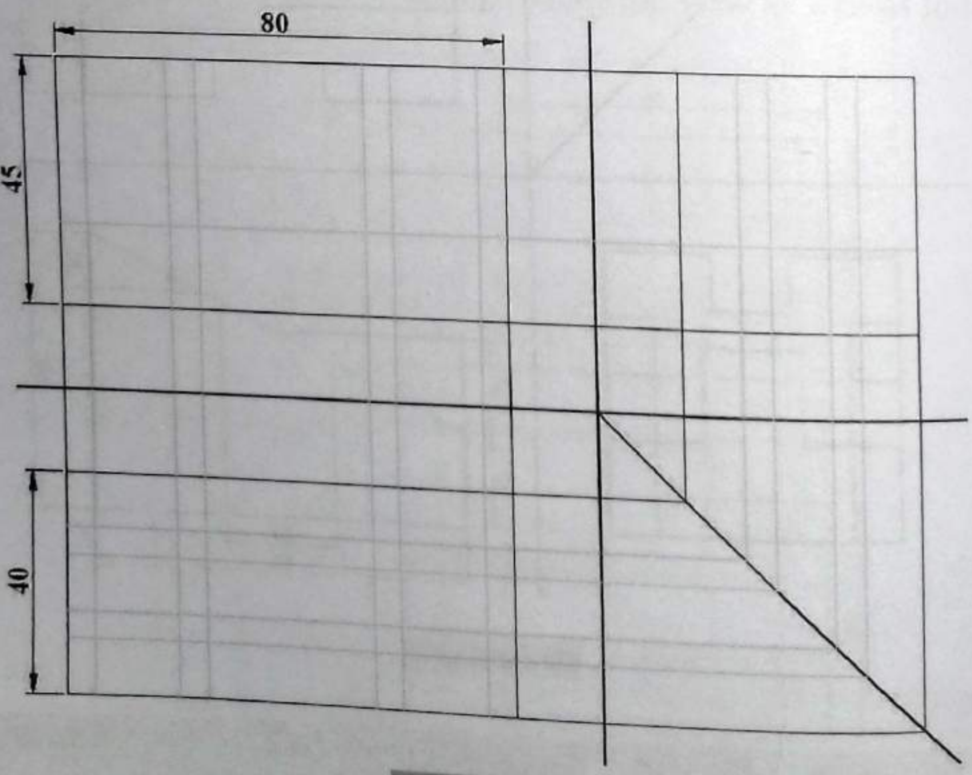


Figure E5.5(a)

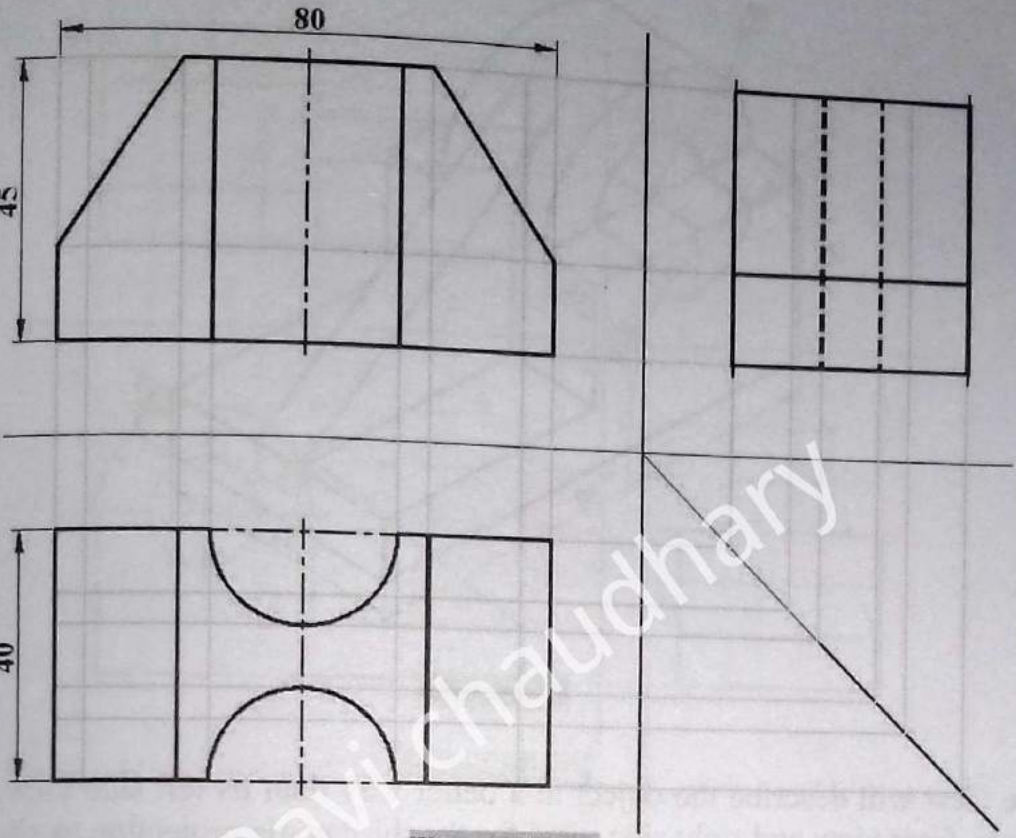


Figure E5.5(b)

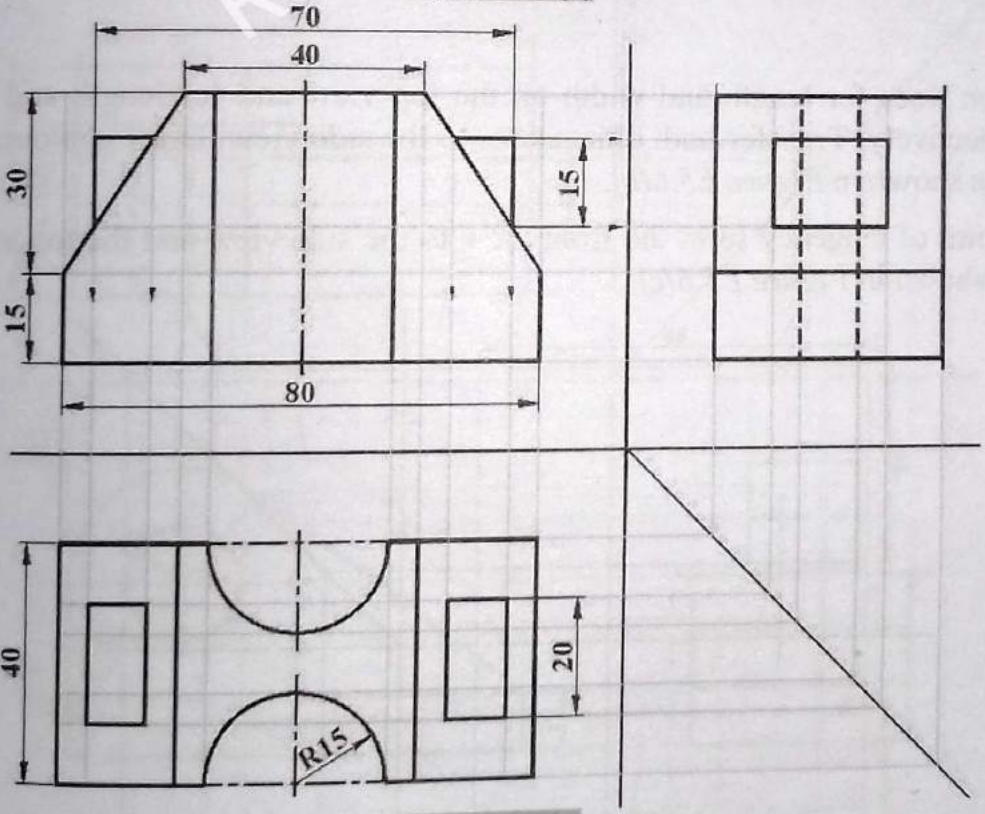


Figure E5.5(c)

Example 5.6

Draw orthographic views of the object shown in *Figure E5.6*. Use the third angle system of projection.

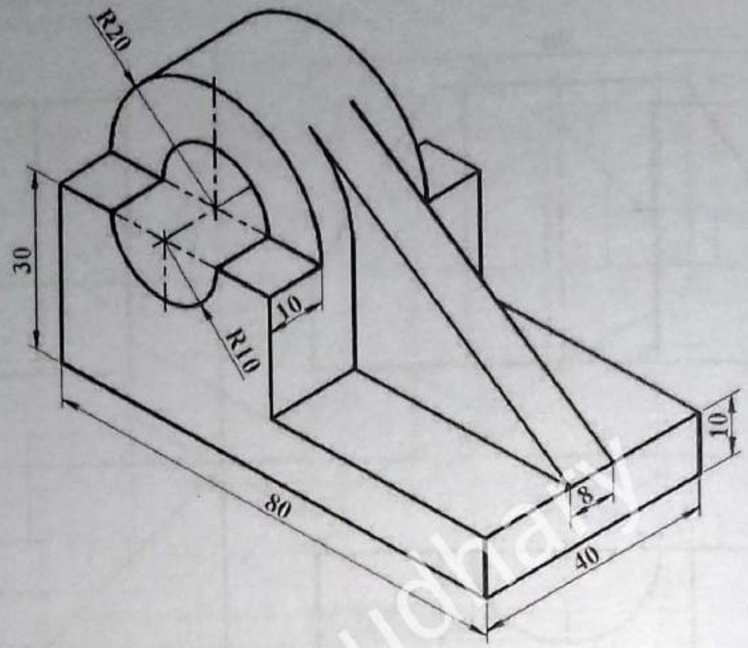


Figure E5.6

Solution

The right side view will describe the object in a better way than its left side view. Draw layout for the top view, front view and right side view for the third angle projection as shown in *Figure E5.6(a)*.

Draw projection lines for length and width on the top view and for length and height on the front view respectively. Transfer both dimensions to the side view. Draw contours of the object on each view as shown in *Figure E5.6(b)*.

Transfer the point of tangency form the front view to the side view and the top view. Place the dimensions as shown in *Figure E5.6(c)*.

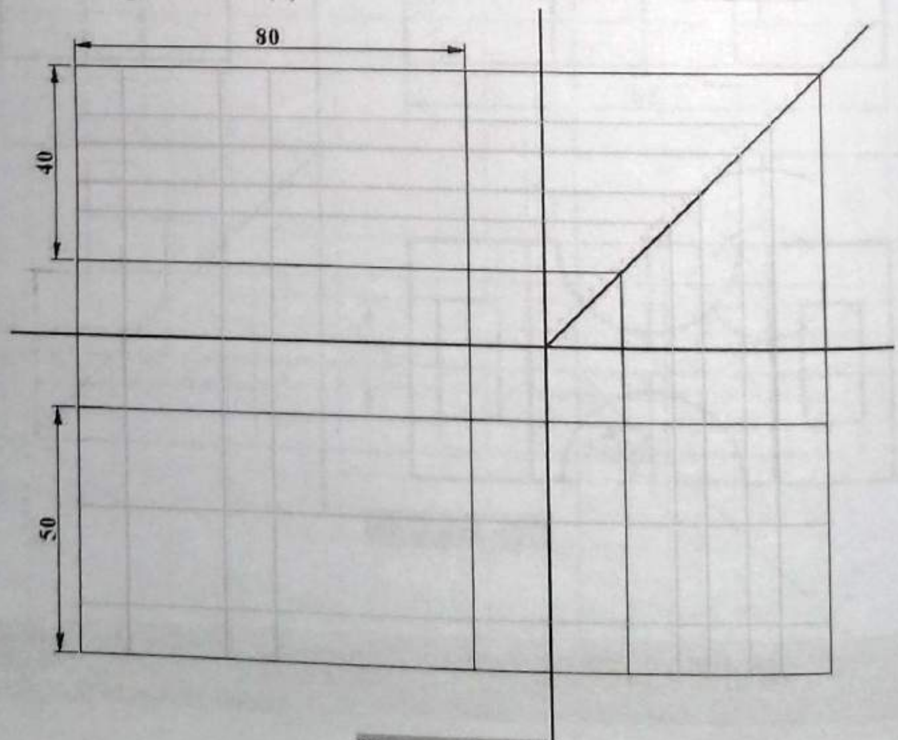


Figure E5.6(a)

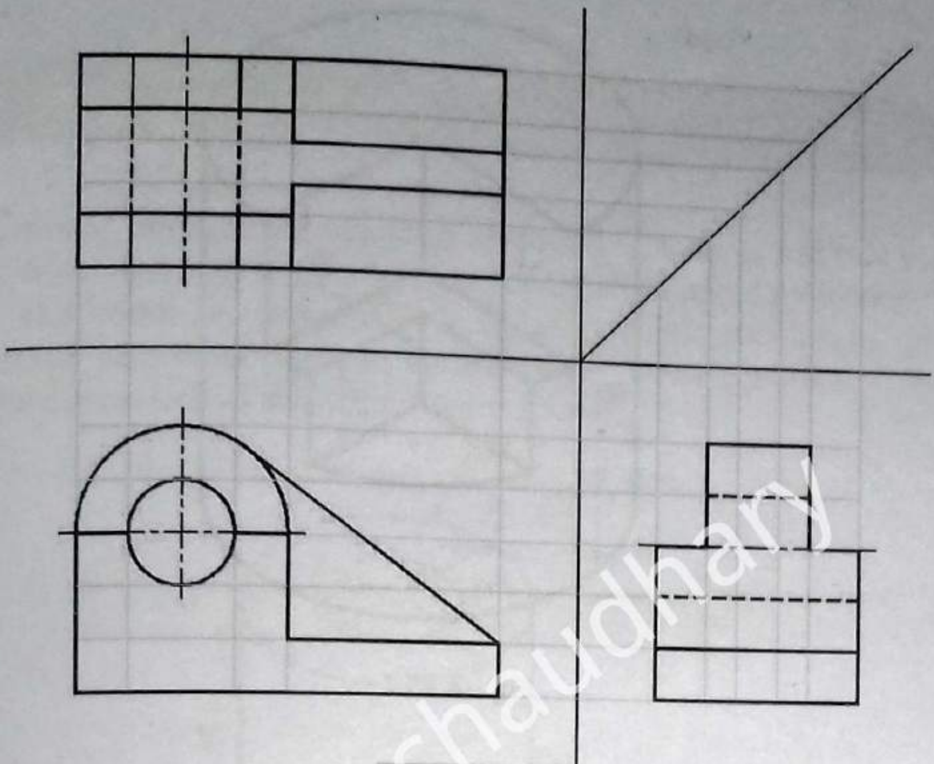


Figure E5.6(b)

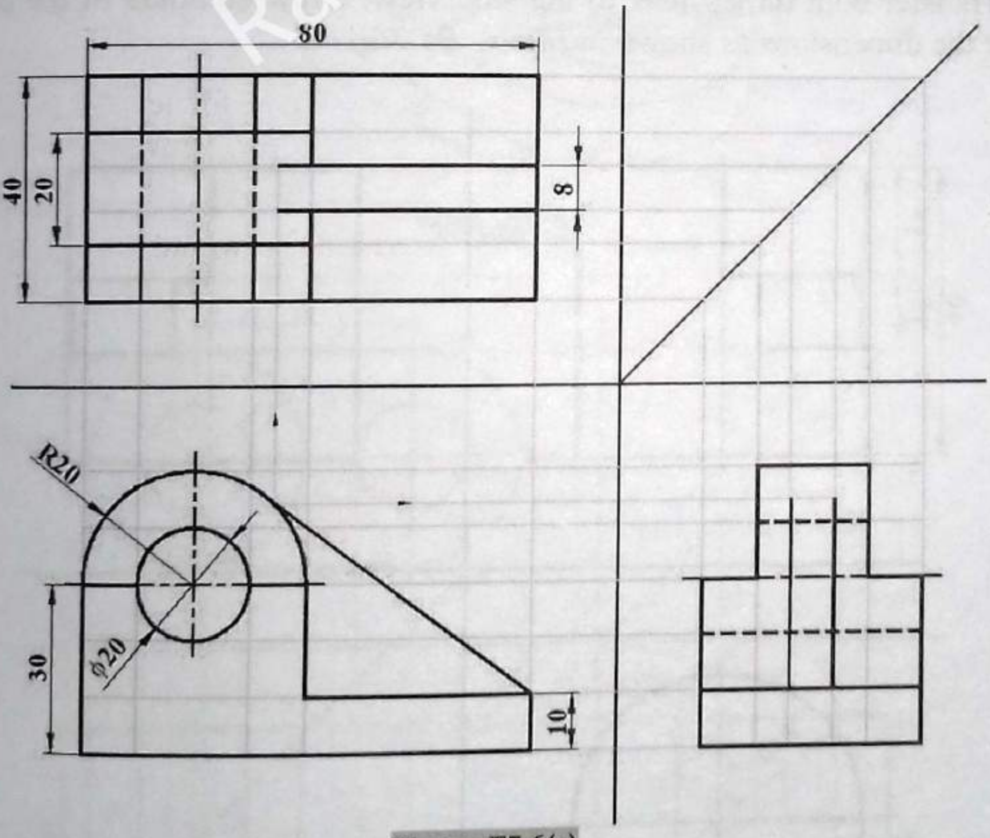


Figure E5.6(c)

Example 5.7

Draw orthographic views of the object shown in *Figure E5.7*. Use the first angle system of projection.

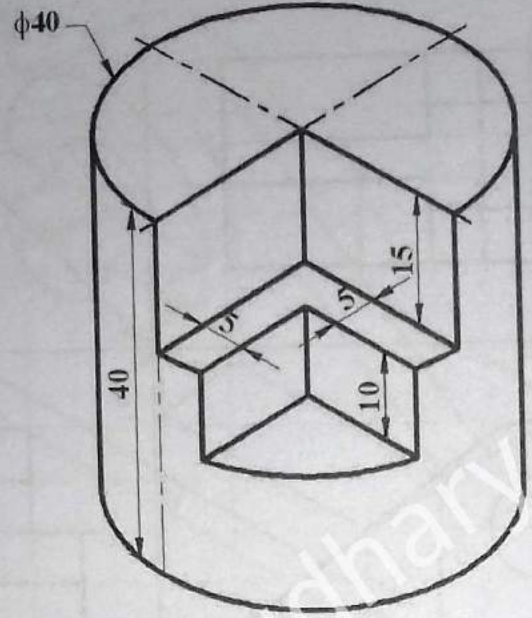


Figure E5.7

Solution

Draw layout for the top view, front view and left side view for the first angle projection. Draw projection lines for length and width on the top view and for length and height on the front view respectively. Transfer both dimensions to the side view. Draw contours of the object on each view and place the dimensions as shown in *Figure E5.7(a)*.

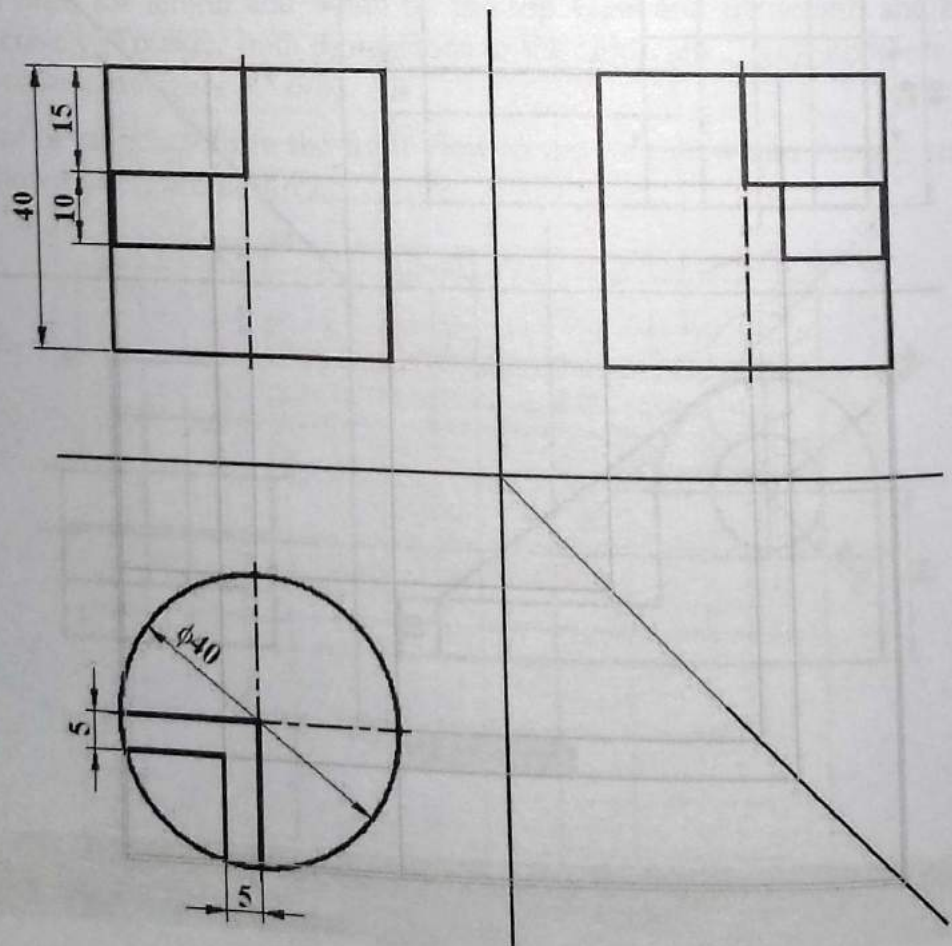


Figure E5.7(a)

Example 5.8

Draw orthographic views of the object shown in *Figure E5.8*. Use the third angle system of projection.

Solution

The right side view will describe the object in a better way than its left side view. Draw layout for the top view, front view and right side view for the third angle projection. Draw projection lines for length and width on the top view and for length and height on the front view respectively. Transfer both dimensions to the side view. Draw contours of the object on each view and place the dimensions as shown in *Figure E5.8(a)*.

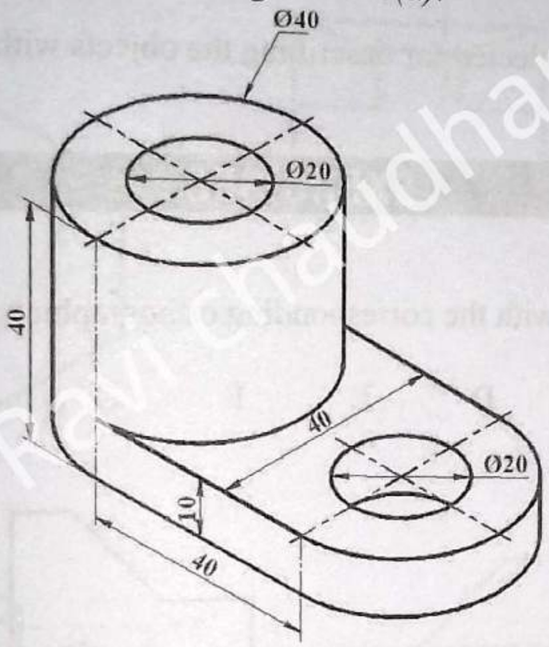


Figure E5.8

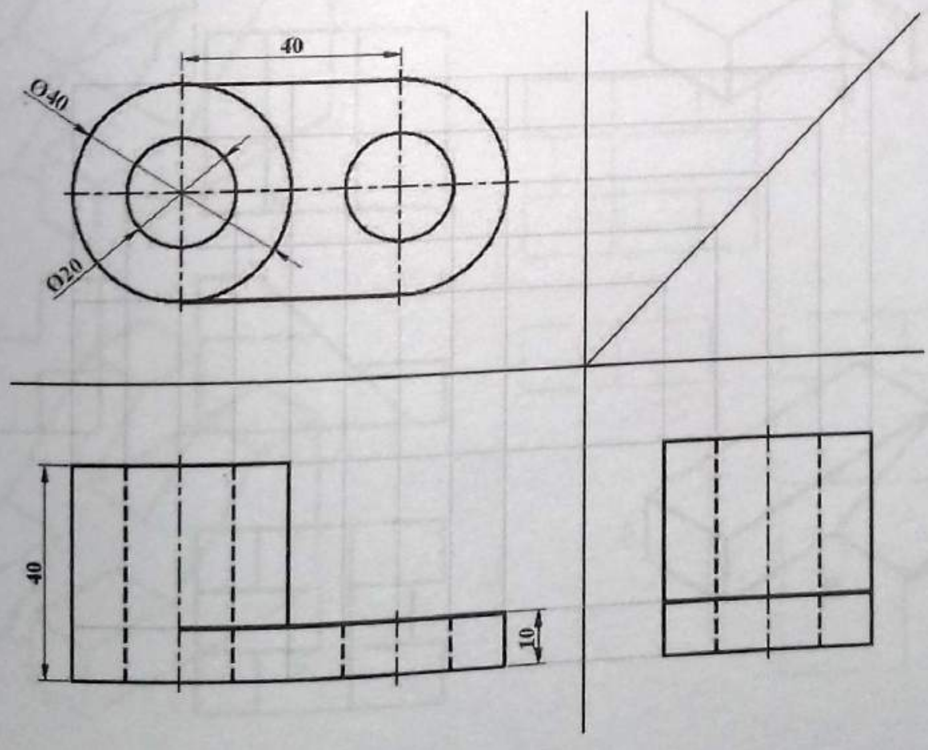


Figure E5.8(a)

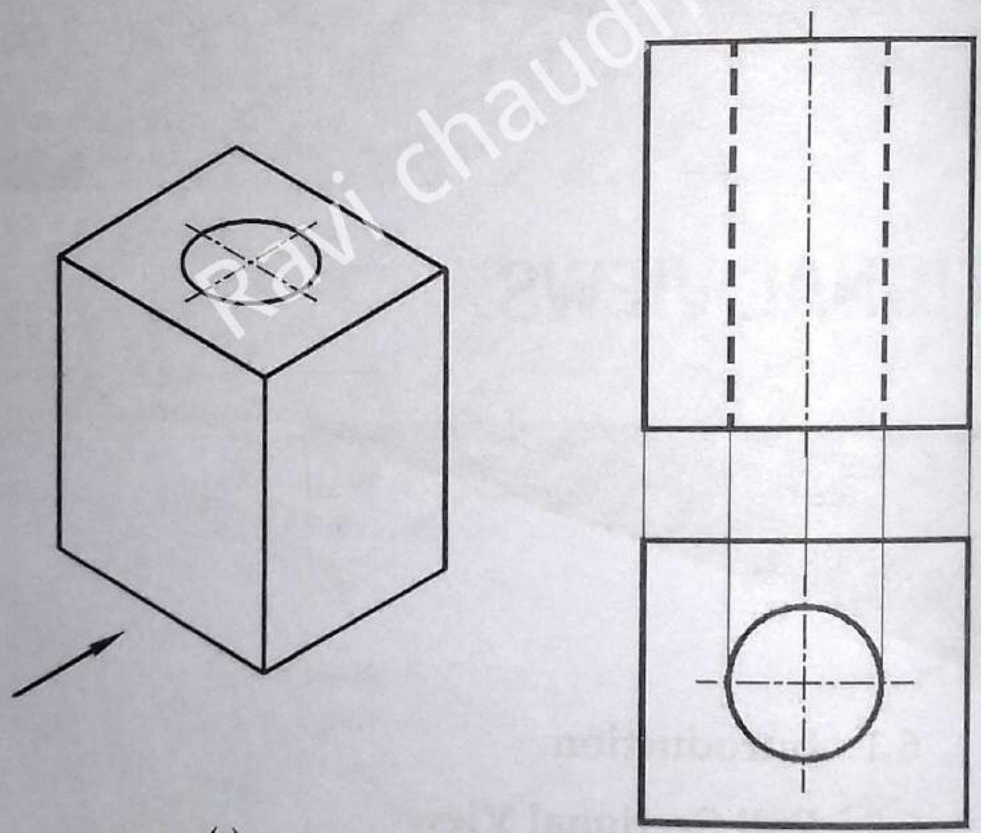
CHAPTER 6

SECTIONAL VIEWS

6.1 Introduction

Although orthographic views of an object describe interior contours of the object by the hidden lines, sometimes it might be confusing when many hidden lines occur in a single orthographic view. In such circumstances, object having complex interior features are assumed to be cut by a plane and orthographic view is then drawn assuming certain portion of the object from the observer's side is removed. View of the remaining part of the object thus obtained is called a sectional view.

For example consider an object having vertical through hole as shown in *Figure 6.1(a)*. The corresponding top view and front view of the object are also shown in *Figure 6.1(b)*. The vertical through hole is represented by a set of hidden line on the front view.



(a) (b)

Figure 6.1: Object with a Vertical Hole and its Orthographic Views

To make the internal curve surface visible on the front view, the object is assumed to be cut by a vertical plane passing through the middle of the hole as shown in *Figure 6.2(a)*. When the front half portion of the object is removed *(Figure 6.2(b))*, the front view of the remaining part of the solid gives the sectional front view of the given object, as shown in *Figure 6.2(c)*.

Sectional view usually replaces the corresponding orthographic view as shown in *Figure 6.1(c)*. In this case front view of the object is replaced by its sectional front view. The plane which cuts the object is called a cutting plane and is specified by the cutting plane line. The portion of the object which is touched by the cutting plane is represented by the hatching lines.

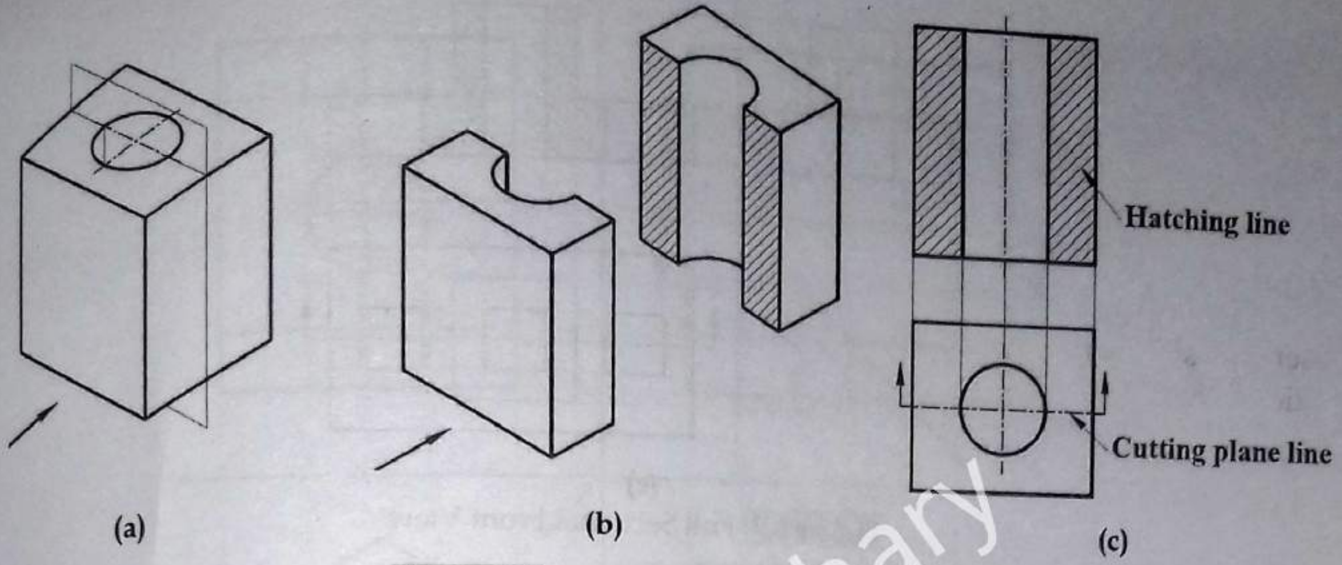


Figure 6.2: Sectional Front View of the Object

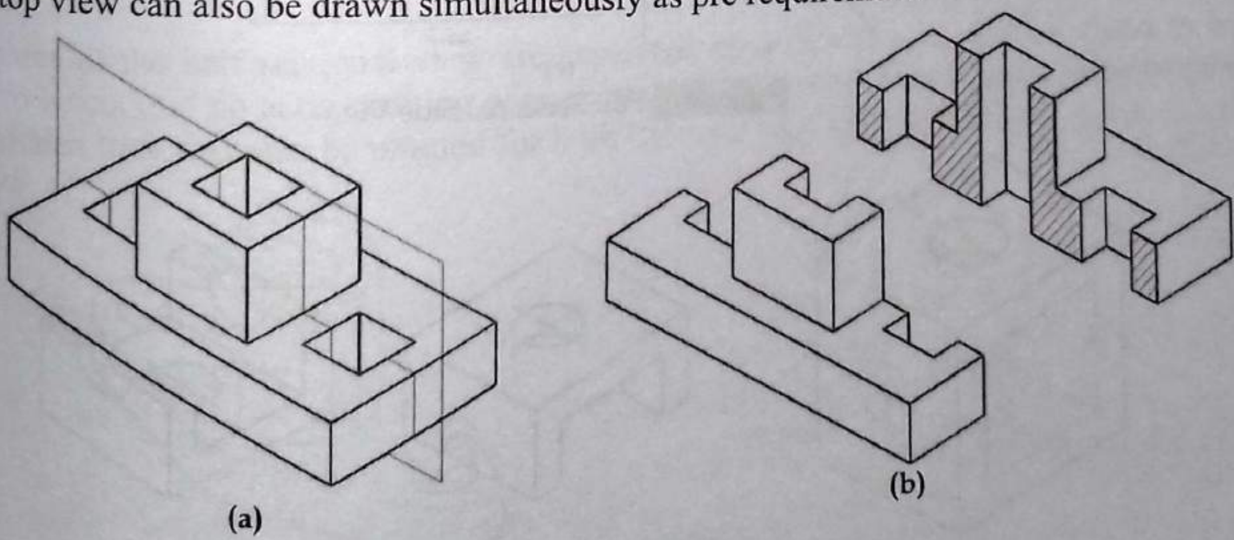
Any number of sectional views can be drawn for the given object as per requirements, therefore a sectional view drawn to show any particular internal feature should not include hidden details of other features.

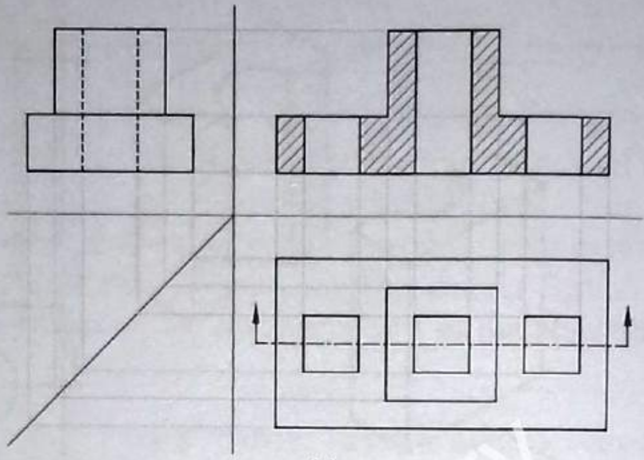
Depending upon the nature of the object as well requirements, different types of sectional views can be drawn such as full sectional view, half sectional view, offset sectional view, partial or broken section, etc.

6.2 Full Sectional View

When half portion of the object is assumed to be removed, it affects the view parallel to the cutting plane. The sectional view of the object thus obtained is called a full sectional view.

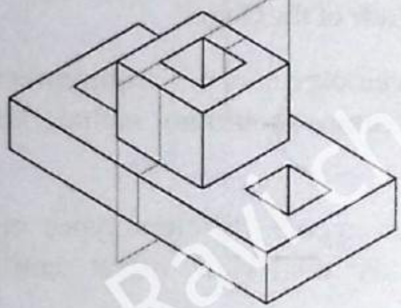
Figure 6.3 shows the full sectional front view of an object when it is cut by a longitudinal vertical section. *Figure 6.4* shows the full sectional side view of an object when it is cut by a transverse vertical section. Similarly *Figure 6.5* shows the full sectional top view of an object when is cut by a horizontal cutting plane. However sectional front view, sectional side view and sectional top view can also be drawn simultaneously as pre requirement.



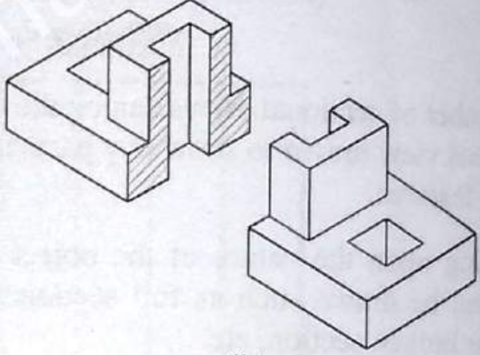


(c)

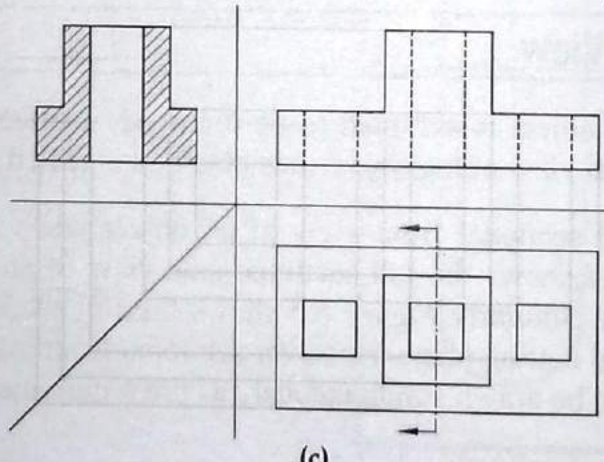
Figure 6.3: Full Sectional Front View



(a)

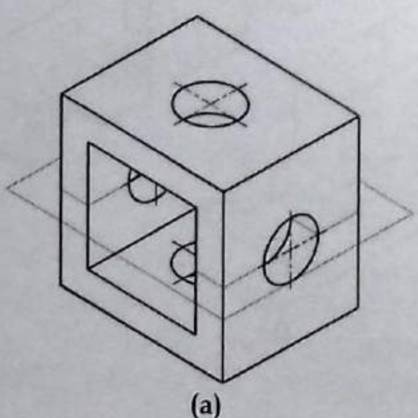


(b)

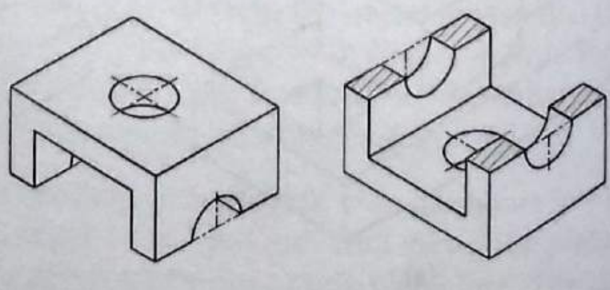


(c)

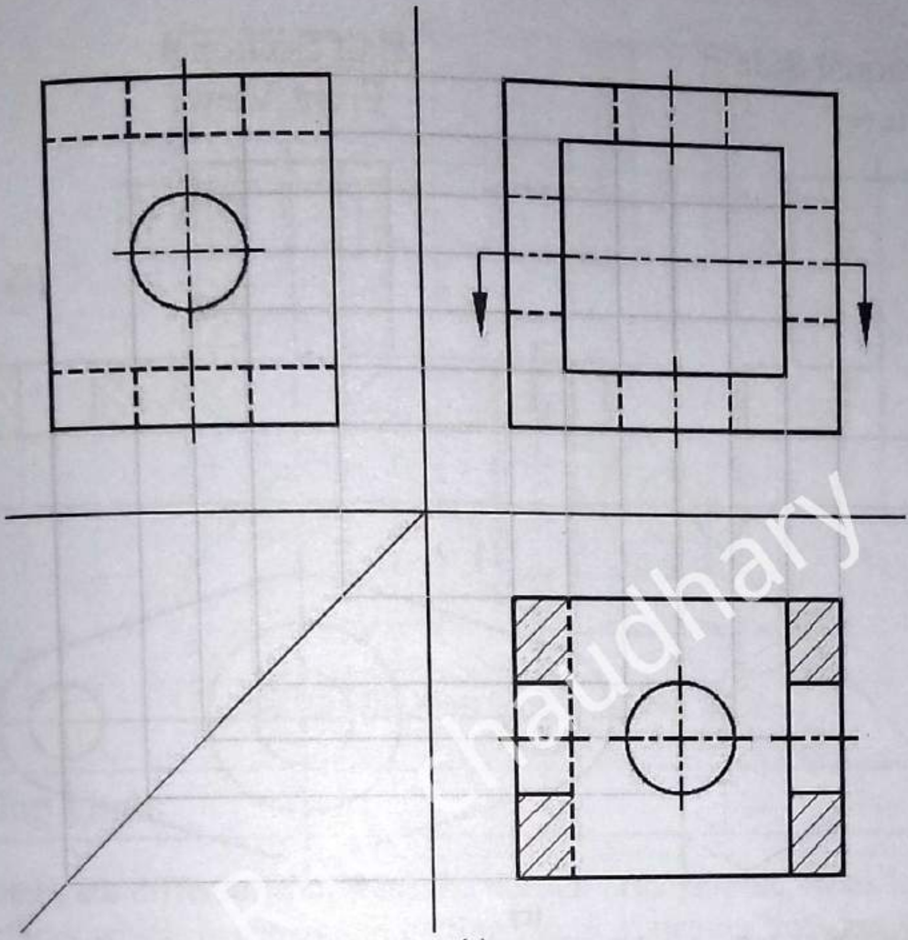
Figure 6.4: Full Sectional Side View



(a)



(b)



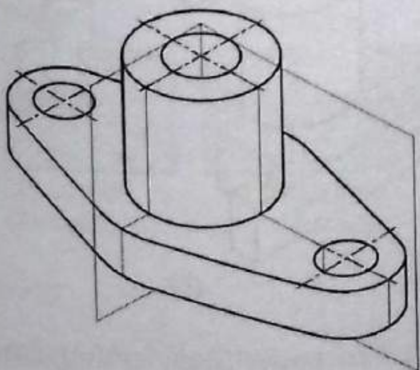
(c)

Figure 6.5: Full Sectional Top View

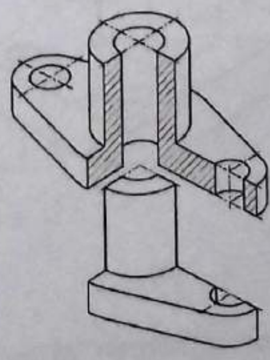
6.3 Half Sectional View

For symmetrical objects both internal and external features of the object can be shown on a single view by removing one quarter of the object. When one quarter of the object is removed, it affects on half portion of the corresponding view, and therefore, view thus obtained is called a half sectional view. *Figure 6.6* shows half sectional front view and half section side view of the object when one quarter of the object is removed by the two mutually perpendicular cutting planes.

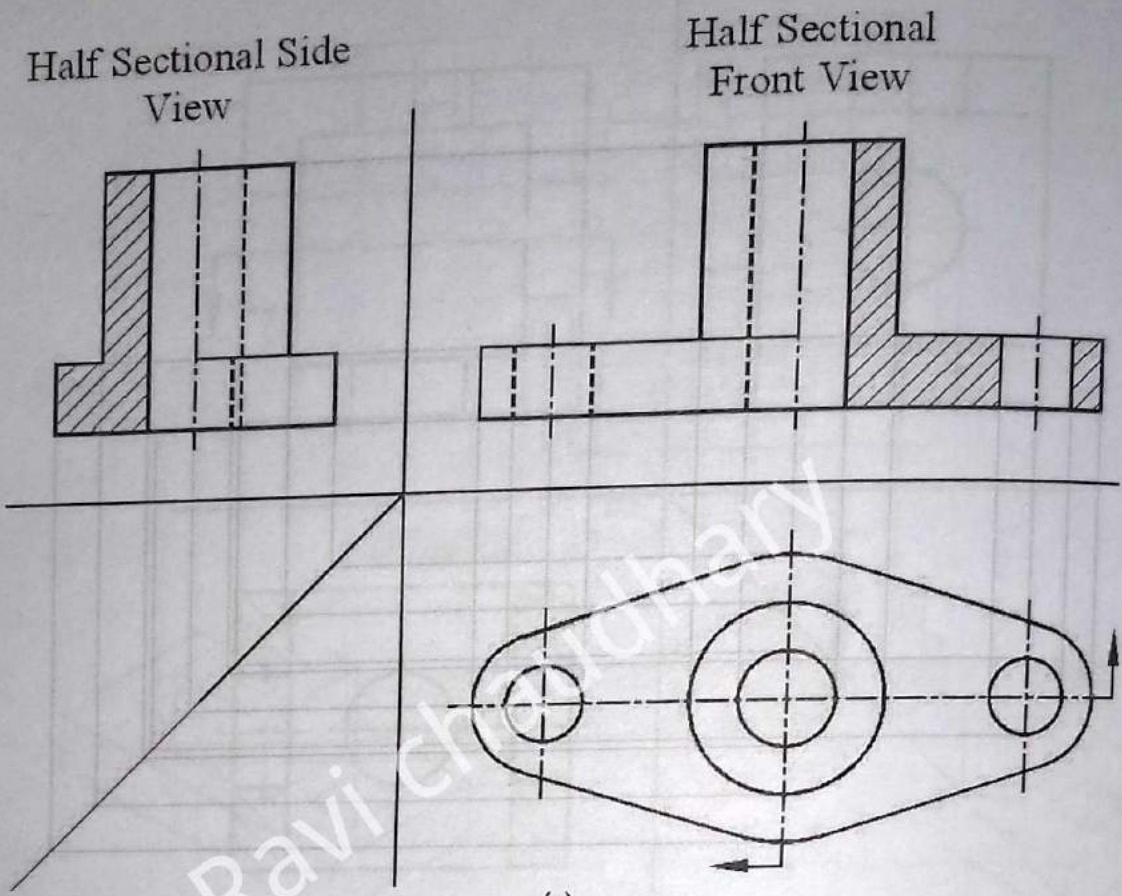
Two portions of the half sectional view are separated by a center line. As the object is only assumed to be cut and no actual cutting edge exists, use of a solid line will not be appropriate. Usually hidden lines can also be omitted for both halves of the view unless they are absolutely necessary.



(a)



(b)

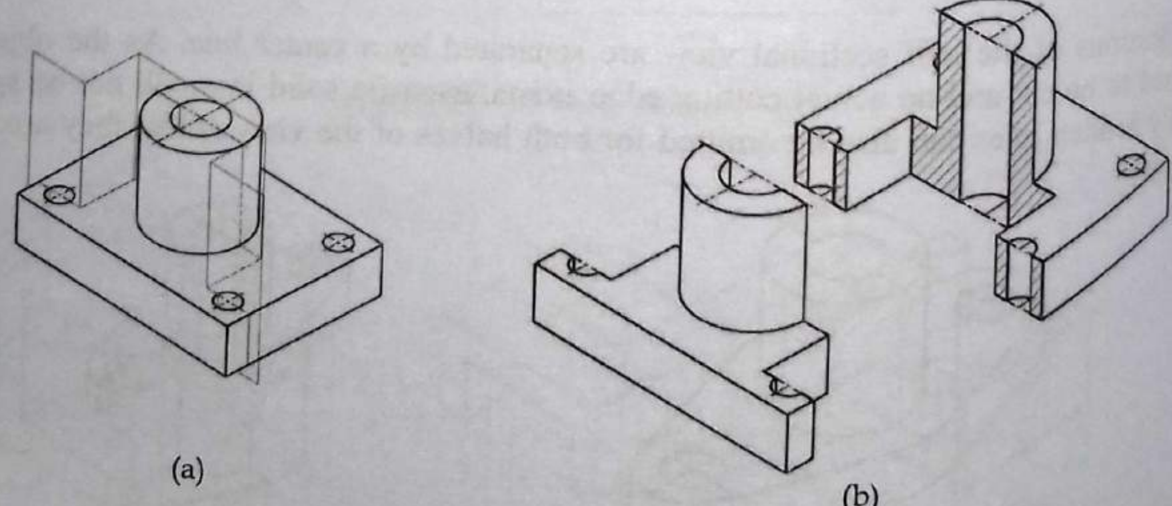


(c)

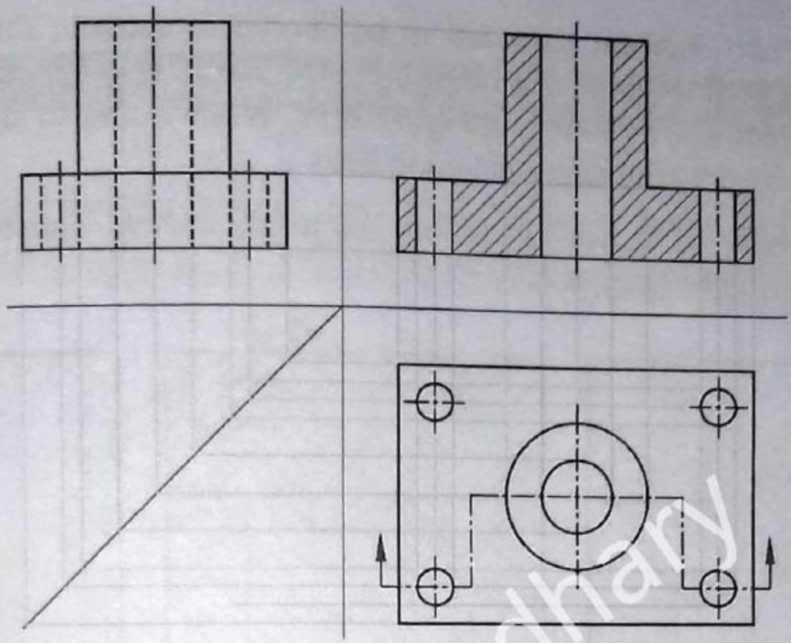
Figure 6.6: Half Sectional Front View and Half Sectional Side View

6.4 Offset Sectional View

When a single cutting plane could not show all the internal features of the object, an appropriate combination of different parallel cutting planes can be used as shown in *Figure 6.7(a)* and *Figure 6.7(b)*. The sectional view thus obtained is called an offset section. *Figure 6.7(c)* shows offset full section of the object.



(a) (b)



(c)

Figure 6.7: Offset Sectional Front View

6.5 Hatching Lines

The sectional views are differentiated from the normal orthographic views by drawing hatching lines on the surfaces which is cut by the cutting plane. Hatching lines are thin and continuous lines usually inclined at 45^0.

Following point should be noted while drawing hatching lines on any section:

(a) Hatching lines should be drawn at 45^0 to the horizontal line unless they are parallel or perpendicular to the visible outlines of the object, as shown in *Figure 6.8.*

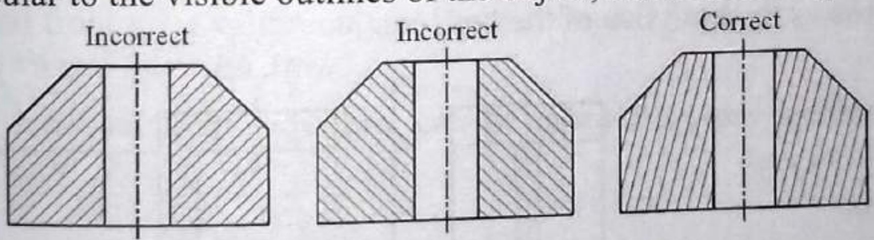


Figure 6.8

(b) Hatching lines should be have uniform thickness and should be equally spaced, as shown in *Figure 6.9.* Overrun of the hatching lines should be avoided.

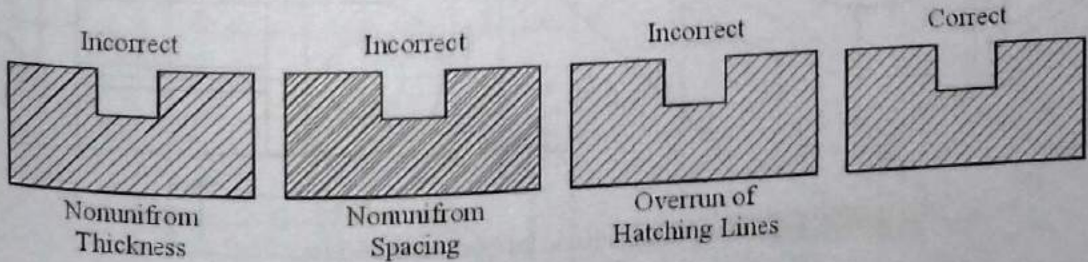


Figure 6.9

(c) In assembly drawings, when two adjacent parts are to be shown in section, hatching lines should be drawn in opposite directions, as shown in *Figure 6.10.*

(d) If more than two adjacent parts are to be shown in section, third part should have hatching lines inclined at angles other than 45^0 (usually 30^0 or 60^0) should be drawn in opposite directions, as shown in *Figure 6.11*. While doing so smaller regions can be differentiated by drawing dense hatching lines.

(e) If large area is to be shown in section, the hatching lines can be limited to a small portion as shown in *Figure 6.12*.

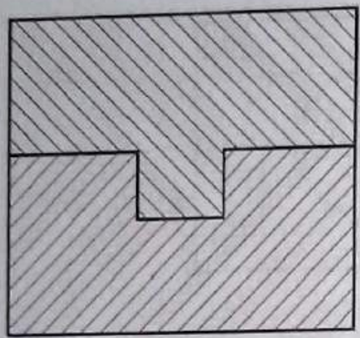


Figure 6.10

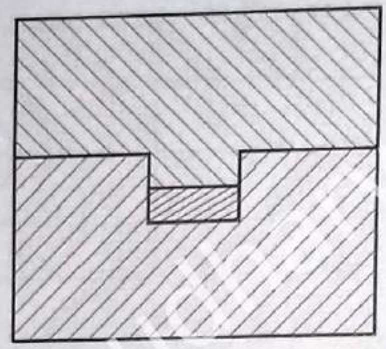


Figure 6.11

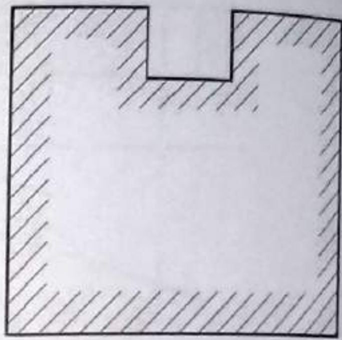
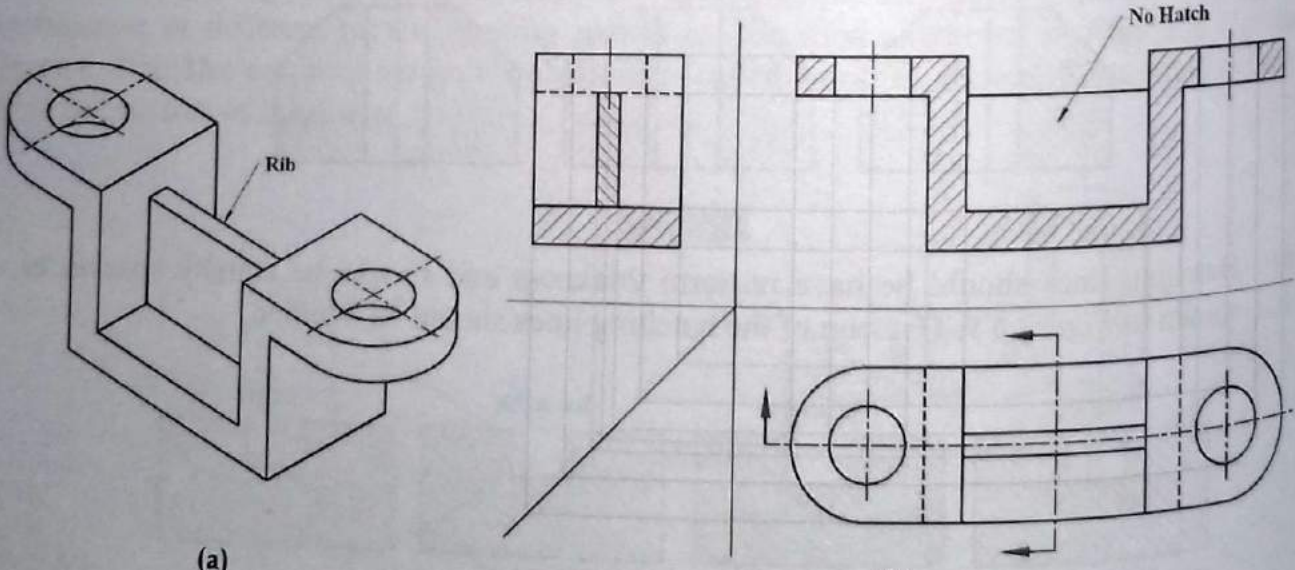


Figure 6.12

6.6 Exceptional Rules of Sectional Views

Objects such as solid shafts, bolts, keys, webs, ribs spokes of wheel etc which not have any internal details are not shown in section even if the cutting plane passes longitudinally through theses components as shown in *Figure 6.13*.

These parts are, however, cut and shown in section when the cutting plane passes through transverse direction. Occasionally, small areas of the shaft may be shown in section to serve some particular purpose as to show size of the key, etc.



(a) (b)

Figure 6.13: Exceptional Rule of Sectional Views

WORKOUT EXAMPLES

Example 6.1

Draw orthographic views of the object shown in *Figure E6.1* with full sectional front view. Use the first angle system of projection.

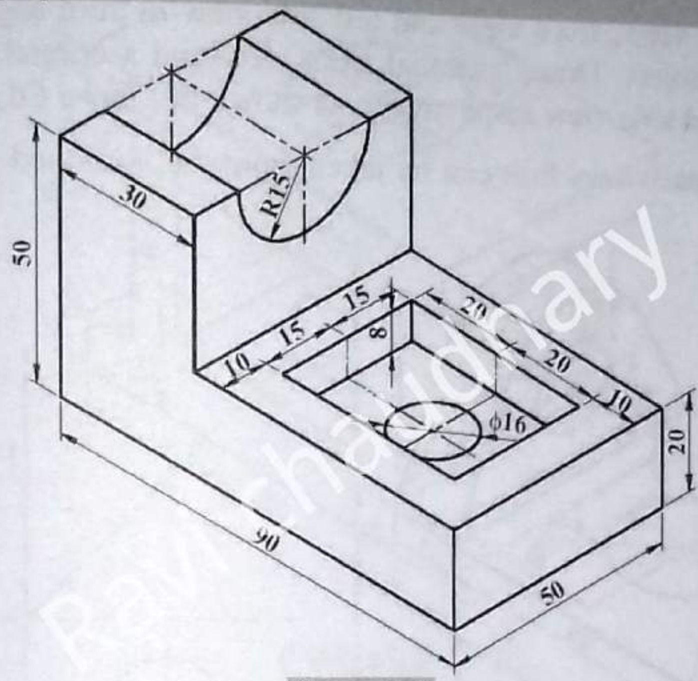


Figure E6.1

Solution

Draw layout for the top view, front view and right side view in first angle system of projection. Draw top view and front view of the object. Draw sectional front view of the object instead of front view, as shown in *Figure E6.1(a)*.

To complete the sectional front view hint can be taken from the sectioned object shown in *Figure E6.1(b)*.

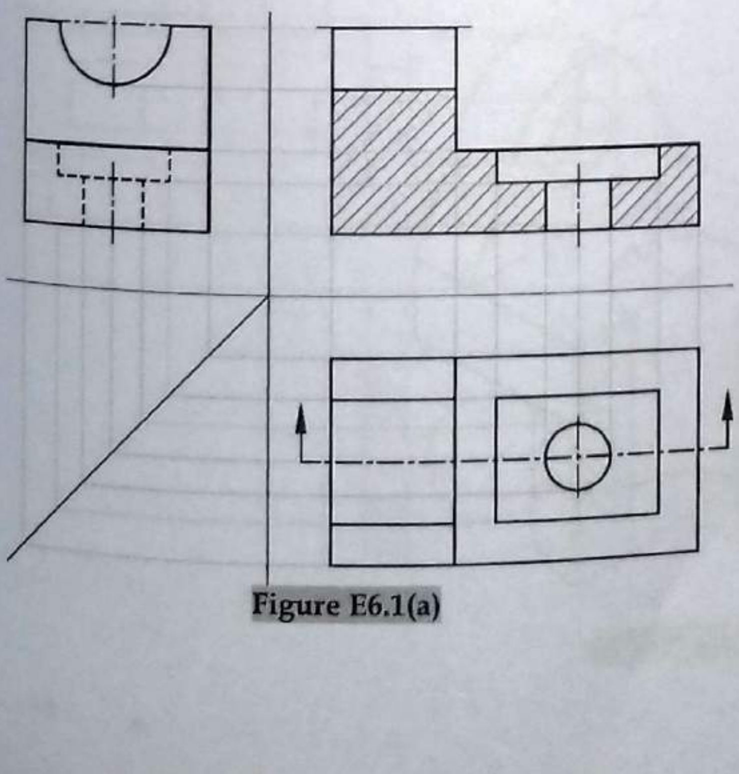


Figure E6.1(a)

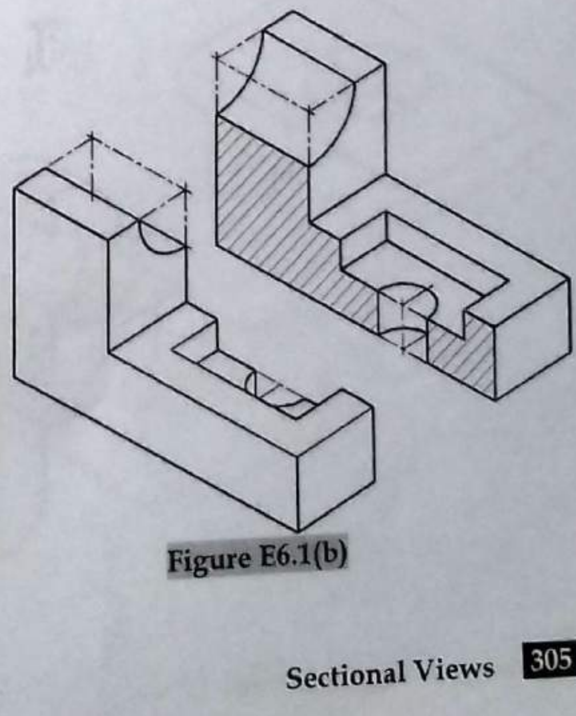


Figure E6.1(b)

Example 6.2

Draw orthographic views of the object shown in *Figure E6.2* with half sectional front view and half sectional side view. Use the third angle system of projection.

Solution

Draw layout for the top view, front view and left side view in third angle system of projection. Draw top view of the object. Draw sectional front view and sectional side view of the object instead of front view and side view respectively, as shown in *Figure E6.2(a)*.

To complete the sectional views hint can be taken from the sectioned object shown in **Figure E6.2(b)**.

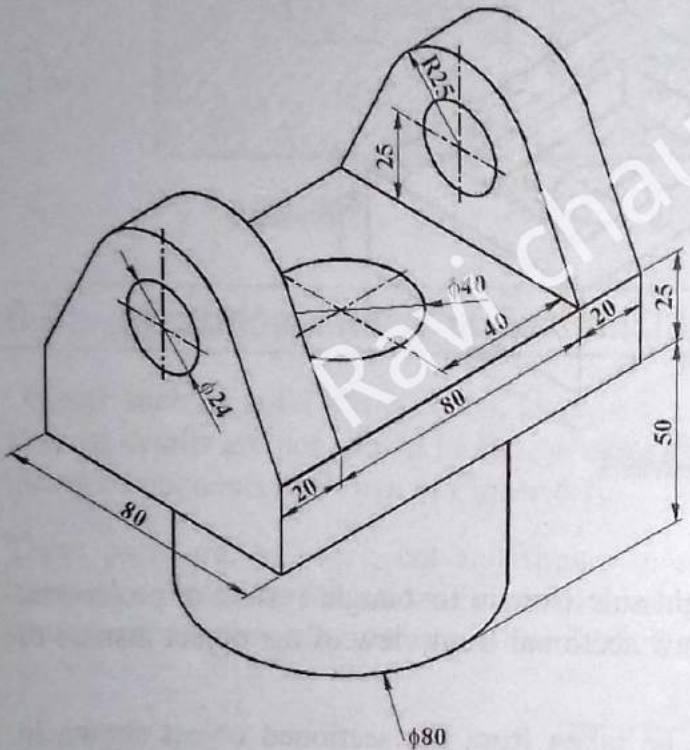


Figure E6.2

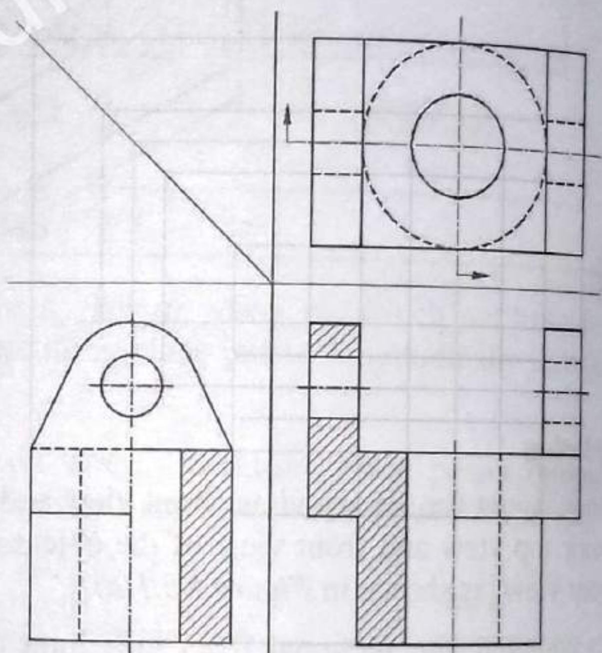


Figure E6.2(a)

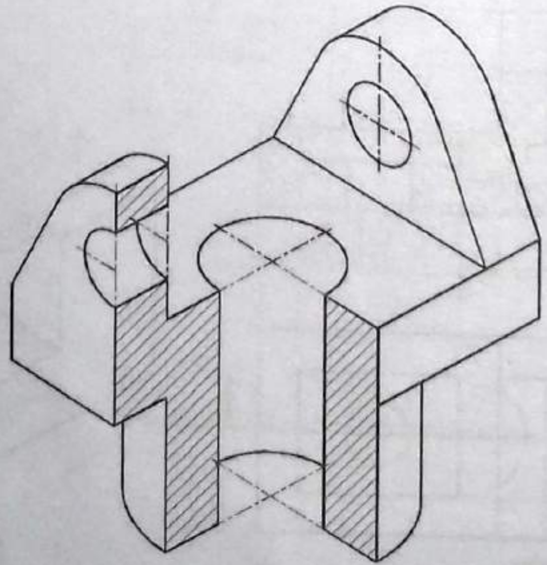


Figure E6.2(b)

Example 6.3

Draw orthographic views of the object shown in *Figure E6.3* with full sectional top view. Use the first angle system of projection.

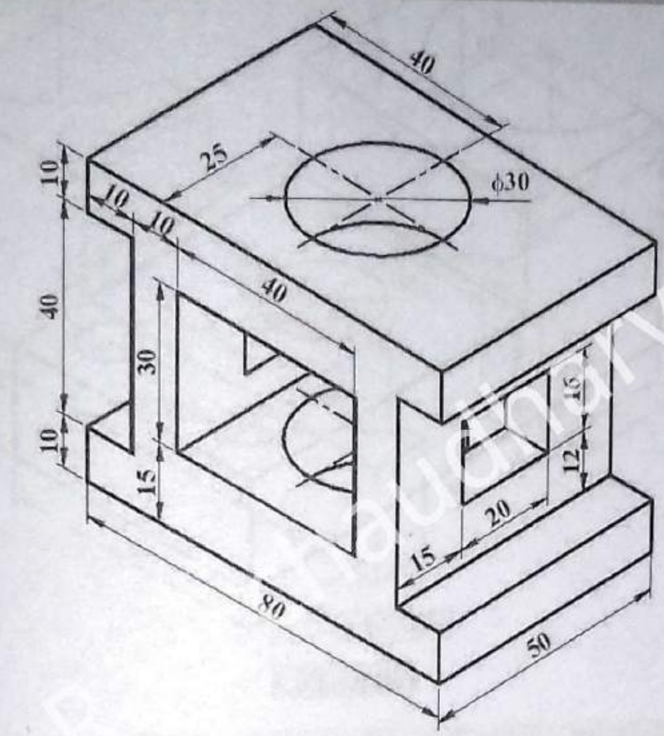


Figure E6.3

Solution

Draw layout for the top view, front view and right side view in first angle system of projection. Draw front view and side view of the object. Draw sectional top view of the object instead of top view, as shown in *Figure E6.3(a)*.

To complete the sectional front view hint can be taken from the sectioned object shown in *Figure E6.3(b)*.

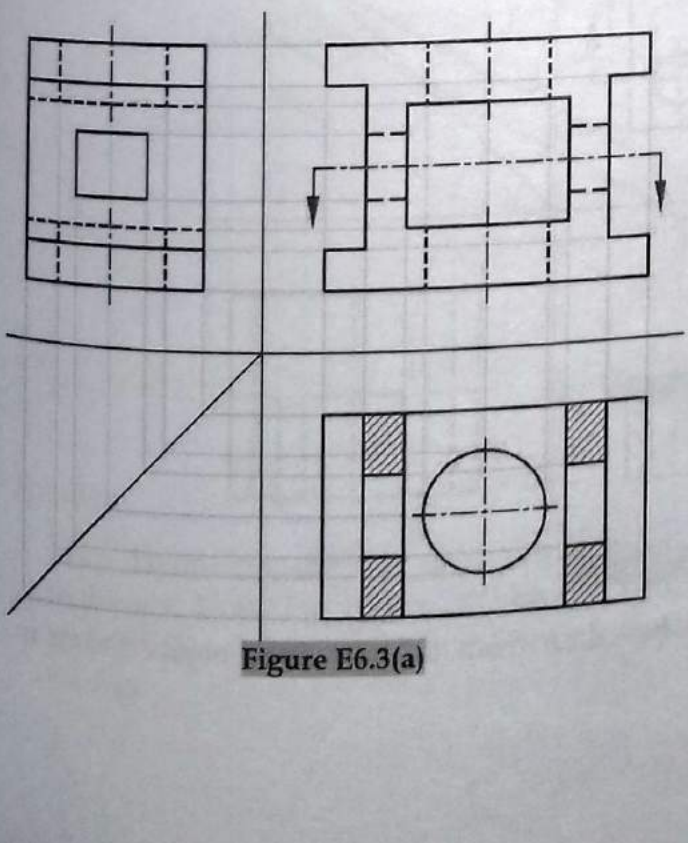


Figure E6.3(a)

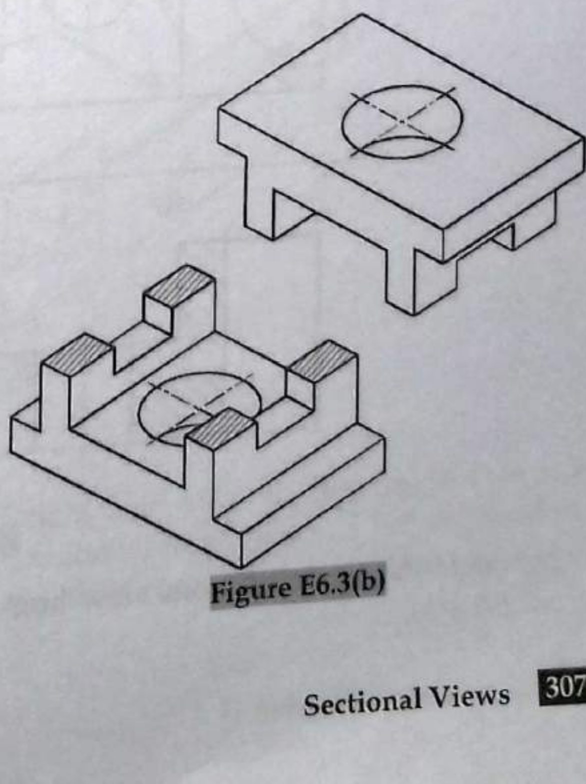


Figure E6.3(b)

Example 6.4

Draw orthographic views of the object shown in *Figure E6.4* with offset sectional front view. Use the third angle system of projection.

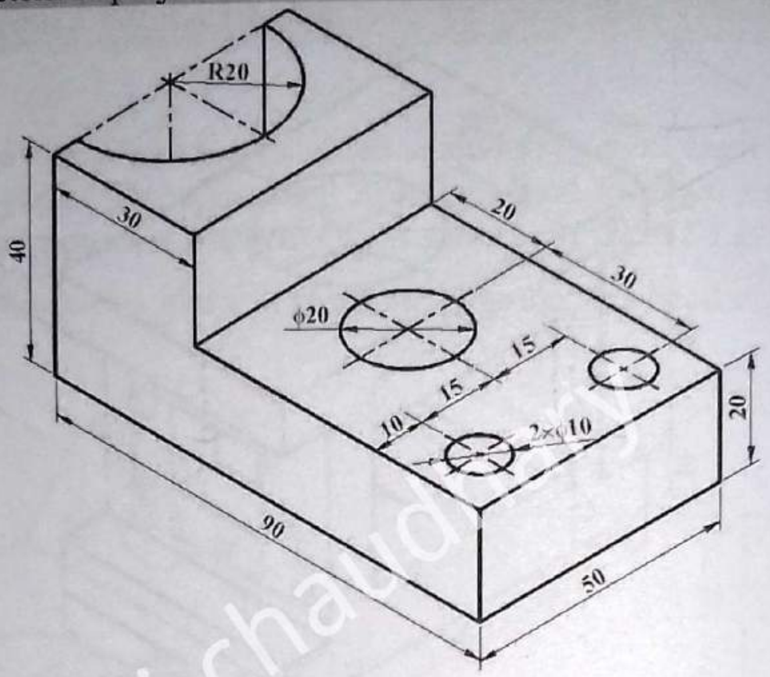


Figure E6.4

Solution

Draw layout for the top view, front view and right side view in third angle system of projection. Draw top view and side view of the object. Draw sectional front view of the object instead of front view, as shown in *Figure E6.4(a)*.

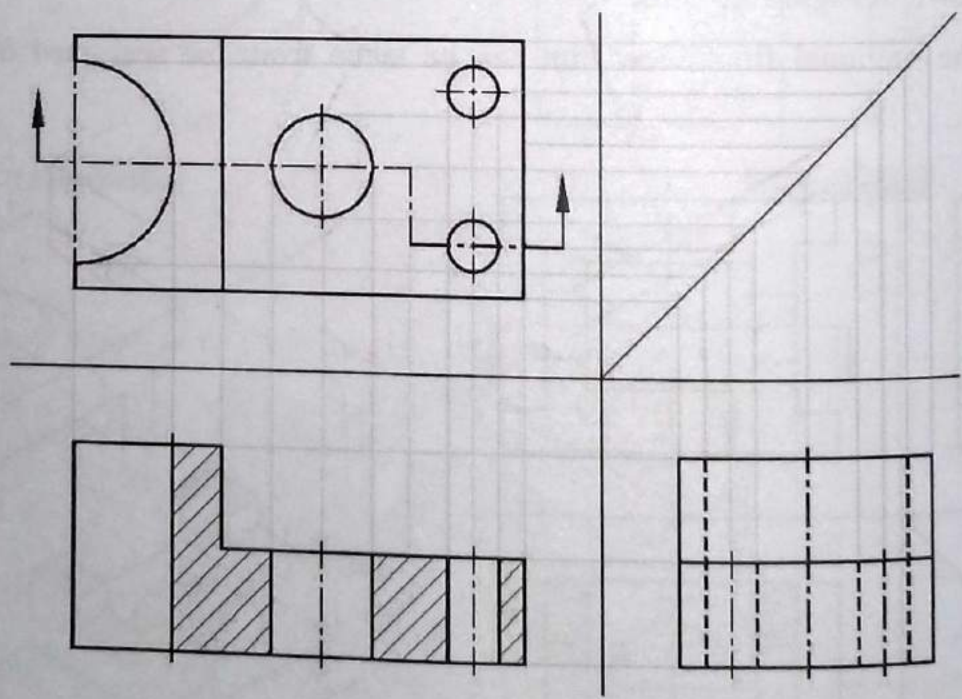


Figure E6.4(a)

To complete the sectional front view hint can be taken from the sectioned object shown in *Figure E6.4(b)*.

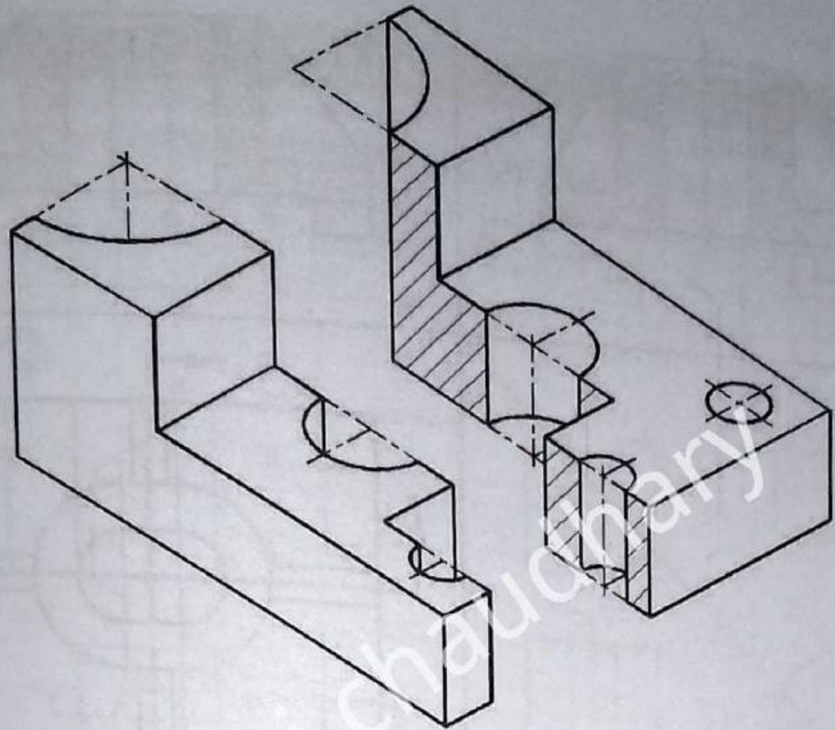


Figure E6.4(b)

Example 6.5

Draw orthographic views of the object shown in *Figure E6.5* with full sectional front view and full sectional side view. Use the first angle system of projection.

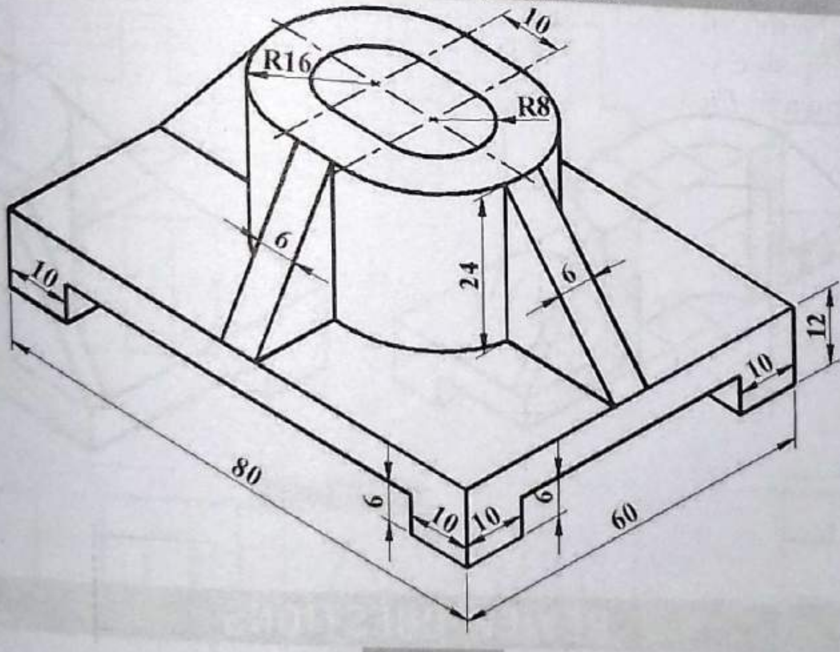


Figure E6.5

Solution

Draw layout for the top view, front view and right side view in first angle system of projection. Draw top view of the object. Draw full sectional front view and full sectional side view of the object instead of front view and side view respectively, as shown in *Figure E6.5(a)*.

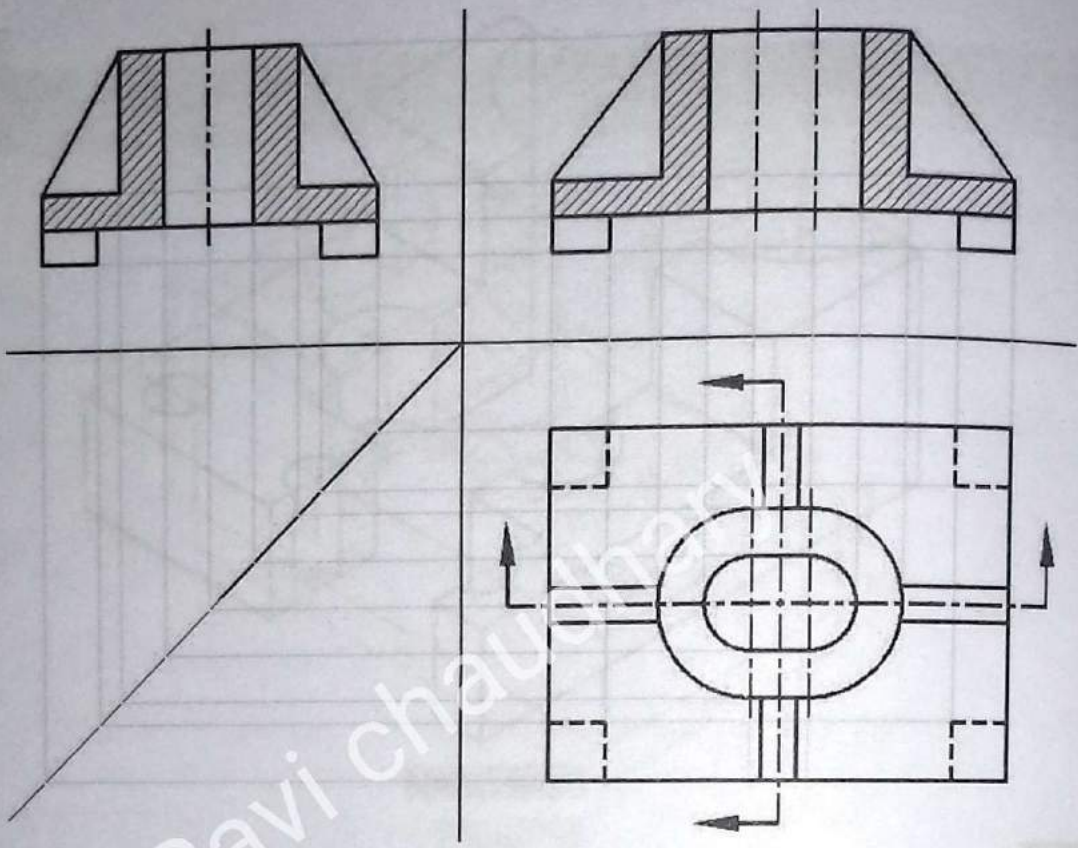


Figure E6.5(a)

To complete the sectional views hint can be taken from the sectioned objects shown in *Figure E6.5(b)*.

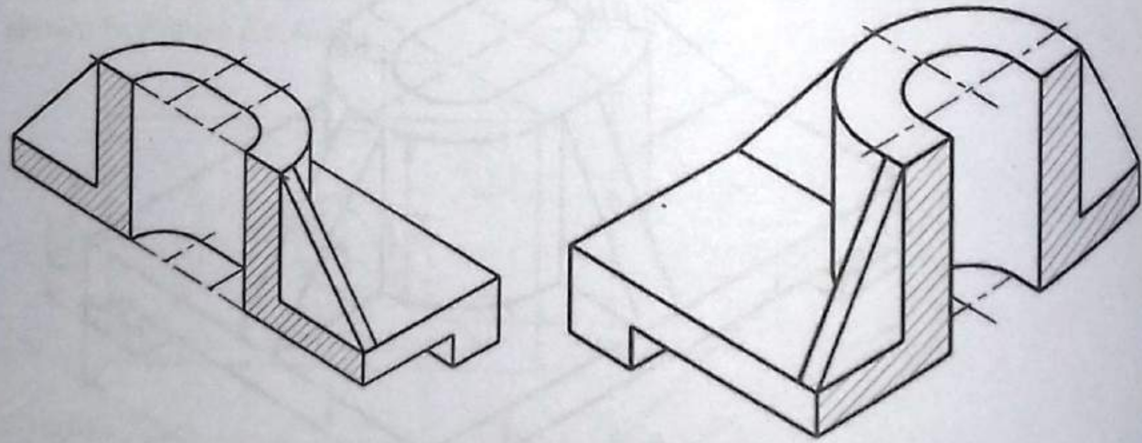


Figure E6.5(b)

REVIEW QUESTIONS

1. What is a sectional view? Why sectional views of the object are drawn?
2. What are hatching lines and cutting plane line?
3. Explain different types of sectional views.
4. Explain general rules of hatching lines.
5. Explain exceptional rules of sectioning.

CHAPTER 7

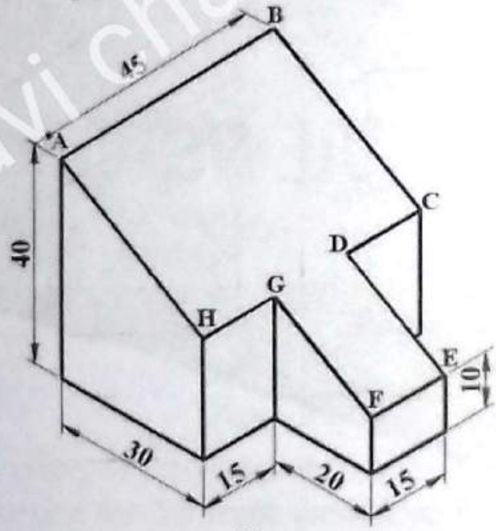
AUXILIARY VIEWS

7.1 Introduction

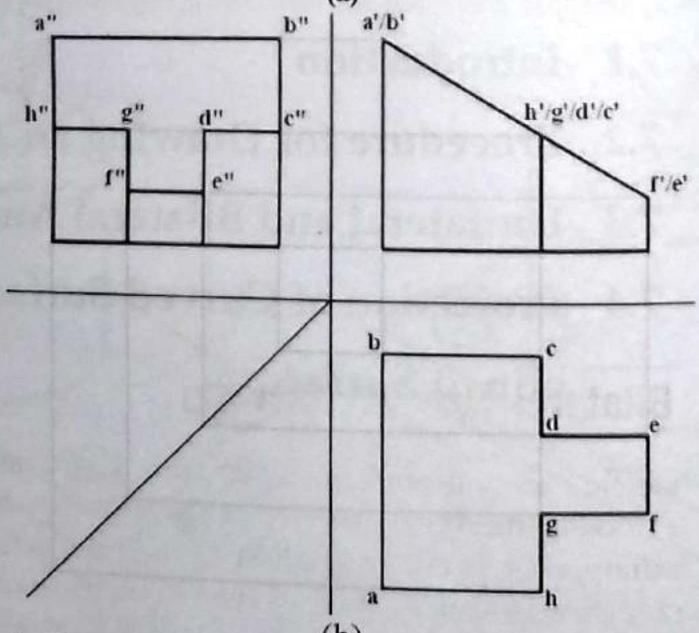
When an object consists of inclined surfaces, none of the principal views show their true shapes and sizes. To show true shape of such a surface, the surface is projected into a projection plane which is parallel to the surface of concern. The projection plane used for this purpose is called an auxiliary plane and the view obtained on the plane is called an auxiliary view. Auxiliary view is usually a partial view showing only the oblique surface. Sometimes, partial auxiliary view is essential to complete some features of the principal view.

7.2 Procedure for Drawing an Auxiliary View

Consider an object consisting of an inclined surface as shown in *Figure 7.1(a)*. Its orthographic views are also shown in *Figure 7.1(b)*. The inclined surface **AB...GH** appears in distorted forms both in the top view and side view. To draw the true shape or auxiliary view of the inclined surface, follow the following procedure.



(a)



(b)

Figure 7.1: Object Consisting of an Inclined Surface

- Draw a reference line **R/L** at the edge view of the front face of the object on the top view.
- Draw a reference line **R'/L'** parallel to the edge view **a'b'...g'h'** of the inclined surface on the front view. Draw projection lines passing through each points **a'**, **b'**, ... **g'** and **h'** on the front view and perpendicular to the reference line **R'/L'**.
- Measure the distance of each point on the front view from the reference line **R/L** and transfer it into the respective projection lines from the reference line **R'/L'**.
- Join the points on the proper sequence to get the true shape $\mathbf{a_1b_1...g_1h_1}$ of the inclined surface as shown in *Figure 7.2.*

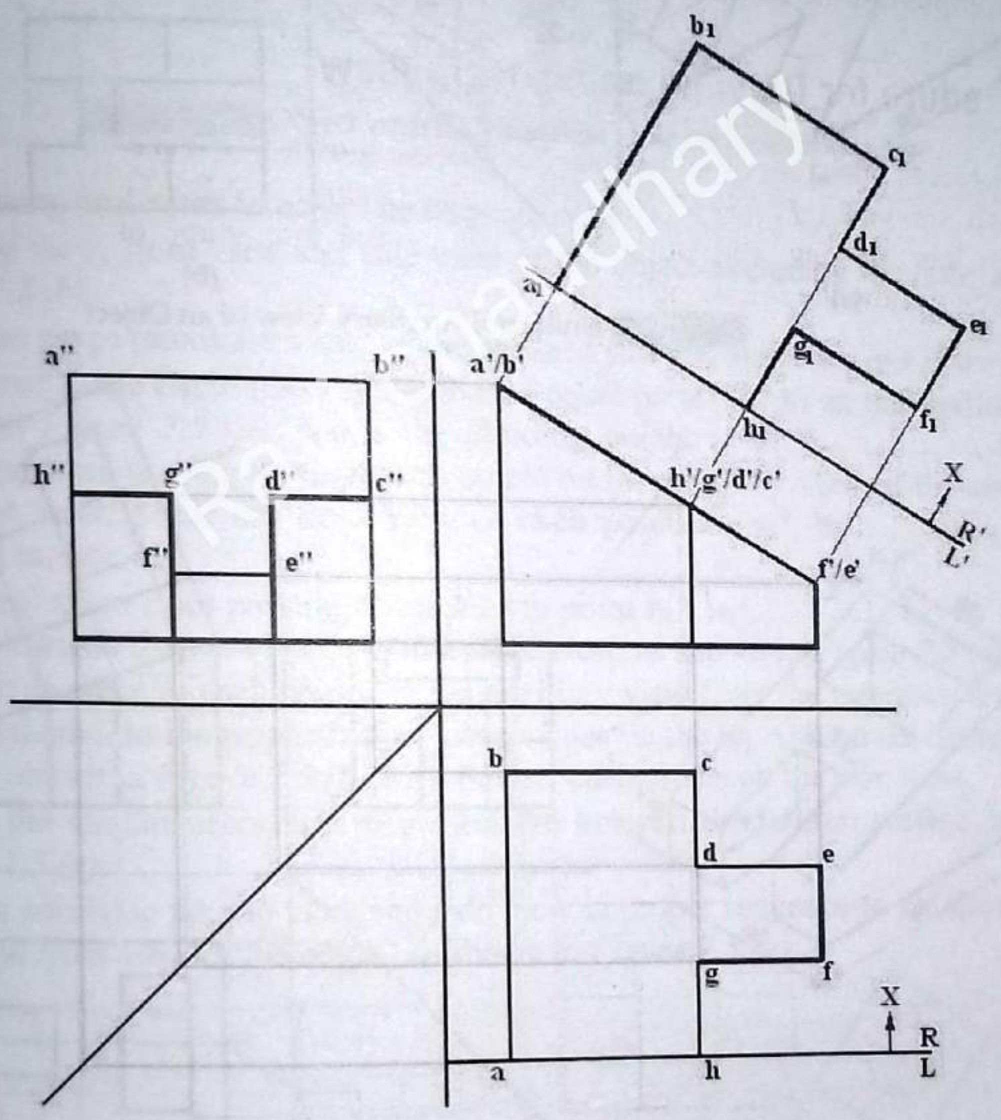
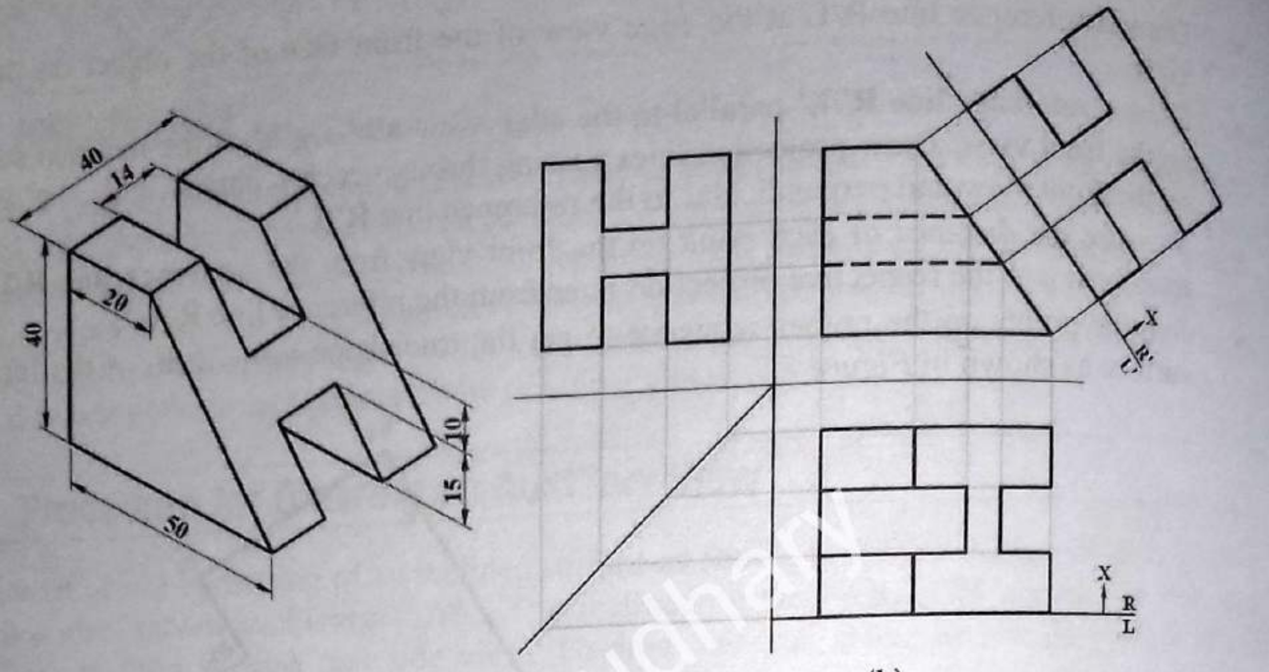


Figure 7.2: Auxiliary View of an Object

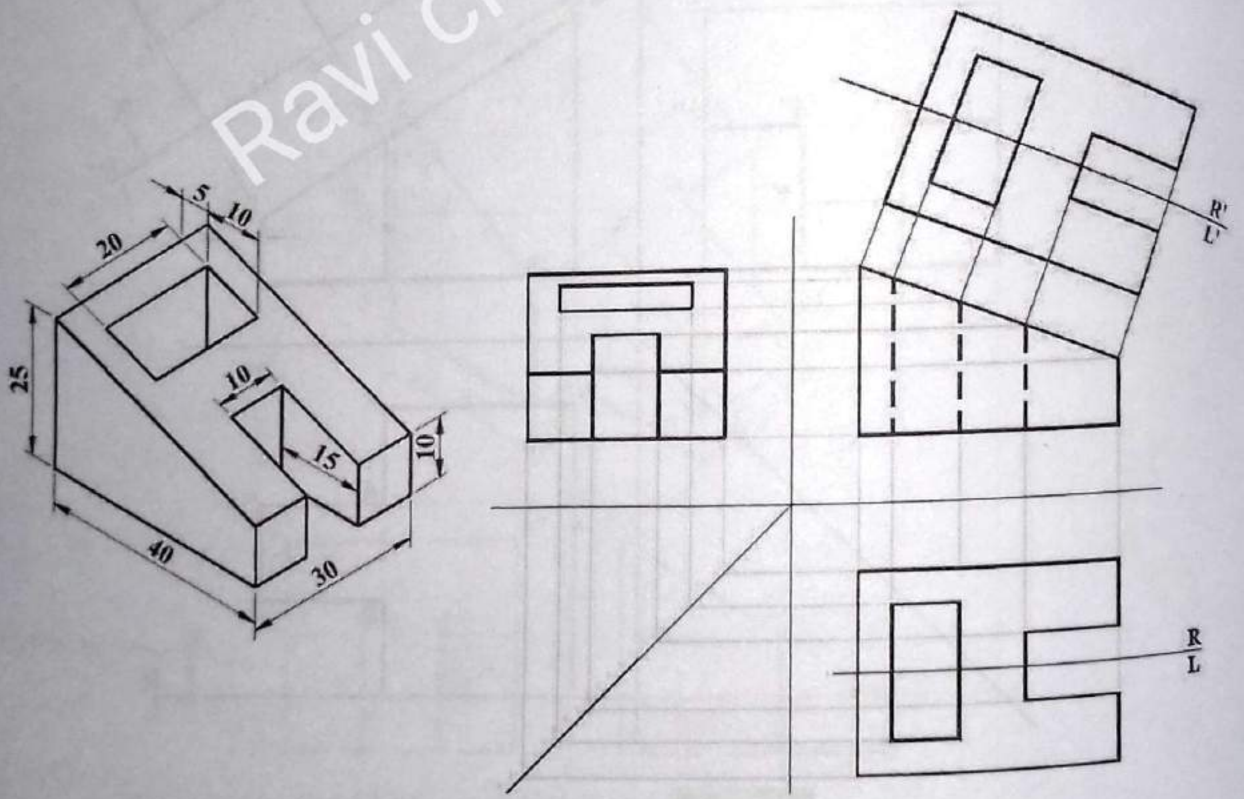
7.3 Unilateral and Bilateral Auxiliary Views

Auxiliary view can be classified as unilateral auxiliary view and bilateral auxiliary view. A unilateral auxiliary view is drawn entirely on one side of the reference line as shown in *Figure 7.3* whereas a bilateral auxiliary view is drawn on both sides of the reference line as shown in *Figure 7.4*. Bilateral auxiliary view is usually used for the symmetrical objects and the reference line is located at the middle of the view.



(a) (b)

Figure 7.3: Unilateral Auxiliary View of an Object



(a) (b)

Figure 7.4: Bilateral Auxiliary View of an Object

7.4 Projection of Curved Surfaces on an Inclined Surface

Figure 7.5(a) shows pictorial view of an object containing a cylindrical hole on the inclined surface. The axis of the cylindrical hole is perpendicular to the inclined surface. As the circular section on the inclined surface appears in distorted form both on the top view and the side view, these vies cannot be completed without its true shape.

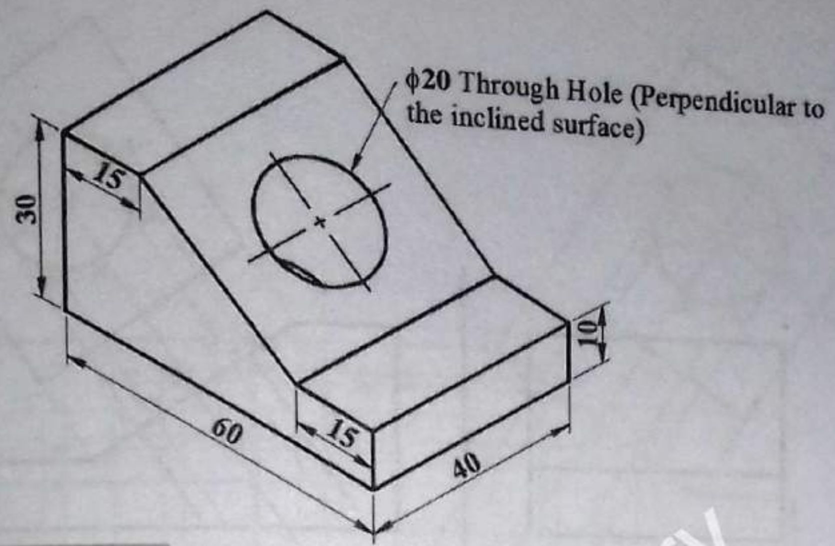


Figure 7.5(a): Object with a Cylindrical Hole on Inclined Surface

Follow the following procedure to complete its projection.

- Draw top view, front view and side view of the object excluding the hole, as shown in *Figure 7.5 (b)*.
- Draw true shape (auxiliary view) of the inclined surface, as shown in *Figure 7.5 (c)*.
- Divide true shape circle into any number of equal parts (say 8) on the auxiliary view, as shown in *Figure 7.5 (d)*. Name the dividing points as $\mathbf{a_1}$, $\mathbf{b_1}$, ………, $\mathbf{h_1}$. Draw projection lines passing through each points on the auxiliary view of the circle towards the front view. Mark the front view of each points as $\mathbf{a_1'}$, $\mathbf{b_1'}$, ………, $\mathbf{h_1'}$ on the inclined surface.
- Draw projection lines passing through each point $\mathbf{a_1'}$, $\mathbf{b_1'}$, ………, $\mathbf{h_1'}$ on the inclined surface towards the top view as well as side view, as shown in *Figure 7.5 (e)*.
- Measure distance of each points on the auxiliary view from the reference line **R'/L'** and transfer them into the respective projection lines on the top view from the reference line **R/L**, as shown in *Figure 7.5 (f)*. Also project each points on the side view.
- Repeat the similar procedure to project the hole on the bottom surface, as shown in *Figure 7.5 (g)*.
- Join the points on the top view and side view in proper sequence to get the required top view and front view of the object, as shown in *Figure 7.5 (h)*.

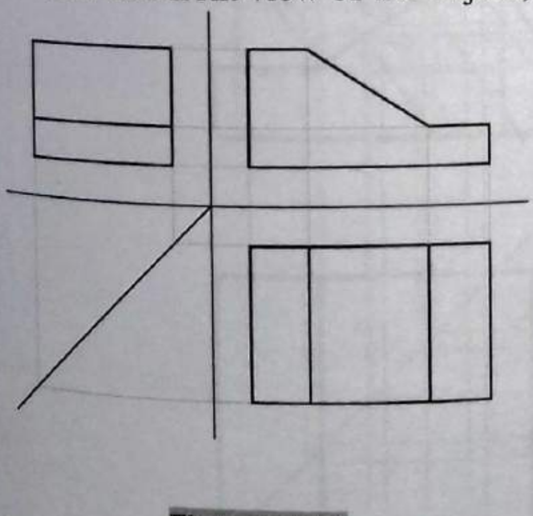


Figure 7.5 (b)

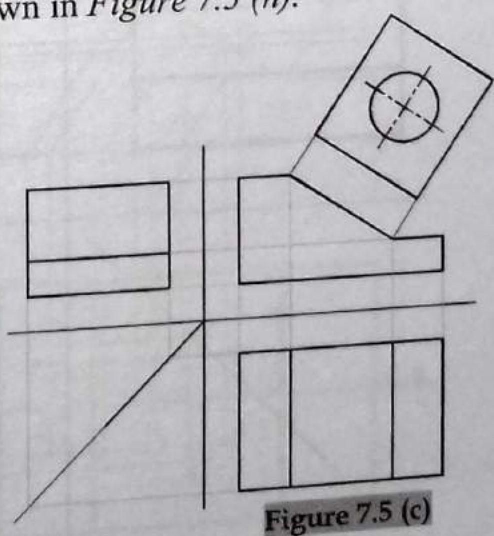


Figure 7.5 (c)

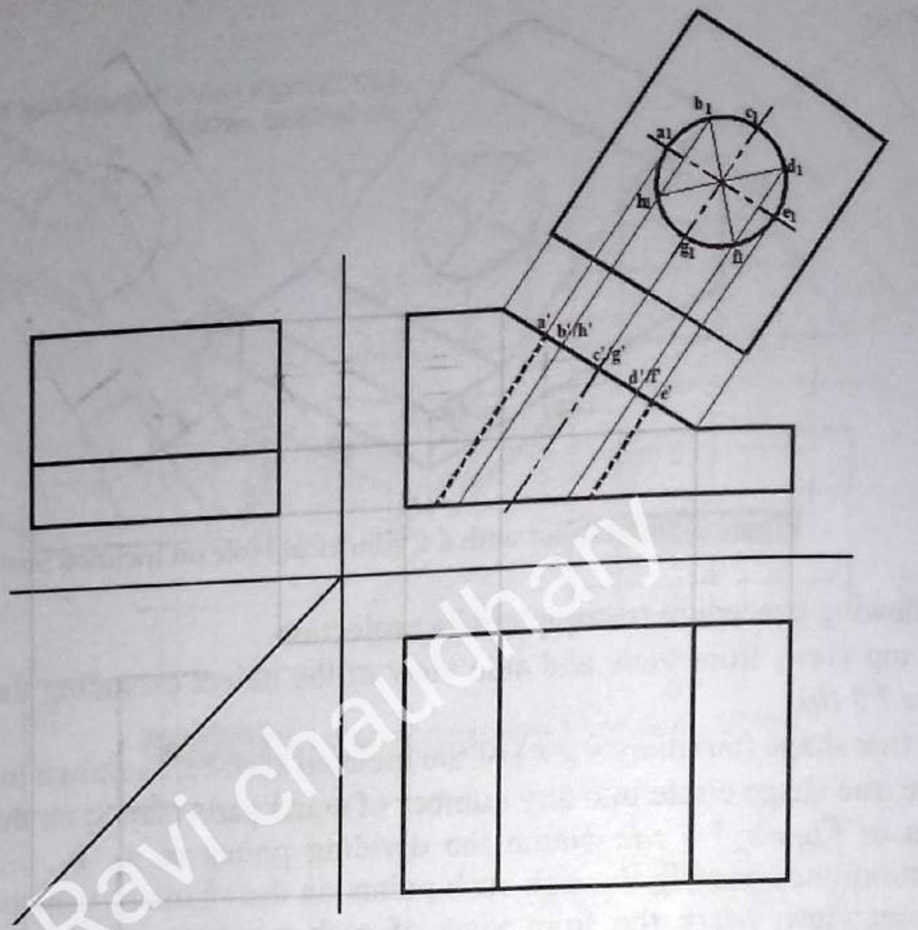


Figure 7.5 (d)

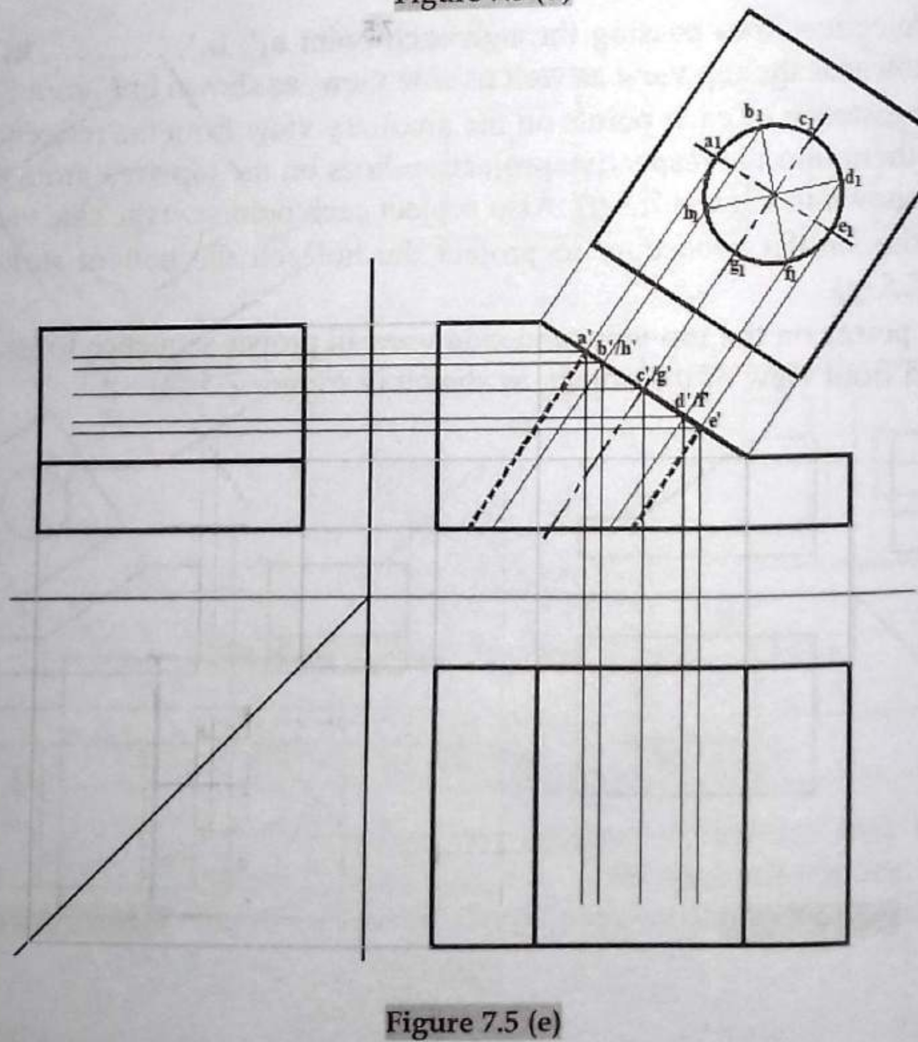


Figure 7.5 (e)

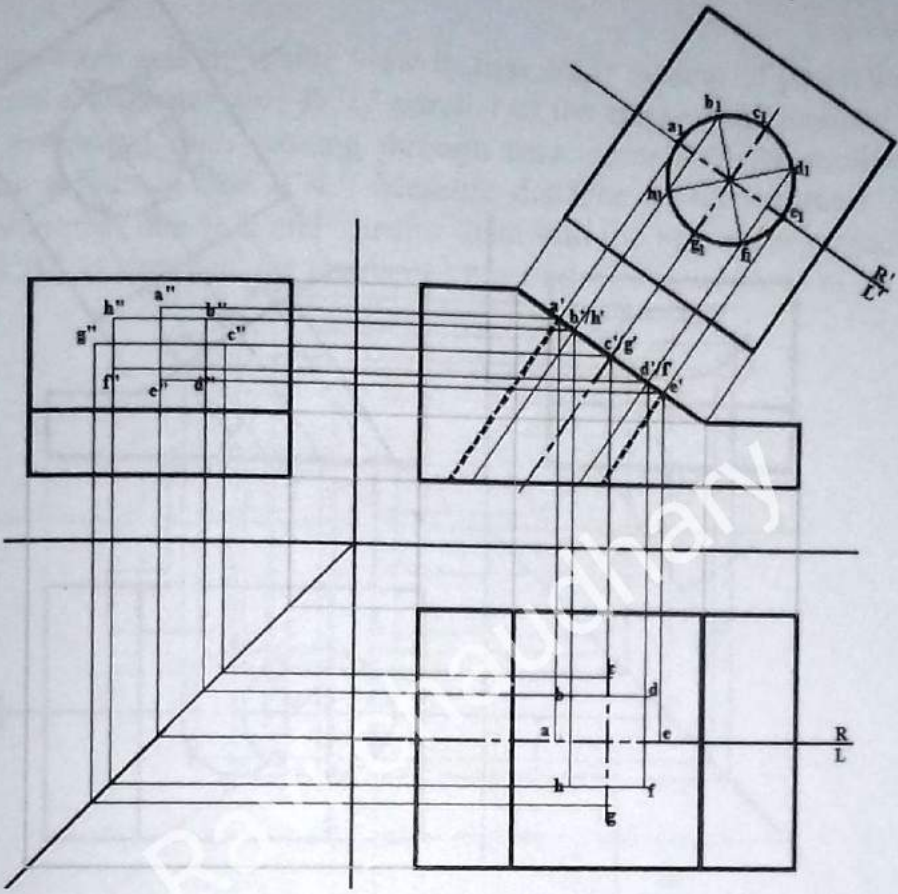


Figure 7.5 (f)

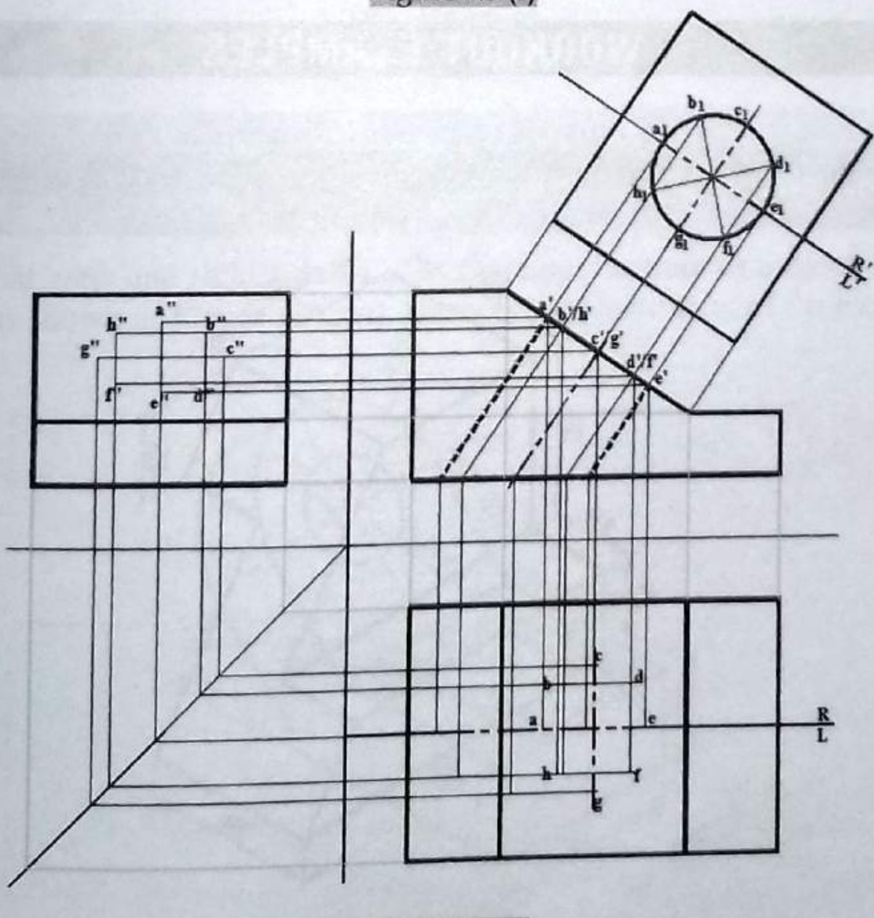


Figure 7.5 (g)

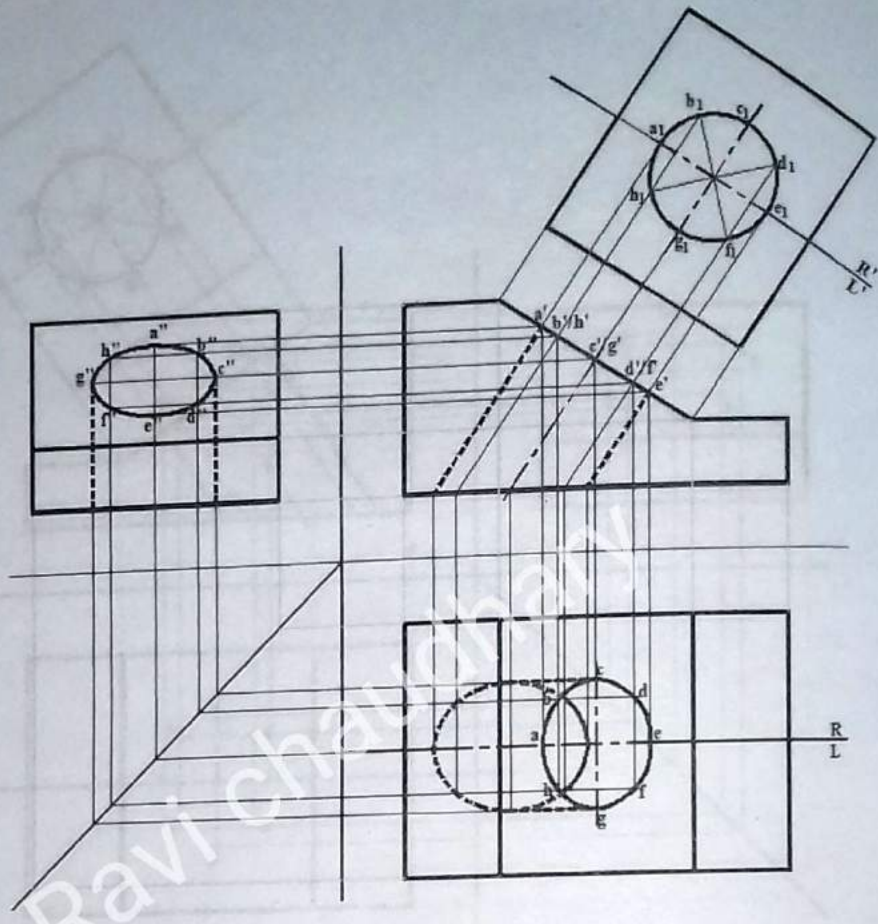


Figure 7.5 (h)

WORKOUT EXAMPLES

Example 7.1

Draw orthographic views of the object shown in *Figure E7.1*. Also draw true shape of the inclined surface.

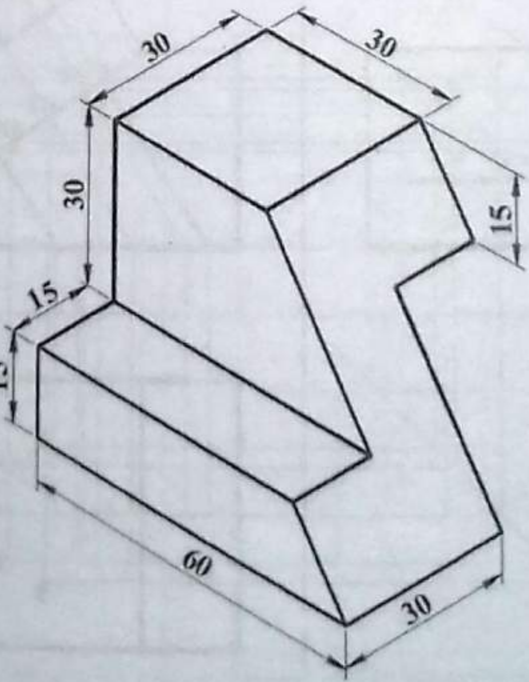


Figure E7.1

Solution

Draw top view, front view and right side view in first angle system of projection as shown in *Figure E7.1(a)*. Draw a reference line **R'/L'** parallel to the edge of the inclined surface on the front view. Draw projection lines passing through each corners of the inclined surface and perpendicular to the reference line **R'/L'**. Measure distance of each corners of the inclined surface from the reference line **R/L** and transfer them into the respective projection lines from the reference line **R'/L'** to complete the auxiliary view of the inclined surface.

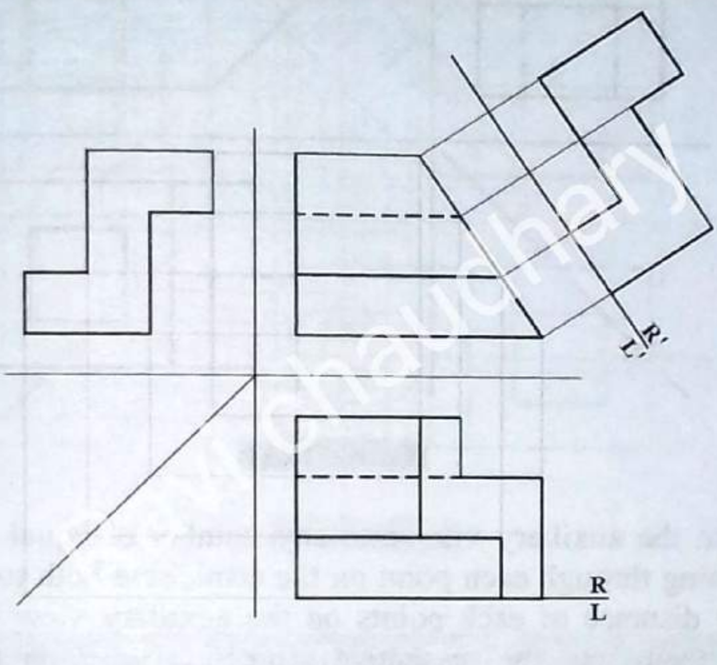


Figure E7.1(a)

Example 7.2

Complete orthographic views of the object shown in *Figure E7.2* with the help of an auxiliary view.

Solution

Draw top view, front view and right side view in first angle system of projection excluding the circular contours, as shown in *Figure E7.2(a)*. Draw an auxiliary view of the inclined surface.

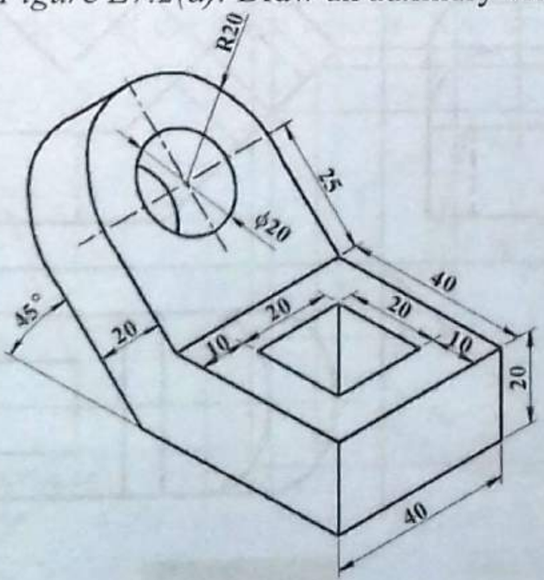


Figure E7.2

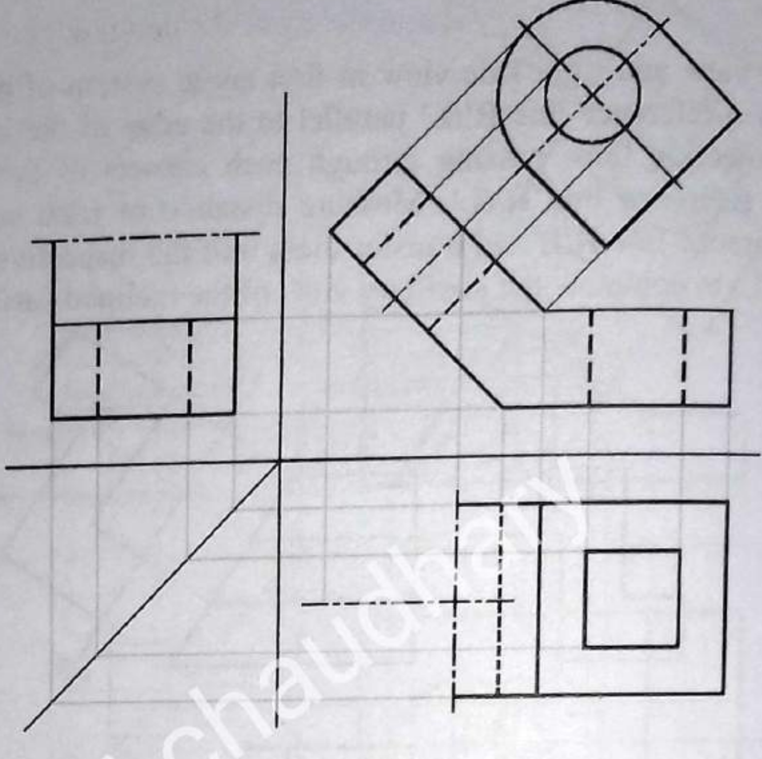


Figure E7.2 (a)

Divide semicircle on the auxiliary view into any number of equal parts (say 4) and draw projection lines passing through each point on the semicircle both towards the top view and side view. Measure distance of each points on the auxiliary view from the reference line **R'/L'** and transfer them into the respective projection lines on the top view from the reference line **R/L**, as shown in *Figure E7.2 (b)*. Join the points on the proper sequence to get the top view of the semicircle. Also project each point to the side view to get the side view of the semicircle.

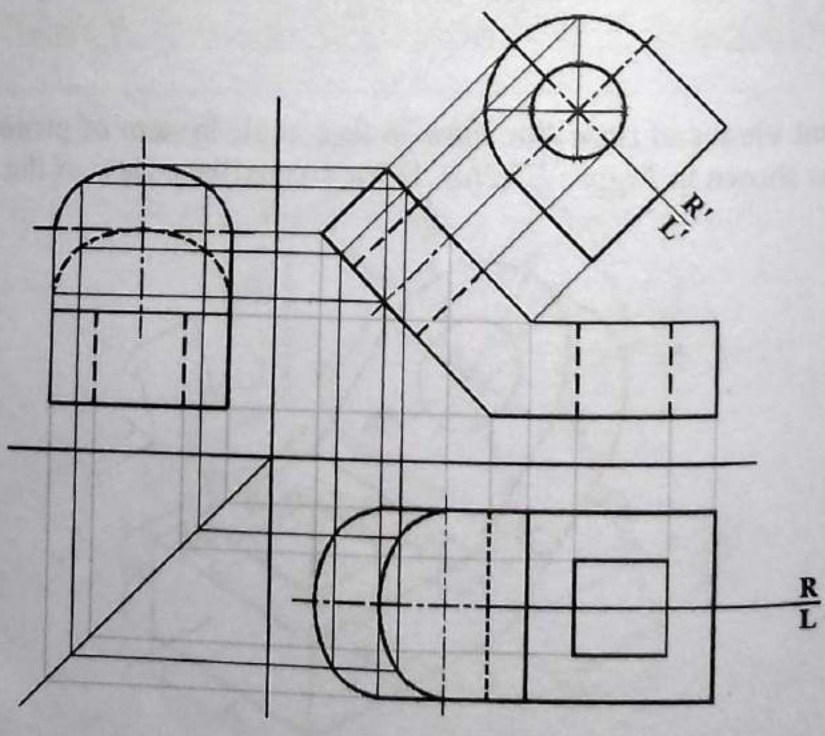


Figure E7.2 (b)

Repeat the similar procedure to complete the top view and side view cylindrical hole as shown in *Figure E7.2(c)*.

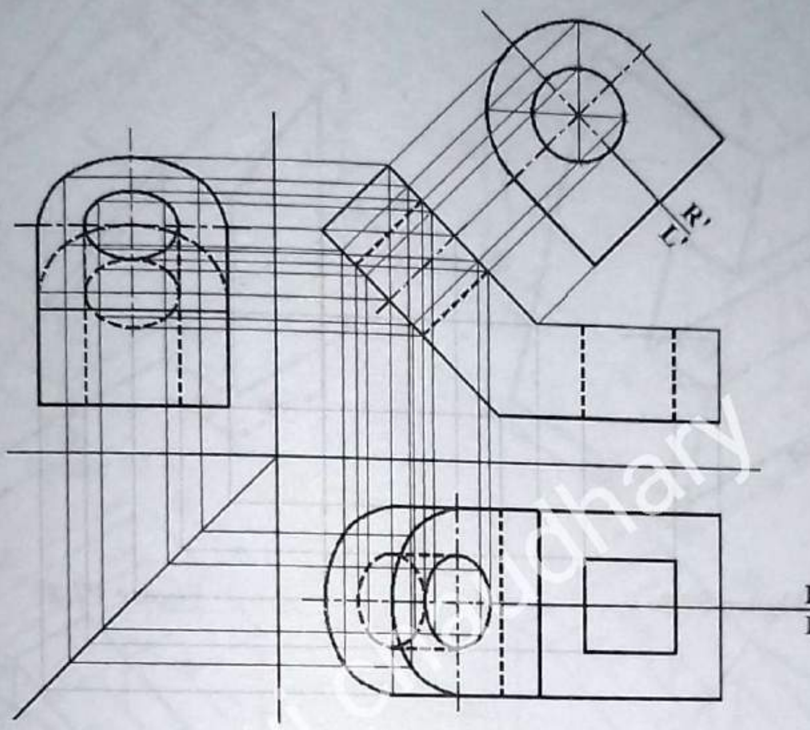


Figure E7.2 (c)

REVIEW QUESTIONS

1. Define an auxiliary plane and auxiliary view.
2. Explain why an auxiliary view is necessary
3. Differentiate between unilateral and bilateral auxiliary views.

EXERCISES

1. to 12. Draw orthographic views of the given objects. Also draw an auxiliary view of the inclined section.

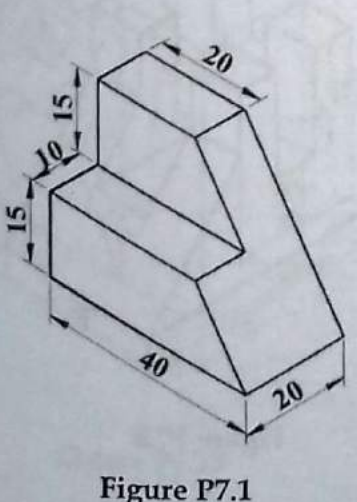


Figure P7.1

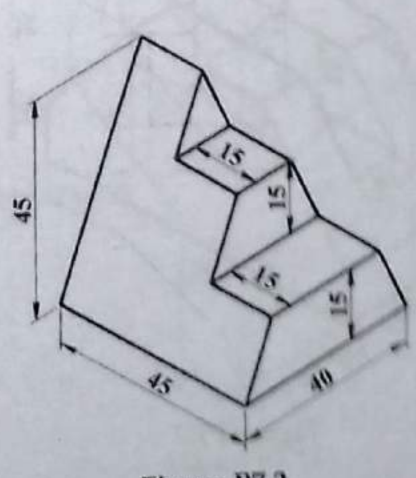


Figure P7.2

CHAPTER 8

DEVELOPMENT OF SURFACES

8.1 Introduction

Development of surfaces is the process of constructing a single two-dimensional pattern that covers the surfaces of the given solid completely. Development of surfaces can also be considered as unwrapping the imaginary surfaces enclosing a given solid such that the resulting pattern is a single two-dimensional pattern as shown in *Figure 8.1*.

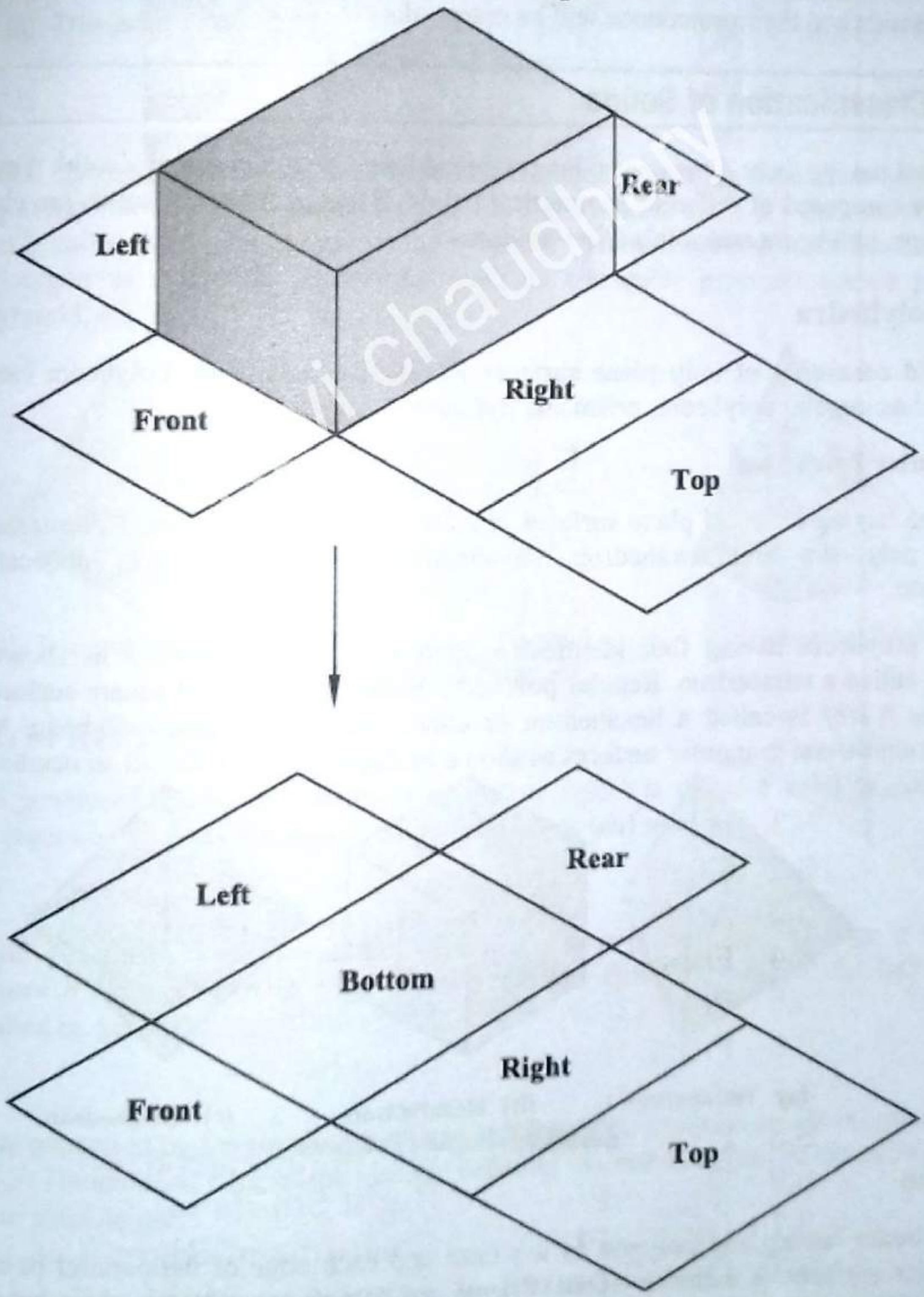


Figure 8.1: Surface Development of a Solid

Hence development of any solid can be done by constructing true shape and size of each surfaces of the solid in series with their common edges. The development of the surfaces except the top and bottom surfaces of the solid is called lateral surface development.

The development of surfaces is essential for many design and manufacturing process such as sheet metal work, pattern making, etc.

To have a clear idea about the development of surfaces of solids, the knowledge about different types of solids and their projections will be essential.

8.2 Classification of Solids

Any object having three dimensions: length, breadth and height is called a solid. Any real object is usually composed of different geometrical solids. Basic geometrical solids are classified into two groups: polyhedra and solids of revolution.

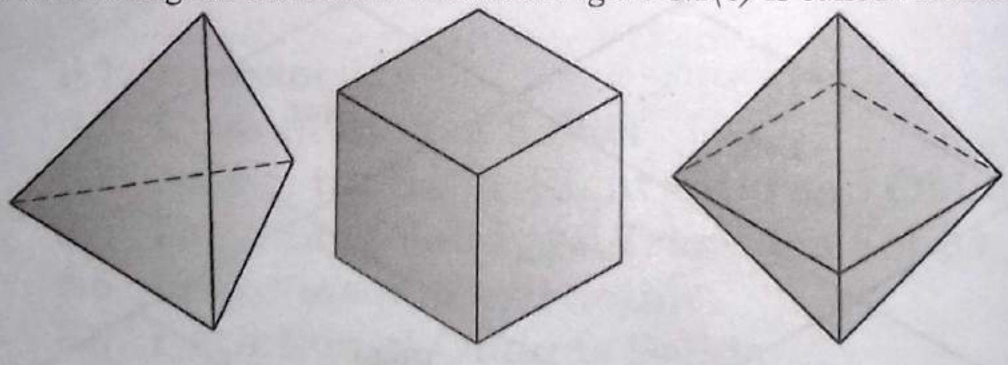
8.2.1 Polyhedra

Any solid consisting of only plane surfaces is called a polyhedra. Polyhedra can further be classified as: regular polyhedra, prism and pyramid.

(a) Regular Polyhedra

Polyhedra having identical plane surfaces is called a regular polyhedra. Common examples of regular polyhedra are: tetrahedron, hexahedron (cube), octahedron, dodecahedron and icoshedron.

Regular polyhedra having four identical equilateral triangular surfaces as shown in *Figure 8.2(a)* is called a tetrahedron. Regular polyhedra having six identical square surfaces as shown in *Figure 8.2(b)* is called a hexahedron or cube. Similarly, regular polyhedra having eight identical equilateral triangular surfaces as shown in *Figure 8.2(c)* is called an octahedron.

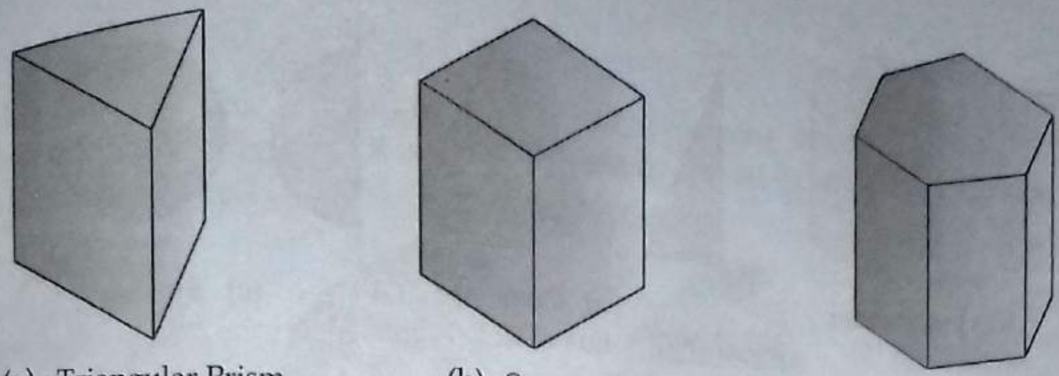


(a) Tetrahedron **(b) Hexahedron** **(c) Octahedron**

Figure 8.2: Regular Polyhedron

(b) Prism

Any poyhedra having any polygon as it s base and each edge of the parallel bases joined the rectangular surfaces is called a prism. Prisms are named according to their base polygon as shown in *Figure 8.3* such as triangular prism, square prism, hexagonal prism, etc.

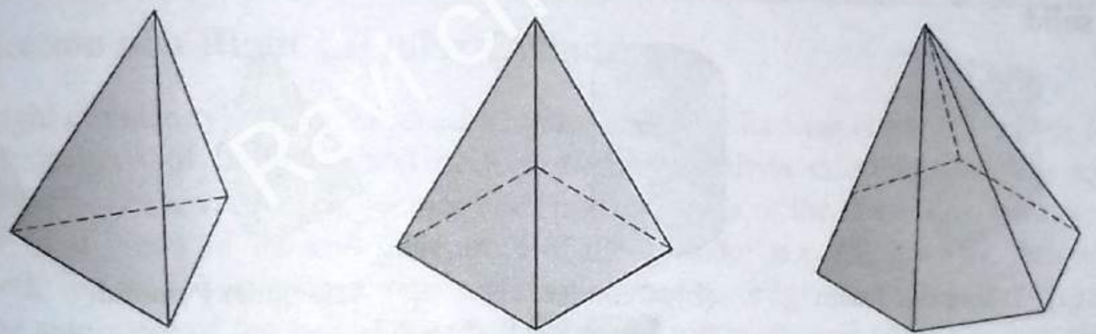


(a) Triangular Prism (b) Square Prism (c) Hexagonal Prism

Figure 8.3: Prisms

(c) Pyramid

Any polyhedral having any polygon as it base and triangular surfaces from each edge of the base meeting at a common point is called a pyramid. The common point at which the triangular surfaces meet is called the vertex or apex of the pyramid. Pyramids are also named according to their base polygon as shown in *Figure 8.4* such as triangular pyramid, square pyramid, hexagonal pyramid, etc.



(a) Triangular Pyramid (b) Square Pyramid (c) Hexagonal Pyramid

Figure 8.4: Pyramids

8.2.2 Solids of Revolution

Curved solid generated by the revolution of any plane figure is called a solid of revolution. Common examples of solids of revolution are cylinder cone and sphere.

(a) Cylinder

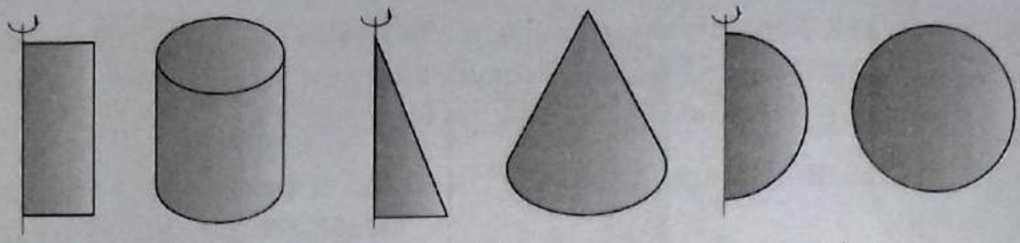
A curved solid generated by the revolution of a rectangular plane about its edge is called a cylinder *(Figure 8.5(a))*. The outer edge of the rectangle defining the material limit of the cylinder is called its generator.

(b) Cone

A curved solid generated by the revolution of a triangular plane about its height is called a cone *(Figure 8.5(b))*. The inclined edge of the triangle defining the material limit of the cone is called its generator or slant height.

(c) Sphere

A curved solid generated by the revolution of a semicircular plane about its diameter is called a sphere *(Figure 8.5(c))*.



(a) Cylinder (b) Cone (c) Sphere

Figure 8.5: Solids of Revolution

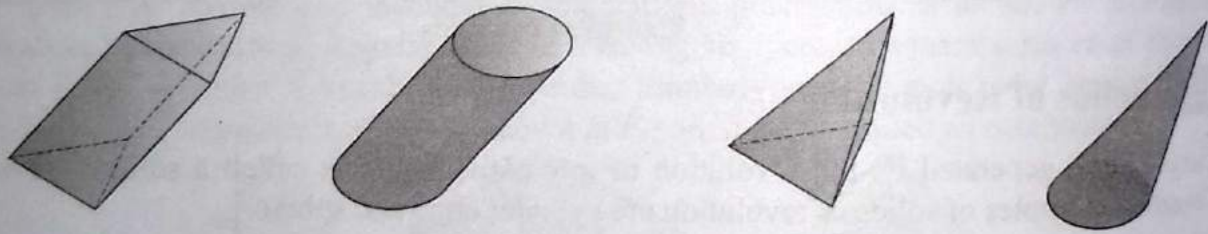
8.3 Axis of the Solid, Right Solid and Oblique Solid

In case of prism and cylinder, the line joining the center points of the parallel bases is called the axis, whereas in case of pyramid and cone, the line joining the center point of the base and the vertex of the solid is called the axis.

If the axis of the solid is perpendicular to its base as shown in *Figure 8.6*, it is called a right solid, whereas if the axis of the solid is inclined to its base as shown in *Figure 8.7*, it is called an oblique solid.

(a) Triangular Prism (b) Cylinder (c) Triangular Pyramid (d) Cone

Figure 8.6: Right Solids

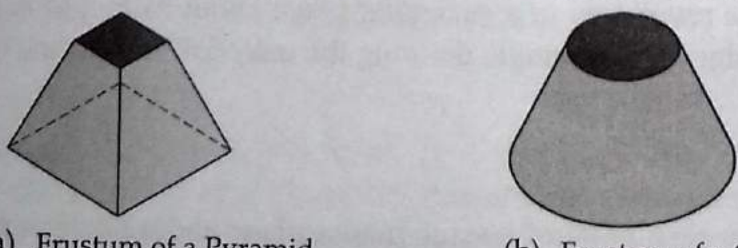


(a) Triangular Prism (b) Cylinder (c) Triangular Pyramid (d) Cone

Figure 8.7: Oblique Solids

8.4 Frustum of Solid and Truncated Solids

When a pyramid or a cone is cut by a plane parallel to its base, the remaining bottom portion of the solid is called its frustum. *Figure 8.8* shows the frustums of a square pyramid and a cone.



(a) Frustum of a Pyramid (b) Frustum of a Cone

Figure 8.8: Frustums of Solids

When a solid is cut by a plane inclined to its base, the remaining bottom portion of the solid is called a truncated solid. *Figure 8.9* shows truncated prism, truncated cylinder, truncated pyramid and truncated cone.

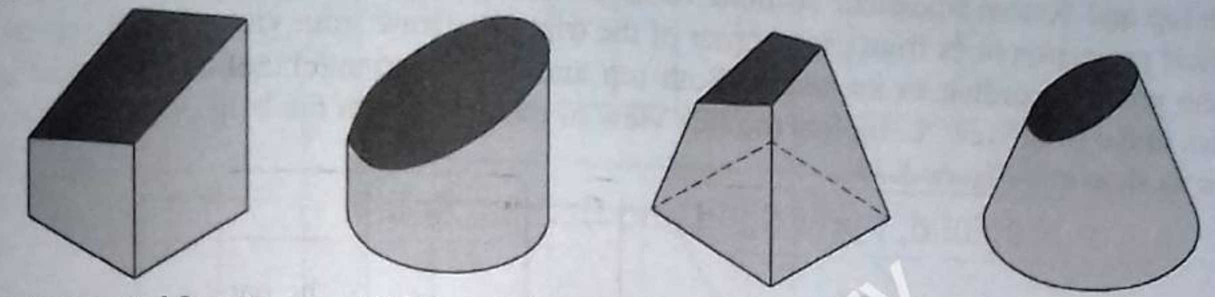


Figure 8.9: Truncated Solids

8.5 Projection of Right Solids

8.5.1 Projection of a Right Circular Cylinder

Consider a right circular cylinder positioned in space such that its base is parallel to the HP. The horizontal projections of both top and bottom circular sections coincide and the top view appears as a circle. Draw vertical projection lines from the ends of the horizontal diameter of the circle. Draw front views of the end generators of the cylinder according to the height of the cylinder. Both top and bottom circular sections appear as edge views in the front view. Complete the side view of the cylinder with the help of top view and front view as shown in *Figure 8.10*.

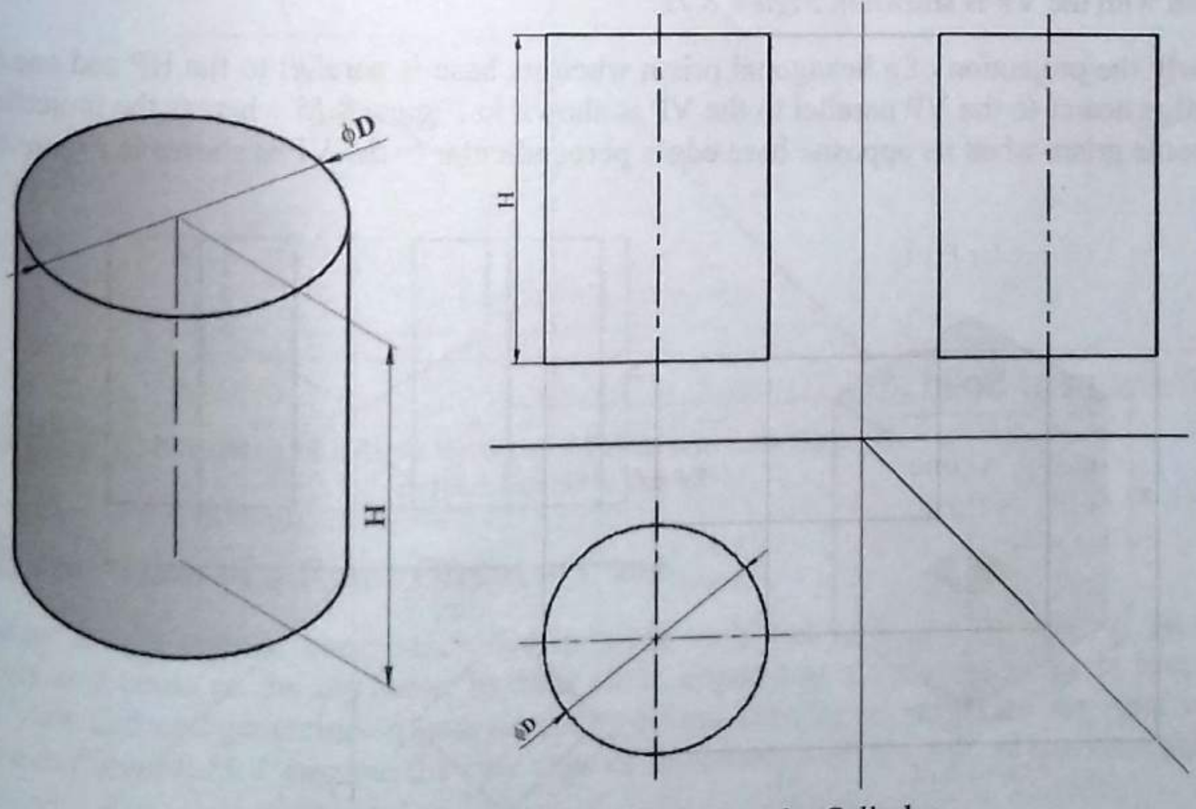


Figure 8.10: Projection of a Right Circular Cylinder

8.5.2 Projections of Right Prisms

Consider a triangular prism positioned in space such that its base is parallel to the HP and its one base edge nearer to the VP is parallel to it. In this case also, the horizontal projections of both top and bottom triangular sections coincide and the top view appears as a triangle. Draw vertical projection lines from each corner of the triangle. Draw front views of the vertical edges of the prism according to its height. Both top and bottom triangular sections appear as edge views in the front view. Complete the side view of the prism with the help of top view and front view as shown in *Figure 8.11*.

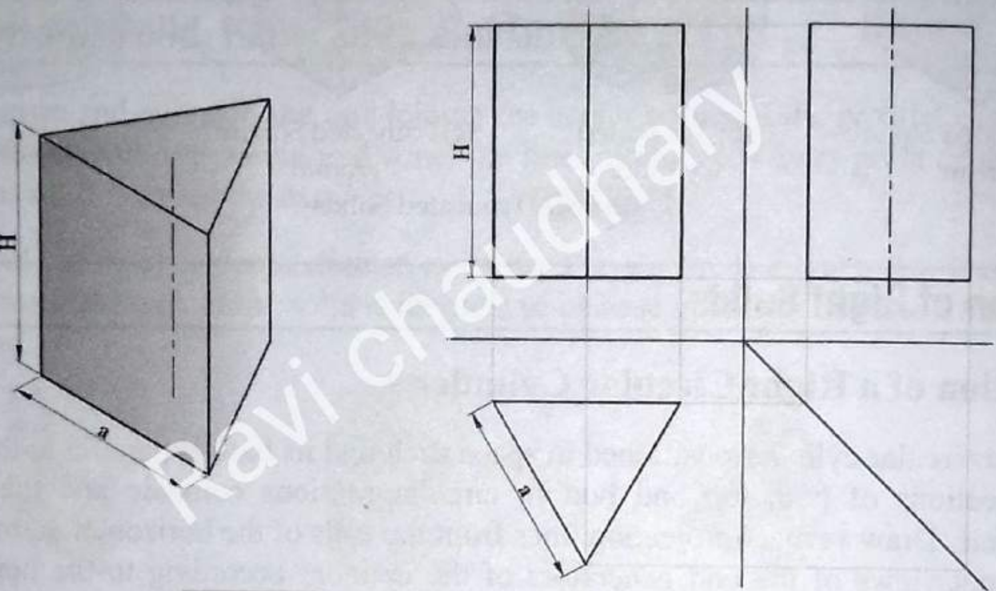


Figure 8.11: Projection of a Right Triangular Prism

The projection of a square prism when its base is parallel to the HP and its base edges equally inclined with the VP is shown in *Figure 8.12*.

Similarly the projection of a hexagonal prism when its base is parallel to the HP and one of its base edge nearer to the VP parallel to the VP is shown in *Figure 8.13* whereas the projection of a the same prism when its opposite base edges perpendicular to the VP is shown in *Figure 8.14*.

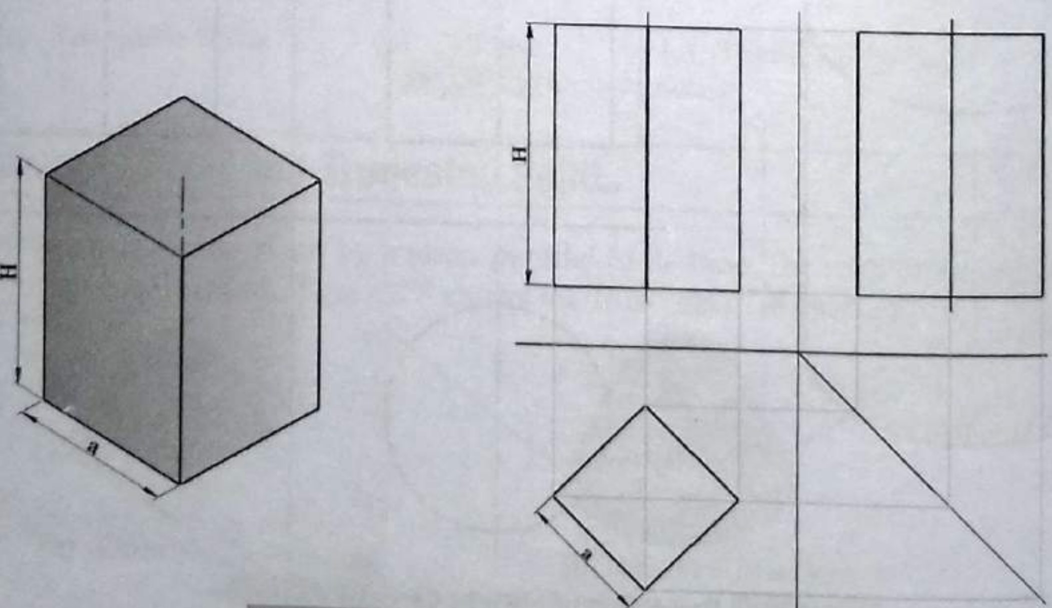


Figure 8.12: Projection of a Right Square Prism

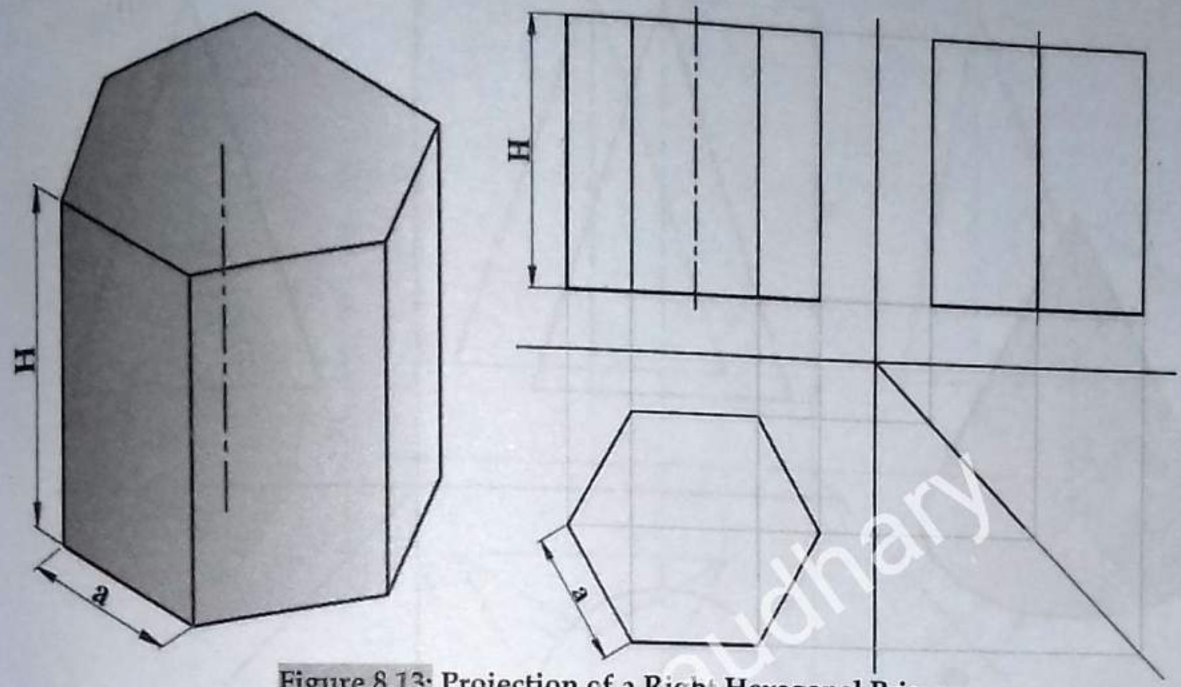


Figure 8.13: Projection of a Right Hexagonal Prism

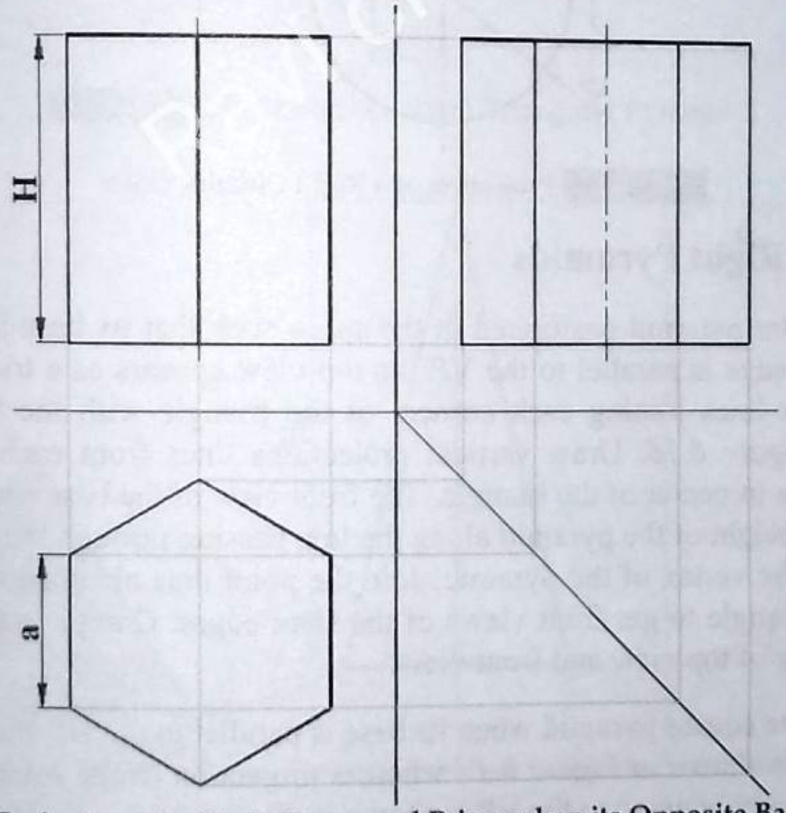


Figure 8.14: Projection of a Right Hexagonal Prism when its Opposite Base Edges are Perpendicular to the VP

8.5.3 Projection of a Right Circular Cone

Consider a right circular cone positioned in space such that its base is parallel to the HP. It appears as a circle on the top view. Its base circle appears as a horizontal straight line on the front view and end generators appear as inclined lines meeting at vertex on the front view as shown in *Figure 8.15*. Complete the side view of the corner with the help of top view and front view.

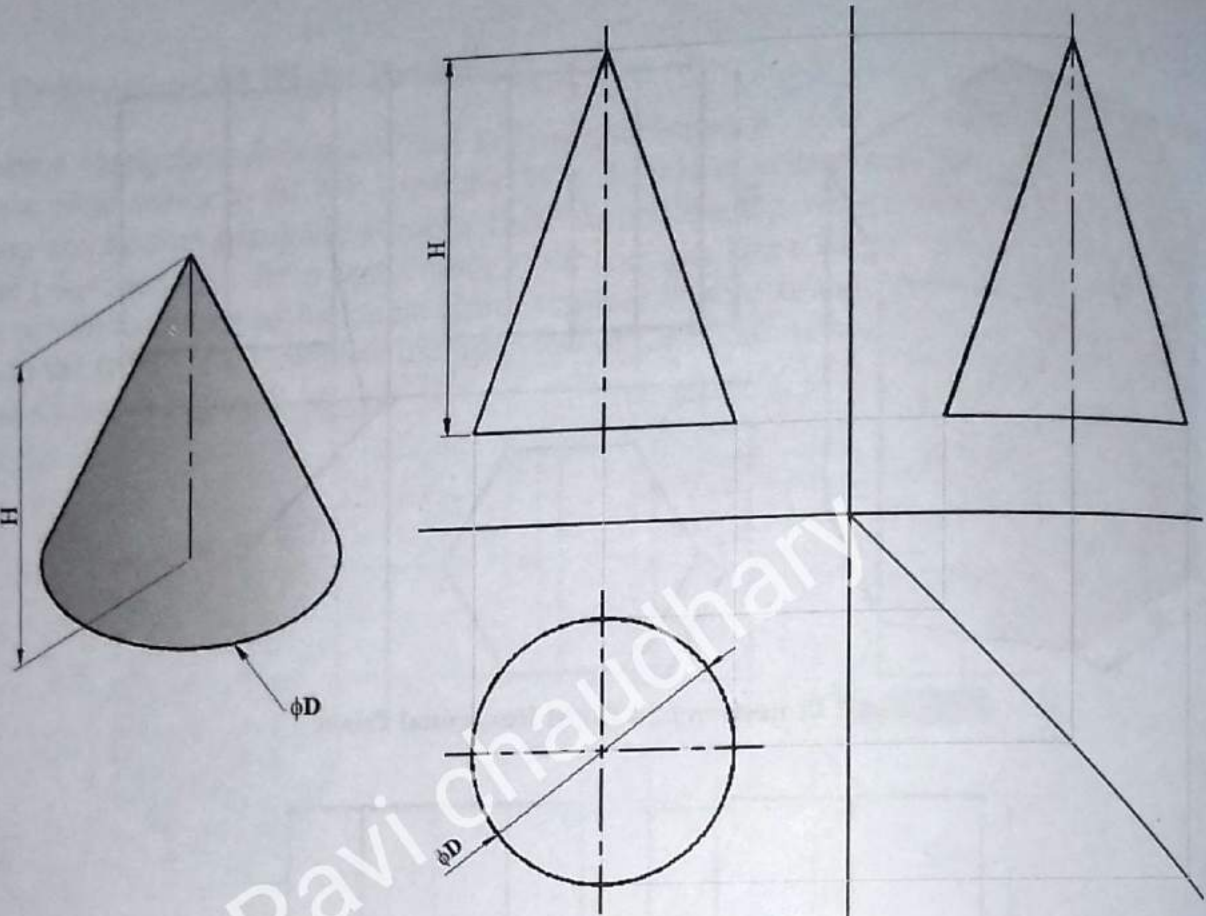


Figure 8.15: Projection of a Right Circular Cone

8.5.4 Projections of Right Pyramids

Consider a right triangular pyramid positioned in the space such that its base is parallel to the HP and one of the base edge is parallel to the VP. Its top view appears as a triangle. The slant edges appear as straight lines joining each corners of the triangle with the in-center of the triangle as shown in *Figure 8.16*. Draw vertical projections lines from each corners of the triangle and also from the in-center of the triangle. The front view of the base triangle appears as an edge view. Mark off height of the pyramid along the line passing through the in-center which gives the front view of the vertex of the pyramid. Join the point thus obtained with the ends of edge view of the base triangle to get front views of the slant edges. Complete the side view of the pyramid with the help of top view and front view.

Similarly projection of the square pyramid when its base is parallel to the HP and one of its base edge parallel to the VP is shown in *Figure 8.17* whereas projection of the same pyramid when its all base edges are equally inclined to the VP is shown in *Figure 8.18*.

Similarly projection of the hexagonal pyramid when its base is parallel to the HP and one of its base edge parallel to the VP is shown in *Figure 8.19* whereas projection of the same pyramid when its opposite base edges are perpendicular to the VP is shown in *Figure 8.20*.

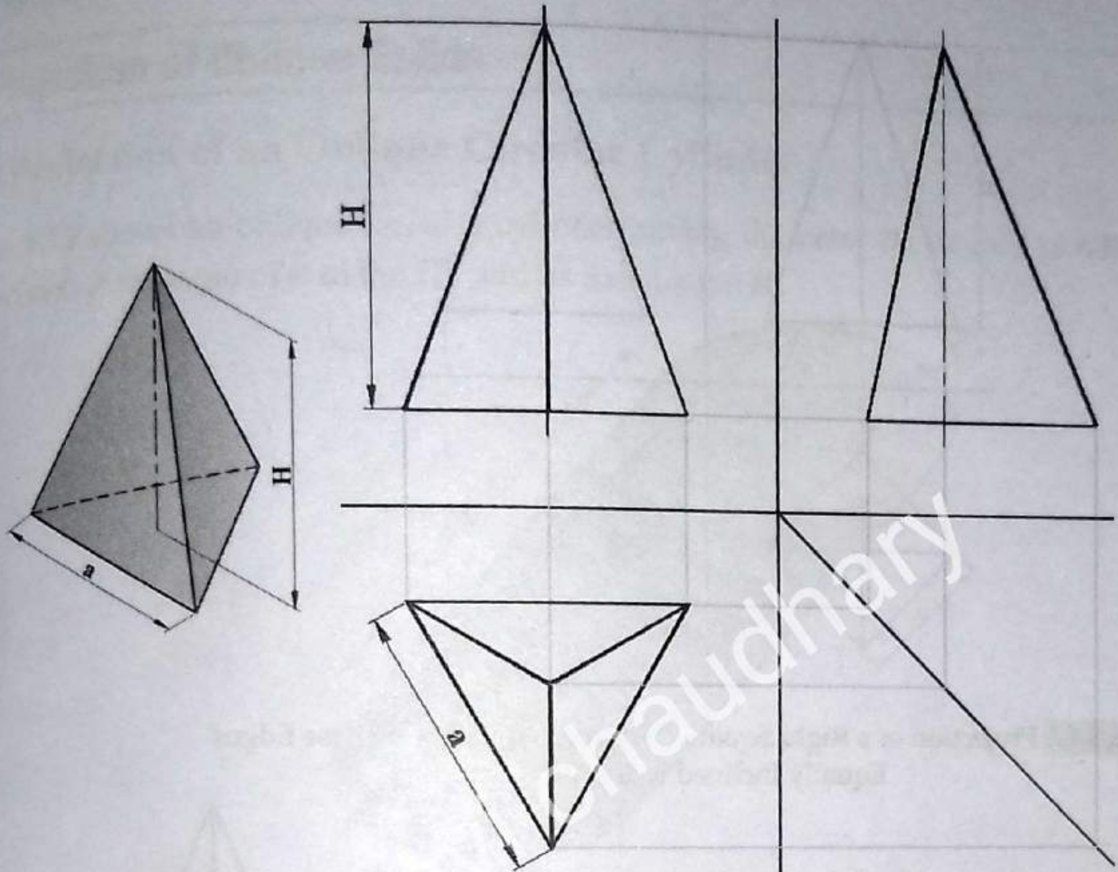


Figure 8.16: Projection of a Right Triangular Pyramid

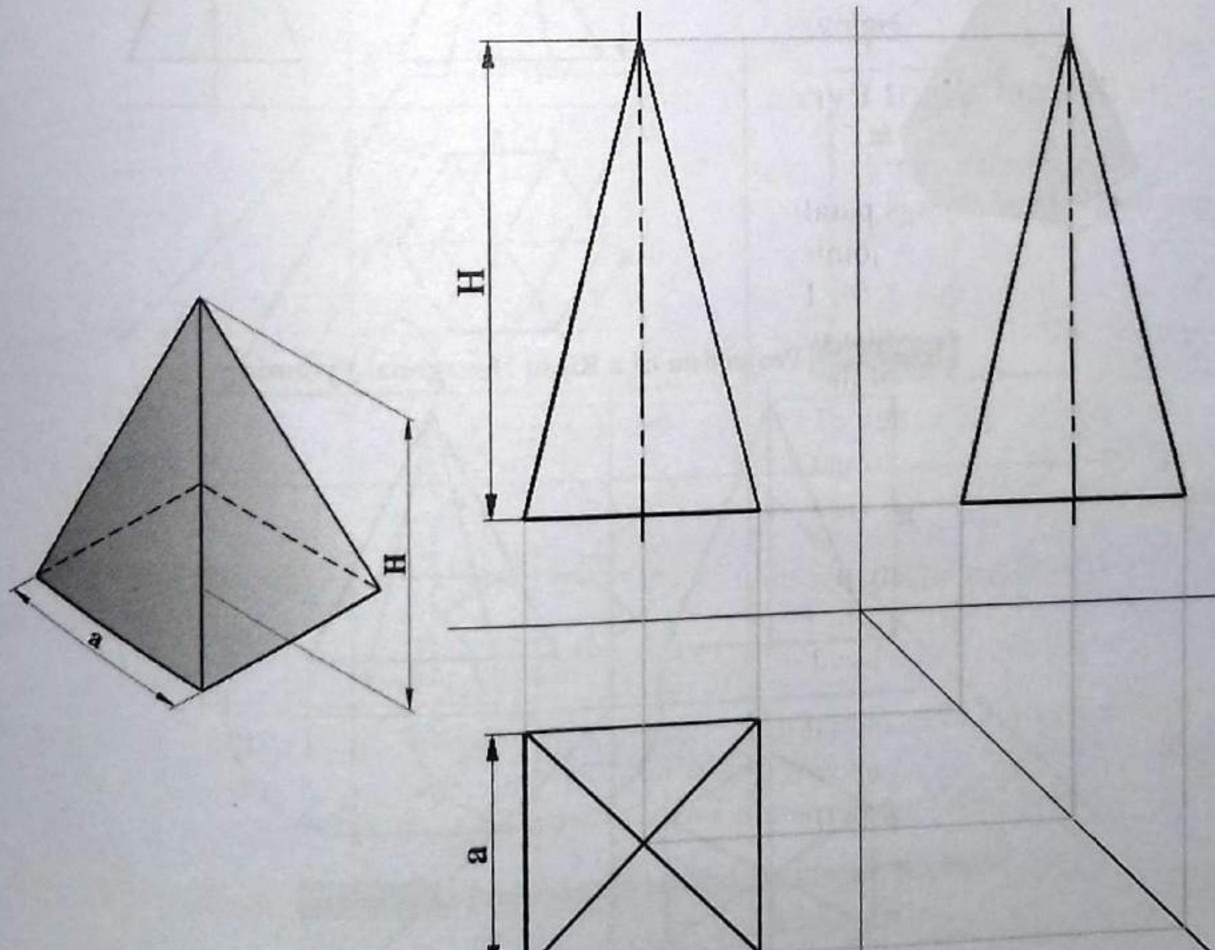


Figure 8.17: Projection of a Right Square Pyramid

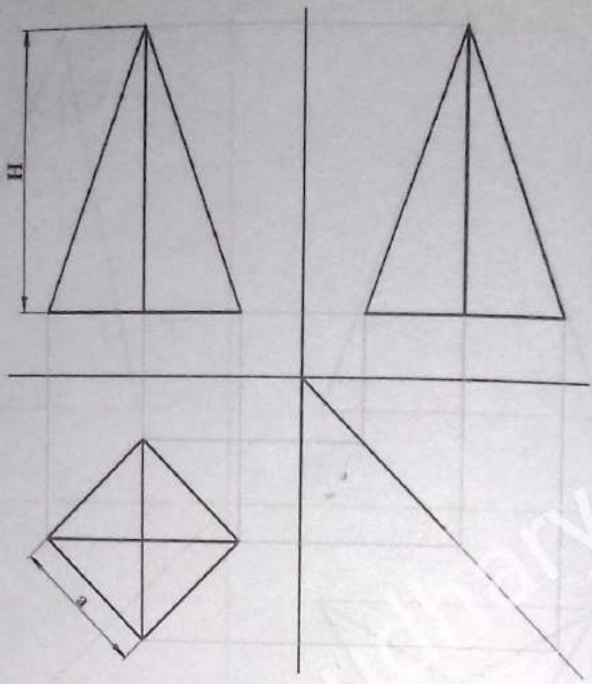


Figure 8.18: Projection of a Right Square Pyramid with all of its Base Edges Equally Inclined to the VP

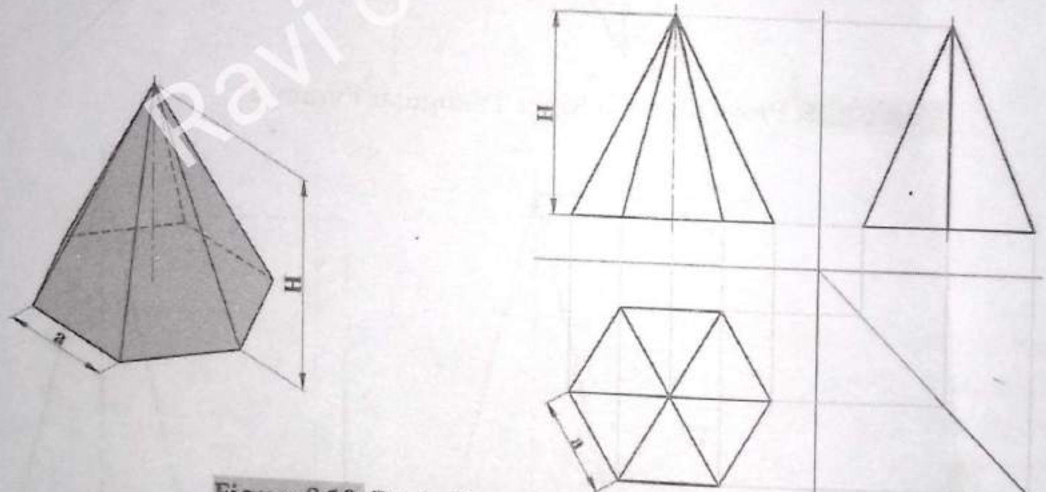


Figure 8.19: Projection of a Right Hexagonal Pyramid

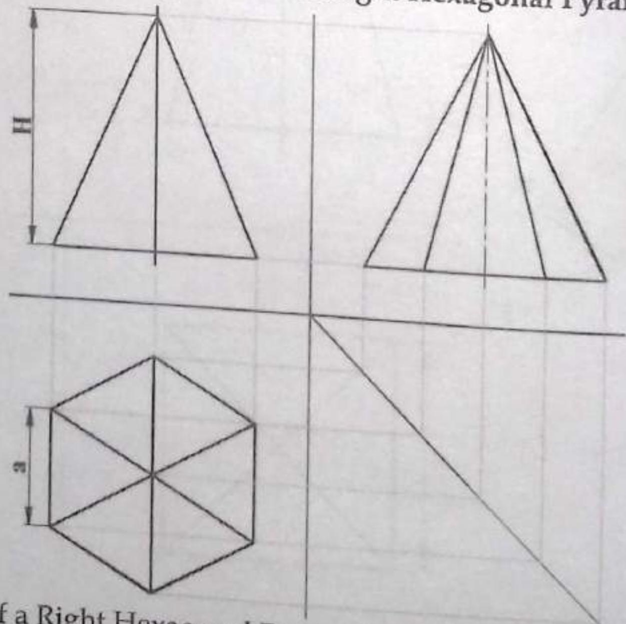


Figure 8.20: Projection of a Right Hexagonal Pyramid with is Opposite Base Edges Perpendicular to the VP

8.6 Projection of Oblique Solids

8.6.1 Projection of an Oblique Circular Cylinder

Figure 8.21 shows an oblique circular cylinder having diameter D, its axis parallel to the VP and inclined at an angle of α to the HP and its axis length H.

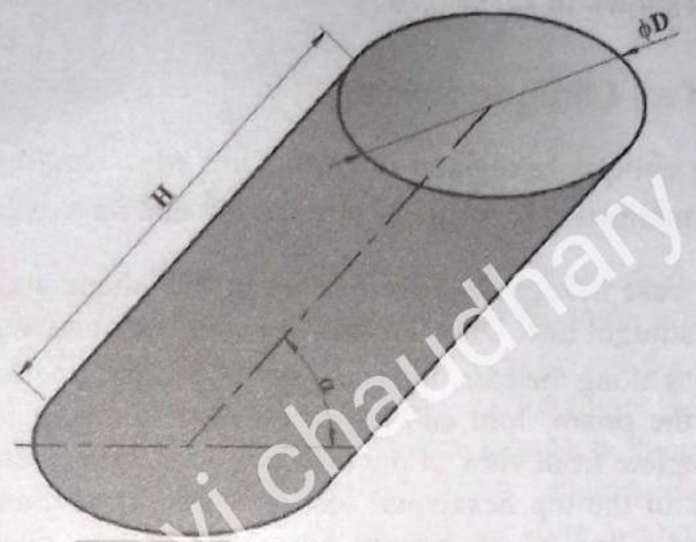


Figure 8.21: An Oblique Circular Cylinder

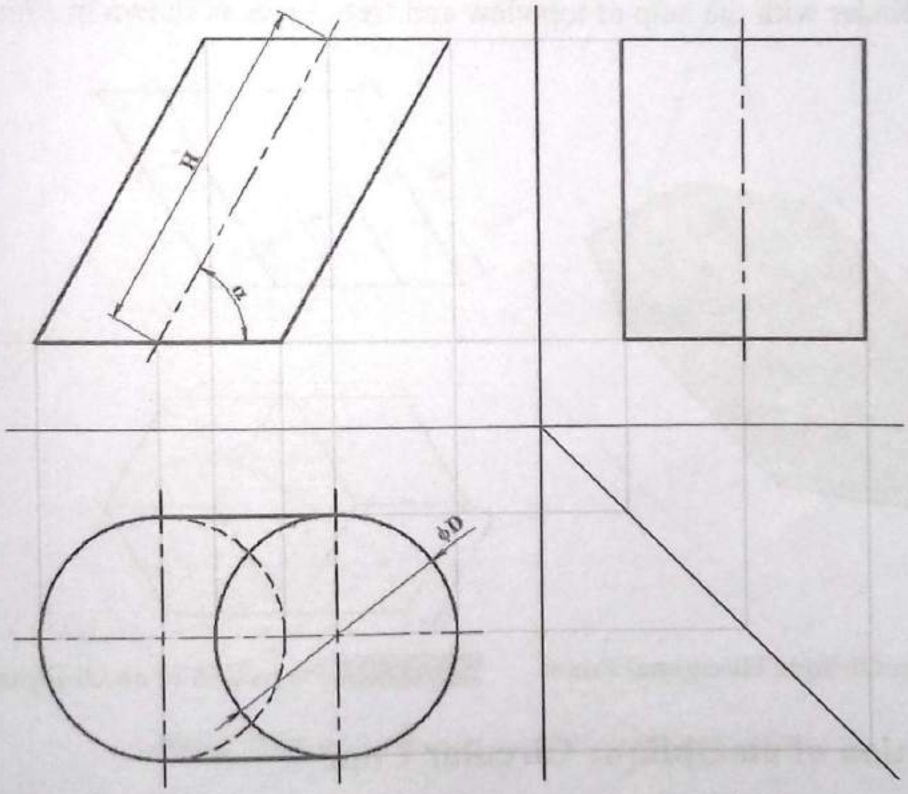


Figure 8.22: Projection of an Oblique Circular Cylinder

Draw top view of the base circle which appears in true shape and size. Draw front view of the base circle as a straight line. Draw an axis line inclined at an angle α to the base and mark off the given axis length along the axis line and draw horizontal line for the front view of the top circular section of the cylinder. Join end points of edge views of top and bottom circular sections to get the complete front view of the oblique cylinder. Draw vertical projection lines from the ends of edge view of the top circular section. The intersection of these projection lines with the extended diameter of the bottom circular section gives the end points of the diameter for the top view of the top circular section. Draw tangent lines to the circles to get the complete top view of the oblique cylinder. Complete the side view of the cylinder with the help of top view and front view as shown in *Figure 8.22*.

8.6.2 Projection of an Oblique Prism

Figure 8.23 shows an oblique hexagonal prism having edge length of base a, its axis its axis parallel to the VP and inclined at an angle of α to the HP and its axis length H.

Draw top view of the base hexagon which appears in true shape and size. Draw front view of the base hexagon as a straight line. Draw an axis line inclined at an angle α to the base and mark off the given axis length along the axis line and draw horizontal line for the front view of the top hexagonal section of the prism. Join end points of edge views of top and bottom hexagonal sections to get the complete front view of the oblique prism. Draw vertical projection lines from the ends of edge view of the top hexagonal section. The intersection of these projection lines with the extended center line of the bottom hexagonal section gives the end points for the opposite corners for the top view of the top hexagonal section. Join the corresponding corners point of the hexagon to get the top view of the inclined edge of the hexagon. Complete the side view of the cylinder with the help of top view and front view as shown in *Figure 8.24*.

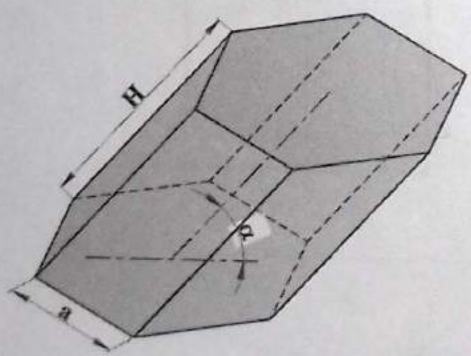


Figure 8.23: An Oblique Hexagonal Prism

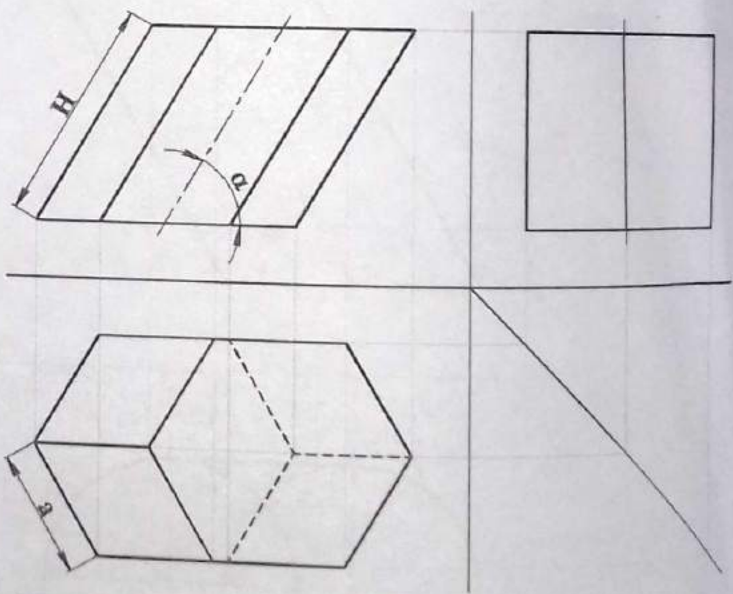


Figure 8.24: Projection of an Oblique Hexagonal Prism

8.6.3 Projection of an Oblique Circular Cone

Figure 8.25 shows an oblique circular cone having diameter D, its axis paralle to the VP and inclined at an angle of α to the HP and its axis length H.

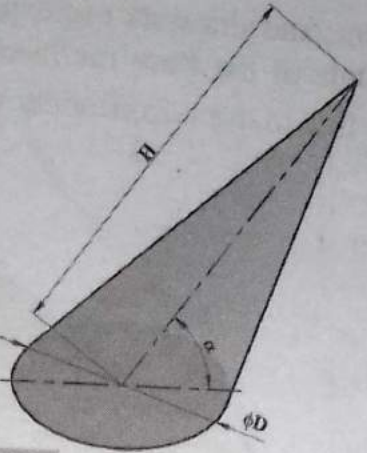


Figure 8.25: An Oblique Circular Cylinder

Draw top view of the base circle which appears in true shape and size. Draw front view of the base circle as a straight line. Draw an axis line inclined at an angle α to the base and mark off the given axis length along the axis line and draw front view (v') of the vertex of the cone. Join v' with the end points of edge views of bottom circular section to get the complete front view of the oblique cone. Draw vertical projection line v' which intersects the extended diameter of the bottom circular section on the top view at point v. Draw tangent lines to the circle passing through point v to get the complete top view of the oblique cone. Complete the side view of the cylinder with the help of top view and front view as shown in *Figure 8.26.*

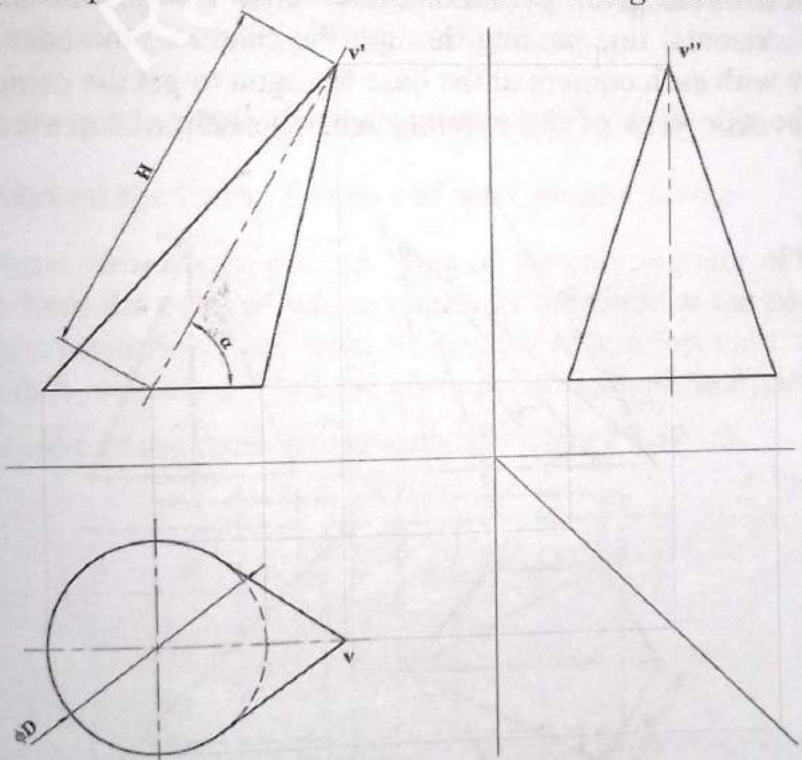


Figure 8.26: Projection of an Oblique Circular Cone

8.6.4 Projection of an Oblique Pyramid

Figure 8.27 shows an oblique hexagonal pyramid having edge length of base a, its axis its axis parallel to the VP and inclined at an angle of α to the HP and its axis length H.

Draw top view of its base hexagon and draw its corresponding front view. Draw an axis line from the midpoint of the edge view of the base inclined at an angle α to the horizontal line. Mark axis length H along the axis line to draw front view v' of the vertex V of the pyramid.

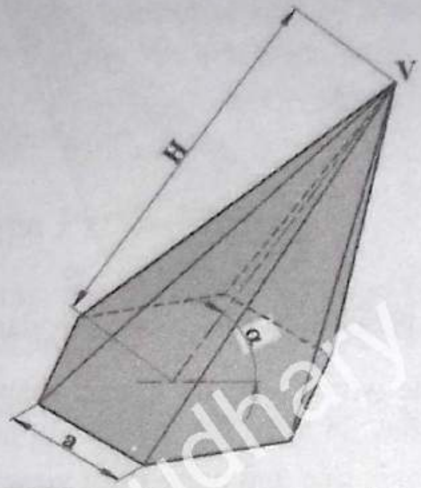


Figure 8.27: An Oblique Hexagonal Pyramid

Draw vertical projection lines from each corners of the hexagon from the top view to get their respective front views. Join v' with the front views of each corners of the base hexagon to get the complete front view of the given pyramid. Draw vertical projection line from the point v' which intersects the horizontal line passing through the center of the base hexagon on the top view at point v. Join v with each corners of the base hexagon to get the complete top view of the pyramid. Complete the side view of the pyramid with the help of top view and front view as shown in *Figure 8.28.*

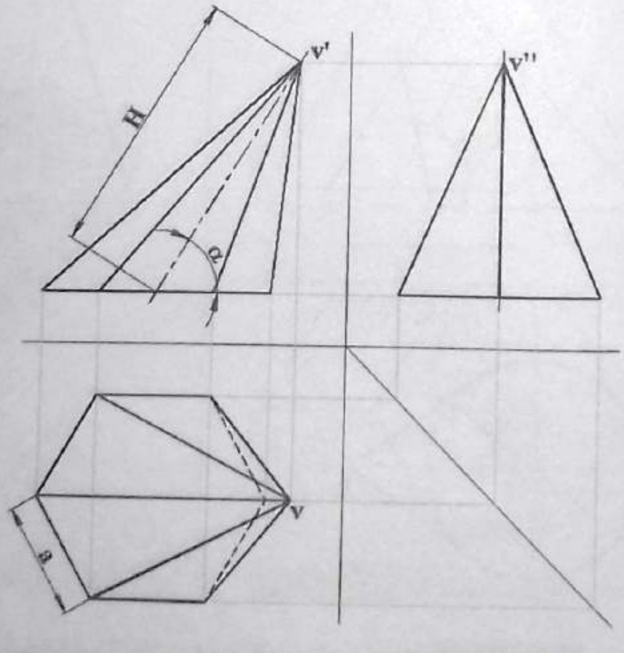


Figure 8.28: Projection of an Oblique Hexagonal Pyramid

8.7 Projections of a Sphere

A sphere appears as circle in every view as shown in *Figure 8.29* whatever be its position and orientation.

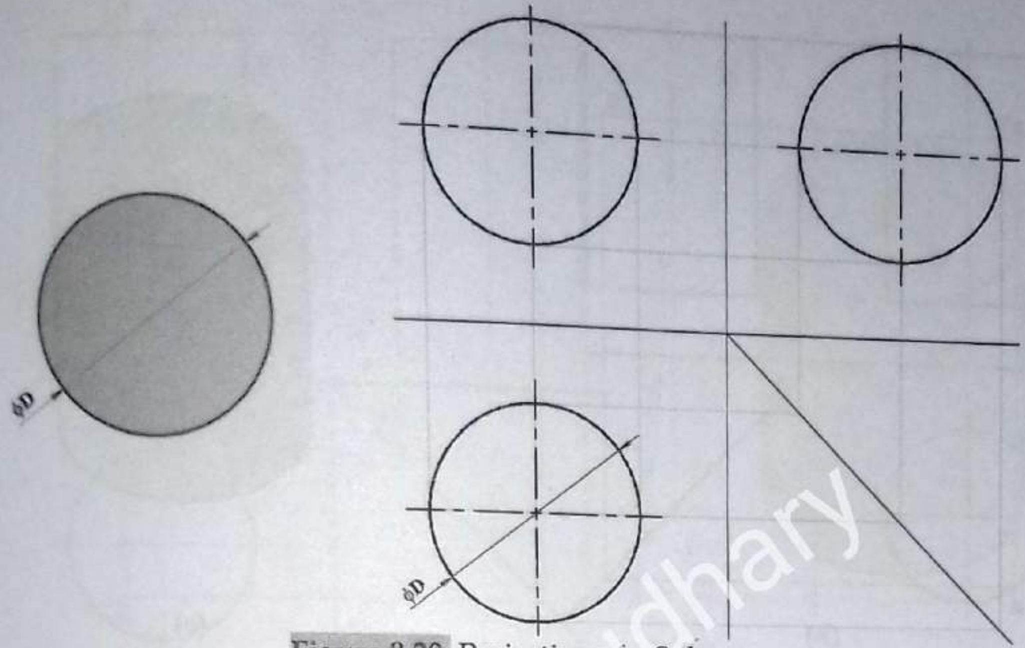


Figure 8.29: Projection of a Sphere

8.8 Projection of Points on the Surfaces of the Solids

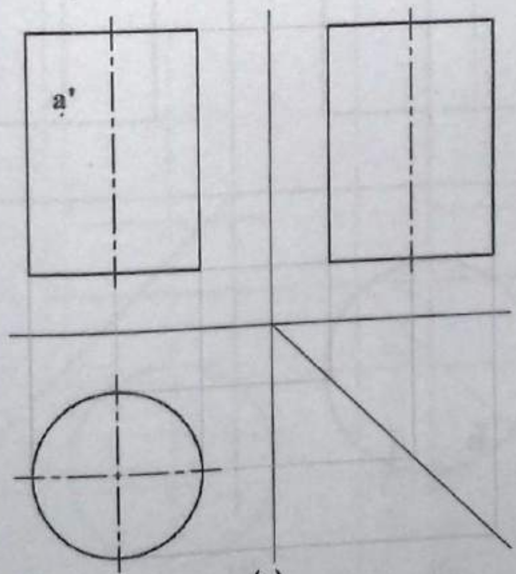
The development of surfaces of any solids or intersection of surfaces of different solids can be completed by the projection of points that define the surface or the intersection curve. Transfer of points lying on the surfaces of the solids is explained below.

8.8.1 Projection of Points on the Surfaces of a Right Circular Cylinder

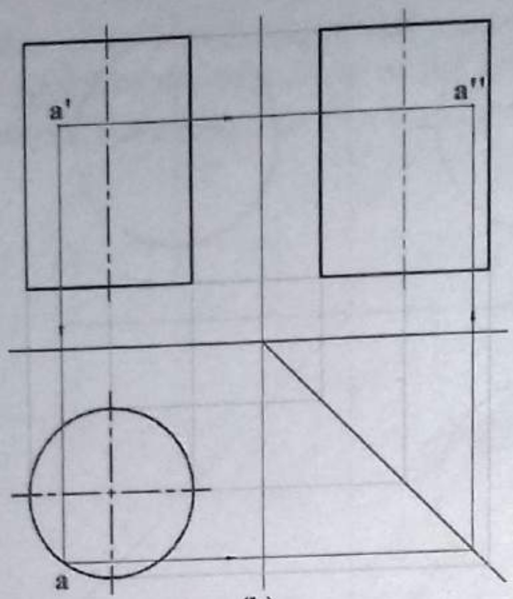
(a) Front View of a Point on the Curve Surface of the Cylinder Given

Figure 8.30(a) shows front view **a'** of a point **A** lying on the curve surface of the cylinder. Draw vertical projection line from the point **a'** which intersects the circle at the point **a**, which is the required top view. Draw projection lines from front view **a'** and top view **a** towards the side view of the cylinder to draw side view **a''** of the given point as shown in *Figure 8.30(b)*.

For reference, pictorial view of the point is also shown in **Figure 8.30 (c)**.



(a)



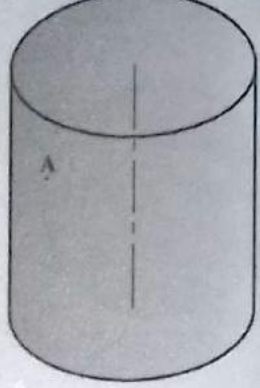
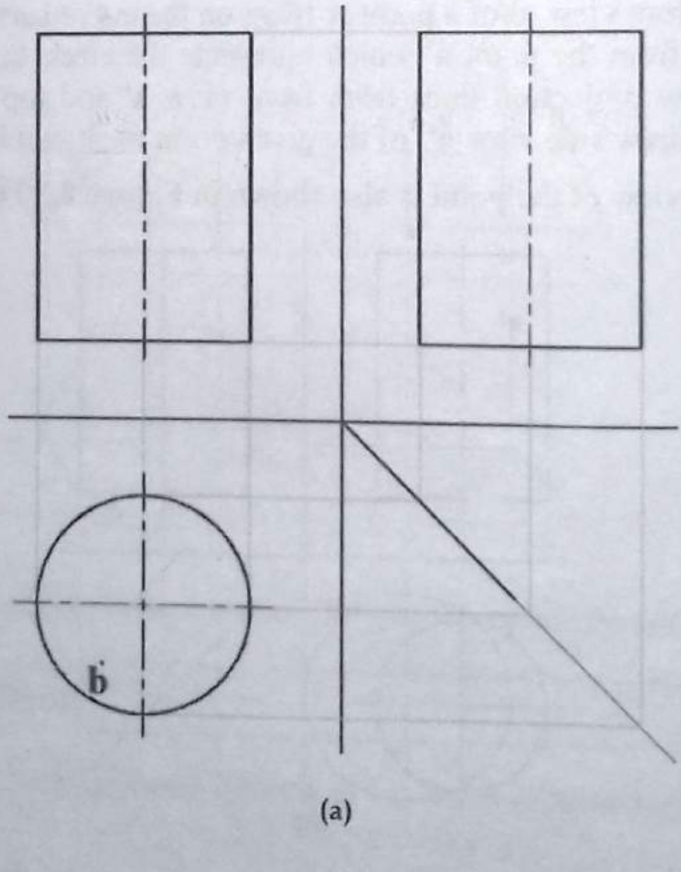


Figure 8.30: Transfer of a point A on the Surface of a Cylinder from the Front View to other Views

(b) Top View of a Point on the Top Surface of the Cylinder Given

Figure 8.31(a) shows top view **b** of a point **B** lying on the top circular section of the cylinder. Draw vertical projection line from the point **b** which intersects the front view of the top circular section at the point **b'**, which is the required front view. Draw projection lines from front view **b'** and top view **b** towards the side view of the cylinder to draw side view **b''** of the given point as shown in *Figure 8.31(b)*.

For reference, pictorial view of the point is also shown in *Figure 8.31 (c)*.



(a)

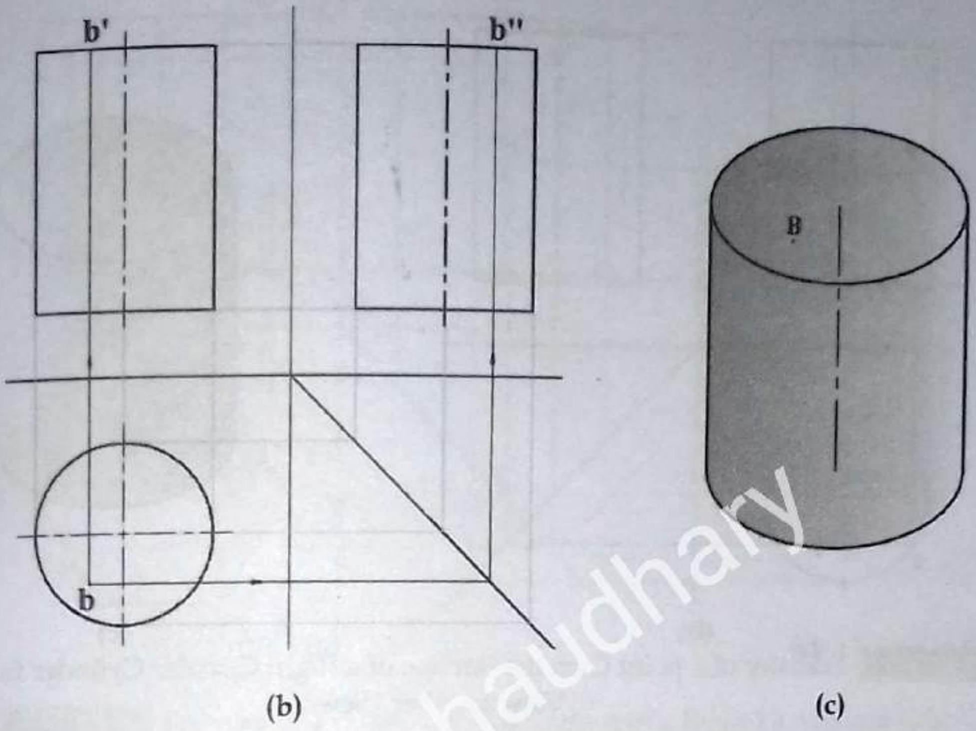
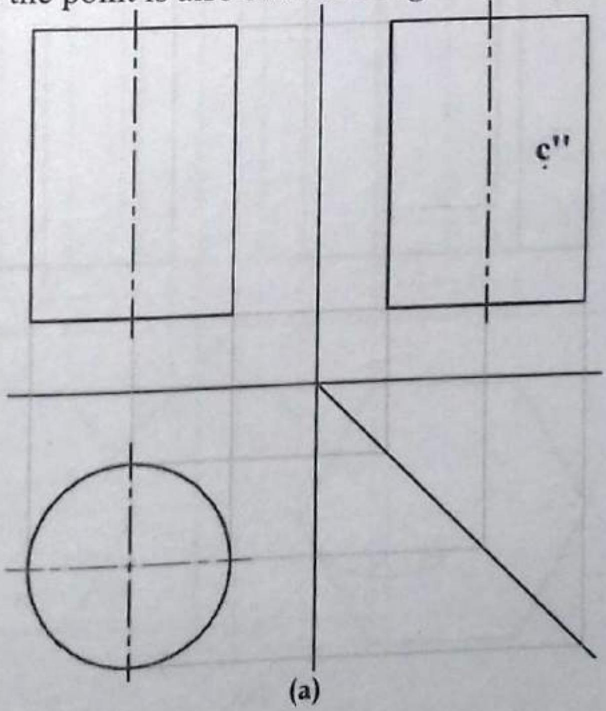


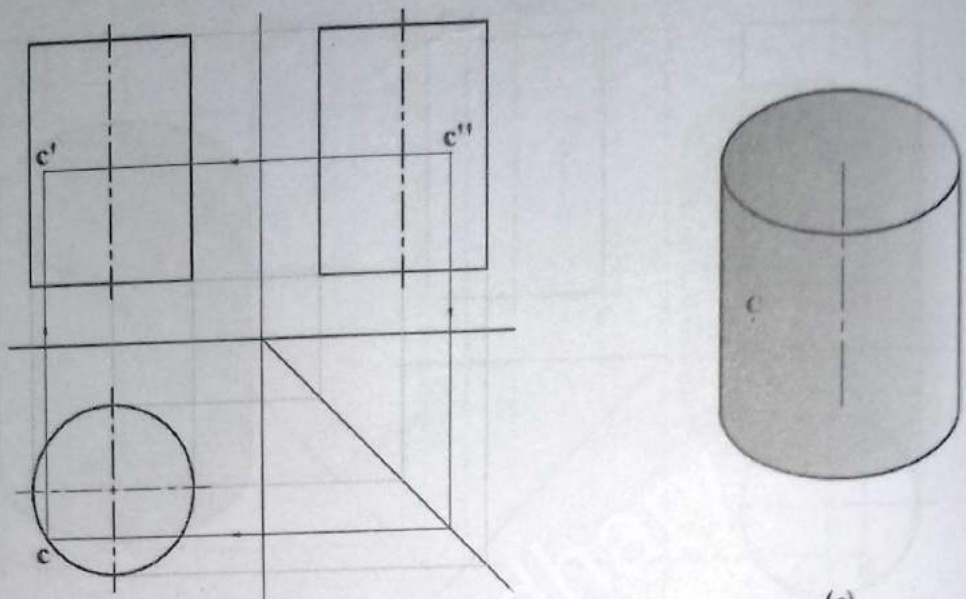
Figure 8.31: Transfer of a point B on the Surface of a Right Circular Cylinder from the Top View to other Views

(c) Side View of a Point on the Curve Surface of the Cylinder Given

Figure 8.32(a) shows side view **c''** of a point **C** lying on the curve surface of the cylinder. Draw vertical projection line from the point **c** towards the top view which intersects the circle at the point **c**, which is the required top view. Draw projection lines from side view **c''** and top view **c** towards the front view of the cylinder to draw front view **c'** of the given point as shown in *Figure 8.32(b)*.

For reference, pictorial view of the point is also shown in *Figure 8.32 (c)*.





(b) (c)

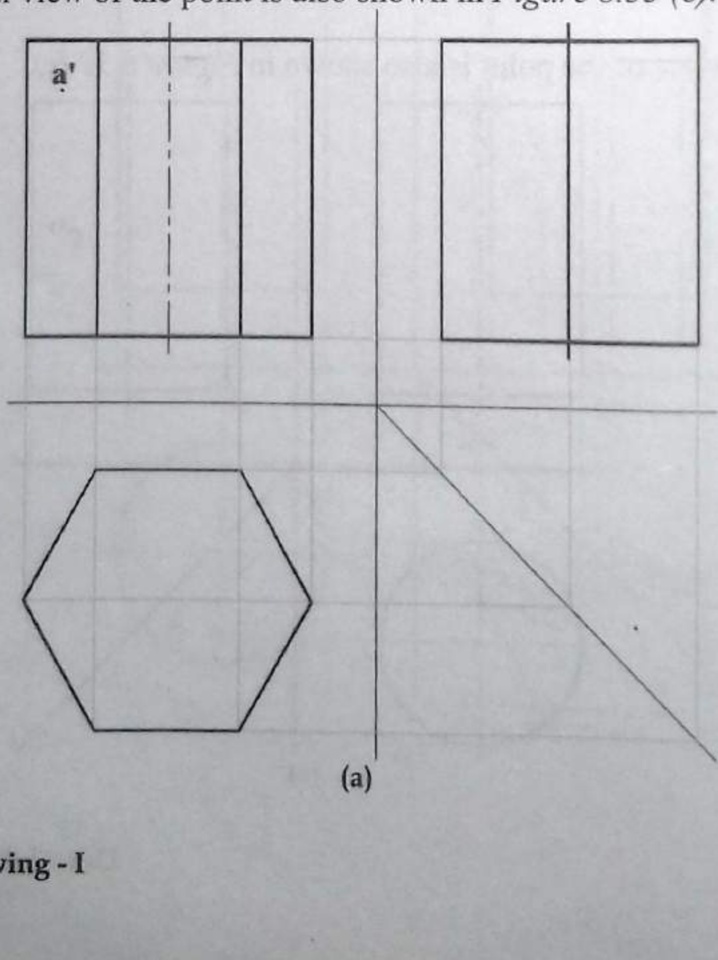
Figure 8.32: Transfer of a point C on the Surface of a Right Circular Cylinder from the Side View to other Views

8.8.2 Projection of Points on the Surfaces of a Right Prism

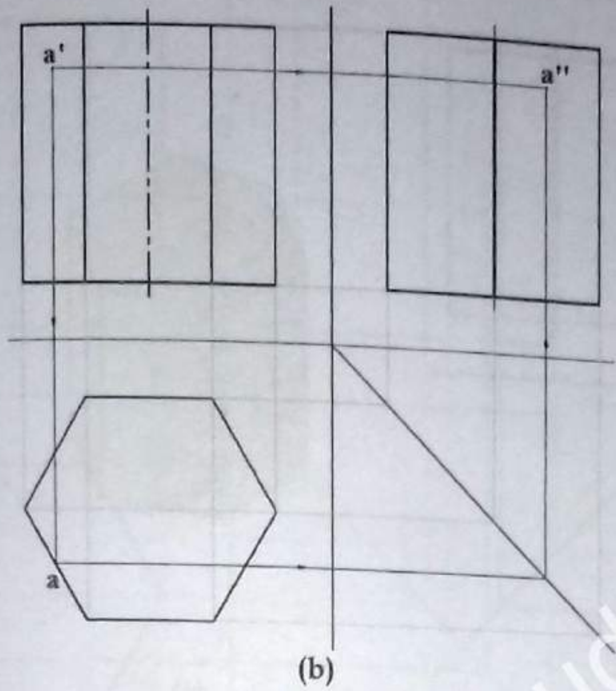
(a) Front View of a Point on the Rectangular Surface of the Prism Given

Figure 8.33(a) shows front view **a'** of a point **A** lying on the rectangular surface of the hexagonal prism. Draw vertical projection line from the point **a'** which intersects the base hexagon on the top view at the point **a**, which is the required top view. Draw projection lines from front view **a'** and top view **a** towards the side view of the prism to draw side view **a''** of the given point as shown in *Figure 8.33(b)*.

For reference, pictorial view of the point is also shown in *Figure 8.33 (c)*.



(a)



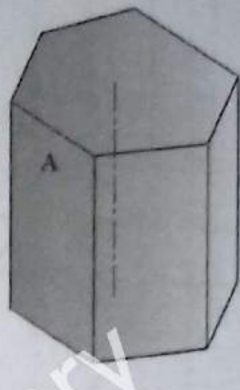
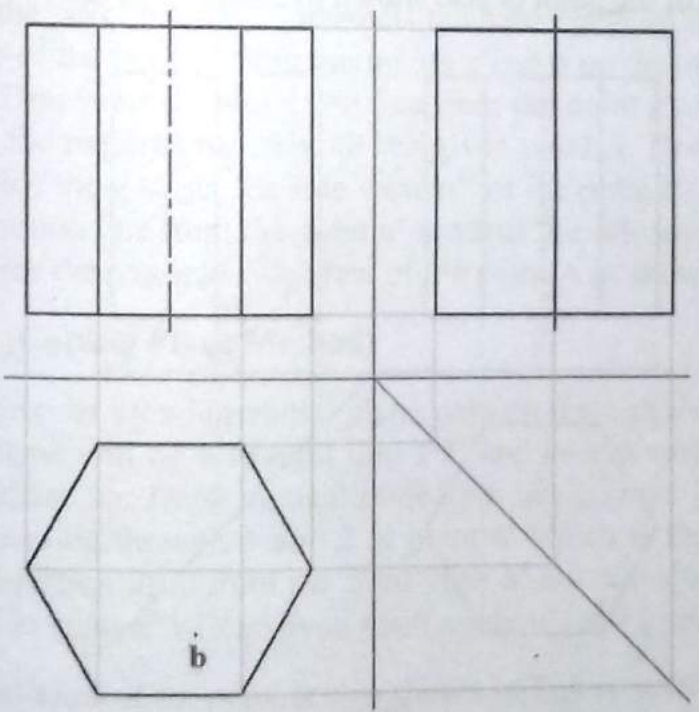


Figure 8.33: Transfer of a point A on the Surface of a Right Hexagonal Prism from the Front View to other Views

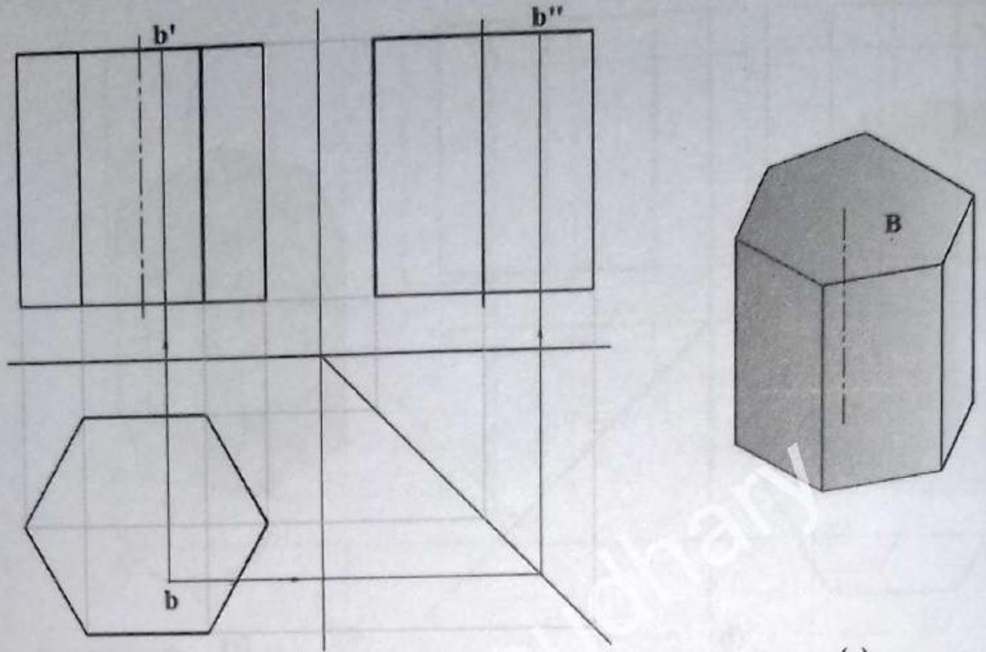
(b) Top View of a Point on the Top Surface of the Prism Given

Figure 8.34(a) shows top view **b** of a point **B** lying on the top hexagonal section of the prism. Draw vertical projection line from the point **b** which intersects the front view of the top hexagonal section at the point **b'**, which is the required front view. Draw projection lines from front view **b'** and top view **b** towards the side view of the prism to draw side view **b''** of the given point as shown in *Figure 8.34(b)*.

For reference, pictorial view of the point is also shown in *Figure 8.34 (c)*.



(a)



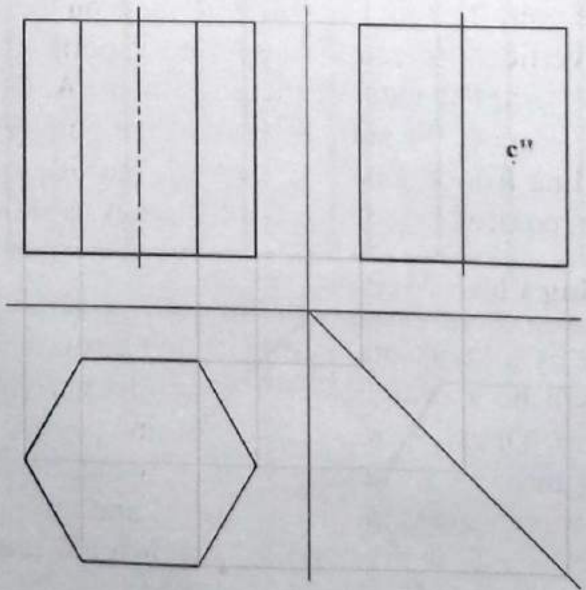
(b) (c)

Figure 8.34: Transfer of a point B on the Surface of a Right Hexagonal Prism from the Top View to other Views

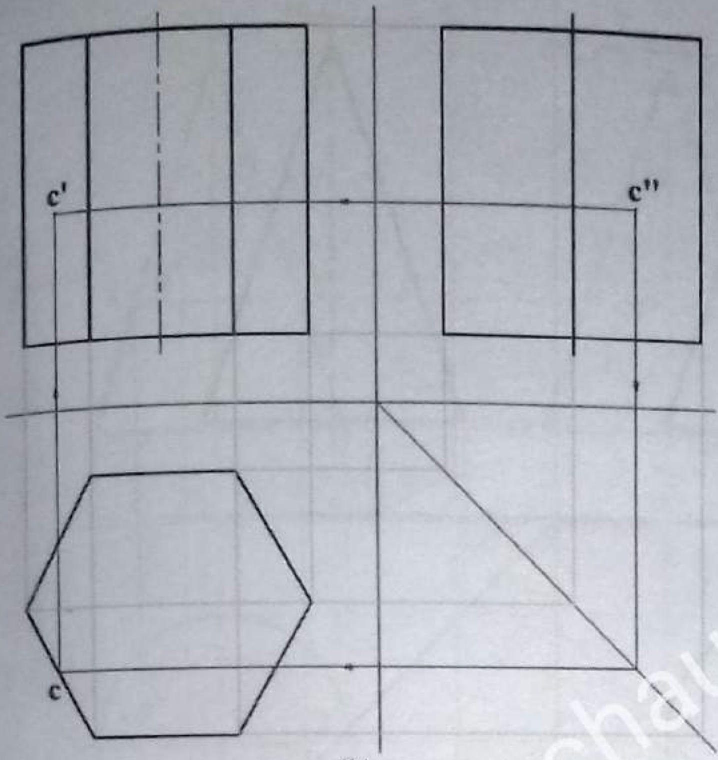
(c) Side View of a Point on the Rectangular Surface of the Prism Given

Figure 8.35(a) shows side view **c''** of a point **C** lying on the curve surface of the cylinder. Draw vertical projection line from the point **c** towards the top view which intersects the circle at the point **c**, which is the required top view. Draw projection lines from side view **c''** and top view **c** towards the front view of the cylinder to draw front view **c'** of the given point as shown in *Figure 8.35(b)*.

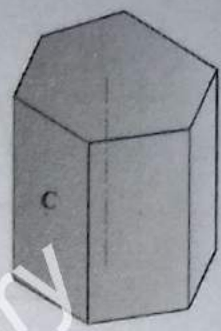
For reference, pictorial view of the point is also shown in *Figure 8.35 (c)*.



(a)



(b)



(c)

Figure 8.35: Transfer of a point C on the Surface of a Right Hexagonal Prism from the Side View to other Views

8.8.3 Projection of Points on the Surfaces of a Right Circular Cone

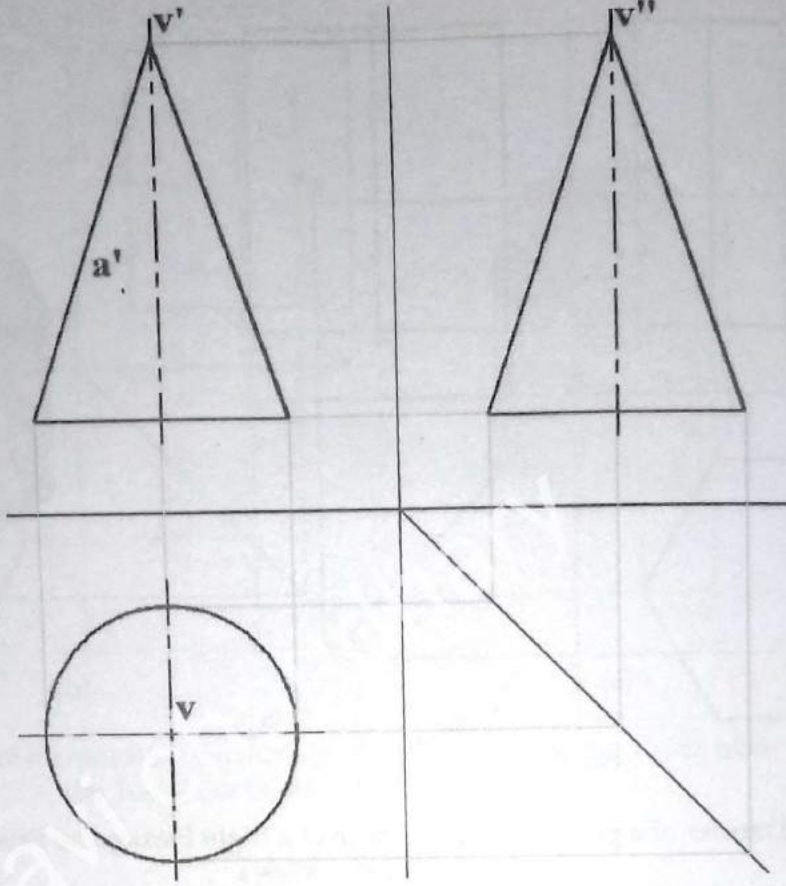
(a) Front View of a Point on the Surface of the Cone Given

Figure 8.36(a) shows front view **a'** of a point **A** lying on the curve surface of the cone. Join front view of the vertex **v'** and the given point **a'** and extend it to the front view of the base circle of the cone to get the front view **p'** of the point **P** at the base of the cone. Draw vertical projection line from the point **p'** which intersects the base circle on the top view at the point **p**, which is the top view of the point **P**. Join the points **v** and **p** on the top view to get the top view of the generator **VP**. Draw vertical projection line from the point **a'** which intersects the line **vp** at the point **a**, which the required top view of the given point **A**. Draw projection line from the point **p** towards the side view to get the side view **p''** of the point **P**. Join the points **v''** and **p''**. Draw horizontal projection line from the point **a'** towards the side view which intersects the line **v''p''** at point **a''**, which the required side view of the point **A** as shown in *Figure 8.36(b)*.

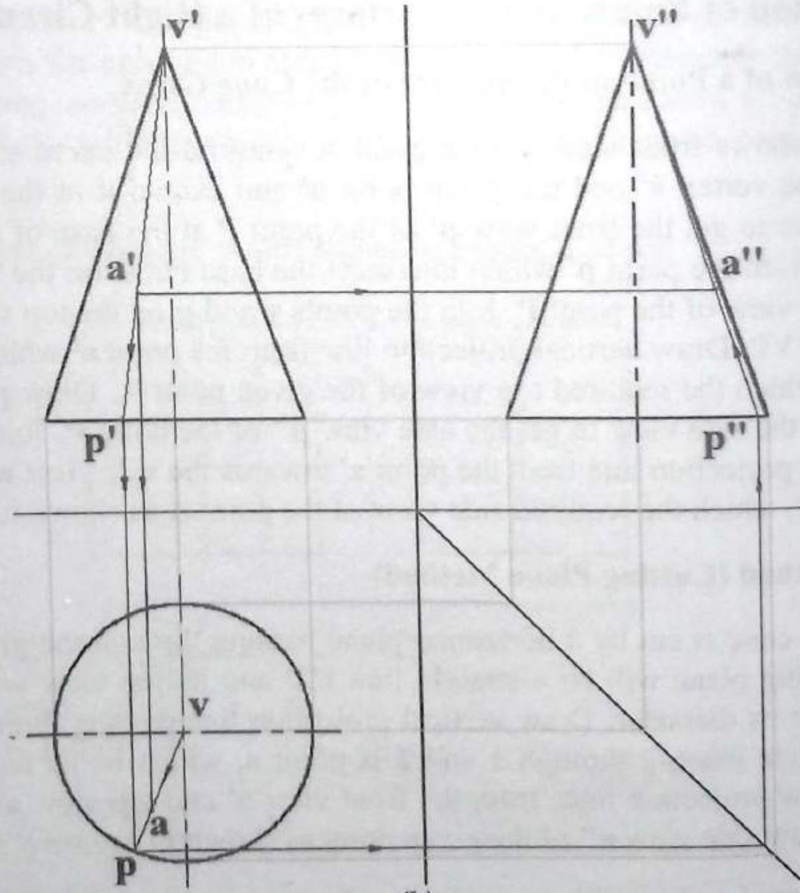
Alternative Method (Cutting Plane Method)

Assume that the cone is cut by a horizontal plane passing through the given point **A**. The front view of the cutting plane will be a straight line **1'2'** and its top view will be a circle with the points **1** and **2** as its diameter. Draw vertical projection line passing through the point **a'** which intersects the circle passing through **1** and **2** at point **a**, which is the required top view of the given point. Draw projection lines from the front view **a'** and top view **a** towards the side view of the cone to draw side view **a''** of the given point as shown in *Figure 8.36(c)*.

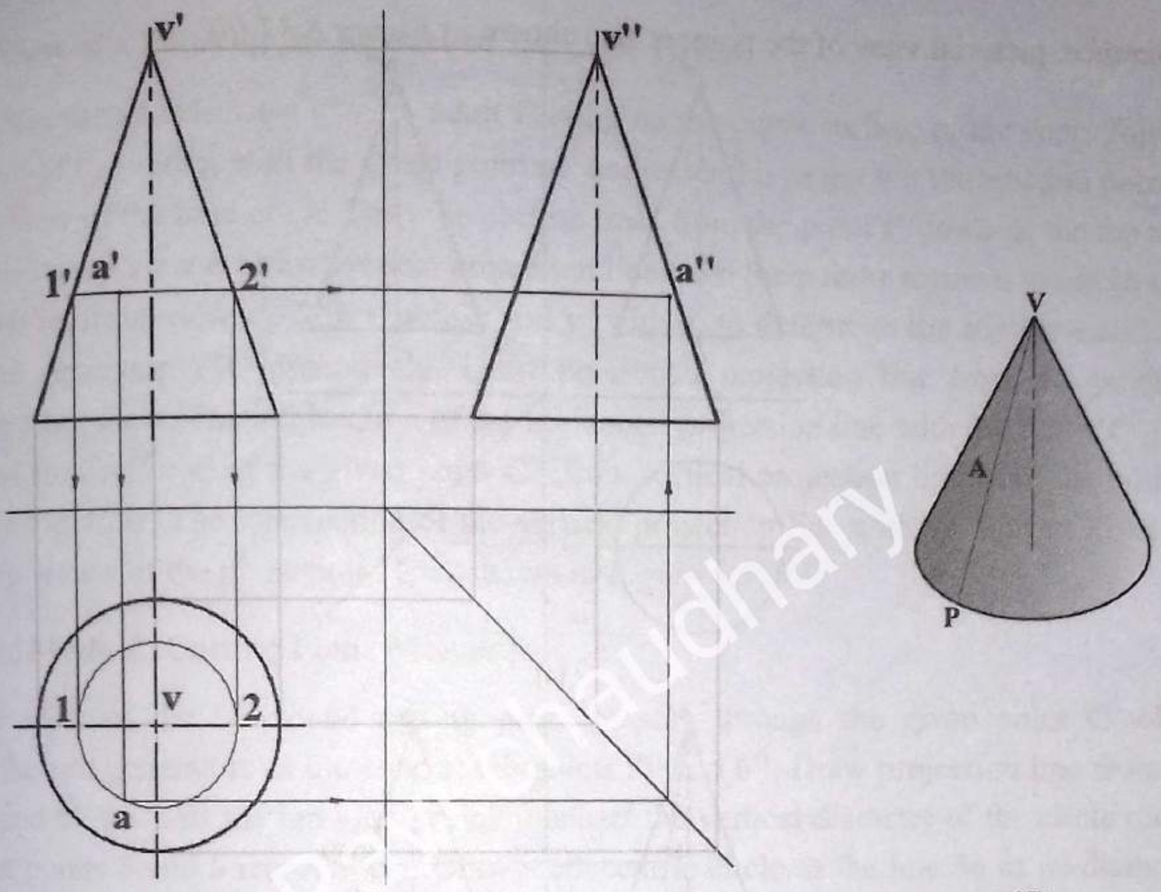
For reference, pictorial view of the point is also shown in *Figure 8.36 (d)*.



(a)



(b)



(c) (d)

Figure 8.36: Transfer of a point A on the Surface of a Right Circular Cone from the Front View to other Views

(b) Top View of a Point on the Surface of the Cone Given

Figure 8.37(a) shows top view **b** of a point **B** lying on the curve surface of the cone. Join the top view **v** of the vertex with the given point **b** and extend it to get the intersection point **q** on the circumference of the circle. Draw projection lines from the point **q** towards the front view and side view to get its front view **q'** and side view **q''** respectively. Join **v'** with **q'** and **v''** with **q''** to get the front view and side view of the generator **VQ** respectively. Draw vertical projection line from the point **b** towards the front view which intersect the line **v'q'** at point **b'**, which is the required front view of the given point. Draw horizontal projection line from the point **b'** towards the side view which intersects the line **v''b''** at point **b''**, which the required side view of the point **B** as shown in *Figure 8.37(b)*. As the point **B** is not visible in the side view, its side view is represented by **(b'')**.

Alternative Method (Cutting Plane Method)

Draw a concentric circle passing through the point **b** and mark the ends of the horizontal diameter of the circle as points **3** and **4**. Draw vertical projection lines from the points **3** and **4** towards the front view to determine their front views **3'** and **4'** respectively. Join **3'** and **4'** to get the front view of the horizontal cutting plane passing through the given point **B**. Draw vertical projection line from the point **b** which intersects the line **3'4'** at the point **b'**, which is the required front view of the given point. Draw projection lines from the front view **b'** and top view **b** towards the side view of the cone to draw side view **b''** of the given point as shown in *Figure 8.37(c)*.

For reference, pictorial view of the point is also shown in *Figure 8.37 (d)*.

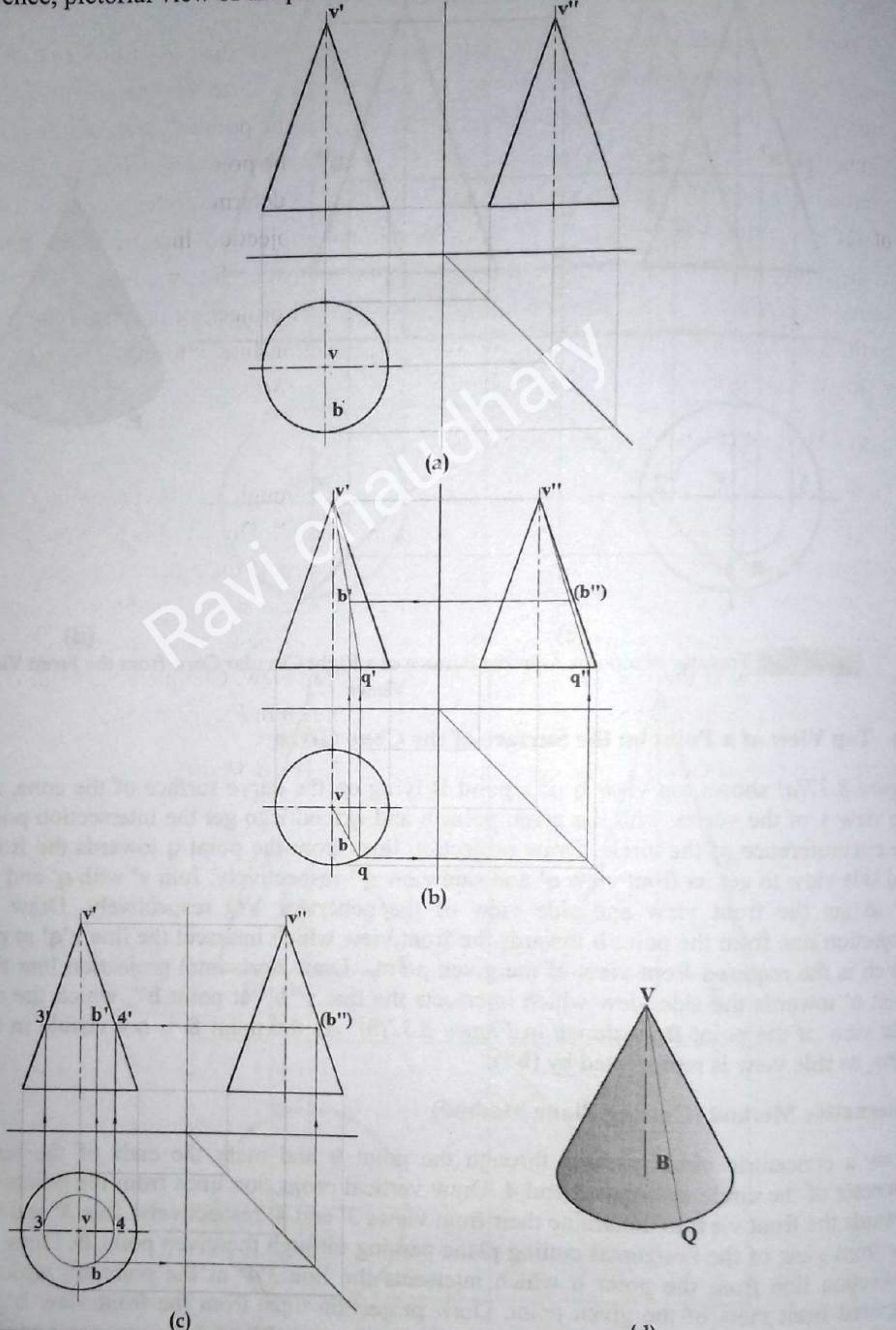


Figure 8.37: Transfer of a point A on the Surface of a Right Circular Cone from the Top View to other Views

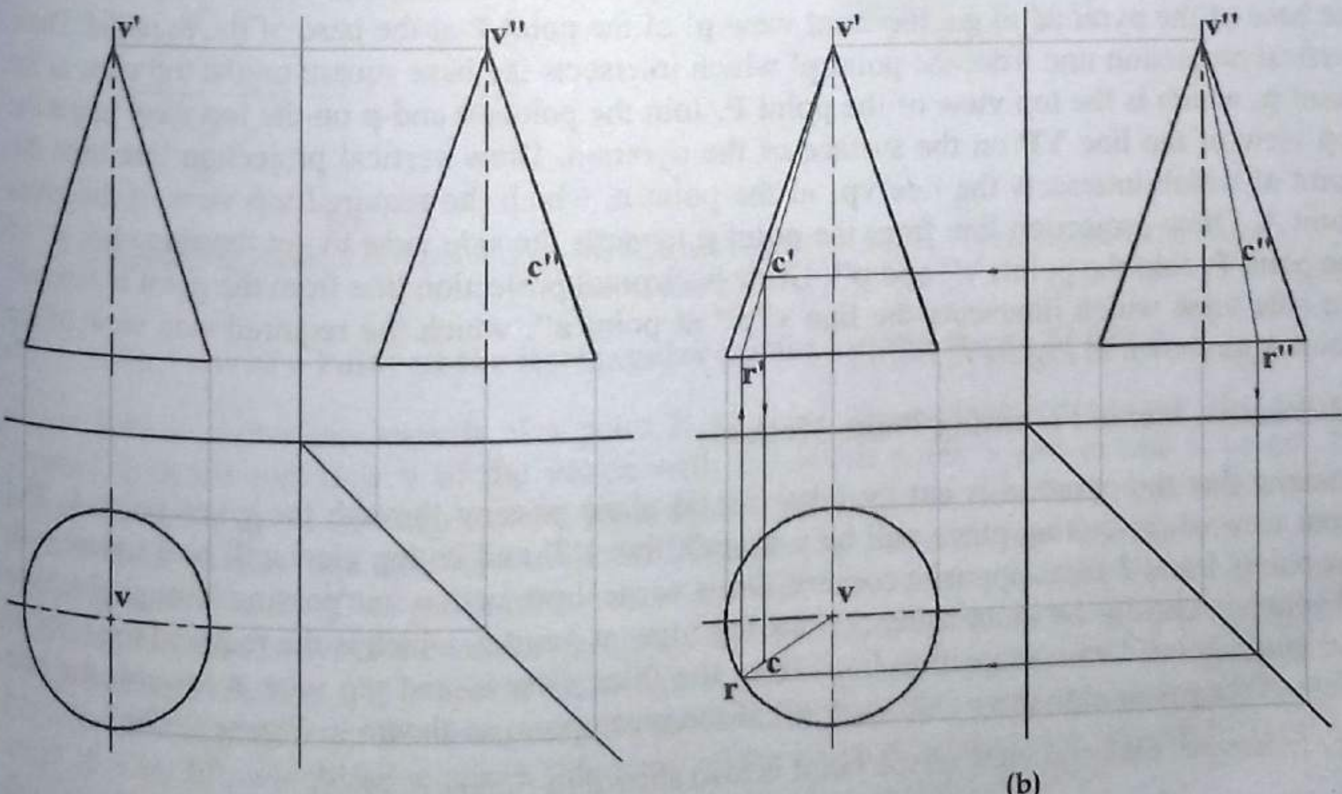
(c) Side View of a Point on the Surface of the Cone Given

Figure 8.38(a) shows side view **c''** of a point **C** lying on the curve surface of the cone. Join the side view **v''** of the vertex with the given point **b''** and extend it to get the intersection point **r''** on the side view of the base circle. Draw projection lines from the point **r''** towards the top view to determine its top view **r**. Draw vertical projection line from the point **r** towards the front view to determine its front view **r'**. Join **v** with **r** and **v'** with **r'** to determine the top view and side view of the generator **VR** respectively. Draw horizontal projection line from the point **c''** towards the front view. The intersection of the horizontal projection line with the line **v'r'** gives the required front view **c'** of the given point **C**. Draw vertical projection line from the point **c'** towards the top view. The intersection of the vertical projection line with the line **vr** gives the required top view **c** of the given point **C** as shown in *Figure 8.38(b)*.

Alternative Method (Cutting Plane Method)

Draw side view of the horizontal cutting plane passing through the given point **C** which intersects the end generators of the cone at the points **5''** and **6''**. Draw projection line from the points **5''** and **6''** towards the top view which intersect the vertical diameter of the circle on the top view at points **5** and **6** respectively. Draw a concentric circle as the line **56** as its diameter. Draw projection line from the given point **c''** towards the top view which intersects the inner circle on the top view at the point **c**, which is the required top view. Complete the front view of the point **c'** with the help of its top view and side view as shown in *Figure 8.38(c)*.

For reference, pictorial view of the point is also shown in *Figure 8.38 (d)*.



(a) (b)

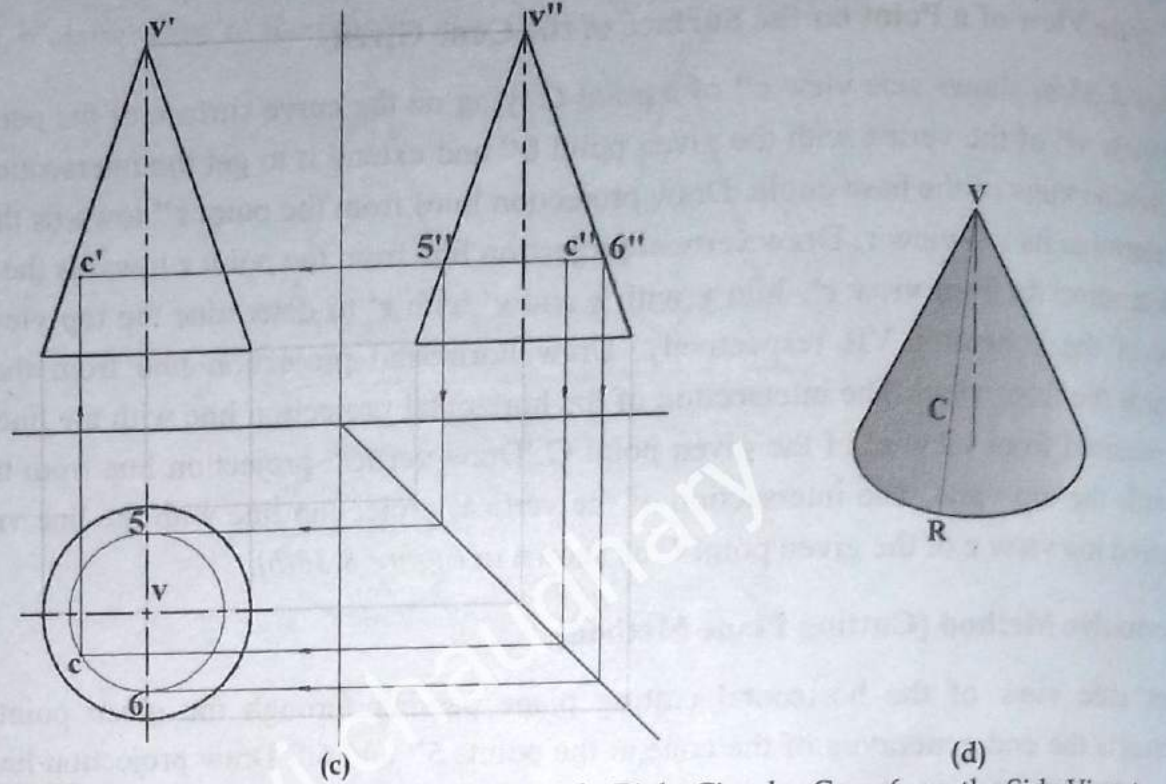


Figure 8.38: Transfer of a point A on the Surface of a Right Circular Cone from the Side View to other Views

8.8.4 Projection of Points on the Surfaces of a Right Pyramid

(a) Front View of a Point on the Rectangular Surface of the Pyramid Given

Figure 8.39(a) shows front view **a'** of a point **A** lying on the rectangular surface of the square pyramid. Join front view of the vertex **v'** and the given point **a'** and extend it to the front view of the base of the pyramid to get the front view **p'** of the point **P** at the base of the pyramid. Draw vertical projection line from the point **p'** which intersects the base square on the top view at the point **p**, which is the top view of the point **P**. Join the points **v** and **p** on the top view to get the top view of the line **VP** on the surface of the pyramid. Draw vertical projection line from the point **a'** which intersects the line **vp** at the point **a**, which the required top view of the given point **A**. Draw projection line from the point **p** towards the side view to get the side view **p''** of the point **P**. Join the points **v''** and **p''**. Draw horizontal projection line from the point **a'** towards the side view which intersects the line **v''p''** at point **a''**, which the required side view of the point **A** as shown in *Figure 8.39(b)*.

Alternative Method (Cutting Plane Method)

Assume that the pyramid is cut by a horizontal plane passing through the given point **A**. The front view of the cutting plane will be a straight line **1'2'** and its top view will be a square with the points **1** and **2** as its opposite corners. Draw vertical projection line passing through the point **a'** which intersects the inner square on the top view at point **a**, which is the required top view of the given point. Draw projection lines from the front view **a'** and top view **a** towards the side view of the pyramid to draw side view **a''** of the given point as shown in *Figure 8.39(c)*.

For reference, pictorial view of the point is also shown in *Figure 8.39 (d)*.

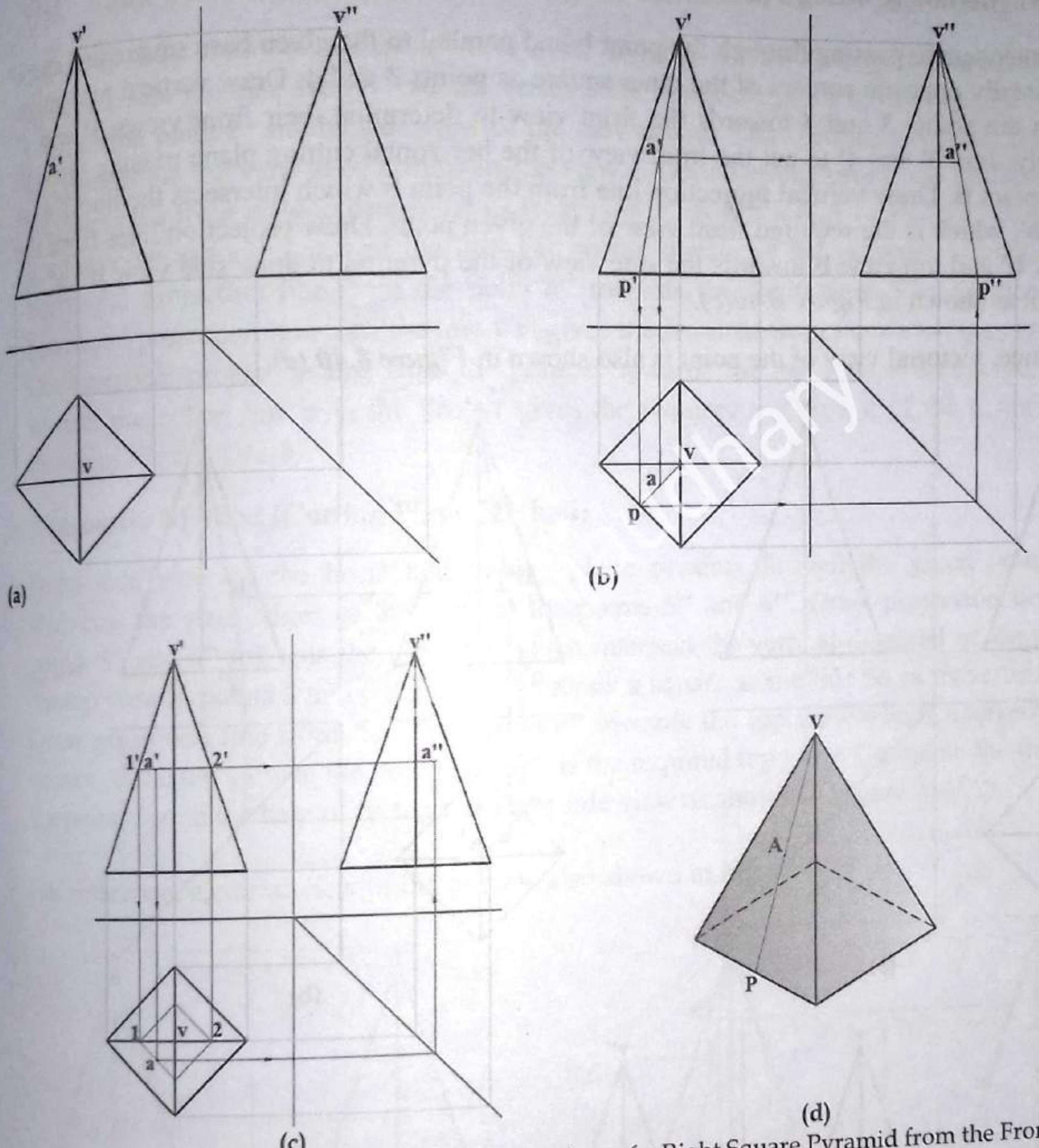


Figure 8.39: Transfer of a point A on the Surface of a Right Square Pyramid from the Front View to other Views

(b) Top View of a Point on the Rectangular Surface of the Pyramid Given

Figure 8.40(a) shows top view **b** of a point **B** lying on the rectangular surface of a square pyramid. Join the top view **v** of the vertex with the given point **b** and extend it to get the intersection point **q** on the edge of the base square. Draw projection lines from the point **q** towards the front view and side view to get its front view **q'** and side view **q''** respectively. Join **v'** with **q'** and **v''** with **q''** to get the front view and side view of the line **VQ** on the surface of the pyramid respectively. Draw vertical projection line from the point **b** towards the front view which intersect the line **v'q'** at point **b'**, which is the required front view of the given point. Draw horizontal projection line from the point **b'** towards the side view which intersects the line **v''q''** at point **b''**, which the required side view of the point **B** as shown in *Figure 8.40(b)*. As the point **B** is not visible in the front view, its front view is represented by **(b')**.

Alternative Method (Cutting Plane Method)

Draw another square passing through the point **b** and parallel to the given base square and mark the horizontally opposite corners of the inner square as points **3** and **4**. Draw vertical projection lines from the points **3** and **4** towards the front view to determine their front views **3'** and **4'** respectively. Join **3'** and **4'** to get the front view of the horizontal cutting plane passing through the given point **B**. Draw vertical projection line from the point **b** which intersects the line **3'4'** at the point **b'**, which is the required front view of the given point. Draw projection lines from the front view **b'** and top view **b** towards the side view of the pyramid to draw side view **b''** of the given point as shown in *Figure 8.40(c)*.

For reference, pictorial view of the point is also shown in ***Figure 8.40 (d)***.

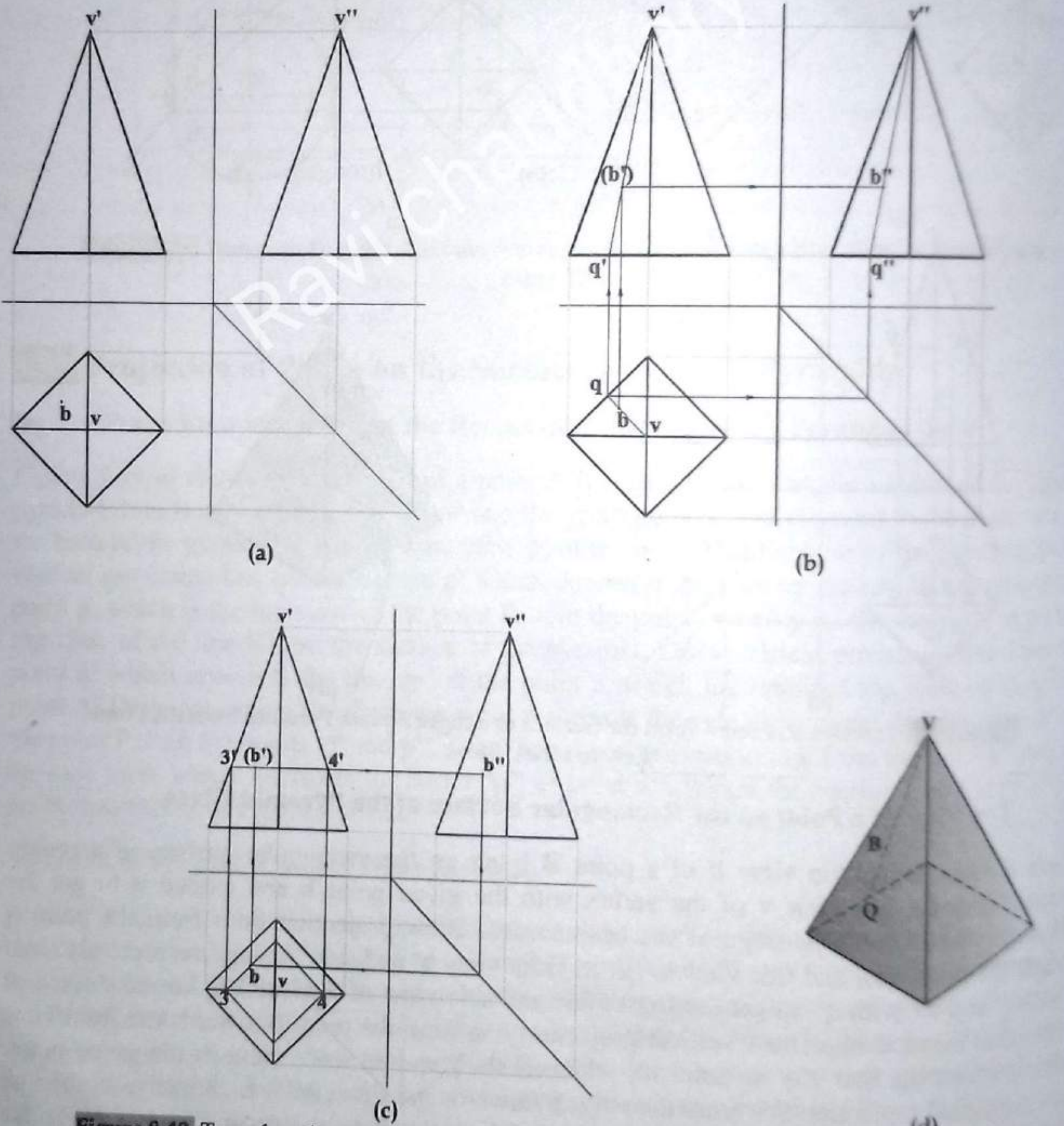


Figure 8.40: Transfer of a point B on the Surface of a Right Square Pyramid from the Top View to other Views

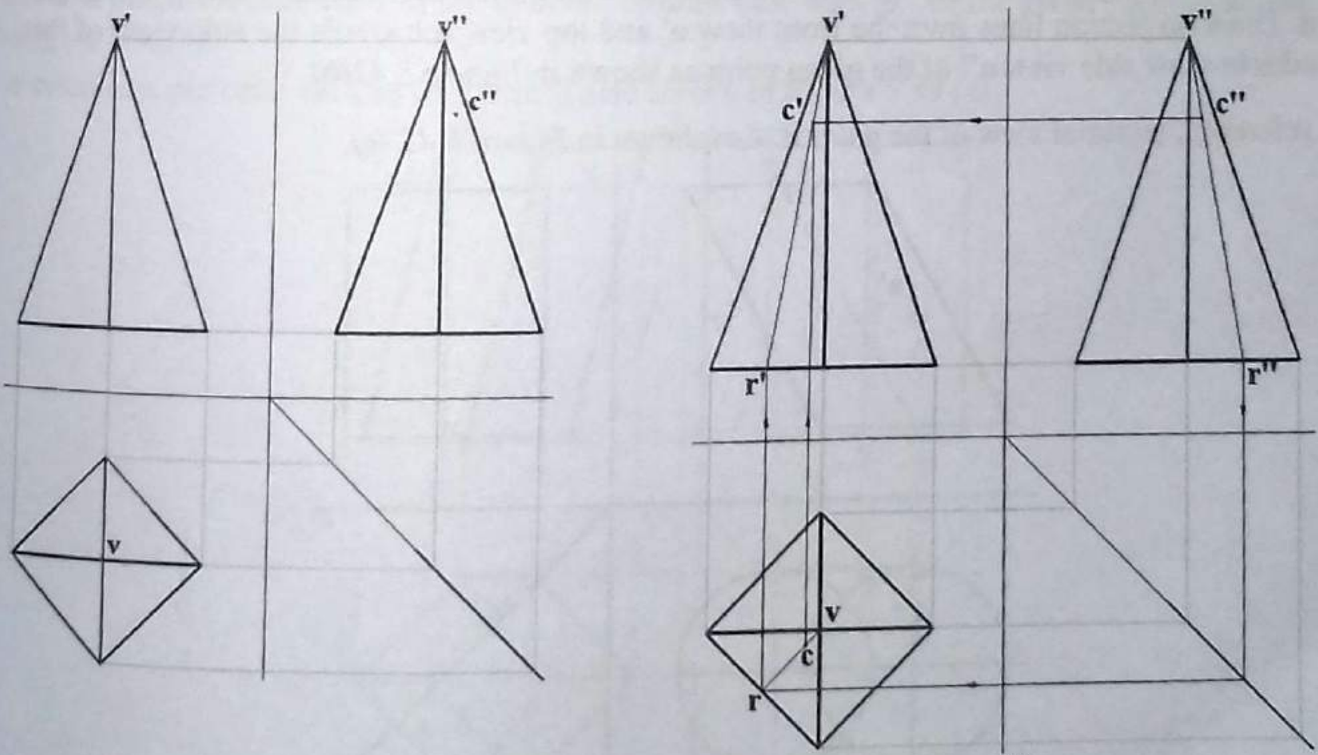
(c) Side View of a Point on the Rectangular Surface of the Pyramid Given

Figure 8.41(a) shows side view **c''** of a point **C** lying on the rectangular surface of a square pyramid. Join the side view **v''** of the vertex with the given point **b''** and extend it to get the intersection point **r''** on the side view of the base square. Draw projection lines from the point **r''** towards the top view to determine its top view **r**. Draw vertical projection line from the point **r** towards the front view to determine its front view **r'**. Join **v** with **r** and **v'** with **r'** to determine the top view and side view of the line **VR** on the surface of the pyramid respectively. Draw horizontal projection line from the point **c''** towards the front view. The intersection of the horizontal projection line with the line **v'r'** gives the required front view **c'** of the given point **C**. Draw vertical projection line from the point **c'** towards the top view. The intersection of the vertical projection line with the line **vr** gives the required top view **c** of the given point **C** as shown in *Figure 8.41(b)*.

Alternative Method (Cutting Plane Method)

Draw side view of the horizontal cutting plane passing through the given point **C** which intersects the slant edges of the cone at the points **5''** and **6''**. Draw projection line from the points **5''** and **6''** towards the top view which intersect the vertical diagonal of the pyramid on the top view at points **5** and **6** respectively. Draw a square as the line **56** as its vertical diagonal. Draw projection line from the given point **c''** towards the top view which intersects the inner square on the top view at the point **c**, which is the required top view. Complete the front view of the point **c'** with the help of its top view and side view as shown in *Figure 8.41(c)*.

For reference, pictorial view of the point is also shown in *Figure 8.41 (d)*.



(a) (b)

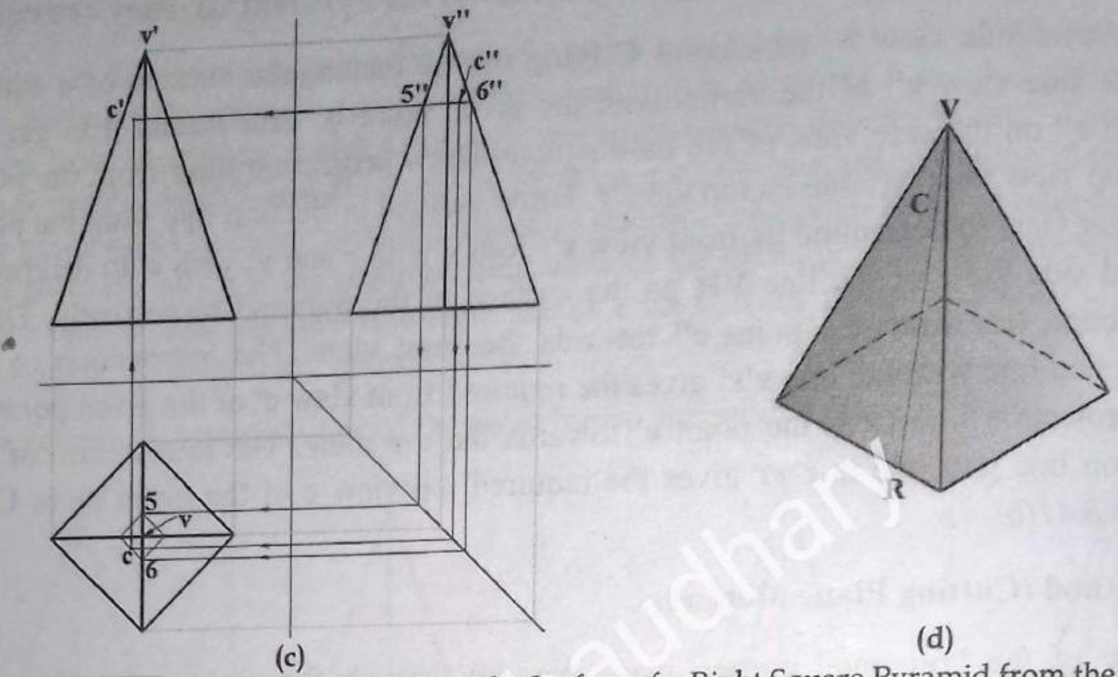
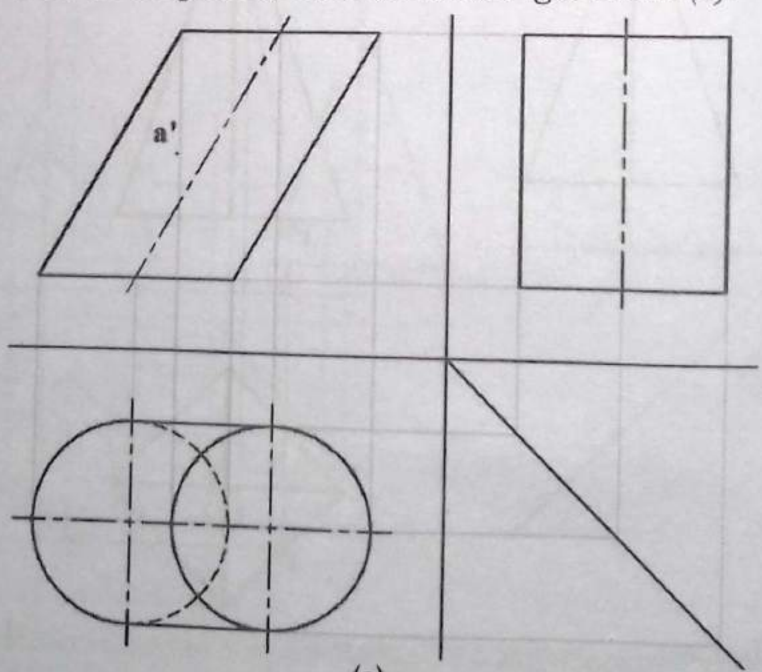


Figure 8.41: Transfer of a point A on the Surface of a Right Square Pyramid from the Side View to other Views

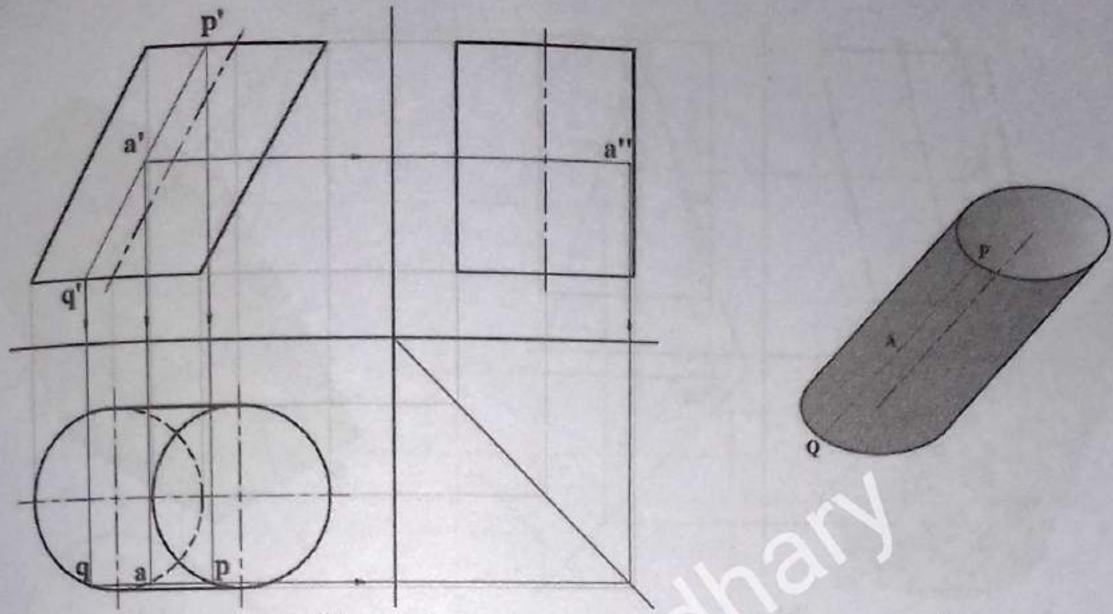
8.8.5 Projection of Points on the Surface of an Oblique Circular Cylinder

Figure 8.42(a) shows front view **a'** of a point **A** lying on the curve surface of the oblique circular cylinder. To transfer the point to other views, draw line passing through the given point **a'** and parallel to the axis which intersect the edge views of the top and bottom circular sections at the point **p'** and **q'**. Draw vertical projection lines from the points **p'** and **q'** to get their respective top views **p** and **q** respectively. Draw vertical projection line from the point **a'** which intersects the line **pq** on the top view at the point **a**, which the required top view of the given point. Draw projection lines from the front view **a'** and top view **a** towards the side view of the cylinder to draw side view **a''** of the given point as shown in *Figure 8.42(b)*.

For reference, pictorial view of the point is also shown in *Figure 8.42 (c)*.



(a)



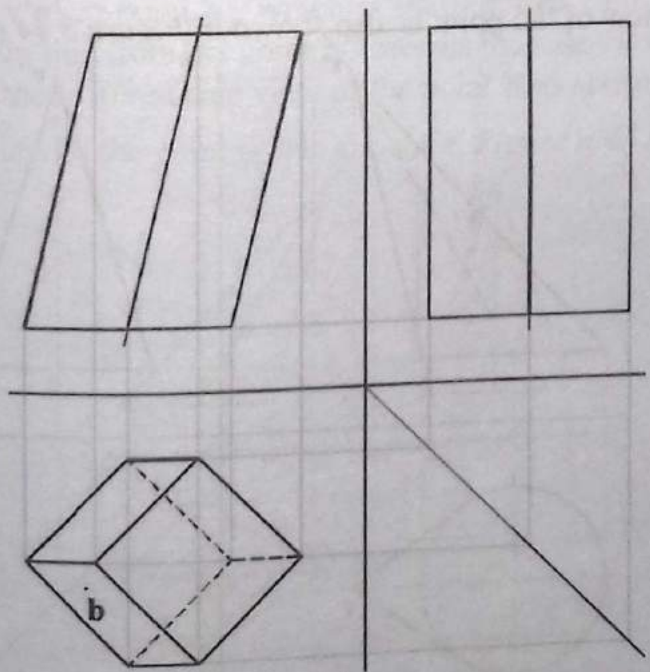
(b) (c)

Figure 8.42: Transfer of a point A on the Surface of an Oblique Circular Cylinder from the Front View to other Views

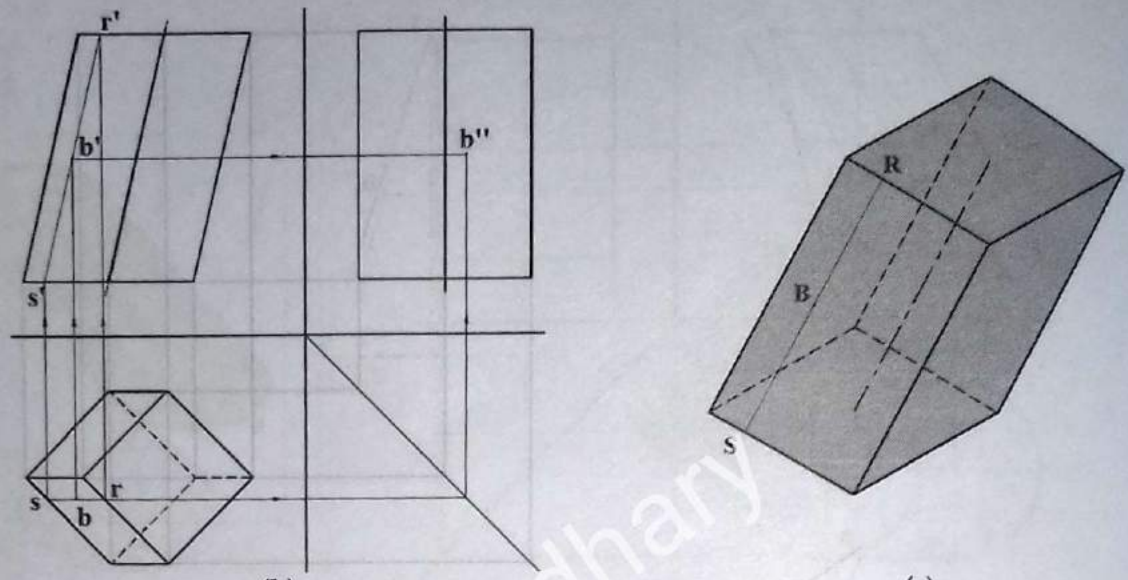
8.8.6 Projection of Points on the Surface of an Oblique Prism

Figure 8.43(a) shows top view **b** of a point **B** lying on the rectangular surface of the oblique square prism. To transfer the point to other views, draw line passing through the given point **b** and parallel to the top view of the inclined edges (i.e. parallel to the axis) which intersect the edges of the base squares at the point **r** and **s**. Draw vertical projection lines from the points **r** and s to get their respective front views **r'** and **s'** respectively. Draw vertical projection line from the point **b** which intersects the line **r's'** on the front view at the point **b'**, which the required front view of the given point. Draw projection lines from the front view **b'** and top view b towards the side view of the cylinder to draw side view **b''** of the given point as shown in *Figure 8.43(b)*.

For reference, pictorial view of the point is also shown in *Figure 8.43 (c)*.



(a)



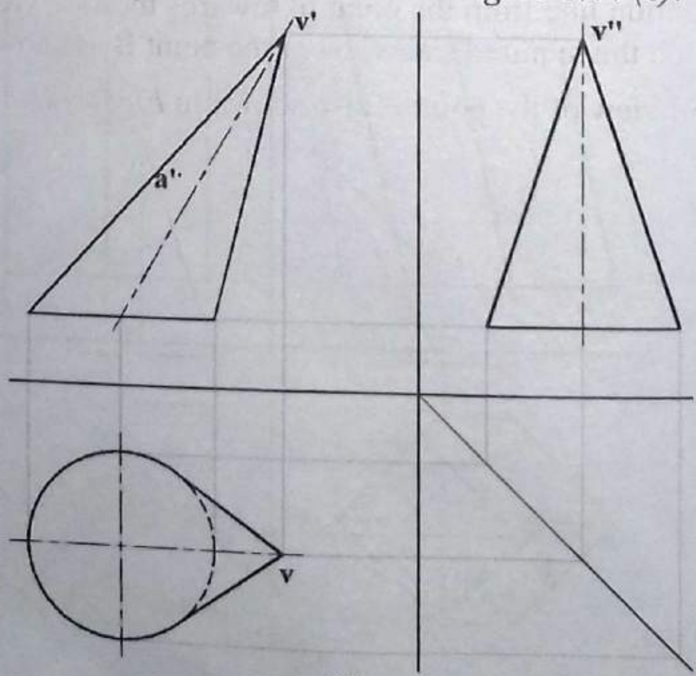
(b) (c)

Figure 8.43: Transfer of a point B on the Surface of an Oblique Square Prism from the Top View to other Views

8.8.7 Projection of Points on the Surface of an Oblique Circular Cone

Figure 8.44(a) shows front view **a'** of a point **A** lying on the curve surface of an oblique cone. Join front view of the vertex **v'** and the given point **a'** and extend it to the front view of the base circle of the cone to get the front view **m'** of the point **M** at the base of the cone. Draw vertical projection line from the point **M'** which intersects the base circle on the top view at the point **M**, which is the top view of the point **M**. Join the points **v** and **m** on the top view to get the top view of the generator **MP**. Draw vertical projection line from the point **a'** which intersects the line **mp** at the point **a**, which the required top view of the given point **A**. Draw projection line from the point **m** towards the side view to get the side view **m''** of the point **M**. Join the points **v''** and **m''**. Draw horizontal projection line from the point **a'** towards the side view which intersects the line **v''m''** at point **a''**, which the required side view of the point **A** as shown in *Figure 8.44(b)*.

For reference, pictorial view of the point is also shown in *Figure 8.44 (c)*.



(a)

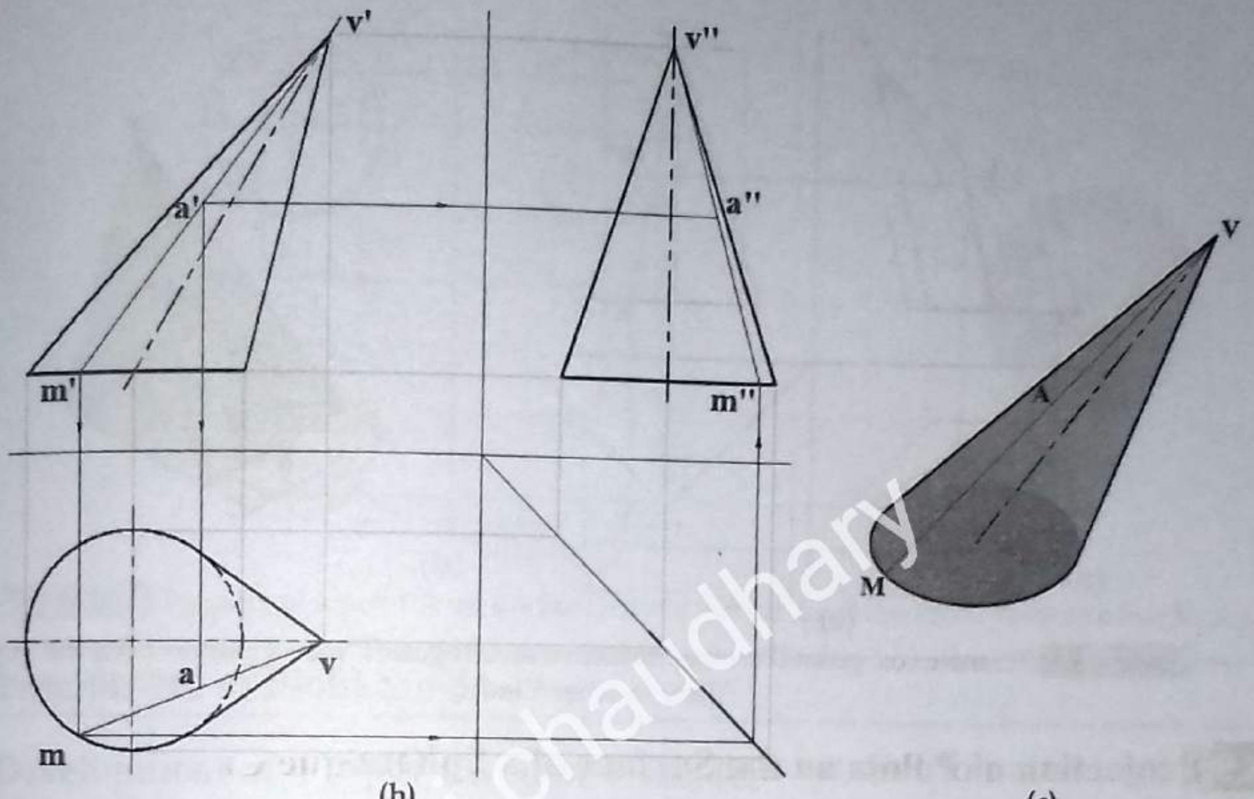
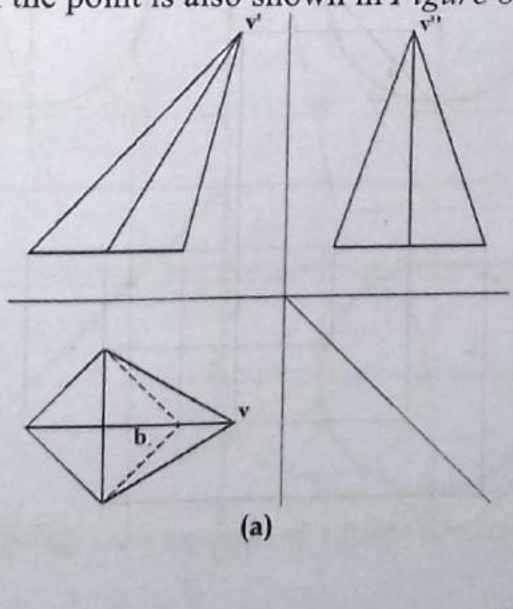


Figure 8.44: Transfer of a point B on the Surface of an Oblique Circular Cone from the Front View to other Views

8.8.8 Projection of Points on the Surface of an Oblique Pyramid

Figure 8.45(a) shows top view **b** of a point **B** lying on the rectangular surface of an oblique square pyramid. Join the top view **v** of the vertex with the given point **b** and extend it to get the intersection point **N** on the edge of the base square. Draw projection lines from the point **n** towards the front view and side view to get its front view **n'** and side view **n''** respectively. Join **v'** with **n'** and **v''** with **n''** to get the front view and side view of the line **VN** on the surface of the pyramid respectively. Draw vertical projection line from the point **b** towards the front view which intersect the line **v'n'** at point **b'**, which is the required front view of the given point. Draw horizontal projection line from the point **b'** towards the side view which intersects the line **v''n''** at point **b''**, which the required side view of the point **B** as shown in *Figure 8.45(b)*.

For reference, pictorial view of the point is also shown in *Figure 8.45 (c)*.



(a)

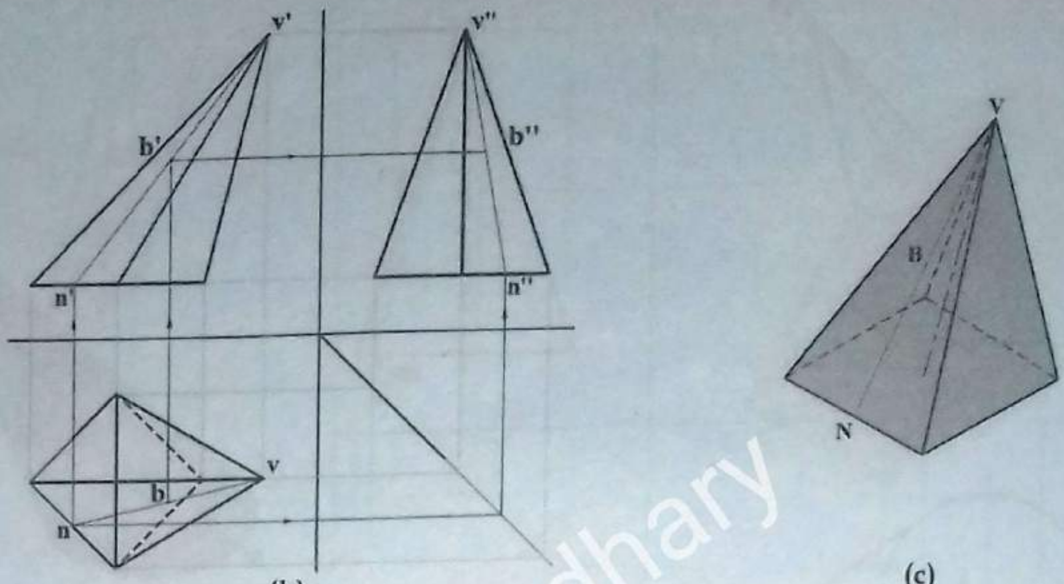
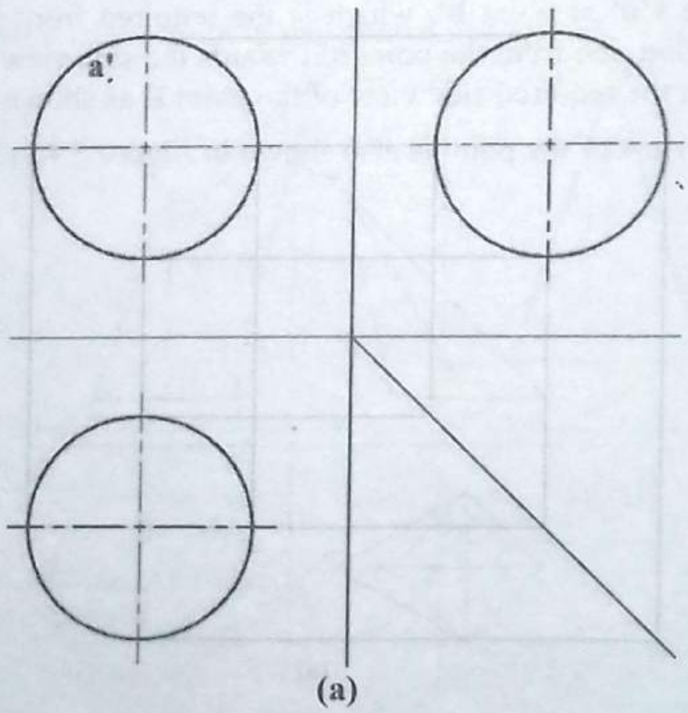


Figure 8.45: Transfer of a point B on the Surface of an Oblique Square Pyramid from the Top View to other Views

8.8.9 Projection of Points on the Surface of a Sphere

Figure 8.46(a) shows front view **a'** of a point **A** lying on the curve surface of the sphere. To transfer the point to other views, assume that the sphere is cut by a horizontal plane passing through the given point A. The front view of the cutting plane will be a straight line **1'2'** and its top view will be a circle with the points **1** and **2** as its diameter. Draw vertical projection line passing through the point **a'** which intersects the circle passing through **1** and **2** at point **a**, which is the required top view of the given point. Draw projection lines from the front view **a'** and top view **a** towards the side view of the sphere to draw side view **a''** of the given point as shown in *Figure 8.46(b)*.

For reference, pictorial view of the point is also shown in **Figure 8.46 (c)**.



(a)

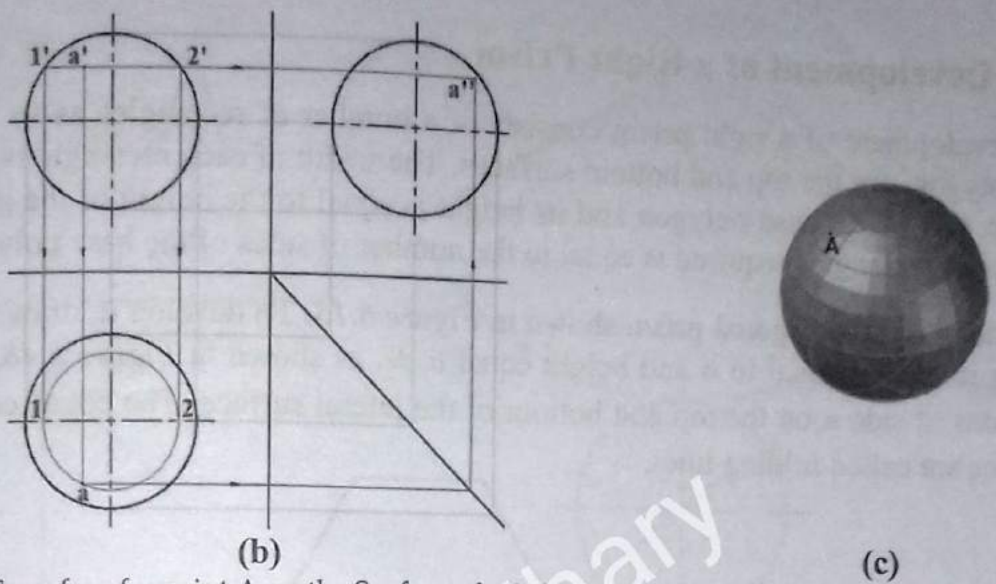


Figure 8.46: Transfer of a point A on the Surface of a Sphere from the Front View to other Views

8.9 Development of Right Solids

8.9.1 Development of a Right Circular Cylinder

The surface development of a right circular cylinder consists of a rectangle with its length equal to the circumference of the base circle and height equal to the height of the given cylinder for its lateral surface and two circles with diameter equal to the diameter of the given base circle for its top and bottom surfaces.

Consider a right circular cylinder shown in *Figure 8.10.* To develop it, draw a rectangle with its length equal to πD and height equal to H, as shown in *Figure 8.47.* Draw circles with diameter equal to D and tangent to both the upper and bottom edges of the rectangle.

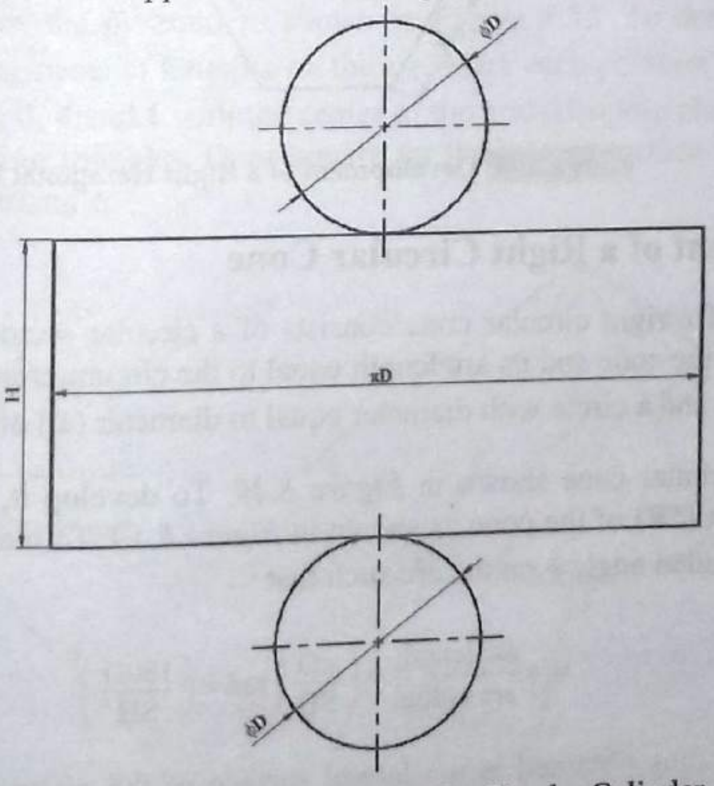


Figure 8.47: Development of a Right Circular Cylinder

8.9.2 Development of a Right Prism

The development of a right prism consists of a number of rectangles as its lateral surface and two polygons for the top and bottom surfaces. The width of each rectangle is equal to the length of each side of the base polygon and its height is equal to the height of the given prism and the number of rectangles required is equal to the number of sides of the base polygon.

Consider a right hexagonal prism shown in *Figure 8.13*. To develop it, draw six rectangles each having its height equal to a and height equal to H, as shown in *Figure 8.48*. Draw two regular hexagons of side a on the top and bottom of the lateral surface. The edges common to adjacent surfaces are called folding lines.

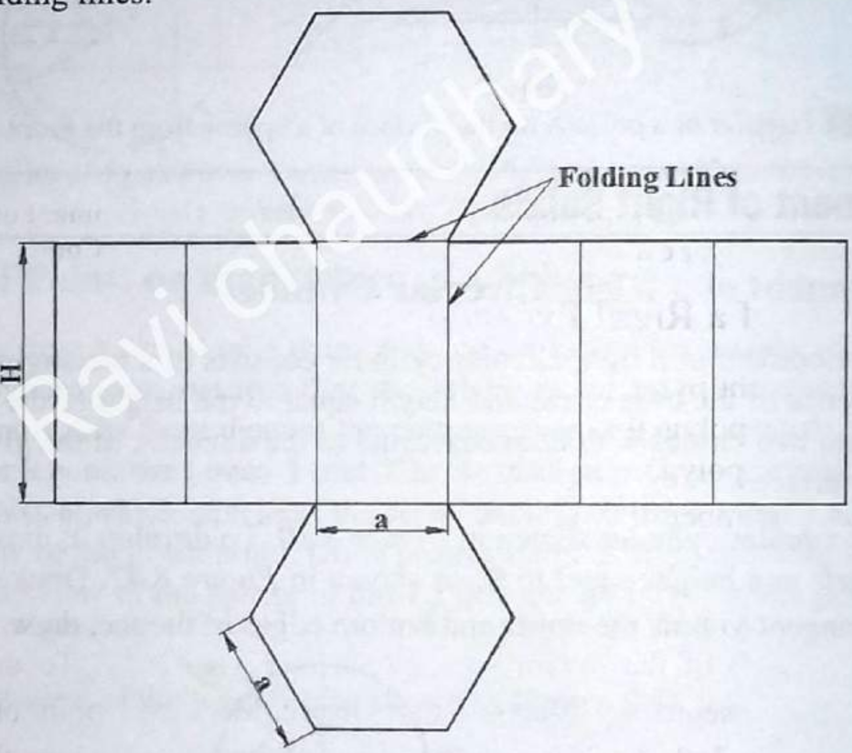


Figure 8.48: Development of a Right Hexagonal Prism

8.9.3 Development of a Right Circular Cone

The development of a right circular cone consists of a circular sector with its radius equal to slant height (SH) of the cone and its arc length equal to the circumference (πD) of the base circle as its lateral surface and a circle with diameter equal to diameter (D) of the base circle.

Consider a right circular cone shown in *Figure 8.49*. To develop it, draw an arc with radius equal to slant height (SH) of the cone as shown in *Figure 8.50*. To mark the required arc length of πD, draw an included angle θ on the arc such that

$$\theta = \frac{\text{arc length}}{\text{arc radius}} = \left(\frac{\pi D}{SH}\right) \text{rad} = \left(\frac{180D}{SH}\right)^0$$

The circular sector thus obtained is the lateral surface of the given cone. Draw a circle with diameter of D and tangent to the circular sector at any point for the bottom face of the cone.

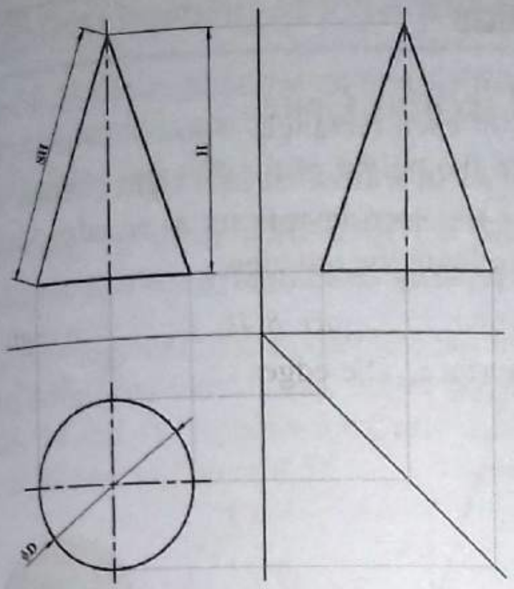


Figure 8.49: Orthographic Projection of a Right Circular Cone

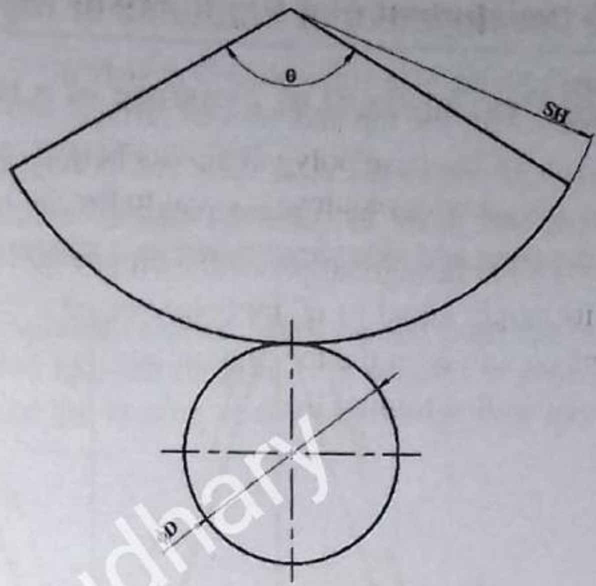


Figure 8.50: Development of a Right Circular Cone

8.9.4 Development of a Right Pyramid

The development of a right pyramid consists of a number of isosceles triangular surfaces as its lateral surface and a base polygon as its bottom surface. The base of each triangle is equal to the length of each side of the polygon (*a*) and other two sides have length equal to slant height (***SH***) of the pyramid and the number of triangular surfaces is equal to the number of sides of the base polygon.

Consider a right square pyramid shown in *Figure 8.51*. To develop it, draw an arc with radius equal to slant height (***SH***) of the pyramid as shown in *Figure 8.52*. To draw four isosceles triangles, mark four chord segments of length *a* on the arc. Mark each point on the arc as **1**, **2**, **3**, **4** and **1**. Join each point **1**, **2**, **3**, **4** and **1** with the center of the arc. Also join chords **1-2**, **2-3**, **3-4** and **4-1** to get the required four triangles. Draw square for the bottom surface with its one edge common to base of any one triangle.

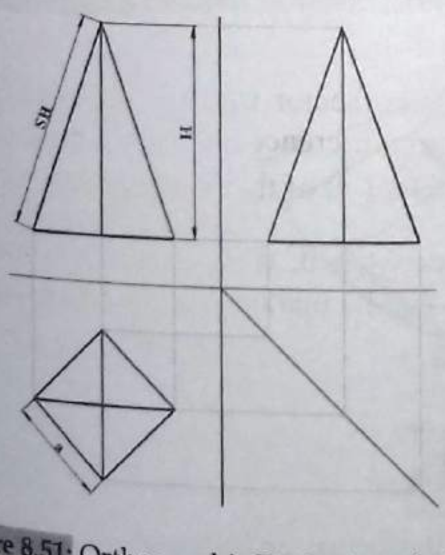


Figure 8.51: Orthographic Projection of a Right Square Pyramid

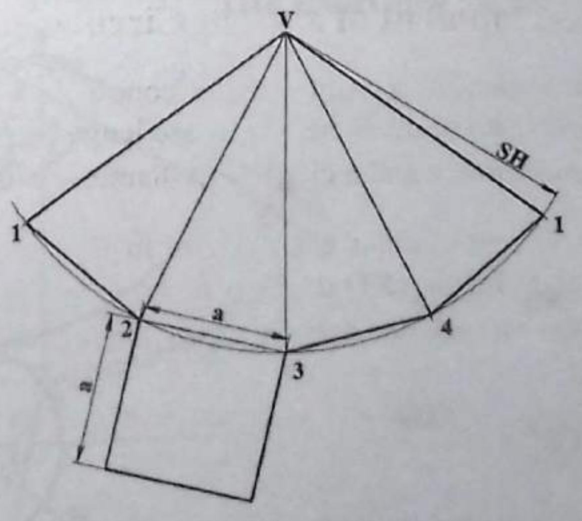
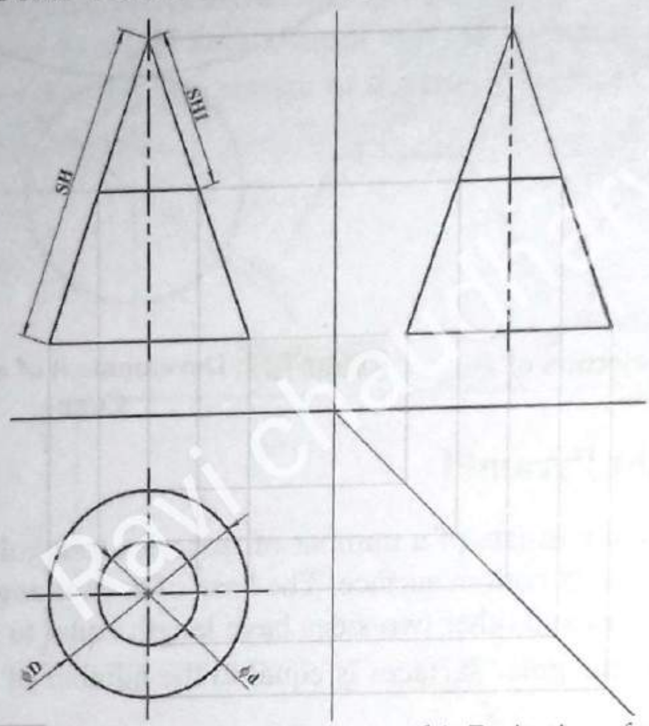


Figure 8.52: Development of a Right Square Pyramid

8.10 Development of Frustums of Right Solids

8.10.1 Development of Frustum of a Right Circular Cone

Figure 8.53 shows pictorial view and orthographic views of a frustum of a right circular cone. When a cone is cut by a plane parallel to its base, the top section appears as an edge view on both the front and side views whereas it appears as a circle on the top view.



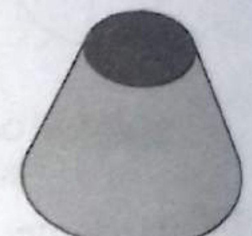


Figure 8.53: Pictorial view and Orthographic Projection of a Frustum of a Right Circular Cone

To develop it, some portion of the lateral surface should be removed from the complete development of the cone and small circular section should be added for the top surface.

Draw development of the complete cone as explained earlier. With **V** as center and **SH1** as radius draw a circular arc on the developed view to draw the lateral surface of the remaining part of the cone as shown in *Figure 8.54*. Draw a circle with a diameter **d** and tangent at any point to the arc with radius **SH1**.

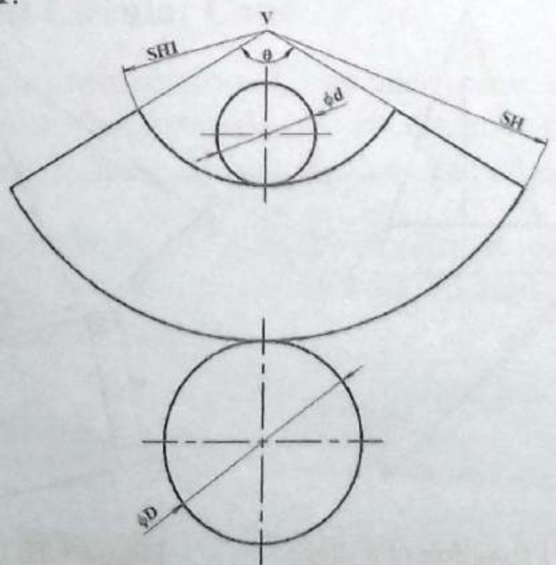


Figure 8.54: Development of Frustum of a Right Circular Cone

8.10.2 Development of Frustum of a Right Pyramid

Figure 8.55 shows pictorial view and orthographic views of a frustum of a right square pyramid. When a pyramid is cut by a plane parallel to its base, the top section appears as an edge view on both the front and side views whereas it appears as a square on the top view.

To develop it, some portion of the lateral surface should be removed from the complete development of the pyramid and small square section should be added for the top surface.

Draw development of the complete pyramid as explained earlier. Mark a point **p** on the line **V1** of the developed view. Draw edges **pq**, **qr**, **rs** and **sp** such that they are parallel to base edges **12**, **23**, **34** and **41** respectively. Draw square for the top section as any upper edge (say **qr**) as its base as shown in *Figure 8.56*.

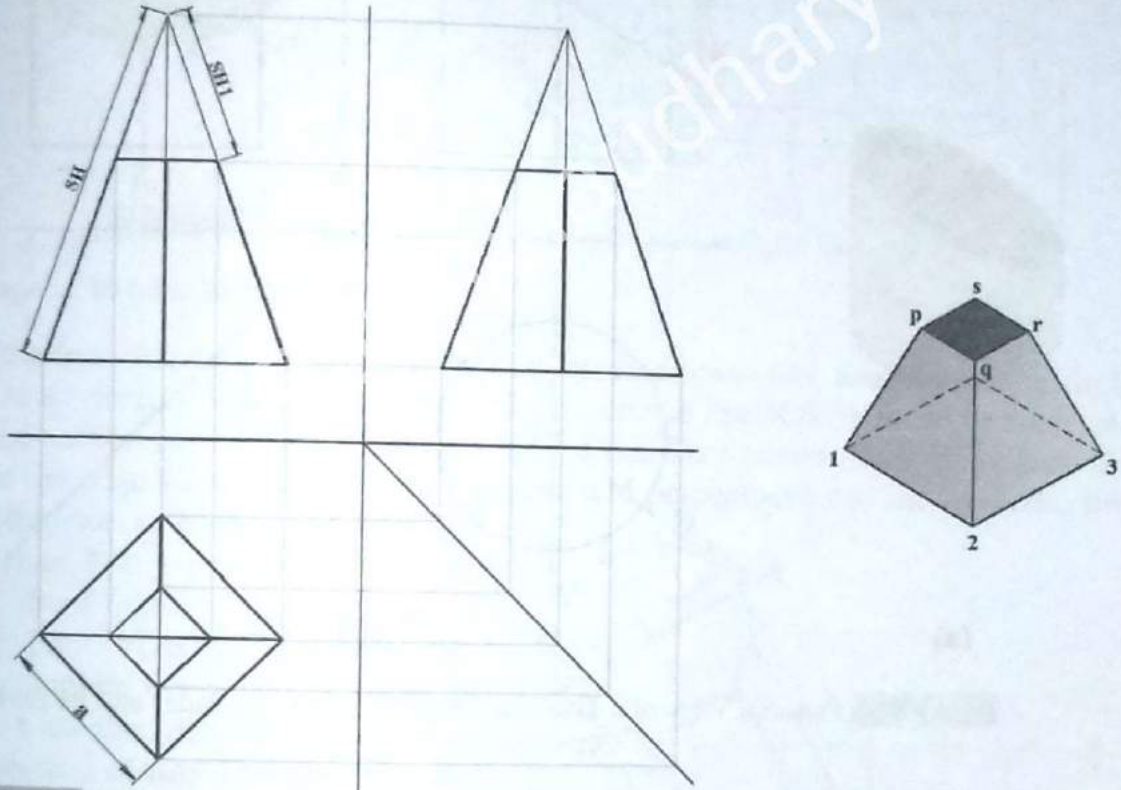


Figure 8.55: Pictorial view and Orthographic Projection of a Frustum of a Right Square Pyramid

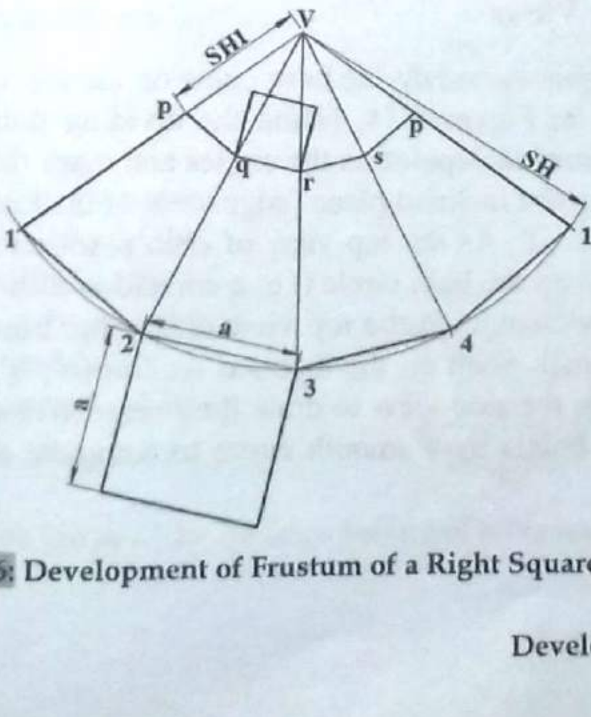


Figure 8.56: Development of Frustum of a Right Square Pyramid

8.11 Development of Truncated Right Solids

8.11.1 Development of a Truncated Right Circular Cylinder

Figure 8.57 shows the pictorial view of a truncated right circular cylinder and its corresponding front view, top view and partial side view.

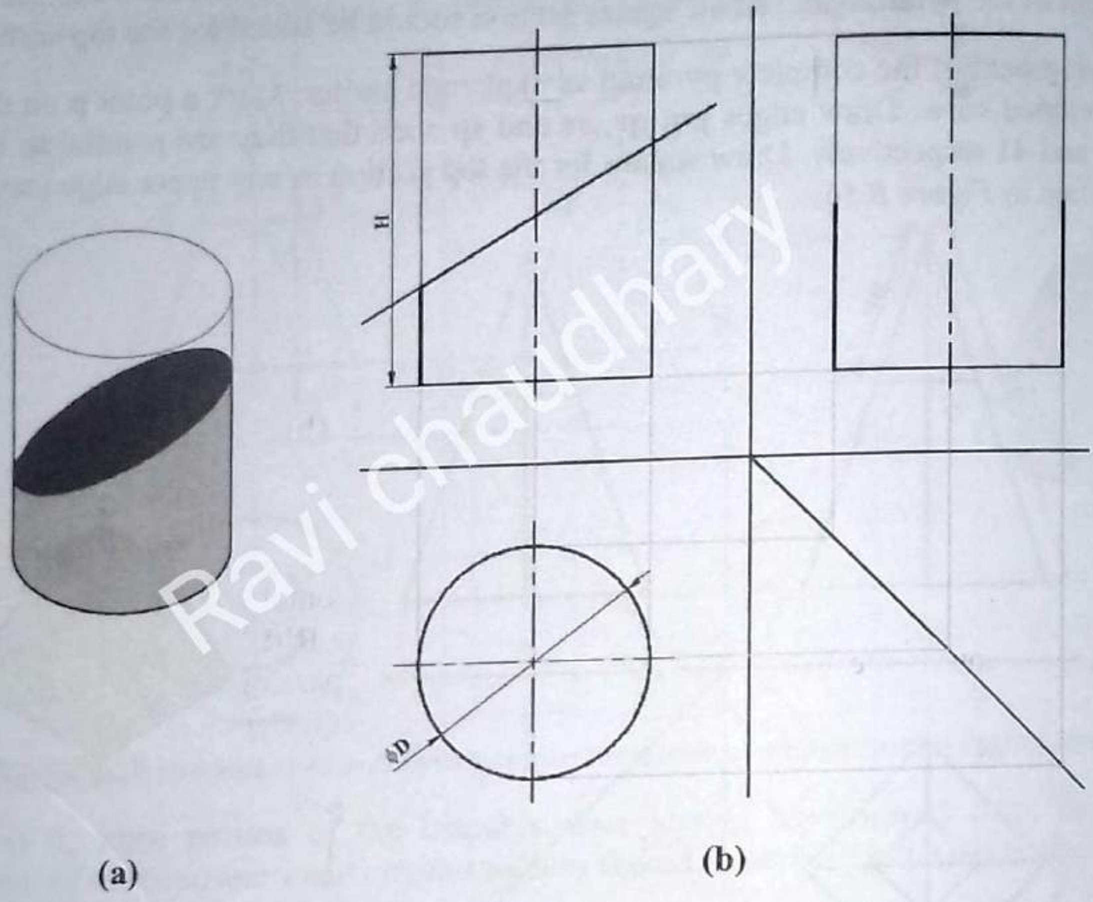


Figure 8.57: Pictorial View of a Truncated Right Circular Cylinder and its Partial Orthographic Views

Complete Orthographic Views

To complete its orthographic views, divide base circle on the top view into any number of equal parts (say **12**) as shown in *Figure 8.58*. Name the dividing points as **1**, **2**, ……., **12**. Draw vertical projection lines from each point on the circles and mark the pints of intersection of these projections line with the given inclined plane (edge view of the inclined elliptical section) on the front view as **a'**, **b'**, …….., **l'**. As the top view of each point on the elliptical section coincide with corresponding points on the base circle (i.e. **a** coincides with **1**, **b** coincides with **2**, ..and so on), there will not be any change on the top view due to the truncation. Draw projection lines from the front views of each point on the inclined section (**a'**, **b'**, …, **l'**) and top view of each point (**a**, **b**, …, **l**) towards the side view to draw their respective side views (**a''**, **b''**, ……, **l''**). Join side views of each points by a smooth curve to complete the side view of the truncated cylinder.

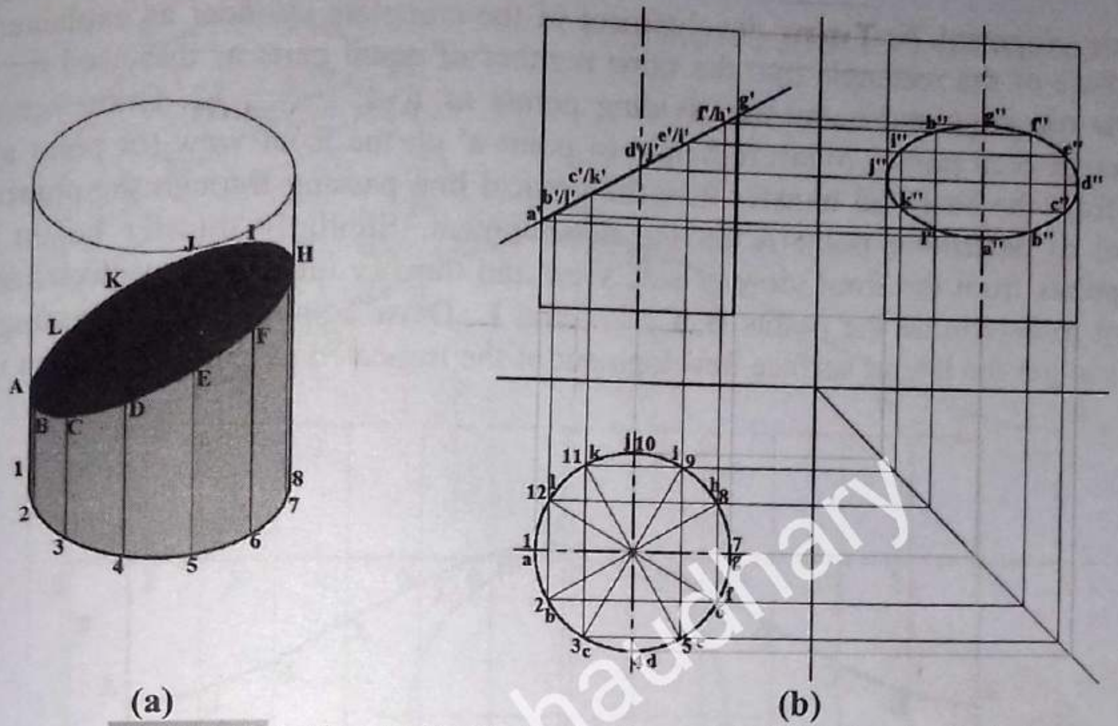


Figure 8.58: Orthographic Views of a Truncated Right Circular Cylinder

True Shape of the Inclined Section

To draw the true shape of the inclined section, assume horizontal diameter of the circle on the top view as a reference line **R/L**. Draw another reference line **R'/L'** parallel to the edge view of the inclined section on the front view. Draw projection lines passing through each points (**a'**, **b'**, ..., **l'**) on the edge view of the inclined section and perpendicular to the reference line **R'/L'**. Measure distance of each point from the reference line **R/L** on the top view and transfer them into the respective projection lines from the reference line **R'/L'** to complete the auxiliary views of each points on the inclined section. Join each points thus obtained on the auxiliary view by a smooth curve to get the true shape of the inclined section as shown in *Figure 8.59.*

Development

To develop the truncated cylinder, some portion of the lateral surface should be removed from the complete surface development. Similarly, the top circular section is not required and instead of it inclined section should be covered by its true shape.

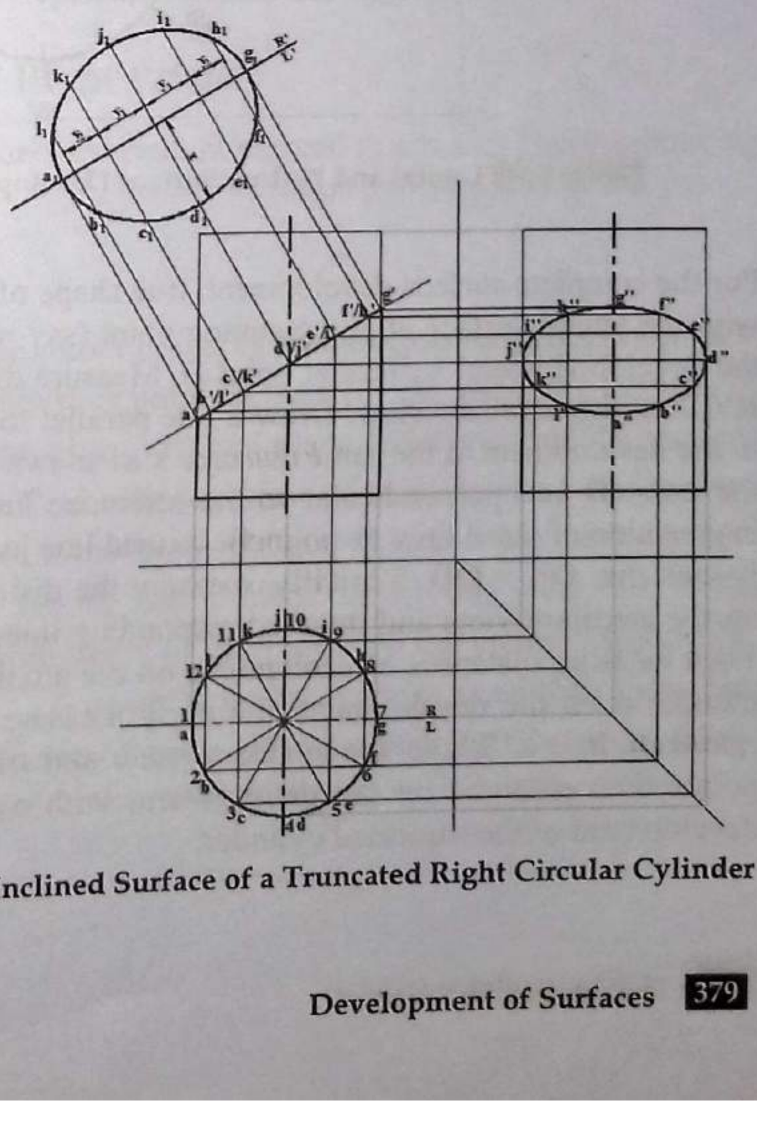


Figure 8.59: True Shape of the Inclined Surface of a Truncated Right Circular Cylinder

So for the development, first draw development of the complete cylinder as explained earlier. Divide the base of the rectangle into the same number of equal parts as that used for the base circle on the top view and name the dividing points as **1**, **2**, ……, **12**. Draw vertical lines passing through each point. Measure height of point **a'** on the front view (or point **a''** on the side view) from the base and transfer it to the vertical line passing through the point **1** on the development to determine point **A** on the development. Similarly transfer height of other remaining points from the front view or side view and transfer into the respective lines on the development to determine the points **B**, **C**, …., and **L**. Draw a smooth curve passing through these points to get the lateral surface development of the truncated cylinder, as shown in *Figure 8.60.*

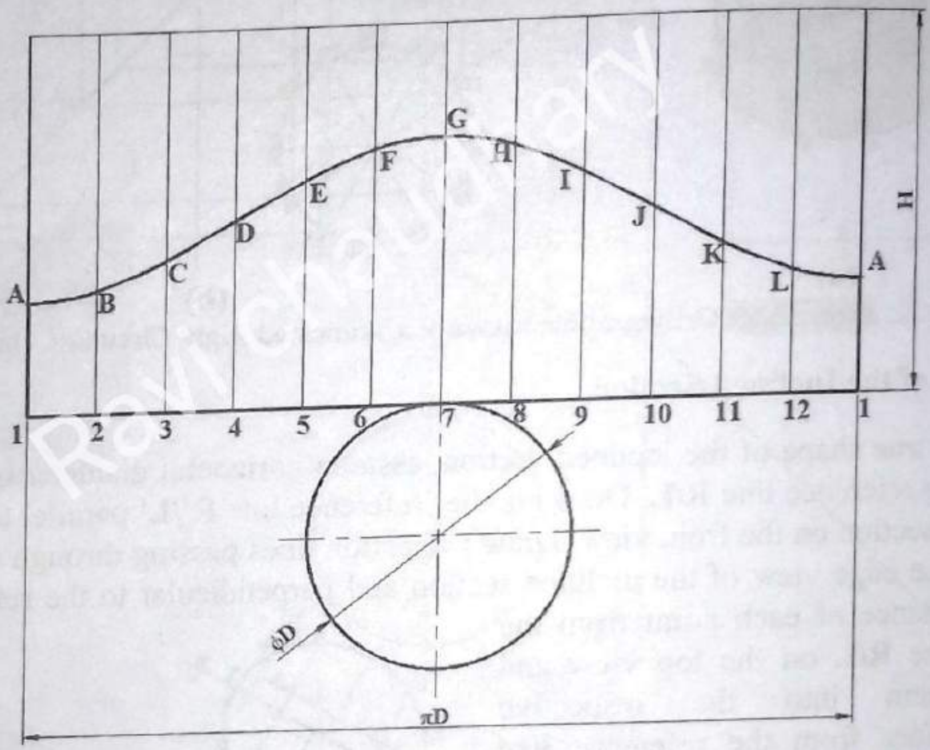


Figure 8.60: Lateral and Bottom Surface Development of a Truncated Right Circular Cylinder

For the complete surface development, true shape of the inclined section should also be attached with the lateral surface at any common point (say point **d**). Draw a line tangent to the curve of the developed lateral surface at point **D**. Measure distance **x** of point $\mathbf{d_1}$ from the reference line **R'/L'** on the auxiliary view. Draw a line parallel to the tangent line drawn through the point **D** in the development at the same distance **x** as shown in *Figure 8.61*. Draw a line passing through the point **D** and perpendicular to the reference line **R'/L'** on the development and mark the intersection of these lines as point **O**. Extend line joining the points **O** and **D** and mark the point $\mathbf{j_1}$ such that $\mathbf{Oj_1 = OD}$. Similarly, measure the distances $\mathbf{y_1}$ and $\mathbf{y_2}$ between the projection lines on the auxiliary view and draw corresponding lines on the development at the same distances. Then measure distances of each points on the auxiliary view from the reference line **R'/L'** and transfer it on the development. Similarly measure the distance of points $\mathbf{a_1}$ and $\mathbf{g_1}$ along the reference line **R'/L'** on the auxiliary view and transfer it into the development. Join all the points thus obtained on the development with a smooth curve to get the complete surface development of the truncated cylinder.

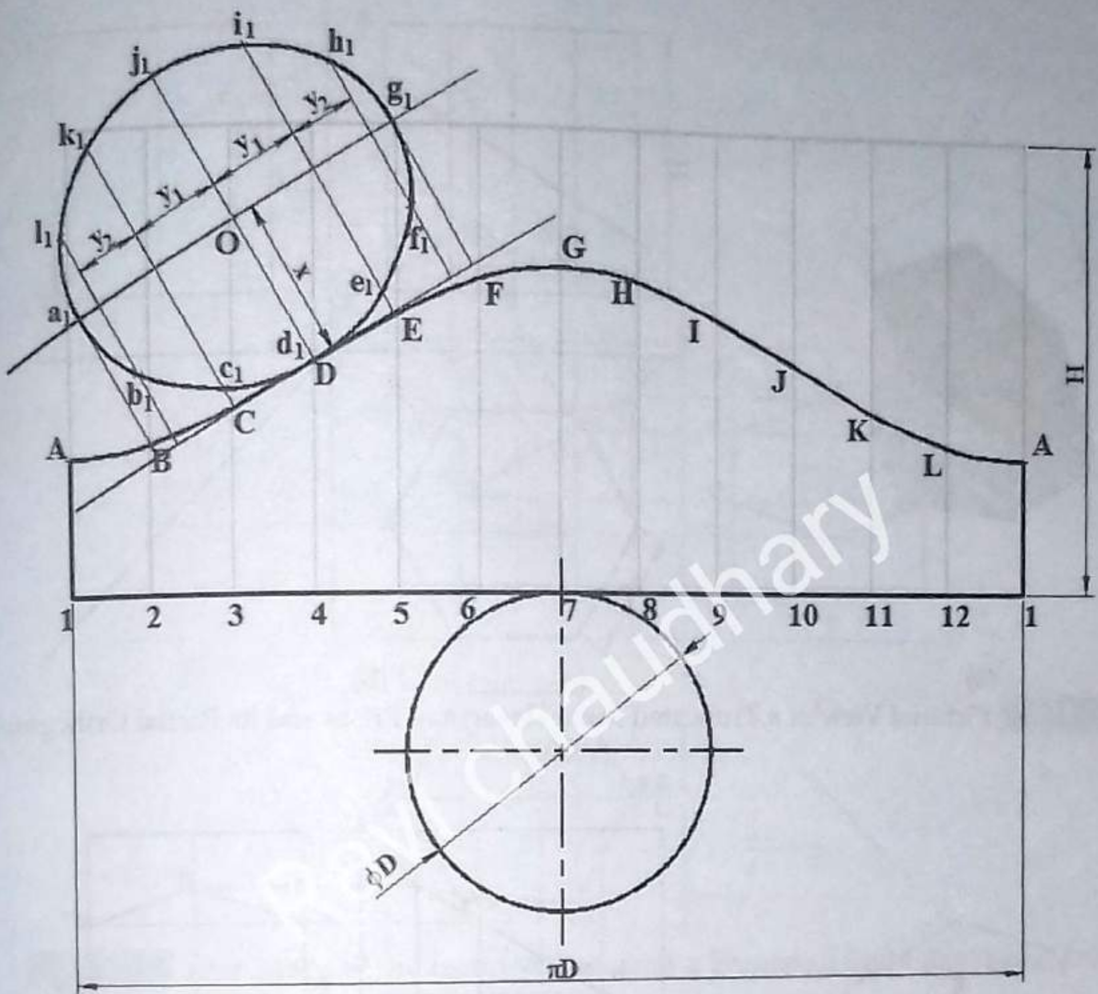


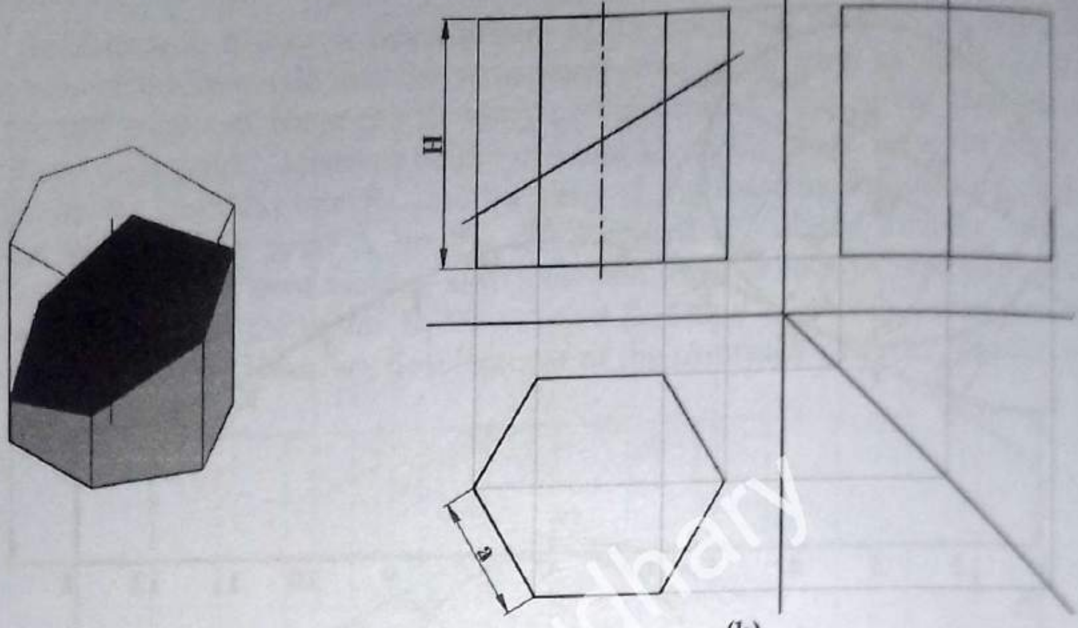
Figure 8.61: Complete Surface Development of a Truncated Right Circular Cylinder

8.11.2 Development of a Truncated Right Prism

Figure 8.62 shows the pictorial view of a truncated right hexagonal prism and its corresponding front view, top view and partial side view.

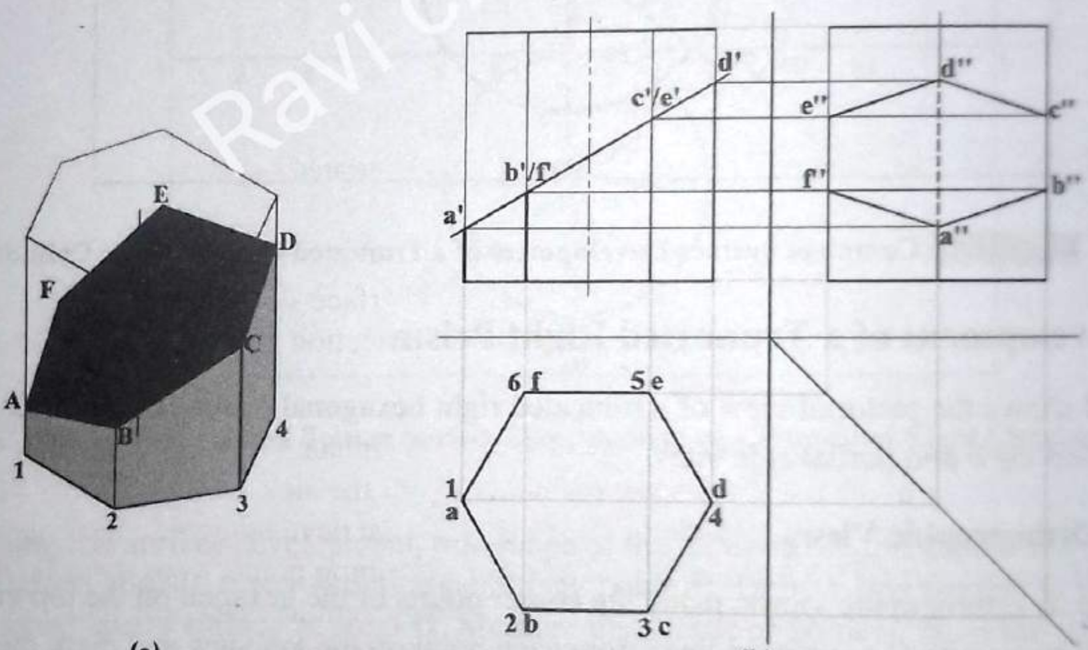
Complete Orthographic Views

To complete its orthographic views, name the corner points of the hexagon on the top view as **1**, **2**, ……., **6**. Draw vertical projection lines from each point on the top view and mark the pints of intersection of these projections line with the given inclined plane (edge view of the inclined section) on the front view as **a'**, **b'**, …….., **f'**. As the top view of each point on the inclined section coincide with corresponding points on the base hexagon (i.e. **a** coincides with **1**, **b** coincides with **2**, ..and so on), there will not be any change on the top view due to the truncation. Draw projection lines from the front views of each point on the inclined section (**a'**, **b'**, …, **f'**) and top view of each point (**a**, **b**, …, **l**) towards the side view to draw their respective side views (**a''**, **b''**, ……, **l''**). Join side views of each point by straight line segments to complete the side view of the truncated prism.



(a) (b)

Figure 8.62: Pictorial View of a Truncated Right Hexagonal Prism and its Partial Orthographic Views



(a) (b)

Figure 8.63: Orthographic Views of a Truncated Right Hexagonal Prism

True Shape of the Inclined Section

To draw the true shape of the inclined section, assume horizontal line from the middle of the hexagon on the top view as a reference line **R/L**. Draw another reference line **R'/L'** parallel to the edge view of the inclined section on the front view. Draw projection lines passing through each points (**a'**, **b'**, ..., **f'**) on the edge view of the inclined section and perpendicular to the reference line **R'/L'**. Measure distance of each point from the reference line **R/L** on the top view and transfer them into the respective projection lines from the reference line **R'/L'** to complete the auxiliary view of each points on the inclined section. Join each point thus obtained on the auxiliary view by straight line segments to get the true shape of the inclined section as shown in *Figure 8.64*.

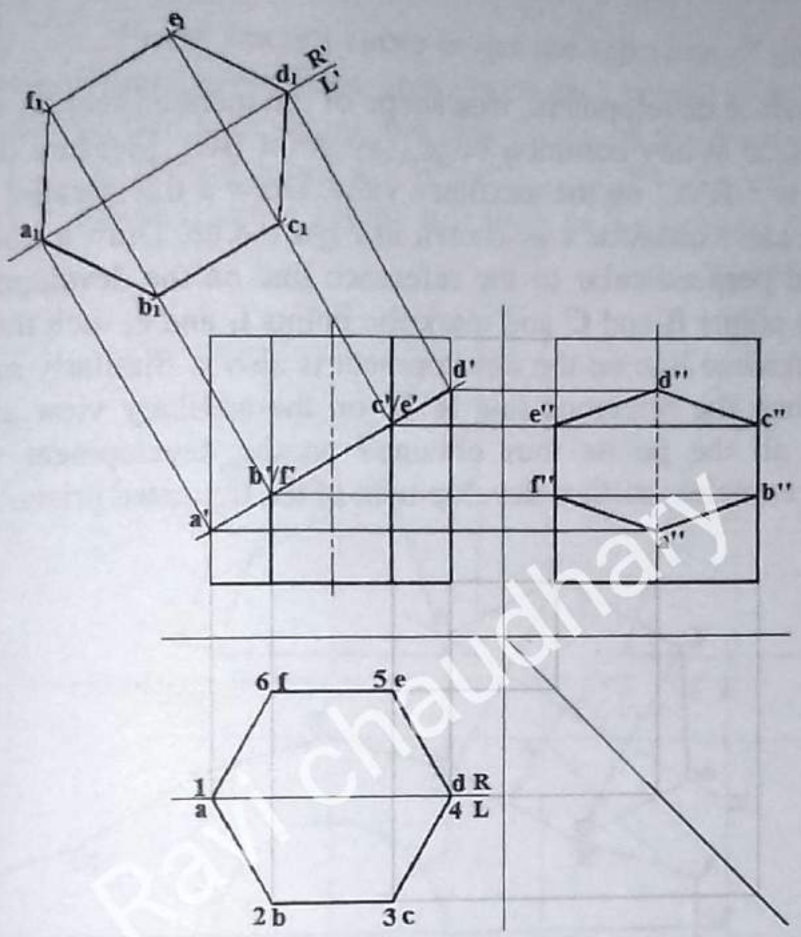


Figure 8.64: True Shape of the Inclined Surface of a Truncated Right Hexagonal Prism

Development

To develop the truncated prism, some portion of the lateral surface should be removed from the complete surface development. Similarly, the top hexagonal section is not required and instead of it inclined section should be covered by its true shape.

So for the development, first draw development of the complete prism as explained earlier. Measure height of point **a'** on the front view (or point **a''** on the side view) from the base and transfer it to the vertical line passing through the point **1** on the development to determine point A on the development. Similarly transfer height of other remaining points from the front view or side view and transfer into the respective lines on the development to determine the points **B**, **C**, ..., and **F**. Draw a straight line segments passing through these points to get the lateral surface development of the truncated prism, as shown in *Figure 8.65.*

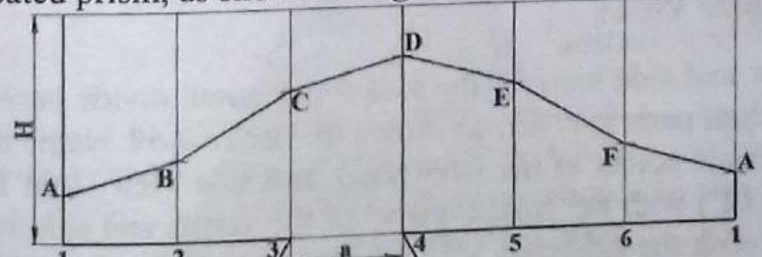


Figure 8.65: Lateral and Bottom Surface Development of a Truncated Right Hexagonal Prism

For the complete surface development, true shape of the inclined section should also be attached with the lateral surface at any common edge (say point **BC**). Measure distance **x** of edge b_1c_1 from the reference line **R'/L'** on the auxiliary view. Draw a line parallel to the edge **BC** in the development at the same distance **x** as shown in *Figure 8.66.* Draw a lines passing through the points **B** and **C** and perpendicular to the reference line on the development. Extend the lines passing through the points **B** and **C** and mark the points f_1 and e_1 such the distance between the line f_1e_1 and the reference line on the development is also **x**. Similarly measure the distance of points a_1 and f_1 along the reference line **R'/L'** on the auxiliary view and transfer it into the development. Join all the points thus obtained on the development with the straight line segments to get the complete surface development of the truncated prism.

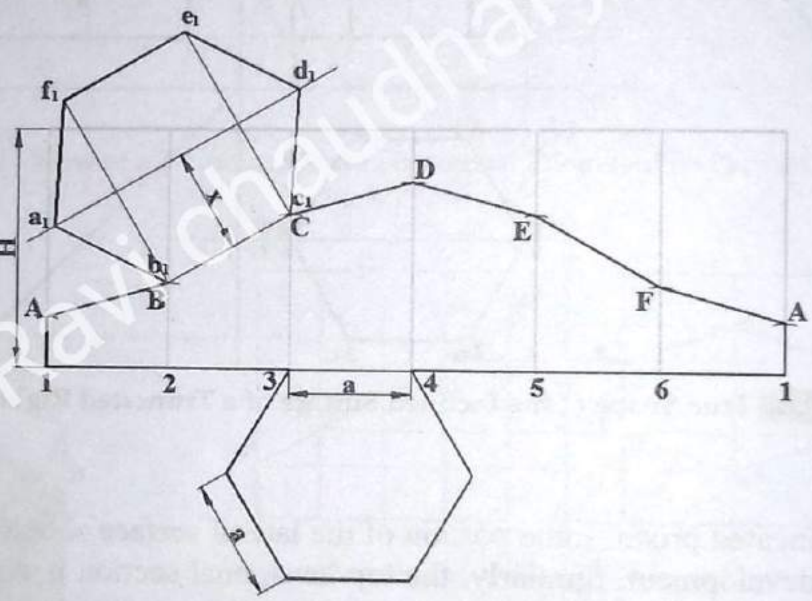


Figure 8.66: Complete Surface Development of a Truncated Right Hexagonal Prism

8.11.3 Development of a Truncated Right Circular Cone

Figure 8.67 shows the pictorial view of a truncated right circular cone and its corresponding front view, partial top view and partial side view. *Figure 8.68* shows the same cone with generators.

Complete Orthographic Views

To draw the top view and side view of the inclined section, divide base circle on the top view into any number of equal parts (say **12**) as shown in *Figure 8.69*. Name the dividing points as **1**, **2**, ……., **12**. Transfer all points to the front view and side view. Join front views of all these points (**1'**, **2'**, ……., **12'**) with the front view **v'** of the vertex and side views of all these points (**1''**, **2''**, ……., **12''**) with the side view **v''** of the vertex. Mark intersections of the edge view of the inclined surface and the generators on the front view as point **a'** on **v'1'**, **b'** on **v'2'**, **c'** on **v'3'**, and so on. Draw vertical projection lines from each point (**a'**, **b'**, **c'**, ….., **l'**) on the front view of the inclined section intersecting the top views of the corresponding generators **v1**, **v2**, **v3**, ……, **v12** on points **a**, **b**, **c**, ……, **l** respectively. To get top views of the points **D** and **J**, either use cutting plane method or transfer them first to the side view and then to the top view.

Join the points **a**, **b**, **c**, ……, **l** by a smooth curve to get the top view of the inclined elliptical section. Similarly, draw horizontal projections lines from each point (**a'**, **b'**, **c'**, ….., **l'**) on the front view of the inclined section intersecting the side views of the corresponding generators v''1'', v''2'', v''3'', ……, v''12'' on points **a''**, **b''**, **c''**, ……, **l''** respectively. Join the points a'', b'', c'', ……, l'' by a smooth curve to get the side view of the inclined elliptical section.

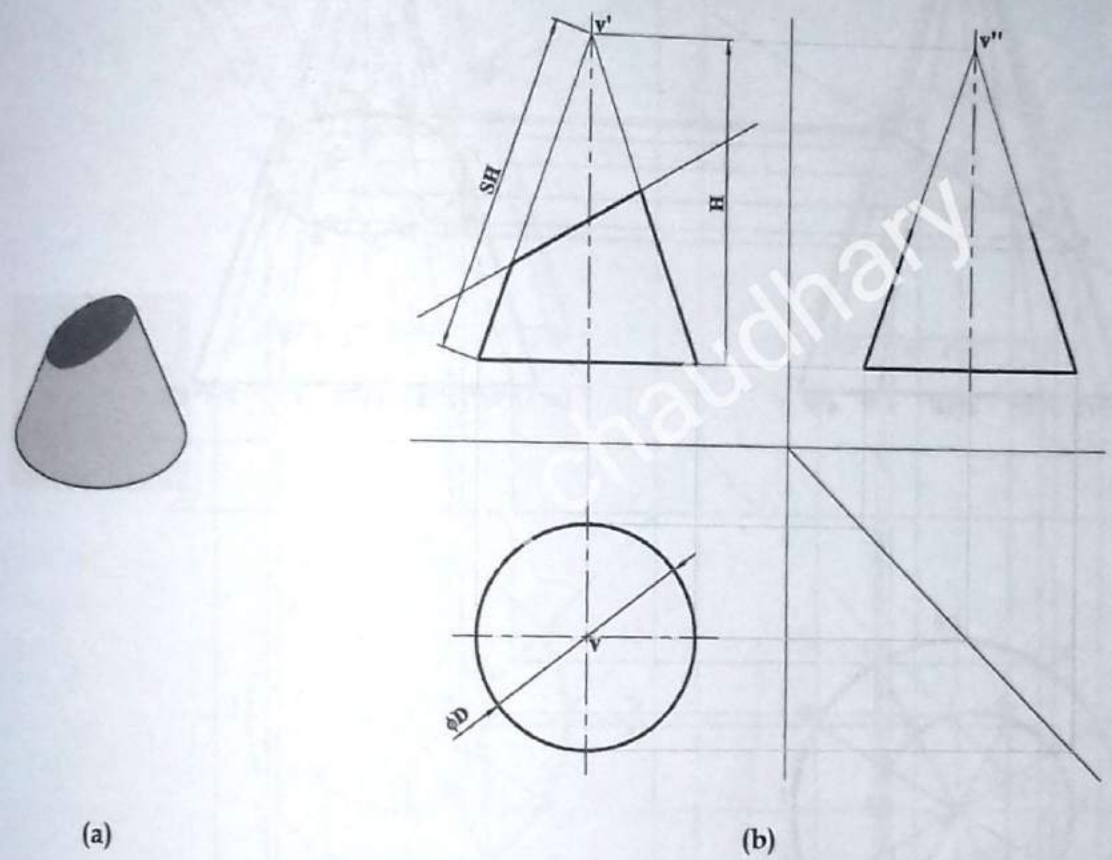


Figure 8.67: Pictorial View of a Truncated Right Circular Cone and its Partial Orthographic Views

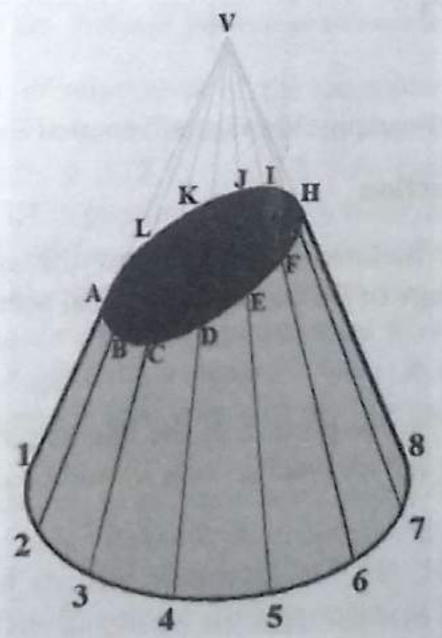


Figure 8.68: Pictorial View of a Truncated Right Circular Cone with Generators

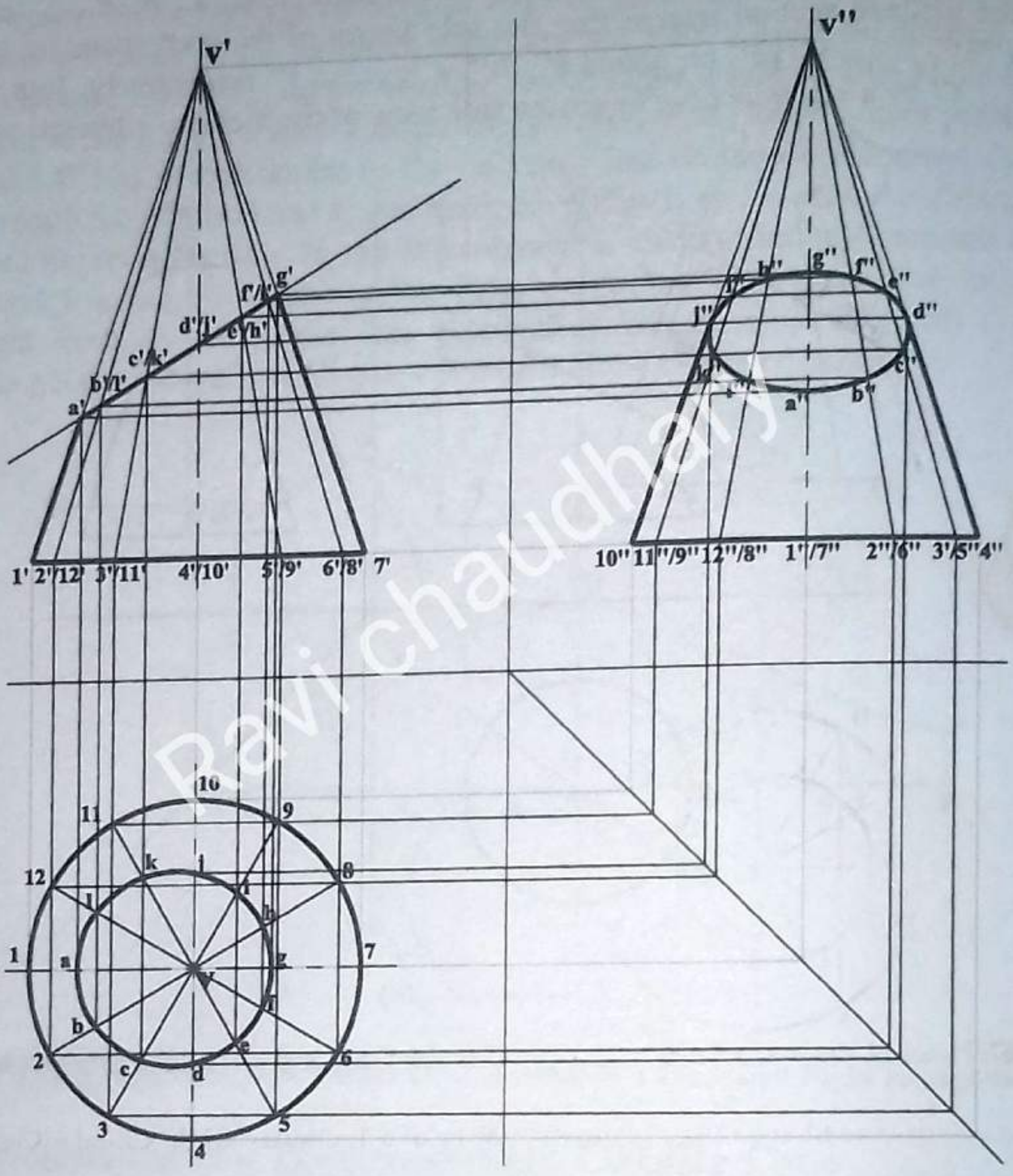


Figure 8.69: Orthographic Views of a Truncated Right Circular Cone

True Shape of the Inclined Section

To draw the true shape of the inclined section, follow the same prouder as explained for the truncated cylinder. The true shape of the inclined elliptical section is shown in *Figure 8.70.*

Development

To develop the truncated cone, some portion of the lateral surface should be removed from the complete surface development and inclined section should be covered by its true shape.

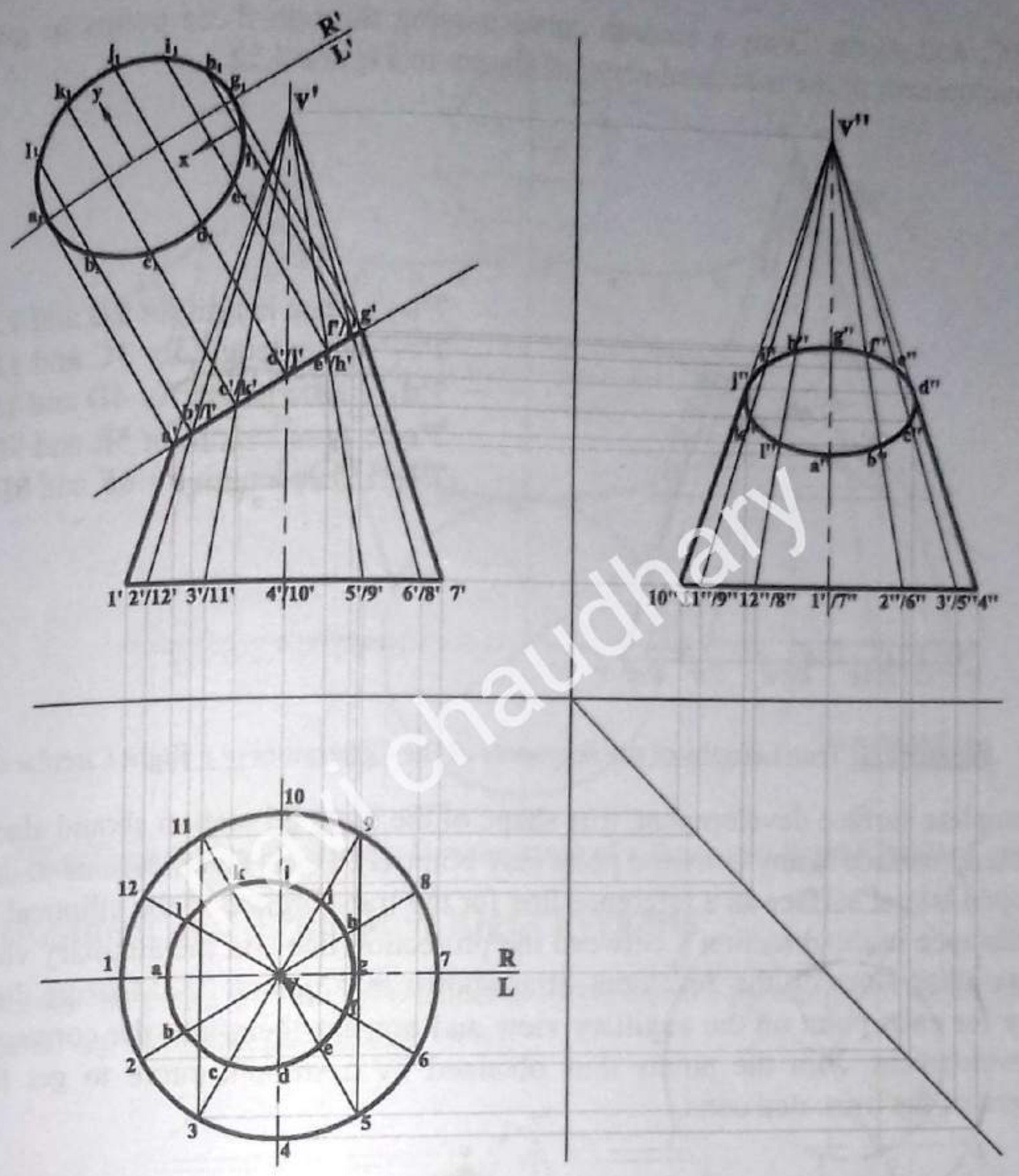
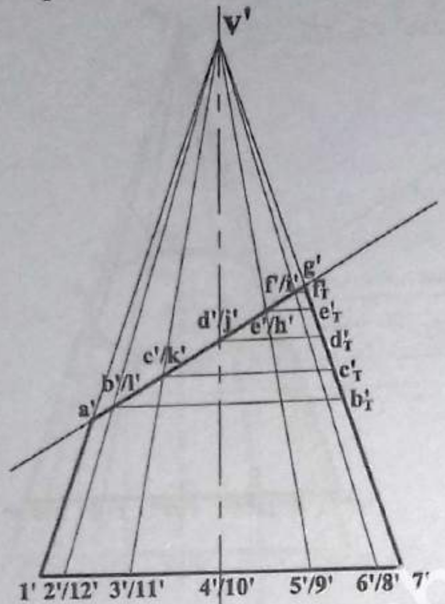


Figure 8.70: True Shape of the Inclined Surface of a Truncated Right Circular Cone

So for the development, first draw development of the complete cone as explained earlier. Divide the circular arc into the same number of equal parts as that used for the base circle on the top view and name the dividing points as **1, 2, ……., 12**. Join each point on the development with the vertex **V**. As the generator **V1** is parallel to the VP, it appears in true length in the front view. Therefore, measure true length of the segment of the generator **1A** in the front view **1'a'** and transfer it into the development along the line **V1** to mark the point **A** in the development. Point **G** can also be marked into the development because its front view **7'g'** also appears in true length. But the other remaining generators do not appear in true length in either of the principal views. Therefore to determine the true length of the line segment **2B** draw a horizontal passing through the point **b'** on the front view intersecting **v'7'** at the point **b_T'** as shown in *Figure 8.71*. Then **7'b_T'** gives the true length of the line segment **2B**. Transfer this length along the line **2B** in the development to mark the point **B** in the development. Follow the similar procedure to find the true lengths of the line segments **3C, 4D, 5E, 6F** and transfer into the respective line in the development. True length for the line segment **12L** will be same as that for the line segment **2B**, true length for the line segment **11K** will be same as that for the line segment 3C, and so on. Draw a smooth curve passing through these points to get the lateral surface development of the truncated cone, as shown in **Figure 8.72**.



$7'b_T'$: True length for **2B** and **12L**
$7'c_T'$: True length for **3C** and **11K**
$7'd_T'$: True length for **4D** and **10J**
$7'e_T'$: True length for **5E** and **9I**
$7'f_T'$: True length for **6F** and **8H**

Figure 8.71: True Lengths of the Segments of the Generators of a Right Circular Cone

For the complete surface development, true shape of the inclined section should also be attached with the lateral surface at any common point (say point **G**). Draw a vertical line at the point **D** of the developed lateral surface as a reference line for the transference of the elliptical true section. Measure distance in the direction **x** between the projection lines on the auxiliary view and draw the corresponding lines on the development as shown in *Figure 8.73*. Measure distance in the direction **y** for each point on the auxiliary view and transfer them into the corresponding lines on the development. Join the points thus obtained by a smooth curve to get the complete development of the truncated cone.

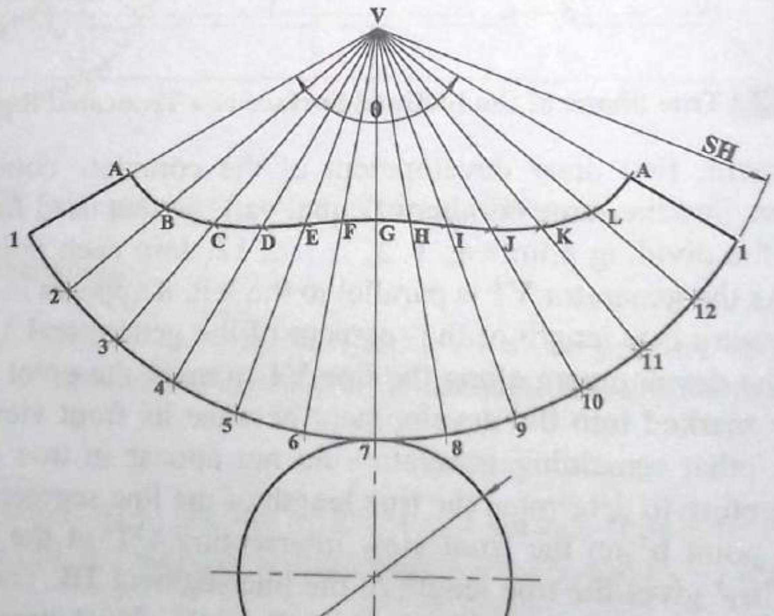


Figure 8.72: Lateral and Bottom Surface Development of a Truncated Right Circular Cone

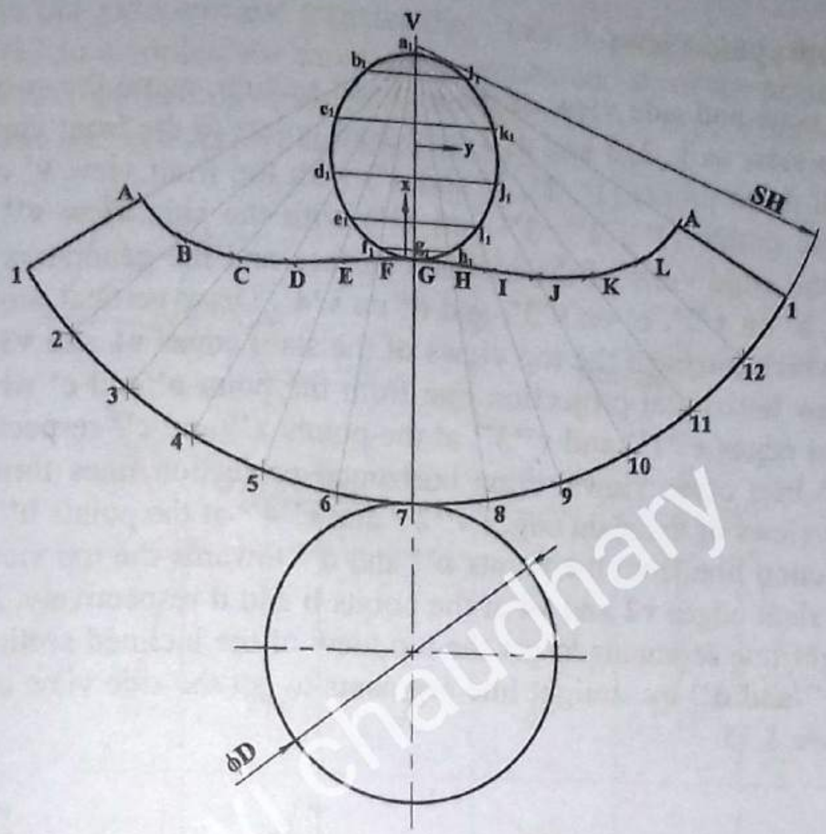


Figure 8.73: Complete Surface Development of a Truncated Right Circular Cone

8.11.4 Development of a Truncated Right Pyramid

Figure 8.74 shows the pictorial view of a truncated right square pyramid and its corresponding front view, partial top view and partial side view.

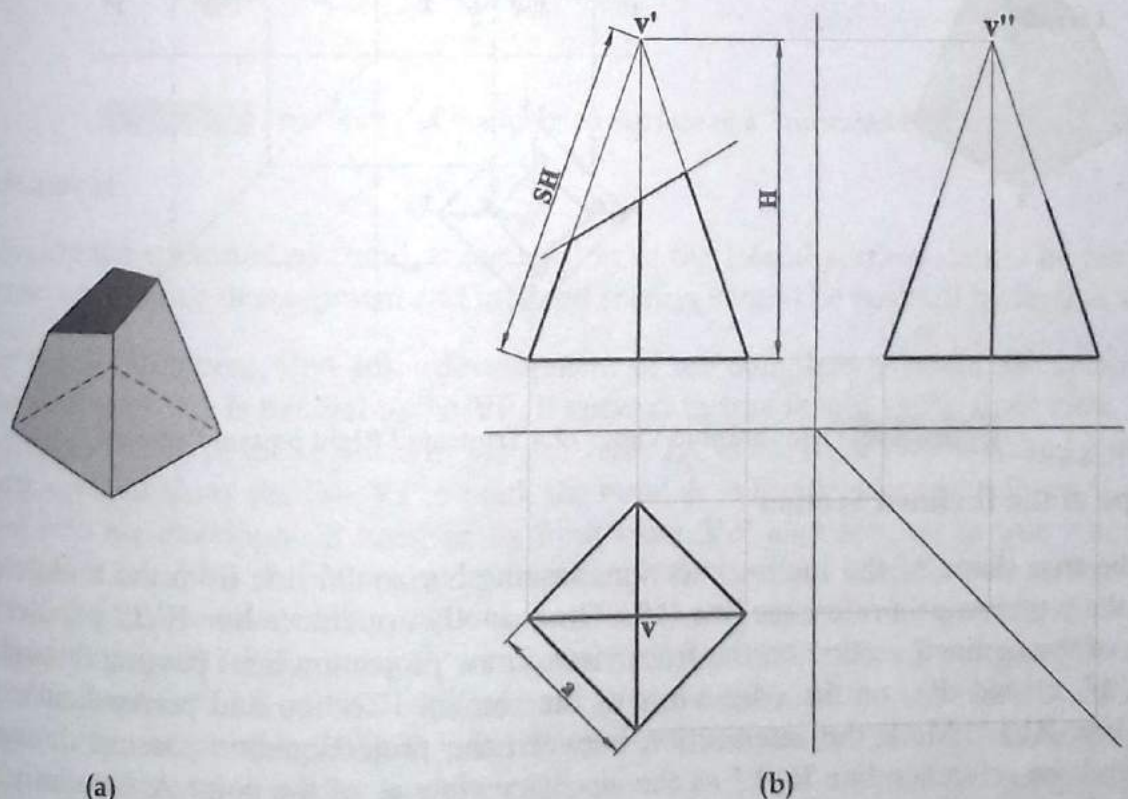
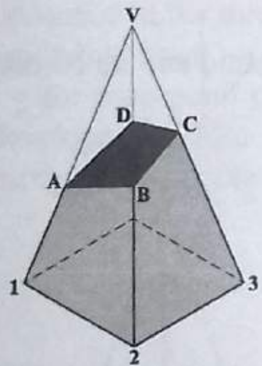


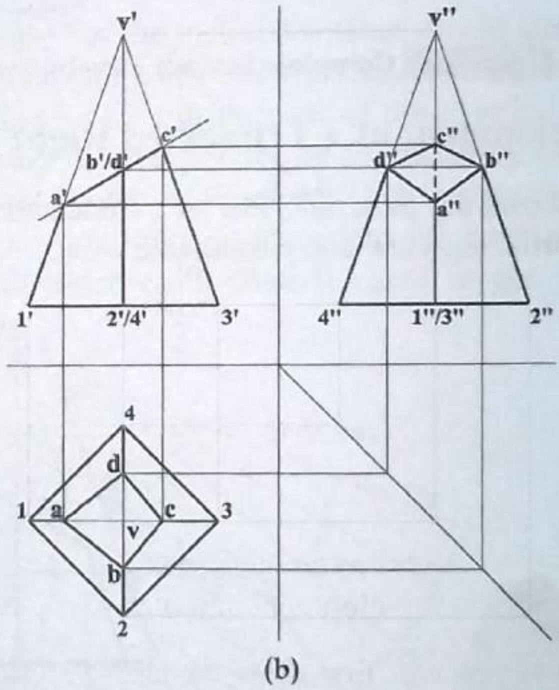
Figure 8.74: Pictorial View of a Truncated Right Square Pyramid and its Partial Orthographic Views

Complete Orthographic Views

To draw the top view and side view of the inclined section, name the corner points of the base square on the top view as **1, 2, 3** and **4**. Transfer all points to the front view and side view. Join front views of all these points (**1', 2', 3'** and **4'**) with the front view **v'** of the vertex and side views of all these points (**1'', 2'', 3''** and **4''**) with the side view **v''** of the vertex. Mark intersections of the edge view of the inclined surface and the generators on the front view as point **a'** on **v'1'**, **b'** on **v'2'**, **c'** on **v'3'**, and **d'** on **v'4'**. Draw vertical projection lines from the point **a'** and **c'** which intersect the top views of the slant edges **v1** and **v3** at the points **a** and **c** respectively. Draw horizontal projection line from the point **a'** and **c'** which intersect the side views of the slant edges **v''1''** and **v''3''** at the points **a''** and **c''** respectively. To transfer the points **b'** and **d'** into other views, draw horizontal projection lines form these points which intersect the side views of the slant edges **v''2''** and **v''4''** at the points **b''** and **d''** respectively. Then draw projection line from the points **b''** and **d''** towards the top view which intersect the top views of the slant edges **v2** and **v4** at the points **b** and **d** respectively. Join the points **a, b, c** and **d** by a straight line segments to get the top view of the inclined section. Similarly, join the points **a'', b'', c''** and **d''** by straight line segments to get the side view of the inclined section as shown in *Figure 8.75.*



(a)



(b)

Figure 8.75: Orthographic Views of a Truncated Right Square Pyramid

True Shape of the Inclined Section

To draw the true shape of the inclined section, assume horizontal line from the middle of the square on the top view as a reference line **R/L**. Draw another reference line **R'/L'** parallel to the edge view of the inclined section on the front view. Draw projection lines passing through each points (**a'**, **b'**, **c'** and **d'**) on the edge view of the inclined section and perpendicular to the reference line **R'/L'**. Mark the intersection between the projection line passing through the points **a'** and the reference line **R'/L'** as the auxiliary view $\mathbf{a_1}$ of the point **A**. Similarly, mark the intersection between the projection line passing through the points **c'** and the reference line **R'/L'** as the auxiliary view $\mathbf{c_1}$ of the point **C**. Measure distance of points **b** and **d** from the reference line **R/L** on the top view and transfer them into the respective projection lines from the reference line **R'/L'** to complete the auxiliary views $\mathbf{b_1}$ and $\mathbf{d_1}$ of the points **B** and **D** on the inclined section. Join each point thus obtained on the auxiliary view by straight line segments to get the true shape of the inclined section as shown in *Figure 8.76.*

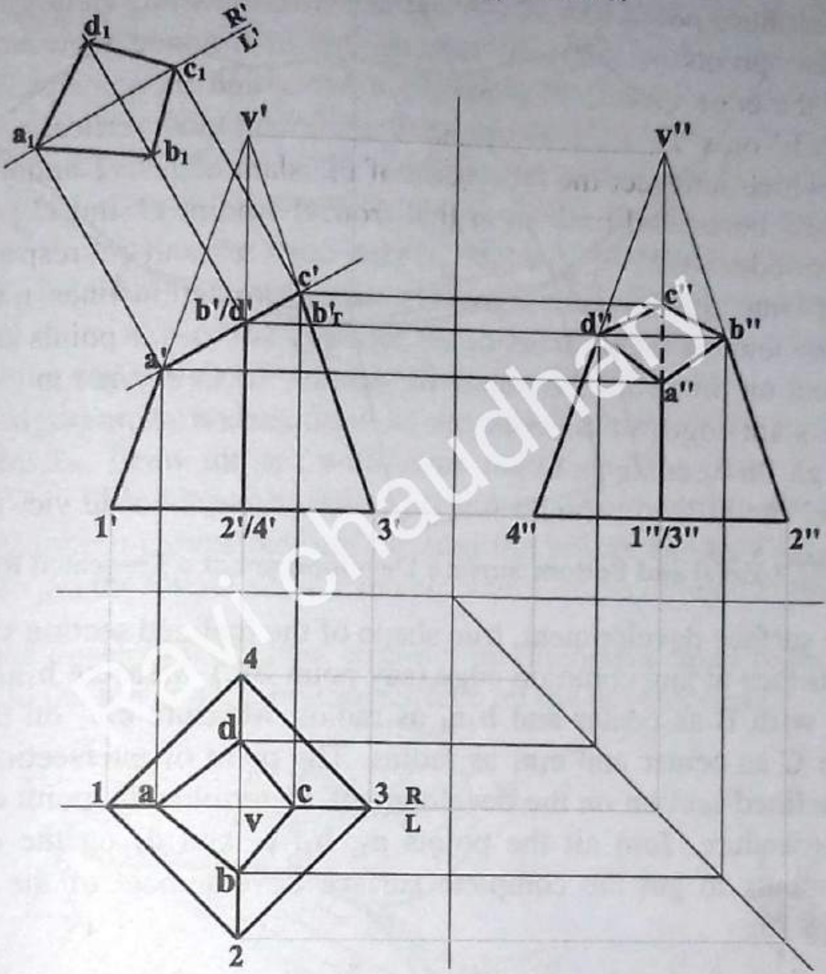


Figure 8.76: True Shape of the Inclined Surface of a Truncated Right Square Pyramid

Development

To develop the truncated pyramid, some portion of the lateral surface should be removed from the complete surface development and inclined section should be covered by its true shape.

So for the development, first draw development of the complete pyramid as explained earlier. As the slant edge **V1** is parallel to the VP, it appears in true length in the front view. Therefore, measure true length of the segment of the generator **1A** in the front view **1'a'** and transfer it into the development along the line **V1** to mark the point **A** in the development. Point **C** can also be marked into the development because its front view **3'c'** also appears in true length. But the slant edges **V2** and **V4** do not appear in true length front view. Therefore to determine the true length of the line segment **2B** draw a horizontal passing through the point **b'/d'** on the front view intersecting **v'3'** at the point $\mathbf{b_T}$**'** as shown in *Figure 8.76.* Then **3'**$\mathbf{b_T}$**'** gives the true length for the line segments **2B** and **4D**. Transfer these true lengths along the line **2B** and **4D** in the development to mark the points **B** and **D** in the development. Join all these points by the straight line segments to get the lateral surface development of the truncated pyramid, as shown in *Figure 8.77.*

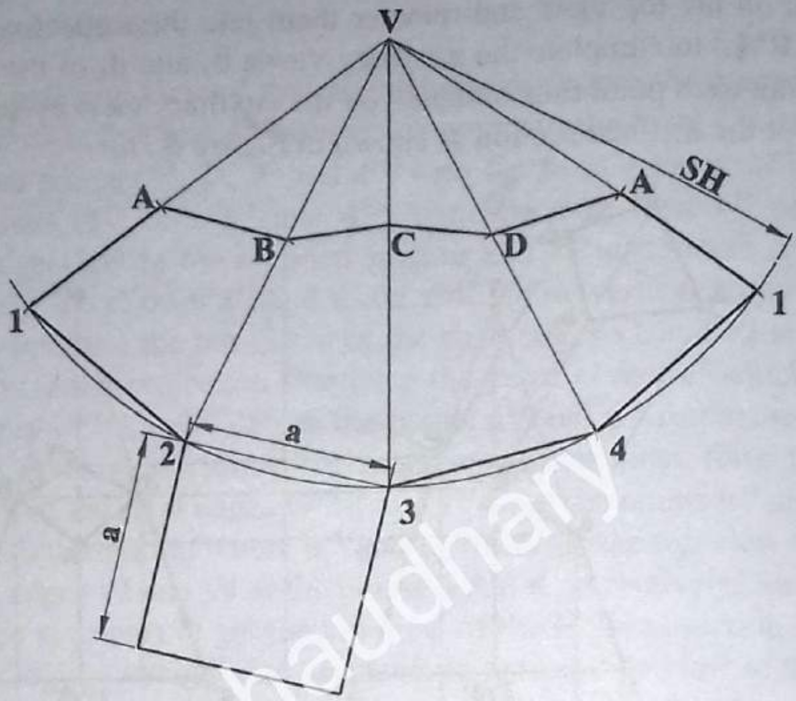


Figure 8.77: Lateral and Bottom Surface Development of a Truncated Right Square Pyramid

For the complete surface development, true shape of the inclined section should also be attached with the lateral surface at any common edge (say point **BC**). Measure $\mathbf{b_1a_1}$ on the auxiliary view and draw an arc with **B** as center and $\mathbf{b_1a_1}$ as radius. Measure $\mathbf{c_1a_1}$ on the auxiliary view and draw an arc with **C** as center and $\mathbf{c_1a_1}$ as radius. The point of intersection of theses arcs gives point $\mathbf{a_1}$ of the inclined section on the development. Determine the point $\mathbf{d_1}$ on the development by the similar procedure. Join all the points $\mathbf{a_1}$, $\mathbf{b_1}$, $\mathbf{c_1}$ and $\mathbf{d_1}$ on the development with the straight line segments to get the complete surface development of the truncated pyramid as shown in *Figure 8.78.*

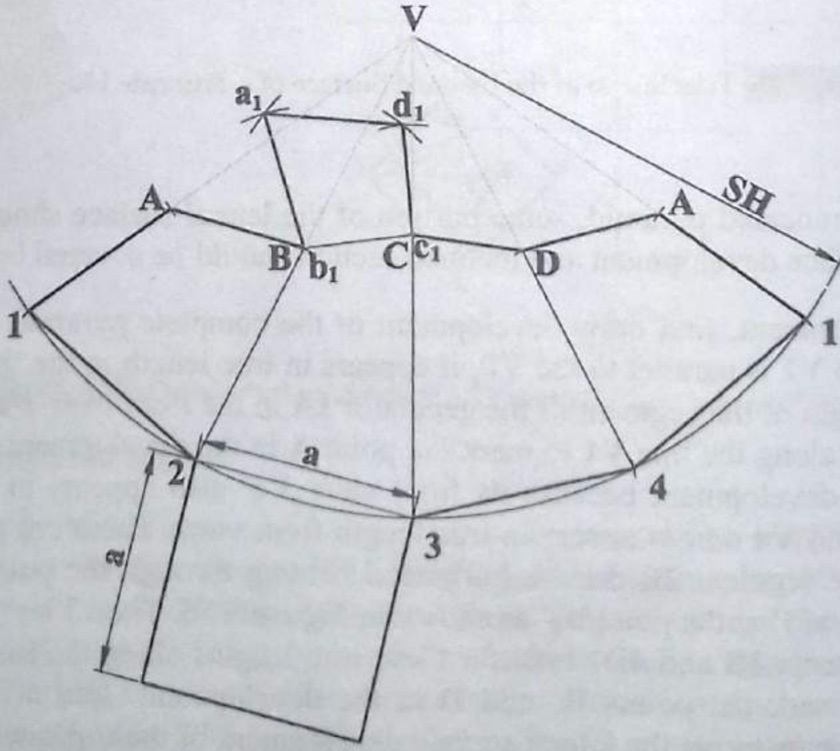


Figure 8.78: Complete Surface Development of a Truncated Right Square Pyramid

8.12 Development of Oblique Solids

8.12.1 Development of an Oblique Circular Cylinder

Consider an oblique circular cylinder shown in *Figure 8.21* and **8.22**. To develop it, divide top view of the bottom circular section into any number of equal parts (say **12**) and name the dividing points as **1**, **2**, **3**, ……, **12** as shown in **Figure 8.79**. Transfer each point on the circle into the front view to get their corresponding front views **1'**, **2'**, **3'**, ……., **12'**. Draw lines passing through the points **1'**, **2'**, **3'**, ……., **12'** and parallel to the axis to get the front views of the generator. All the generators appear in true length in the front view. Draw straight lines passing through ends of each generators and perpendicular to the generators in the front view. Mark any line $\mathbf{1_B}$ and $\mathbf{1_T}$ between the lines passing through ends of the generator passing though the point **1'** and parallel to the axis of the cylinder. With $\mathbf{1_B}$ as the center and arc length between the points **1** and **2** in the top (= **x**) (if number divisions of the circle is more chord length approximates the arc length) as the radius, draw an arc which intersects the line passing through the point **2'** at the point $\mathbf{2_B}$. Draw the arc with same radius and the point $\mathbf{1_T}$ as the center to determine the point $\mathbf{2_T}$. By following the similar procedure determine the points $\mathbf{3_B}$, $\mathbf{3_T}$, $\mathbf{4_B}$, $\mathbf{4_T}$, ……, $\mathbf{12_B}$, $\mathbf{12_T}$. Draw smooth curves passing through the points $\mathbf{1_B}$, $\mathbf{2_B}$, ….. , $\mathbf{12_B}$ and $\mathbf{1_T}$, $\mathbf{2_T}$, ….. , $\mathbf{12_T}$ respectively to get the lateral surface development of the oblique cylinder.

For the complete surface development, draw circles with diameters equal to the diameter of the given cylinder tangent to the top and the bottom surface on the development.

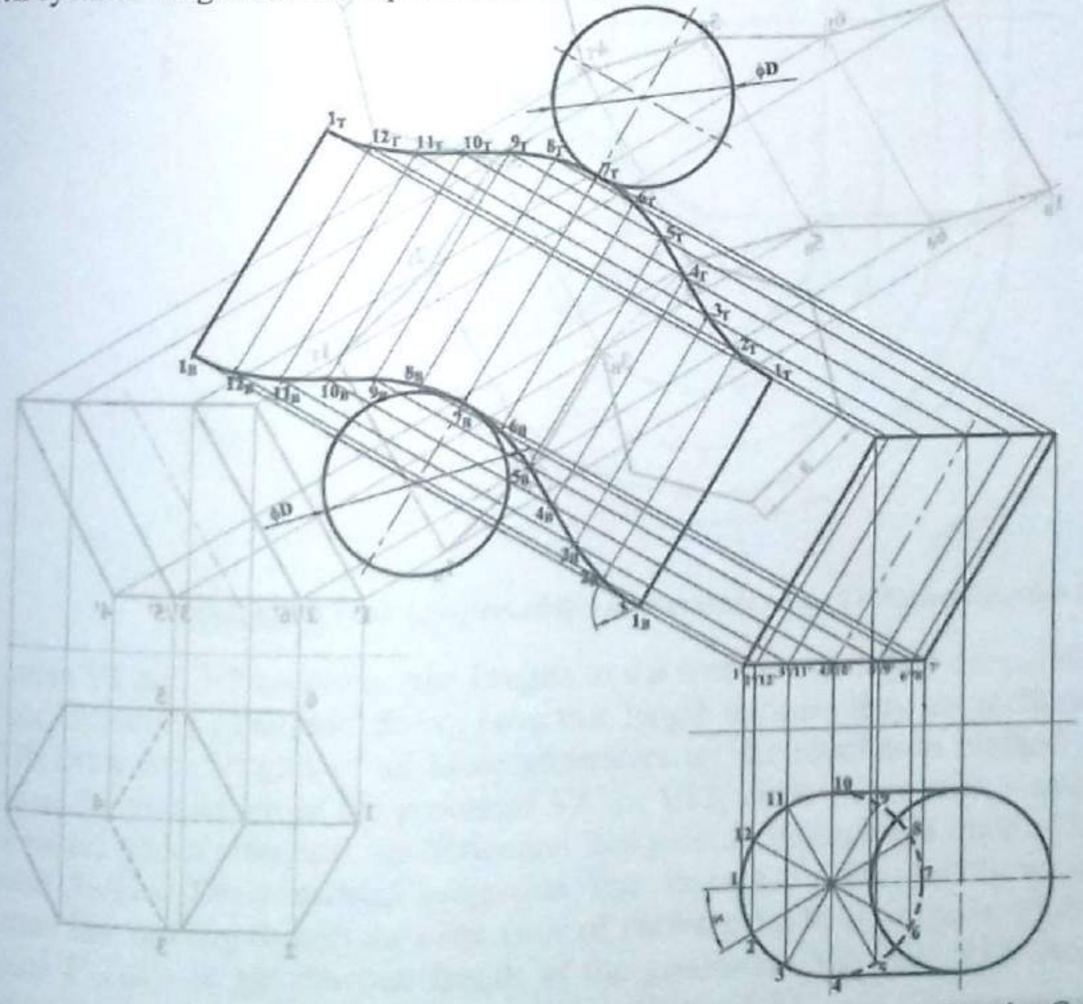


Figure 8.79: Complete Surface Development of an Oblique Circular Cylinder

8.12.2 Development of an Oblique Prism

Consider an oblique hexagonal prism shown in **Figure 8.23** and **8.24**. To develop it, name the corner points of the top view of the bottom hexagonal surface as **1**, **2**, **3**, ……, **6** and mark their corresponding front views **1'**, **2'**, **3'**, ……., **6'** as shown in *Figure 8.80*. All the inclined edges of the prism appear in true length in the front view. Draw straight lines passing through ends of each inclined edges and perpendicular to the axis in the front view. Mark any line $\mathbf{1_B}$ and $\mathbf{1_T}$ between the lines passing through ends of the inclined edges passing though the point **1'** and parallel to the axis of the prism. With $\mathbf{1_B}$ as the center and edge length of the base **a** as the radius, draw an arc which intersects the line passing through the point **2'** at the point $\mathbf{2_B}$. Draw the arc with same radius and the point $\mathbf{1_T}$ as the center to determine the point $\mathbf{2_T}$. By following the similar procedure determine the points $\mathbf{3_B}$, $\mathbf{3_T}$, $\mathbf{4_B}$, $\mathbf{4_T}$, ……, $\mathbf{6_T}$. Join the points $\mathbf{1_B}$, $\mathbf{2_B}$, ….., $\mathbf{6_B}$ and $\mathbf{1_T}$, $\mathbf{2_T}$, ….. , $\mathbf{6_T}$ respectively by the straight line segments to get the lateral surface development of the oblique prism.

For the complete surface development, draw regular hexagons with edge length equal to **a** on the top and the bottom surface on the development with any one edge common.

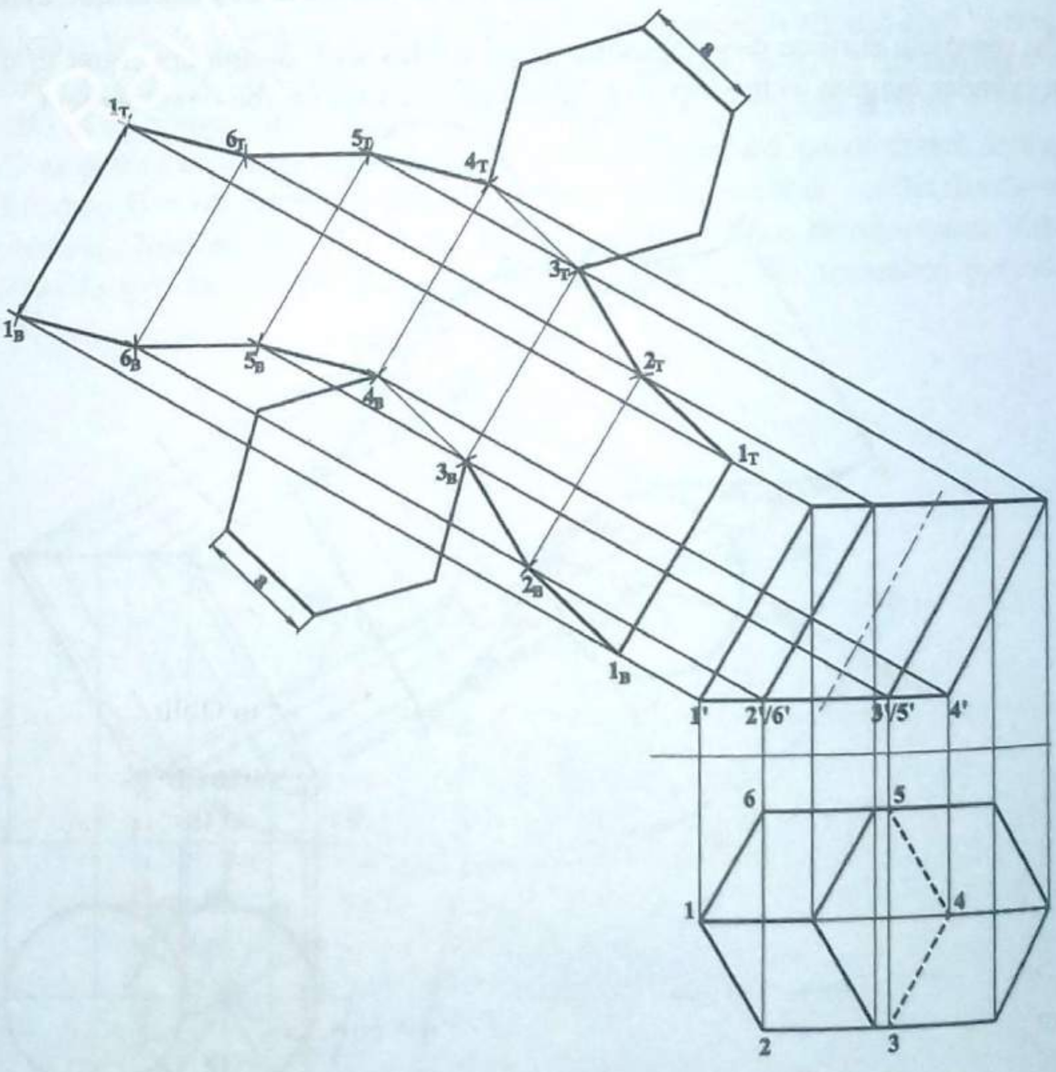


Figure 8.80: Complete Surface Development of an Oblique Hexagonal Prism

8.12.3 Development of an Oblique Circular Cone

Consider an oblique circular cone shown in ***Figure 8.25*** and *8.26*. To develop it, divide top view of the bottom circular section into any number of equal parts (say **12**) and name the dividing points as **1, 2, 3, ……, 12** as shown in *Figure 8.81*. Join each point **1, 2, 3, ……, 12** on the circle with the top view **v** of the vertex to get the top view of the generators. Transfer each point on the circle into the front view to get their corresponding front views **1', 2', 3', ……., 12'**. Join all the points **1', 2', 3', ……., 12'** on the edge view of the base cycle with the front view v' of the vertex to get the front views of the generator.

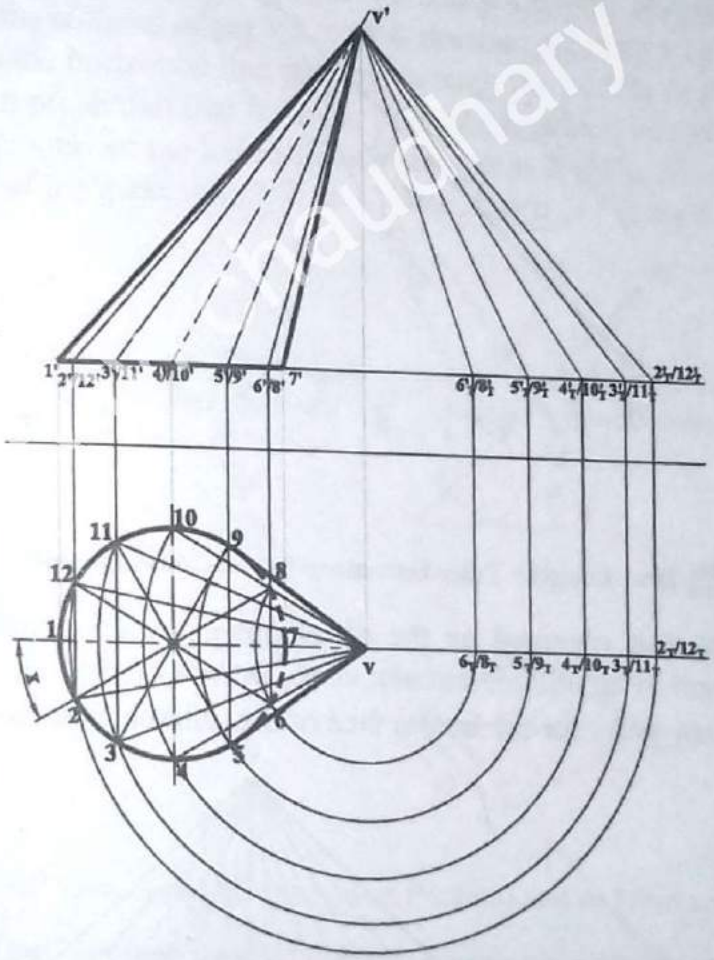


Figure 8.81: True Lengths of the Generators of an Oblique Circular Cone

Generators **V1** and **V7** appear in true lengths in the front view as they are parallel to the VP but the other remaining generators do not have true length because they are inclined to both the VP and HP. Draw true lengths of all these generators by the revolution method. For example to determine the true length of the generator **V2** or **V12**, draw an arc with **v** as center and **v2** or **v12** as radius which intersects the horizontal line passing through the cater of the base circle at the point $2_T/12_T$. Draw vertical projection line from the point $2_T/12_T$ which intersect the horizontal line passing though the edge view of the base circle at the point $2'_T/12'_T$. Join **v'** and the point $2'_T/12'_T$ to get the true length of the generators **V2** and **V12**. Repeat the similar procedure for the remaining generators as shown *Figure 8.81*.

For the development, draw any straight line **V1** having length equal to the true length of the generator **V1** i.e. **v'1'**. Draw an arc with the point **1** on the development as center and the arc length between the point **1** and **2** i.e. **x** as radius. Draw another arc with the point **V** as center and the true length of the generator **V2** i.e. $v'2'_T/12'_T$ as radius. The intersection of these arcs gives point **2** on the development. Similarly, draw an arc with the point **2** on the development as center and the arc length between the point **2** and **3** i.e. **x** as radius. Draw another arc with the point **V** as center and the true length of the generator **V3** i.e. $v'3'_T/11'_T$ as radius. The intersection of these arcs gives point **3** on the development. Repeat the similar process to determine all the remaining points in the development as shown in *Figure 8.82.*

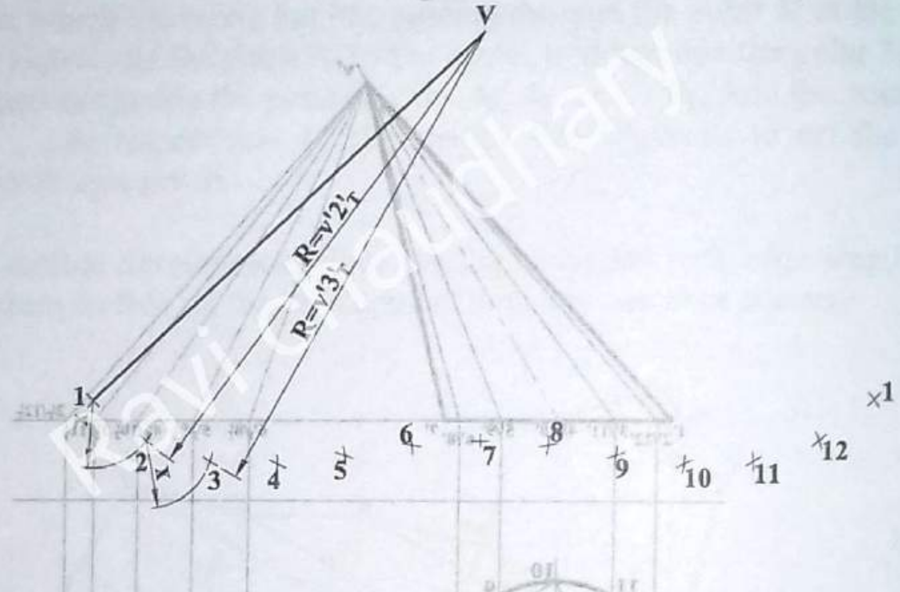


Figure 8.82: True Lengths Transformation to the Development of an Oblique Circular Cone

Join all the points thus obtained on the development by a smoothed curve to get the lateral surface development of an oblique circular cone. Draw a circle with diameter of D and tanget to the base curve at any point for the bootm face of the oblique cone as shown in *Figure 8.83.*

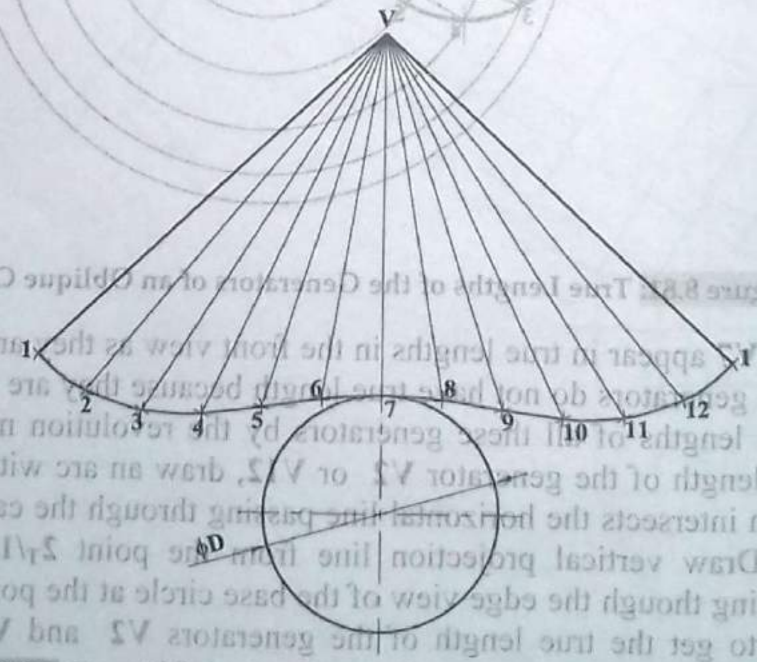


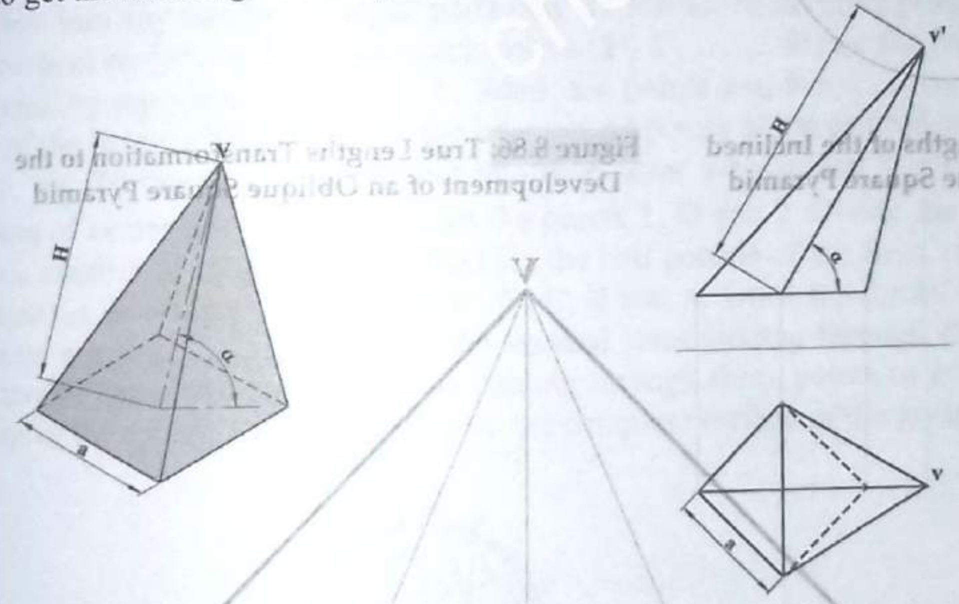
Figure 8.83: Complete Surface Development of an Oblique Circular Cone

8.12.4 Development of an Oblique Pyramid

Figure 8.84(a) shows the pictorial view of an oblique square pyramid and *Figure 8.84(b)* shows its corresponding front view and top view.

To develop it, name the corner points of the base square as **1**, **2**, ……, **4** as shown in *Figure 8.85*. Join each point **1**, **2**, ……, **4** and transfer each point on the square into the front view to get their corresponding front views **1'**, **2'**, ……., **4'**.

Inclined edges **V1** and **V3** appear in true lengths in the front view as they are parallel to the VP but the inclined edges **V2** and **V4** do not have true lengths because they are inclined to both the VP and HP. Draw true lengths of the inclined edges **V2** and **V4** by the revolution method. To determine the true length of the inclined edges **V2** or **V4**, draw an arc with **v** as center and **v2** or **v4** as radius which intersects the horizontal line passing through the middle of the base square at the point $2_T/4_T$. Draw vertical projection line from the point $2_T/4_T$ which intersect the horizontal line passing through the edge view of the base square at the point $2'_T/4'_T$. Join **v'** and the point $2'_T/4'_T$ to get the true length of the generators **V2** and **V4** as shown in *Figure 8.85*.



(a) (b)

Figure 8.84: Pictorial View of an Oblique Square Pyramid and its Front and Top Views

For the development, draw any straight line **V1** having length equal to the true length of the inclined edge **V1** i.e. **v'1'**. Draw an arc with the point **1** on the development as center and the edge length of base square i.e. **a** as radius. Draw another arc with the point **V** as center and the true length of the generator **V2** i.e. $\mathbf{v'2'_T/4'_T}$ as radius. The intersection of these arcs gives point **2** on the development. Similarly, draw an arc with the point **2** on the development as center and the edge length of base square i.e. **a** as radius. Draw another arc with the point **V** as center and the true length of the generator **V3** i.e. **v'3'** as radius. The intersection of these arcs gives point **3** on the development. Repeat the similar process to determine the points **4** and **1** in the development as shown in *Figure 8.86*.

Join all these points by the straight line segments to get the lateral surface development of the truncated pyramid, as shown in *Figure 8.87*. Draw square for the bottom surface with its one edge common to base of any one triangle.

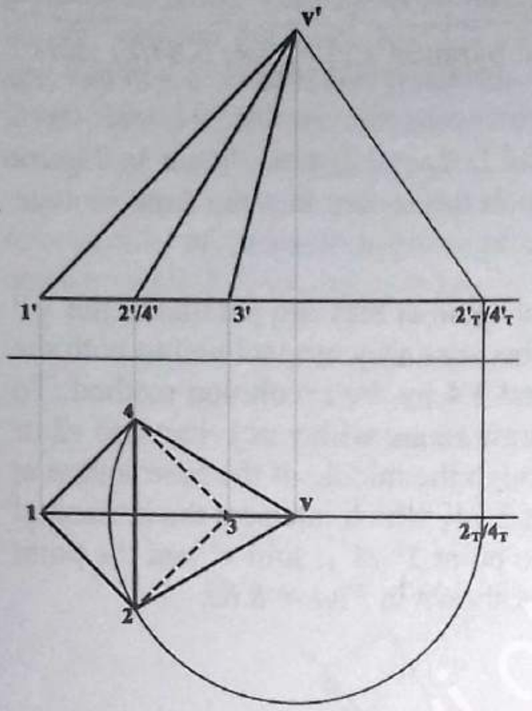


Figure 8.85: True Lengths of the Inclined Edges of an Oblique Square Pyramid

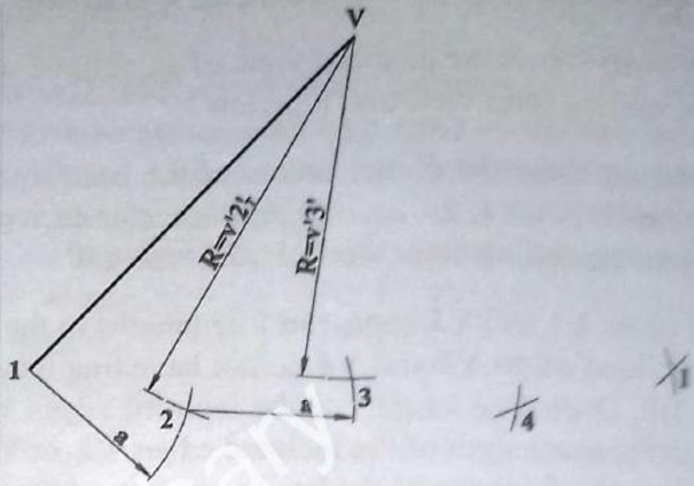


Figure 8.86: True Lengths Transformation to the Development of an Oblique Square Pyramid

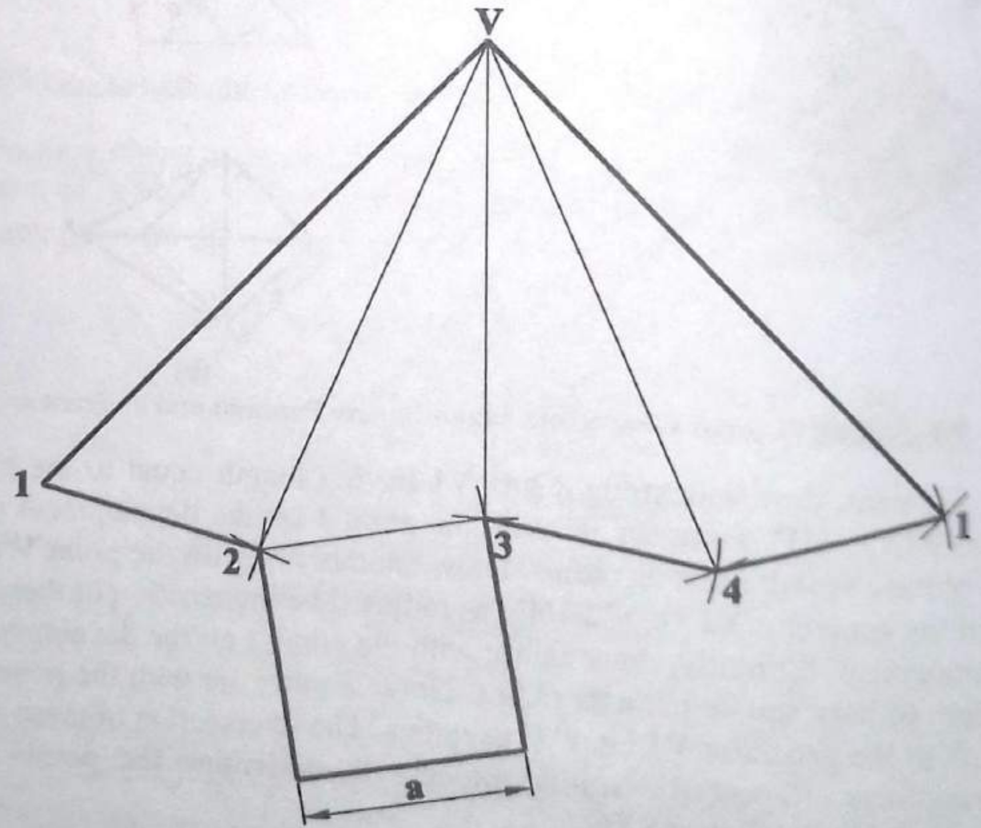


Figure 8.87: Complete Surface Development of an Oblique Square Pyramid

8.13 Development of Sphere

Sphere is a solid consisting of curved surface only. Hence surface development of a sphere can only be approximated. Any spherical surface can be developed by two methods: Lune method and Zone method.

8.13.1 Lune Method

In this method, the given sphere is assumed to be cut by vertical planes passing through the vertical axes so the complete sphere is converted into a certain number of lunes of equal size. Each piece is then assumed as a part of a horizontal cylinder having the same diameter of the sphere. This method is also called polycylindric method or gore method.

To develop a sphere by the Lune method, draw top view and front view of the given sphere as *Figure 8.88*. Divide the circle in the top view into any number of equal parts (say **12**), to get the top views of the lunes. To get intermediate points on the lune, divide the right half portion of the front view into any number of equal parts (say **8**). Name the dividing points as **1'**, **2'**, ……, **9'**. Draw vertical projection lines from each points (**1'**, **2'**, ……. **9'**) on the front views to get their corresponding top views (**1**, **2**, ….., **9**). Mark the points **a**-a, **b**-b, ….., on the top view of two edges of the lune such that they are the intersection points of the projection lines drawn through 1, 2, …. etc. To draw the development of a lune, draw a horizontal line **19** of length πR, as an extension of center line passing through the points **1**, **O** and **9**. Divide the line segment **19** into the same number of equal parts that used for the half portion of the front view circle, i.e. **8** parts and name the dividing points as **A**, **B**, **C**, **D**, **C**, **B** and **A**. Draw horizontal lines passing through the points **a**-a, **b**-b, ….., to intersect the vertical lines passing through the points **A**, **B**, **C**, **D** respectively. Draw two smooth curves passing through these points to get the development of one lune of the sphere. Such 12 lunes give the complete surface of the given sphere.

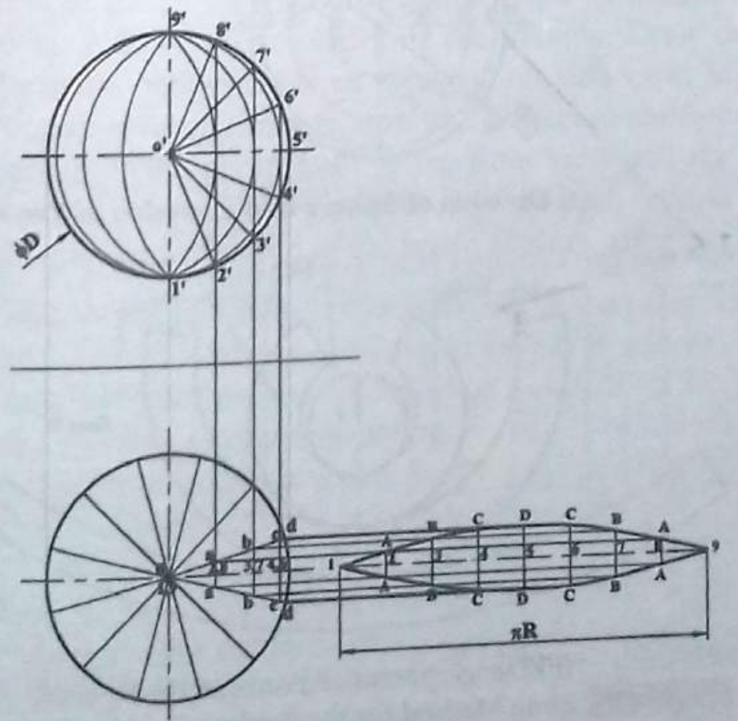
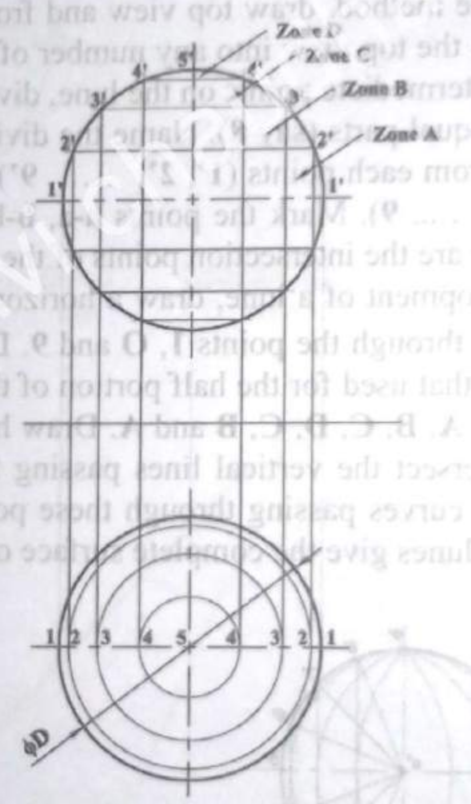


Figure 8.88: Lune Method for the Surface Development of a Sphere

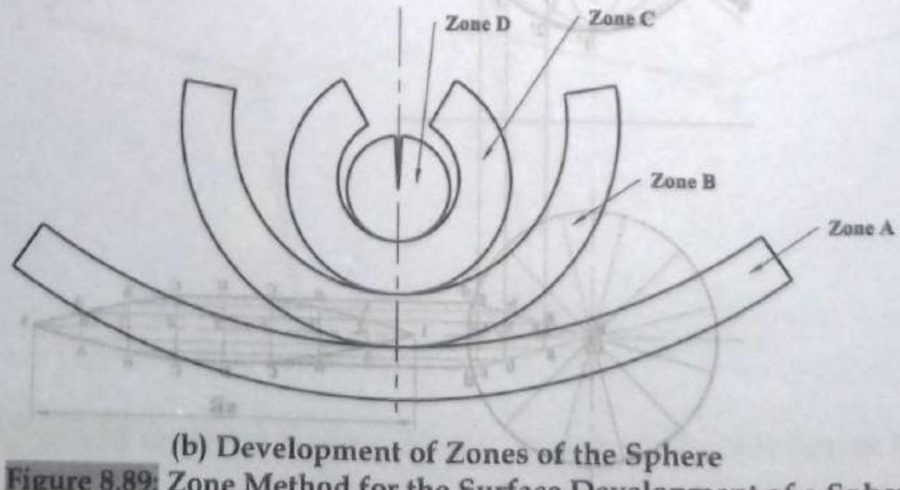
8.13.2 Zone Method

In this method, the given sphere is assumed to be cut by horizontal planes to form a series zones of equal width. Each zone is assumed as a frustum of a right circular cone and is developed accordingly. This method is also called polyconic method.

To develop a sphere by the Zone method, draw top view and front view of the given sphere as *Figure 8.89(a)*. Divide the top half of the front view circle into any number of zones (say 4) having equal widths, i.e. **1'2' = 2'3' = 3'4' = 4'5'**. Assume that each zone is a frustum of a cone inscribed within the sphere, except zone **D** which is full cone. Then develop each zone separately by following the procedure explained earlier for the complete cone and frustum of a cone as shown in *Figure 8.89(b)*. The pattern thus obtained with its symmetrical bottom half give the complete development of the sphere.



(a) Division of Sphere into a number of Zones



(b) Development of Zones of the Sphere

Figure 8.89: Zone Method for the Surface Development of a Sphere

WORKOUT EXAMPLES

Example 8.1

Draw three orthographic views of the following right solids with its base parallel to the HP and their axes perpendicular. Take any convenient distance of the solid from the HP and VP.

(a) A right circular cylinder with base diameter 40 mm and height 60 mm.

(b) A triangular prism edge of the base 40 mm and height 50mm, with its base edge nearer to the VP is parallel to it.

(c) A right hexagonal prism with edge of the base 25 mm and height 60 mm having one of its base edges perpendicular to the VP.

(d) A right circular cone with base diameter 40 mm and height 60 mm.

(e) A right square pyramid with edge of base 40 mm and height 50 mm with its base edge nearer to the VP is parallel to the VP.

(f) A right pentagonal pyramid with edge of the base 30 mm and height 60 mm with one of its edge perpendicular to the VP.

Solution

(a) Draw a circle of diameter 40 mm as a top view of the cylinder. Draw projection lines towards the front view from the ends of the horizontal diameter of the circle. Similarly, draw projection lines towards the side view from the ends of the vertical diameter of the circle. Draw edge views of the bottom circle of the cylinder both in the front view and the side view. Mark a height of 60 mm either on the front view or on the side view and project to another view. Draw edge views of the top circle of the cylinder both in the front view and the side view. Join ends of the edge views of the bottom and top section in both the front and side views to get the complete orthographic projection of the cylinder as shown in *Figure E8.1(a)*.

(b) Draw a straight line **ab** 40 mm long and parallel to the reference line as a top view of one of edge of the base triangle. Draw an equilateral triangle as the line **ab** as its base to complete the top view of the given triangular prism. Draw projection lines towards the front view and side view from each corners of the triangle. Draw edge views of the bottom triangle of the prism both in the front view and the side view. Mark a height of 50 mm either on the front view or on the side view and project to another view. Draw edge views of the top triangle of the prism both in the front view and the side view. Draw front and side views of the vertical edges passing through each corners of the triangle to get the complete orthographic projection of the prism as shown in *Figure E8.1(b)*.

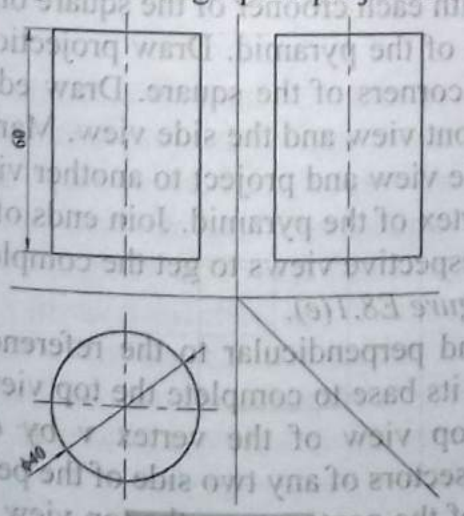


Figure E8.1(a)

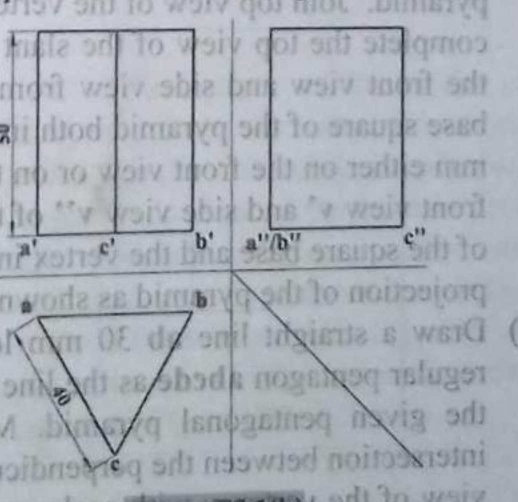


Figure E8.1(b)

(c) Draw a regular hexagon **abcdef** as the top view of the prism with its diagonal **ad** 50 mm long (twice the edge length of the hexagon) and perpendicular to the reference line. Draw projection lines towards the front view and side view from each corners of the hexagon. Draw edge views of the bottom hexagon of the prism both in the front view and the side view. Mark a height of 60 mm either on the front view or on the side view and project to another view. Draw edge views of the top hexagon of the prism both in the front view and the side view. Draw front and side views of the vertical edges passing through each corners of the hexagon to get the complete orthographic projection of the prism as shown in *Figure E8.1(c)*.

(d) Draw a circle of diameter 40 mm as a top view of the cone. Draw projection lines towards the front view from the ends of the horizontal diameter of the circle. Similarly, draw projection lines towards the side view from the ends of the vertical diameter of the circle. Draw edge views of the circular base of the cone both in the front view and the side view. Mark a height of 60 mm either on the front view or on the side view and project to another view to locate the front view and side view of the vertex of the cone. Join ends of the edge views of the circular base and the vertex in the respective views to get the complete orthographic projection of the cone as shown in *Figure E8.1(d)*.

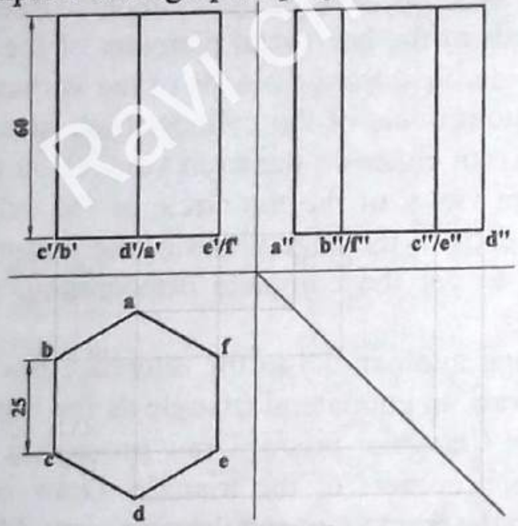


Figure E8.1(c)

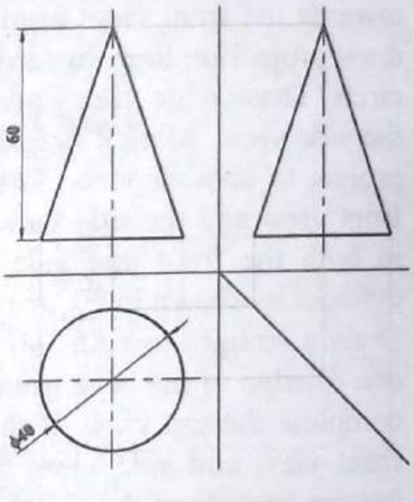


Figure E8.1(d)

(e) Draw a straight line **ab** 40 mm long and parallel to the reference line. Draw a square **abcd** as the line **ab** as its base to complete the top view of the base of the given square pyramid. Join top view of the vertex **v** with each crooner of the square on the top view to complete the top view of the slant edges of the pyramid. Draw projection lines towards the front view and side view from each corners of the square. Draw edge views of the base square of the pyramid both in the front view and the side view. Mark a height of 50 mm either on the front view or on the side view and project to another view to locate the front view **v'** and side view **v''** of the vertex of the pyramid. Join ends of the edge views of the square base and the vertex in the respective views to get the complete orthographic projection of the pyramid as shown in *Figure E8.1(e)*.

(f) Draw a straight line **ab** 30 mm long and perpendicular to the reference line. Draw a regular pentagon **abcde** as the line **ab** as its base to complete the top view of the base of the given pentagonal pyramid. Mark top view of the vertex **v** by determining the intersection between the perpendicular bisectors of any two side of the pentagon. Join top view of the vertex **v** with each crooner of the pentagon on the top view to complete the top view of the slant edges of the pyramid. Draw projection lines towards the front view and side view from each corners of the pentagon. Draw edge views of the base pentagon of the pyramid both in the front view and the side view. Mark a height of 60 mm either on the front view or on the side view and project to another view to locate the front view v' and side view v'' of the vertex of the pyramid. Join views each corners of the base pentagon with the vertex in the respective views to get the complete orthographic projection of the pyramid as shown in *Figure E8.1(f)*. As the slant edges **VA** and **VB** are not visible from the left side draw hidden lines for them in the side view.

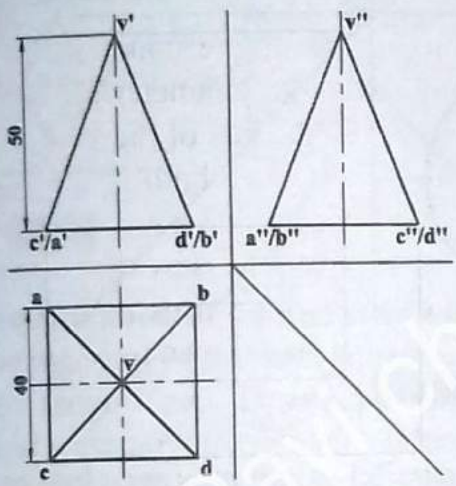


Figure E8.1(e)

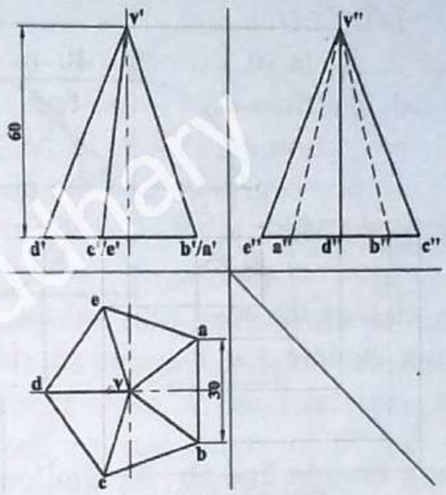


Figure E8.1(f)

Example 8.2

Draw three orthographic views of the following oblique solids with its base parallel to the HP and their axes perpendicular. Take any convenient distance of the solid from the HP and VP.

(a) An oblique circular cylinder with base diameter 40 mm and axis length of 60 mm with its axis parallel to the VP and inclined to the HP at 60^0.

(b) An oblique hexagonal prism with edge of the base 25 mm and axis length 60 mm having one of its base edges parallel to the VP with its axis parallel to the VP and inclined to the HP at 60^0.

(c) An oblique circular cone with base diameter 40 mm and axis length 60 mm with its axis parallel to the VP and inclined to the HP at 60^0.

(d) An oblique square pyramid with edge of base 40 mm and axis length 60 mm with its all base edges equally inclined to the VP and its axis parallel to the VP and inclined to the HP at 45^0.

Solution

(a) Draw a circle of diameter 40 mm as a top view of the bottom section of the oblique cylinder. Draw projection lines towards the front view from the ends of the horizontal diameter of the circle. Draw edge view of the bottom circle of the oblique cylinder in the front view. Draw an axis line inclined at 60^0 to the front view of the bottom circle and passing through its midpoint. Mark an axis length of 60 mm along the inclined axis line. Draw a vertical projection line from the top end of the axis in the front view to determine the center of the top circular section on the top view. Draw top view of the top circular section and its corresponding front view. Complete the side view of the oblique cylinder with the help of its top view and front view as shown in **Figure E8.2(a)**.

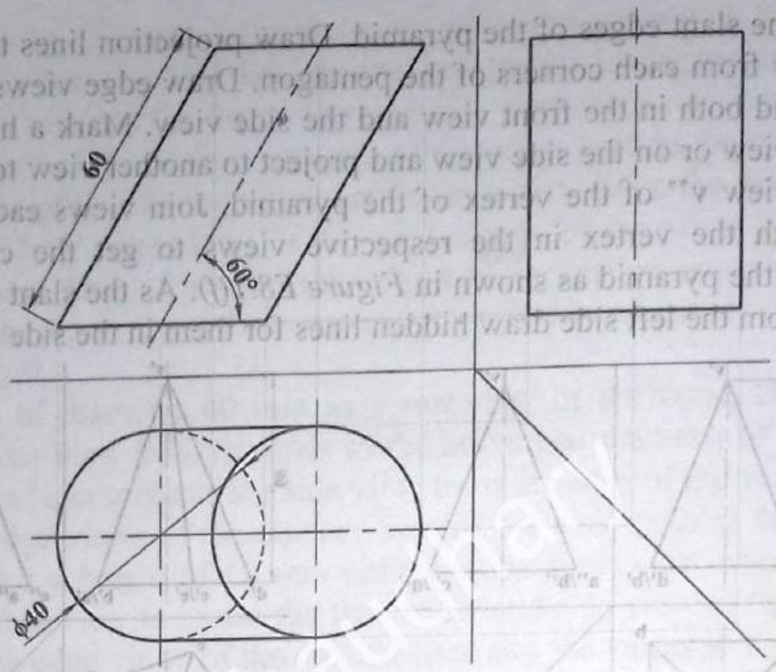


Figure E8.2(a)

(b) Draw a straight line **ab** 25 mm long and parallel to the reference line as a top view of one of edge of the bottom hexagonal section. Draw a regular hexagon **abcdef** as the line **ab** as its base to complete the top view of the bottom hexagonal section of the given oblique hexagonal prism. Draw edge view of the bottom hexagon of the oblique prism in the front view. Draw an axis line inclined at 60^0 to the front view of the bottom hexagon and passing through its midpoint. Mark an axis length of 60 mm along the inclined axis line. Draw front view of inclined edges of the oblique prism passing through front views of each corners of the bottom hexagon and each having length of 60 mm. Join the top ends of the front views of each inclined edges to get the front view of the top hexagonal section of the prism. Draw vertical projection lines from the edge view of the top hexagonal section towards the top view to complete its corresponding top view. Also draw the top views of the inclined edges by joining corresponding corners of the top views of the top and bottom hexagon. Complete the side view of the oblique prism with the help of its top view and front view as shown in *Figure E8.2(b)*.

(c) Draw a circle of diameter 40 mm as a top view of the base circle of the oblique cone. Draw projection lines towards the front view from the ends of the horizontal diameter of the circle. Draw edge view of the base circle in the front view. Draw an axis line inclined at 60^0 to the front view of the base circle and passing through its midpoint. Mark an axis length of 60 mm along the inclined axis line to mark the front view of the vertex. Join front view of the vertex with the ends of the edge view of the base circle in the front view to complete the front view of the oblique cone. Draw a vertical projection line from the front view of the vertex. The intersection of the projection line with the extended horizontal diameter of the base circle gives the top view of the vertex. Draw tangent lines from the top view of the vertex to the top view of the base circle to complete the top view of the oblique cone. Complete the side view of the oblique cone with the help of its top view and front view as shown in *Figure E8.2(c)*.

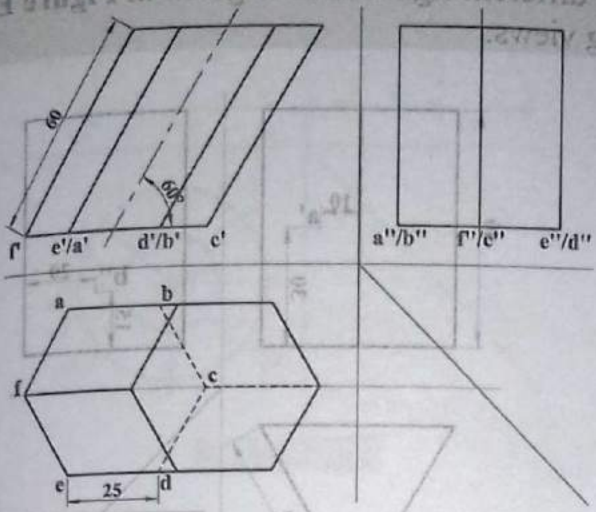


Figure E8.2(b)

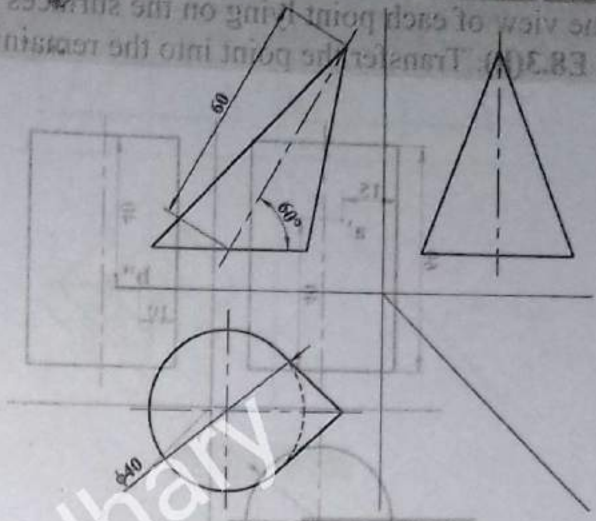


Figure E8.2(c)

(d) Draw a square **abcd** of 40 mm side with its diagonal **db** perpendicular to the reference line as the top view of the base square of the oblique pyramid and draw its corresponding front view (edge view). Draw an axis line inclined at 45^0 to the front view of the base square and passing through its midpoint. Mark an axis length of 60 mm along the inclined axis line to mark the front view of the vertex **v'**. Join the point **v'** with the front views of each corner of the square to the front views of the inclined edges of the pyramid. Draw a vertical projection from the front view of the vertex **v'** to draw it corresponding top view **v**. Join the point **v** with the top views of each corners of the base square to get the complete top view of the oblique pyramid. Complete the side view of the oblique pyramid with the help of its top view and front view as shown in *Figure E8.2(d)*.

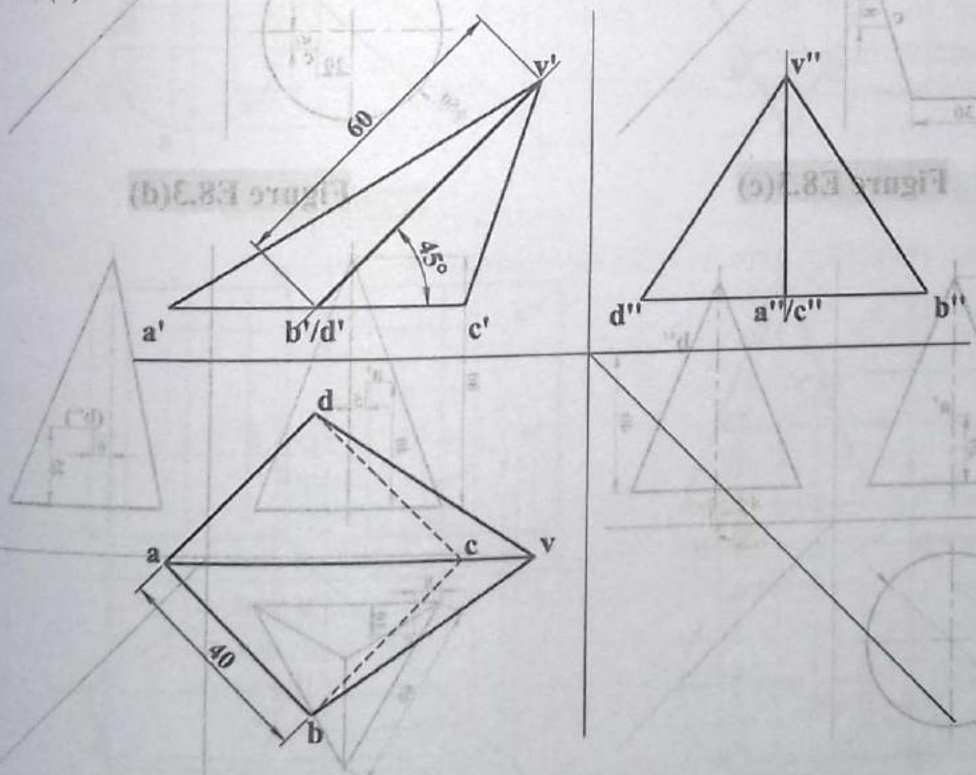


Figure E8.2(d)

Example 8.3

One view of each point lying on the surfaces of different right solids is given in **Figure E8.3(a)** to **E8.3(h)**. Transfer the point into the remaining views.

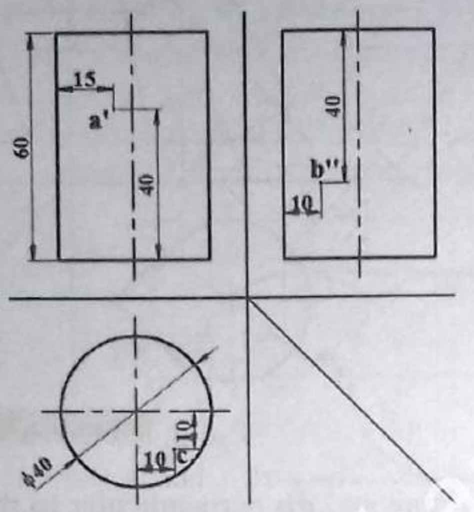


Figure E8.3(a)

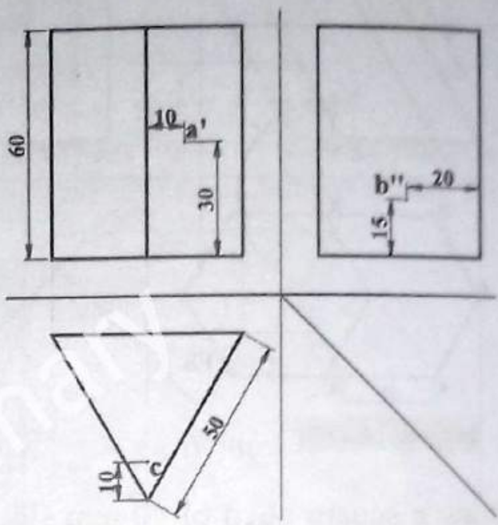


Figure E8.3(b)

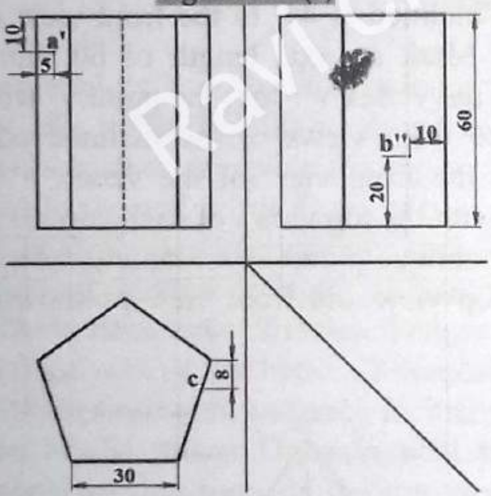


Figure E8.3(c)

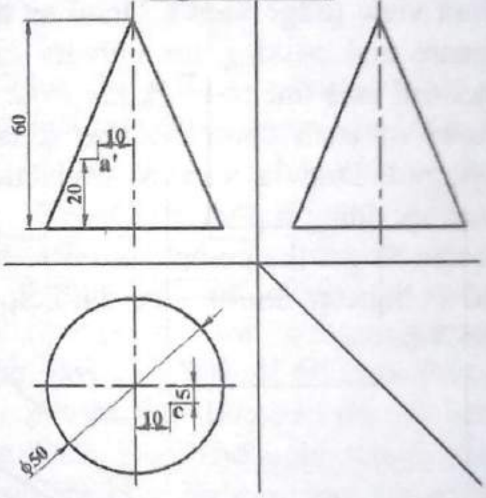


Figure E8.3(d)

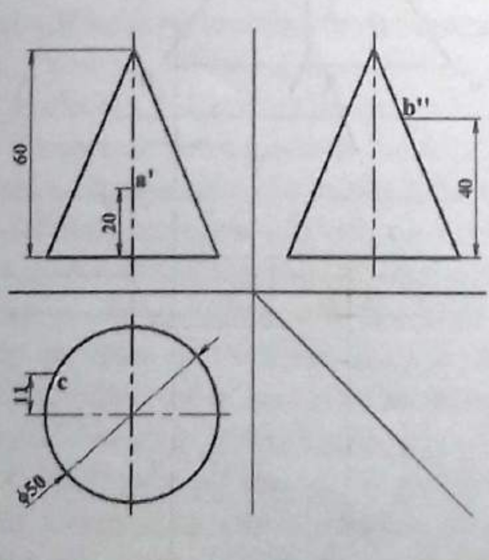


Figure E8.3(e)

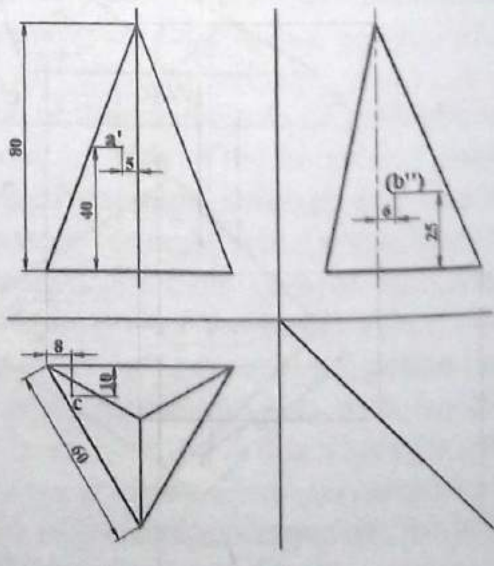


Figure E8.3(f)

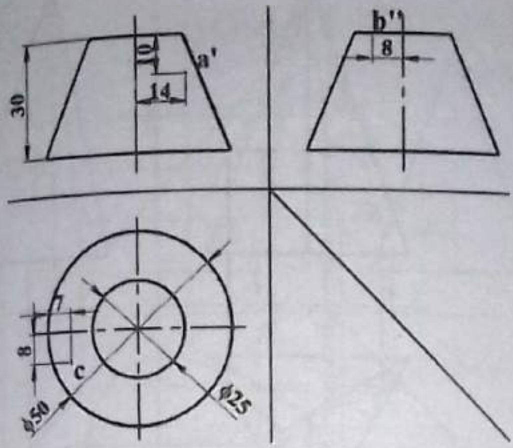


Figure E8.3(g)

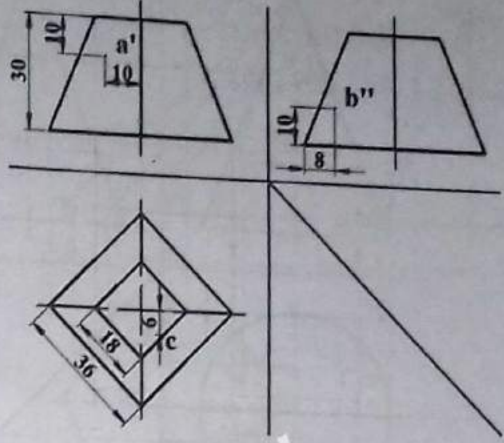


Figure E8.3(h)

Solution

Transfer of each point from the given view to the other remaining view are shown in *Figure E8.3S(a), ….., Figure E8.3S(h)* respectively. The arrow heads show the direction of point transfer from one view to another.

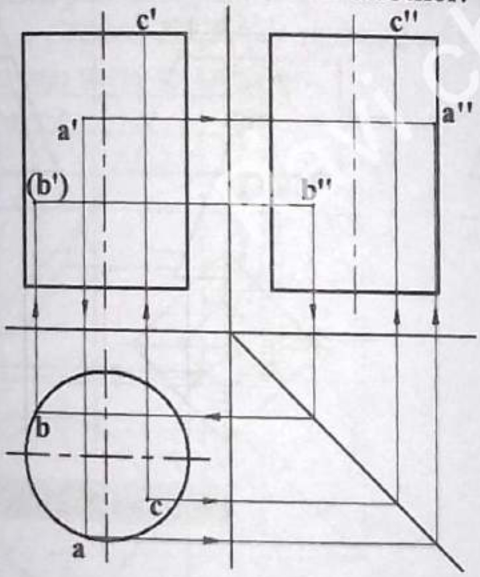


Figure E8.3S(a)

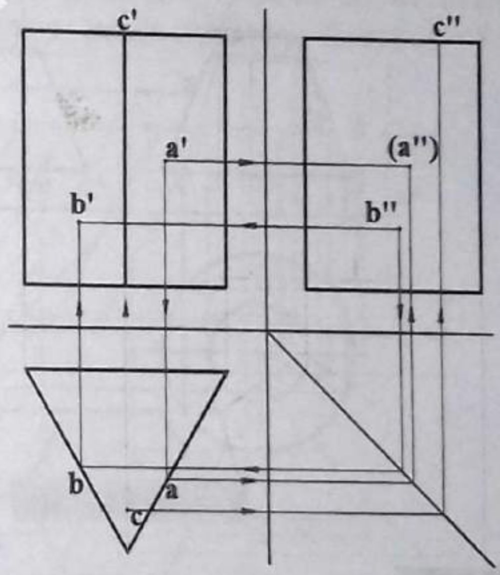


Figure E8.3S (b)

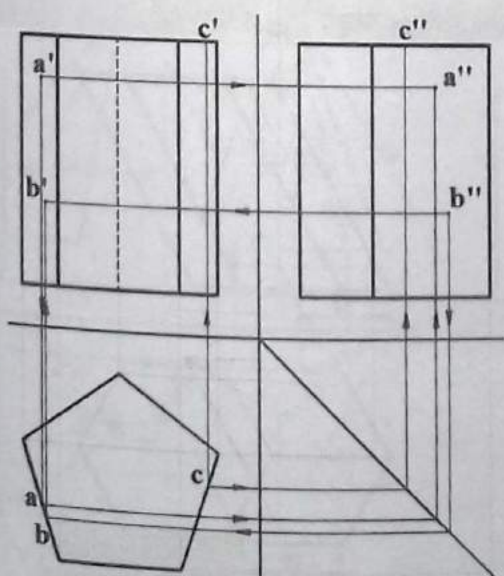


Figure E8.3S(c)

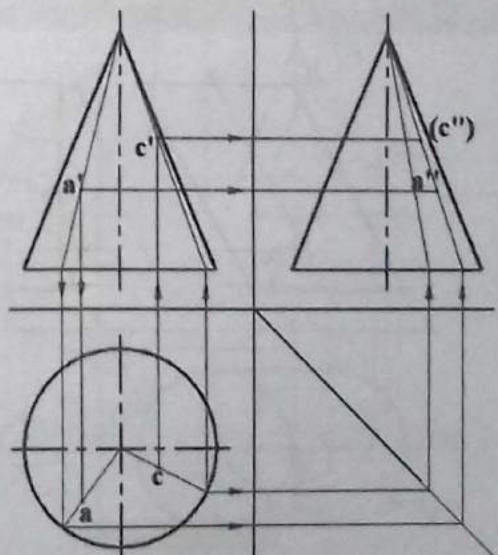


Figure E8.3S(d)

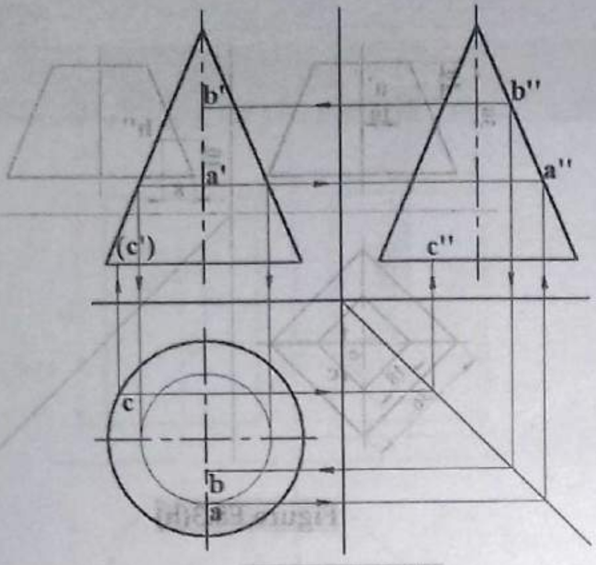


Figure E8.3S(e)

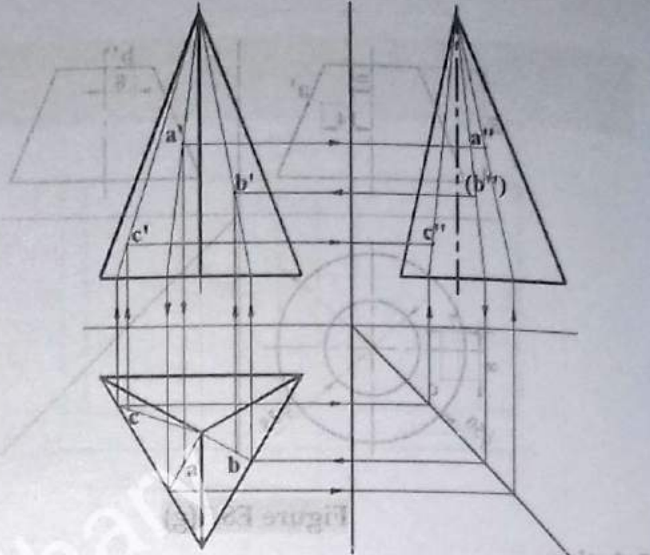


Figure E8.3S(f)

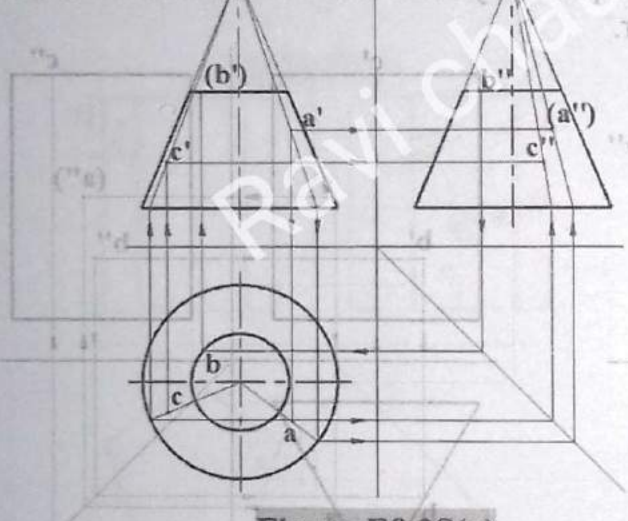


Figure E8.3S(g)

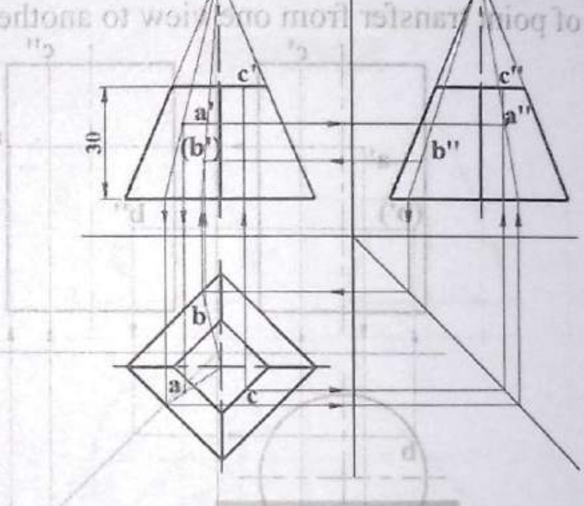


Figure E8.3S(h)

Example 8.4

One view of each point lying on the surfaces of different oblique solids is given in *Figure E8.4(a)* to *Figure E8.4(d)*. Transfer the point into the remaining views.

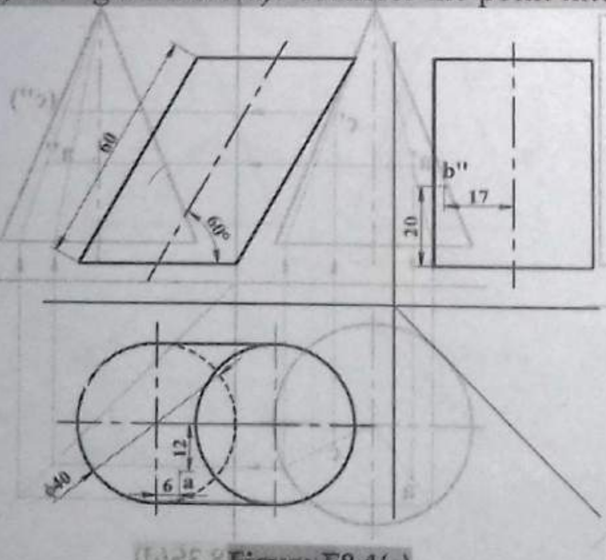


Figure E8.4(a)

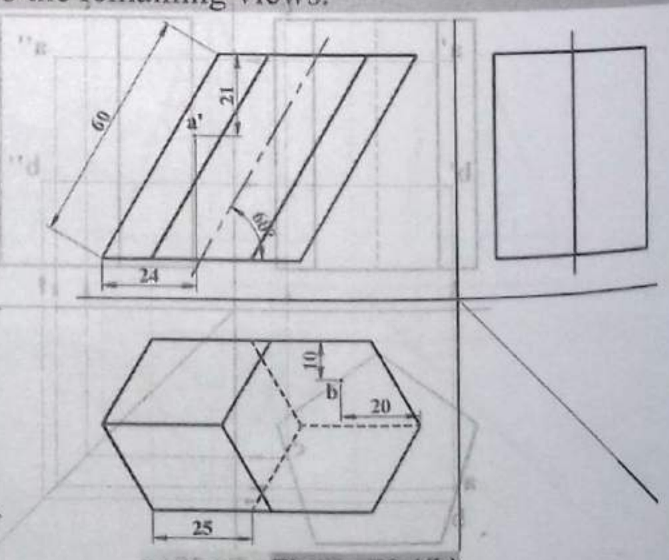


Figure E8.4(b)

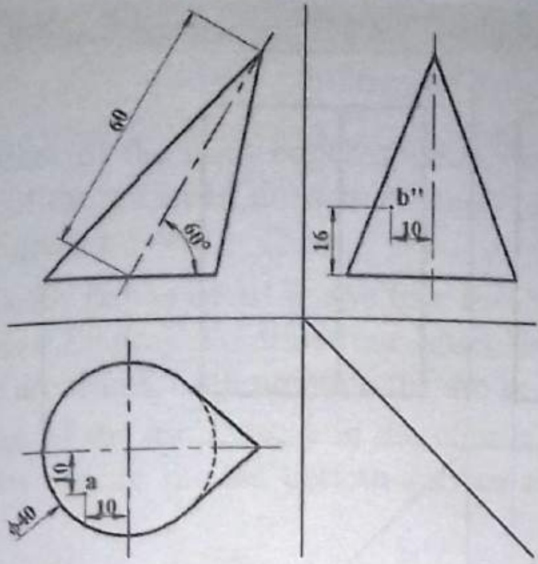


Figure E8.4(c)

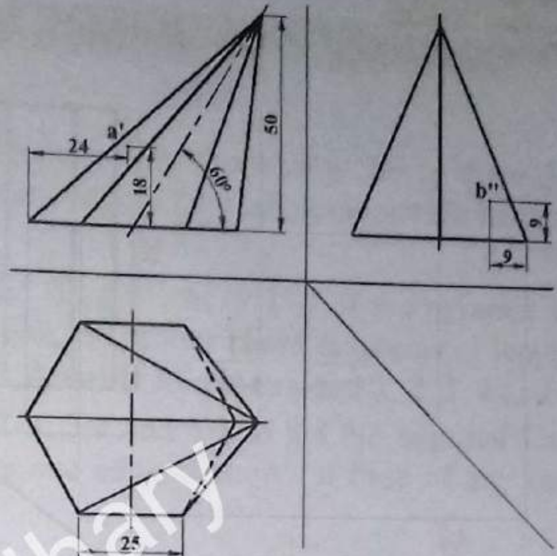


Figure E8.4(d)

Solution

Transfer of each points from the given view to the other remaining view are shown in *Figure E8.4S(a), ….., Figure E8.3S(d)* respectively. The arrow heads show the direction of point transfer from one view to another.

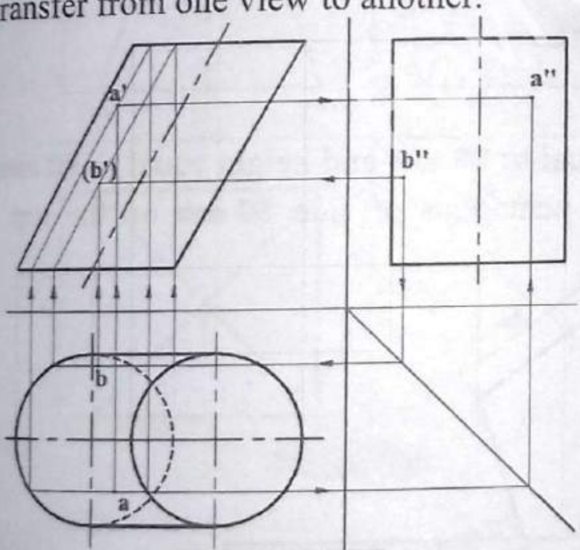


Figure E8.4S(a)

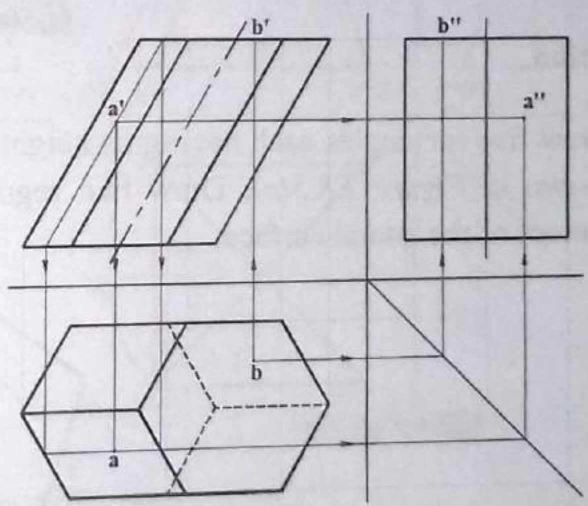


Figure E8.4S(b)

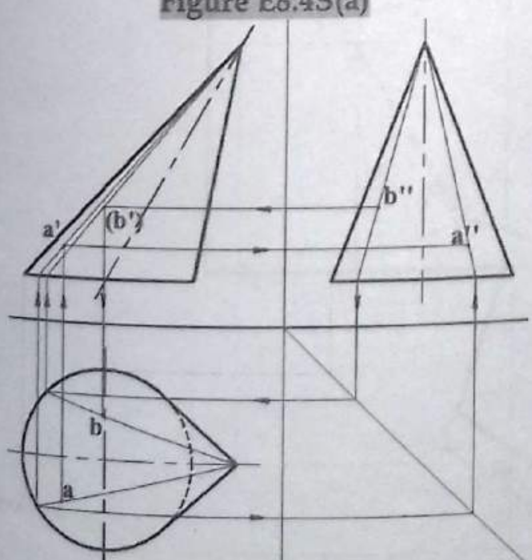


Figure E8.4S(c)

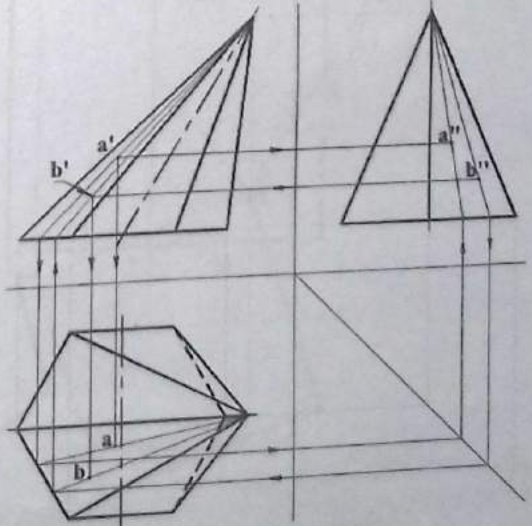


Figure E8.4S(d)

Example 8.5

Draw the complete surface development of a right pentagonal prism shown in **Figure E8.5**.

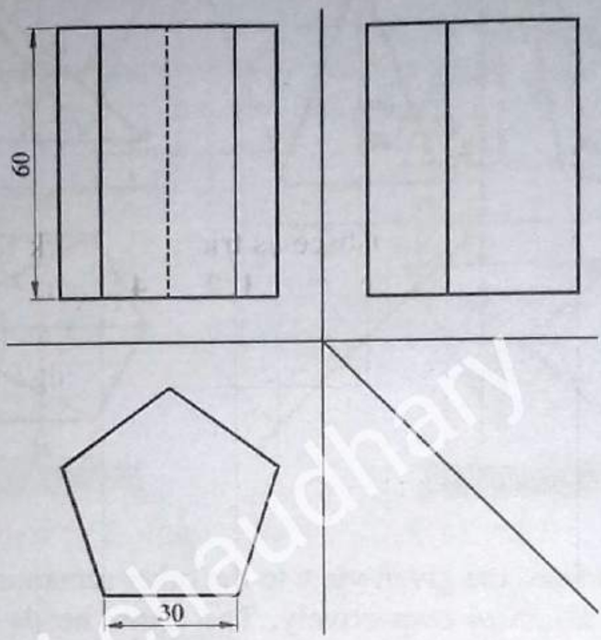


Figure E8.5

Solution

Draw five rectangles each having its height equal to 30 *mm* and height equal to 60 *mm*, as shown in *Figure E8.5(a)*. Draw two regular pentagons of side 30 *mm* on the top and bottom of the lateral surface.

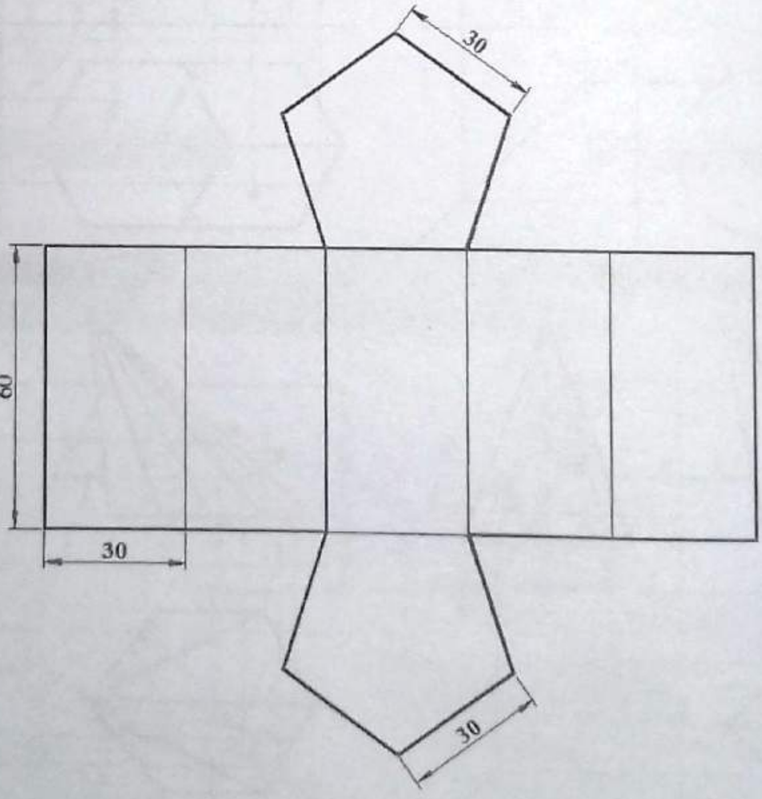


Figure E8.5(a)

Example 8.6

Draw the complete surface development of a right square pyramid shown in Figure E8.6.

Solution

Note that neither of the slant edges appears in true length on the front view. So to draw the development of the surfaces, draw true length of the slant edge **V1** by using revolution method as shown in *Figure E8.6(a)*.

Draw an arc with radius equal to the true length of the slant height ($v'1'_T$) of the pyramid as shown in *Figure E8.6(b)*. To draw four isosceles triangles, mark four chord segments of length 45 mm on the arc. Mark each point on the arc as **1, 2, 3, 4** and **1**. Join each point **1, 2, 3, 4** and **1** with the center of the arc. Also join the chords **1-2, 2-3, 3-4** and **4-1** to get the required four triangles. Draw square for the bottom surface with its one edge common to base of any one triangle.

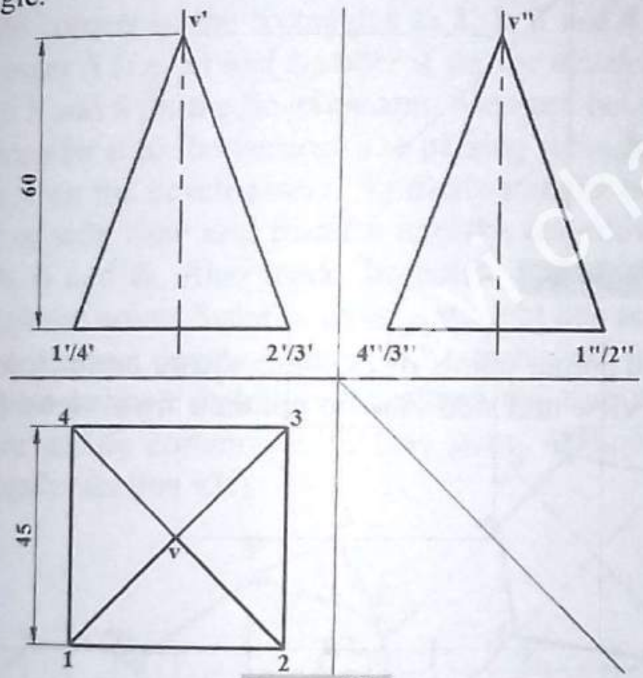


Figure E8.6

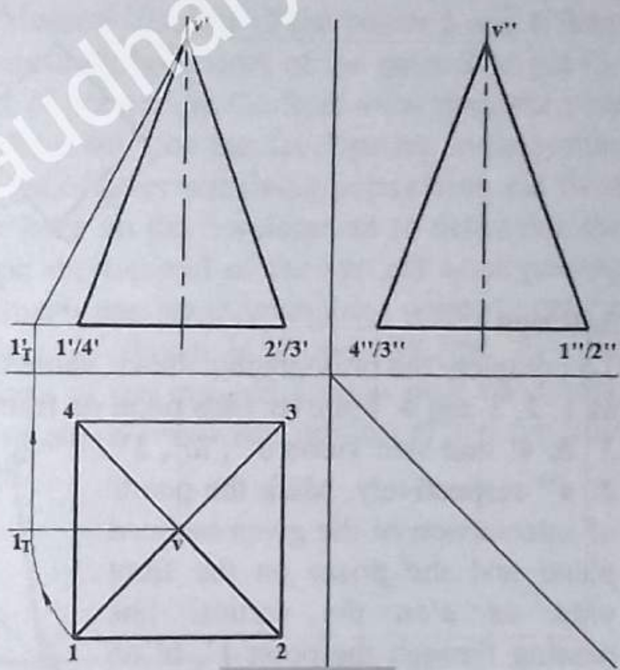


Figure E8.6(a)

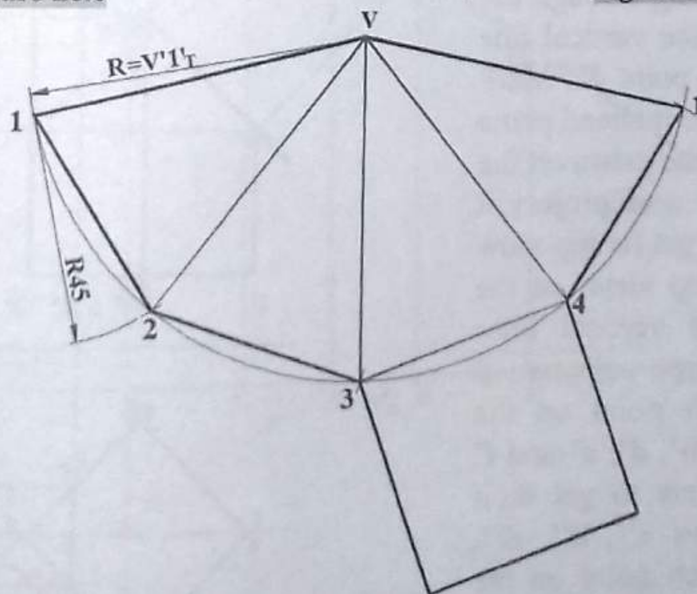


Figure E8.6(b)

Example 8.7

Complete the orthographic views of a truncated right square prism shown in *Figure E8.7* and then develop it.

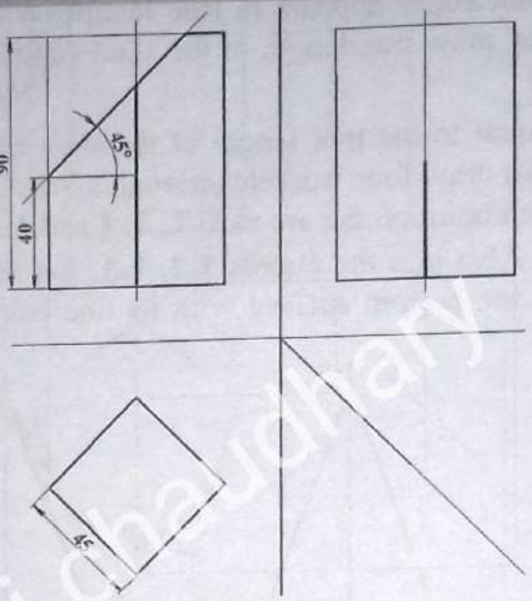


Figure E8.7

Solution

To complete the orthographic views, name the corner points of the base square on the top view as **1**, **2**, **3** and **4**. Transfer each point on front view and side view to get their front views **1'**, **2'**, **3'** & **4'** and side views **1''**, **2''**, **3''** & **4''** respectively. Mark the points of intersection of the given inclined plane and the prism on the front view as **a'** on the vertical line passing through the point **1'**, **b'** on the vertical line passing through the point **2'**, and **d'** on the vertical line passing through the point **4'**. Mark the intersection of the inclined plane with the top face of the prism on the front view as **e'/f'** and project it towards top view to get its top view **ef**. Also mark the top views of the bottom ends of the vertical lines passing through theses points as **5** and **6**. Project each point on the inclined section **a'**, **b'**, **d'**, **e'** and **f'** towards the side view to get their respective side views **a''**, **b''**, **d''**, **e''** and **f''**. Join each point on the side view by the straight line segments to get the complete side view of the truncated prism.

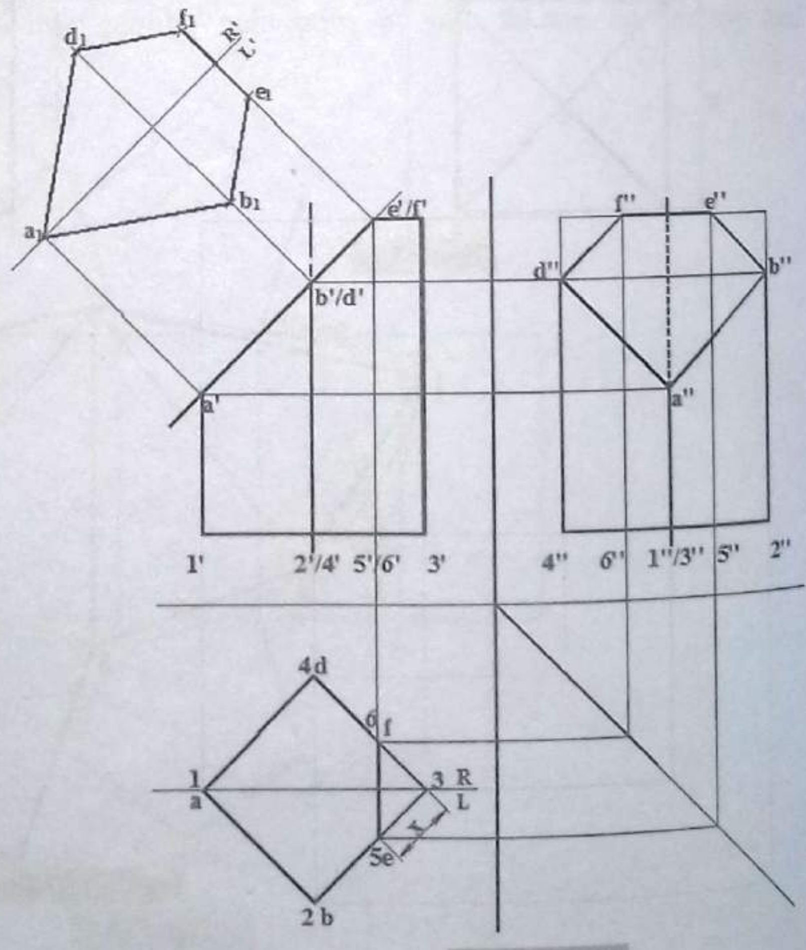


Figure E8.7(a)

To draw the true shape of the inclined section, assume horizontal line from the middle of the square on the top view as a reference line **R/L**. Draw another reference line **R'/L'** parallel to the edge view of the inclined section on the front view. Draw projection lines passing through each points (**a'**, **b'**, **d'**, **e'** and **f'**) on the edge view of the inclined section and perpendicular to the reference line **R'/L'**. Measure distance of each point from the reference line **R/L** on the top view and transfer them into the respective projection lines from the reference line **R'/L'** to complete the auxiliary view of each points on the inclined section. Join each point thus obtained on the auxiliary view by straight line segments to get the true shape of the inclined section as shown in *Figure E8.7(a)*.

To develop it, draw four rectangles each having width of **45 mm** and height of **90 mm**. Name the bottom corners of the rectangles as **1**, **2**, **3** and **4**. Measure distance of the points **5** and **6** from the corner **3** (i.e. **x**) and transfer it on the development on both sides of the point **3** to get the points **5** and **6** on the development. Measure height of point **a'** on the front view from the base and transfer it to the vertical line passing through the point **1** on the development to determine point **A** on the development. Similarly transfer height of other remaining points from the front view or side view and transfer into the respective lines on the development to determine the points **B** and **D**. Also mark the points **E** and **F** on the top end of the vertical lines passing through the points **5** and **6**. Draw a straight line segments passing through these points to get the lateral surface development of the truncated prism, as shown in *Figure E8.7(b)*. For the complete surface development, attach the true shape of the inclined section with the lateral surface at any common edge (say point **AB**). Also attach remaining portion of the top face (triangular section **e3f**).

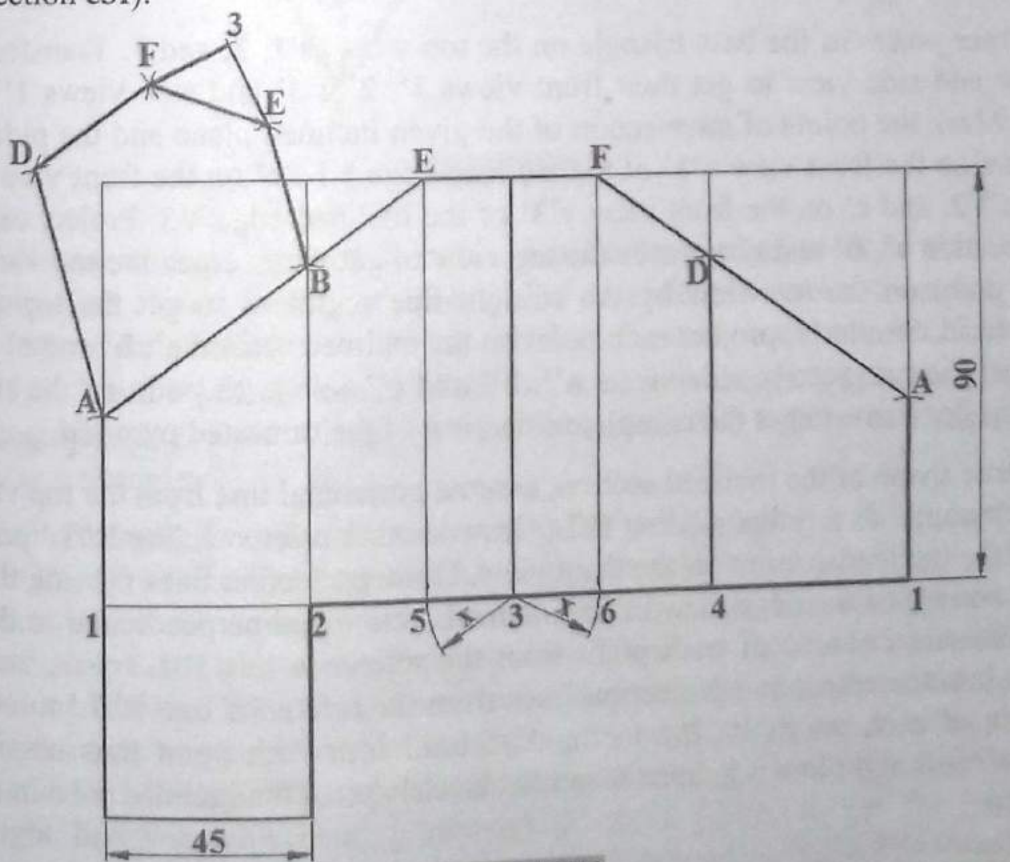


Figure E8.7(b)

Example 8.8

Complete the orthographic views of a truncated right triangular pyramid shown in *Figure E8.8* and then develop it.

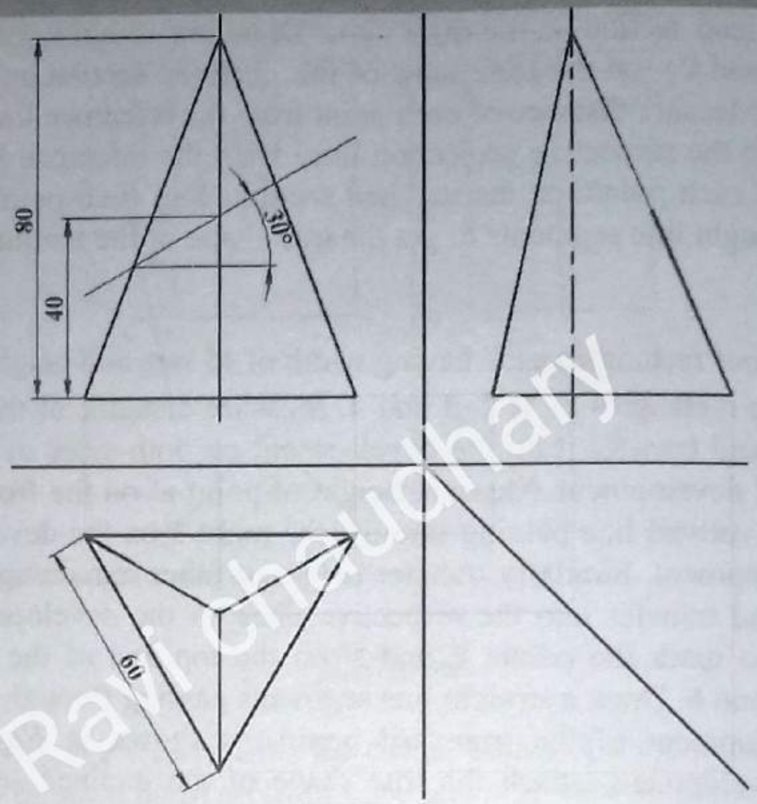


Figure E8.8

Solution

Name the corner points of the base triangle on the top view as **1**, **2**, and **3**. Transfer each point on front view and side view to get their front views **1'**, **2'**& **3'** and side views **1''**, **2''** & **3''** respectively. Mark the points of intersection of the given inclined plane and the pyramid on the front view as **a'**on the front view **v'1'** of the inclined edge **V1** , **b'** on the front view **v'2'** of the inclined edge **V2**, and **c'** on the front view **v'3'** of the inclined edge **V3**. Project each point on the inclined section **a'**, **b'** and **c'** towards the top view to get their respective top views **a**, **b** and **c**. Join each point on the top view by the straight line segments to get the top view of the truncated pyramid. Similarly, project each point on the inclined section **a'**, **b'** and **c'** towards the top view to get their respective side views **a''**, **b''** and **c''** Join each point on the side view by the straight line segments to get the complete side view of the truncated pyramid.

To draw the true shape of the inclined section, assume horizontal line from the top view **v** of the vertex of the pyramid as a reference line **R/L**. Draw another reference line **R'/L'** parallel to the edge view of the inclined section on the front view. Draw projection lines passing through each points (**a'**, **b'** and **c'**) on the edge view of the inclined section and perpendicular to the reference line **R'/L'**. Measure distance of each point from the reference line **R/L** on the top view and transfer them into the respective projection lines from the reference line **R'/L'** to complete the auxiliary view of each points on the inclined section. Join each point thus obtained on the auxiliary view by straight line segments to get the true shape of the inclined section as shown in *Figure E8.8(a)*.

None of the slant edges **V1**, **V2** and **V3** appear in true lengths on the front view. Draw true length **v'1'**$_T$ of the slant edge **V1**.

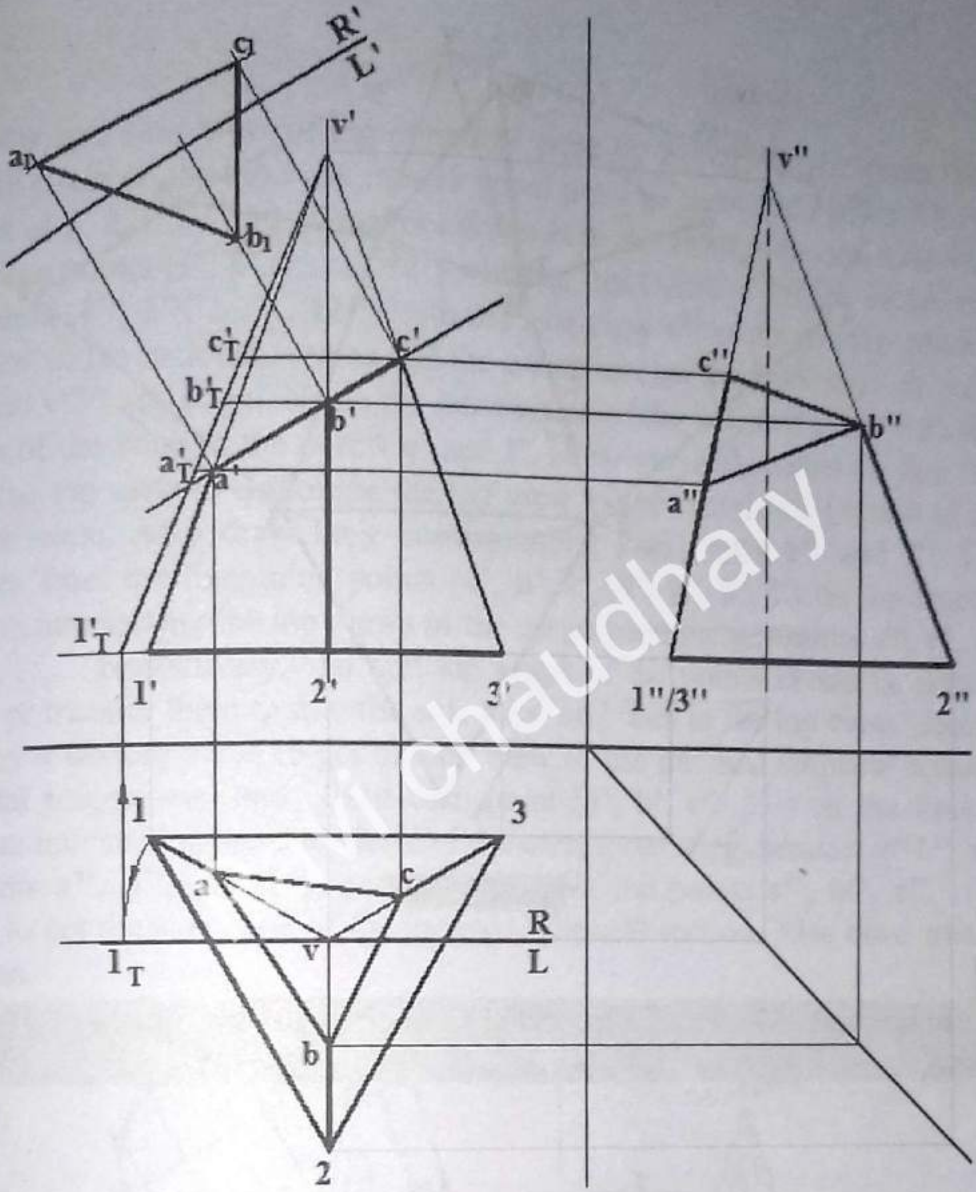


Figure E8.8(a)

For the development, draw an arc with radius equal to true length $v'1'_T$ of the slant edge **V1** of the pyramid, as shown in *Figure E8.8(b)*. To draw four isosceles triangles, mark three chord segments of length 60 mm on the arc. Mark each point on the arc as **1**, **2**, **3** and **1**. Join each point **1**, **2**, **3** and **1** with the center of the arc. Also join chords **1-2**, **2-3** and **3-1** to get the required three triangles. Draw triangle for the bottom surface with its one edge common to base of any one triangle.

To remove the truncated portion from the complete development of the pyramid, measure true length ($1'_Ta'_T$) of the line segment **1A** along the true length line $v'1'_T$ and transfer it along the line **V1** on the development to mark the point **A** on the development. Similarly, measure true length ($1'_Tb'_T$) of the line segment **2B** along the true length line $v'1'_T$ and transfer it along the line **V2** on the development to mark the point **B** on the development. Again, measure true length ($1'_Tc'_T$) of the line segment **3C** along the true length line $v'1'_T$ and transfer it along the line **V3** on the development to mark the point **C** on the development. Draw straight line segments passing through all these points to get the lateral surface development of the truncated pyramid, as shown in *Figure E8.8(b)*. For the complete surface development, attach the true shape of the inclined section with the lateral surface at any common edge (say point **BC**).

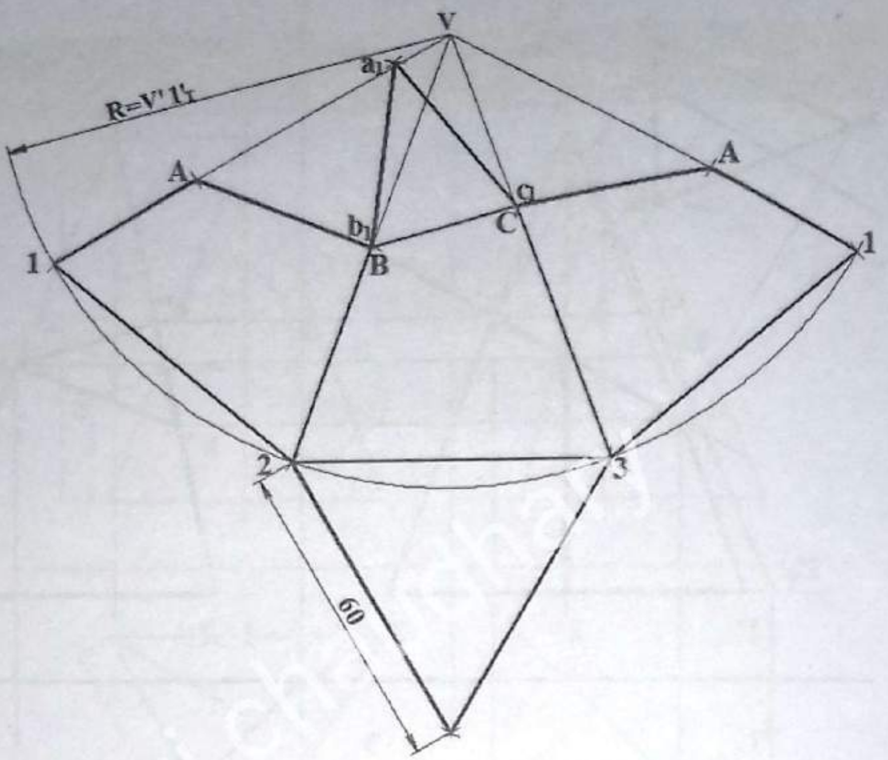


Figure E8.8(b)

Example 8.9

Complete the orthographic views of a truncated right circular cone shown in *Figure E8.9* and then develop it.

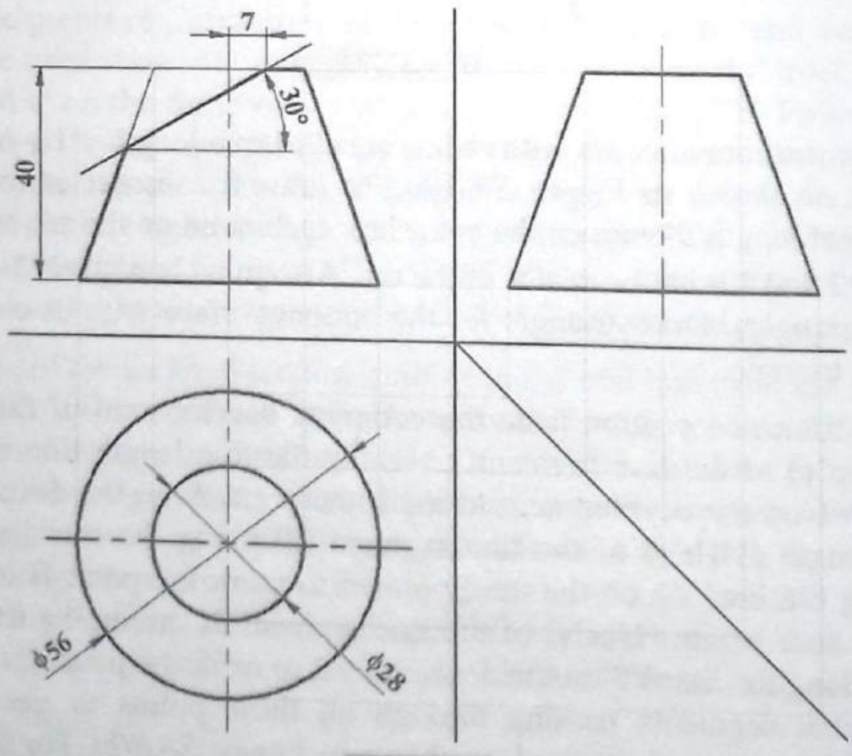


Figure E8.9

Solution

Draw front view and side view of the complete cone by extending the given end generators. Divide the base circle on the top view into 12 equal parts as shown in *Figure E8.9(a)*. Name the dividing points as **1**, **2**, ……., **12**. Transfer all points to the front view and side view. Join front views of all these points (**1'**, **2'**, ……., **12'**) with the front view **v'** of the vertex and side views of all these points (**1''**, **2''**, ……., **12''**) with the side view **v''** of the vertex. Mark intersections of the edge view of the inclined surface and the generators on the front view as point **a'** on **v'1'**, **b'** on **v'2'**, **c'** on **v'3'**, and so on. Name the intersection of the inclined plane with the top section of the frustum of the cone as the points **e'** and **f'**. Draw vertical projection line from the point **e'/f'** towards the top view to determine the top view of the remaining portion of the top of the frustum of the cone. Also draw their corresponding side views **e''** and **f''**. Draw vertical projection lines from the remaining points (**a'**, **b'**, **c'**, **d'**, **g'**, **h'**, **i'**) on the front view of the inclined section intersecting the top views of the corresponding generators **v1**, **v2**, **v3**, ……, on points **a**, **b**, **c**, …… respectively. To get top views of the points **D** and **G**, either use cutting plane method or transfer them first to the side view and then to the top view. Join the points **a**, **b**, **c**, ……, **i** by a smooth curve to get the top view of the inclined elliptical section. Similarly, draw horizontal projections lines from each point (**a'**, **b'**, **c'**, …..) on the front view of the inclined section intersecting the side views of the corresponding generators **v''1''**, **v''2''**, **v''3''**, …… on points **a''**, **b''**, **c''**, …… respectively. Join the points **a''**, **b''**, **c''**, ……, **i''** by a smooth curve to get the side view of the inclined elliptical section. Also draw true shape of the inclined section.

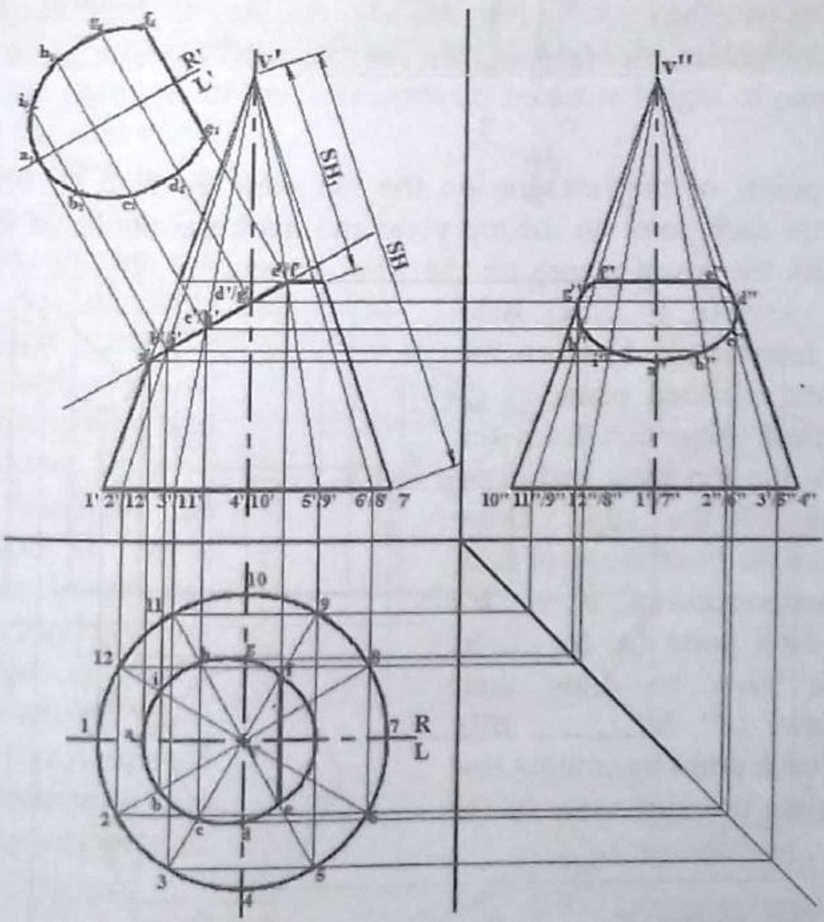
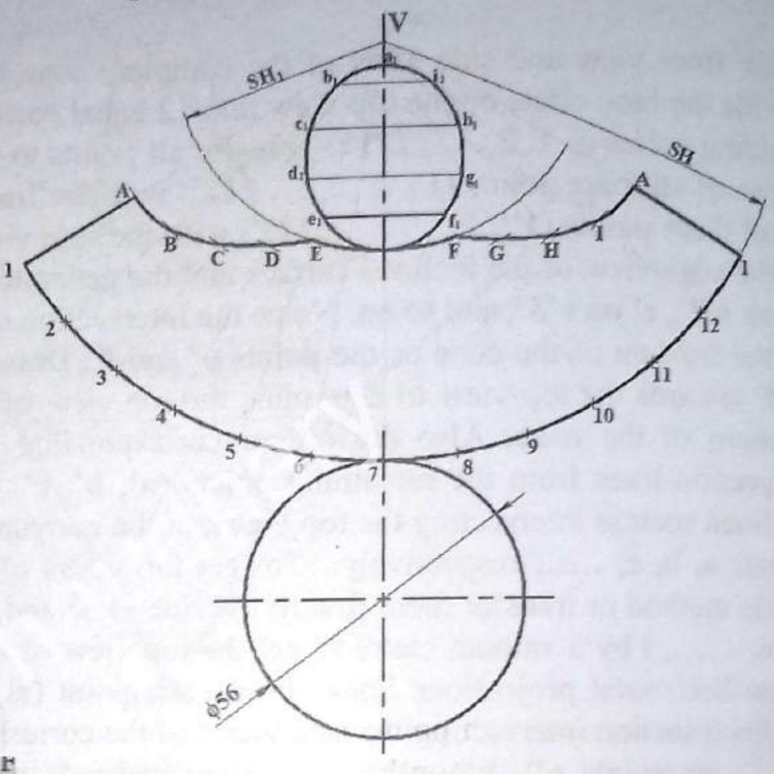


Figure E8.9(a)

To develop the given truanted cone, draw development of the frustum of the cone as shown in *Figure E8.9(b)*. Divide the base arc on the development into 12 number of equal parts and name the dividing points as **1**, **2**, ……., **12**. Mark the remaining part of the frustum (i.e. arc **EF**) on the development. Measure the true length of all the remaining points (**A**, **B**, **C**, **D**, **G**, **H**, **I**) along the true length line **v'7'** on the front view and transfer along the respective lines on the development. Join all the points thus obtained by a smooth curve to get the lateral surface development of the given truncated cone. Attach the part of the remaining portion of the top circular section and the true shape of the inclined section to the lateral surface.



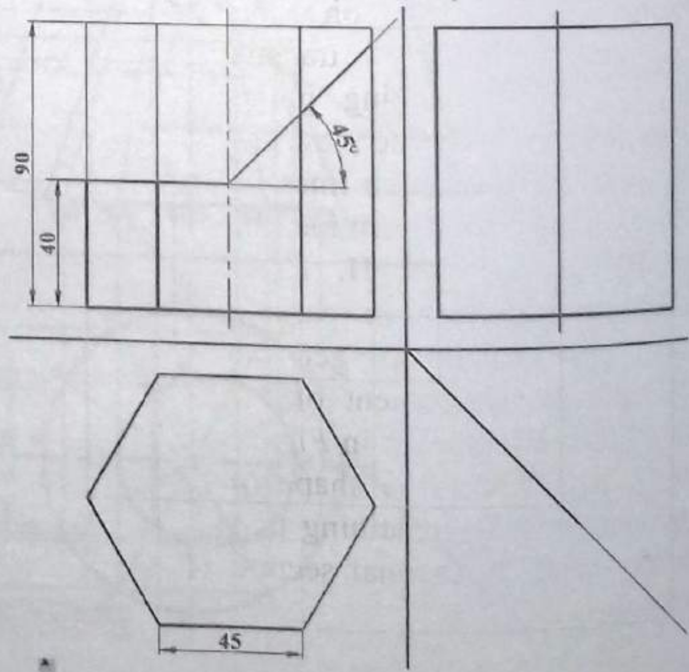
igure E8.9(b)

Example 8.10

A right hexagonal prism shown in **Figure E8.10** is cut by a combination of horizontal and inclined planes. Complete its orthographic views and then develop it.

Solution

Name the corner points of the hexagon on the top view as **1**, **2**, ……., **6**. Draw vertical projection lines from each point on the top view and mark the points of intersection of these projections line with the given planes on the front view as **a'**, **b'**, …….., **f'** and draw their corresponding top views (**a**, **b**, …, **f**). Also mark the edge of intersection between the given horizontal and inclined plane as **g'** and **h'**. Draw vertical projection from the point **g'/h'** towards the top view and draw the top view **gh** of the edge. Draw projection lines from the front views of each point on the inclined section (**a'**, **b'**, …, **h'**) and top view of each point (**a**, **b**, …, **h**) towards the side view to draw their respective side views (**a''**, **b''**, ……, **h''**). Join side views of each point by straight line segments to complete the side view of the truncated prism.



igure E8.10

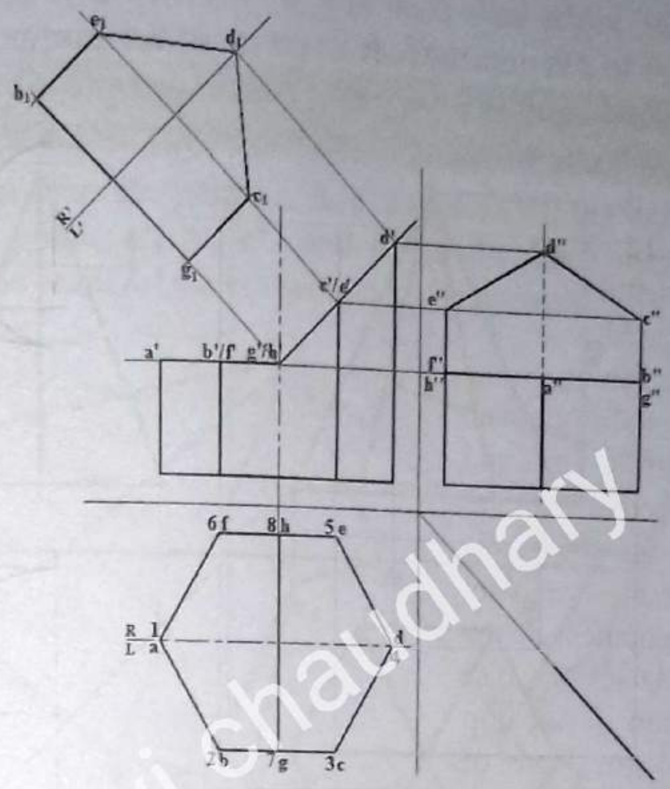


Figure E8.10(a)

Measure distance of each point from the reference line **R/L** on the top view and transfer them into the respective projection lines from the reference line **R'/L'** to complete the auxiliary view of each points on the inclined section. Join each point thus obtained on the auxiliary view by straight line segments to get the true shape of the inclined section as shown in *Figure E8.10(a)*.

For the development, first draw development of the complete prism as explained earlier. Also measure the distance of point **7** and **8** respectively from the corners **2** or **3** and **5** or **6** on the top view and transfer on the base line of the development. Measure height of point **a'** on the front view (or point **a''** on the side view) from the base and transfer it to the vertical line passing through the point **1** on the development to determine point **A** on the development. Similarly transfer height of other remaining points from the front view or side view and transfer into the respective lines on the development to determine the points **B**, **C**, …., and **H**. Draw straight line segments in passing through these points to get the lateral surface development of the truncated prism, as shown in *Figure E8.10(b)*. Attach the true shape of the inclined and the remaining part of the horizontal hexagonal section from the top view.

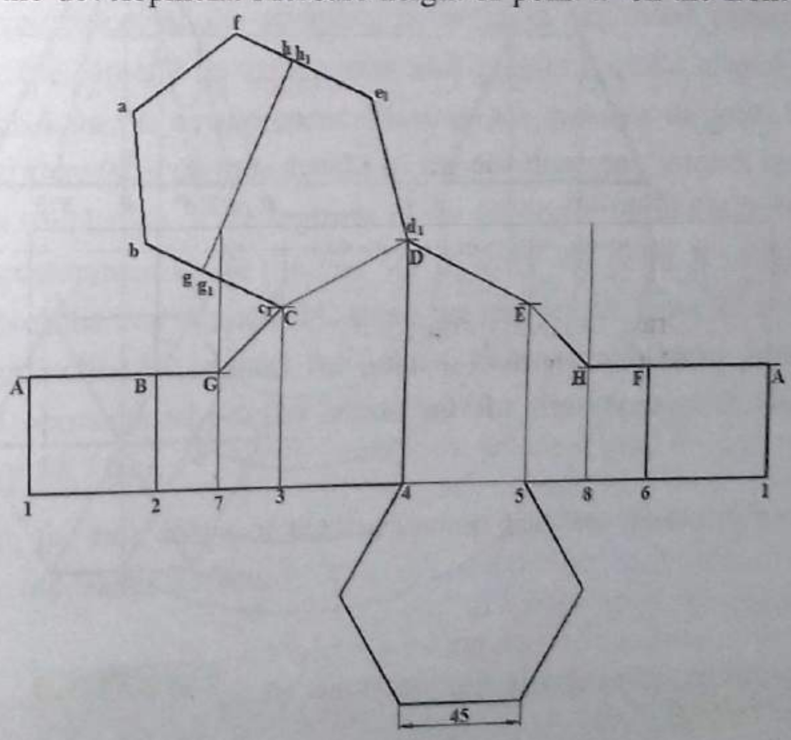


Figure E8.10(b)

Example 8.11

A right hexagonal pyramid shown in *Figure E8.11* is cut by a plane perpendicular to the HP and inclined to the VP. Complete its orthographic views and then develop it.

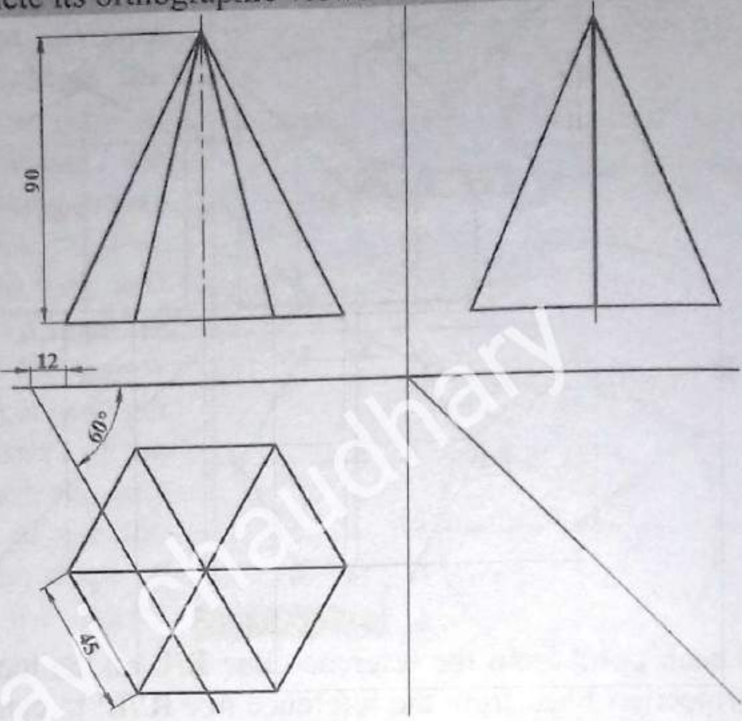


Figure E8.11

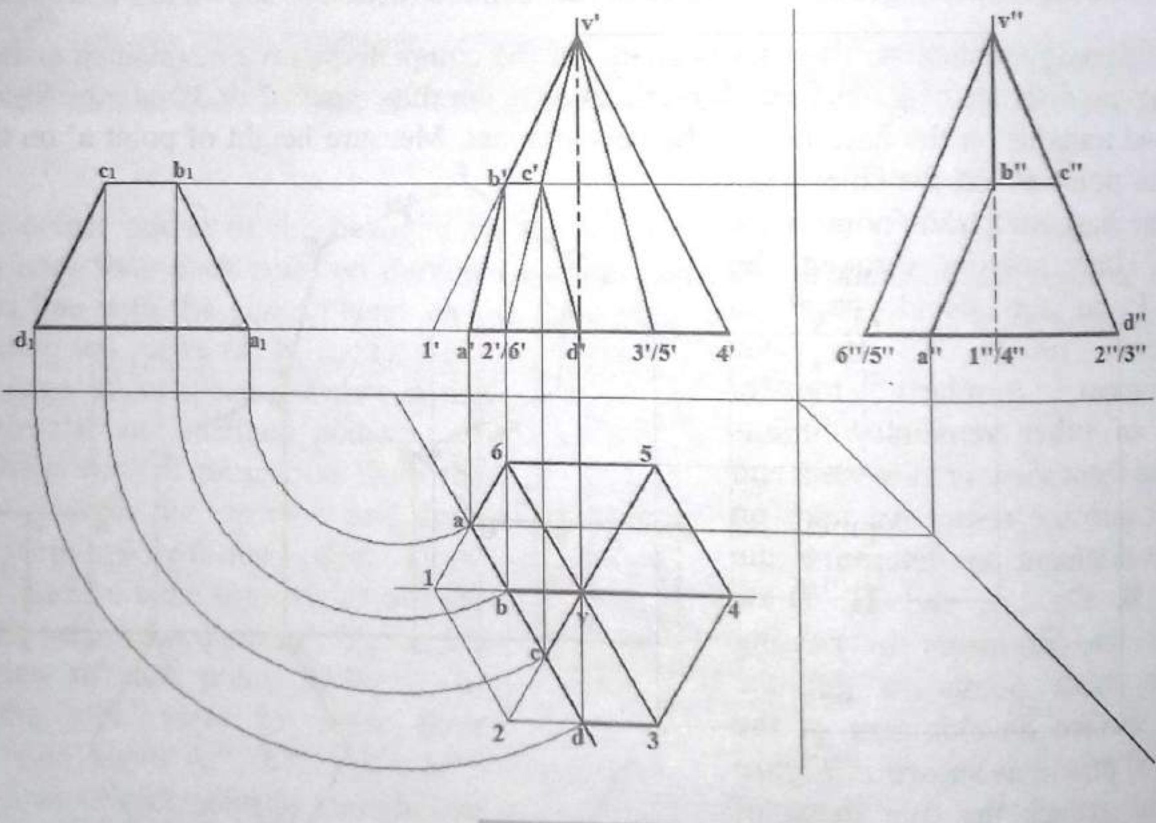


Figure E8.11(a)

Solution

Name the corner points of the base hexagon on the top view as 1, 2, ……., 6 and name the top view of the vertex as v. Mark corresponding front views (1', 2', , ……., 6') and side views (1'', 2'', , ……., 6'') of each point as shown in *Figure E8.11(a)*. Draw front views (v'1', v'2', ……

v'6') and side views (v''1'', v''2'', ….., v''6'') of each slant edges. Mark intersection of the given plane and the given pyramid as **a**, **b**, **c** and **d**. Transfer each of these points to the front view (**a'**, **b'**, **c'** and **d'**) and side view (**a''**, **b''**, **c''** and **d''**). Join the points on the respective view by the straight line segments to get the complete orthographic views of the given pyramid.

Draw radial projection lines from the points **a**, **b**, **c** and **d** on the top view and draw horizontal projection lines from the points **a'**, **b'**, **c'** and **d'** on the front view. Intersections of the projection lines give the true shape of the cut section of the pyramid.

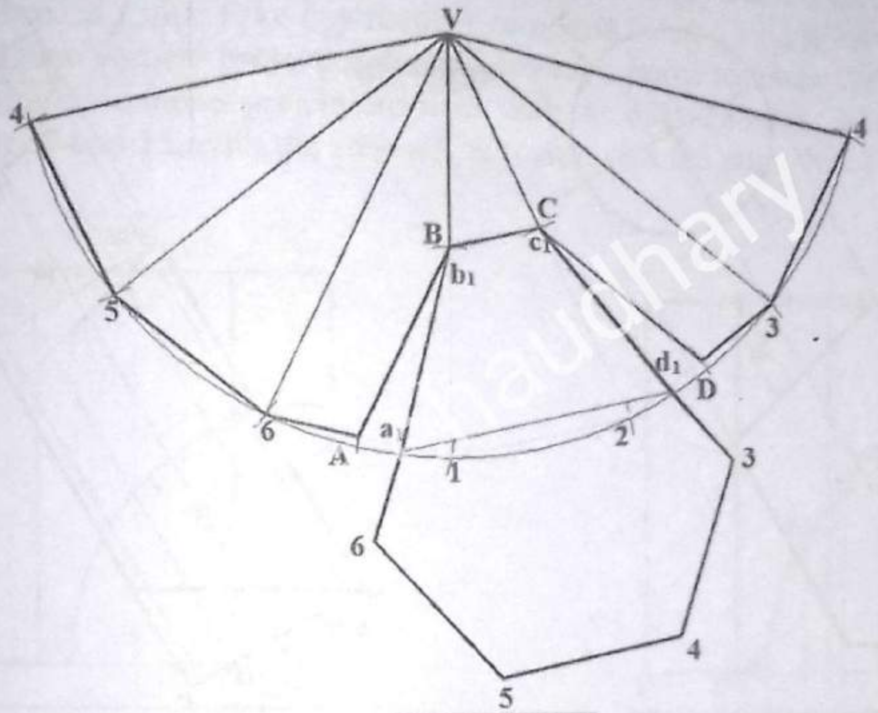


Figure E8.11(b)

For the development, first draw development of the complete pyramid as explained earlier. Measure the distance of point **a** from the corner **1** on the top view and transfer it on the edge **61** of the development to mark the point **A** on the development. Measure the distance of point **d** from the corner **2** on the top view and transfer it on the edge **23** of the development to mark the point **D** on the development. Measure true length of the segment of the generator **1B** in the front view **1'b'** and transfer it into the development along the line **V1** to mark the point **B** in the development. Measure the true length of the line segment **2C** along the true length line **v'1'** and transfer it into the development along the line **V2** to mark the point **C** in the development. Join all these points by the straight line segments to get the lateral surface development of the truncated pyramid, as shown in *Figure E8.11(b)*.

For the complete development attach the true shape of the cut section and true shape of the remaining part of the base hexagon to the lateral surface.

Example E 8.12

Draw lateral surface development of the remaining part of the right circular cylinder shown in *Figure E8.12*.

Solution

Draw complete front view of the given right circular cylinder assuming that its axis inclined to the HP and parallel to the VP as shown in *Figure E8.12(a)*. Divide the circular view of the inclined cylinder into any number of equal parts (say **12**) and name the dividing points as **1**, **2**, ….., **12**. Draw projection lines from each point on the circle towards the front view of the cylinder and draw corresponding front views of the generator **1'**, **2'**, …… , **12'**.

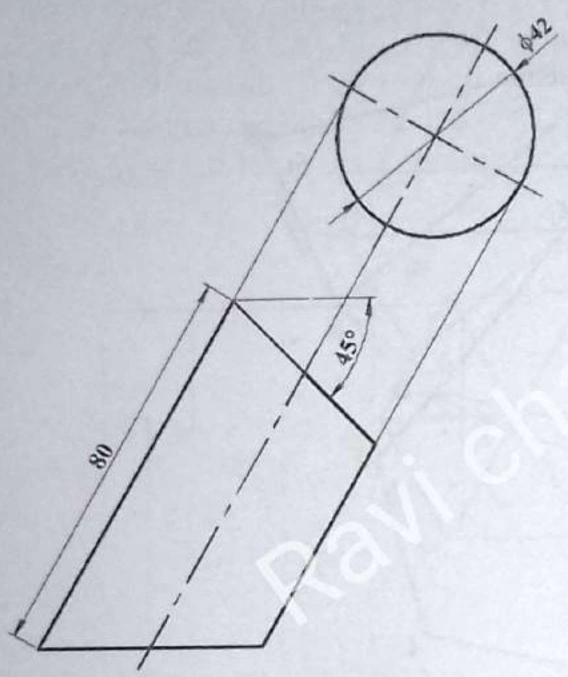


Figure E8.12

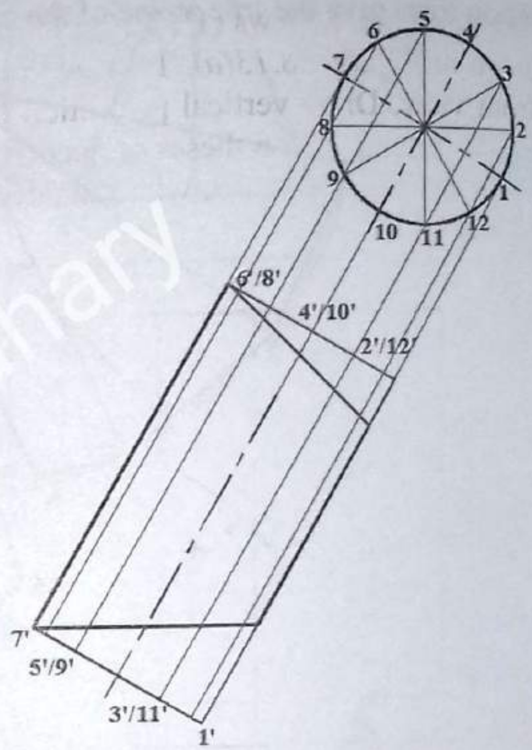


Figure E8.12(a)

Draw rectangle with its base as 132 mm (= $\pi D = \pi \times 42$) and height as 80 mm as lateral surface development of the complete cylinder. Divide the rectangle along its base into **12** equal parts and name the dividing points as **1**, **2**, ….., **12**. Measure the removed portion of each generator both from the bottom end and top end and transfer them into the respective lines on the development. Join all the points thus obtained by a smooth curve to get the lateral surface development of the given cylinder as shown in *Figure E8.12(b)*.

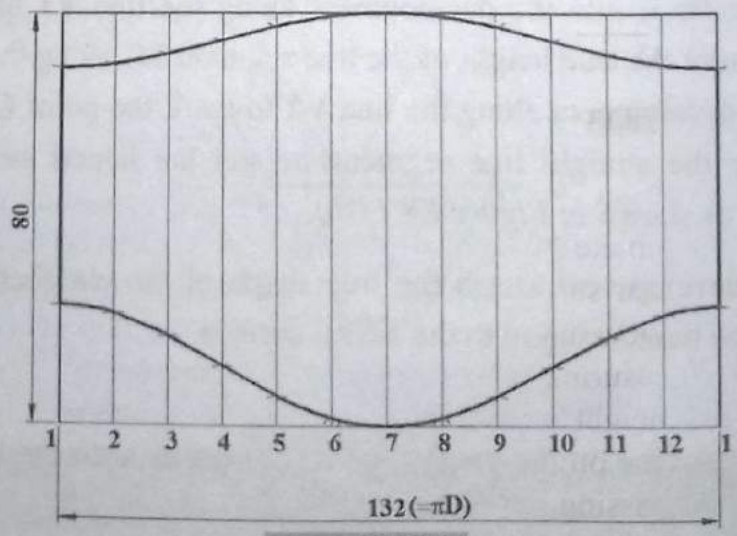


Figure E8.12(b)

Example 8.13

Draw lateral surface development of the remaining part of the right hexagonal prism shown in *Figure E8.13*.

Solution

Name the corner points of the base hexagon on the top view as **1**, **2**, ..., **6**. Mark corresponding front views (**1'**, **2'**, , ..., **6'**) and side views (**1''**, **2''**, , ..., **6''**) of each point as shown in *Figure E8.13(a)*. Take any number of points (**a'**, **b'**, ..., **g'**) on the given curve on the front view. Draw vertical projection lines from each point towards the top view and mark the points of intersection theses projections lines with the edge **23** of the hexagon as **7**, **8**, **9** and **10**, with the edge **34** as **15**, with the edge **45** as **6** and with the edge **56** of the hexagon as **11**, **12**, **13** and **14**.

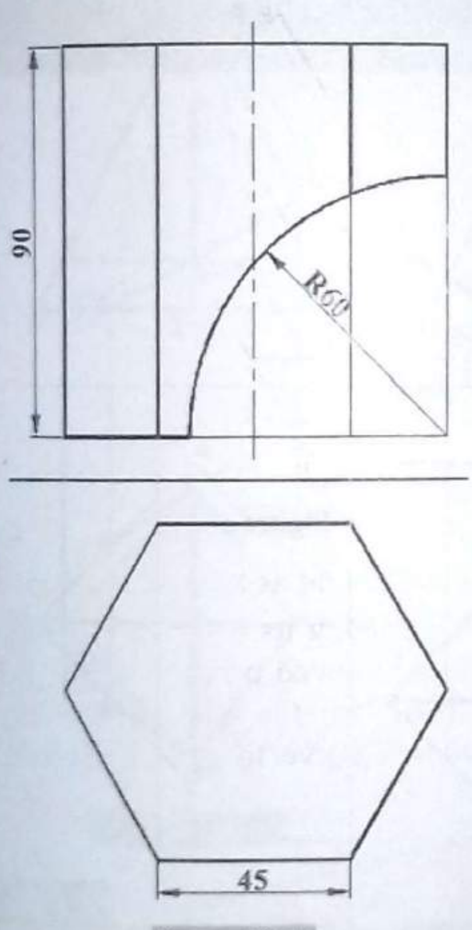


Figure E8.13

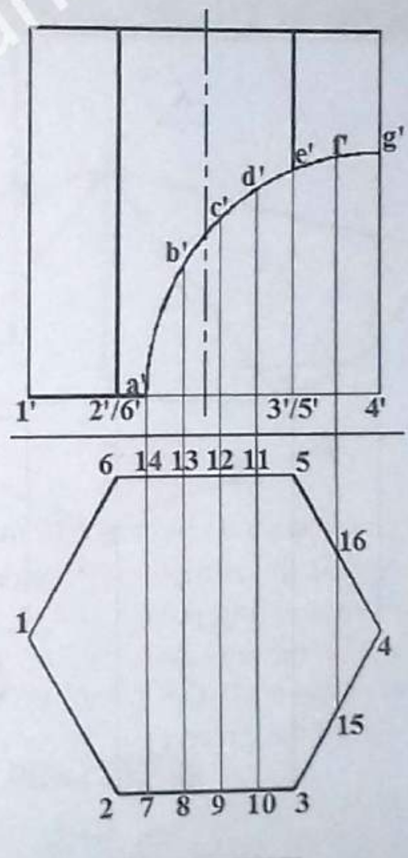


Figure E8.13(a)

For the development of the remaining portion, first draw development of the lateral surface development of the complete hexagonal prism as shown in Figure E8.13 (b). Also name the corners of each rectangular surface as the points 1, 2, ..., 6. Measure distance of each point on the edge 23 on the top view and transfer them into the development. Similarly, transfer all the remaining points by measuring their respective distance on the top and transferring into the development. Measure height of each point on the curve on the front view and transfer them into the corresponding line on the development (i.e., point **A** on the line passing through **7** and **14**, point **B** on the line passing through **8** and **13**, point **C** on the line passing through **9** and **12** and so on). Join all the point thus obtained by a smooth curve to get the required lateral surface development.

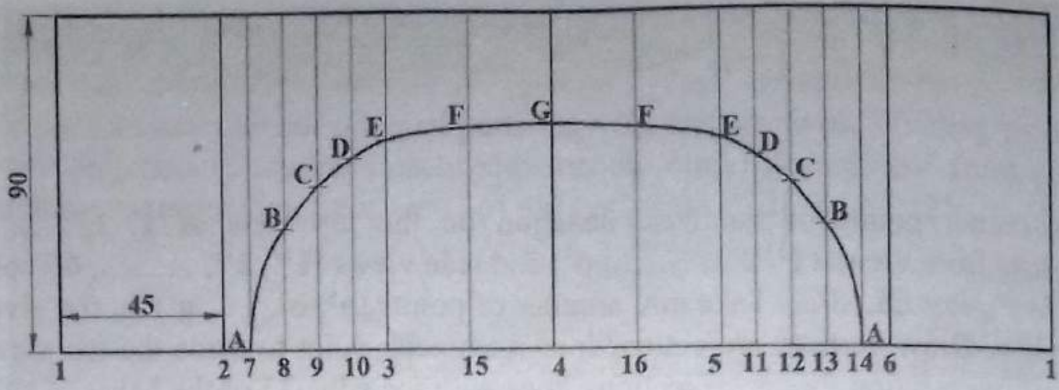


Figure E8.13(b)

Example 8.14

Draw lateral surface development of the remaining part of the right square pyramid shown in *Figure E8.14.*

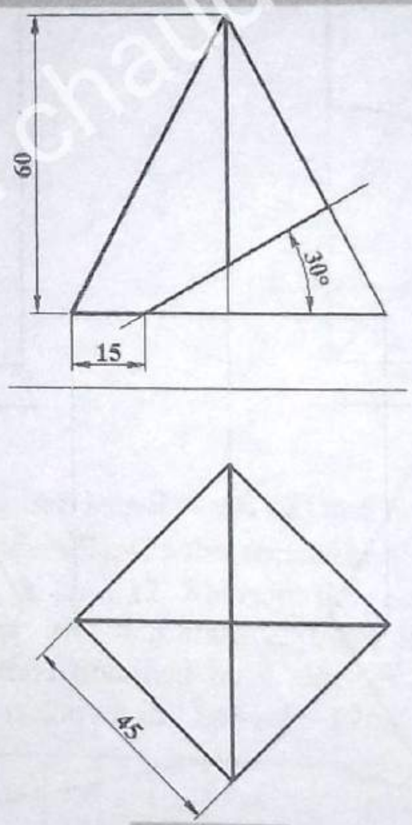


Figure E8.14

Solution

Name the corner points of the base square on the top view as **1**, **2**, **3** and **4** and name the top view of the vertex as **v**. Name the corresponding front views of the corners of the base square as **1'**, **2'**, **3'** and **4'** and that of the vertex as **v'** as shown in *Figure E8.14(a)*. Mark the points of intersection of the given plane and the square pyramid as **a'/e'**, **b'/d'** and **c'**. Transfer each point to the top view to get their corresponding top views **a**, **b**, **c**, **d** and **e**. Join the points on the top view in proper sequence by straight line segments to get the top view of the remaining portion of the given pyramid.

For the development, first draw development of the complete pyramid as explained earlier. Measure the distance of point **a** from the corner **1** on the top view and transfer it on the edge **12** of the development to mark the point **A** on the development. Measure the distance of point **e** from the corner **1** on the top view and transfer it on the edge **14** of the development to mark the point **E** on the development. Measure true length of the segment of the generator **2B** (= **4D**) in the front view along the true length line **v'1'** or **v'3'** and transfer it into the development along the lines **V2** and **V4** to mark the points **B** and **D** in the development. Measure the true length of the line segment **3C** (= **3'c'**) and transfer it into the development along the line **V3** to mark the point **C** in the development. Join all these points by the straight line segments to get the lateral surface development of the truncated pyramid, as shown in *Figure E8.14(b)*.

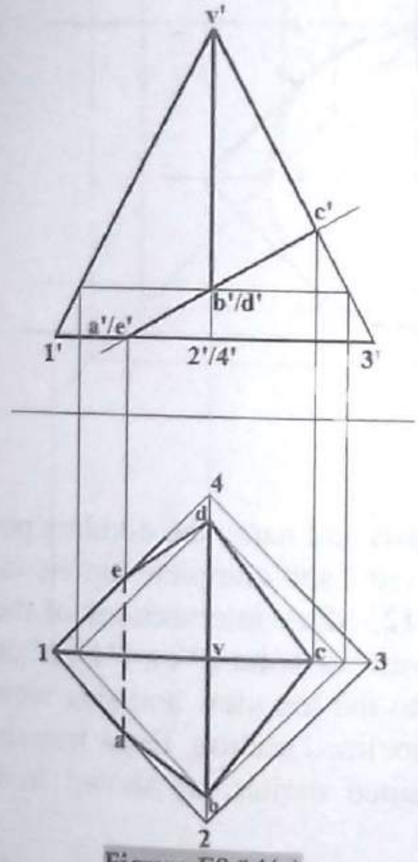


Figure E8.14(a)

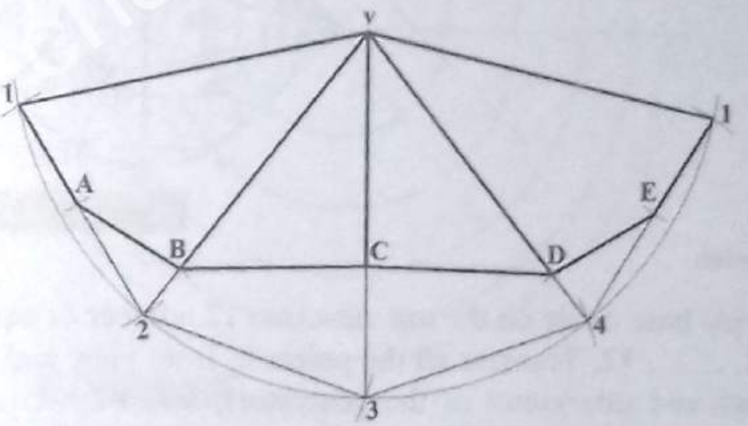


Figure E8.14(b)

Example 8.15

Complete the orthographic views of a truncated oblique circular cone shown in *Figure E8.15* and then develop it.

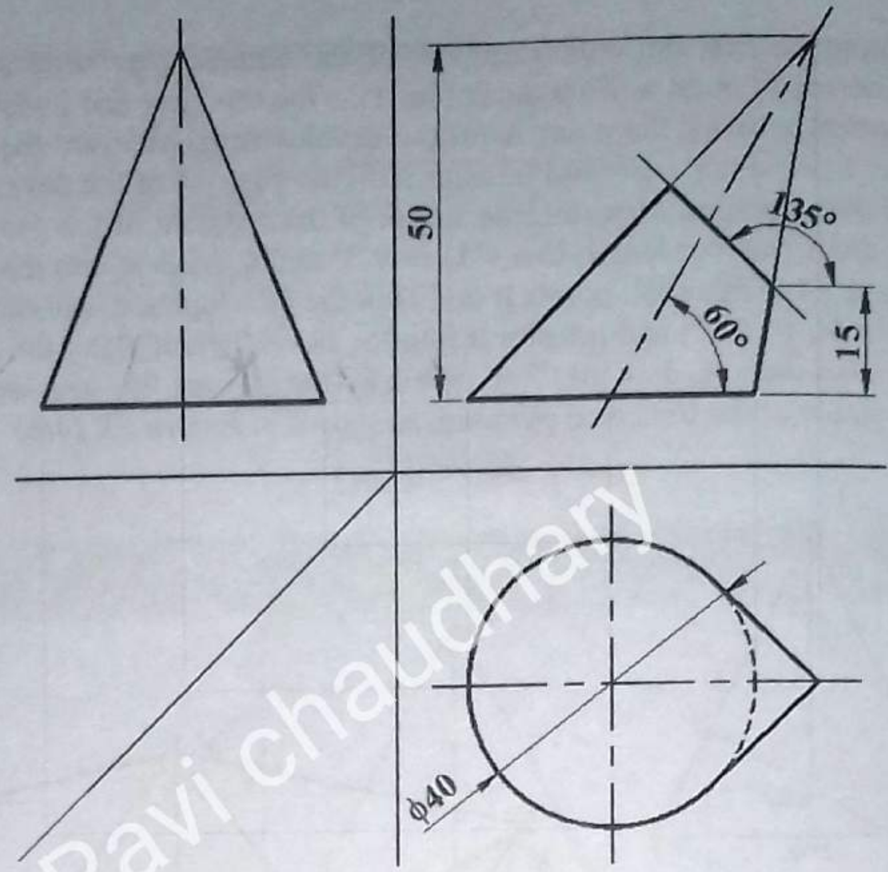


Figure E8.15

Solution

Divide base circle on the top view into 12 number of equal parts and name the dividing points as **1**, **2**, ……, **12**. Transfer all the points to front view and side view and complete top views, front views and side views of the generators, **V1**, **V2**, …….., **V12**. Mark intersections of the edge view of the inclined surface and the generators on the front view as point **a'** on **v'1'**, **b'** on **v'2'**, **c'** on **v'3'**, and so on. Transfer each point (**a'**, **b'**, ……, **l'**) to the top view and side view. Join the points on each view to get the orthographic views of the inclined section. Draw true shape of the inclined section by following the procedure as explained earlier, as shown in *Figure E8.15(a)*.

To get the development, draw true length of generators which are not parallel to the VP and do not appear as true length on the front view and then draw surface development of the complete oblique circular cone. Measure true length (**1'a'**) of the line segment **1A** on the front view and trifler it along the line **V1** on the development to mark point **A**. Similarly, measure true length (**7'g'**) of the line segment **7G** on the front view and trifler it along the line **V7** on the development to mark point **G**. To determine true lengths of the other lines segments (**2B**, **3C**, ….**6F**, **8H**, ….. **12L**), draw horizontal lines form the points (**b'**, **c'**, ……, **f'**, **h'**, …..**l'**) to their respective true length lines ($v'2'_T$, $v'3'_T$, ……. $v'6'_T$, $v'7'_T$, ……, $v'12'_T$). Measure toe lengths of the each line segments (**2B**, **3C**, ….**6F**, **8H**, ….. **12L**) and transfer them on the corresponding lines on the development to get the remaining points **B**, **C**, ….. **F**, **H**, …., **L**. Draw smooth curve passing through all these points to get the development of the truncated oblique cone as shown in *Figure E8.15(b)*. Also attach the true shape of the inclined section to the lateral surface to the complete surface development.

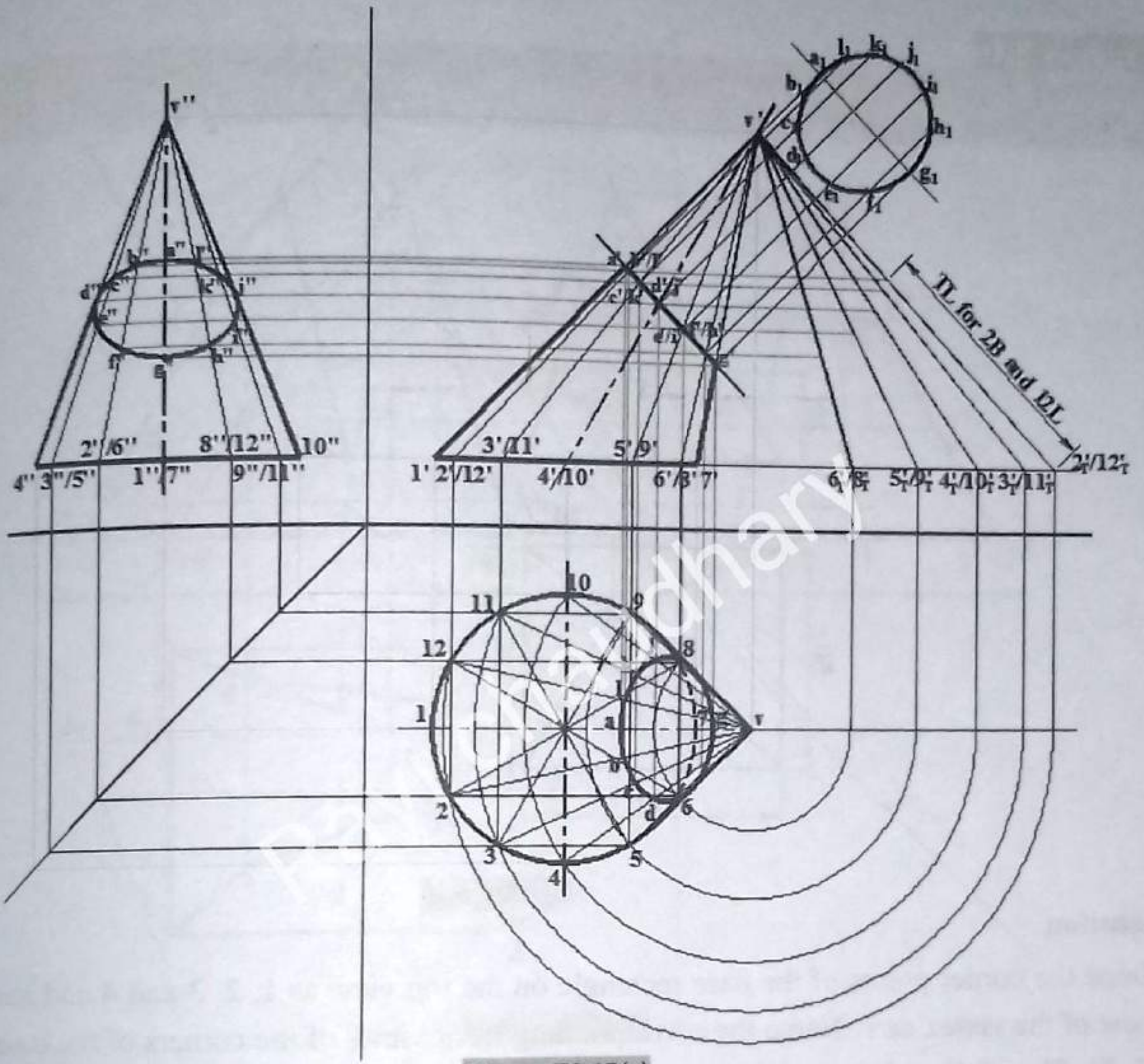


Figure E8.15(a)

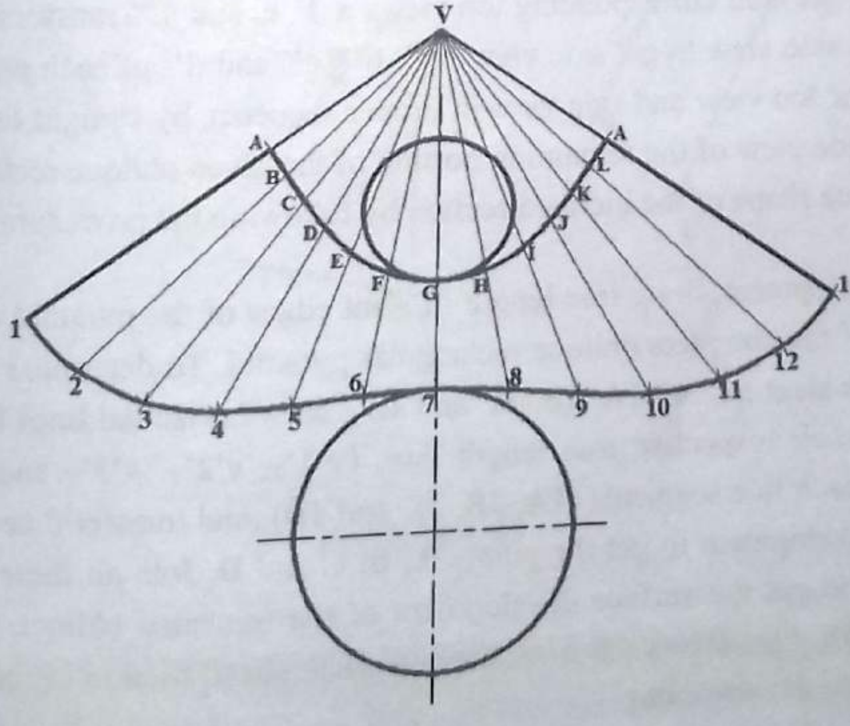


Figure E8.15(b)

Example 8.16

Complete the orthographic views of a truncated oblique rectangular pyramid shown in *Figure E8.16* and then develop it.

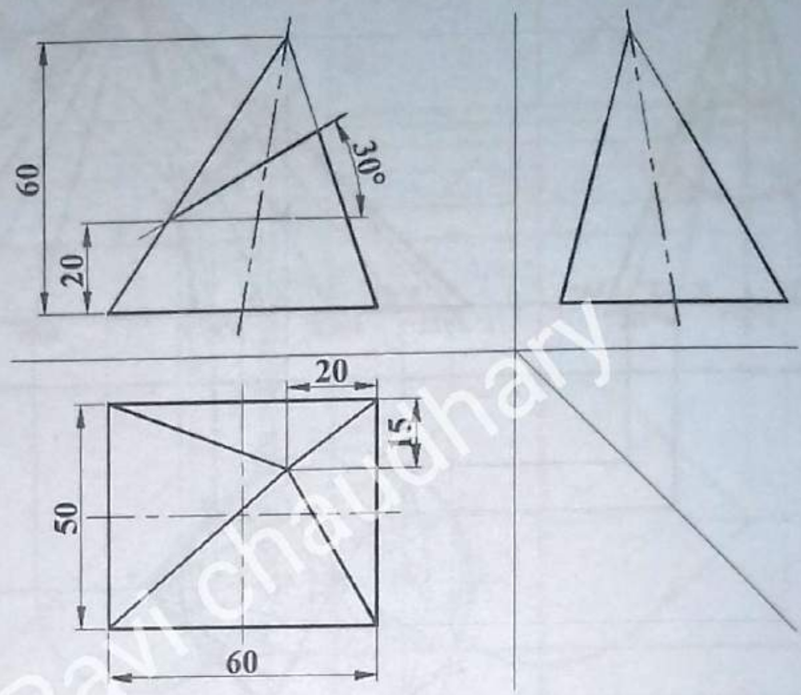


Figure E8.16

Solution

Name the corner points of the base rectangle on the top view as **1**, **2**, **3** and **4** and name the top view of the vertex as **v**. Name the corresponding front views of the corners of the base square as **1'**, **2'**, **3'** and **4'** and that of the vertex as **v'** as shown in *Figure E8.16(a)*. Mark the points of intersection of the given plane and the square pyramid as **a'/d'** and **b'/c'**. Transfer each point to the top view to get their corresponding top views **a**, **b**, **c**, and **d**. Transfer all the points **a'**, **b'**, **c'** and **d'** towards side view to get side views **a''**, **b''**, **c''** and **d''** of each points respectively. Join the points on the top view and side view in proper sequence by straight line segments to get the top view and side view of the remaining portion of the given oblique rectangular pyramid. Also complete the true shape of the inclined section by following the procedure as explained earlier.

To get the development, draw true length of slant edges of the pyramid and then draw surface development of the complete oblique rectangular pyramid. To determine true lengths of the cut segments of the slant edges (**1A**, **2B**, **3C** and **4D**), draw horizontal lines form the points (**a'**, **b'**, **c'** and **d'**) to their respective true length lines (**v'1'$_T$**, **v'2'$_T$**, **v'3'$_T$** and **v'4'$_T$**). Measure true lengths of the each line segments (**1A**, **2B**, **3C** and **4D**) and transfer them on the corresponding lines on the development to get the points **A**, **B**, **C** and **D**. Join all these points by the straight line segments to get the surface development of the truncated oblique pyramid, as shown in *Figure E8.16(b)*. Also attach the true shape of the inclined section to the lateral surface to the complete surface development.

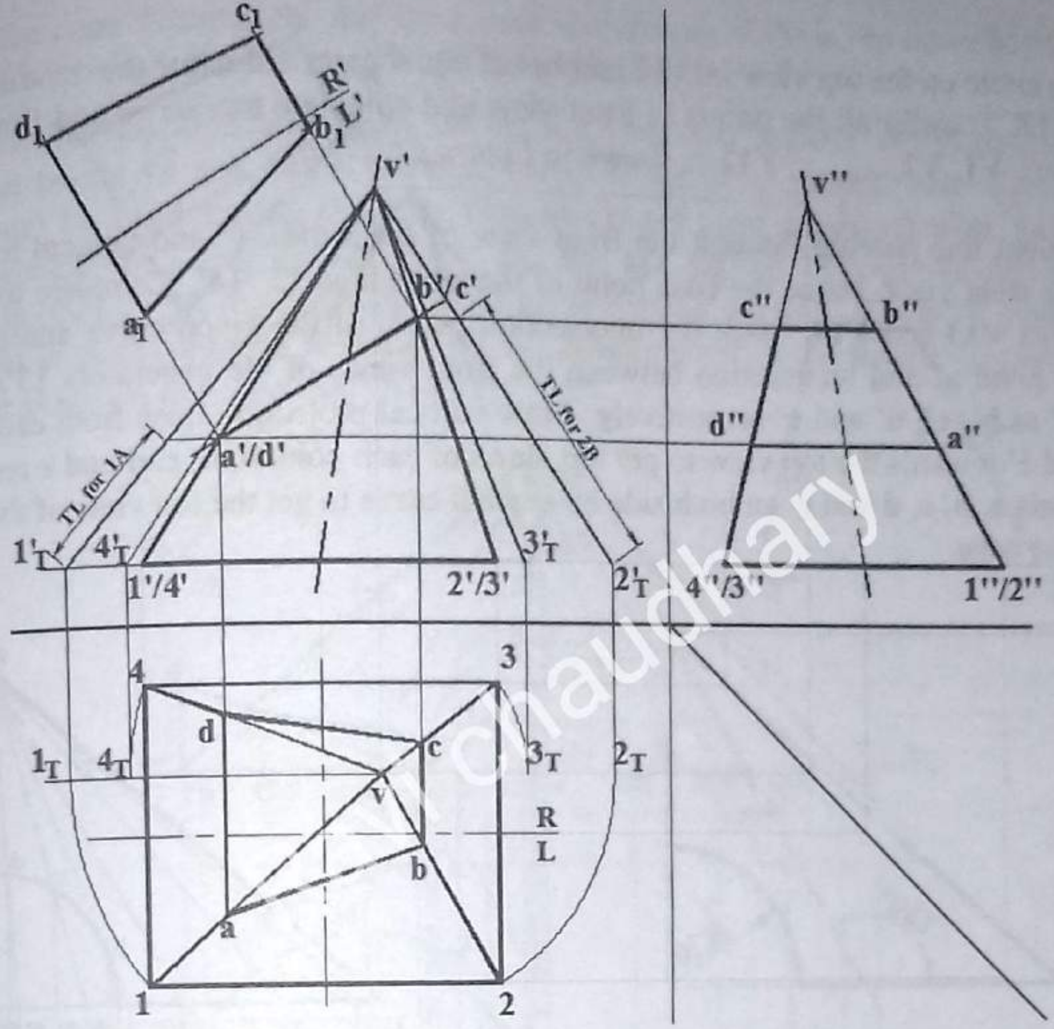


Figure E8.16(a)

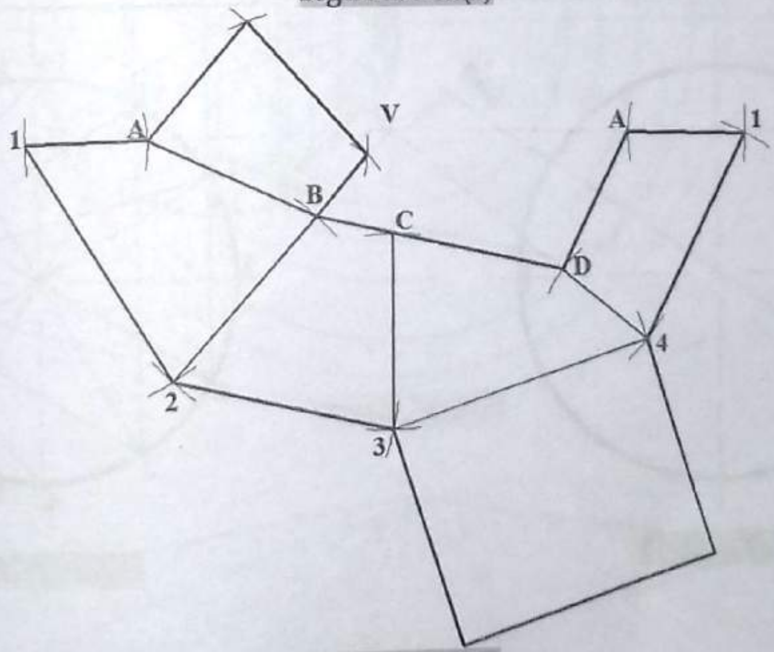


Figure E8.16(b)

Example 8.17

Complete top view of and draw lateral surface development of the remaining part of the oblique cone shown in *Figure E8.17*.

Solution

Divide base circle on the top view into 12 number of equal parts and name the dividing points as **1, 2, ……, 12**. Transfer all the points to front view and complete top views and front views of the generators, **V1, V2, ……..., V12** as shown in *Figure E8.17(a).*

Draw a straight line passing through the front view of the vertex **v'** and tangent to the given curve on the front view. Name the base point of the tangent as **13'/14'**. Complete top views of the generators **V13** and **V14**. Mark the intersection points of the given curve and base of the cone as the point **a'** and intersection between the front views of the generators **13'/14', 5'/9', 6'/8'** and **7'** as **b', c', d'** and **e'** respectively. Draw vertical projection lines from each point **a', b' c', d'** and **e'** towards the top view to get top views of each point **a, b, c, d** and **e** respectively. Join the points **a, b, c, d** and **e** on both side by smooth curve to get the top view of the removed portion of the cone.

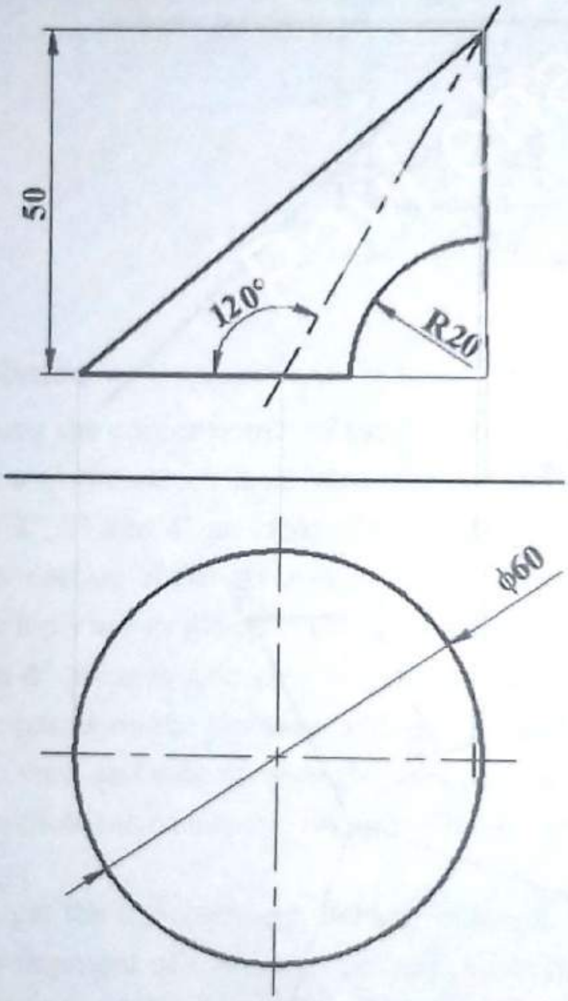


Figure E8.17

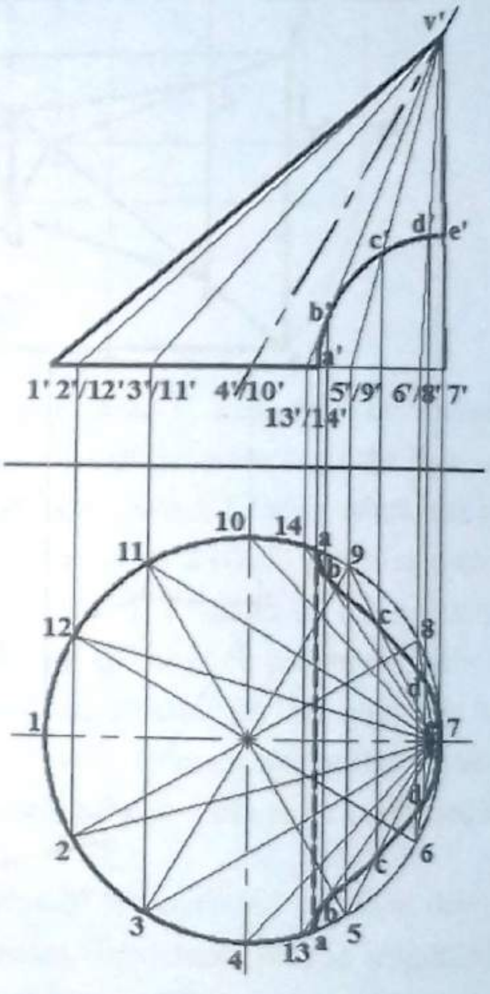


Figure E8.17(a)

To get the development, draw true length of generators which are not parallel to the VP and do not appear as true length on the front view and then draw surface development of the complete oblique circular cone. Draw horizontal lines from each points of the given curve **b' c'** and **d'** to the respective true length lines as shown in *Figure E8.17(b)*.

Measure chord lengths **4-13** and **10-14** on the top view and transfer to the development to determine the points **13** and **14** on the base curve of the development. Also measure chord length **13-a** and **14-a** and transfer to the development to get point **A** on both sides. Measure true lengths of the each line segments (**13B, 14B, 5C, 9C, 6D, 8D** and **7E**) and transfer them on the corresponding lines on the development to get the remaining points **B, C, ….. E**. Draw smooth curve passing through all these points to get the development of the remaining portion of the oblique cone as shown in *Figure E8.17(c)*.

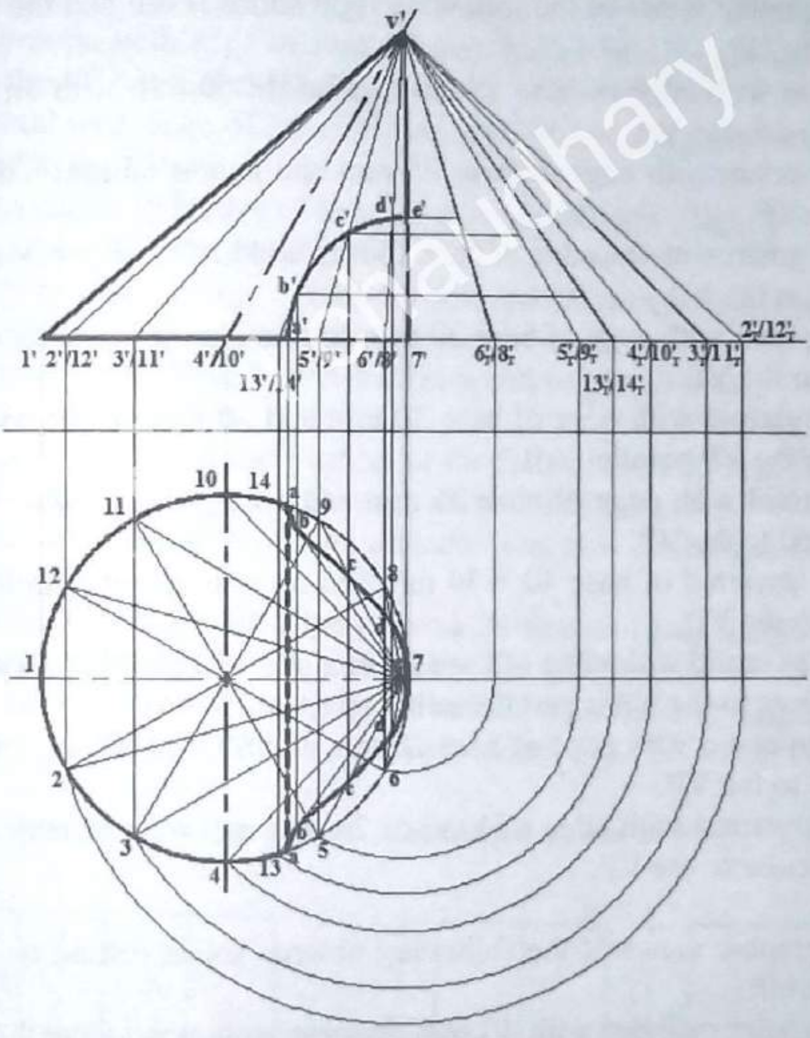


Figure E8.17(b)

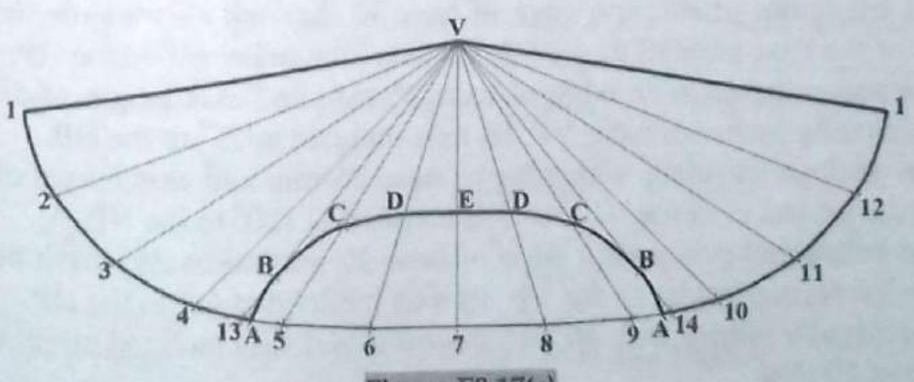


Figure E8.17(c)

CHAPTER 9

INTERSECTION OF SOLIDS

9.1 Introduction

Engineers while designing machine components or structure frequently encounter the intersection of different surfaces or solids. Pipe fittings, boiler fittings, ducts, containers are most common examples for this. To prepare the two dimensional pattern of such cases, the first step is to determine the line or curve intersection between the solids.

Any two geometrical elements are said to be intersected when they have some common portion. For example, two straight lines or a straight line and a plain are said to be intersecting each other if they have a common point and two planes are said to be intersecting each other, if they have a common line. Similarly, when two solids intersect with each other, the locus of points common to both the solids generate a line or curve of intersection, which is called intersection curve or interpenetration curve.

9.2 Classification of Intersection Curves

Intersection between common geometrical solids can be classified into the following three categories with reference to the method of construction to be used.

- (a) Both solids having uniform cross section
- (b) One solid having uniform cross section and another having uniformly varying cross section, and
- (c) Both solids having uniformly varying cross section.

9.2.1 Intersection between Solids with both Solids having Uniform Cross section

When two solids having uniform cross section intersect at right angle to each other, the intersecting points can be marked directly on any two orthographic views and third view can be completed with the help of these two views. For example, consider two intersecting cylinders shown in *Figure 9.1.* The points of intersection can be marked on the top view of the vertical cylinder where as side views of the points of intersection coincide with the side view of the horizontal cylinder. With the help of these two views, intersection curve can be drawn on the front view.

This method can be applied for the intersection of cylinder and cylinder, prism and prism and cylinder and prism.

9.2.2 Intersection between Solids with One Solid having Uniform Cross section and another having uniformly Varying Cross section

When a solid having uniform cross section intersects with a solid having a uniformly varying cross section. The points of intersection can be directly marked in only view and therefore these points should be transferred into other two views to complete the orthographic views of the intersection curve. *Figure 9.2* shows a cone and a cylinder intersecting each other at right angle. In this case, the points of intersection can be marked only on the side view of the horizontal cylinder and two other views should be completed by transferring the points of intersection in the respective views.

This method can be applied for the intersection cylinder and cone, prism and pyramid, prism and cone and cylinder and pyramid.

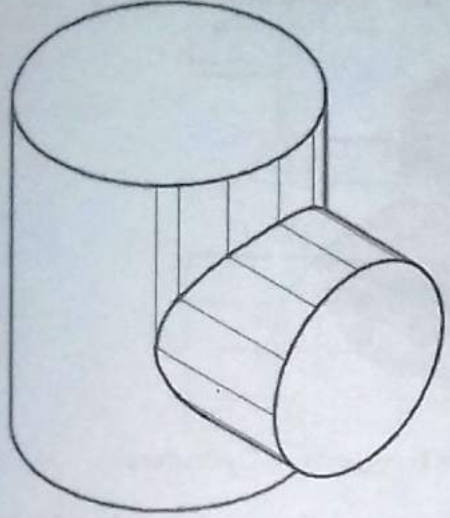


Figure 9.1: Intersection of two Cylinders

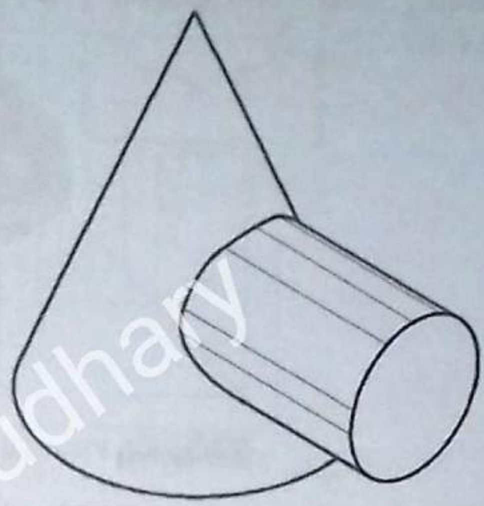


Figure 9.2: Intersection of Cylinder and Cone

9.2.3 Intersection between Solids with both Solids having uniformly Varying Cross section

When two solids both having uniformly varying cross section intersect with each other at right angles, the points of intersection cannot be marked directly in any principal views. In this case, cutting plane or auxiliary view method should be used to complete the orthographic views of the intersection curve. This method can be applied for the intersection of cone and cone, pyramid and pyramid and cone and pyramid.

9.3 Nature of Intersection Curves

Nature of intersection curve and hence method used to complete the orthographic views of the intersection curve depends upon the types of the intersecting solids.

When two solids having only plane surfaces intersect with each other, the intersection generally appears as combination of straights lines. Hence orthographic views of the intersection curve in this case can be completed by the projection of only corners points of end view of one solid.

When two solids having curve surfaces intersect with each other, the intersection generally appears as curve. Hence orthographic views of the intersection curve in this case can be completed by the projection of number of intermediate points of end view of one solid.

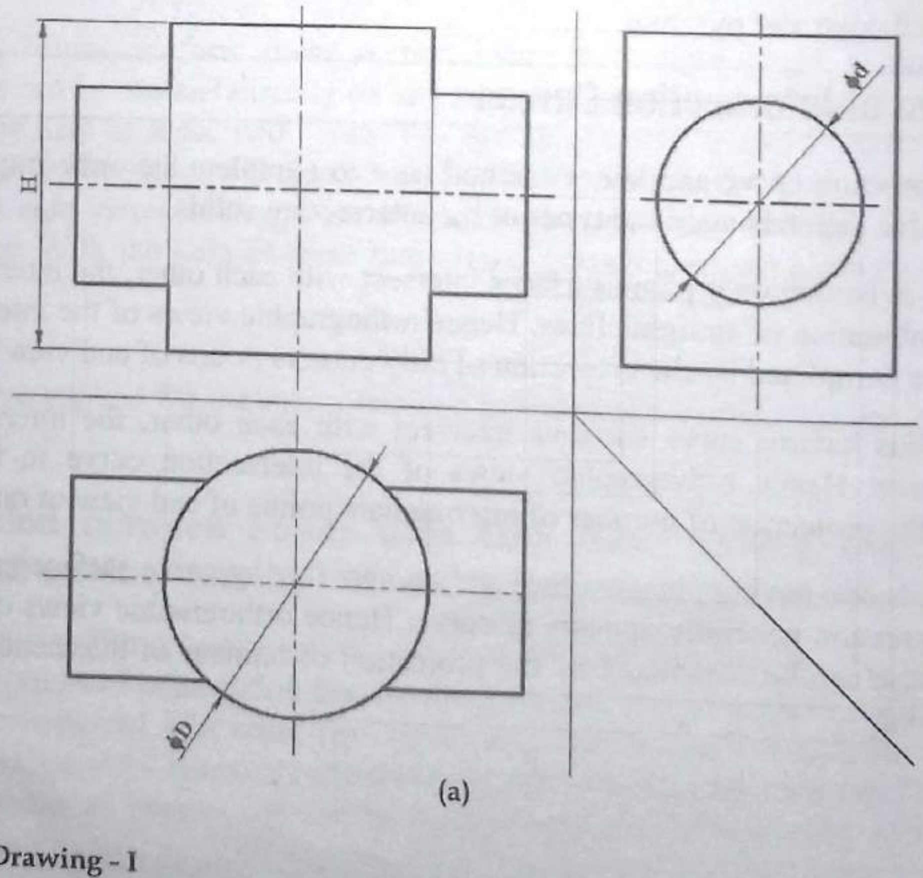
When two solids one having plane surface and another having curve surface intersect with each other, the intersection generally appears as curve. Hence orthographic views of the intersection curve in this case can be completed by the projection of number of intermediate points of end view of one solid.

9.4 Intersection of Cylinder and Cylinder

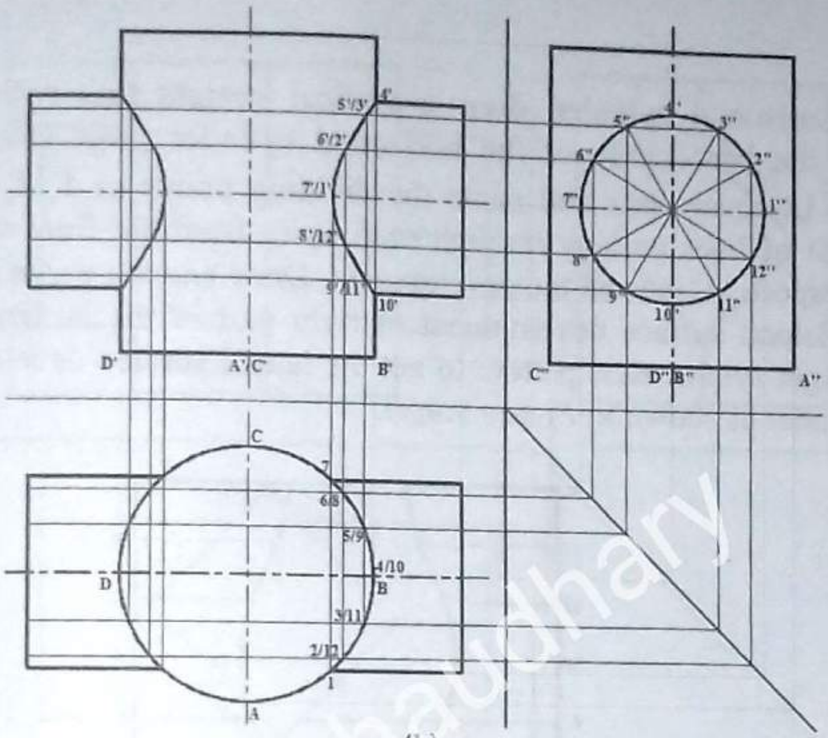
Figure 9.3 shows the pictorial view of two cylinders intersecting each other at right angle. The corresponding three orthographic views without the intersection curve are shown in *Figure 9.4(a)*.

Figure 9.3: Pictorial View of two Intersecting Cylinders

To complete the intersection curve, divide the side view of the horizontal cylinder (circular view) into any number of equal parts (say 12) and name the dividing points as **1'', 2''**, ..., **11''** and **12''** as the side views of the points on the intersection curve. Transfer each of these points into the top view and mark the top view of each point as **1, 2**, ..., **11** and **12** respectively where the projection lines intersect the circumference of the top view (circle) of the vertical cylinder. Draw vertical projection lines through top view of each points and horizontal projection lines through each points on the side view to get the front view as **1', 2'**, ..., **11'** and **12'** of each point. Join the points thus obtained by a smooth curve to get the front view of the intersection curve as shown in *Figure 9.4 (b)*.



(a)



(b)

Figure 9.4: Orthographic View of two Intersecting Cylinders

Development

To develop the vertical cylinder, draw rectangle with its width and height equal to circumference of the base circle of the vertical cylinder (πD) and height of the vertical cylinder (H) respectively as a lateral surface of the cylinder. Divide base line of the development into four equal parts and name the dividing points as **A**, **B**, **C** and **D**. Measure chord length (which approximates arc length) of points **3/11** and **5/9** from the point **B** on the top view and transfer into the development. Similarly measure the chord lengths of the remaining points on the **2/12**, **6/8**, **1** and **7** from the nearer point on the top view and transfer them into the development. Draw vertical lines from each of these points. Measure height of each points from the front view or side view and transfer into the development to get the points **1**, **2**, ………, **11** and **12** on the development. Draw smooth curve passing through these points. Repeat similar process to get the hole of the left side as shown in *Figure 9.4(c)*.

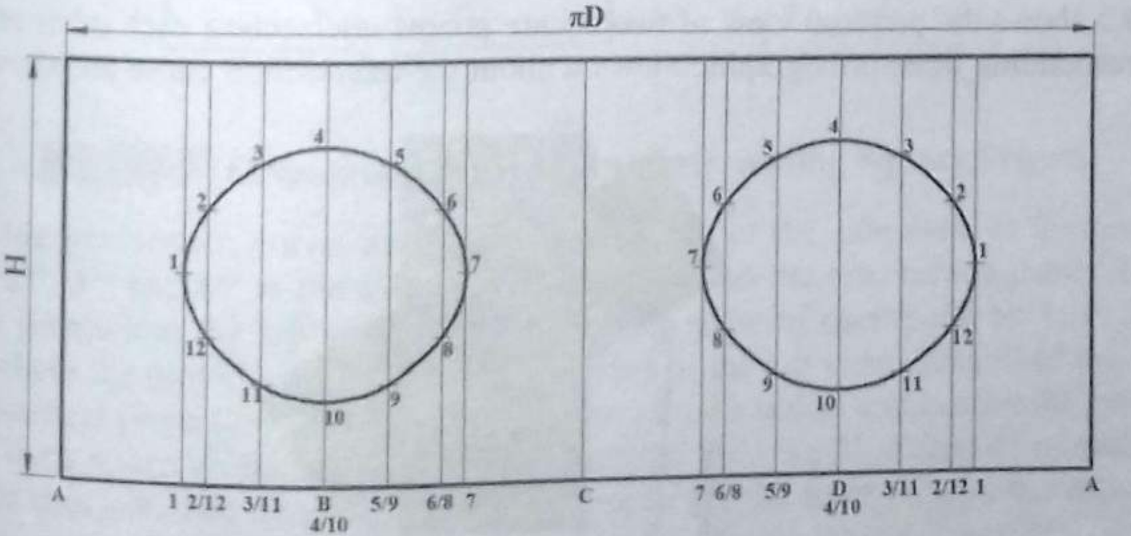


Figure 9.4(c): Development of Vertical Cylinder

To develop the horizontal cylinder, draw a vertical straight line with its length equal to circumference of the base circle of the horizontal cylinder (πd). Divide base line of the development into **12** equal parts and name the dividing points as **1, 2, ………, 11** and **12**. Measure the length of lines passing through each point from the front view or top view and transfer into the respective lines on the development. Draw smooth curve passing through these points to get the lateral surface development of right part of the horizontal cylinder. Repeat similar process to get symmetrical pattern to get the lateral surface development of left part of the horizontal cylinder as shown in *Figure 9.4(d)*.

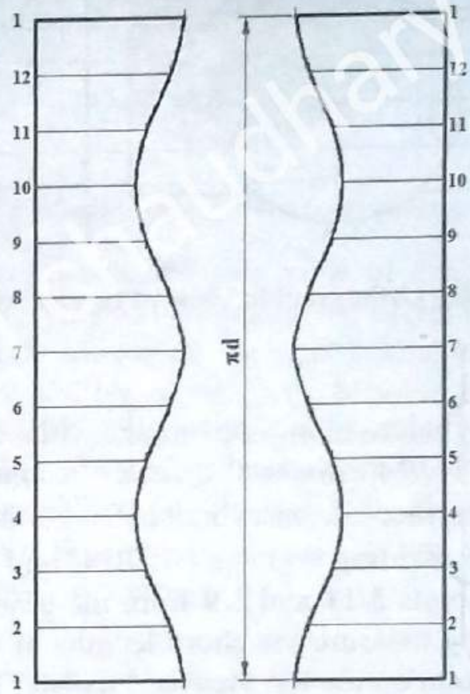


Figure 9.4(d): Development of Horizontal Cylinder

9.5 Intersection of Prism and Prism

Figure 9.5 shows the pictorial view of two square prisms intersecting each other at right angle. The corresponding three orthographic views without the intersection curve are shown in *Figure 9.6(a)*.

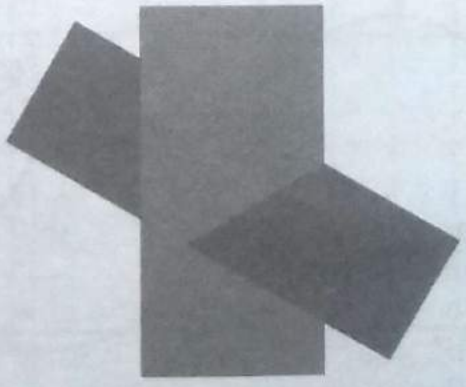


Figure 9.5: Pictorial View of two Intersecting Square Prisms

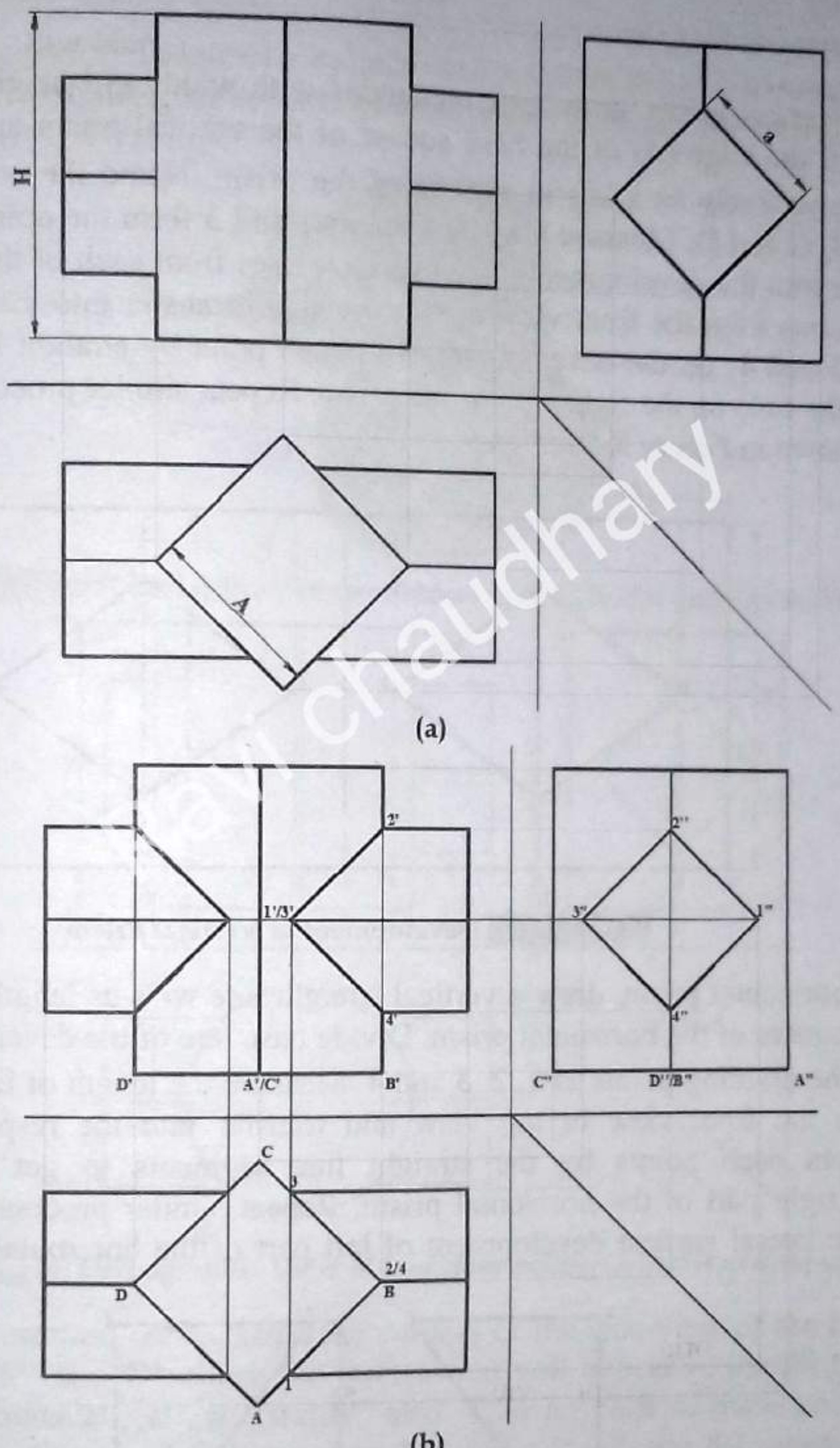


Figure 9.6: Orthographic View of two Intersecting Square Prisms

To complete the intersection curve, name the corner points of the side view of the horizontal prism as **1''**, **2''**, **3''** and **4''** as the side view of the points on the intersection curve. Transfer each of these points into the top view and mark the top view of each point as **1**, **2**, **3** and **4** respectively where the projection lines intersect the edge of the top view (square) of the vertical prism. Draw vertical projection lines through top view of each points and horizontal projection lines through each points on the side view to get the front view **1'**, **2'**, **3'** and **4'** of each point. Join the points thus obtained by straight line segments to get the front view of the intersection curve as shown in *Figure 9.6 (b)*.

Development

To develop the vertical prism, draw four rectangles with width and height of each rectangle equal to length of the edge (A) of the base square of the vertical prism and height (H) of the vertical prism respectively as a lateral surface of the prism. Name the corner points of each rectangle as **A**, **B**, **C** and **D**. Measure length of points **1** and **3** form the point **B (2/4)** on the top view and transfer into the development. Draw vertical lines from each of these points. Measure height of each points from the front view or side view and transfer into the development to get the points **1**, **2**, **3** and **4** on the development. Join each point by straight line segments to get development of the hole on the right side of the prism. Repeat similar process to get the hole of the left side as shown in *Figure 9.6(c)*.

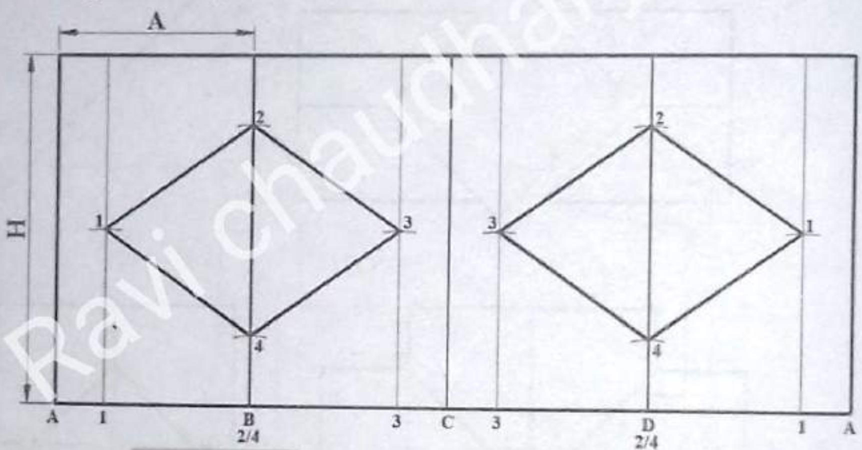


Figure 9.6(c): Development of Vertical Prism

To develop the horizontal prism, draw a vertical straight line with its length equal to perimeter ($4a$) of the base square of the horizontal prism. Divide base line of the development into 4 equal parts and name the dividing points as **1**, **2**, **3** and **4**. Measure the length of lines passing through each point from the front view or top view and transfer into the respective lines on the development. Join each points by the straight line segments to get the lateral surface development of right part of the horizontal prism. Repeat similar process to get symmetrical pattern to get the lateral surface development of left part of the horizontal prism as shown in *Figure 9.6(d)*.

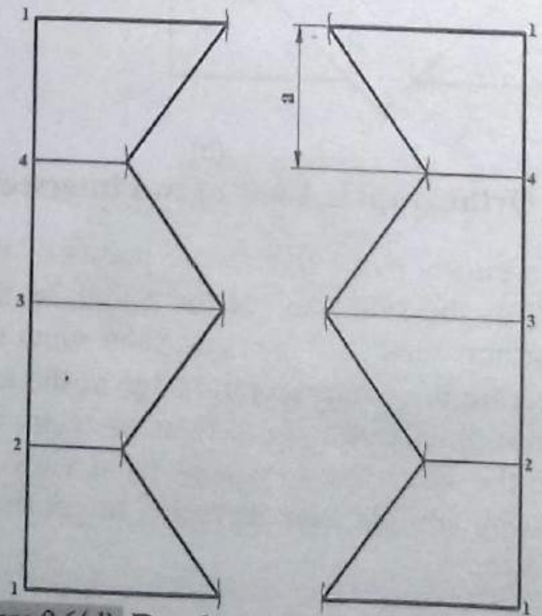


Figure 9.6(d): Development of Horizontal Prism

9.6 Intersection of Cylinder and Prism

Figure 9.7 shows the pictorial view of a cylinder and a square prism intersecting each other at right angle. The corresponding three orthographic views without the intersection curve are shown in *Figure 9.8(a)*.

Figure 9.7: Pictorial View of two Intersecting Cylinder and Square Prism

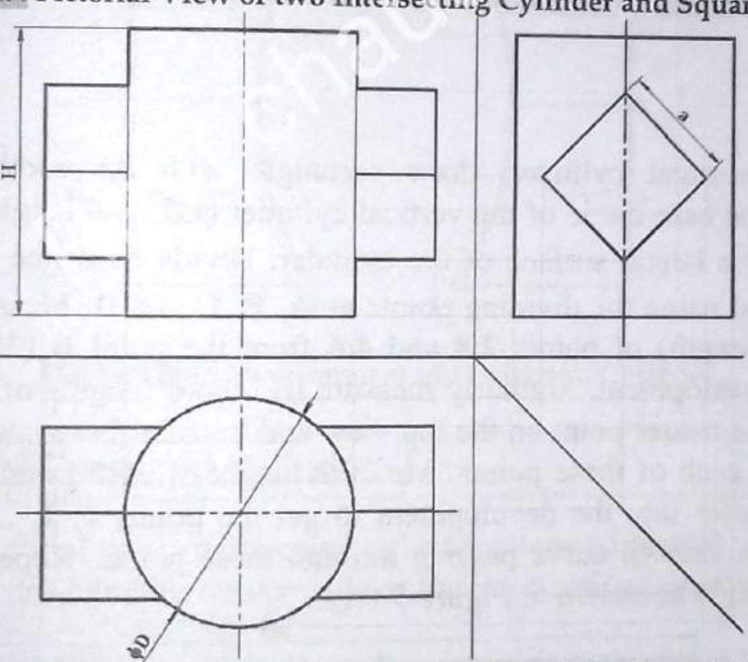


Figure 9.8: (a) Orthographic View of two Intersecting Cylinder and Square Prism

To complete the intersection curve, name the corners of the side view of the horizontal prism (square) as **1'', 3'', 5''** and **7''**. Since the intersection will appear in the form of curve, take some intermediate points **2'', 4'', 6''** and **8''** also. Transfer each of these points into the top view and mark the top view of each point as **1, 2, ………, 7** and **8** respectively where the projection lines intersect the circumference of the top view (circle) of the vertical cylinder. Draw vertical projection lines through top view of each points and horizontal projection lines through each points on the side view to get the front view as **1', 2', ………, 7'** and **8'** of each point. Join the points thus obtained by a smooth curve to get the front view of the intersection curve as shown in *Figure 9.8 (b)*.

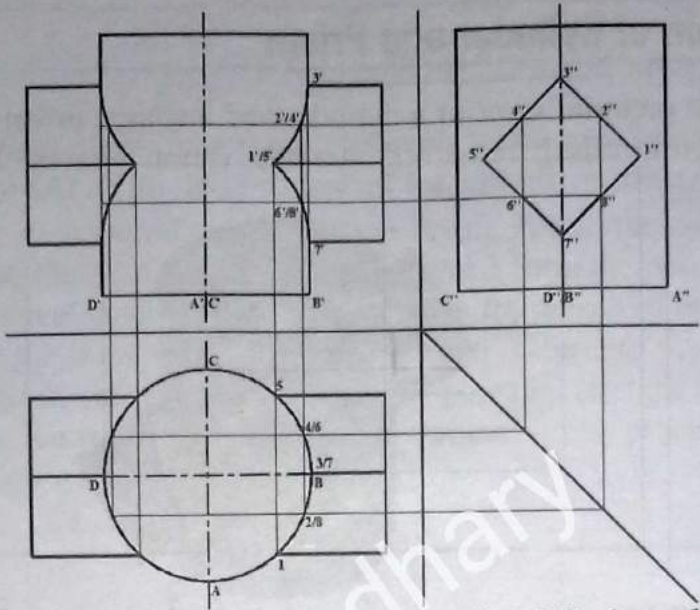


Figure 9.8 (b): Orthographic View of two Intersecting Cylinder and Square Prism

Development

To develop the vertical cylinder, draw rectangle with its width and height equal to circumference of the base circle of the vertical cylinder (πD) and height of the vertical cylinder (H) respectively as a lateral surface of the cylinder. Divide base line of the development into four equal parts and name the dividing points as **A**, **B**, **C** and **D**. Measure chord length (which approximates arc length) of points **2/8** and **4/6** from the point **B (3/7)** on the top view and transfer into the development. Similarly measure the chord lengths of the remaining points on the **1** and **5** from the nearer point on the top view and transfer them into the development. Draw vertical lines from each of these points. Measure height of each points from the front view or side view and transfer into the development to get the points **1**, **2**, ..., **7** and **8** on the development. Draw smooth curve passing through these points. Repeat similar process to get the hole of the left side as shown in *Figure 9.8(c)*.

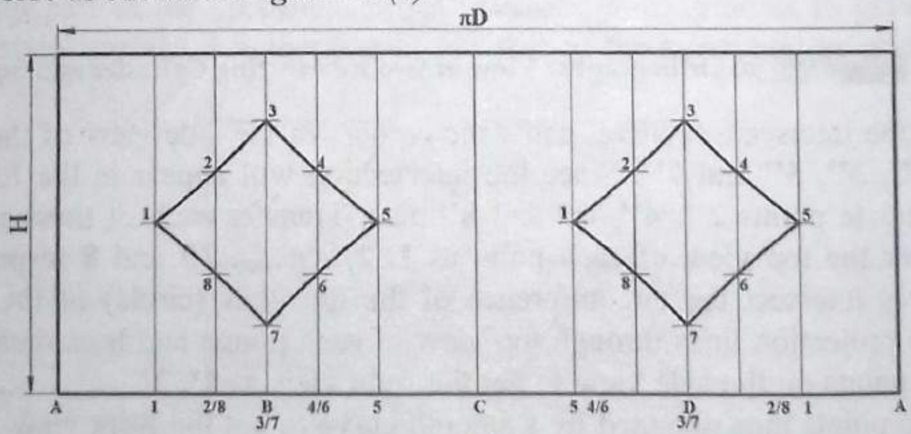


Figure 9.8(c): Development of Vertical Cylinder

To develop the horizontal prism, draw a vertical straight line with its length equal to perimeter ($4a$) of the base square of the horizontal prism. Divide base line of the development into 4 equal parts and name the dividing points as **1**, **3**, **5** and 7. Measure intermediate points **2**, **4**, **6** and **8** and transfer them into the development. Measure the length of lines passing through each point from the front view or top view and transfer into the respective lines on the development. Join each point by a smooth curve to get the lateral surface development of right part of the horizontal prism. Repeat similar process to get symmetrical pattern to get the lateral surface development of left part of the horizontal prism as shown in *Figure 9.8(d)*.

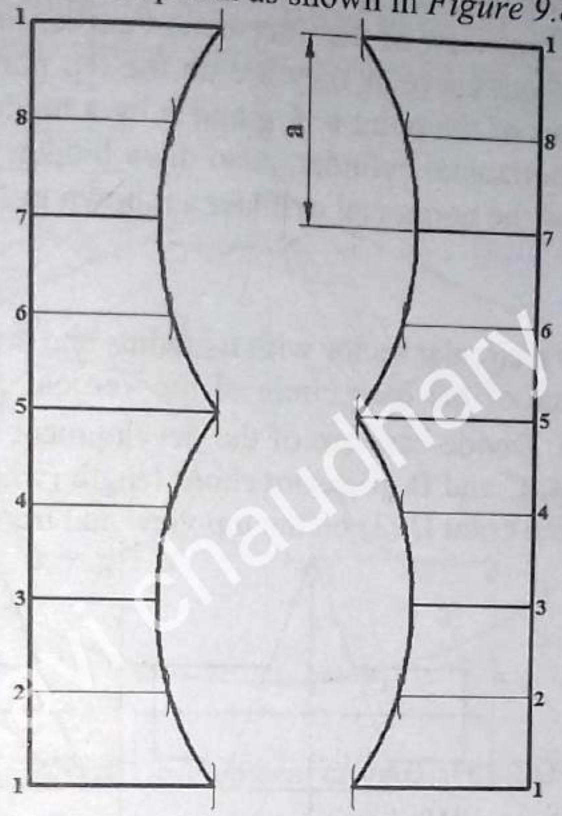


Figure 9.8(d): Development of Horizontal Cylinder

9.7 Intersection of Cone and Cylinder

Figure 9.9 shows the pictorial view of cone and cylinder intersecting each other at right angle. The corresponding three orthographic views without the intersection curve are shown in *Figure 9.10(a)*.

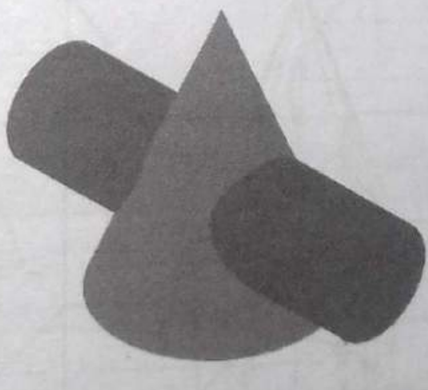


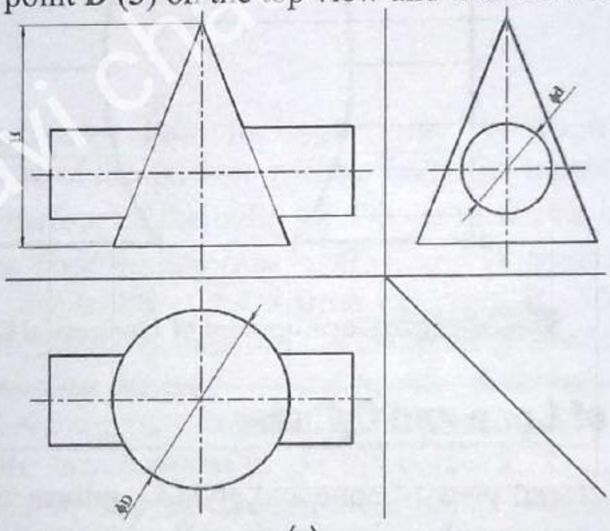
Figure 9.9: Pictorial View of two Intersecting Cone and Cylinder

To complete the intersection curve, draw straight lines passing thorough the side view v'' of the vertex and tangent to the circle on the side view which intersect the edge view of the base circle at points **1''** and **5''**. Transfer these points **1''** and **5''** on both top and front view and complete the top views and side views of the generators **V1** and **V5**. Mark any number of points (**2, 3** and **4**) between points **1** and **5** on the circumference of circle on the top view. Transfer these points

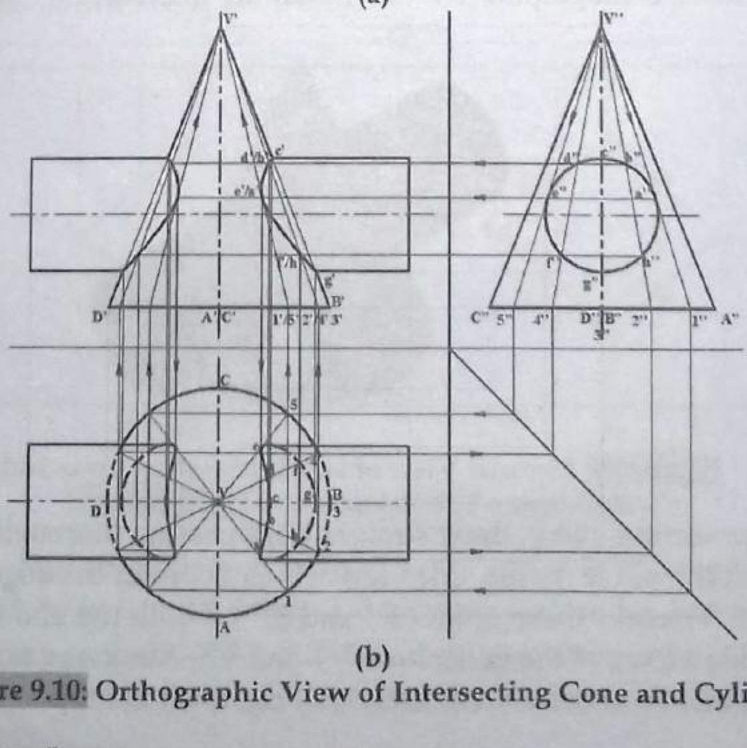
(**2, 3** and **4**) to both the front view and side view and complete the front views and side views of the generators **V2** and **V4**. Name the points of intersection of the side views of the generators and the circle on the side view as **a''**, **b''**, ………., **g''** and **h''**. Transfer all these points to both the front views and top views of all theses points. Join font views (**a'**, **b'**, ………., **g'** and **h'**) by a smooth curve to get the front view of the intersection curve. Join the top views of the point **a**, **b**, **c** and **d** by a visible smooth curve as they are on the top portion of the horizontal cylinder. Similarly, join the top views of the point **e**, **f**, **g** and **h** by a hidden smooth curve as they are on the bottom portion of the horizontal cylinder. Also draw hidden arc for the part of the circle on the top view which is below the horizontal cylinder as shown in *Figure 9.10 (b)*.

Development

To develop the cone, draw a circular sector with its radius and arc length equal to slant height of the cone and circumference of the base circle of the vertical cylinder (πD) respectively as a lateral surface of the cone. Divide base arc of the development into four equal parts and name the dividing points as **A**, **B**, **C** and **D**. Measure chord length (which approximates arc length) of points **2**, **1** and **4**, **5** form the point **D (3)** on the top view and transfer into the development.



(a)



(b)

Figure 9.10: Orthographic View of Intersecting Cone and Cylinder

Joint all these points 1, 2, 3 4 and 5 with the vertex V on the development. Measure true lengths of line segments (1'a', 2'b', 3'c', ..., 3'g' and 2'h') along the end generator and transfer it on the respective lines to get the points (a, b, ..., g and h) on the development. Draw smooth curve passing through these points. Repeat similar process to get the hole of the left side as shown in *Figure 9.10(c).*

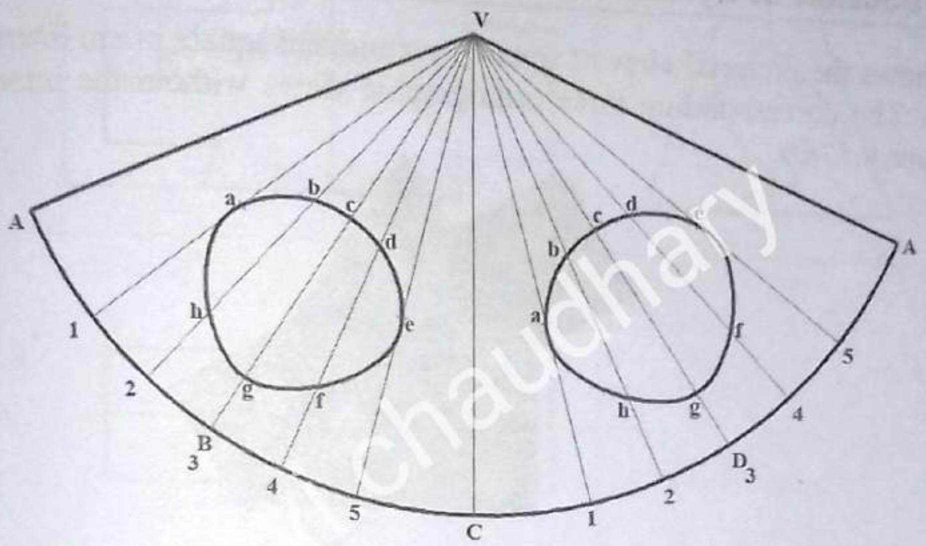


Figure 9.10(c): Development of Vertical Cylinder

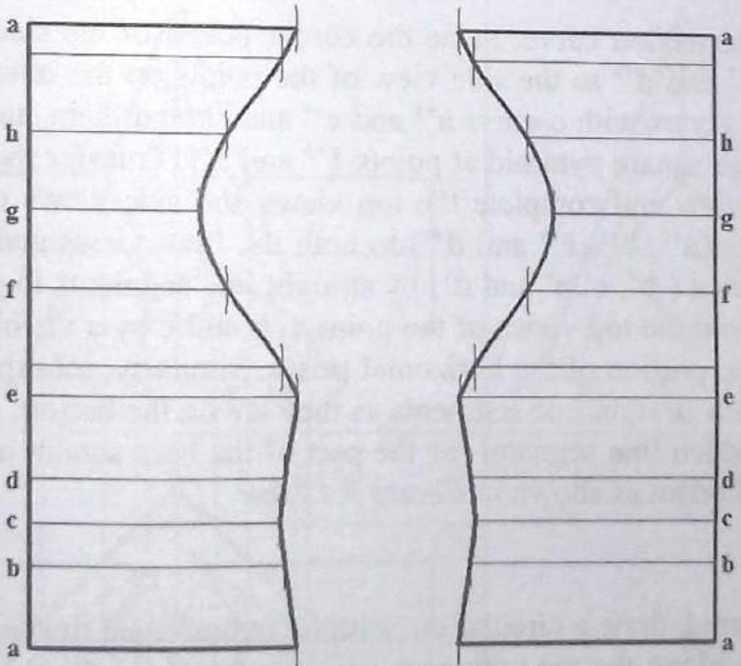


Figure 9.10(d): Development of Horizontal Cylinder

To develop the horizontal cylinder, draw a vertical straight line with its length equal to circumference of the base circle of the horizontal cylinder. Measure relative distance between each point (chord lengths which approximate arc length between each point) (a, b, ..., g and h) on the side view and transfer them along the base line on the development. To minimize error, also transfer the end points of horizontal diameter of the circle on the development. Measure the length of lines passing through each point from the front view or top view and transfer into the respective lines on the development. Draw smooth curve passing through these points to get the lateral surface development of right part of the horizontal cylinder. Repeat similar process to get symmetrical pattern to get the lateral surface development of left part of the horizontal cylinder as shown in *Figure 9.10(d)*.

9.8 Intersection of Pyramid and Prism

Figure 9.11 shows the pictorial view of square pyramid and square prism intersecting each other at right angle. The corresponding three orthographic views without the intersection curve are shown in *Figure 9.12(a)*.

Figure 9.11: Pictorial View of two Intersecting Square Pyramid and Square Prism

To complete the intersection curve, name the corner points of the side view of the horizontal prism as **a''**, **b''**, **c''** and **d''** as the side view of the points on the intersection curve. Join the side view **v''** of the vertex with corners **a''** and **c''** and extend them such that they intersect the edge view of the base square pyramid at points **1''** and **3''**. Transfer these points **1''** and **3''** on both top and front view and complete the top views and side views of the lines **V1** and **V3**. Transfer these points (**a''**, **b''**, **c''** and **d''**) to both the front views and top views of all theses points. Join front views (**b'**, **c'/a'** and **d'**) by straight line segments to get the front view of the intersection curve. Join the top views of the point **a**, **b** and **c** by a visible straight line segments as they are on the top portion of the horizontal prism. Similarly, join the top views of the point **a**, **d** and **b** by a hidden straight line segments as they are on the bottom portion of the horizontal prism. Also draw hidden line segment for the part of the base square on the top view which is below the horizontal prism as shown in *Figure 9.12 (b)*.

Development

To develop the pyramid, draw a circular arc with its radius equal to slant height of the pyramid. Draw four segments along the arc with each segment having a chord length equal to length of edge of the base square (*A*). Name the ends of each chord on the development as **A**, **B**, **C** and **D**. Measure distance of points **1** and **2** form the point **D** (3) on the top view and transfer into the development. Joint all these points **1**, **2** and **3** with the vertex **V** on the development. Inclined edge **V3** appears as true length on the front view. Draw true length ($v'1'_{T}/2'_{T}$) of the line **V1/V2**. Measure true lengths of line segments **3'b'** and **3'd'** along the front view of the inclined edge **v'3'** and transfer it along the line **V3** on the development. Similarly, measure true lengths of line segments **1'a'**and **2'c'** along the true length line $v'1'_{T}/2'_{T}$ and transfer them on the respective lines to get the points **a** and **c** on the development. Join each point by straight line segments to get development of the hole on the right side of the prism. Repeat similar process to get the hole of the left side as shown in *Figure 9.12(c)*.

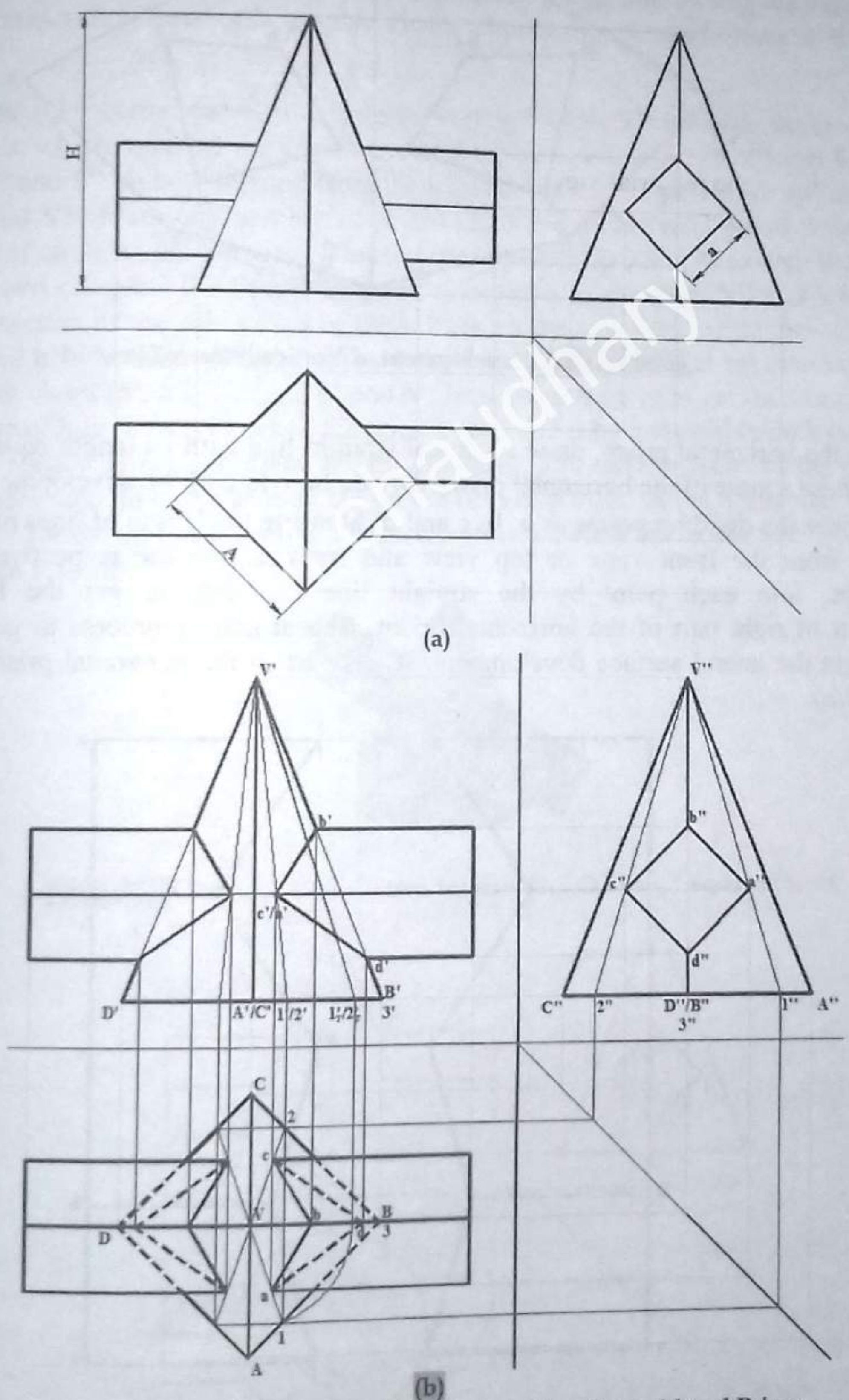


Figure 9.12: Orthographic View of Intersecting Pyramid and Prism

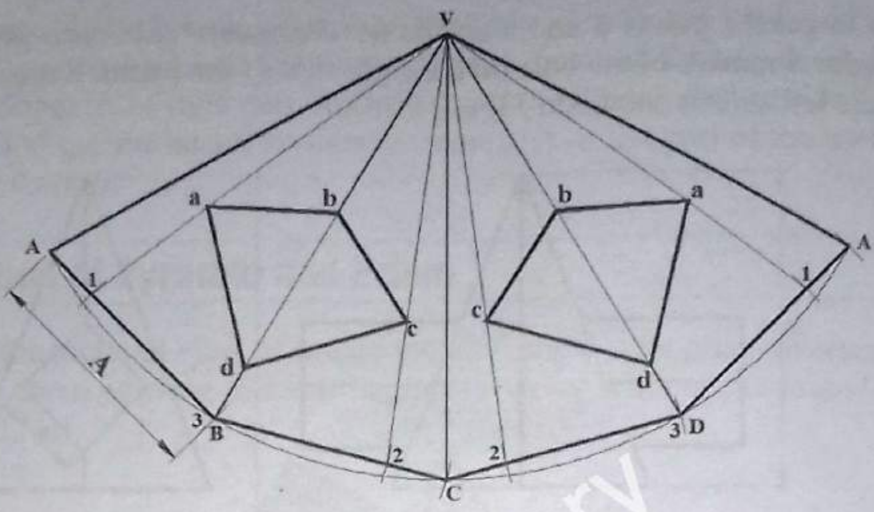


Figure 9.12(c): Development of Vertical Square Pyramid

To develop the horizontal prism, draw a vertical straight line with its length equal to perimeter (4a) of the base square of the horizontal prism. Divide base line of the development into 4 equal parts and name the dividing points as **a**, **b**, **c** and **d**. Measure the length of lines passing through each point from the front view or top view and transfer into the respective lines on the development. Join each point by the straight line segments to get the lateral surface development of right part of the horizontal prism. Repeat similar process to get symmetrical pattern to get the lateral surface development of left part of the horizontal prism as shown in *Figure 9.12(d)*.

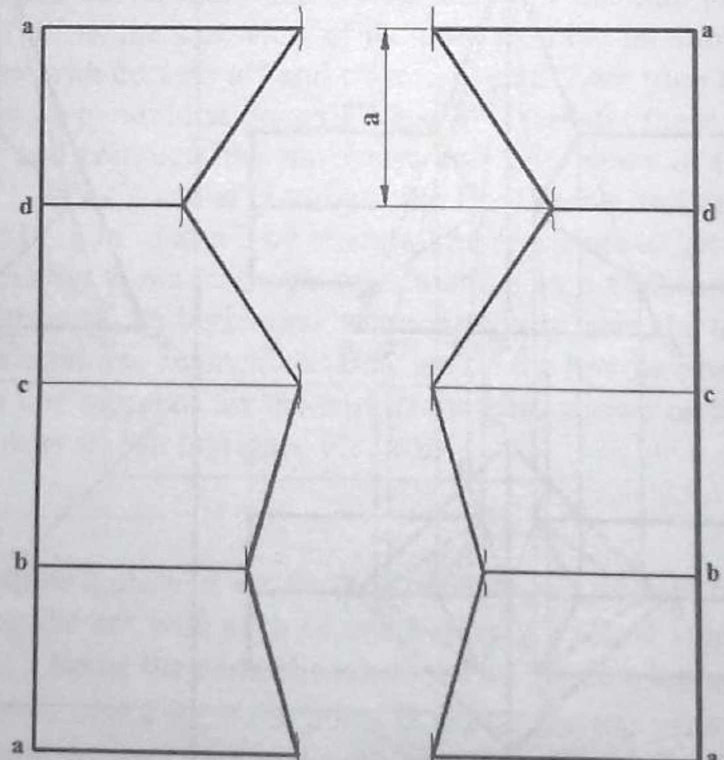


Figure 9.12(d): Development of Horizontal Square Prism

9.9 Intersection of Cone and Prism

Figure 9.13 shows the pictorial view of cone and square prism intersecting each other at right angle. The corresponding three orthographic views without the intersection curve are shown in *Figure 9.14(a)*.

To complete the intersection curve, join the side view **v''** of the vertex and corners **a''** and **e''** on the side view which intersect the edge view of the base circle at points **1''** and **5''**. Transfer these points **1''** and **5''** on both top and front view and complete the top views and side views of the lines **V1** and **V5**. Mark any number of points **(2, 3** and **4)** between points **1** and **5** on the circumference of circle on the top view. Transfer these points (**2, 3** and **4**) to both the front view and side view and complete the front views and side views of the sides **V2** and **V4**. Name the points of intersection of the side views of these lines on the side view of the prism as **a''**, **b''**, ..., **g''** and **h''**. Transfer all these points to both the front views and top views of all theses points. Join font views (**a'**, **b'**, ..., **g'** and **h'**) by a smooth curve to get the front view of the intersection curve. Join the top views of the point **a**, **b**, **c** and **d** by a visible smooth curve as they are on the top portion of the horizontal prism. Similarly, join the top views of the point **e**, **f**, **g** and **h** by a hidden smooth curve as they are on the bottom portion of the horizontal prism. Also draw hidden arc for the part of the circle on the top view which is below the horizontal prism as shown in *Figure 9.14 (b)*.

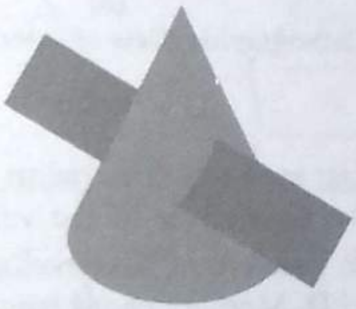
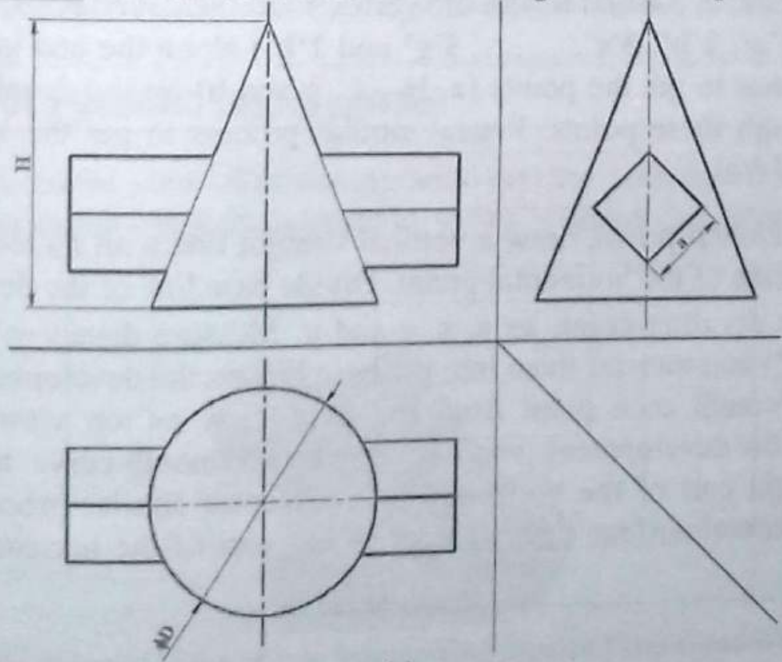
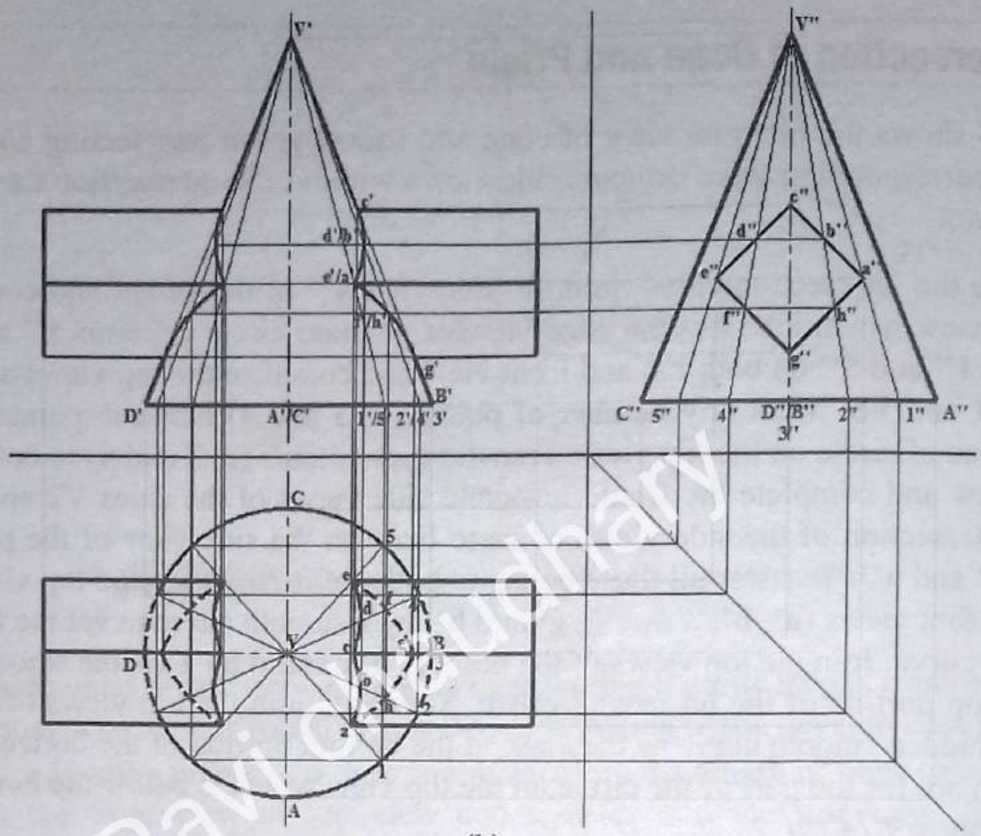


Figure 9.13: Pictorial View of two Intersecting Cone and Square Prism



(a)



(b)

Figure 9.14: Orthographic View of Intersecting Cone and Prism

Development

To develop the cone, draw a circular sector with its radius and arc length equal to slant height of the cone and circumference of the base circle of the vertical cylinder (πD) respectively as a lateral surface of the cone. Divide base arc of the development into four equal parts and name the dividing points as **A**, **B**, **C** and **D**. Measure chord length (which approximates arc length) of points **2**, **1** and **4**, **5** form the point **D** (**3**) on the top view and transfer into the development. Joint all these points **1**, **2**, **3 4** and **5** with the vertex **V** on the development. Measure true lengths of line segments (**1'a'**, **2'b'**, **3'c'**, ……., **3'g'** and **2'h'**) along the end generator and transfer it on the respective lines to get the points (**a**, **b**, …., **g** and **h**) on the development. Draw smooth curve passing through these points. Repeat similar process to get the hole of the left side as shown in *Figure 9.14(c).*

To develop the horizontal prism, draw a vertical straight line with its length equal to perimeter (**4a**) of the base square of the horizontal prism. Divide base line of the development into 4 equal parts and name the dividing points as **a**, **c**, **e** and **g**. Measure distances of intermediate points (**b''**, **d''**, **f''** and **h''**) and transfer them into the base line on the development Measure the length of lines passing through each point from the front view or top view and transfer into the respective lines on the development. Join each point by a smooth curve to get the lateral surface development of right part of the horizontal prism. Repeat similar process to get symmetrical pattern to get the lateral surface development of left part of the horizontal prism as shown in *Figure 9.14(d).*

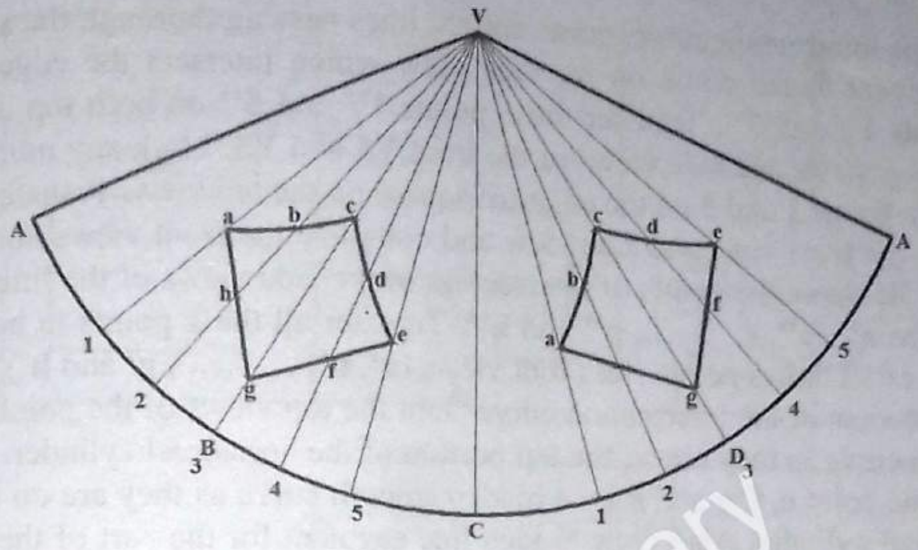


Figure 9.14(c): Development of Vertical Cone

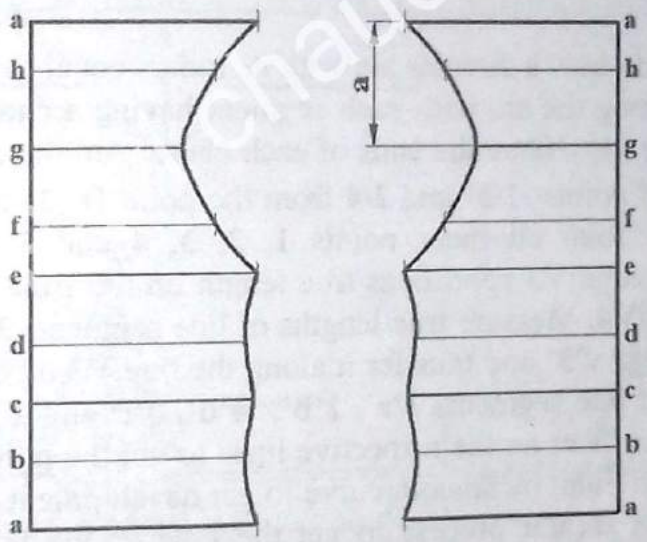


Figure 9.14(d): Development of Horizontal Square Prism

9.10 Intersection of Pyramid and Cylinder

Figure 9.15 shows the pictorial view of square pyramid and cylinder intersecting each other at right angle. The corresponding three orthographic views without the intersection curve are shown in *Figure 9.16(a)*.

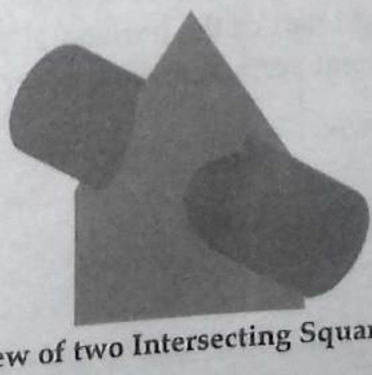


Figure 9.15: Pictorial View of two Intersecting Square Pyramid and Cylinder

To complete the intersection curve, draw straight lines passing thorough the side view **v''** of the vertex and tangent to the circle on the side view which intersect the edge view of the base square at points **1''** and **5''**. Transfer these points **1''** and **5''** on both top and front view and complete the top views and side views of the lines **V1** and **V5**. Mark any number of points (**2, 3** and **4**) between points **1** and **5** on the edges of square on the top view. Transfer these points (**2, 3** and **4**) to both the front view and side view and complete the front views and side views of the lines **V2** and **V4**. Name the points of intersection of the side views of the lines and the circle on the side view as **a''**, **b''**, ………, **g''** and **h''**. Transfer all these points to both the front views and top views of all theses points. Join font views (**a'**, **b'**, ………, **g'** and **h'**) by a smooth curve to get the front view of the intersection curve. Join the top views of the point **a**, **b**, **c** and **d** by a visible smooth curve as they are on the top portion of the horizontal cylinder. Similarly, join the top views of the point **e**, **f**, **g** and **h** by a hidden smooth curve as they are on the bottom portion of the horizontal cylinder. Also draw hidden line segment for the part of the square on the top view which is below the horizontal cylinder as shown in *Figure 9.14 (b)*.

Development

To develop the pyramid, draw a circular arc with its radius equal to slant height of the pyramid. Draw four segments along the arc with each segment having a chord length equal to length of edge of the base square (*A*). Name the ends of each chords on the development as **A**, **B**, **C** and **D**. Measure distance of points **1/5** and **2/4** from the point **D** (3) on the top view and transfer into the development. Joint all these points **1**, **2**, **3**, **4** and **5** with the vertex **V** on the development. Inclined edge **V3** appears as true length on the front view. Draw true lengths of the line **V1/V5** and **V2/V4.** Measure true lengths of line segments **3'g'** and **3'c'** along the front view of the inclined edge **v'3'** and transfer it along the line **V3** on the development. Similarly, measure true lengths of line segments **1'a'**, **2'b'**, **4'd'**, **4'f'** and **2'h'** along the respective true length lines and transfer them on the respective lines to get the points **a b**, **d**, **e**, **f**, and **c** on the development. Join each point by smooth curve to get development of the hole on the right side of the pyramid. Repeat similar process to get the hole of the left side as shown in *Figure 9.16(c)*.

To develop the horizontal cylinder, draw a vertical straight line with its length equal to circumference of the base circle of the horizontal cylinder. Measure relative distance between each point (chord lengths which approximate arc length between each point) (**a**, **b**, …., **g** and **h**) on the side view and transfer them along the base line on the development. To minimize error, also transfer the end points of horizontal diameter of the circle on the development. Measure the length of lines passing through each point from the front view or top view and transfer into the respective lines on the development. Draw smooth curve passing through these points to get the lateral surface development of right part of the horizontal cylinder. Repeat similar process to get symmetrical pattern to get the lateral surface development of left part of the horizontal cylinder as shown in *Figure 9.16(d)*.

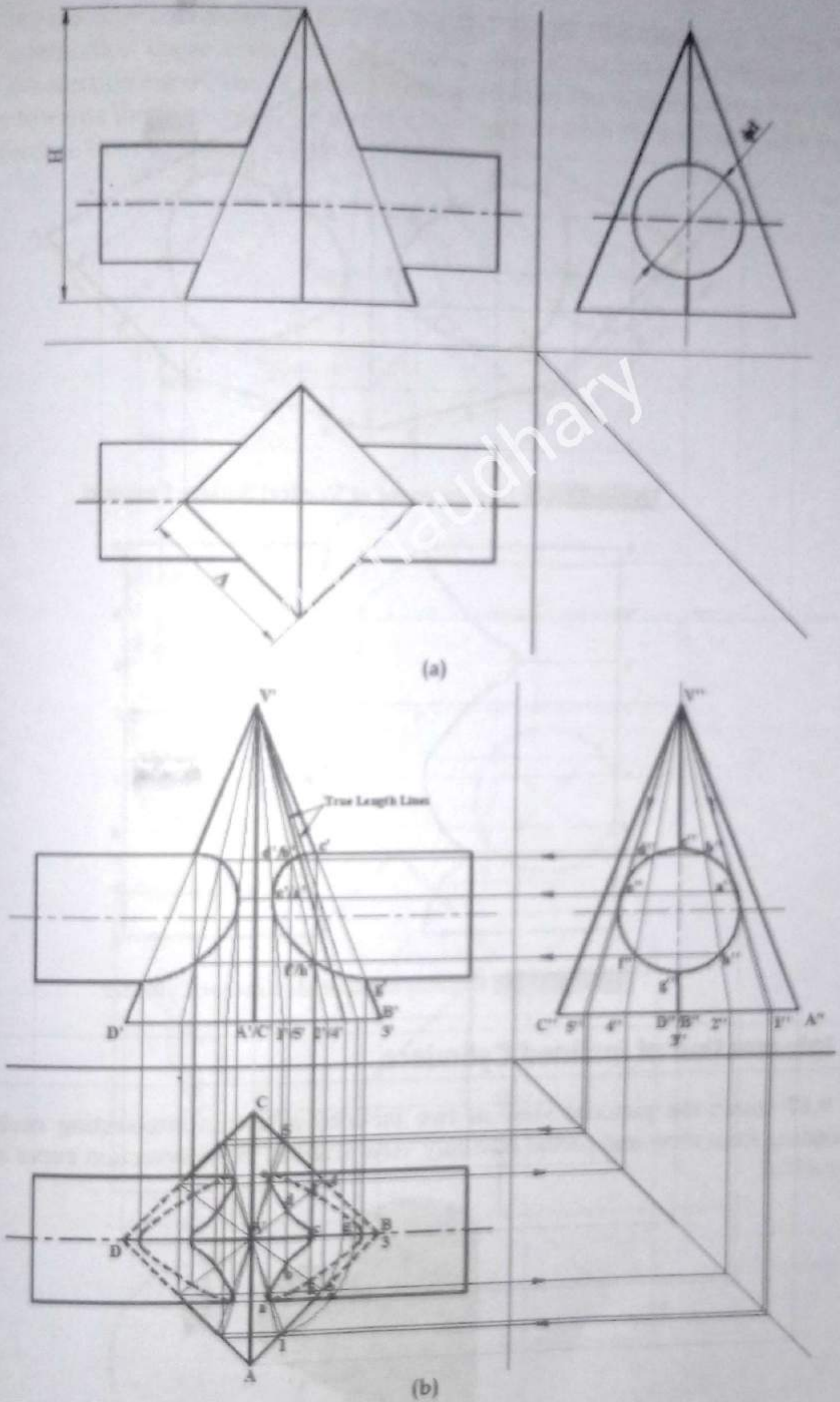


Figure 9.16: Orthographic View of Intersecting Square Pyramid and Cylinder

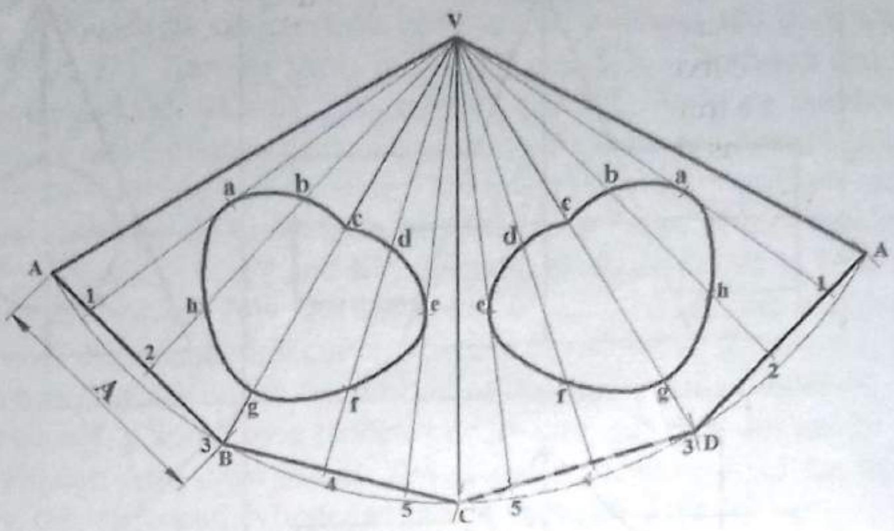


Figure 9.16(c): Development of Vertical Square Pyramid

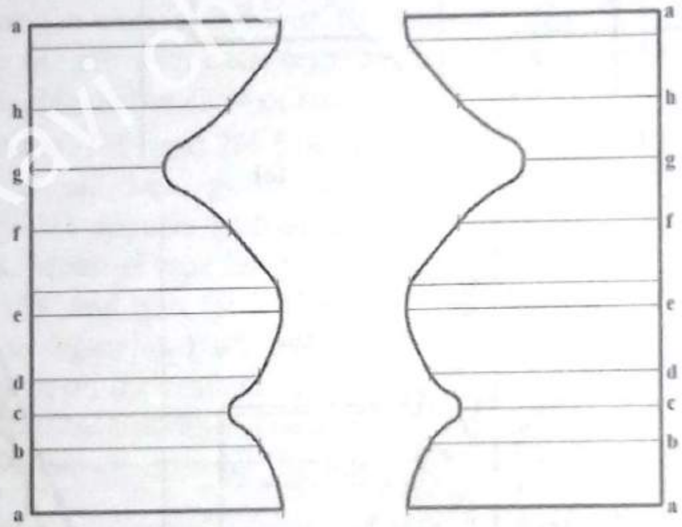


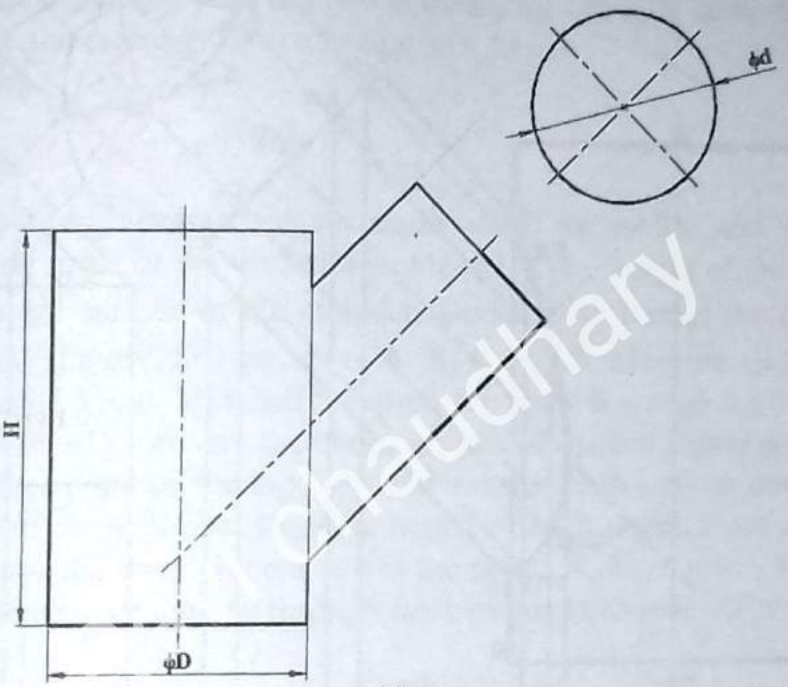
Figure 9.16(d): Development of Horizontal Cylinder

9.11 Intersection of Inclined Cylinders

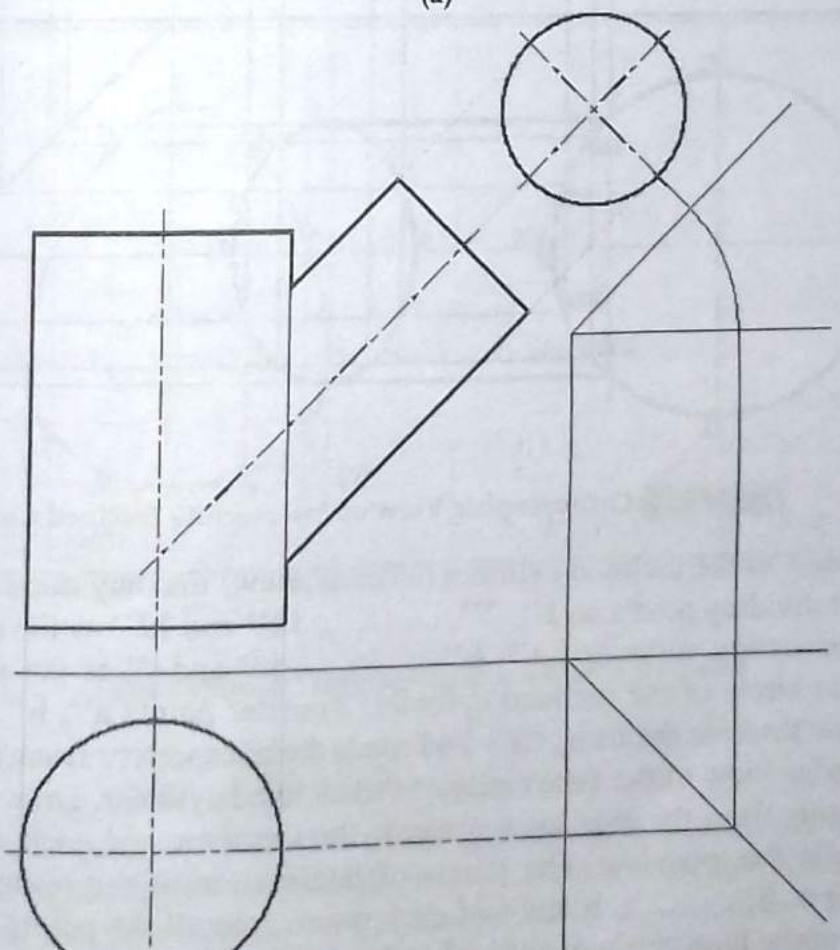
Figure 9.17 shows the pictorial view of two inclined cylinders intersecting each other. The corresponding front view and partial auxiliary view without the intersection curve are shown in *Figure 9.18(a)*.

Figure 9.17: Pictorial View of Intersecting Inclined Cylinders

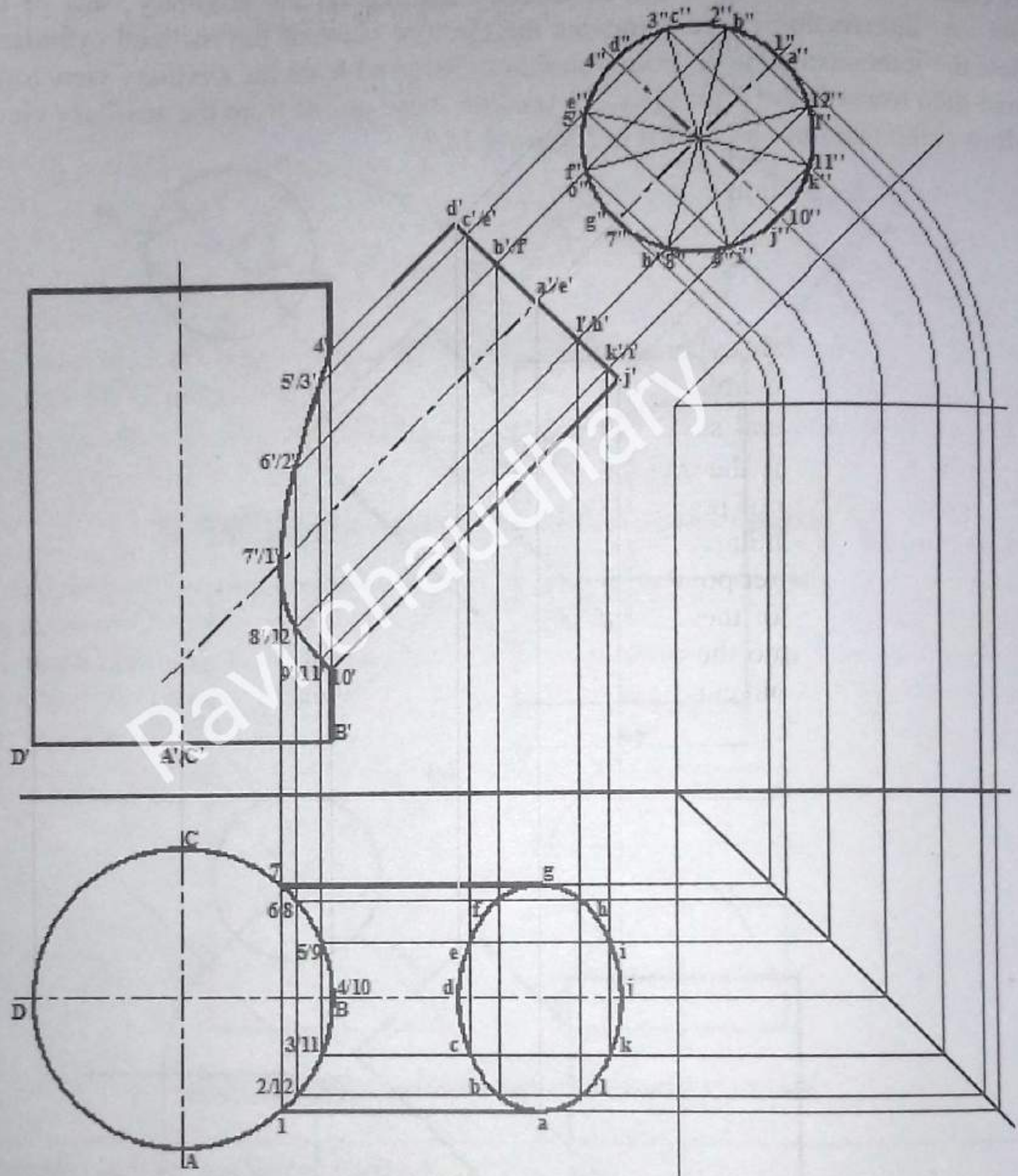
In this case, intersection curve can be directly marked on the auxiliary view of the inclined cylinder i.e., intersection curve coincides the circular view of the inclined cylinder. Hence, to complete the intersection curve, points should transferred from the auxiliary view toward the top view and then towards the front view. To transfer these points from the auxiliary view to the top view draw reference lines as shown in *Figure 9.18(b)*.



(a)



(b)



(c)

Figure 9.16: Orthographic View of Intersecting Inclined Cylinders

Divide the side view of the inclined cylinder (circular view) into any number of equal parts (say 12) and name the dividing points as **1''**, **2''**, ..., **11''** and **12''** as the auxiliary views of the points on the intersection curve and **a''**, **b''**, ..., **k''** and **l''** as the auxiliary views of the points on the base circle of the inclined cylinder. Transfer points **a''**, **b''**, ..., **k''** and **l''** from the side view towards the front view and mark their respective front views **a'**, **b'**, ..., **k'** and **l'** on the edge view of the base circle of the inclined cylinder. Draw projection lines from each of these points from the side view towards the top view and each of these points on the front view towards the top view. The points of intersection of the respective projection lines give the top view **a**, **b**, ..., **k** and **l** of each point. Join all the points by a smooth curve to get the top view of the base circle of the inclined cylinder.

Similarly, transfer each of the points 1'', 2'', ………., 11'' and 12'' from the auxiliary view into the top view and mark the top view of each point as 1, 2, ………., 11 and 12 respectively where the projection lines intersect the circumference of the top view (circle) of the vertical cylinder. Draw vertical projection lines through top view of each points and horizontal projection lines through each points on the auxiliary view to get the front view as 1', 2', ………., 11' and 12' of each point. Join the points thus obtained by a smooth curve to get the front view of the intersection curve as shown in *Figure 9.18 (c)*.

Development

To develop the vertical cylinder, draw rectangle with its width and height equal to circumference of the base circle of the vertical cylinder (πD) and height of the vertical cylinder (H) respectively as a lateral surface of the cylinder. Divide base line of the development into four equal parts and name the dividing points as **A, B, C** and **D**. Measure chord length (which approximates arc length) of points **3/11** and **5/9** from the point **B** on the top view and transfer into the development. Similarly measure the chord lengths of the remaining points on the **2/12**, **6/8**, **1** and **7** from the nearer point on the top view and transfer them into the development. Draw vertical lines from each of these points. Measure height of each points from the front view or side view and transfer into the development to get the points **1**, **2**, ………., **11** and **12** on the development. Draw smooth curve passing through these points as shown in *Figure 9.18(d)*.

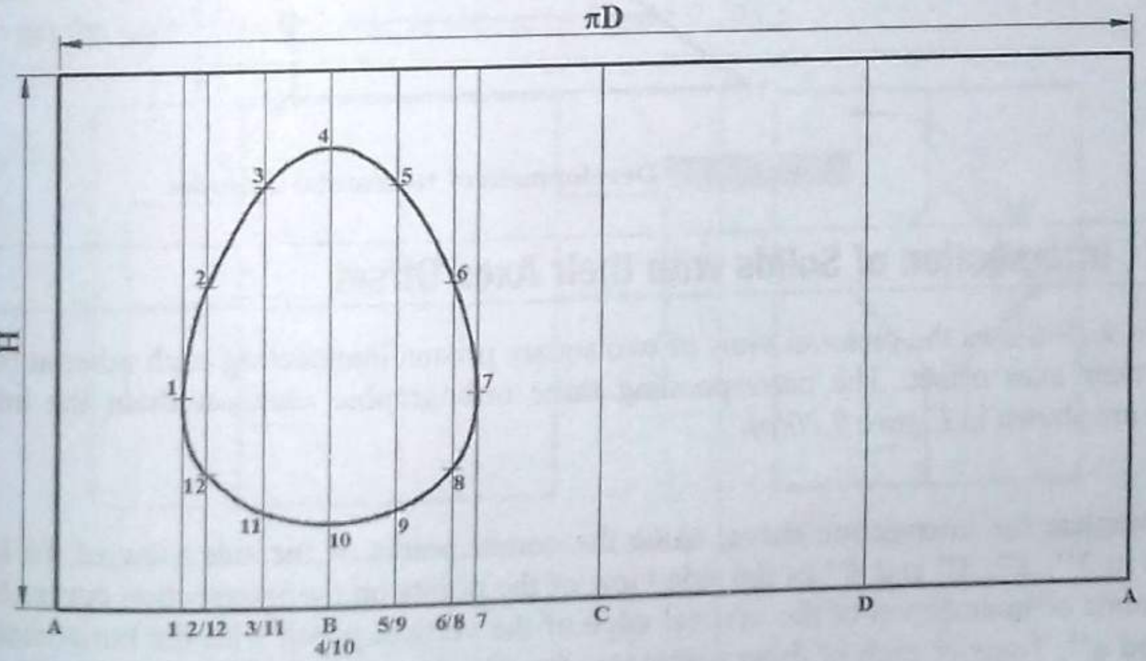


Figure 9.18(d): Development of Vertical Cylinder

To develop the inclined cylinder, draw a vertical straight line with its length equal to circumference of the base circle of the horizontal cylinder (πd). Divide base line of the development into **12** equal parts and name the dividing points as **a**, **b**, ………., **k** and **l**. Measure the length of lines passing through each point from the front view or top view and transfer into the respective lines on the development to get the points **1**, **2**, ………., **11** and **12** on the development. Draw smooth curve passing through these points to get the lateral surface development of the inclined cylinder as shown in *Figure 9.18(e)*.

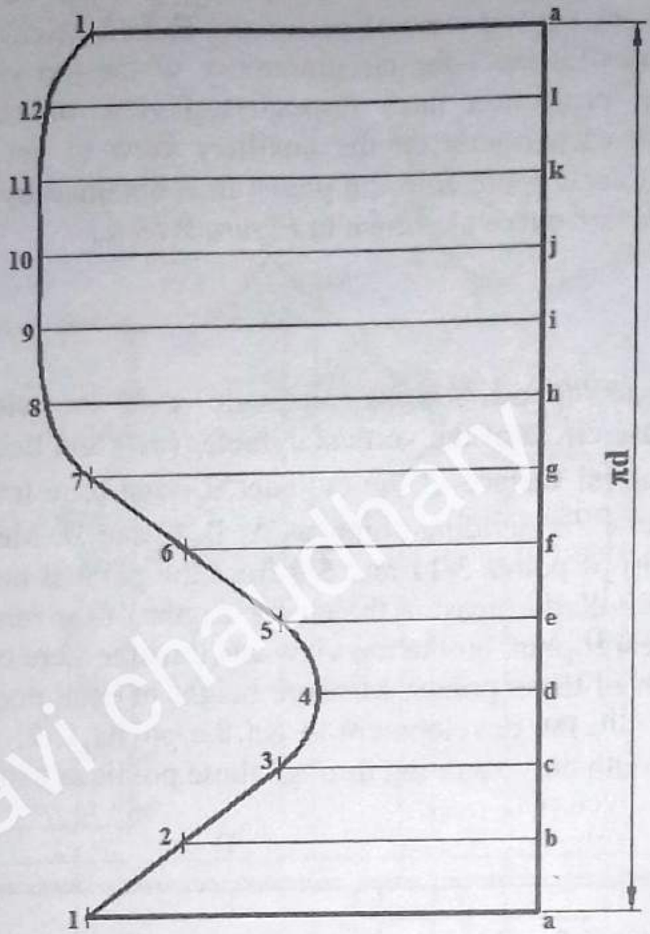


Figure 9.18(d): Development of Horizontal Cylinder

9.12 Intersection of Solids with their Axes Offset

Figure 9.19 shows the pictorial view of two square prisms intersecting each other at right angle with their axes offset. The corresponding three orthographic views without the intersection curve are shown in *Figure 9.20(a)*.

To complete the intersection curve, name the corner points of the side view of the horizontal prism as **1''**, **2''**, **3''** and **4''** as the side view of the points on the intersection curve. Also mark the points of intersection of the vertical edge of the vertical prism with the horizontal prism as **p''** and **q''**. Transfer each of these points into the top view and mark the top view of each point as **1**, **2/4**, **3** and **p/q** respectively where the projection lines intersect the edge of the top view (square) of the vertical prism. Draw vertical projection lines through top view of each points and horizontal projection lines through each points on the side view to get the front view **1'**, **2'**, **3'**, **4'**, **p'** and **q'** of each point. Join the points thus obtained by straight line segments in sequence as named on the side view, i.e., **1' – 2' – p' – 3' – q' – 4' – 1'** to get the front view of the intersection curve. Draw visible lines for the front portion **(2' – 4' – 1')** of the intersection and hidden lines from the rear portion **(2' – p' – 3' – q' – 4')** of the intersection as shown in *Figure 9.20 (b)*.

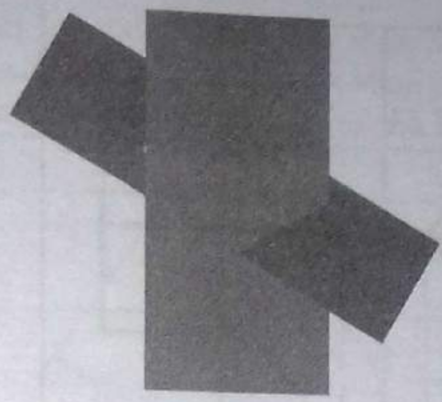
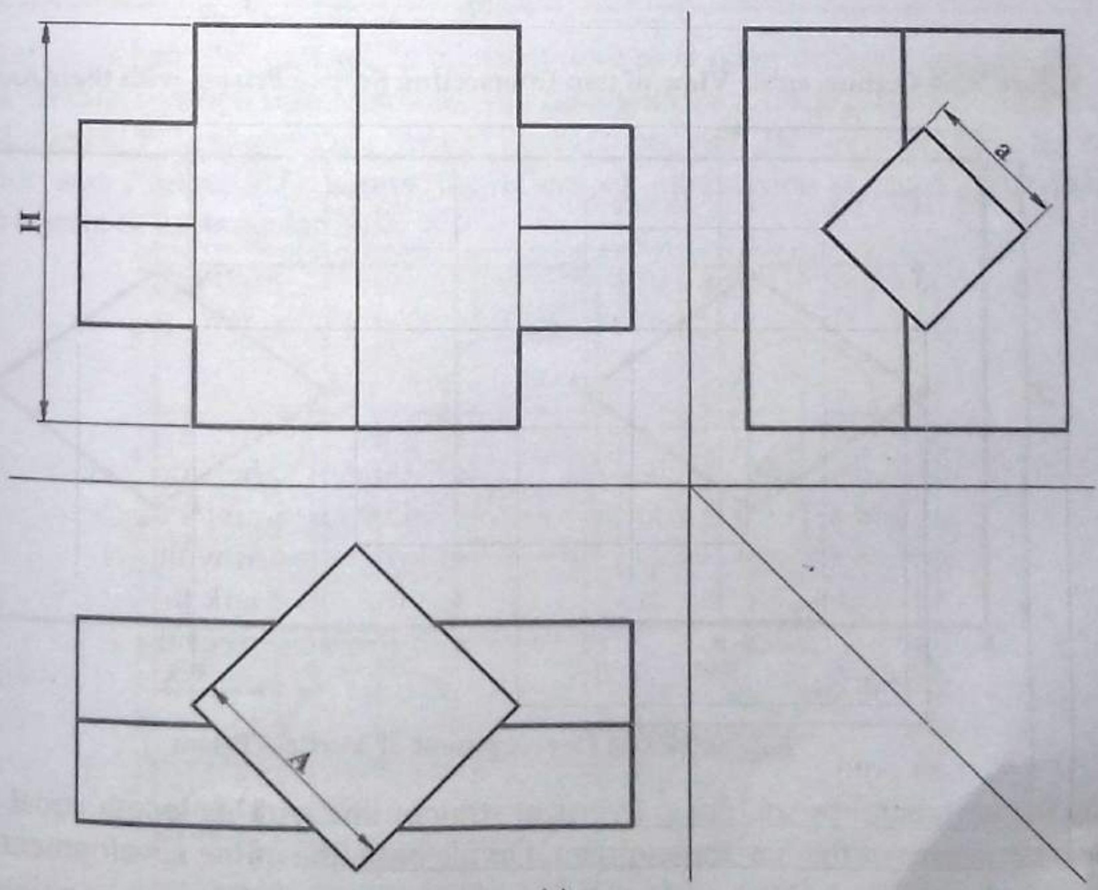


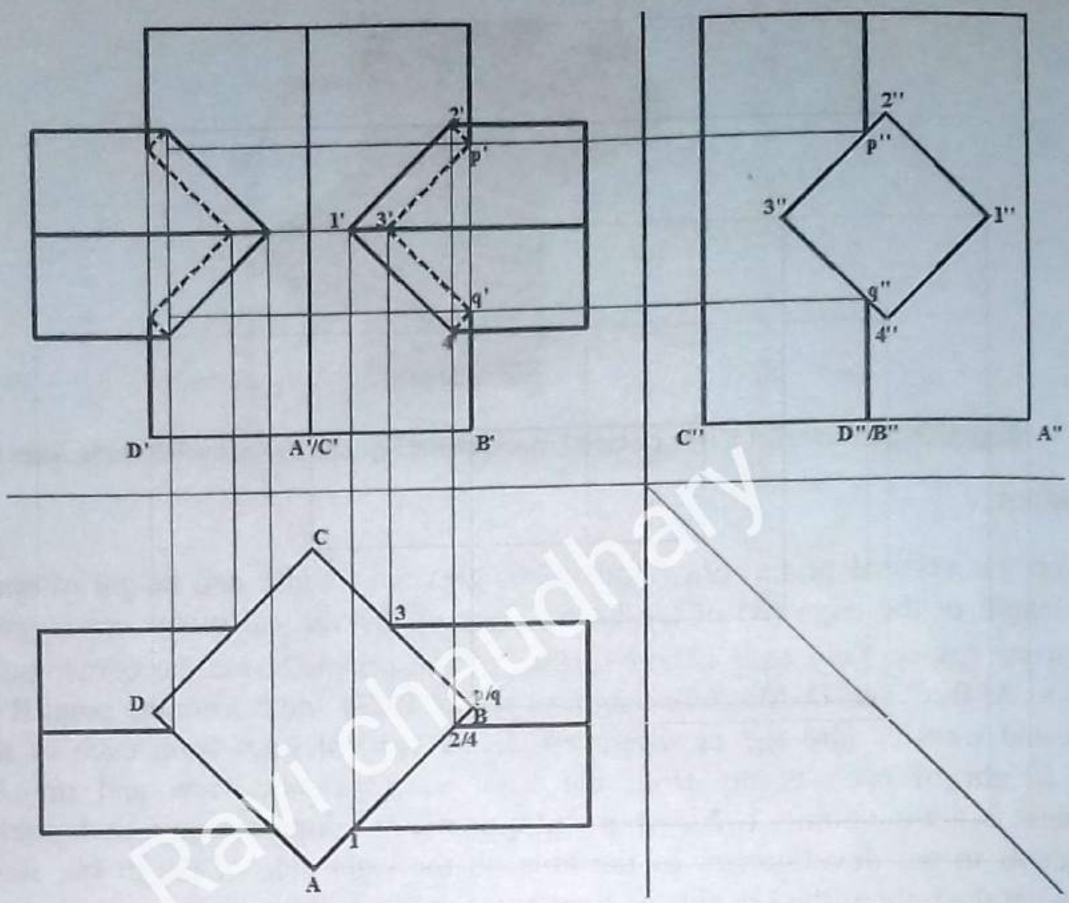
Figure 9.19: Pictorial View of two Intersecting Square Prisms with their Axes Offset

Development

To develop the vertical prism, draw four rectangles with width and height of each rectangle equal to length of the edge (*A*) of the base square of the vertical prism and height (*H*) of the vertical prism respectively as a lateral surface of the prism. Name the corner points of each rectangle as **A**, **B**, **C** and **D**. Measure length of points **1**, **2/4** and **3** form the point **B (p/q)** on the top view and transfer into the development. Draw vertical lines from each of these points. Measure height of each points from the front view or side view and transfer into the development to get the points **1**, **2**, **3**, **4**, **p** and **q** on the development. Join each point by straight line segments to get development of the hole on the right side of the prism. Repeat similar process to get the hole of the left side as shown in *Figure 9.20(c)*.



(a)



(b)

Figure 9.20: Orthographic View of two Intersecting Square Prisms with their Axes Offset

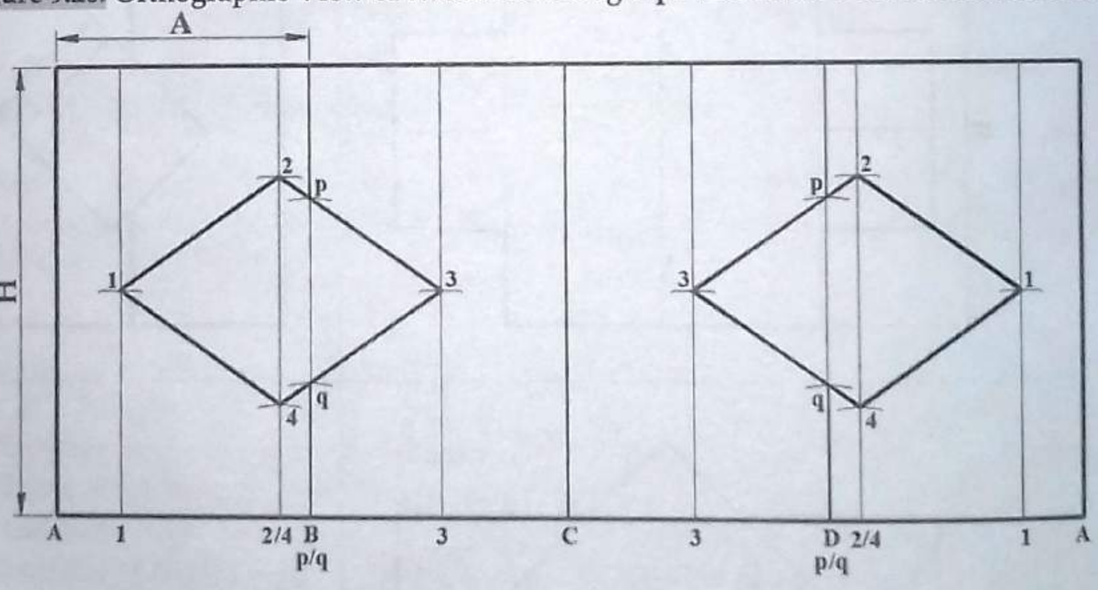


Figure 9.20(c): Development of Vertical Prism

To develop the horizontal prism, draw a vertical straight line with its length equal to perimeter (4*a*) of the base square of the horizontal prism. Divide base line of the development into 4 equal parts and name the dividing points as **1**, **2**, **3** and **4**. Also measure distances of points **p''** and **q''** form the points **2''** and **3''** and transfer them into the development. Measure the length of lines passing through each point from the front view or top view and transfer into the respective lines on the development. Join each point by the straight line segments to get the lateral surface development of right part of the horizontal prism. Repeat similar process to get symmetrical pattern to get the lateral surface development of left part of the horizontal prism as shown in *Figure 9.20(d)*.

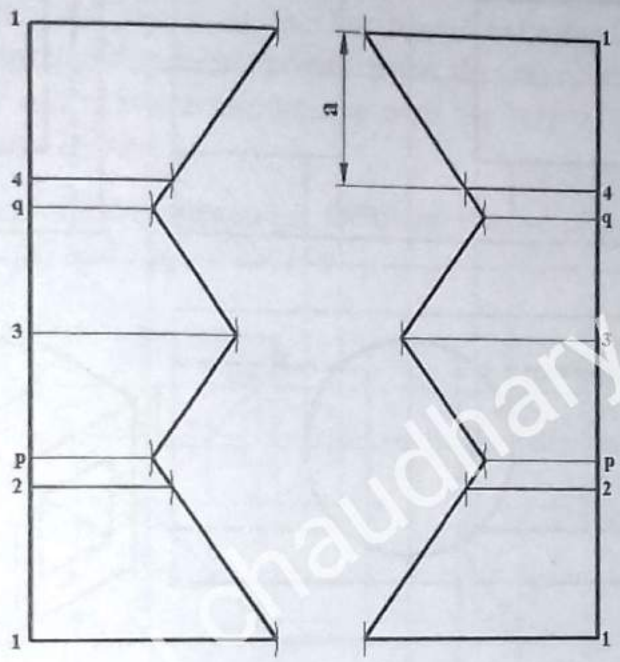


Figure 9.5(d): Development of Horizontal Prism

9.13 Effect of Intersection of any Object

As described earlier, when different solids intersect with each other different intersection curves will be formed. Similarly when different holes intersect with each other different hollow contour will be formed. *Figure 9.21* shows the effect of intersection when two cylindrical holes intersect with each other and *Figure 9.22* shows the effect of intersection when a cylindrical hole intersects with square or rectangular hole.

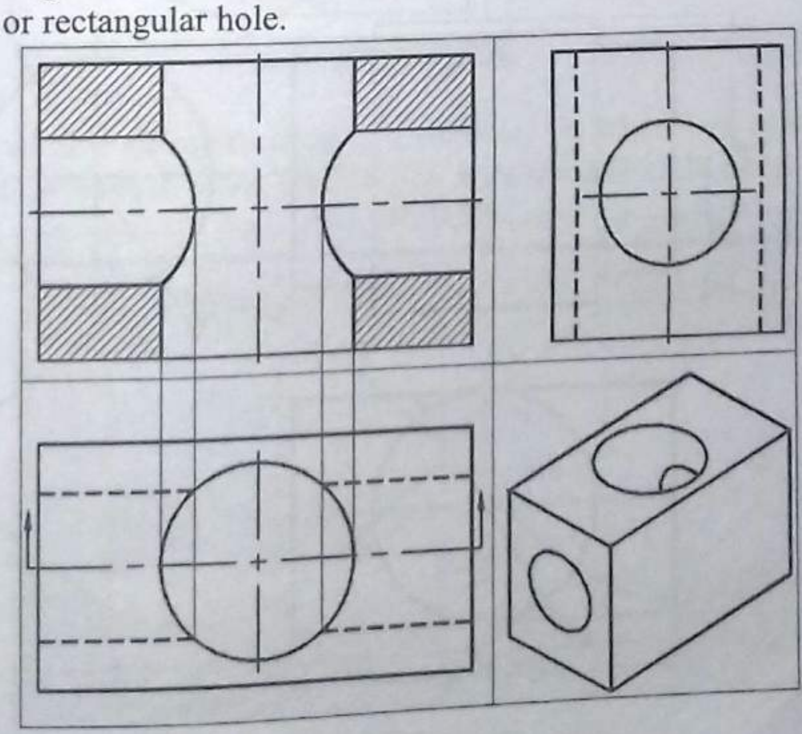


Figure 9.21: Effect of Intersection of Cylindrical Holes

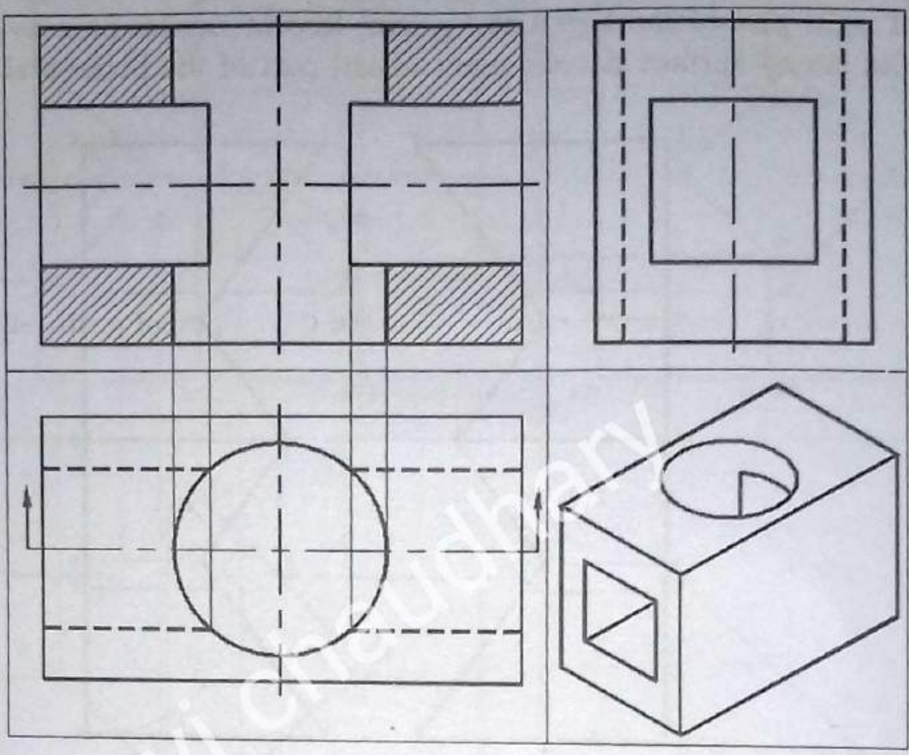


Figure 9.27: Effect of Intersection of Cylindrical Hole and Square Hole

WORKOUT EXAMPLES

Example 9.1

Complete the intersection curve between the given cylinders shown in *Figure E9.1* and develop the surfaces of both cylinders.

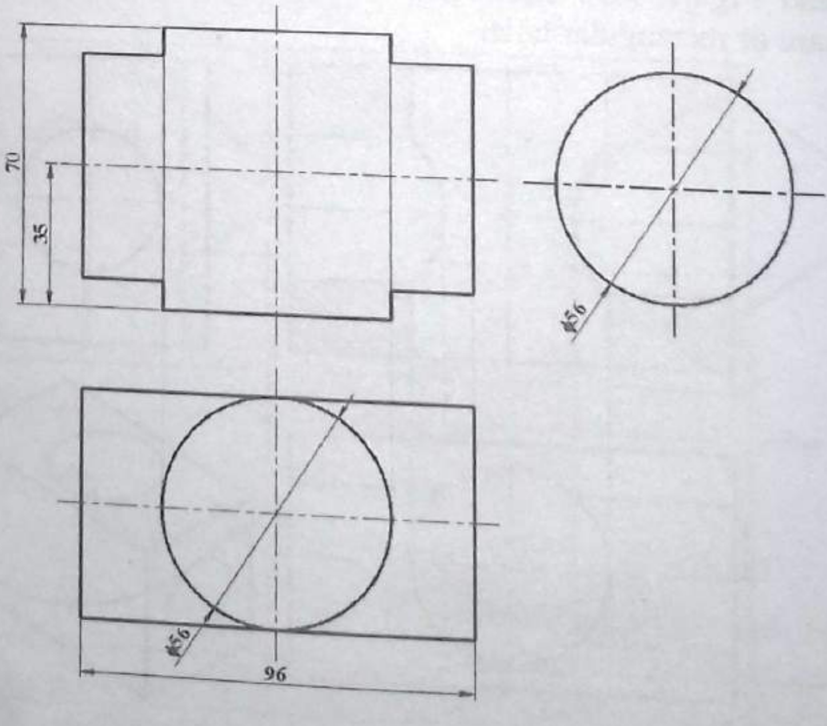


Figure E9.1

Solution

Since both the given solids have curved surfaces and uniform cross section, take any number of points on the side view of the horizontal cylinder as the side view of the intersection curve. Then transfer these points from the side view to the top view and complete the front view of the intersection curve with the help of top views and side views of each point as shown in *Figure E9.1(a)*.

Follow the procedure explained earlier to get the development of the surfaces of both solids as shown in *Figure E9.1(b)* and *Figure E9.1(c)*.

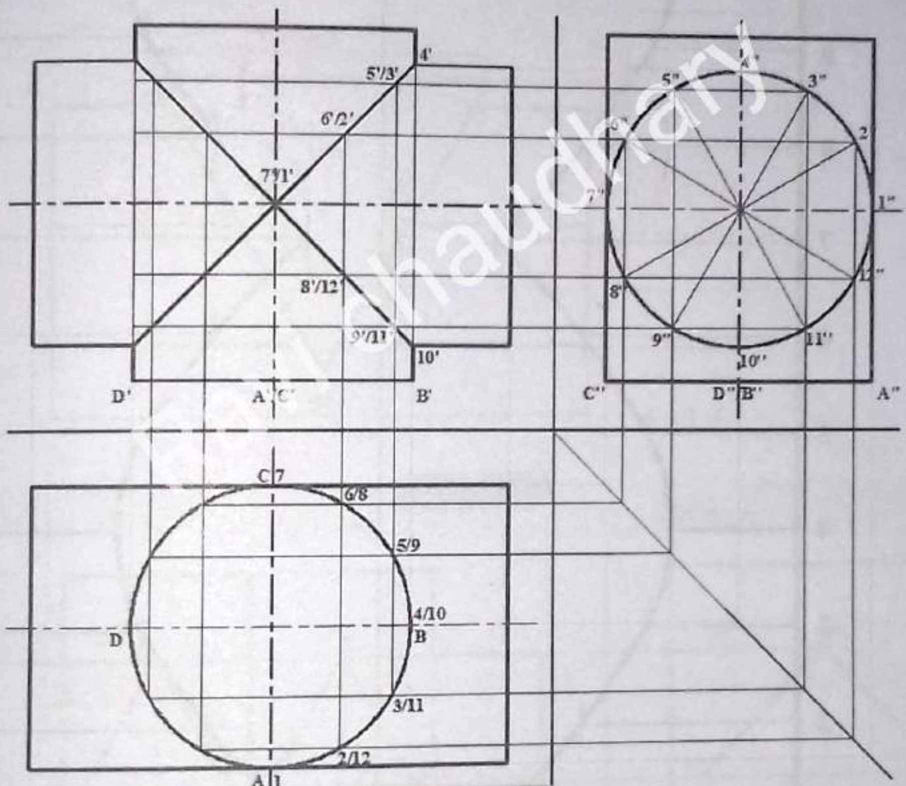


Figure E9.1(a)

It is an exceptional case of intersection of cylinders, i.e. when two cylinders having equal diameters intersect with each other, intersection appears as straight lines.

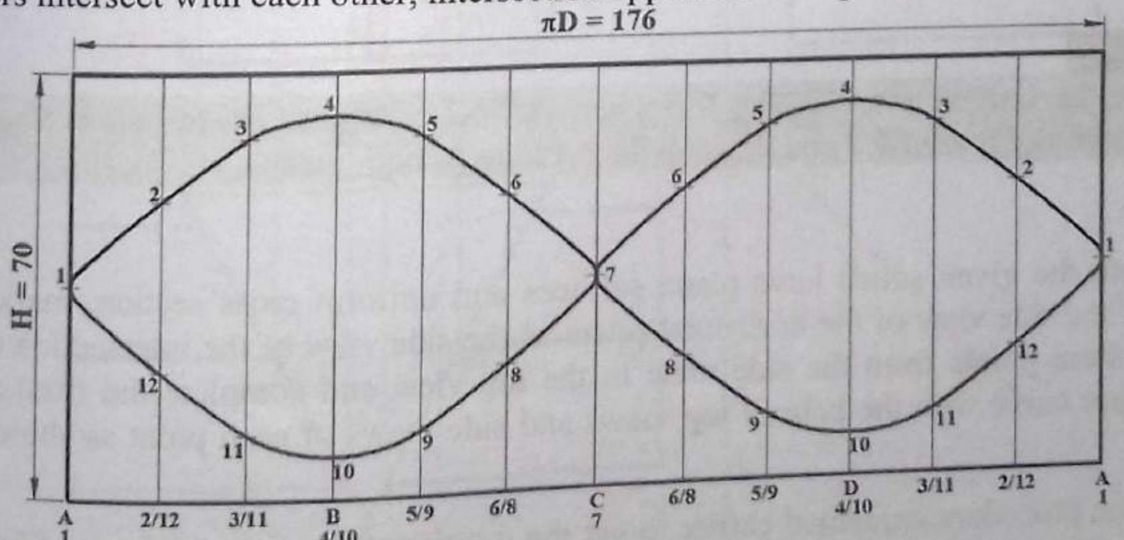


Figure E9.1(b)

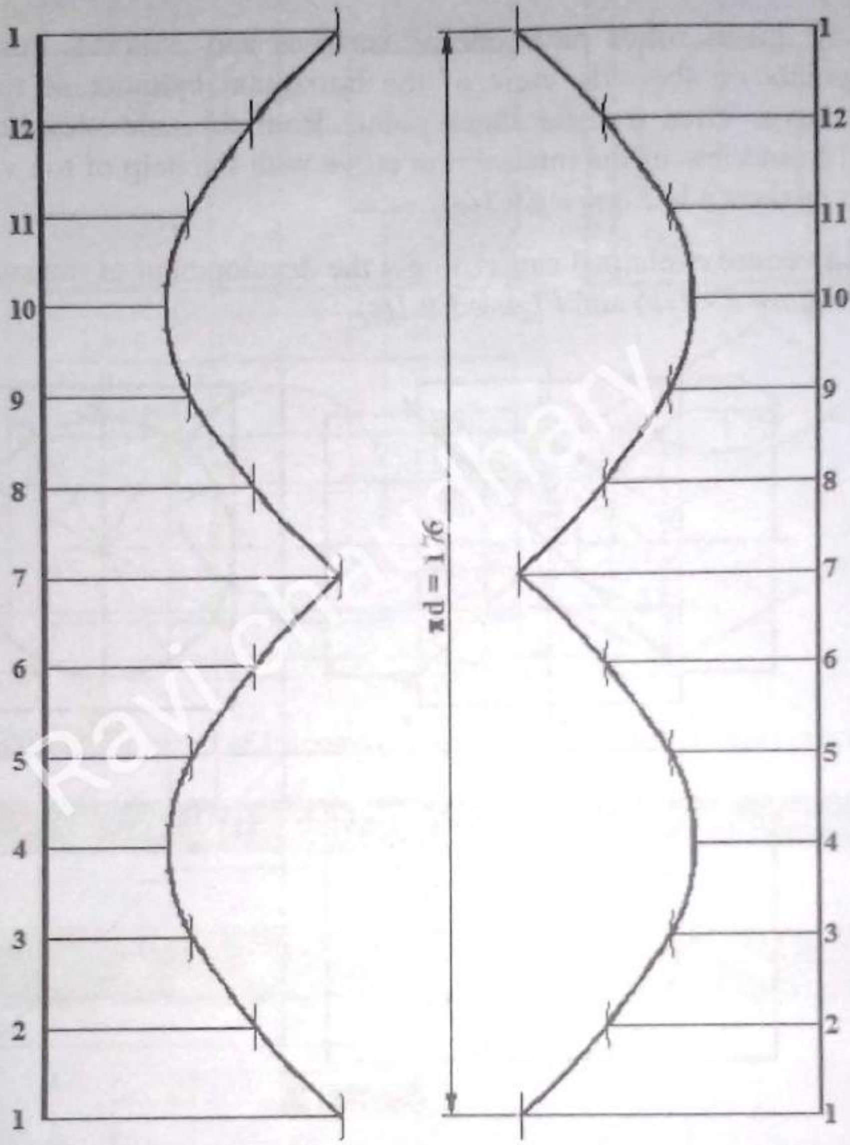


Figure E9.1(c)

Example 9.2

Complete the intersection between the given vertical triangular prism and horizontal square prism shown in *Figure E9.2* and develop the surfaces of both prisms.

Solution

Since both the given solids have plane surfaces and uniform cross section, mark the corner points of the side view of the horizontal prism as the side view of the intersection curve. Then transfer these points from the side view to the top view and complete the front view of the intersection curve with the help of top views and side views of each point as shown in *Figure E9.2(a)*.

Follow the procedure explained earlier to get the development of the surfaces of both solids as shown in *Figure E9.2(b)* and *Figure E9.2(c)*.

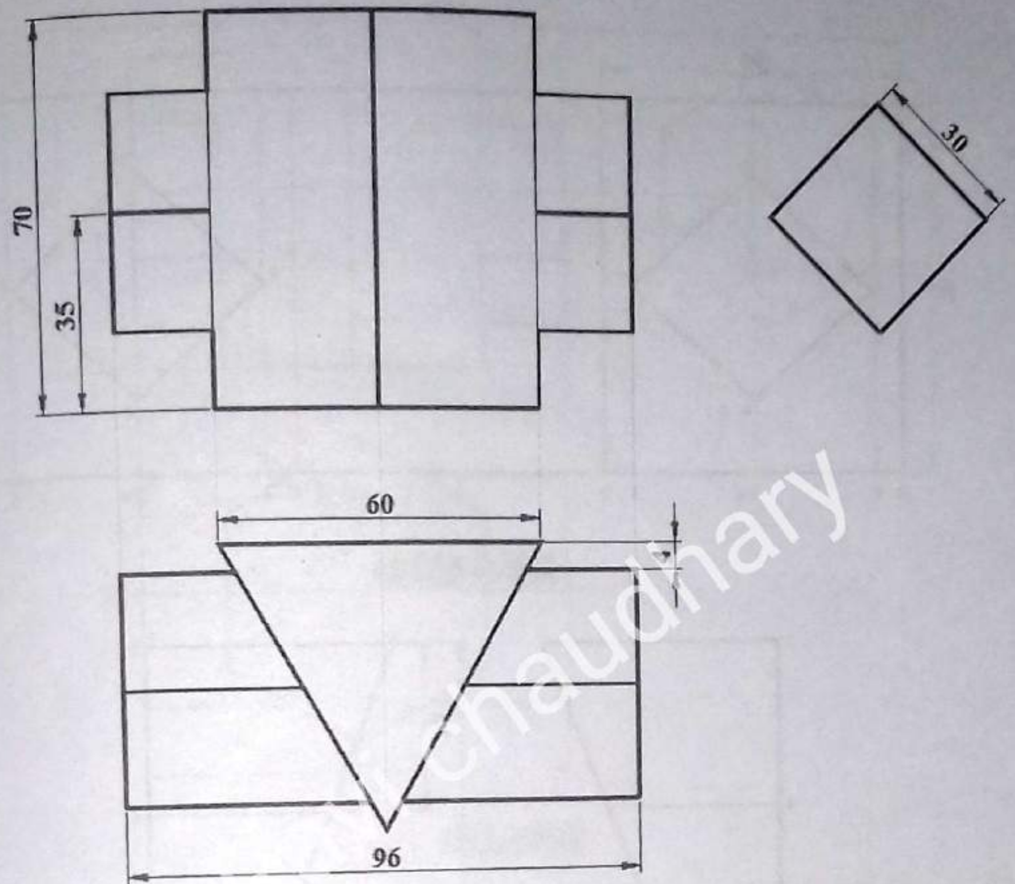


Figure E9.2

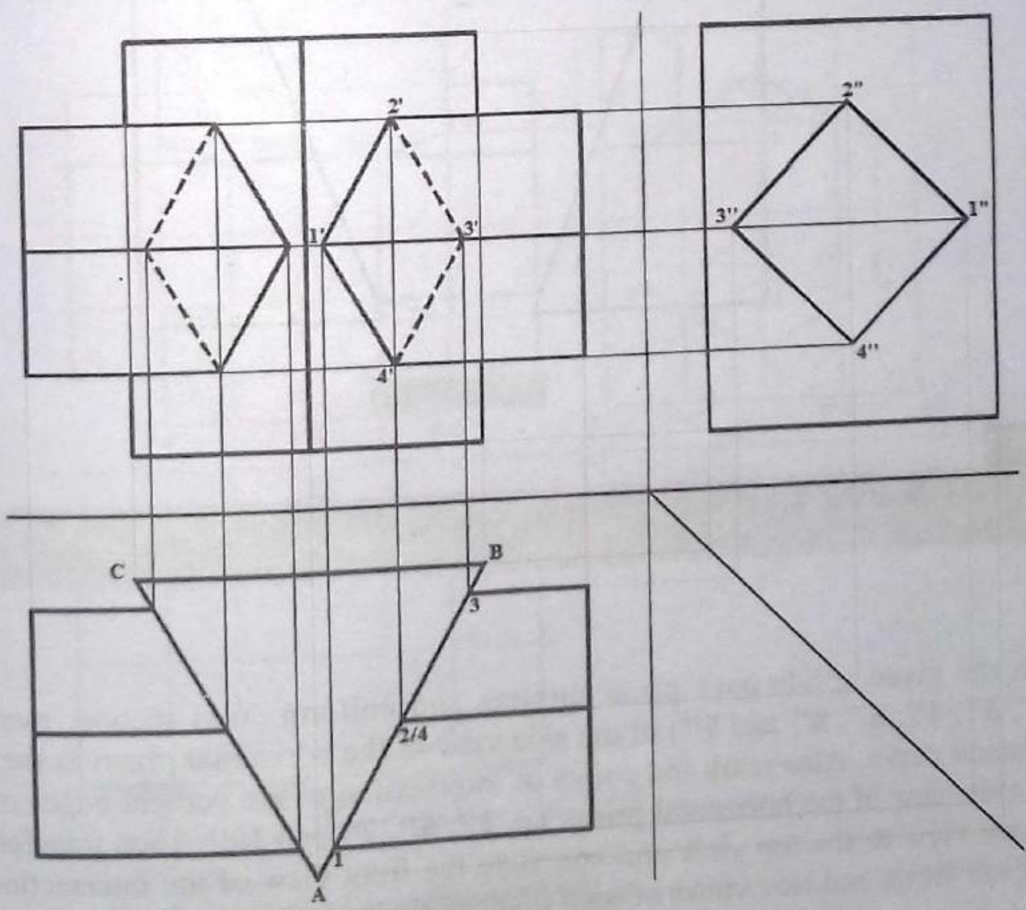


Figure E9.2(a)

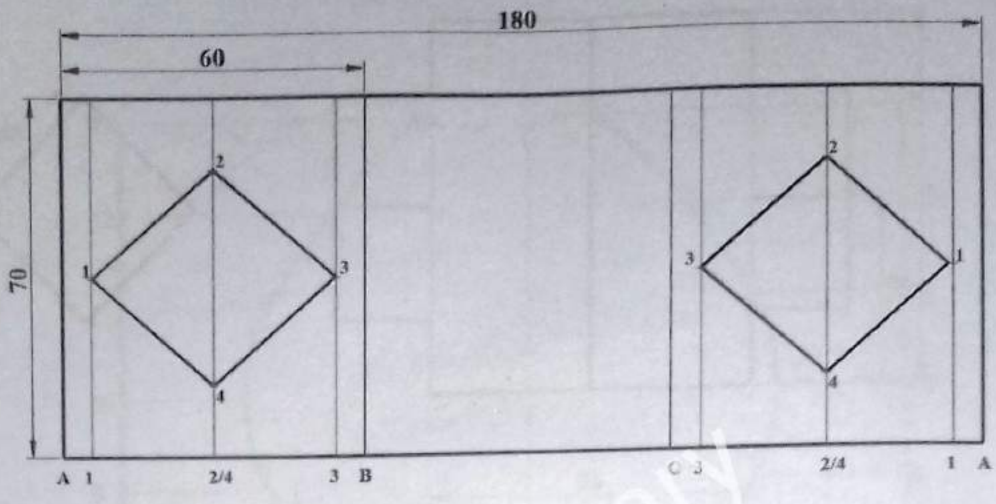


Figure E9.2(b)

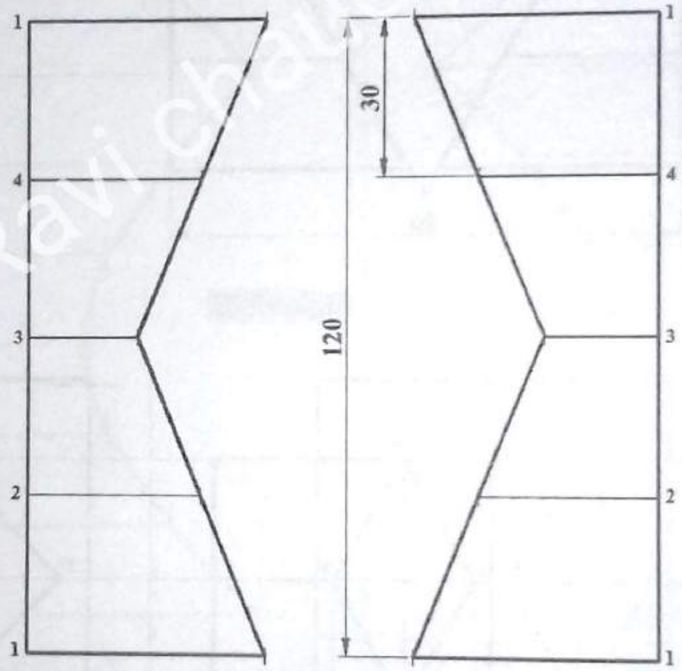


Figure E9.2(c)

Example 9.3

Complete the intersection between the given hexagonal prisms shown in *Figure E9.3* and develop the surfaces of both prisms.

Solution

Since both the given solids have plane surfaces and uniform cross section, mark the corner points (**1'', 3'', 4'', 6'', 8''** and **9''**) of the side view of the horizontal prism as the side view of the intersection curve. Also mark the points of intersection of the vertical edges of the vertical prism and side view of the horizontal prism, i.e. **2'', 5'', 7''** and **10''**. Then transfer these points from the side view to the top view and complete the front view of the intersection curve with the help of top views and side views of each point as shown in *Figure E9.3(a)*.

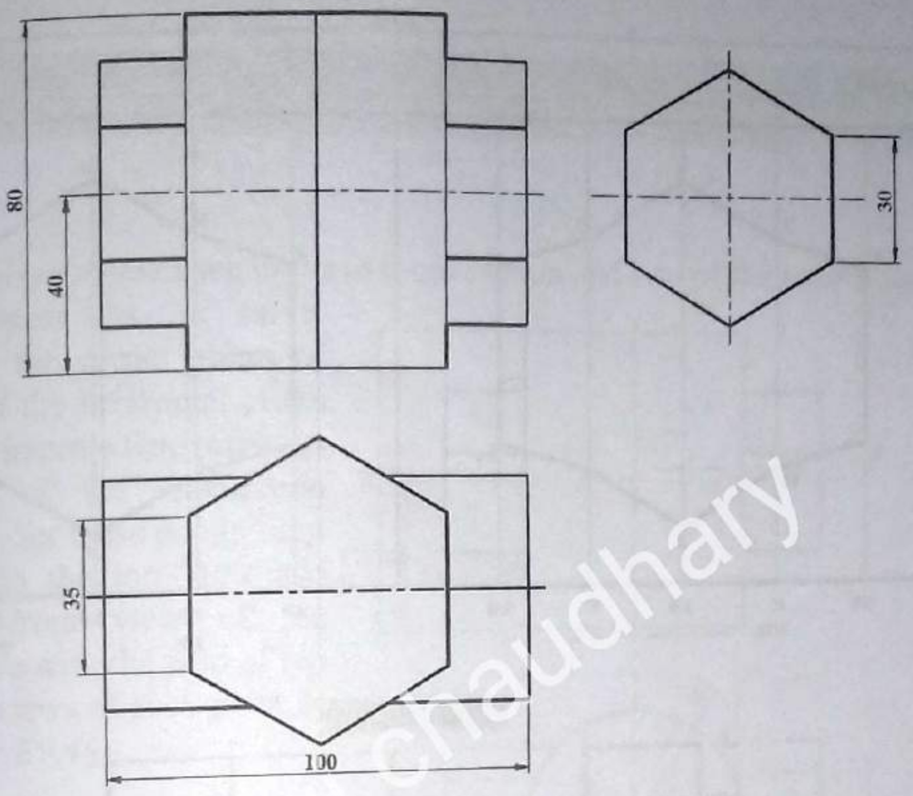


Figure E9.3

Follow the procedure explained earlier to get the development of the surfaces of both solids as shown in *Figure E9.3(b)* and *Figure E9.3(c)*.

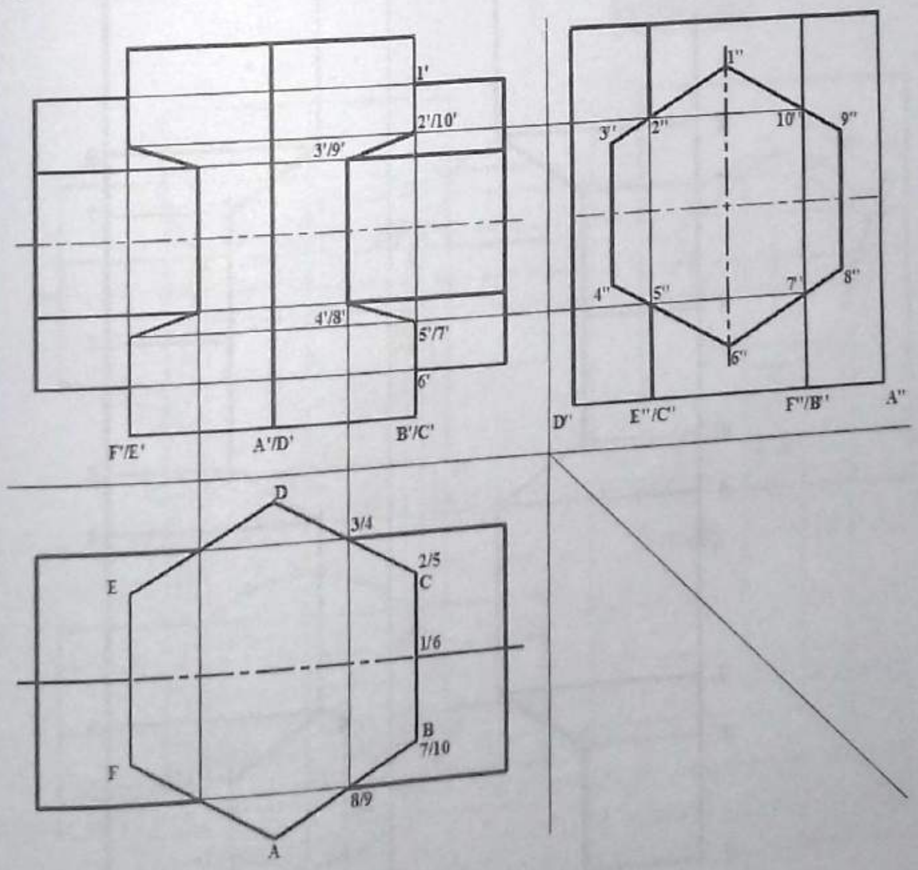


Figure E9.3(a)

Scanned by CamScanner

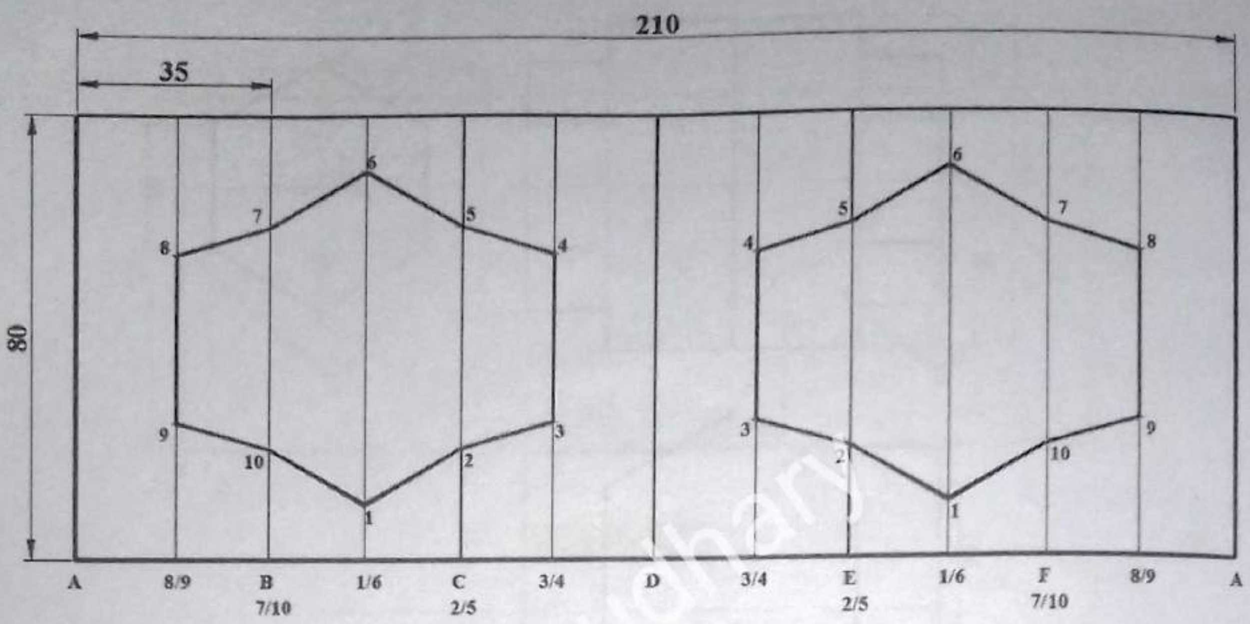


Figure E9.3(b)

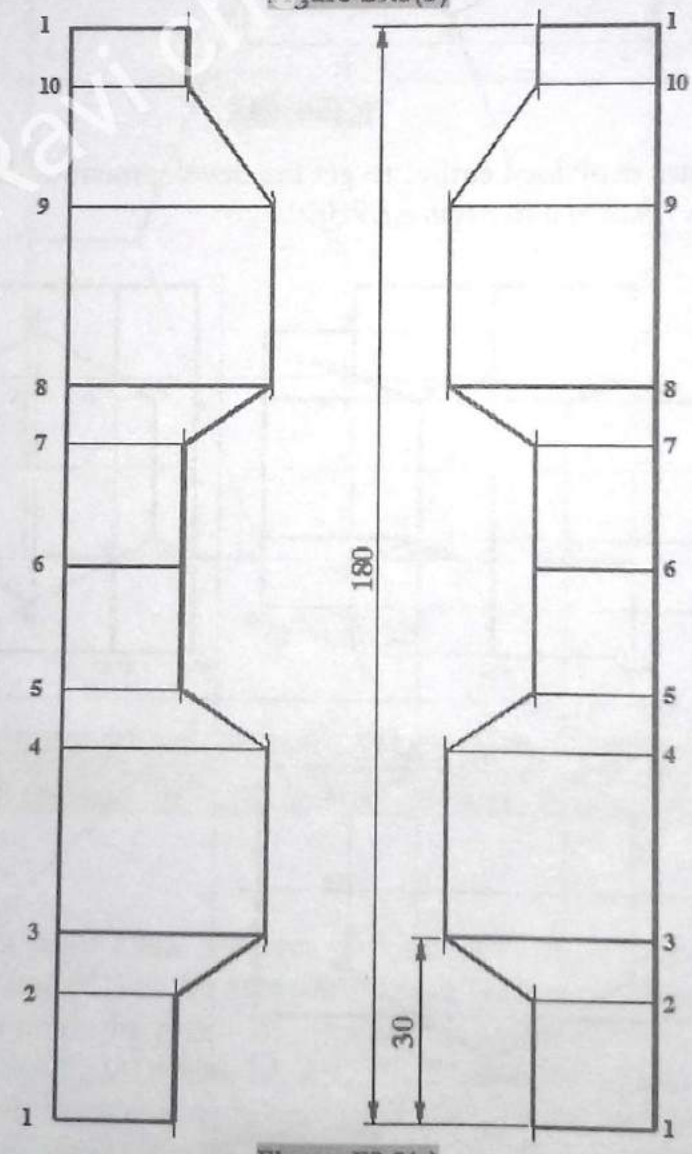


Figure E9.3(c)

Example 9.4

Complete the intersection between the given vertical cylinder and horizontal pentagonal prism shown in *Figure E9.4* and develop the surfaces of both solids.

Solution

Since both the given solids have uniform cross section and one of the solid has curved surface, intersection appear as a curve. Therefore mark the corner points of the side view of the horizontal prism as well as some intermediate points as the side view of the intersection curve. Then transfer these points from the side view to the top view and complete the front view of the intersection curve with the help of top views and side views of each point as shown in *Figure E9.4(a)*.

Follow the procedure explained earlier to get the development of the surfaces of both solids as shown in *Figure E9.4(b)* and *Figure E9.4(c)*.

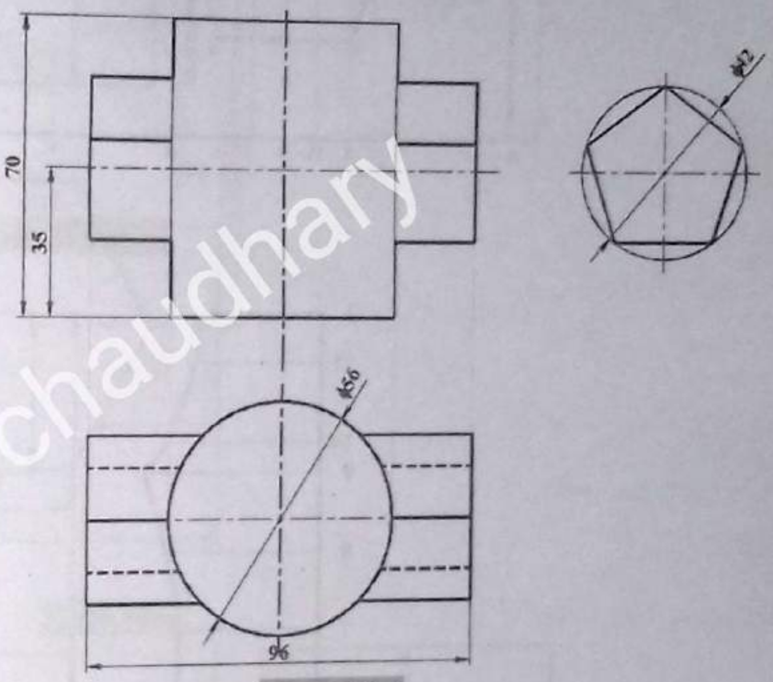


Figure E9.4

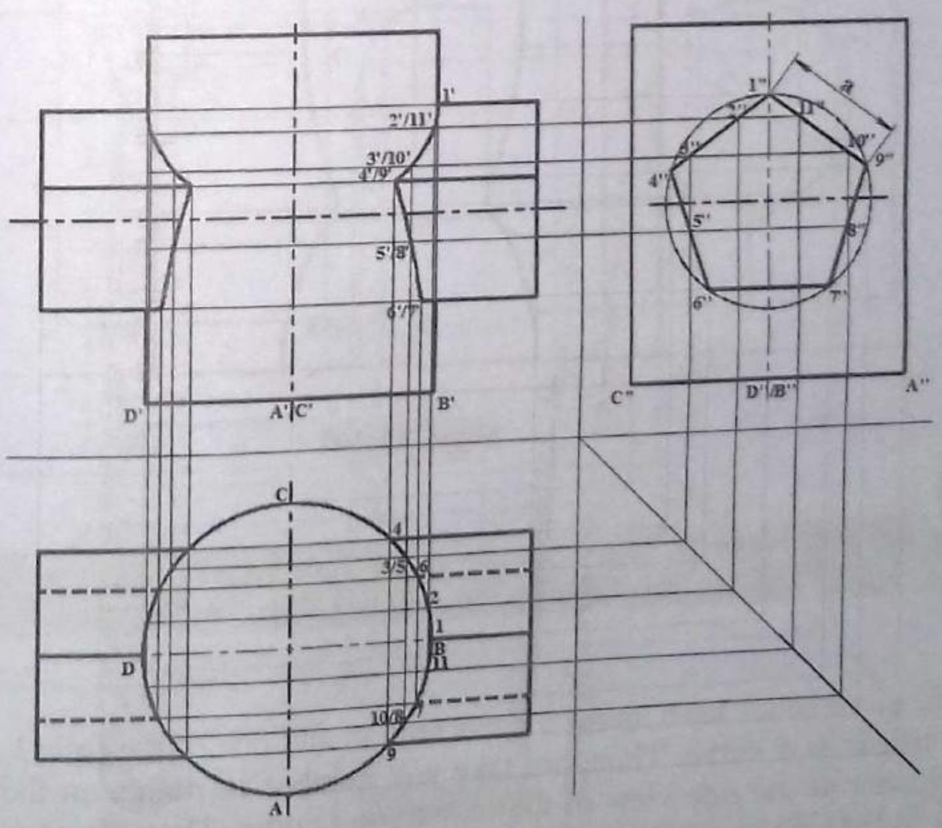


Figure E9.4(a)

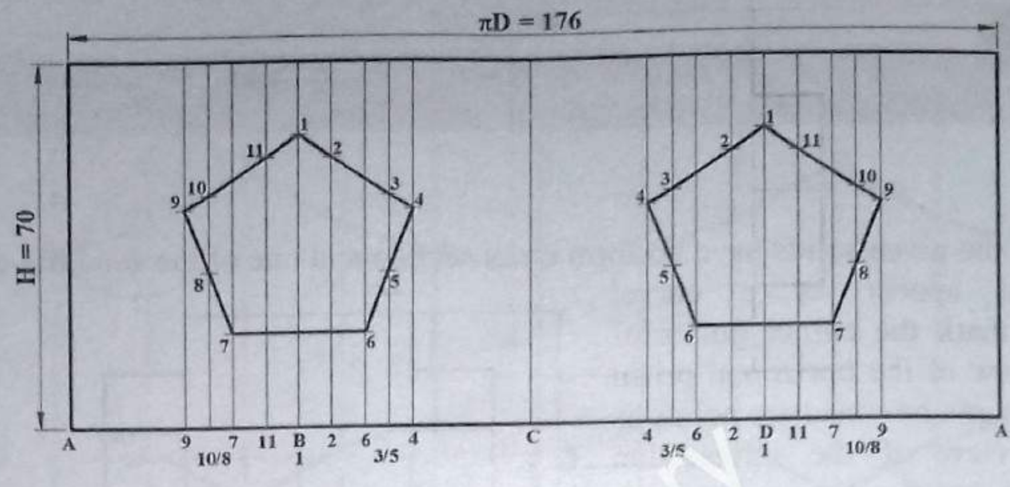


Figure E9.4(b)

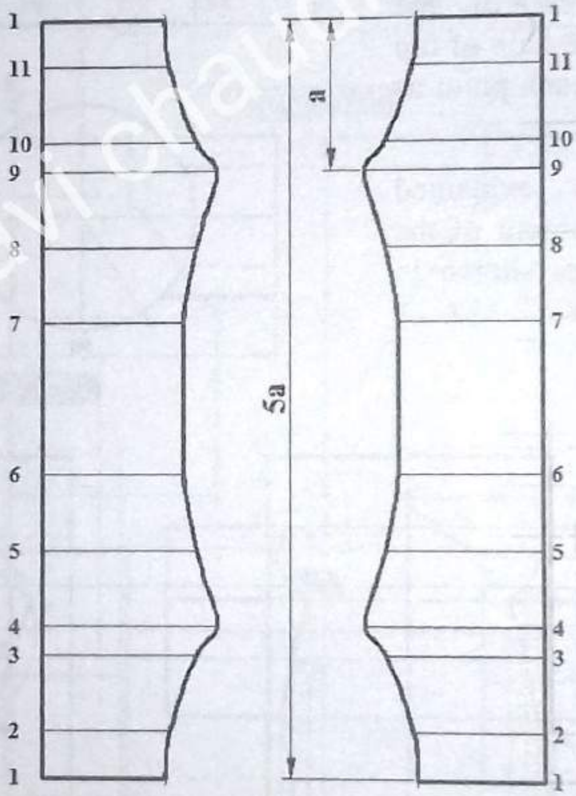


Figure E9.4(c)

Example 9.5

Complete the intersection between the given vertical hexagonal prism and horizontal cylinder shown in *Figure E9.5* and develop the surfaces of both solids.

Solution

Since both the given solids have uniform cross section and one of the solid has curved surface, intersection appear as a curve. Therefore take any number of points on the side view of the horizontal cylinder as the side view of the intersection curve. Then transfer these points from the side view to the top view and complete the front view of the intersection curve with the help of top views and side views of each point as shown in *Figure E9.5(a)*.

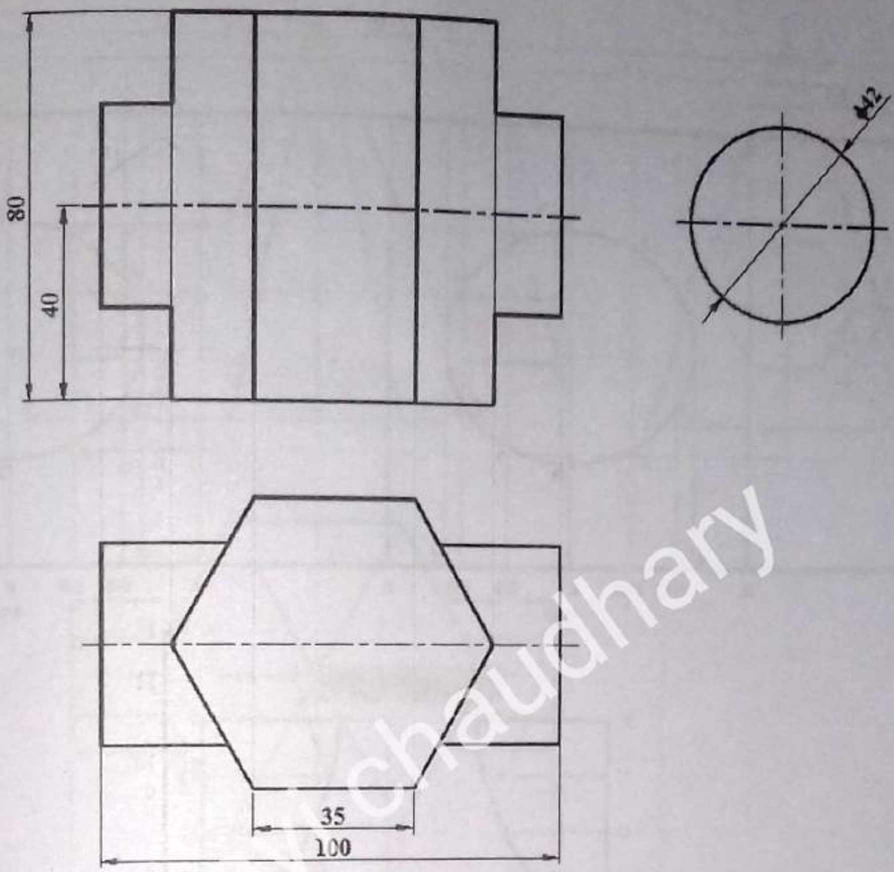


Figure E9.5

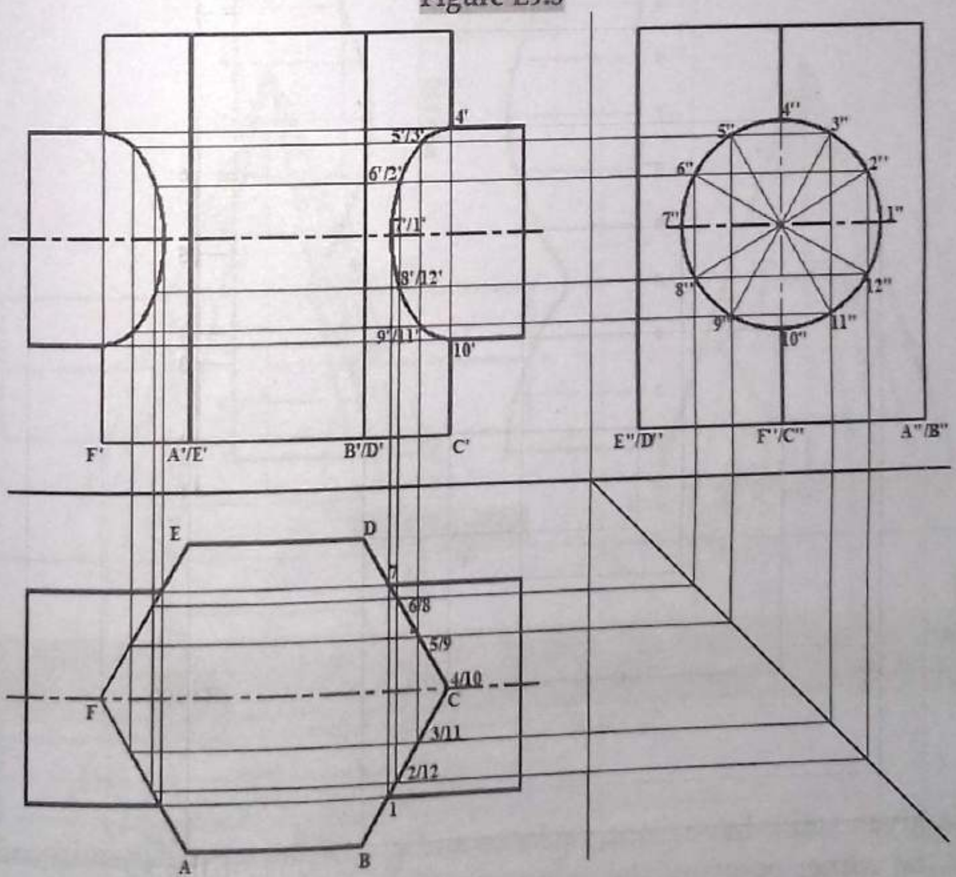


Figure E9.5(a)

Follow the procedure explained earlier to get the development of the surfaces of both solids as shown in *Figure E9.5(b)* and *Figure E9.5(c)*.

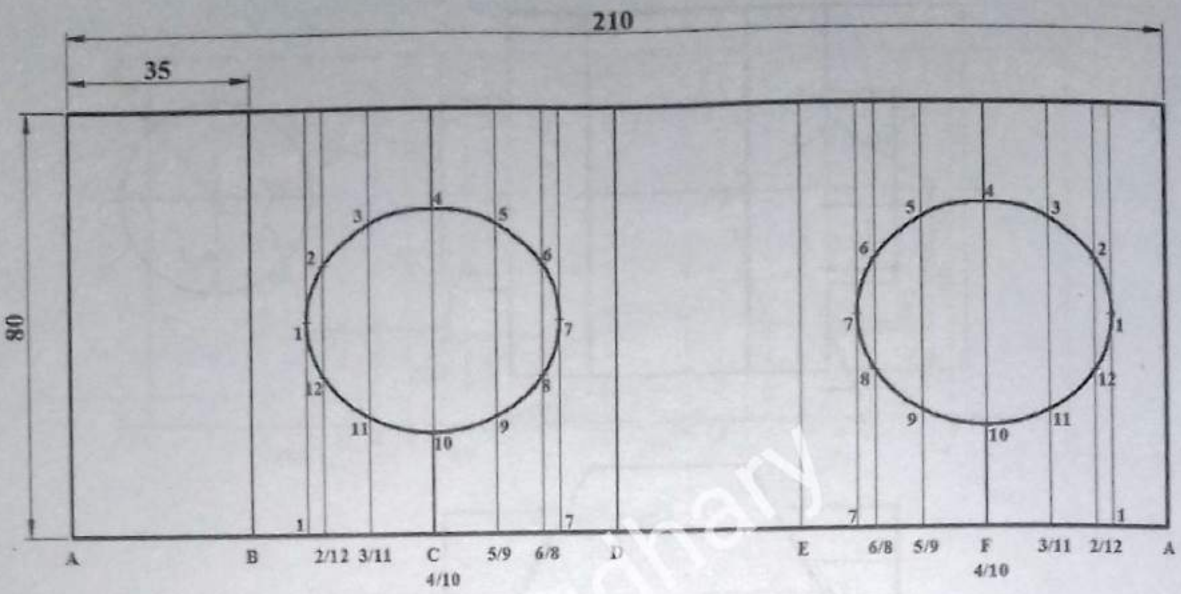


Figure E9.5(b)

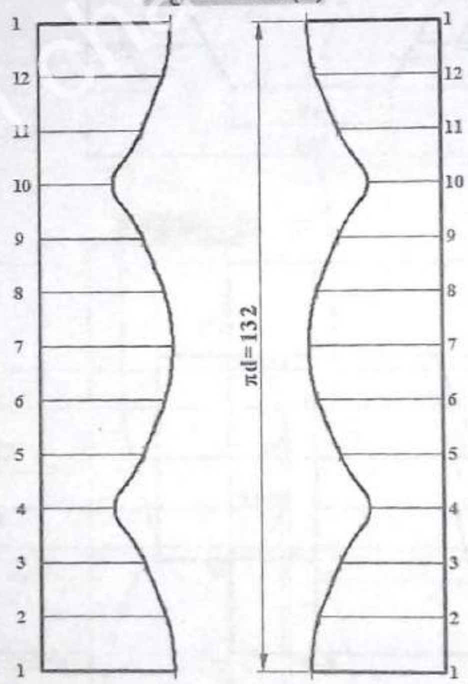


Figure E9.5(c)

Example 9.6

Complete the intersection between the given vertical hexagonal pyramid and horizontal hexagonal prism shown in *Figure E9.6.*

Solution

Since both the given solids have plane surfaces and one of the solid has uniformly varying cross section, mark the corner points of the side view of the horizontal prism as the side view of the intersection curve. Then transfer these points from the side view to the top view and front view to complete the corresponding top view and front view of the intersection curve as shown in *Figure E9.6(a).*

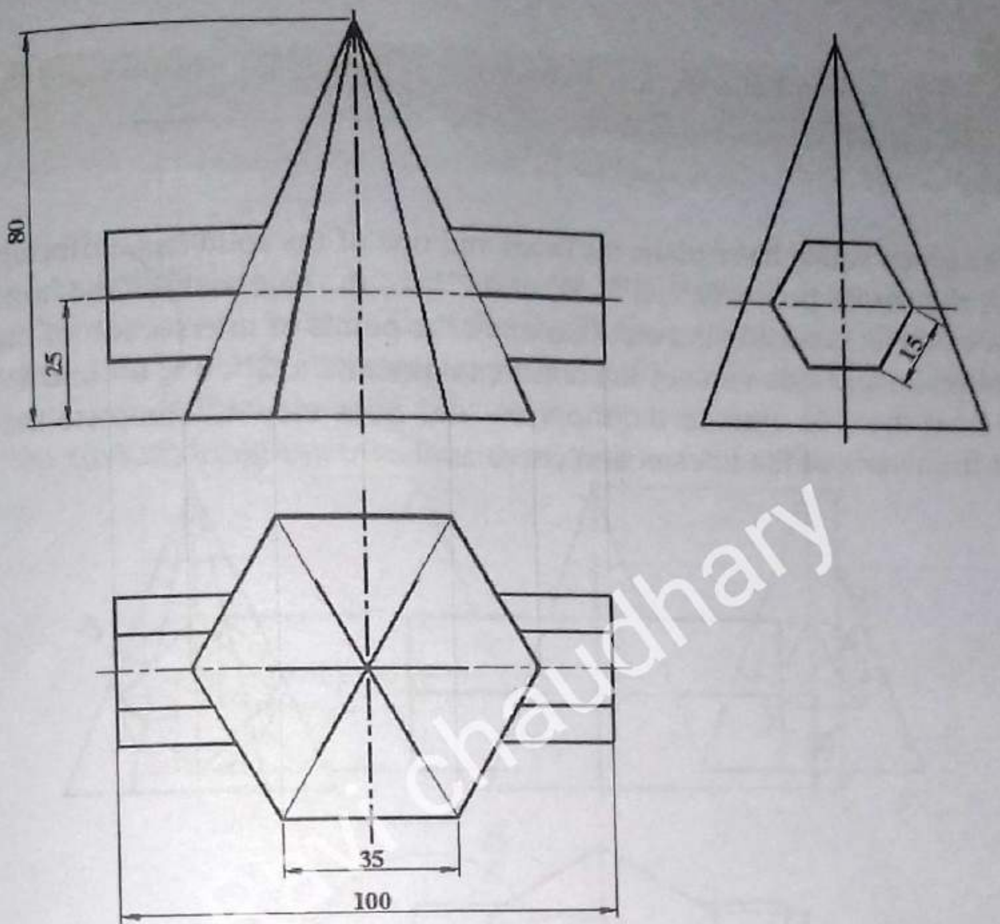


Figure E9.6

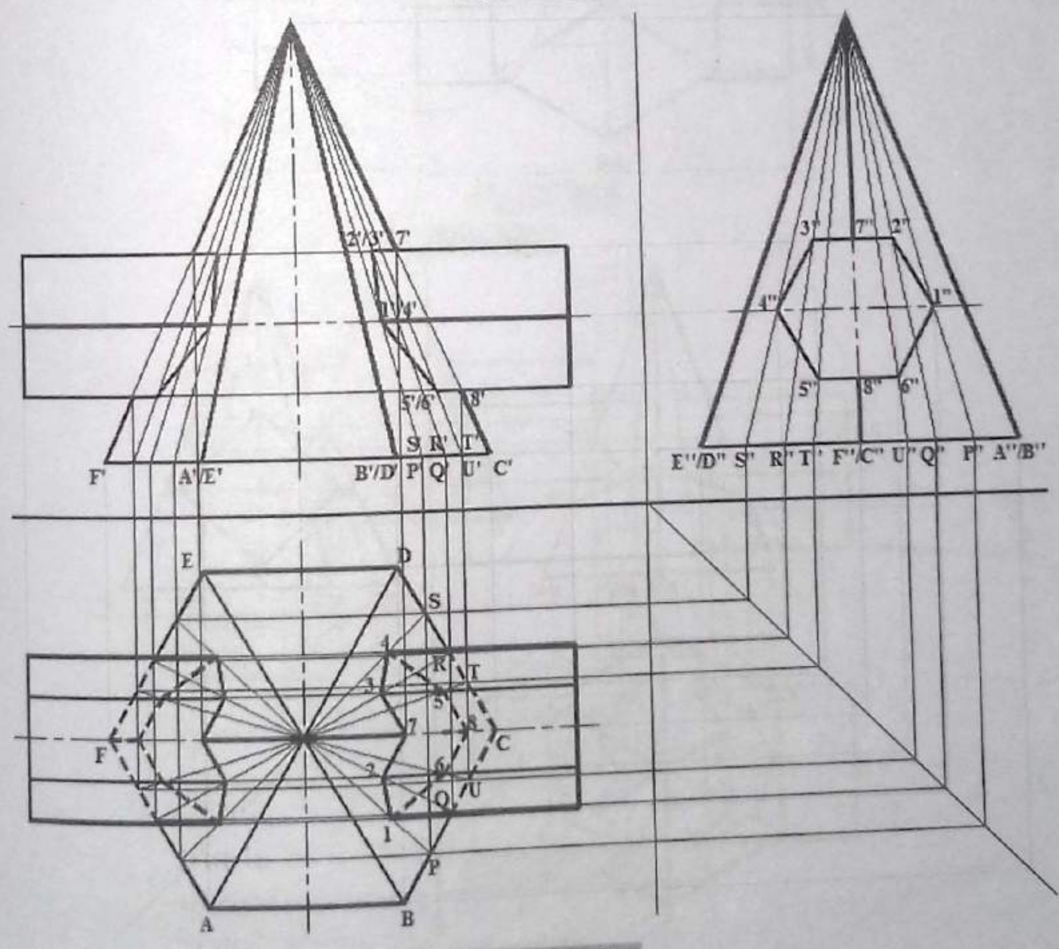


Figure E9.6(a)

Example 9.7

Complete the intersection between the given vertical hexagonal pyramid and horizontal square prism shown in **Figure E9.7**.

Solution

Since both the given solids have plane surfaces and one of the solid has uniformly varying cross section, mark the corner points (**1''**, **3''**, **5''** and **7''**) of the side view of the horizontal prism as the side view of the intersection curve. Also mark the points of intersection of the vertical edges of the vertical prism and side view of the horizontal prism, i.e. **2''**, **4''**, **6''** and **8''**. Then transfer these points from the side view to the top view and front view to complete the corresponding top view and front view of the intersection curve as shown in *Figure E9.7(a)*.

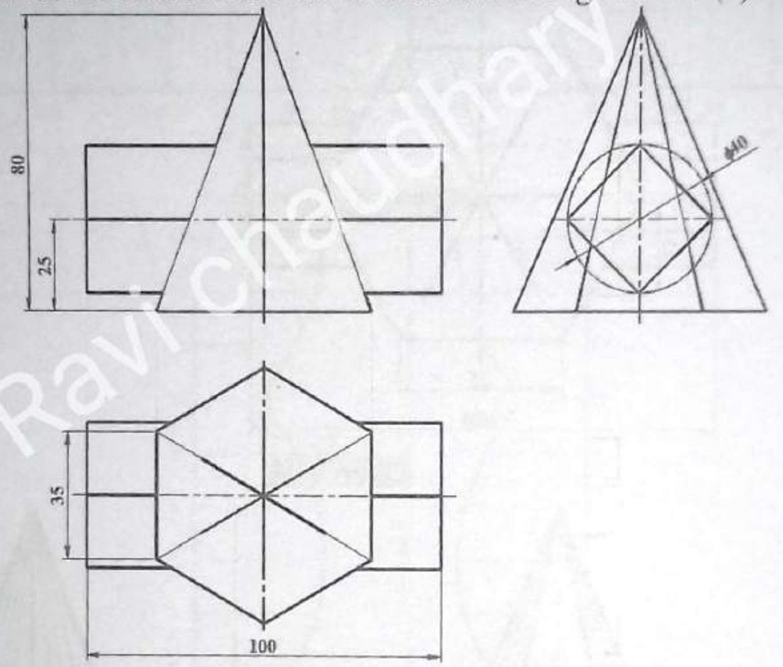


Figure E9.7

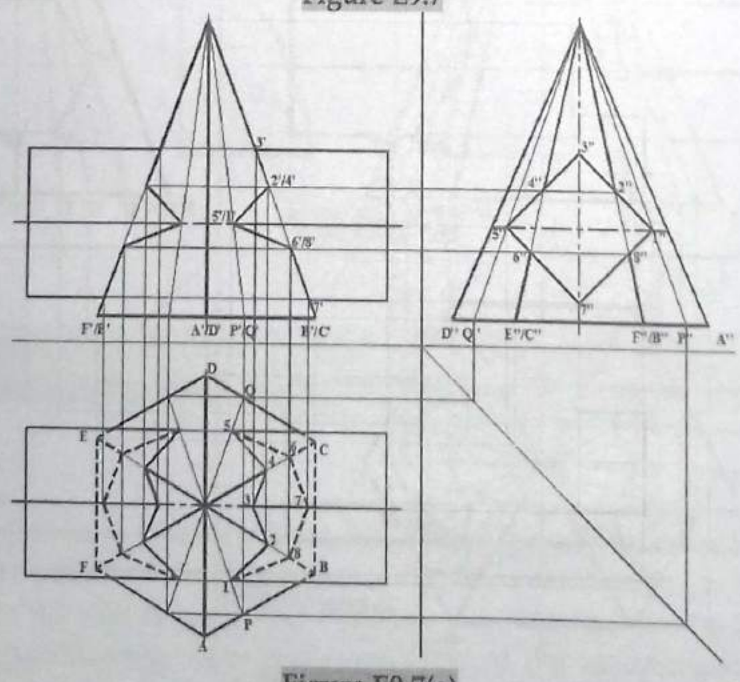


Figure E9.7(a)

Example 9.8

Complete the intersection between the given vertical cone and horizontal triangular prism shown in *Figure E9.8.*

Solution

Since one the given solid has plane surface and one of the solid has uniformly varying cross section, intersection appear as a curve. Therefore mark the corner points of the side view of the horizontal prism as well as some intermediate points as the side view of the intersection curve. Then transfer these points from the side view to the top view and front view to complete the corresponding top view and front view of the intersection curve as shown in *Figure E9.8(a).*

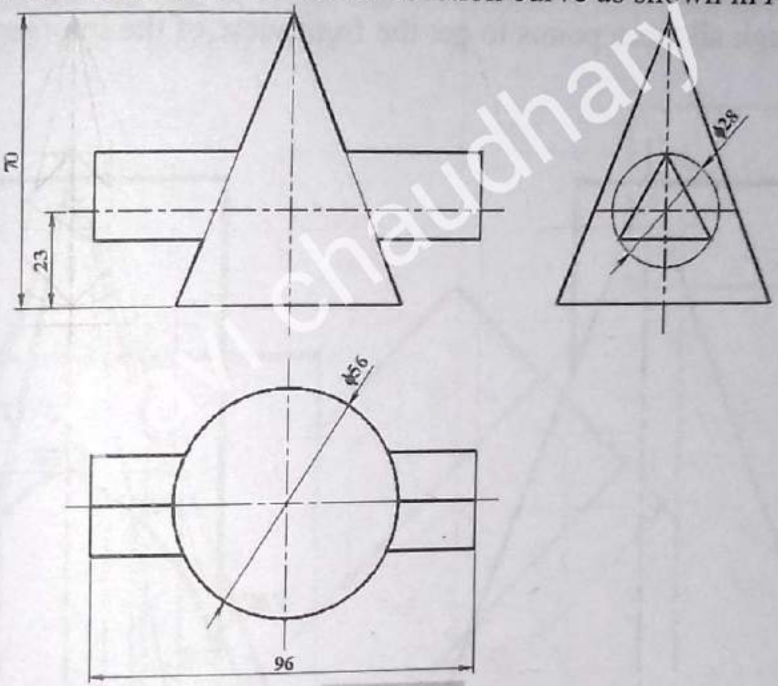


Figure E9.8

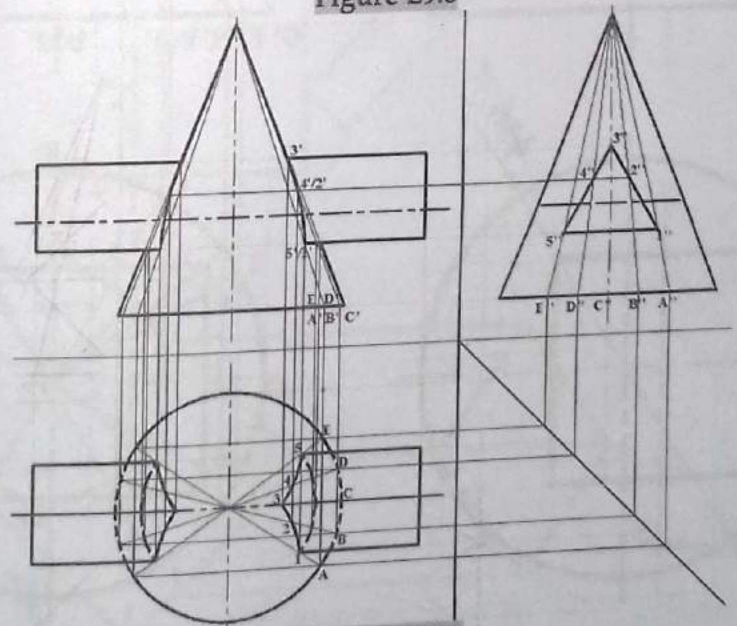


Figure E9.8(a)

Example 9.9

Draw front view of the intersection curve between the given cone and square prism shown in *Figure E9.9*.

Solution

Since one the given solid has plane surface and one of the solid has uniformly varying cross section, intersection appear as a curve. Divide base circle of the vertical cone into any number of equal parts, say 12 and name the dividing points as **A**, **B**, **C**, …….. **L**. Join each points with top view **V** of the vertex of the cone. Name points of intersection of the top views of the generators **VA**, **VB**, … **VL** with the top view of the given prism as **1**, **2**, … **12**. Transfer each of these points to the front view to get front views of each point **1'**, **2'**, …. **12'**. Draw smooth curve passing through all these points to get the front view of the intersection curve, as shown in *Figure E9.9(a)*.

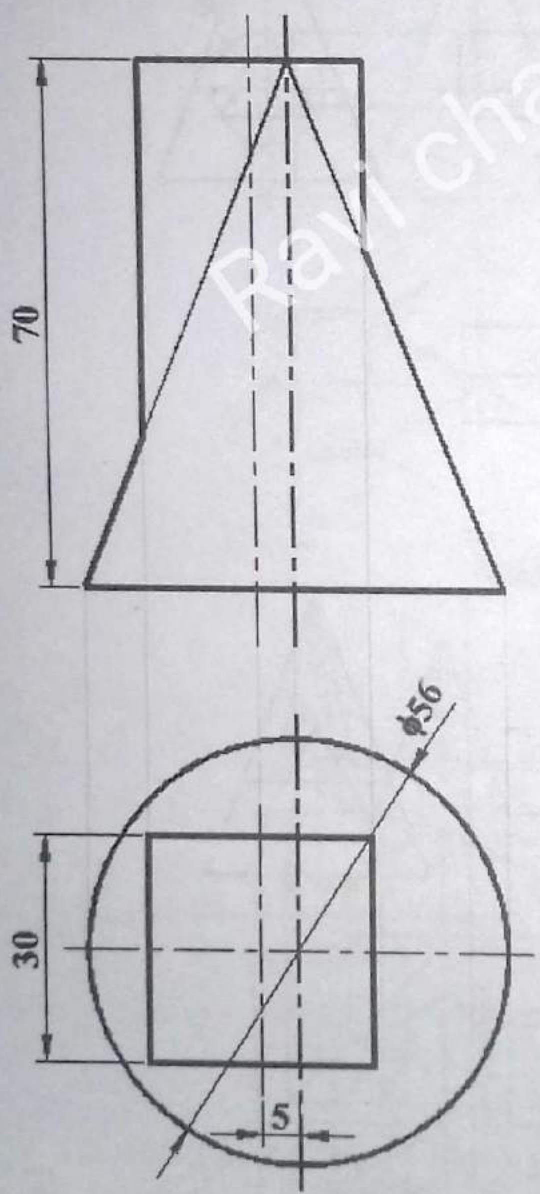


Figure E9.9

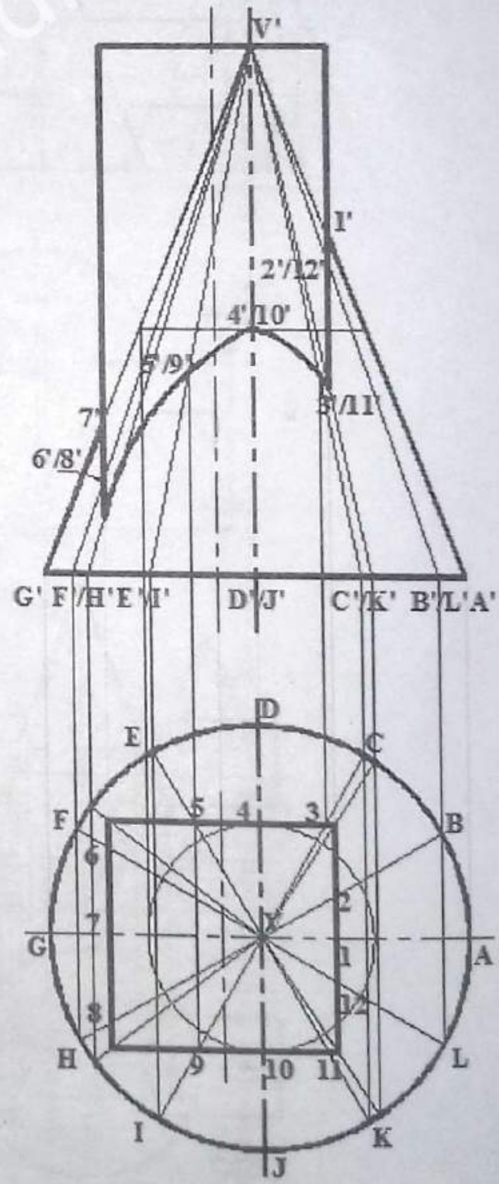


Figure E9.9(a)

Example 9.10

Complete the intersection between the given vertical square prism and inclined square shown in *Figure E9.10* and develop the surfaces of both solids.

Solution

Since both the given solids have plane surfaces and uniform cross section, intersection appears as a combination of straight lines. Therefore mark the corner points of the auxiliary view of the inclined prism as the auxiliary view of the intersection curve. Then transfer these points from the auxiliary view to the top view and complete the front view of the intersection curve with the help of top views and auxiliary views of each point as shown in *Figure E9.10(a)*.

Follow the procedure explained earlier to get the development of the surfaces of both solids as shown in *Figure E9.10(b)* and *Figure E9.10(c)*.

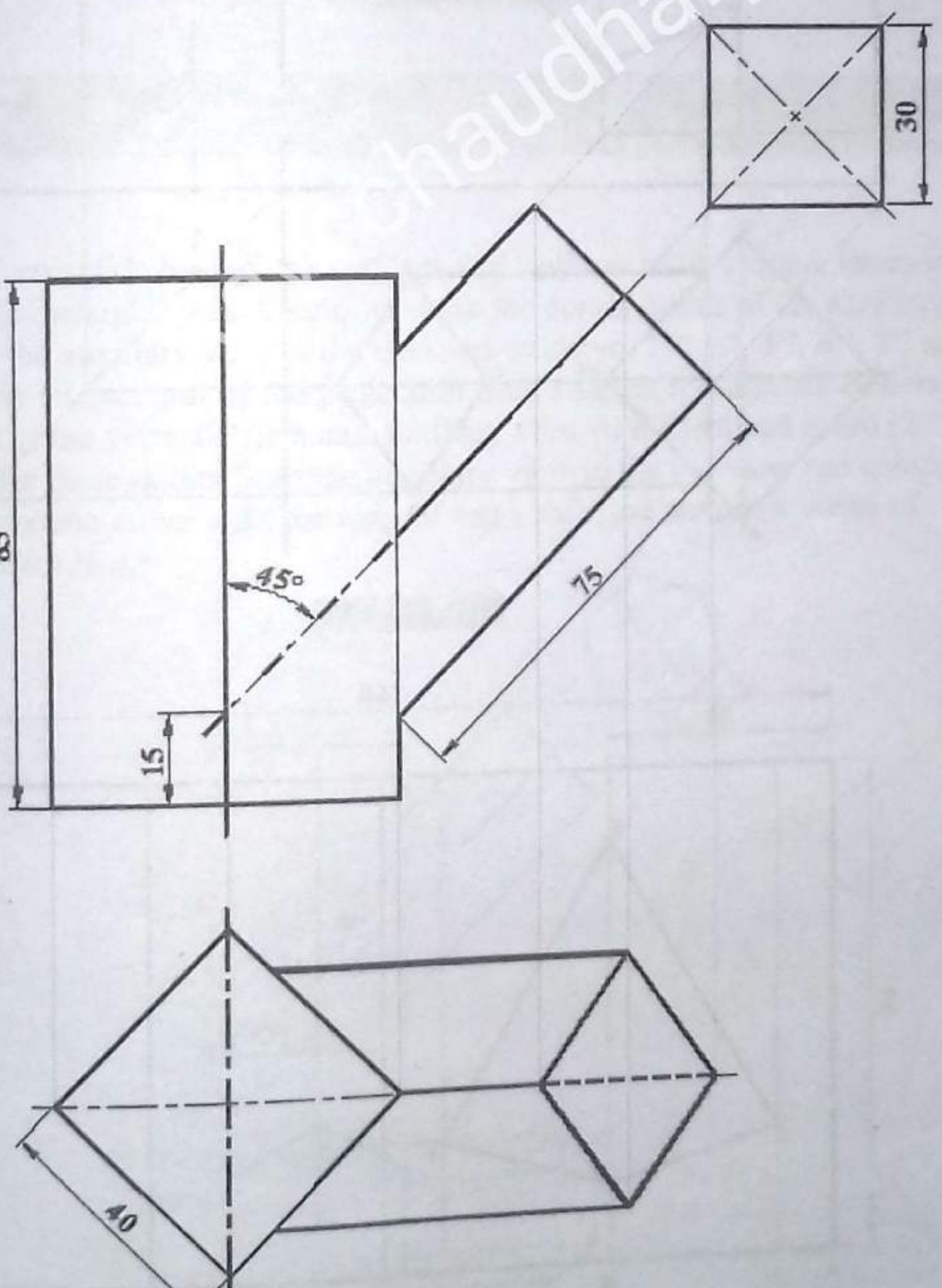


Figure E9.10

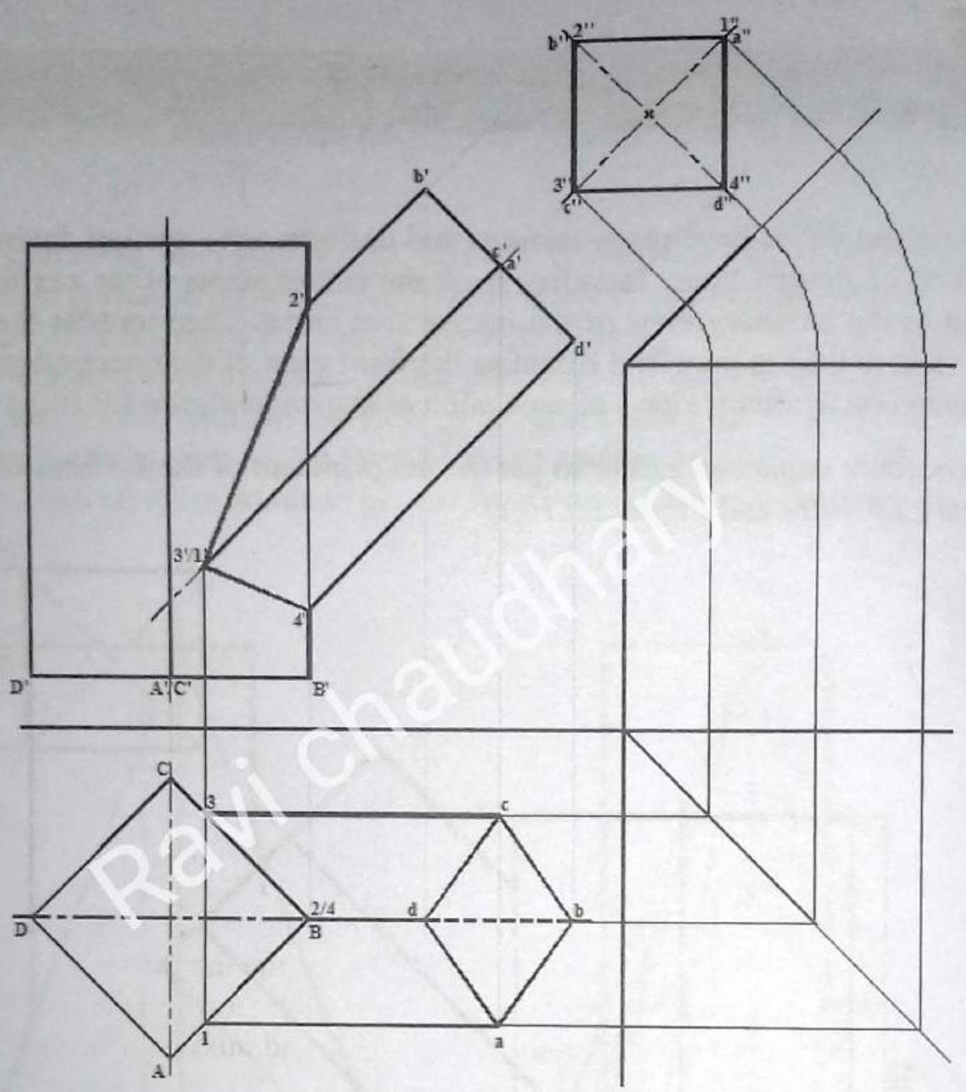


Figure E9.10(a)

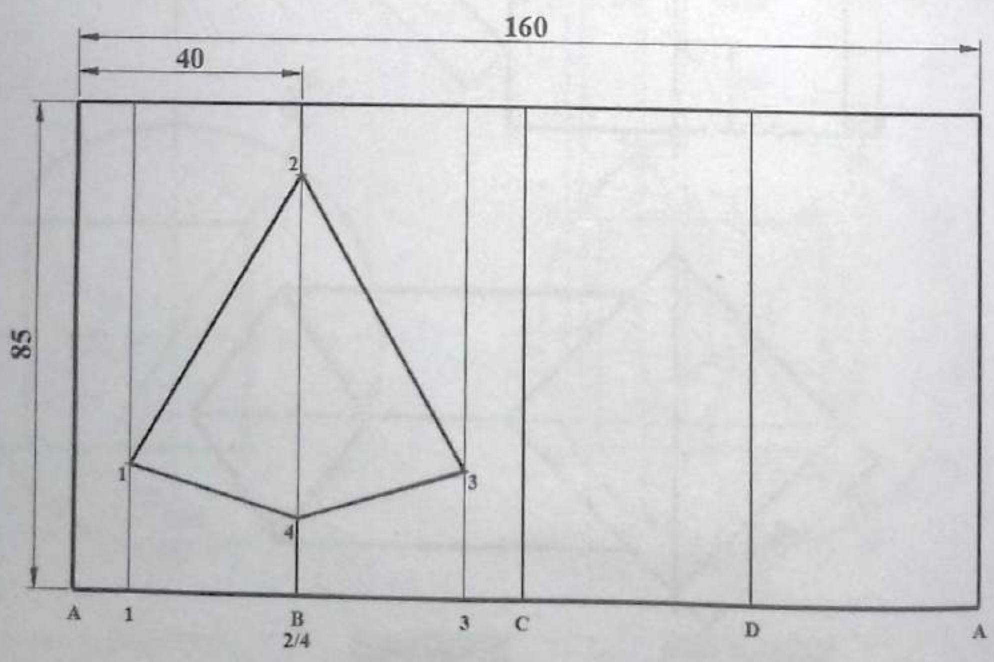


Figure E9.10(b)

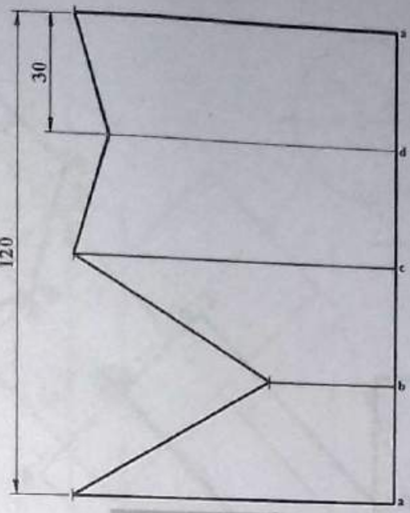


Figure E9.10(c)

Example 9.11

Complete the intersection between the given vertical hexagonal prism and inclined hexagonal prism shown in *Figure E9.11.*

Solution

Since both the given solids have plane surfaces and uniform cross section, intersection appears as a combination of straight lines. Therefore mark the corner points of the auxiliary view of the inclined prism as the auxiliary view of the intersection curve (**1'', 3'', 4'', 6'', 8''** and **9''**). Also mark the points of intersection of the projection lines passing through the corners **B** and **C** of the base hexagon of the vertical prism and auxiliary view of the inclined prism (**2'', 5'', 7''** and **10''**). Then transfer these points from the auxiliary view to the top view and complete the front view of the intersection curve with the help of top views and auxiliary views of each point as shown in *Figure E9.11(a).*

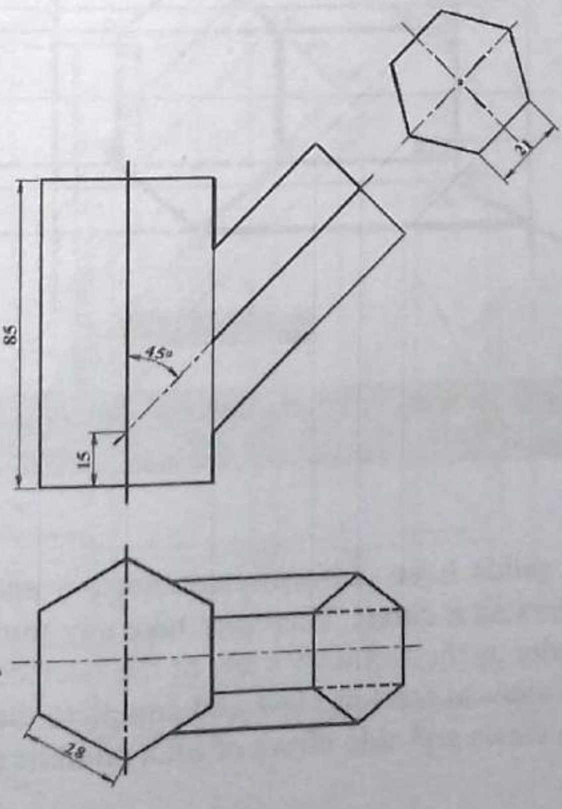


Figure E9.11

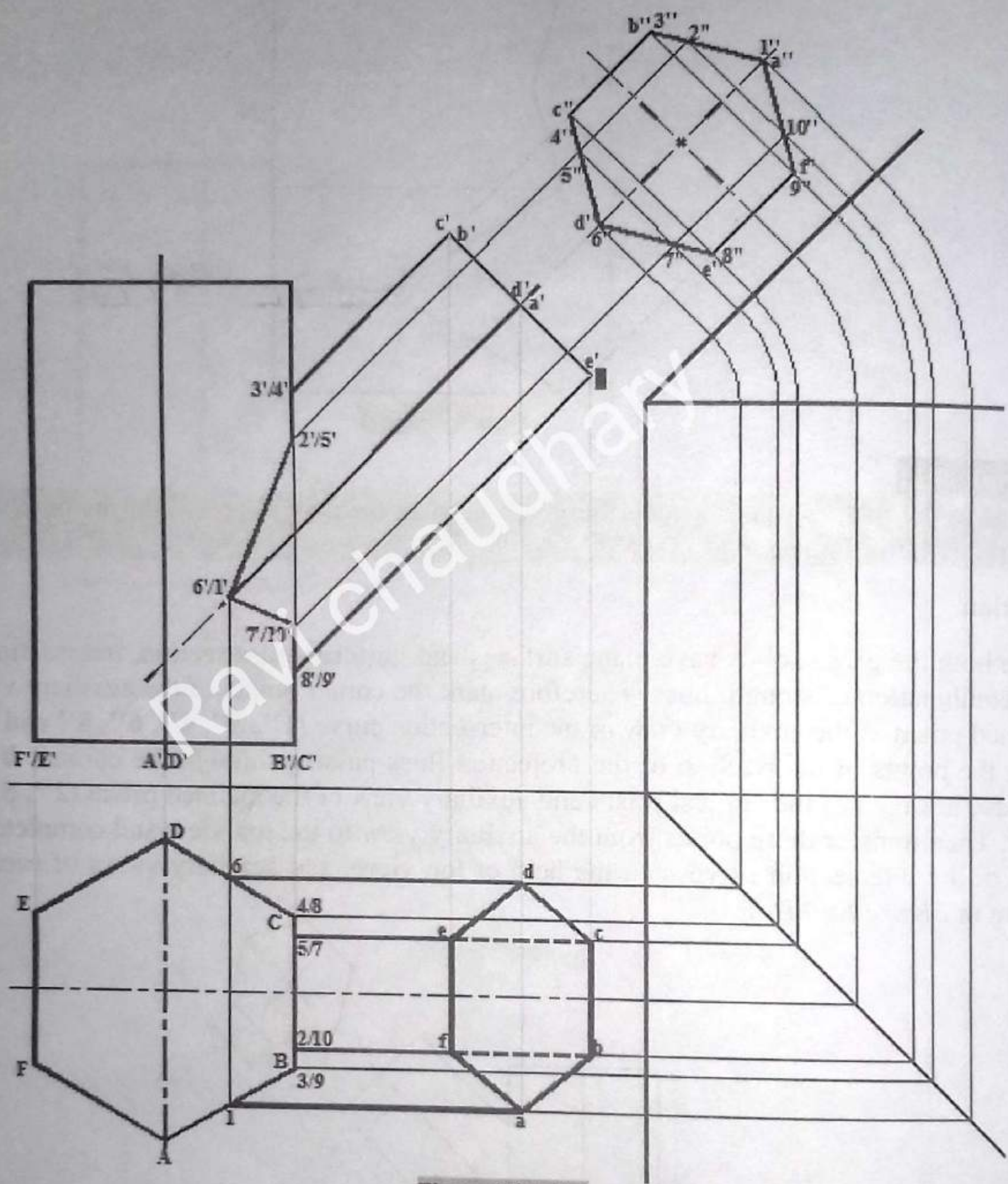


Figure E9.11(a)

Example 9.12

Complete the intersection between the given vertical hexagonal prism and inclined cylinder shown in *Figure E9.12*.

Solution

Since both of the given solids have uniform cross section and one of the solid has curved surface, intersection appears as a curve. Therefore take any number of points on the auxiliary view of the inclined cylinder as the auxiliary view of the intersection curve. Then transfer these points from the auxiliary view to the top view and complete the front view of the intersection curve with the help of top views and side views of each point as shown in *Figure E9.12(a)*.

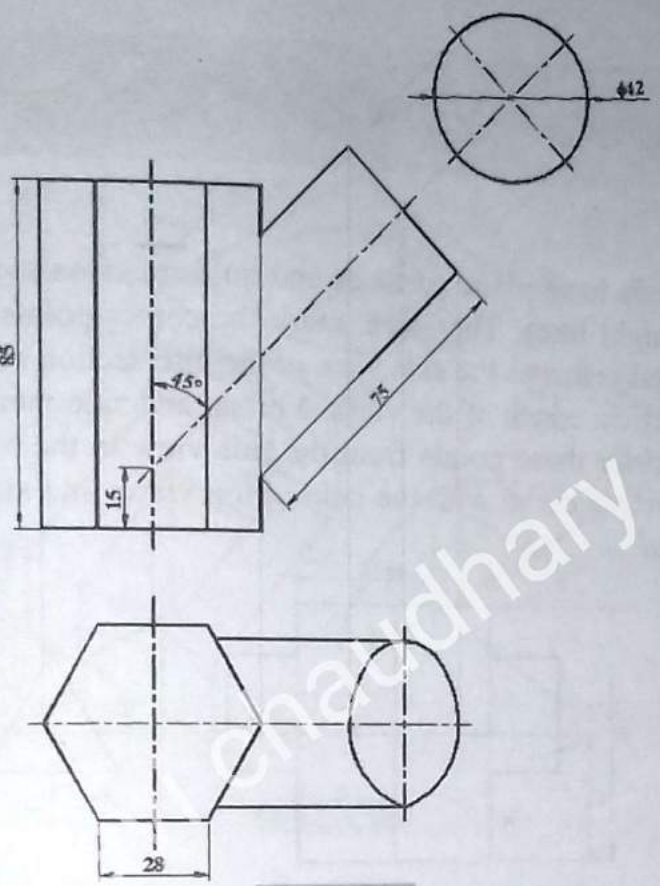


Figure E9.12

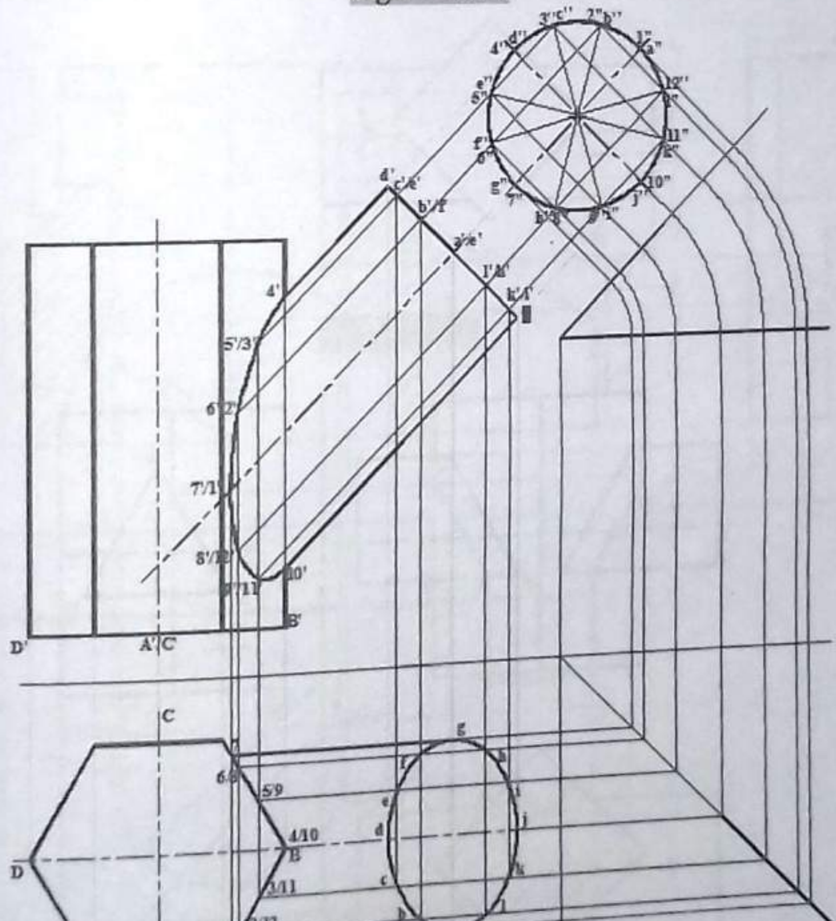


Figure E9.12(a)

Example 9.13

Complete the intersection between the given prisms with their axes offset as shown in *Figure E9.13.*

Solution

Since both the given solids have plane surfaces and uniform cross section, intersection appears as a combination of straight lines. Therefore, mark the corner points (**1''**, **3''**, and **5''**) of the side view of the horizontal prism as the side view of the intersection curve. Also mark the points of intersection of the vertical edges of the vertical prism and side view of the horizontal prism, i.e. **2''**and **4''**. Then transfer these points from the side view to the top view and complete the front view of the intersection curve with the help of top views and side views of each point as shown in *Figure E9.13(a)*.

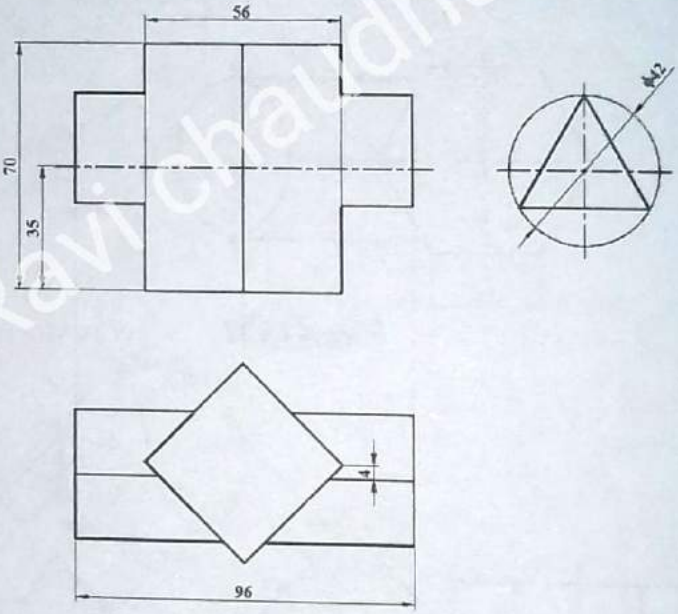


Figure E9.12

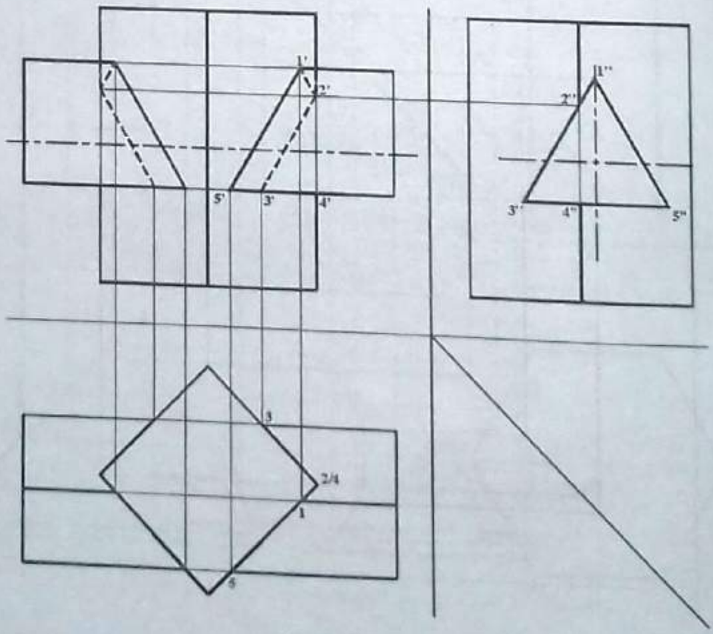


Figure E9.13(a)

Example 9.14

Draw orthographic views of an object shown in *Figure E9.14* with sectional front view. Also show the effects of intersection.

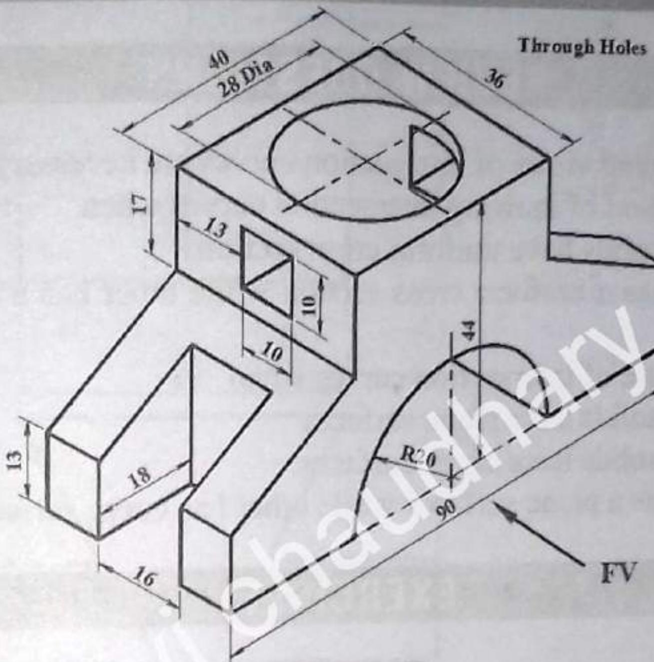


Figure E9.14

Solution

Orthographic views of the given object with sectional front view is shown in *Figure E9.14(a)*.

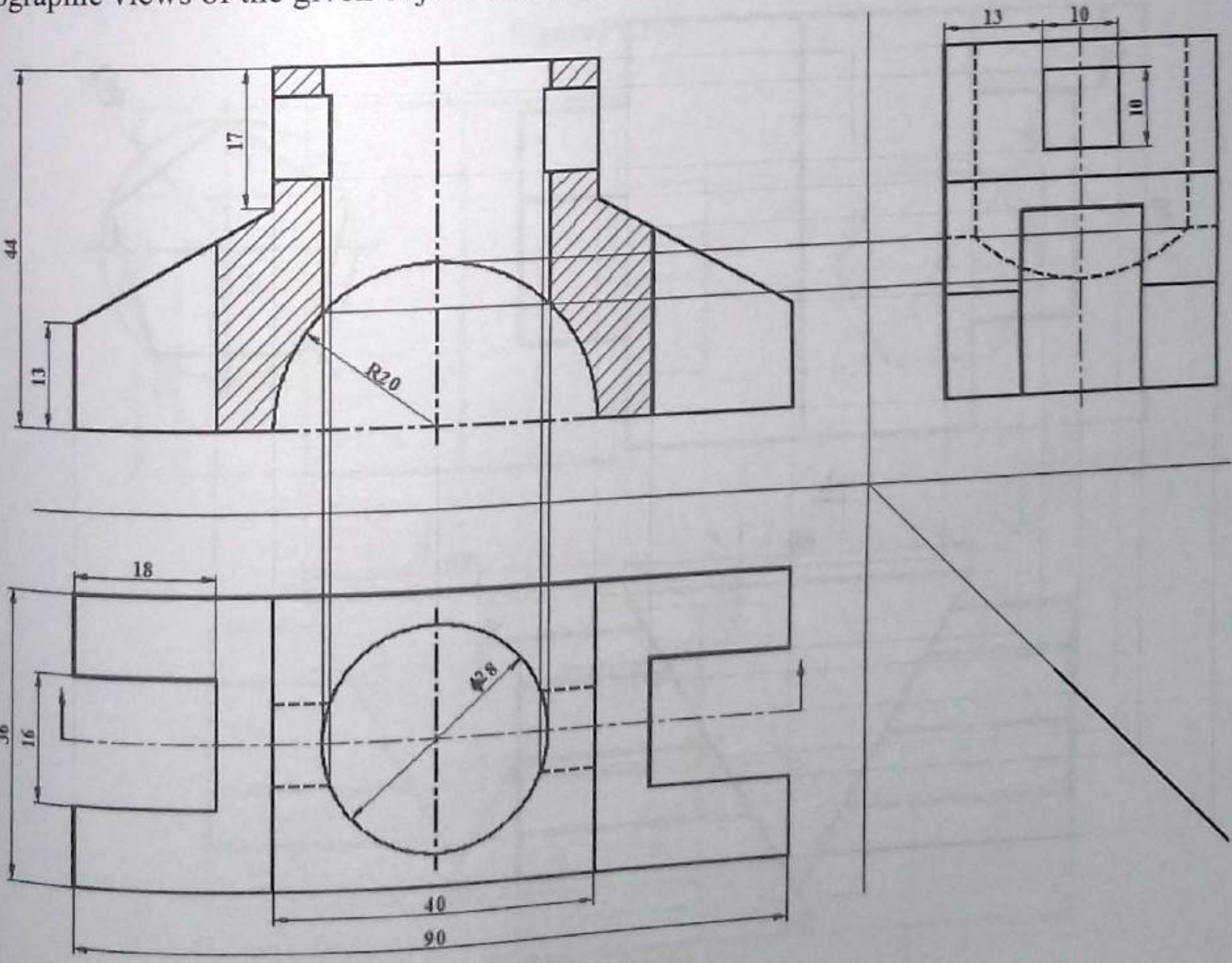


Figure E9.14(a)

The effect of intersection of horizontal square hole and vertical cylindrical hole is shown on the front view whereas the effect of intersection of the vertical and horizontal cylindrical holes is shown on the side view.

REVIEW QUESTIONS

1. Explain why orthographic views of intersection curves are necessary in engineering.
2. Discuss about the method of drawing intersection curves when
   (a) both intersecting solids have uniform cross section.
   (b) one of the solid has a uniform cross section while other has a uniformly varying cross section.
3. Discuss about the nature of intersection curves when
   (a) both intersecting solids have plane surfaces.
   (b) both intersecting solids have curve surfaces.
   (c) one of the solid has a plane surfaces while other has curve surface.

EXERCISES

1. Complete the intersection between the given two prisms and develop the surfaces of both solids.

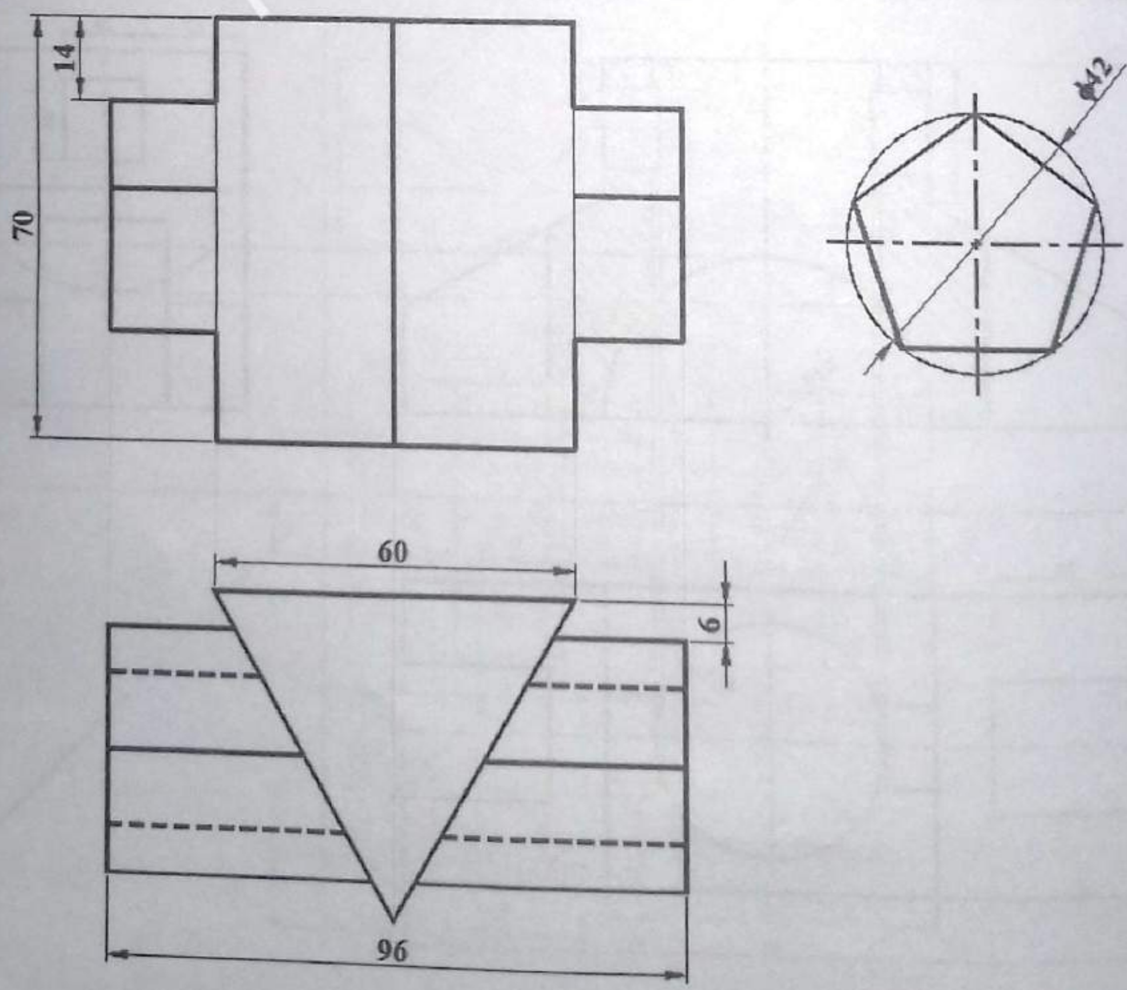


Figure P9.1(a)