Engineering Physics

Bachelor of Engineering

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Chapter 1

WAVE AND OSCILLATIONS

1.1 SIMPLE HARMONIC MOTION

Any motion that repeats itself at regular intervals is called *periodic motion* or *harmonic motion*.

A particle may be said to execute a *simple harmonic motion (S.H.M.)* if its acceleration is proportional to its displacement from its equilibrium position or any other fixed point in its path and is always directed towards it.

The differential equation of the simple harmonic motion is

$$\frac{d^2y}{dt^2} + \omega^2 y = 0 \qquad 1.1$$

where, y is the displacement of the particle of mass m.

$\omega = \sqrt{\frac{k}{m}}$ is often called angular velocity of object executing S.H.M..

k is a positive constant, called force constant.

The mechanical energy of the S.H.M. is constant and independent of time. The total energy E is proportional to square of the amplitude of the oscillation A.

i.e., $E = kA^2$ 1.2

1.2 EXAMPLES OF S.H.M.

i) The block spring system

The block spring system executes a simple harmonic motion with an angular frequency ω and time period of oscillation T as;

$$\left.\begin{aligned} \omega &= \sqrt{\frac{k}{m}} \\ T &= \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} \end{aligned}\right\} \qquad 1.3$$

ii) Simple pendulum

A simple pendulum is an idealized system in which a point mass is suspended at one end of an inextensible weightless string whose other

end is fixed in a rigid support. This fixed point is referred as the poin
suspension.

Within the small angle approximation, simple pendulum exec
S.H.M. with angular frequency,

$$\omega = \sqrt{\frac{g}{l}}$$

where, g is acceleration due to gravity

l is length of pendulum being measured from the point
suspension to the centre of mass of the bob.

iii) **Physical pendulum**

A physical pendulum or compound pendulum or rigid pendulum i
rigid body, of whatever of shape, capable of oscillating about a horizon
axis passing through it.

The point in which vertical plane passing through the c. g. of t
pendulum meets the axis of rotation is called its centre of suspension a
the distance between the point of suspension and c. g. of the pendulu
measures the length of the pendulum.

The physical pendulum executes S.H.M. with time period,

$$T = 2\pi\sqrt{\frac{I}{mgl}} = 2\pi\sqrt{\frac{m(k^2+l^2)}{mgl}} = 2\pi\sqrt{\frac{(k^2+l^2)}{gl}}$$

i.e., $$T = 2\pi\sqrt{\frac{\frac{k^2}{l}+l}{g}} = 2\pi\sqrt{\frac{L}{g}}$$

where, $I = m(k^2 + l^2)$ is moment of inertia of the pendulum about t
axis of suspension

k is the radius of gyration about an axis passing through the c.
of the pendulum

The equation (1.6) implies that the time period of the physical pendulu
is same as that of the simple pendulum of the length $L = \frac{k^2}{l} + l$. Th
length is, therefore, called the length of an equivalent simple pendulu
or reduced length of the physical pendulum.

iv) **Torsional pendulum**

A heavy body like a disc or a cylinder, fastened at its mid-point to a fairl
long and thin wire, suspended from a rigid support, con
torsional pendulum.

f the disc or the cylinder is Turned in its own plane *i.e.*, in the
orizontal plane, to twist the wire a little and then released, it executes
orsional vibrations or oscillations about wire as axis. That's why, this
endulum is called torsional pendulum.

The disc or the cylinder executes an angular S.H.M. with time period as;

$$T = 2\pi\sqrt{\frac{I}{C}} \qquad 1.7$$

where, $C = \frac{\pi\eta R^4}{2L}$ is torsional couple per unit twist of the wire or
torsional constant.

R is the radius of the wire

L is the length of the wire

η is the modulus of rigidity of its material

The time period of a torsional pendulum remains unaffected even if the
amplitude be large, provided that the elastic limit of the suspension wire
is not exceeded.

1.3 FREE, FORCED, DAMPED AND RESONANT OSCILLATIONS

Free Oscillation

When a body capable of oscillation is displaced from its equilibrium
position and then left free, it begins to oscillate with a definite amplitude
and frequency. If the body is not restricted by any kind of friction, the
motion continues. Such oscillation is called free oscillation. *Example:*
When a bob of simple pendulum is displaced from its mean position and
left free, it executes a free oscillation.

Forced Oscillation

When a body is maintained in a state of oscillation by an external
periodic force of frequency other than the natural frequency of the body,
the oscillation is called forced oscillation. A body can be forced to
oscillate with any frequency depending upon that of the applied periodic
force. The oscillation dies out as soon as the applied force is removed.

Damped Oscillation

The oscillation whose amplitude goes on decreasing with time is called
damped oscillation. Practically all oscillations are damped. An oscillation
whose amplitude remains constant with time is called undamped oscillation.

Resonant Oscillation

When a body is maintained in a state of oscillation by a periodic f having the same frequency as the natural frequency of the body, oscillation is called resonant oscillation. The phenomenon of produ resonant oscillation is called resonance.

1.4 QUALITY FACTOR

Mathematically, quality factor Q is defined as 2π times the rati energy store in the system to the energy cycle. It is sometimes ca figure of merit of the harmonic oscillator. The figure of merit is def as the ratio of the frequency at velocity resonance to the full bandw at half maximum power. It measures the sharpness of the resonance.

1.5 SOLVED EXAM QUESTIONS

1. **Define damped oscillation and derive the differential equa of damped oscillation of a mechanical system. Also deriv frequency of the oscillation. [T.U. 2061 Baish**

Solution:

Damped Oscillation:

The oscillation whose amplitude goes on decreasing with tin called damped oscillation. Practically all oscillations are dam Energy of such a damped oscillator decreases with time.

Whenever a pendulum oscillates in the air, the energy of pendulum dissipated in each oscillation. After a long time vibration die out. The dissipative force is proportional to velo of particle at that time.

i.e., $F \propto \frac{dx}{dt}$

or, $F = -b\frac{dx}{dt}$

where, b is the damping constant. The differential equatio motion of in this case will be;

$$F = -kx - b\frac{dx}{dt} = m\frac{d^2x}{dt^2}$$

or, $m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$

i.e., $\frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = 0$

This is known as differential equation of damped oscillation of a mechanical system. The solution of this equation is

$$x = ae^{-\left(\frac{b}{2m}\right)t}\sin(\omega' t) \quad 1.10$$

where, ω' is called damped angular frequency and given by;

$$\omega' = \sqrt{\left(\frac{k}{m} - \frac{b^2}{4m^2}\right)} \quad 1.11$$

Thus, the frequency of damped oscillation is;

$$f' = \frac{\omega'}{2\pi} = \frac{1}{2\pi}\sqrt{\left(\frac{k}{m} - \frac{b^2}{4m^2}\right)} \quad 1.12$$

The damped angular frequency ω' is less than natural angular frequency ω_0.

A damped oscillator has mass 250 g, spring constant 85 N/m and damping constant 70 g/s.

i) **How long does it take for the amplitude of the damped oscillator to drop to half of its initial value?**

ii) **How long does it take for the mechanical energy to drop to half of its initial value?**

Solution:

Here,

Mass of damped oscillator, $(m) = 250\ g = 0.25\ kg$

Spring constant, $(k) = 85\ Nm^{-1}$

Damping constant, $(b) = 70\ gs^{-1} = 0.07\ kgs^{-1}$

i) We have,

The amplitude variation of damped oscillator with time,

$$A = A_0 e^{-\left(\frac{b}{2m}\right)t} \quad 1.12$$

When amplitude of the damped oscillator drops to half of its initial value, *i.e.,* $A = \frac{A_0}{2}$, the equation (1.12) leads us to;

$$\frac{A_0}{2} = A_0 e^{-\left(\frac{b}{2m}\right)t}$$

or, $\frac{1}{2} = e^{-\left(\frac{b}{2m}\right)t}$

or, $ln\left(\frac{1}{2}\right) = -\left(\frac{b}{2m}\right)t$

or, $-0.693 = -\left(\frac{0.07}{2 \times 0.25}\right)t$

$\therefore\ t = 4.95\ s$

The damped oscillator takes 4.95 s for the amplitude to drop half of its initial value.

ii) We have,
At any point the mechanical energy of the damped oscillator be calculated using the expression,

$$E = E_0 e^{-\left(\frac{b}{m}\right)t}$$

When mechanical energy of the damped oscillator drops to half its initial value, *i.e.*, $E = \frac{E_0}{2}$, the equation (1.13) leads us to

$$\frac{1}{2} = e^{-\left(\frac{b}{m}\right)t}$$

or, $ln\left(\frac{1}{2}\right) = -\left(\frac{b}{m}\right)t$

or, $-0.693 = -\frac{0.07}{0.25} \times t$

$\therefore\ t = 2.48\ s$

The damped oscillator takes 2.48 s for the mechanical energ drop half of its initial value.

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3. **A uniform rod 1 m in length oscillates about a horizontal perpendicular to its length along the vertical plane. Find position of the points about which the time period is minim If g = 9.8 ms⁻², Find the minimum period of oscillation.**
[T.U. 2061 Ashw

Solution:
Let $l_1\ m$ be the distance from centre of suspension to the centr gravity, then, $(1 - l_1)\ m$ be the distance from centre of oscilla to the centre of gravity.
The time period of pendulum will be minimum when its leng is equal to its radius of gyration k.

i.e., $l_1 = k$

or, $l_1^2 = k^2 = l_1(1 - l_1)$

or, $2l_1 = 1$

$\therefore\ l_1 = 0.5\ m$

Hence, the centre of suspension and centre of oscillation are at distance of 0.5 m from the centre of gravity.
The minimum time period of oscill

$$T_{min} = 2\pi\sqrt{\frac{(k^2 + l_1^2)}{gl_1}} = 2\pi\sqrt{\frac{(l_1^2 + l_1^2)}{gl_1}} = 2\pi\sqrt{\frac{2l_1}{g}}$$

$$= 2\pi\sqrt{\frac{2 \times 0.5}{9.8}} = 2.01\ s$$

Define the terms: frequency and amplitude of the simple harmonic motion. Derive the expression for the time period of (i) physical pendulum and (ii) spring mass system. **[T.U. 2062 Baishakh]**

olution:

Amplitude and frequency of the simple harmonic motion

The maximum displacement of the particle on either side of the equilibrium position of S.H.M. is called amplitude of S.H.M.
The number of the complete oscillations per second is called frequency of S.H.M.

Physical pendulum

A physical pendulum or compound pendulum or rigid pendulum is a rigid body, of whatever of shape, capable of oscillating about a horizontal axis passing through it.
The point in which vertical plane passing through the c.g. of the pendulum meets the axis of rotation is called its centre of suspension and the distance between the point of suspension and c.g. of the pendulum measures the length of the pendulum.

Figure: A physical pendulum

Figure depicts the vertical section of a rigid body, *i.e.*, a physical pendulum, free to rotate about a horizontal axis passing through the point called centre of suspension S. In its normal position of rest, its g. G, naturally lies vertically below S, the distance between S and G giving the length l of the pendulum.
When the pendulum is displaced through a small angle θ so that its c. g. takes up new position G', where $SG' = l$. The weight of the pendulum is acting vertically downward at G' and its reaction at the point of suspension S' constitutes the couple tending to bring back into its original position.
The moment of restoring couple $= -mgl\sin\theta$

i.e., $I\dfrac{d^2\theta}{dt^2} = -mgl\ \sin\theta$

where, I is the moment of inertia of the pendulum about th

of suspension and $\dfrac{d^2\theta}{dt^2}$ is the angular acceleration of the pendu

For a small angular displacement, we write,

$$\frac{d^2\theta}{dt^2} + \frac{mgl}{I}\theta = 0$$

The pendulum thus executes a simple harmonic motion an time period is,

$$T = 2\pi\sqrt{\frac{I}{mgl}}$$

The moment of inertia of the pendulum about an axis pa through S and perpendicular to its plane;

$$I = mk^2 + ml^2 = m(k^2 + l^2)$$

Thus,

$$T = 2\pi\sqrt{\frac{m(k^2+l^2)}{mgl}} = 2\pi\sqrt{\frac{(k^2+l^2)}{gl}}$$

i.e., $T = 2\pi\sqrt{\dfrac{\frac{k^2}{l}+l}{g}} = 2\pi\sqrt{\dfrac{L}{g}}$

where, k is the radius of gyration about an axis passing thro the c. g. of the pendulum.

The equation (1.18) implies that the time period of the phy pendulum is same as that of the simple pendulum of the le $l = \dfrac{k^2}{l} + l$. This length is, therefore, called the length of equivalent simple pendulum or reduced length of the phy pendulum.

Spring Mass System

Consider a block of mass is oscillating at the end of a mass less spring as shown in figure. Assume that the net force acting on the block is that exerted by the spring which is obtained from the Hook's law.

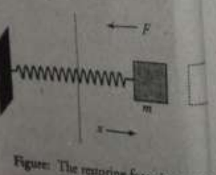

Figure: The restoring force in a sprin mass system

i.e., $F = -kx$

where, x is displacement from equilibrium position and k is force constant

From Newton's law,

$$a = \frac{F}{m} = -\frac{k}{m}x$$

i.e., $\dfrac{d^2x}{dt^2} + \dfrac{k}{m}x = 0$ 1.19

This equation shows that spring mass system executes a simple harmonic motion. Comparing equation (1.19) with differential equation of S.H.M., angular frequency ω is,

$$\omega = \sqrt{\frac{k}{m}}$$

The time period of spring mass system is,

$T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{m}{k}}$ 1.20

Give the necessary theory of forced vibration and deduce the condition for amplitude and velocity resonance. Explain the sharpness of resonance and relate it with quality factor.

[T.U. 2063 Baishakh]

lution:

Forced Oscillation

When a body is maintained in a state of oscillation by an external periodic force of frequency other than the natural frequency of the body, the oscillation is called forced oscillation. A body can be forced to oscillate with any frequency depending upon that of the applied periodic force. The oscillation dies out as soon as the applied force is removed.

Assume that the applied force is sinusoidal and represented as;

$$F_{ext} = F_0 \sin\omega t$$

where, ω is applied angular frequency. Newton's second law in such a forced oscillator yields,

$m\dfrac{d^2x}{dt^2} + b\dfrac{dx}{dt} + kx = F_0\sin\omega t$ 1.21

The energy dissipated by damping is balanced with the external energy. The solution of such differential equation is;

$x = A_0\sin(\omega t + \phi_0)$ 1.22

where, ϕ_0 is the phase angle between the displacement x and the external force F_{ext}. On substituting the value of x into equation

(1.21), we obtain the amplitude of oscillation and phase angle ϕ_0 i terms of frequency of external force ω and natural frequency ω_0 a

$$A_0 = \frac{F}{m\sqrt{\left[(\omega^2-\omega_0^2)^2+\left(\frac{b\omega}{m}\right)^2\right]}}$$

$$\phi_0 = tan^{-1}\left\{\frac{m(\omega_0^2-\omega^2)}{\omega b}\right\}$$

Figure depicts that the amplitude becomes large when the appl frequency ω is near the natural frequency ω_0. When damping small, the amplitude near $= \omega_0$, will become very large. Th condition is known as resonance. Thus, when a periodic force frequency equal to natural frequency of a body, is applied, t body gains amplitude slowly and finally begins to vibrate w large amplitude. Hence resonance is defined as the phenomenon of setting a body into vibrations of its natural frequency by the application of an external periodic force of the same frequency. Such frequency is called resonant frequency. Resonance is also said to be state of vibration with maximum amplitude. This is also termed as amplitude resonance.

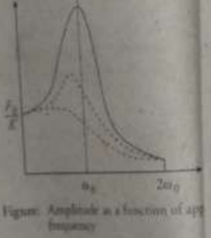

Figure: Amplitude as a function of app frequency

The Sharpness of the resonance refers to fall in amplitude v change in frequency on each side of maximum amplitude. As frequency of the applied force ω is increased or decreased from resonant value ω_0, the value of the amplitude always decrea When the amplitude at resonance falls rapidly as the frequenc the applied force is changed slightly from its resonant value, resonance is said to be sharp.

The sharpness of resonance is inversely proportional to damping constant b and can be represented in terms of qu factor Q. Quality factor is defined as 2π times the ratio of en store in the system to the energy cycle. It is sometimes ca

figure of merit of the harmonic oscillator. The figure of merit is defined as the ratio of the frequency at velocity resonance to the full bandwidth at half maximum power. It measures the sharpness of the resonance.

Quality factor, $Q = 2\pi\left(\frac{\omega_0}{\omega_1-\omega_2}\right)$

If ω_1 and ω_2 be the frequencies at which amplitude becomes $\frac{A_0}{\sqrt{2}}$, the quantity $(\omega_1-\omega_2)$ is called width of the resonance peak. A larger value of quality factor represents a system of high quality with narrow resonance peak.

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6. For a compound pendulum prove that both the length of point of suspension and point of oscillation from centre of gravity equal to radius of gyration. **[T.U. 2063 Baishakh]**

Solution:

The time period of the compound pendulum is;

$$T = 2\pi\sqrt{\frac{(k^2+l^2)}{gl}} = 2\pi\sqrt{\frac{\frac{k^2}{l}+l}{g}}$$

where, k is radius of gyration about centre of gravity and l is the distance of the point of suspension from centre of gravity. Squaring this equation, we obtain,

$$T^2 = \frac{4\pi^2}{g}\left(\frac{k^2}{l}+l\right)$$

Differentiating with respect to l, we obtain,

$$2T\frac{dT}{dl} = \frac{4\pi^2}{g}\left(-\frac{k^2}{l^2}+1\right)$$

Obviously, T will be a maximum or a minimum when $\frac{dT}{dl} = 0$.

i.e., $l^2 = k^2$

$\therefore$ $l = k$

If l' be the distance of the point of oscillation from centre of gravity, then,

$l^2 = k^2 = ll'$

i.e., $l = l'$

$\therefore$ $l = k = l'$

Thus, the length of point of suspension and point of oscillation from the centre of gravity are equal to radius of gyration.

7. **A spring is hanging vertically and loaded with a mass of 100 g and allowed to oscillate. Calculate (i) time period of oscillation and (ii) frequency of oscillation if the spring is further loaded with 200 g producing an extension of 10 cm. [T.U. 2063 Baishakh]**

Solution:

When additional 200 g is loaded on spring, it produces the extension of 10 cm. Thus,

$$mg = kx$$

$$\text{or,}\quad k = \frac{mg}{x}$$

$$\therefore\quad k = 0.2\,kg \times \frac{9.8\,ms^{-2}}{0.1\,m} = 19.6\,N/m$$

i) Time period of oscillation,

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.1\,kg}{19.6\,N/m}} = 0.45\,s$$

ii) Frequency of oscillation,

$$f = \frac{1}{T} = \frac{1}{0.45\,s} = 2.22\,Hz$$

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8. **What is S.H.M.? Discuss the theory of a simple spring mass system and derive an expression for its period and frequency. [T.U. 2064 Poush]**

Solution:

Simple Harmonic Motion [S.H.M.]

A particle may be said to execute a *simple harmonic motion [S.H.M.]* if its acceleration is proportional to its displacement from its equilibrium position or any other fixed point in its path and is always directed towards it.

Spring mass system

See the solution of Q. No. 4 on page no. 7

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9. **Show for a bar pendulum that the time period is minimum points of suspension and oscillation are equidistant from C.G. [T.U. 2064 Poush]**

Solution:

The time period of the compound pendulum is,

$$T = 2\pi\sqrt{\frac{(k^2 + l^2)}{gl}} = 2\pi\sqrt{\frac{\frac{k^2}{l} + l}{g}}$$

where, k is radius of gyration about centre of gravity and l is the distance of the point of suspension from centre of gravity. Squaring this equation, we obtain,

$$T^2 = \frac{4\pi^2}{g}\left(\frac{k^2}{l} + l\right).$$

Differentiating with respect to l, we obtain,

$$2T\frac{dT}{dl} = \frac{4\pi^2}{g}\left(-\frac{k^2}{l^2} + 1\right)$$

Obviously, T will be a maximum or a minimum when $\frac{dT}{dl} = 0$.

$$\text{i.e.,}\quad l^2 = k^2$$

$$\therefore\quad l = k$$

The negative value of k is simply meaningless. Since $\frac{d^2T}{dl} > 0$, it is obvious that time period T is a minimum when $l = k$, *i.e.*, the time period of a compound pendulum is the minimum when its length is equal to its radius of gyration about an axis through its C.G.

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10. **Explain the terms free vibration, damped vibration, forced vibration and resonance. Develop the differential equation of a particle executing damped vibrations in a medium. Explain the physical meaning of each term and each constant in the equation. [T.U. 2065 Shrawan]**

Solution:

Free Vibration

When a body capable of vibration is displaced from its equilibrium position and then left free, it begins to oscillate with a definite amplitude and frequency. If the body is not restricted by any kind of friction, the motion continues. Such vibration is called free vibration. *Example:* When a bob of simple pendulum is displaced from its mean position and left free, it executes a free vibration.

Damped Vibration

The vibration whose amplitude goes on decreasing with time is called damped vibration. Practically all vibrations are damped. A vibration whose amplitude remains constant with time is called undamped vibration.

Forced Vibration

When a body is maintained in a state of vibration by an external periodic force of frequency other than the natural frequency of the body, the vibration is called forced vibration. A body can be forced to vibrate with any frequency depending upon that of the applied periodic force. The vibration dies out as soon as the applied force is removed.

Resonance

When a body is maintained in a state of vibration by a periodic force having the same frequency as the natural frequency of a body, the vibration is called resonant vibration. The phenomenon of producing resonant vibration is called resonance.

The differential equation of a particle executing damped vibrations in medium

The dissipative force of damped vibration is proportional to velocity of particle at that instant and velocity exponentially in the time,

i.e., $F = -b\dfrac{dx}{dt}$

where, b is damping constant and is measured in kg/s. The differential equation of motion for damped vibration will be,

$$\begin{aligned} F &= -kx - b\frac{dx}{dt} \\ &= m\frac{d^2x}{dt^2} \end{aligned}$$

$$\text{or,}\quad m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$

$$\text{i.e.,}\quad \frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = 0$$

This is known as differential equation of damped oscillation of mechanical system. The solution of this equation is;

$$x = ae^{-\left(\frac{b}{2m}\right)t}\sin(\omega' t)$$

where, ω' is called damped angular frequency and given by;

$$\omega' = \sqrt{\left(\frac{k}{m} - \frac{b^2}{4m^2}\right)}$$

Thus, the frequency of damped oscillation is;

$$f' = \frac{\omega'}{2\pi} = \frac{1}{2\pi}\sqrt{\left(\frac{k}{m} - \frac{b^2}{4m^2}\right)}$$

The damped angular frequency ω' is less than natural angular frequency ω_0. For the exponential relation of displacement, the amplitude decreases to zero in long time. The time duration in which the amplitude drops to $\frac{1}{e}$ times of its initial value is called mean life time of oscillation.

11. What is compound pendulum? Deduce the expression for the time period of a compound pendulum and formulate the equivalent length of the simple pendulum. **[T.U. 2065 Chaitra]**

Solution: See the solution of Q. No. 4 on page no. 7

12. Differentiate between free oscillation and forced oscillation. **[T.U. 2065 Kartik]**

Solution:

Free Oscillation

When a body capable of oscillation is displaced from its equilibrium position and then left free, it begins to oscillate with a definite amplitude and frequency. If the body is not restricted by any kind of friction, the motion continues. Such oscillation is called free oscillation. *Example:* When a bob of simple pendulum is displaced from its mean position and left free, it executes a free oscillation.

Forced Oscillation

See the solution of Q. No. 5 on page no. 9

13. What is torsional pendulum? Find the time period for torsional pendulum. Also write its significance. **[T.U. 2065 Kartik]**

Solution:

Torsional pendulum

A heavy body like a disc or a cylinder, fastened at its mid-point to a fairly long and thin wire, suspended from a rigid support, constitutes a torsional pendulum.

If the disc or the cylinder is turned in its own plane *i.e.*, in the horizontal plane, to twist the wire a little and then released, it executes torsional vibrations or oscillations about wire as axis. That's why, this pendulum is called torsional pendulum.

Figure: A torsional pendulum

When the disc or the cylinder is turned through an angle θ, the suspension wire gets twisted through the same angle and this give rise to torsional couple $-C\theta$ in it, tending to bring it back into its original condition.

Here,

Torsional couple per unit twist of the wire or torsional constant is;

$$C = \frac{\pi\eta R^4}{2L} \quad \dots (i)$$

R is the radius of the wire.
L is the length of the wire.
η is the modulus of rigidity of its material.
If I be the moment of inertia of the disc about the wire as an axis, the couple acting on it is equal to $I\frac{d^2\theta}{dt^2}$, where, $\frac{d^2\theta}{dt^2}$ is the angular acceleration of disc. Thus, we can write,

$$I\frac{d^2\theta}{dt^2} = -C\theta$$

i.e., $$\frac{d^2\theta}{dt^2} + \frac{C}{I}\theta = 0$$

The disc or the cylinder executes an angular S.H.M. with time period as:

$$T = 2\pi\sqrt{\frac{I}{C}}$$

Notice that there is no approximation whatever has been used to determine time period. The time period of a torsional pendulum, remains unaffected *i.e., oscillations remain isochronous*, even if the amplitude be large. The elastic limit of the suspension wire, however, is not exceeded.

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14. A spring is hung vertically and loaded with a mass of 75 g and allowed to oscillate. Calculate (i) time period and (ii) frequency of oscillation, when the spring is further loaded with 100 g producing an extension of 5 cm. **[T.U. 2065 Kartik]**

Solution:

When additional 100 g is loaded on spring, it produces the extension of 5 cm. Thus,

$$mg = kx$$

or, $$k = \frac{mg}{x}$$

$\therefore$ $$k = 0.1\,kg \times \frac{9.8\,ms^{-2}}{0.05\,m}$$

$$= 19.6\,Nm^{-1}$$

i) Time period of oscillation.

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.075\,kg}{19.6\,Nm^{-1}}} = 0.39\,s$$

ii) Frequency of oscillation,

$$f = \frac{1}{T} = \frac{1}{0.39\,s} = 2.56\,Hz$$

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15. Show that there are four collinear points within compound pendulum having same time period. Give their physical significance. **[T.U. 2067 Ashadh]**

Solution:

The time period of the compound pendulum is;

$$T = 2\pi\sqrt{\frac{(k^2+l^2)}{gl}} = 2\pi\sqrt{\frac{\frac{k^2}{l}+l}{g}}$$

where, k is the radius gyration and l is the length of pendulum from point of suspension to centre of gravity of the pendulum. If l' be the distance of the point of oscillation from centre of gravity, the time period of pendulum is;

$$T' = 2\pi\sqrt{\frac{\frac{k^2}{l'}+l'}{g}}$$

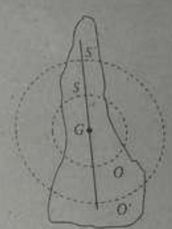

Figure: Centre of suspensions and oscillations in compound pendulum

We have,

$$k^2 = ll'$$

Thus,

$$\frac{k^2}{l} = l'$$

and $$\frac{k^2}{l'} = l$$

This implies,

$$\frac{\frac{k^2}{l}+l}{g} = \frac{\frac{k^2}{l'}+l'}{g} = l + l'$$

$\therefore$ $T = T'$

i.e., the time period of the pendulum is same about the point of suspension S and point of oscillation O. This implies that these points are interchangeable.

There are two other points on either side of G about which the time period of the pendulum is same as S and O. If we sketch the two circles about centre G with the radii l and l' respectively, the SG produced will cut S and S' above and O and O' below G. From figure,
We have,

$SG = GO' = l$

and $GO = GS' = \frac{k^2}{l} = l'$

Thus,

$O'S' = O'G + GS' = l + l' = OS$

Hence, there are four collinear points, *i.e.*, S, S', O and O', colline[...] with c. g. of pendulum G about which its time period is same.

---

16. **What is a torsional pendulum? Obtain an expression for its tim[...] period and explain why, unlike a simple or a compoun[...] pendulum the time period in this case remains unaffected even [...] the amplitude be large?** [T.U. 2067 Mangsi[...]

Solution: See the solution of Q. No. 13 on page no. 15

---

17. **A meter stick suspended from one end swings as a physica[...] pendulum (a) what is the period of oscillation? (b) What woul[...] be the length of the simple pendulum that would have the sam[...] period?** [T.U. 2067 Mangsi[...]

Solution:

The moment of inertia of thin rod about an axis through one en[...]

$$I = \frac{1}{3}ml^2$$

Since the $c.g.$ is at the centre,

$$l' = \frac{l}{2}$$

We have, time period of the pendulum,

$$T = 2\pi\sqrt{\frac{I}{mgl}}$$

or, $T = 2\pi\sqrt{\frac{\frac{1}{3}ml^2}{mg\frac{l}{2}}}$

$\therefore$ $T = 2\pi\sqrt{\frac{2l}{3g}} = 2\pi\sqrt{\frac{2 \times 1\,m}{3 \times 9.8\,ms^{-2}}} = 1.64\,s$

A simple pendulum must have a length L in order to have sam[...] time period of compound pendulum. Thus,

$$T = 2\pi\sqrt{\frac{L}{g}}$$

or, $L = \frac{gT^2}{4\pi^2} = \frac{9.8\,ms^{-2} \times (1.64\,s)^2}{4\pi^2}$

$\therefore$ $L = 0.67\,m$

---

18. **List the common pendulums in practice. Which of them is a physical pendulum and why? Show that point of suspension and point of oscillations are interchangeable.** [T.U. 2068 Shrawan]

Solution:

The common pendulums are;

i) Mass spring system
ii) Simple pendulum
iii) Compound pendulum
iv) Torsional pendulum
v) LC circuit
vi) Helmholtz resonator

A compound pendulum is a physical pendulum. It is a rigid body, of whatever of shape, capable of oscillating about a horizontal axis passing through it.

For the remaining part

See the solution of Q. No. 15 on page no. 17

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19. **Explain the theory of simple mass spring system. Develop the relation for time period and frequency of two springs having spring constants K_1 and K_2 supporting a mass M between them on a frictionless horizontal table.** [T.U. 2068 Shrawan]

Solution:

Spring mass system

See the solution of Q. No. 4 on page no. 8

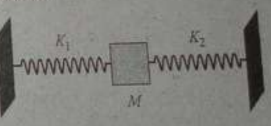

Figure: Two spring-coupled mass

Now, if the mass M is displaced to one side or the other through distance x, one spring gets extended and other compressed. Both of them exerting a restoring force on the mass in the same direction, tending to bring it back to its original position.

If F_1 and F_2 be the restoring forces due to the two springs respectively, the resultant restoring force,

$$F = F_1 + F_2 = -K_1x - K_2x = -(K_1 + K_2)x$$

This implies that the effecting force constant K is the combinations of two springs.

i.e., $K = (K_1 + +K_2)$

The time period of the oscillating mass M is, therefore,

$$T = 2\pi\sqrt{\frac{M}{K}} = 2\pi\sqrt{\frac{M}{(K_1+K_2)}}\sqrt{\frac{m}{k}}$$

The electrical analogue of this system is a series arrangement two capacitors.

---

20. **A thin straight, uniform rod of length $l = 1\ m$ and mass $m = 1$ gm hangs from a pivot at one end. (i) What is its period for sm amplitude oscillation? (ii) What is the length of a simp pendulum that will have the same period? [T.U. 2064 Pous**

Solution:

The moment of inertia of thin rod about an axis through one end

$$I = \frac{1}{3}ml^2$$

Since the c. g. is at the centre,

$$l' = \frac{l}{2}$$

Time period of the pendulum,

$$T = 2\pi\sqrt{\frac{I}{mgl}}$$

$$\therefore\ T = 2\pi\sqrt{\frac{\frac{1}{3}ml^2}{mg\frac{l}{2}}}$$

$$= 2\pi\sqrt{\frac{2l}{3g}} = 2\pi\sqrt{\frac{2\times 1\ m}{3\times 9.8\ ms^{-2}}}$$

$$= 1.64\ s$$

A simple pendulum must have a length L in order to have sa time period of compound pendulum. Thus,

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$i.e.,\ L = \frac{gT^2}{4\pi^2} = \frac{9.8\ ms^{-2}\times(1.64\ s)^2}{4\pi^2} = 0.67\ m$$

The equivalent length of simple pendulum, $L = 0.67\ m$.

---

21. **Define simple harmonic motion. Derive the relationship for ti period of (i) bar pendulum and (ii) torsional pendulum.[P.U..20**

Solution:

Simple Harmonic Motion (S.H.M.)

See the solution of Q. No. 8 on page no. 12

Compound pendulum

See the solution of Q. No. 4 on page no. 7

Torsional pendulum

See the solution of Q. No. 13 on page no. 15

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22. **What is an elastic restoring force? Obtain an expression for the time period of a compound pendulum and show that the centers of suspension and oscillations are interchangeable. [P.U. 2004]**

Solution:

Elastic restoring force

An elastic restoring force is defined as property by the virtue of which elastic bodies regains its original shape after removing deforming force.

Compound pendulum

See the solution of Q. No. 4 on page no. 7

Interchangeability of point of suspension and point of oscillation

See the solution of Q. No. 15 on page no. 17

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23. **Define harmonic motion, elastic restoring force and Hook's law. Find an expression for time period of torsional pendulum.[P.U. 2005]**

Solution:

A particle may be said to execute a *harmonic motion* if its acceleration is proportional to its displacement from its equilibrium position or any other fixed point in its path and is always directed towards it.

An elastic restoring force is defined as property by the virtue of which elastic bodies regains its original shape after removing deforming force.

A Hook's law states that "within an elastic limit, elastic stress is directly proportional to elastic strain."

Torsional pendulum

See the solution of Q. No. 13 on page no. 15

---

24. **Define simple harmonic motion and state its characteristics. Describe with necessary theory how you will determine the value of the modulus of rigidity of a metal in laboratory by using torsional pendulum. [P.U. 2006]**

Solution:

Simple harmonic motion

See the solution of Q. No. 8 on page no. 12

Notice that there is no approximation whatever has been used to determine time period. The time period of a torsional pendulum remains unaffected *i.e., oscillations remain isochronous,* even if the amplitude are large. The elastic limit of the suspension wire, however, is not exceeded.

Practically, if one regular body of given moment of inertia I_1 is placed on the disc. The time period of circular disc, T_1

$$T_1 = 2\pi\sqrt{\frac{I_1}{C}} \quad \dots \text{(ii)}$$

Time period of circular disk and ring is;

$$T_2 = 2\pi\sqrt{\frac{I_1 + I_2}{C}} \quad \dots \text{(iii)}$$

Thus,

$$I_1 = \left(\frac{T_1^2}{T_2^2 - T_1^2}\right) I_2 \quad \dots \text{(iv)}$$

This gives the moment of inertia of a circular disc.

The moment of inertia of circular ring.

$$I_2 = \frac{1}{2} m(r_1^2 + r_2^2) \quad \dots \text{(v)}$$

where, m is the mass of a ring, r_1 is the radius of ring and r_2 is the radius of external ring.

Form the equations (i), (ii) and (iii), we obtain,

$$\eta = \frac{8\pi l I_2}{r^4(T_2^2 - T_1^2)} \quad \dots \text{(vi)}$$

This is the required expression for the modulus of rigidity of a wire. Measuring time periods T_1 and T_2, moment of inertia I_2, length of suspension wire l and radius of suspension wire r, the modulus of rigidity can be determined.

---

25. Differentiate between S.H.M. and periodic motion. **[P.U. 2007]**

Solution:

A particle may be said to execute a *simple harmonic motion (S.H.M.)* if its acceleration is proportional to its displacement from its equilibrium position or any other fixed point in its path and is always directed towards it.

Any motion that repeats itself at regular interval of time is said to be periodic motion.

---

26. What is torsional pendulum? Find the time period for torsional pendulum. **[P.U. 2007]**

Solution: See the solution of Q. No. 13 on page no. 15

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27. Define simple harmonic motion. Why are really S.H.M.'s rare? Describe with necessary theory how you will determine the value of the modulus of rigidity of a metal in laboratory by using torsional pendulum. **[P.U. 2008]**

Solution:

Simple harmonic motion

See the solution of Q. No. 8 on page no. 12

Simple harmonic motion is based ideal assumptions, for example, in case of simple pendulum, the string attached to bob of pendulum is must be weightless and angle of displacement must be small during performing experiment. These assumptions are only ideal concept and in practice these are not possible.

Torsional pendulum

See the solution of Q. No. 13 on page no. 15

For the remaining part

See the solution of Q. No. 24 on page no. 21

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28. Define elastic restoring force. Derive an expression for the time period of a compound pendulum and prove that the centers of oscillation and suspension are interchangeable. **[P.U. 2010]**

Solution: See the solution of Q. No. 22 on page no. 21

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29. Define angular harmonic motion. Derive an expression for the time period of a compound pendulum and prove that the centers of oscillation and suspension are interchangeable. **[P.U. 2011]**

Solution:

Simple Harmonic Motion (S.H.M.)

See the solution of Q. No. 8 on page no. 12

For the remaining part

See the solution of Q. No. 28 on page no. 23

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30. A spiral spring 3 m long from the ceiling. When a mass of 1 kg is suspended from the spring it lengthens by 40 cm. The mass is then pulled and released. Compute the frequency of oscillation. **[P.U. 2002]**

Solution:

When 1 kg is loaded on spring, it produces the extension of 40 cm = 0.4 m. Thus,

$$mg = kx$$

$$\text{or, } k = \frac{mg}{x}$$

$$\therefore \quad k = 1\,kg \times \frac{9.8\,ms^{-2}}{0.4\,m} = 24.5\,N/m$$

The frequency of oscillation,

$$f = \frac{1}{2\pi}\sqrt{\frac{m}{k}} = \frac{1}{2\pi}\sqrt{\frac{1\,kg}{24.5\,Nm^{-1}}} = 3.21 \times 10^{-2}\,Hz$$

---

31. **The moment of inertia of a disc used in a tutorial pendul about the suspension wire is 0.2 kgm^2. It oscillates with a peri of 2 s. Another disc is placed over the first one and the ti period of the system becomes 2.5 s. Find the moment of inertia the second disc about the wire.** [P.U. 200

Solution:

Here,

Moment of inertia of first disc, $(I_1) = 0.2\,kg\,m^2$

Time periods, $(T_1) = 2\,s$

$(T_2) = 2.5\,s$

Moment of inertia of second disc, $(I_2) = ?$

We have,

$$I_1 = \left(\frac{T_1^2}{T_2^2 - T_1^2}\right) I_2$$

$$\text{or,} \quad I_2 = I_1\left(\frac{T_1^2}{T_2^2 - T_1^2}\right)^{-1} = 0.2\,kg\,m^2\left[\frac{(2\,s)^2}{(2.5\,s)^2 - (2\,s)^2}\right]^{-1} = 0.11\,kg\,m^2$$

Moment of inertia of second disc, $(I_2) = 0.11\,kg\,m^2$

1.6 ADDITIONAL SOLVED PROBLEMS

1. **A mass of 5 kg stretches a spring 0.5 m from its equilibrium position. The mass is removed and another body of mass 1 kg is hanged from the spring. What would be the period of motion if the spring is now stretched and released?**

Solution:

When 5 kg is loaded on spring, it produces the extension of 0.5 m. Thus,

$$mg = kx$$

$$\therefore \quad k = \frac{mg}{x} = 5\,kg \times \frac{9.8\,ms^{-2}}{0.5\,m} = 96\,N/m$$

Time period of oscillation,

$$T = 2\pi\sqrt{\frac{m'}{k}} = 2\pi\sqrt{\frac{1\,kg}{96\,Nm^{-1}}} = 0.64\,s$$

---

2. **A simple pendulum of one meter length is hanged at one end. Considering oscillations to be of small displacements, find period of oscillations if mass of pendulum is 2.0 kg.**

Solution:

Here,

Length of the pendulum, $(l) = 1\,m$

Mass of the pendulum, $(m) = 2\,kg$

We have,

Time period of the oscillation,

$$(T) = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{1}{9.8}}$$

$$= 2.01\,s$$

---

3. **An oscillatory motion of a body is represented by $y = ae^{i\omega t}$, where y is the displacement in time t, a is its amplitude and ω is its angular frequency. Show that the motion is simple harmonic.**

Solution:

Here,

$$y = ae^{i\omega t}$$

On differentiating with respect to t, we obtain,

$$\frac{dy}{dt} = i\omega ae^{i\omega t}$$

Again,

$$\frac{d^2y}{dt^2} = -\omega^2 ae^{i\omega t} = -\omega^2 y$$

$$\text{i.e.,} \quad \frac{d^2y}{dt^2} + \omega^2 y = 0$$

This is the differential equation of S.H.M. Hence, $y = ae^{i\omega t}$ represents a S.H.M.

---

4. **Show that the velocity of simple harmonic oscillator at any instant leads the displacement by a phase angle of $\frac{\pi}{2}$. Hence find the general expression for the velocity of a simple harmonic oscillator.**

Solution:

The displacement of a simple harmonic oscillator at any instant of time t can be written as;

$$y = a\sin(\omega t + \phi)$$

The velocity v is defined as rate of chance of displacement, *i.e.,*

$$v = \frac{dy}{dt} = a\omega\cos(\omega t + \phi) = a\omega\sin\left(\frac{\pi}{2} + \omega t + \phi\right)$$

Comparing the equation fro displacement velocity, the velocity of simple harmonic oscillator at any instant t leads the displacement by a phase difference $\frac{\pi}{2}$. However, the velocity varies simply harmonically with the same frequency.

As, $\sin(\omega t + \phi) = \frac{y}{a}$

or, $\cos(\omega t + \phi) = \sqrt{1 - \frac{y^2}{a^2}} = \frac{\sqrt{a^2 - y^2}}{a}$

Thus,

$$v = a\omega\cos(\omega t + \phi) = a\omega\frac{\sqrt{a^2 - y^2}}{a}$$

$$= \omega\sqrt{a^2 - y^2}$$

The velocity will be maximum if the displacement is zero.

The maximum velocity,

$v_{max} = \omega a$

5. **Two identical springs each of force constant k are connected in series. Calculate the force constant and time period of the system.**

Solution:

Two identical springs are connected to a mass m as shown in the figure.

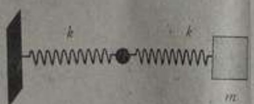

Figure: Two spring-coupled mass

Let x_1 and x_2 be the extensions produced in the two springs respectively as the mass m displaced outwards, the total extension,

$x = x_1 + x_2$

Since the restoring force due to each spring is same. We write, the restoring force,

$F = -kx_1 = -kx_2$

i.e., $x_1 = -\frac{F}{k} = x_2$

$\therefore\ x = x_1 + x_2 = -\frac{F}{k} - \frac{F}{k} = -2\frac{F}{k}$

i.e., $F = -\frac{k}{2}x$

Indicating that the effective force constant of the combination of two identical springs $K = \frac{k}{2}$.

The time period of the oscillating mass system,

$$T = 2\pi\sqrt{\frac{m}{K}} = 2\pi\sqrt{\frac{2m}{k}}$$

This is analogous to parallel combination of two capacitors of equal capacitance.

6. **What is the frequency of a simple pendulum 2.0 m long? Assuming small amplitude, what would its frequency be in an elevator accelerating upward at the rate of 2 ms^{-2}? What would its frequency be in free fall?**

Solution:

We have,

Time period of a simple pendulum,

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Thus, frequency of a simple pendulum,

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{9.8\ ms^{-2}}{2\ m}} = 0.35\ Hz$$

While moving elevator upwards with an acceleration a, the effective weight of the pendulum is $mg\left(1 + \frac{a}{g}\right) = m(g + a)$. In this case, the effective value of $g' = (g + a)$.

The frequency pendulum in the elevator,

$$f' = \frac{1}{2\pi}\sqrt{\frac{g'}{l}} = \frac{1}{2\pi}\sqrt{\frac{g + a}{l}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{(9.8 + 2)\ ms^{-2}}{2\ m}}$$

$$= 0.39\ Hz$$

In the free fall of pendulum, the effective weight of the pendulum is $mg\left(1 - \frac{g}{g}\right) = 0$, i.e., the effective value of $g = g' = 0$. The time period of pendulum,

$$T = 2\pi\sqrt{\frac{l}{g}} = \infty$$

i.e., there is no oscillation of the pendulum at all. Its frequency is, therefore, zero.

7. **A uniform circular disc of radius R oscillates in a vertical plane about a horizontal axis. Find the distance of the axis of rotation from centre for which period is minimum. What is the value of time period?**

Solution:

Here, the circular disc oscillates as a compound pendulum of length l, whose time period is;

$$T = 2\pi\sqrt{\frac{(k^2+l^2)}{gl}}$$

$$= 2\pi\sqrt{\frac{\frac{k^2}{l}+l}{g}}$$

where, k is radius of gyration about an axis through its c. g., parallel to its axis of suspension. We know that, the time period of compound pendulum is minimum if its length is equal to its radius of gyration about its c. g., *i.e.*, $l = k$.

Therefore,

$$T = 2\pi\sqrt{\frac{\frac{k^2}{k}+k}{g}}$$

$$= 2\pi\sqrt{\frac{2k}{g}}$$

Since the moment of inertia of a disc about an axis perpendicular to its plane and passing through its centre,

$$I = \frac{1}{2}MR^2 = Mk^2$$

This implies,

$$k = \frac{R}{\sqrt{2}}$$

where, M is the mass of the circular disc and R is its radius.

Hence, the disc will oscillate with the minimum time period when the distance of the axis of rotation form its centre is $\frac{R}{\sqrt{2}}$.

The minimum time period,

$$T_{min} = 2\pi\sqrt{\frac{2\frac{R}{\sqrt{2}}}{g}}$$

$$= 2\pi\sqrt{\frac{1.41R}{g}}$$

8. **A uniform circular disc of diameter 20 *cm* vibrates about a horizontal axis perpendicular to its plane and at a distance of 5 *cm* from the centre. Calculate the time period of oscillation and the equivalent length of the simple pendulum.**

Solution:

Here, the circular disc oscillates as a compound pendulum of length l, whose time period is;

$$T = 2\pi\sqrt{\frac{(k^2+l^2)}{gl}} = 2\pi\sqrt{\frac{\frac{k^2}{l}+l}{g}}$$

where, k is radius of gyration about an axis through its c. g., parallel to its axis of suspension. We know that, the time period of compound pendulum is minimum if its length is equal to its radius of gyration about its c. g., *i.e.*, $l = k$.

Therefore,

$$T = 2\pi\sqrt{\frac{\frac{k^2}{k}+k}{g}} = 2\pi\sqrt{\frac{2k}{g}}$$

Since the moment of inertia of a disc about an axis perpendicular to its plane and passing through its centre,

$$I = \frac{1}{2}MR^2 = Mk^2$$

This implies,

$$k = \frac{R}{\sqrt{2}}$$

where, M is the mass of the circular disc and R is its radius.

Hence, the disc will oscillate with the minimum time period when the distance of the axis of rotation form its centre is $\frac{R}{\sqrt{2}}$.

The minimum time period,

$$T_{min} = 2\pi\sqrt{\frac{2\frac{R}{\sqrt{2}}}{g}} = 2\pi\sqrt{\frac{1.41\,R}{g}} = 2\pi\sqrt{\frac{1.41 \times 0.1\,m}{9.8\,ms^{-2}}} = 0.75\,s$$

To have the same time period, a simple pendulum must have a length L.

Thus,

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Implies,

$$L = \frac{gT^2}{4\pi^2} = \frac{9.8\,ms^{-2} \times (0.75\,s)^2}{4\pi^2} = 0.14\,m$$

9. A solid sphere of radius 0.3 m executes torsional oscillations of time period $2\pi\sqrt{12}$ s at the end of a suspension wire whose upper end is fixed in a rigid support. If torsional constant of the wire be 6×10^{-3} Nm/rad, calculate the mass of the sphere.

Solution:

Here,

The moment of inertia of a solid sphere about any diameter or about any axis passing through its centre,

$$I = \frac{2}{5}MR^2$$

where, M is the mass of the solid sphere and R is its radius

Given that,

Torsional constant, $(C) = 6 \times 10^{-3}$ Nm/rad

Radius of solid sphere, $(R) = 0.3$ m

Time period of oscillations, $(T) = 2\pi\sqrt{12}$ s

We have,

$$T = 2\pi\sqrt{\frac{I}{C}}$$

$$\text{or, } 2\pi\sqrt{12} = 2\pi\sqrt{\frac{I}{C}}$$

$$\text{or, } I = 12\,C$$

$$\text{or, } \frac{2}{5}MR^2 = 12\,C$$

$$\therefore M = \frac{30\,C}{R^2} = \frac{30 \times 6 \times 10^{-3}\ Nm\ rad^{-1}}{(0.3)^2} = 2.0\ kg$$

The mass of the solid sphere, $(M) = 2.0\ kg$.

10. A wire has a torsional constant 2 Nm/rad. A disc of radius 5 cm and mass 100 gm is suspended at its centre. What is the frequency of torsional oscillations?

Solution:

Here,

The moment of inertia of a disc,

$$I = \frac{1}{2}MR^2$$

where, M is the mass of a disc and R is its radius

The time period of a torsional pendulum,

$$T = 2\pi\sqrt{\frac{I}{C}}$$

Thus, the frequency of the pendulum,

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{C}{I}} = \frac{1}{2\pi}\sqrt{\frac{2C}{MR^2}}$$

Given that,

Torsional constant, $(C) = 2\ Nm/rad$

Radius of a disc, $(R) = 5\ cm = 0.05\ m$

Mass of a disc, $(M) = 100\ gm = 0.1\ kg$

$$\therefore f = \frac{1}{2\pi}\sqrt{\frac{2 \times 2\ Nm/rad}{0.1\ kg \times (0.05\ m)^2}} = 20.13\ Hz$$

11. A mass of 1 kg is suspended from a spring of spring constant 25 Nm. If the undamped frequency is $\frac{2}{\sqrt{3}}$ times of the damped frequency, what will be the damping factor?

Solution:

We have,

Damped frequency,

$$f' = \frac{1}{2\pi}\sqrt{\frac{k}{m} - \frac{b^2}{4m^2}} \qquad \text{...(i)}$$

where, b is a damping factor and k is force constant of spring

In absence of damping, i.e., when $b = 0$, the undamped frequency,

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad \text{...(ii)}$$

Dividing equation (i) by equation (ii), we obtain,

$$\frac{f}{f'} = \frac{\sqrt{\frac{k}{m}}}{\sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}}$$

$$\text{or, } \frac{2}{\sqrt{3}} = \sqrt{\frac{k}{k - \frac{b^2}{4}}} \qquad [\because m = 1\ kg]$$

$$\text{or, } \frac{4}{3} = \frac{25}{25 - \frac{b^2}{4}}$$

$$\text{or, } 100 - b^2 = 75$$

$$\therefore b = 5\ kg\ s^{-1}$$

Hence the damping factor is $5\ kg\ s^{-1}$.

Chapter 2

WAVE MOTION

2.1 WAVE AND WAVE MOTION

A *wave* is a continuous transfer of disturbance from one part of medium to another through successive vibrations of the particles of th medium about their mean positions. In wave motion, the energy an momentum are carried from one region to another region of th medium. If there is no transfer of energy, it is not a wave but a oscillation there is no transfer of energy.

Wave motion is a form of disturbance which travels through a mediu due to the repeated periodic motion of the particles of the medium abo their mean position.

A wave that can't travel without material medium is said to *mechanical wave*. E.g., water waves, earthquake, tsunami, sound wav etc. A wave that can travel without material medium is said to *electromagnetic wave*. E.g., light wave or radio signals.

A wave in which the particles of a medium vibrate about their mea position perpendicular to the direction of propagation of the wave is sai to be *transverse wave*. An electromagnetic wave is a transverse wave. wave in which the particles of a medium vibrate along the direction o propagation of the wave is said to be *longitudinal wave*. A sound wave is longitudinal wave.

2.2 WAVE CHARACTERISTICS

i) Wave motion is the disturbance travelling through a medium.
ii) When a disturbance is produced in a medium, the disturbe particles vibrate about their mean positions.
iii) Particles transfer their energy to neighboring particles through th disturbances but their net displacement over one period is zero.
iv) As the disturbance reaches to a particle, it starts to vibrate. Th disturbance is communicated to the next neighbor a little later, s there is a phase difference in the vibratory motion of th consecutive particles.

v) The energy transference in the medium takes place with a constant speed, $v = f \times \lambda$ and depends on nature of the medium.
vi) The wave velocity is different form particle velocity of a medium.
vii) The wave motion is possible in a medium which possesses the property of elasticity and inertia.
viii) Particle velocity is a function of a time such that it depends in different points of a displacement whereas wave velocity is constant in a medium.
ix) Vibrating particles of the medium possess both kinetic and potential energies.

2.3 PROGRESSIVE WAVE

A wave that travels from one region of a medium to another is said to be progressive wave. In such wave, the disturbance travels forward and is transferred to neighboring particle after a certain time. All the particles vibrate in same amplitude and frequency but the vibration begins a little later than the particle immediately before it. A progressive wave may be transverse wave or longitudinal wave.

The equation of the progressive wave is,

$$y = a\, sin \frac{2\pi}{\lambda}(vt - x) \qquad 2.1$$

If a wave is travelling from right to left, the wave equation is

$$y = a\, sin \frac{2\pi}{\lambda}(vt + x) \qquad 2.2$$

where, v, a and λ are velocity, amplitude and wavelength of the progressive wave

The differential equation of wave motion is;

$$\frac{d^2y}{dt^2} = v^2 \frac{d^2y}{dx^2} \qquad 2.3$$

2.4 RELATION BETWEEN PARTICLE VELOCITY AND WAVE VELOCITY

The equation of progressive wave motion,

$$y = a\, sin \frac{2\pi}{\lambda}(vt - x)$$

where, y is the displacement of a particle of a medium at distance x from the origin at instant t, a is its amplitude and v is a wave velocity. Differentiating this expression for y with respect to t, we obtain particle velocity as,

$$U = \frac{dy}{dt} = \frac{2\pi v}{\lambda} a\, cos \frac{2\pi}{\lambda}(vt - x)$$

Again, differentiating the same expression for y with respect to x, we obtain slope of the displacement curve as;

$$\frac{dy}{dx} = -\frac{2\pi}{\lambda}\, a \cos\frac{2\pi}{\lambda}(vt - x)$$

An insertion of equation (2.5) into equation (2.4) leads us to;

$$U = \frac{dy}{dt} = -v\frac{dy}{dx}$$

i.e., particle velocity at a point = − (wave velocity) × (slope of displacement curve at that point)

2.5 ENERGY DENSITY, POWER AND INTENSITY OF A PROGRESSIVE WAVE

The energy density ε of a plane progressive wave is defined as total energy (sum of potential energy and kinetic energy) per unit volume of the medium through which wave is travelling.

i.e., $\varepsilon = 2\pi^2\rho a^2 f^2$

where, ρ is density of medium and f is a frequency of wave

The rate of transfer of energy is defined as power P of wave.

i.e., $P = \frac{E}{t} = 2\pi^2 A\, v\rho a^2 f^2$

where, A is the cross sectional area through which wave travels

The average power transferred across unit area perpendicular to the direction of energy flow, is called intensity I of a wave.

i.e., $I = 2\pi^2 v\rho a^2 f^2$

2.6 SOLVED EXAM QUESTIONS

1. **What is energy density of a wave? Write down its expression.**

Solution: [T.U. 2061 Baishakh]

The energy density ε of a plane progressive wave is defined as total energy (sum of potential energy and kinetic energy) per unit volume of the medium through which wave is travelling.

i.e., $\varepsilon = 2\pi^2\rho a^2 f^2$

where, ρ is density of medium, a is amplitude and f is a frequency of wave.

2. **Derive an expression for velocity of wave in a stretched string.**

Solution: [TU 2062 Baishakh]

Consider an idealized uniform string which is perfectly flexible. Suppose that the pulse height is so small that the tension on the

string is not change by it. The pulse moves to the right with the speed v. Any small segment AB be treated asa circular arc of radius R. Let θ be the angel between the vertical line and radial line to A or B. The length of arc AB is;

$$2\theta = \frac{Arc\ AB}{R}$$

i.e., $Arc\ AB = 2\theta R$

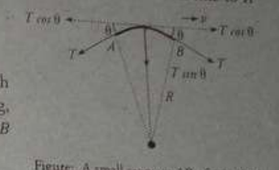

Figure: A small segment AB of stretched string that moves towards right with the speed v

If μ is the mass per unit length of the material of the string, the mass of the segment AB will be;

$$m = 2\mu R\theta$$

The tension T in the string must provide the centripetal force needed for circular motion. The tangents at these points A and B make the same angel as the normal to them make at the centre, from geometry. The horizontal component of this tension $T\cos\theta$ cancels because of the opposite directions. Therefore, the net force in the segment is $2T\sin\theta$ vertically downward. This force balances the centripetal force. Thus, we write;

$$2T\sin\theta = \frac{mv^2}{R}$$

Using the small angle approximation, we may write,

$$2T\theta = \frac{mv^2}{R} = \frac{(2\mu R\theta)v^2}{R}$$

i.e., $$v = \sqrt{\frac{T}{\mu}}$$

It is obvious that the velocity wave along a string depends only on the tension applied to the string and mass per unit length of the string. It is independent of shape and amplitude of the hump and the displacement initially produced in it, *i.e.*, the velocity of a wave along a string is quite independent of the actual wave form.

3. **Derive the differential equation of transverse wave of a stretched string with applied tension T and mass per unit length m. Also find the velocity of the wave propagating through the string.**

Solution: [T.U. 2063 Baishakh]

Consider a portion PQ of the string of length displaced slightly in the vertical plane into the position $P'Q'$ from its equilibrium position along the axis of x. The displacement is very small su that $PQ = P'Q' = \delta x$. The string is supposed to be perfec flexible such that tension T is same at all points on it, both t positions, whether it is displaced and is not displaced. Assum that it is acting tangentially at P and Q to the end portions of P of the string as shown in figure. End points P and Q are incline at angles θ and $\theta + d\theta$ respectively.

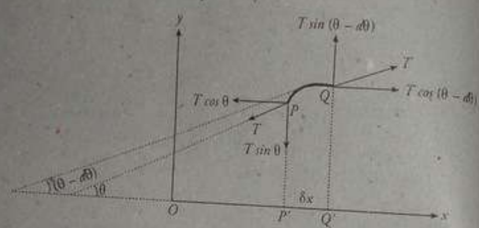

Figure: A small segment PQ of a transverse wave along a string

The horizontal and vertical components of the tension are also shown in the figure. The element δx of the string and its displacement being infinitesimally small, the horizontal and vertical components of T at P and Q are taken along horizontal and vertical line respectively, though of course in opposite directions of each other.

Thus, the resultant horizontal force on element δx of the string:

$$= T\cos(\theta - d\theta) - T\cos\theta = 0 \qquad [\because d\theta \text{ is very small}]$$

and the resultant vertical force on element δx of the string:

$$= T\sin(\theta - d\theta) - T\sin\theta = T\tan(\theta - d\theta) - T\tan\theta$$

$$[\because \theta \text{ and } (\theta - d\theta) \text{ are small}]$$

i.e., the resultant downward force on element δx of the string:

$$= T[\text{slope of the curve at } P - \text{slope of the curve at } Q]$$

$$= T\left[\frac{dy}{dx} - \left(\frac{dy}{dx} - \frac{d^2y}{dx^2}\delta x\right)\right]$$

If the acceleration of the element δx downwards be $\frac{d^2y}{dt^2}$, the downward force also equal to mass × acceleration $= m\delta x\frac{d^2y}{dt^2}$.

where, m is mass per unit length

Therefore, we write,

$$m\delta x\frac{d^2y}{dt^2} = T\left[\frac{dy}{dx} - \left(\frac{dy}{dx} - \frac{d^2y}{dx^2}\delta x\right)\right] = T\frac{d^2y}{dx^2}\delta x$$

i.e., $$\frac{d^2y}{dt^2} = \frac{T}{m}\frac{d^2y}{dx^2}$$

This is the differential equation of transverse of a stretched string.

The velocity of wave propagating through the string is;

$$v = \sqrt{\frac{T}{m}}$$

It is obvious that the velocity wave along a string depends only on the tension applied to the string and mass per unit length of the string. It is independent of shape and amplitude of the hump and the displacement initially produced in it, *i.e.*, the velocity of a wave along a string is quite independent of the actual wave form.

4. **What do you mean by particle velocity and wave velocity? Obtain the relationship between these two quantities. [T.U. 2064 Poush]**

Solution:

Particle velocity and wave velocity

The particle velocity of medium is defined as the rate of change of displacement y. The particle velocity is given by;

$$U = \frac{dy}{dt} = \frac{2\pi v}{\lambda}\, a\cos\frac{2\pi}{\lambda}(vt - x)$$

The distance travelled by a wave in a second is called wave velocity. The wave velocity is given by;

$$v = f \times \lambda$$

The equation of progressive wave motion,

$$y = a\sin\frac{2\pi}{\lambda}(vt - x)$$

where, y is the displacement of a particle of a medium at distance x from the origin at instant t, a is its amplitude and v is a wave velocity

Differentiating this expression for y with respect to t, we obtain particle velocity as,

$$U = \frac{dy}{dt} = \frac{2\pi v}{\lambda}\, a\cos\frac{2\pi}{\lambda}(vt - x)$$

Again, differentiating the same expression for y with respect to x, we obtain slope of the displacement curve as;

$$\frac{dy}{dx} = -\frac{2\pi}{\lambda}\, a\cos\frac{2\pi}{\lambda}(vt - x) = -v\frac{dy}{dx}$$

$$i.e.,\ U = \frac{dy}{dt} = -v\frac{dy}{dx}$$

i.e., particle velocity at a point = – (wave velocity) × (slope of displacement curve at that point)

---

5. In the progressive wave show that the potential energy and kinetic energy of every particle will change with time, but the average kinetic energy per unit volume and potential energy per unit volume remains constant. **[T.U. 2065 Shrawan]**

Solution:

In the plane progressive wave, there is a continuous transfer of energy in the direction of propagation. The energy is sum of kinetic energy and potential energy. For a particle executing the simple harmonic motion, particle has maximum velocity at its equilibrium position and will be zero at extreme positions *i.e.*, since kinetic energy is proportional to square of velocity, it is maximum at mean position and zero at extreme points. Consequently, particles have maximum potential energy at extreme position and zero at mean position.

A work is said to be done against the acceleration of particle of progressive wave if the particle move from its mean position to a distance y. For a small displacement dy, work done is;

$$dW = F\,dy$$

If ρ is the density of medium, the work done per unit volume is;

$$dw = \rho\, a_p\, dy$$

$$i.e.,\ dw = \rho\left\{\frac{4\pi^2 a v^2}{\lambda^2}\sin\frac{2\pi}{\lambda}(vt - x)\right\}dy$$

The total work done for displacement y is;

$$w = \int_0^y dw = \rho\frac{4\pi^2 v^2}{\lambda^2}\int_0^y a\sin\frac{2\pi}{\lambda}(vt - x)\,dy$$

The work done per unit volume is stored as potential energy per unit volume,

$$i.e.,\ V = \frac{4\pi^2\rho v^2}{\lambda^2}\frac{a^2}{2}\sin^2\frac{2\pi}{\lambda}(vt - x)$$

$$= \frac{2\pi^2 a^2\rho v^2}{\lambda^2}\sin^2\frac{2\pi}{\lambda}(vt - x)$$

and kinetic energy per unit volume will become;

$$T = \frac{1}{2}\rho\left[\frac{2\pi a v}{\lambda}\cos\frac{2\pi}{\lambda}(vt - x)\right]^2$$

$$= \frac{2\pi^2 a^2\rho v^2}{\lambda^2}\cos^2\frac{2\pi}{\lambda}(vt - x)$$

The total energy per unit volume is;

$$E = T + V = \frac{2\pi^2 a^2\rho v^2}{\lambda^2}\left\{\sin^2\frac{2\pi}{\lambda}(vt - x) + \cos^2\frac{2\pi}{\lambda}(vt - x)\right\}$$

$$= \frac{2\pi^2 a^2\rho v^2}{\lambda^2}$$

We have,

$$v = f\lambda$$

$$\therefore\ E = 2\pi^2 a^2\rho f^2$$

This shows that the average kinetic energy per unit volume and potential energy per unit volume are equal and is equal to half of total energy per unit volume.

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6. Deduce analytically the velocity of transverse wave along a stretched string depends on the tension and mass per unit length of the string. **[T.U. 2065 Shrawan]**

Solution: Proceed as the solution of Q. No. 3 on page no. 35

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7. Explain the term "wave motion". Show that for a plane progressive wave, on the average, half the energy is kinetic and half potential. **[T.U. 2065 Chaitra]**

Solution:

Wave motion

It is a form of disturbance which travels through a medium due to the repeated periodic motion of the particles of the medium about their mean position.

In the plane progressive wave, there is a continuous transfer of energy in the direction of propagation. The energy is sum of kinetic energy and potential energy. For a particle executing the simple harmonic motion, particle has maximum velocity at its equilibrium position and will be zero at extreme positions *i.e.*, since kinetic energy is proportional to square of velocity, it is maximum at mean position and zero at extreme points. Consequently, particles have maximum potential energy at extreme position and zero at mean position.

A work is said to be done against the acceleration of particle of progressive wave if the particle move from its mean position to a distance y. For a small displacement dy, work done is;

$$dW = F\,dy$$

If ρ is the density of medium, the work done per unit volume is

$$dw = \rho\, a_p\, dy$$

i.e., $$dw = \rho \left\{\frac{4\pi^2 a v^2}{\lambda^2} \sin\frac{2\pi}{\lambda}(vt - x)\right\} dy$$

The total work done for displacement y is:

$$w = \int_0^y dw = \rho \frac{4\pi^2 v^2}{\lambda^2} \int_0^y a \sin\frac{2\pi}{\lambda}(vt - x)\, dy$$

The work done per unit volume is stored as potential energy per unit volume, i.e.,

$$V = \frac{4\pi^2 \rho v^2}{\lambda^2} \frac{a^2}{2} \sin^2\frac{2\pi}{\lambda}(vt - x)$$

$$= \frac{2\pi^2 a^2 \rho v^2}{\lambda^2} \sin^2\frac{2\pi}{\lambda}(vt - x)$$

and kinetic energy per unit volume will become;

$$T = \frac{1}{2}\rho \left[\frac{2\pi a v}{\lambda} \cos\frac{2\pi}{\lambda}(vt - x)\right]^2 = \frac{2\pi^2 a^2 \rho v^2}{\lambda^2} \cos^2\frac{2\pi}{\lambda}(vt - x)$$

The total energy per unit volume is;

$$E = T + V = \frac{2\pi^2 a^2 \rho v^2}{\lambda^2}\left\{\sin^2\frac{2\pi}{\lambda}(vt - x) + \cos^2\frac{2\pi}{\lambda}(vt - x)\right\}$$

$$= \frac{2\pi^2 a^2 \rho v^2}{\lambda^2}$$

We have,

$$v = f\lambda$$

$$\therefore E = 2\pi^2 a^2 \rho f^2$$

This shows that the average kinetic energy per unit volume and potential energy per unit volume are equal and is equal to half of total energy per unit volume.

8. What is meant by superposition of waves? Point out the difference between transverse wave and mechanical wave and derive an expression for the speed of travelling wave in a stretched string. [P.U. 2002]

Solution:

When two waves of same frequency having constant phase difference traverse in same region of the medium simultaneously meets and produce the resultant wave. Such phenomenon is called superposition of waves.

A transverse wave can travel without material medium whereas mechanical medium requires material medium to travel from one plane to another place.

An electromagnetic wave is an example of transverse wave. Sound is an example of mechanical wave.

Speed of travelling wave in a stretched string

Proceed as the solution of Q. No. 3 on page no. 35

9. Derive an expression for the velocity of transverse waves in a stretched string. [P.U. 2002]

Solution: See the solution of Q. No. 2 on page no. 34

10. Derive an expression for the velocity of transverse waves in a stretched string. [P.U. 2011]

Solution: Proceed as the solution of Q. No. 2 on page no. 34

2.6 ADDITIONAL SOLVED PROBLEMS

1. One end of a string is fixed. It hangs over a pulley and has a block of mass 2 kg attached to the other end. The horizontal part has a length of 1.6 m and mass 20 gm. What is the speed of the transverse pulse on the string?

Solution:

Here,

Mass of the block, $(m) = 2\,kg$

Tension on the string, $(T) = mg = 2\,kg \times 9.8\,ms^{-2} = 19.6\,N$

Since 1.6 m of a string has a mass of 20 gm,

Mass per unit length of the string,

$$\mu = \frac{20 \times 10^{-3}\,kg}{1.6\,m} = 1.25 \times 10^{-2}\,kgm^{-1}$$

We have,

Speed of transverse pulse on string,

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{19.6\,N}{1.25 \times 10^{-2}\,kgm^{-1}}} = 39.61\,ms^{-1}$$

2. A train of simple harmonic waves is travelling in a gas along the positive direction of an x-axis with an amplitude 2 cm, velocity 300 ms^{-1} and frequency 400 Hz. Calculate the displacement, particle velocity and particle acceleration at a distance of 4 cm

from the origin after an interval of 5 s. What will be the maximum speed of a particle?

Solution:

We have, the displacement of a particle,

$$y = a\sin\frac{2\pi}{\lambda}(vt - x)$$

where, x is the distance of a particle from the origin, after time t, a is amplitude of a wave, λ is its wavelength and v is its velocity

Given that,

$a = 2\,cm = 0.02\,m$

$v = 300\,ms^{-1}$

$f = 400\,Hz$

At $t = 5\,s$

$x = 4\,cm = 0.04\,m$

Wavelength, $\lambda = \frac{v}{f} = \frac{300\,ms^{-1}}{400\,Hz} = 0.75\,m$

Now,

$$y = 0.02\sin\frac{2\pi}{0.75}(300 \times 5 - 0.04) = 0.02\sin 12566^{\circ}$$
$$= -1.12 \times 10^{-2}\,m$$

Hence, the distance of the particle at assistance of $0.04\,m$ from the origin, after an interval of $5\,s = -1.12 \times 10^{-2}\,m$.

Particle velocity,

$$U = \frac{dy}{dt} = \frac{2\pi v}{\lambda}a\cos\frac{2\pi}{\lambda}(vt - x)$$
$$= \frac{2\pi \times 300}{0.75} \times 0.02\cos\frac{2\pi}{0.75}(300 \times 5 - 0.04)$$
$$= 41.67\,ms^{-1}$$

Particle acceleration,

$$\frac{d^2y}{dt^2} = -\left(\frac{2\pi v}{\lambda}\right)^2 a\sin\frac{2\pi}{\lambda}(vt - x)$$
$$= -\left(\frac{2\pi \times 300}{0.75}\right)^2 0.02\sin\frac{2\pi}{0.75}(300 \times 5 - 0.04)$$
$$= 7.07 \times 10^4\,m\,s^{-2}$$

The particle speed will be maximum if $\cos\frac{2\pi}{\lambda}(vt - x) = 1$.

Thus, the maximum particle speed $= \frac{2\pi v}{\lambda}a = \frac{2\pi \times 300}{0.75} \times 0.02$

$$= 50.27\,ms^{-1}$$

5. **A wave of frequency 500 Hz has a phase velocity 360 m/s. How far apart are two points 60° out of phase? What is the phase difference between two displacements at a certain point at times 10^{-3} s apart?**

Solution:

We have, the displacement of a particle,

$$y = a\sin\frac{2\pi}{\lambda}(vt - x)$$

where, x is the distance of a particle from the origin, after time t, a is amplitude of a wave, λ is its wavelength and v is its velocity and $\frac{2\pi}{\lambda}(vt - x)$ is the phase angle of a point distance x from origin at any instant of time t.

Thus, phase angle of a point distance x_1 from origin at any instant of time $t = \frac{2\pi}{\lambda}(vt - x_1)$ phase angle of a point distance x_2 from origin at any instant of time $t = \frac{2\pi}{\lambda}(vt - x_2)$

The phase difference between two points;

$$= \frac{2\pi}{\lambda}(vt - x_1) - \frac{2\pi}{\lambda}(vt - x_2) = \frac{2\pi}{\lambda}(x_2 - x_1)$$
$$= \frac{2\pi v}{\lambda}\left(\frac{x_2 - x_1}{v}\right) = 2\pi f\left(\frac{x_2 - x_1}{v}\right) \qquad \left(\because f = \frac{v}{\lambda}\right)$$

Given that,

The phase difference between two points $= 60^{\circ} = \frac{\pi}{3}$ radians

i.e. $\frac{\pi}{3} = 2\pi f\left(\frac{x_2 - x_1}{v}\right)$

or, $\frac{1}{3} = 2 \times 500\left(\frac{x_2 - x_1}{360}\right)$

$\therefore x_2 - x_1 = 0.12\,m$

The two points are $0.12\,m$ apart.

Phase angle of a point distance x from origin at any instant of time $t_1 = \frac{2\pi}{\lambda}(vt_1 - x)$

Phase angle of a point distance x from origin at any instant of time $t_2 = \frac{2\pi}{\lambda}(vt_2 - x)$

In this case,

Phase difference,

$$\phi = \frac{2\pi}{\lambda}(vt_1 - x) - \frac{2\pi}{\lambda}(vt_2 - x) = 2\pi f(t_2 - t_1)$$
$$= 2\pi \times 500 \times 10^{-3} = \pi^c$$

4. The displacement equation of a transverse plane wave at an instant is given by;
$y = 0.3\sin(3\pi t - 0.05\pi x)$
where, x and t are in meter and seconds
Calculate wave length frequency and velocity of the wave. Also calculate phase difference between two particles 0.25 m apart at same instant.

Solution:

Here,
The displacement equation of a transverse plane wave is;

$$y = 0.3\sin(3\pi t - 0.05\pi x) \quad \ldots (i)$$

The general displacement equation of a transverse plane wave is;

$$y = a\sin\frac{2\pi}{\lambda}(vt - x) = a\sin\left(\frac{2\pi v}{\lambda}t - \frac{2\pi}{\lambda}x\right)$$

$$= a\sin\left(2\pi ft - \frac{2\pi}{\lambda}x\right) \quad \ldots (ii)$$

Comparing equations (i) and (ii), we obtain,

$$2\pi f = 3\pi$$
$$\therefore f = 1.5\,Hz$$
$$\frac{2\pi}{\lambda} = 0.05\pi$$
$$\therefore \lambda = 40\,m$$

Velocity of transverse wave,

$$v = f \times \lambda = 1.5\,Hz \times 40\,m = 60\,ms^{-1}.$$

For the two particles 0.25 m apart, the phase difference,

$$\phi = \frac{2\pi}{\lambda}x = 0.05\pi \times 0.25 = (0.0125\pi)^c$$

---

5. Check whether the following expressions are a solution of one dimensional wave equation or not.
   i) $y = x^2 + v^2t^2$
   ii) $y = \sin 2x \cos vt$
   iii) $y = 2\sin x \cos vt$
   iv) $y = (x - vt)^2$
   v) $y = x^2 - v^2t^2$.

Solution:

i) Here,

$$y = x^2 + v^2t^2$$

On differentiating with respect to t, we obtain,

$$\frac{dy}{dt} = 2v^2t$$

$$\text{or, } \frac{d^2y}{dt^2} = 2v^2$$

Again, differentiating with respect to x, we obtain,

$$\frac{dy}{dx} = 2x$$

$$\text{or, } \frac{d^2y}{dx^2} = 2$$

Obviously,

$$\frac{d^2y}{dt^2} = 2v^2 = v^2\frac{d^2y}{dx^2}$$

Hence, $y = x^2 + v^2t^2$ is the solution of one dimensional wave equation, $\frac{d^2y}{dt^2} = v^2\frac{d^2y}{dx^2}$.

ii) Here,

$$y = \sin 2x \cos vt$$

On differentiating with respect to t, we obtain,

$$\frac{dy}{dt} = v\sin 2x \sin vt$$

$$\text{or, } \frac{d^2y}{dt^2} = -v^2\sin 2x \cos vt = -v^2y$$

Again, differentiating with respect to x, we obtain,

$$\frac{dy}{dx} = 2\cos 2x \cos vt$$

$$\text{or, } \frac{d^2y}{dx^2} = -4\sin 2x \cos vt = -4y$$

Obviously,

$$\frac{d^2y}{dt^2} = -v^2y \neq v^2\frac{d^2y}{dx^2}.$$

Hence, $y = \sin 2x \cos vt$ is the solution of one dimensional wave equation.

iii) Here,

$$y = 2\sin x \cos vt$$

On differentiating with respect to t, we obtain,

$$\frac{dy}{dt} = 2v\sin x \sin vt$$

$$\text{or, } \frac{d^2y}{dt^2} = -2v^2\sin x \cos vt = -v^2y$$

Again, differentiating with respect to x, we obtain,

$$\frac{dy}{dx} = 2\cos x \cos vt$$

or, $$\frac{d^2y}{dx^2} = -2\sin x \cos vt = -y$$

Obviously,

$$\frac{d^2y}{dt^2} = -v^2 y = v^2\frac{d^2y}{dx^2}$$

Hence, $y = 2\sin x \cos vt$ is the solution of one dimensional wave equation, $\frac{d^2y}{dt^2} = v^2\frac{d^2y}{dx^2}$.

iv) Here,

$$y = (x - vt)^2$$

On differentiating with respect to t, we obtain,

$$\frac{dy}{dt} = -2xv + 2v^2t$$

or, $$\frac{d^2y}{dt^2} = 2v^2$$

Again, differentiating with respect to x, we obtain,

$$\frac{dy}{dx} = 2x - 2vt$$

or, $$\frac{d^2y}{dx^2} = 2$$

Obviously,

$$\frac{d^2y}{dt^2} = 2v^2 = v^2\frac{d^2y}{dx^2}$$

Hence, $y = (x - vt)^2$ is the solution of one dimensional wave equation, $\frac{d^2y}{dt^2} = v^2\frac{d^2y}{dx^2}$.

v) Here,

$$y = x^2 - v^2t^2$$

On differentiating with respect to t, we obtain,

$$\frac{dy}{dt} = -2v^2t$$

or, $$\frac{d^2y}{dt^2} = -2v^2$$

Again, differentiating with respect to x, we obtain,

$$\frac{dy}{dx} = 2x$$

or, $$\frac{d^2y}{dx^2} = 2$$

Obviously,

$$\frac{d^2y}{dt^2} = -2v^2 \neq v^2\frac{d^2y}{dx^2}$$

Hence, $y = x^2 - v^2t^2$ is the solution of one dimensional wave equation.

---

6. If the intensity of a seismic wave is $10^6\ Wm^{-2}$ at 200 km from the source, what will be the intensity at 500 km the source?

Solution:

The intensity of wave decreases as square of the distance from the source. Therefore, at 500 km the intensity should be;

$$\frac{I_2}{I_1} = \left(\frac{r_1}{r_2}\right)^2$$

i.e., $$I_2 = \left(\frac{r_1}{r_2}\right)^2 I_1 = \left(\frac{200\ km}{500\ km}\right)^2 \times 10^6\ Wm^{-2} = 1.6 \times 10^5\ Wm^{-2}$$

---

7. A source of sound has a frequency of has a frequency of 256 Hz and amplitude of 0.50 cm. Calculate the energy flow across a square cm per sec. the velocity of sound in air is 330 m/s and its density is 1.29 kgm^{-3}.

Solution:

The total energy transferred in a square cm per second is;

$$I = \frac{E}{A \times t} = 2\pi^2 v\rho f^2 a^2$$

$$= 2\pi^2 \times 330\ ms^{-1} \times 1.29\ kg\ m^{-3} \times (256\ Hz)^2 (0.50 \times 10^{-2} m)^2$$

$$= 1.38 \times 10^4\ Jm^{-2}s^{-1}$$

---

8. The equation of a transverse wave travelling along a stretched string is given by;

$y = 10 \sin \pi (2t - 0.05\, x)$

where y and x are expressed in cm and t in seconds. Find the amplitude, velocity, and wavelength of the wave. What will be the maximum speed of a particle in the string?

Solution: Proceed as the solution of Q. No. 4 on page no. 44

9. **By how much would intensity level at a given place change wh intensity of sound produced by a source at that place is doubled**

Solution:

The intensity level of sound,

$$I_L = 10\ log_{10}\left(\frac{I}{I_0}\right)$$

For the intensity level of sound I_{L_1}, let I_1 be the initial intensity sound. Thus,

$$I_{L_1} = 10\ log_{10}\left(\frac{I_1}{I_0}\right)$$

Given that, final intensity,

$$I_2 = 2I_1$$

Let final intensity level of sound be I_{L_2}. Thus, we write,

Final intensity level,

$$I_{L_2} = 10\ log_{10}\left(\frac{I_2}{I_0}\right) = 10\ log_{10}\left(\frac{2I_1}{I_0}\right)$$

$$= 10\ log_{10}2 + 10\ log_{10}\left(\frac{I_1}{I_0}\right)$$

$$= (3.01 + I_{L_1})\ dB$$

Therefore, change in intensity level $= I_{L_2} - I_{L_1} = 3.01\ dB$

Chapter 3

ACOUSTICS PHENOMENA

3.1 ACOUSTICS OF BUILDINGS

A branch of physics or engineering that deal with process of production, transmission and reception of sound is called acoustics. Some of the important fields of acoustics are (i) design of acoustical instruments (ii) electro acoustics relating to method of sound production and recording (microphones, amplifiers, loudspeakers etc.) (iii) architecture acoustics dealing with the designs and construction of buildings, cinema halls, auditoriums, recording rooms in broadcasting stations.

Acoustics becomes very important while designing an auditorium hall for speed or music purposes. In designing an acoustically good building, the sound must be uniformly distributed inside the room with every single person hearing sound distinctly. An auditorium hall is said to be acoustically good if it satisfies the following conditions,

i) Uniform distribution of sound throughout auditorium hall with proper loudness and quality.
ii) The quality of music or speech should be unchanged.
iii) No echoes or resonances.
iv) No overlapping of syllables.
v) No external and internal noise disturbances.
vi) Having an optimum reverberation time.

Reverberation, loudness, focusing, echo, resonance, noise etc. are the factors affecting the acoustics of buildings.

3.2 REVERBERATION

Reverberation is the undue prolongation of sound in a particular space. A reverberation is created in an enclosed space causing a large number of reflections to build up and decay slowly as the sound is absorbed by walls and air.

The duration for which the sound can be heard after the source has ceased to produce sound is called reverberation time. It is found that the sound become inaudible when its intensity falls to one millionth of average intensity. The duration between the production of sound and inaudibility is called *time of reverberation*.

One can manipulate the reverberation time in an auditorium halls by varying the absorption of sound. The amount of absorption of sound depends upon the nature of materials and their surface area.

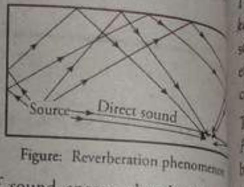

Figure: Reverberation phenomenon

An *absorption coefficient* is the ratio of sound energy absorbed by the given surface to the sound energy absorbed by an equal area of a perfect absorber. Alternatively, an absorption coefficient of sound is defined as the reciprocal of the area which absorbs the same sound energy as absorbed by a unit area of a perfect absorber.

3.3 SABINE'S RELATION

W.C. Sabine (1980) found that the reverberation time is directly proportional to dimensions of rooms or halls and inversely proportional to absorption coefficient. He assumed that;

i) There is uniform distribution of sound inside the room or hall or auditorium hall.

ii) The energy is not lost in hall. The energy is lost only due to absorption of materials of the walls and ceiling and also due to escape through the windows and ventilations.

The Sabine's relations are;

$$T = \frac{0.158\,V}{A\alpha} \quad \text{[In S.I. units; velocity of sound in air} = 350\ m/s]$$

$$T = \frac{0.05\,V}{A\alpha} \quad \text{[In F.P.S.; velocity of sound in air} = 1120\ ft/s]$$

where, T = time of reverberation

V = dimension or volume of hall

A = area of walls

α = absorption coefficient of absorbing surface

Sabine established that there is an *optimum reverberation time* which is larger for large halls and it is also greater for music than that for speech. Below the optimum time, the intensity of sound is weak whereas above the optimum value the distinctness of syllables is hard.

3.4 ULTRASONIC WAVES

The sound waves having their frequencies lying between 20 Hz and 20 kHz can be detected by human ear and are called ***audible waves***. Audible sound waves originate in the vibrating strings (sonometer, violin, guitar, etc.), vibrating membrane (drums, loudspeaker, etc.) vibrating air column (flute, clarinet, etc.) and so forth.

The sound waves of frequency lower than audible range, *i.e.*, below 20 Hz are called *infrasonics*. Such waves are produced by large vibrating bodies, e.g., earthquake waves, vibration of pendulums. These waves are insensitive to human ears.

The sound waves of frequency higher than audible range *i.e.*, above 20 kHz are called *ultrasonic waves*. Animals like dogs, birds and bats are sensitive to particular range of these waves. Bats are almost blind but can locate objects and navigate obstacles by producing ultrasonic waves and receiving echoes. These waves travel in air with same speed as that of audible waves; exhibit the properties of audible sound waves and some new phenomena. e.g., they travel in air with the velocity of 345 m/s at 25°C. Because of high frequency, their wavelength is extremely small. For a lower limit of frequency of 20 kHz, the wavelength of ultrasonic waves at room temperature is $\frac{345\ m/s}{20\ kHz} = 1.8\ cm$ and becomes much less at higher frequencies (0.35 cm at 1 MHz). This wavelength is comparable to X–ray wavelength (10^{-8} to $10^{-11}m$). Because of such a small wavelength, ultrasonic waves have great penetrating power. The ultrasonic waves are not electromagnetic waves.

Generation of Ultrasonic Waves

i. Mechanical Generation

It is divided into two groups. They are;

a) **Gas Driven**

These are simple devices to produce the ultrasonic waves of frequencies up to 30 kHz, such as Galton's whistle, siren, etc.

b) **Liquid Driven**

These are transducers type of devices which converts energy from one form to another, such as vibrating blade transducer and hydrodynamic oscillators.

ii. Electrical Generation

It is widely used for producing ultrasonic waves and is divided into two categories. They are;

a) Piezoelectric Generators

When certain crystals (such as Quartz, Tourmaline and Rochelle salt) subjected to pressure, they exhibit opposite electrical charges on their opposite faces resulting in potential difference. This phenomenon is known as *piezoelectric effect*. Conversely, if the opposite sides of these crystals are maintained at different electrical potentials by applying a voltage, then the crystal slice expands or contracts. This phenomenon is known as *converse piezoelectric effect* is utilized in production of ultrasonic waves of high frequency.

b) Magnetostriction Oscillators

Magnetostriction is the property by which ferromagnetic materials such as nickel, iron, cobalt, etc. in the form of rod changes in length when placed in magnetic field parallel to its length. This phenomenon can be used to produce ultrasonic waves.

Properties of Ultrasonic waves

i) These are the sound wayes having frequencies greater than 20 kHz.

ii) Optical properties like reflection, refraction, diffraction, etc. are observed with ultrasonic waves.

iii) As the wavelength of ultrasonic waves is very small, the energy associated with these waves is enormous.

iv) Ultrasonic waves can penetrate through metals and other materials which are opaque to electromagnetic waves.

v) Ultrasonic waves produce cavitation effect when made to pass from some solids.

vi) Ultrasonic waves produce emulsion when applied at interface between two liquids.

vii) When ultrasonic waves meet surface of separation between two media, they undergo reflection.

viii) Ultrasonic waves of high frequencies produce chemical changes.

ix) Ultrasonic waves can be propagated through the elastic bodies such as liquid, solids, gases or vapors.

x) Ultrasonic luminescence is observed in liquids like water in certain special conditions.

xi) The velocity of ultrasonic waves depends on temperature of the medium through which they propagate.

Applications of Ultrasonic Waves

i) Detection of flaws or cracks in metals

Ultrasonic waves can be used in detecting cracks and cavities in metals. Due to the presence of cracks or cavi[illegible]nic waves propagating

through metals get reflected. The speed of these waves through the cavity in the metals will be different from that in solid region of the metals. Therefore, when ultrasonic waves pass through cracks or cavities inside it, a large amount of reflections occur. Some reflections even take place from bake surface of the metals. The reflected pulses from cavities and back surfaces are received by a receiving transducer and are amplified, and applied to one set plate of cathode ray oscilloscope (CRO). The transmitted signals and reflected signals form from the flaws and back surface of metal produced different peaks. The position of peaks on the time base of CRO determines the position of flaws from the surface of the metals.

ii) Formation of alloys

Various constituents of the alloys have different densities. The constituents of alloys can be mixed uniformly by beam of ultrasonic waves.

iii) Soldering and metal cutting

Ultrasonic waves can be used for drilling, cutting, soldering and room temperature welding of metals. To solder aluminum, ultrasonic waves along with electrical soldering iron are used.

iv) In metallurgy

The grain size can be refined and trapped gases can be removed by irradiating melt with ultrasonic waves during the process of cooling.

v) Ultrasonic mixing

A colloidal solution, an emulsion of two non-miscible liquids, can be mixed by simultaneously subjecting to ultrasonic radiation. Most of the emulsions like polishes, food products and pharmaceutical preparations are prepared by using ultrasonic mixing.

vi) Effect on coagulation and crystallization process

The suspended particles in liquids can be brought quite close to each other by ultrasonic waves, so that coagulation may occur. Also, the crystallization rate is improved by ultrasonic waves. When the crystallizing solution is kept for crystallization, the size of the can be made smaller and more uniform by the use of ultrasonic waves.

vii) Signaling

Due to the smaller wavelength, the ultrasonic waves can be concentrated into a sharp beam. Thus, these waves can be used in signaling in particular direction.

viii) In processing, testing and communications

In processing applications, ultrasonic vibrations at higher densities, is used to produce physical or chemical change in material.

The focused ultrasound is used for surgery. Echo-sounding techniques are used for testing materials. The high frequency elastic properties of materials can be measured by ultrasonic waves with great precision and give information about its structure.

The most important communications application is in ultrasonic delay lines, which are useful in radar and data handling system.

ix) SONAR (Sound Wave Navigation and Ranging)

SONAR is used for the detection of submarines, icebergs and other objects in the ocean. One can determine presence of submerged submarines, ships, icebergs, rocks or enemies aircrafts in the sky by this technique. In this technique, a sharp ultrasonic beam is directed in various directions. After reflection, it is picked by ultrasonic detector. The reflected waves are due to presence of some reflecting medium such as submarines, rocks, icebergs, etc. in the sea. The idea of the position of the body is obtained by the time interval between generation of ultrasonic waves and their return after reflection. If the body is moving there will be change in frequency of echo signals. It helps to determine the velocity of body and predicts its direction.

x) Depth of the sea

The ultrasonic waves are highly energetic and show a negligible diffraction effect. They can be used to determine depth of the sea. The time interval between sending and receiving ultrasonic waves from the sea surface is recorded. As the velocity of wave is known, depth of the sea can be estimated.

xi) Medical applications

The ultrasonic waves have been widely used for examining the shape and movement of organs within the body. The images of ultrasonic waves that are reflected from the boundaries of the organs are obtained and is known as ultrasound scanning. The ultrasonic waves reflected from moving objects like RBC exhibit a Doppler shift frequency that is used to measure the rate of flow of blood.

Ultrasonic waves are used by dentists for the proper extraction of broken teeth or decayed teeth.

xii) Biological use

Ultrasonic waves are used to lame smaller animals like rats, fish, frogs, etc. and kill bacteria. The reproductory power of yeast is loosed if they are exposed to ultrasonic waves.

xiii) Cleaning and clearing

Ultrasonic waves can be used for cleaning machine parts, utensils, washing clothes and removing dust and soot from chimneys.

3.5 SOLVED EXAM QUESTIONS

1. What are ultrasonic waves? How do they differ from ordinary sound? What are their uses? **[T.U. 2061 Baishakh]**

Solution:

Ultrasonic waves

The sound waves of frequency higher than audible range *i.e.*, above 20 *kHz*, are called ultrasonic waves. Animals like dogs, birds and bats are sensitive to particular range of these waves. Bats are almost blind but can locate objects and navigate obstacles by producing ultrasonic waves and receiving echoes.

The sound waves having their frequencies lying between 20 *Hz* and 20 *kHz* can be detected by human ear and are called *audible waves*. Audible sound waves originate in the vibrating strings (sonometer, violin, guitar, etc.), vibrating membrane (drums, loudspeaker, etc.) vibrating air column (flute, clarinet, etc.) and so forth.

Ultrasonic waves have lower wavelength than audible waves and is comparable to wavelength of X-rays.

Applications of Ultrasonic Waves

i) Detection of flaws or cracks in metals

Ultrasonic waves can be used in detecting cracks and cavities in metals. Due to the presence of cracks or cavities, ultrasonic waves propagating through metals get reflected. The speed of these waves through the cavity in the metals will be different from that in solid region of the metals. Therefore, when ultrasonic waves pass through cracks or cavities inside it, a large amount of reflections occur. Some reflections even take place from bake surface of the metals. The reflected pulses from cavities and back surfaces are received by a receiving transducer and are amplified, and applied to one set plate of cathode ray oscilloscope (CRO). The transmitted signals and reflected signals form from the flaws an back surface of metal produced different peaks. The position o peaks on the time base of CRO determines the position of flaw from the surface of the metals.

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xiii) Cleaning and clearing

Ultrasonic waves can be used for cleaning machine parts, utensils, washing clothes and removing dust and soot from chimneys.

2. **What is the reverberation time for a hall with length 12 m breadth 11 m and height 9 m if the coefficient of absorption of walls, ceiling and floor are 0.02, 0.04 and 0.08 respectively? [T.U. 2061 Baishakh]**

Solution:

Here,

The dimension of the hall $(l \times b \times h) = 12\,m \times 11\,m \times 9\,m$

Volume of the hall $(V) = 1188\,m^3$

Coefficient of absorption of walls $(\alpha_1) = 0.02$

Coefficient of absorption of ceiling $(\alpha_2) = 0.04$

Coefficient of absorption of floor $(\alpha_3) = 0.08$

Area of four walls, $(S_1) = 2h(l + b)$

$$= 2 \times 9\,m(12\,m + 11\,m)$$

$$= 414\,m^2$$

Area of ceiling (S_2) = Area of floor (S_3) $= l \times b = 12\,m \times 11\,m$

$$= 132 m^2$$

We know that,

The reverberation time of hall:

$$(T) = \frac{0.158\,V}{\sum_i \alpha_i S_i} = \frac{0.158 \times 1188}{0.02 \times 414 + 0.04 \times 132 + 0.08 \times 132} = 7.78\,s$$

3. **Define ultrasonic waves. Describe piezoelectric method for their production. How are these used for the distance measurement? [T.U. 2061 Ashwin]**

Solution:

Ultrasonic waves

See the solution of Q. No. 1 on page no. 55

These waves travel in air with same speed as that of audible wave exhibit the properties of audible sound waves and some new phenomena. E.g., they travel in air with the velocity of 345 m/s at 25°. Because of high frequency, their wavelength is extremely small. For a lower limit of frequency of 20 kHz, the wavelength of ultrasonic waves at room temperature is $\frac{345\,m/s}{20\,kHz} = 1.8\,cm$ and becomes much less at higher frequencies (0.35 cm at 1MHz)

Piezoelectric generation of ultrasonic waves

When certain crystals (such as Quartz, Tourmaline and Rochelle salt) subjected to pressure, they exhibit opposite electrical charges on their opposite faces resulting in potential difference. This phenomenon is known as piezoelectric effect. Conversely, if the

opposite sides of these crystals are maintained at different electrical potentials by applying a voltage, then the crystal slice expands or contracts. This phenomenon is known as converse piezoelectric effect is utilized in production of ultrasonic waves of high frequency. If an electric field is applied across two faces of the crystal, compression or extension will occur across the other pair of faces. The magnitude of compression or extension is proportional to the potential difference between the faces. This effect is used to produce high frequency oscillations in quartz. Tin foil sheets cover the opposite faces of a slab of quartz crystal. The application of alternating potential difference of same frequency to foils produce oscillations in quartz crystal that can be communicated to liquid. If natural frequency of mechanical vibration of the quartz crystal coincides with that of the applied potential difference, resonance occurs and vibrations of large amplitude are set up in the quartz crystal and hence produce ultrasonic waves.

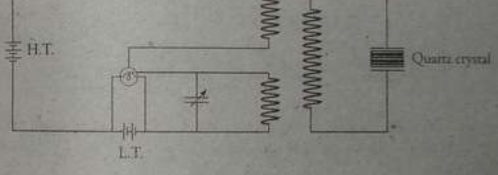

Figure: Piezoelectric generation of ultrasonic waves

The ultrasonic waves are highly energetic and show a negligible diffraction effect. They can be used to determine distance of objects. The time interval between sending and receiving ultrasonic waves from the particular object is recorded. As the velocity of wave is known, distance of an object can be estimated.

4. **Calculate the reverberation time of a small hall in BICC of 1500 m^3 having seating capacity 120 persons when (i) the hall is empty and (ii) with full capacity of the audience for the following data.**

Surface	Areas	Coefficient of absorption
Plastered walls	112 m^2	0.03
Wooden floor	130 m^2	0.06
Plastered ceiling	170 m^2	0.04

Wooden doors	20 m^2	0.06
Cushioned chairs	120	0.05
Audience	120	0.44

[T.U. 2063 Baishakh]

Solution:

Here,

Volume of BICC hall $(V) = 1500\ m^3$

i) Reverberation time of an empty hall,

$$T = \frac{0.158\,V}{\sum_i \alpha_i S_i}$$

$$= \frac{0.158 \times 1500}{0.03\times112+0.06\times130+0.04\times170+0.06\times20+0.05\times120}$$

$$= 9.42\ s$$

ii) Reverberation time with full capacity of audience,

$$T = \frac{0.158V}{\sum_i \alpha_i S_i}$$

$$= \frac{0.158\times1500}{0.03\times112+0.06\times130+0.04\times170+0.06\times20+0.05\times120+0.44\times120}$$

$$= 3.04\ s$$

The reverberation time is significantly lowers with audiences in hall.

5. **What do you understand by reverberation? Formulate Sabine's reverberation equation and discuss its significance.[T.U. 2061 Poush]**

Solution:

Reverberation

Reverberation is the undue prolongation of sound in a particular space. A reverberation is created in an enclosed space causing large number of reflections to build up and decay slowly as the sound is absorbed by walls and air.

Sabine's reverberation equation

W.C. Sabine found that the reverberation time is directly proportional to dimensions of rooms or halls and inversely proportional to absorption coefficient. He assumed that;

i) There is uniform distribution of sound inside the room/hall or auditorium hall.

ii) The energy is not lost in hall. The energy is lost only due to absorption of materials of the walls and ceiling and also due to escape through the windows and ventilations.

Assume that I is the average intensity of sound in a room at any given instant and δI is the fall in intensity due to absorption in a small interval of time δt. The fall in intensity is,

$$\delta I = -\alpha n I \delta t$$

where, α is the mean absorption coefficient of all absorbing surfaces in the room and n is number of reflections of sound per second

The negative sign indicates the loss in intensity as time elapses.

By statistical means, Jaeger proved that sound travels an average distance of $\frac{4V}{S}$ between successive reflections; where, V is the volume of the hall or auditorium and S is the total surface area of the absorbing surfaces in the hall or auditorium. Therefore, the time taken between two successive reflection is $\frac{4V}{Sv}$; where, v is the velocity of sound.

The number of reflection made $= \frac{t}{\frac{4V}{Sv}} = \frac{Sv}{4V}t$

Hence, the number of reflections per second would be,

$$n = \frac{Sv}{4V}$$

Thus, we can write,

$$\delta I = -\alpha \frac{Sv}{4V} I \delta t$$

$$\text{or, } \frac{\delta I}{\delta t} = -\alpha \frac{Sv}{4V} I$$

In the limit, we can rearrange the terms to provide

$$\frac{dI}{I} = -\alpha \frac{Sv}{4V} dt$$

On integrating, we obtain,

$$\ln\left(\frac{I_t}{I_0}\right) = -\alpha \frac{Sv}{4V} t$$

where, I_0 and I_t are intensity at $t = 0$ and $t = t$ respectively.

$$\text{i.e., } I_t = I_0 e^{-\alpha \frac{Sv}{4V} t}$$

By definition of reverberation time T,

$$I_t = \frac{I_0}{10^6}$$

$$\text{or, } I_0 e^{-\alpha \frac{Sv}{4V} T} = \frac{I_0}{10^6}$$

$$\text{or, } e^{-\alpha \frac{Sv}{4V} T} = 10^{-6}$$

$$\text{i.e., } T = \frac{6 \ln 10 \times 4V}{\alpha S v}$$

The velocity of sound in air is $350\ m/s$. Thus, we obtain,

$$T = \frac{0.158\,V}{aS}$$

where, V is the volume of the hall or auditorium in m^3, S is the surface area in m^2 and a is the absorption coefficient of the material in the hall. The absorption coefficient for the standard building materials is given by Sabine's formula,

$$T = \frac{0.158\,V}{\sum_i a_i S_i}$$

The reverberation time is directly proportional to the volume of auditorium halls and inversely proportional to area of walls, ceiling, floor, etc and absorption plus transmission through open surfaces, In order to decrease the reverberation time, the walls of the auditorium are covered with high absorption coefficient materials. The surface area of walls is increased to decrease the reverberation time in good acoustic halls.

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6. **What is reverberation? Explain the causes of reverberation in a theatre and how can it be reduced? Deriving the necessary equation for reverberation time, discuss the factors affecting the acoustics of buildings.** [T.U. 2065 Shrawan]

Solution:

Reverberation

Reverberation is defined as the persistence of the sound even after source is cut off. A reverberation is created in an enclosed space causing a large number of reflections to build up and decay slowly as the sound is absorbed by walls and air.

The reverberation is due to the multiple reflections on smooth surfaces of theatre and small number of openings like ventilation windows and doors. The reverberation can be reduced by increasing the absorption of sound in the theatre using sound absorbing materials such as using heavy curtains, hanging maps and pictures in the walls, covering the wall and ceiling with absorbing material like mineral wools, celotex, felt, flooring with carpets, arranging seats having cushions at their backs which provide a high degree of absorption in themselves. A large number of audiences in the theatre significantly reduce the reverberation.

Sabine's reverberation equation

See the solution of Q. No. 5 on page no. 60

The reverberation time is directly proportional to the volume of auditorium halls and inversely proportional to area of walls, ceiling, floor, etc and absorption plus transmission through open surfaces.

Factors affecting acoustics of building are;

i) control of reverberation
ii) shapes of walls and ceilings
iii) concave shapes and balconies
iv) floor plans with diverging side walls
iv) seats
v) loudness, resonance and noise.

---

7. **A lecture hall with a volume of 4500 m^3 is found to have a reverberation time of 1.5 s. What is the total absorbing power of all the surfaces in the hall? If the area of the sound absorbing surface is 1600 m^2, calculate the average absorption coefficient.** [T.U. 2065 Chaitra]

Solution:

Here,

Volume of the lecture hall $(V) = 4500\ m^3$

Reverberation time $(T) = 1.5\ s$

We have,

$$T = \frac{0.158\,V}{\alpha S}$$

or, $\alpha S = \frac{0.158\,V}{T} = \frac{0.158 \times 4500}{1.5} = 474$ S.I. units

Hence, the absorbing power of all the surfaces of the hall is 474 S.I. units.

Area of absorbing surface $(S) = 1600\ m^2$

Thus,

$\alpha \times 1600 = 474$

$\therefore \alpha = 0.30$

i.e., the average absorption coefficient = 0.30

---

8. **Define absorption coefficient. Derive Sabine's reverberation formula and also explain its importance in our daily life?** [T.U. 2065 Kartik]

Solution:

Absorption coefficient (α)

Absorption coefficient is defined as ratio of sound energy absorbed by the surface to that of the total sound energy incident on the surface. Its unit is open window units (OWU).

i.e., $\alpha = \frac{\text{Sound energy absorbed by the surface}}{\text{Total energy incident on the surface}}$

Sabine's reverberation formula

See the solution of Q. No. 5 on page no. 60

The reverberation time is directly proportional to the volume of auditorium halls and inversely proportional to area of walls, ceiling, floor, etc and absorption plus transmission through open surfaces. This relation provides the conditions for designing acoustically good auditorium halls and designs the rooms with pleasant talking environments.

9. **Define absorption coefficient of sound. Derive a relation between reverberation time and absorption coefficient for acoustically good hall.** [T.U. 2067 Ashadh]

Solution: See the solution of Q. No. 8 on page no. 63

10. **Derive a necessary equation for reverberation time and mention the factor affecting the acoustics of building.** [T.U. 2061 Baishakh]

Solution: See the solution of Q. No. 6 on page no. 62

11. **Distinguish between ultrasonic and infrasonic waves? Describe a method for the production of ultrasonic waves. Mention some important applications of ultrasonic waves.** [P.U. 2008]

Solution:

The sound waves of frequency lower than audible range, i.e. below 20 *Hz*, are called *infrasonics*. Such waves are produced by large vibrating bodies, e.g., earthquake waves, vibration of pendulums. These waves are insensitive to human ears.

The sound waves of frequency higher than audible range i.e. above 20 *kHz*, are called *ultrasonic waves*. Animals like dogs, birds and bats are sensitive to particular range of these waves. Bats are almost blind but can locate objects and navigate obstacles by producing ultrasonic waves and receiving echoes. These waves travel in air with same speed as that of audible waves; exhibit the properties of audible sound waves and some new phenomena. e.g. they travel in air with the velocity of 345 *m/s* at 25°. Because of high frequency, their wavelength is extremely small.

Piezoelectric generation of ultrasonic waves

See the solution of Q. No. 3 on page no. 58

Applications of Ultrasonic Waves

See the solution of Q. No. 1 on page no. 55

12. **Distinguish between ultrasonic and infrasonic waves? Describe a method for the production of ultrasonic waves. Discuss briefly the application of ultrasonic waves finding of focal length of the depth of sea and signaling.** [P.U. 2005 B]

Solution: See the solution of Q. No. 11 on page no. 64

3.6 ADDITIONAL SOLVED PROBLEMS

1. **A hall of floors is $(15 \times 30)\ m^2$ along with height of 6 *m*, in which 500 people occupy upholstered seat and wooden chairs. Optimum reverberation time for orchestral music is 1.36 *s* and absorption coefficient per person is 0.44 OWU.**
   i) **Calculate the coefficient of absorption to be provided by the walls, floor and ceiling when the hall is fully occupied.**
   ii) **Calculate the reverberation time if only the half upholster seats are occupied. (The absorption coefficient of chair is 0.02 OWU).**

Solution:

Here,

Volume of the hall $(V) = 15 \times 30 \times 6 = 2700\ m^3$

Optimum reverberation time $(T) = 1.36\ s$

i) We know that,

Sabine's relation for reverberation time,

$$T = \frac{0.158\ V}{aS}$$

or, $$aS = \frac{0.158\ V}{T} = \frac{0.158 \times 2700}{1.36}$$

$= 313.68$ S.I. units

The absorption due to audience $= 500 \times 0.44 = 220$ S.I. units

Therefore, the absorption provided by walls, floor and ceiling

$= 313.68 - 220 = 93.68$ S.I. units

ii) When the hall is only half filled the absorption will be provided by vacant wooden seats in addition to the absorption by the audience.

Absorption by audience $= 250 \times 0.44$

$= 110$ S.I. units

Absorption by vacant wooden seats $= 250 \times 0.02$

$= 5$ S.I. units

$\therefore$ Total absorption of the hall $= 93.68 + 110 + 5$
$= 208.68$ S.I. units

The reverberation time,

$$T = \frac{0.158\,V}{aS} = \frac{0.158 \times 2700}{208.68} = 2.04\ s$$

2. **Calculate the total absorption coefficient of auditorium hall with reverberation time 1.7 s whose volume is 1500 m^3.**

Solution:

Here,

Reverberation time or auditorium hall $(T) = 1.7\ s$

Volume of the auditorium hall $(V) = 1500\ m^3$

We know that,

$$T = \frac{0.158\,V}{aS}$$

or, $aS = \frac{0.158\,V}{T} = \frac{0.158 \times 1500}{1.7} = 139.41$ S.I. units

The total absorption coefficient of the auditorium hall is 139.41 S.I. units.

3. **A piezoelectric crystal plate has a thickness of 1.6 mm. If the velocity of propagation of sound waves is 5700 m/s, calculate the fundamental frequency of the crystal.**

Solution:

Here,

Thickness of the crystal plate $(t) = 1.6\ mm = 1.6 \times 10^{-3}\ m$

Velocity of sound waves $(v) = 5700\ m/s$

In the lowest mode of vibration, the distance between two faces of the crystal of thickness t will be $\frac{\lambda}{2}$. Therefore,

$$t = \frac{\lambda}{2}$$

i.e., $\lambda = 2t = 2 \times 1.6 \times 10^{-3}\ m = 3.2 \times 10^{-3}\ m$

We have,

$$f = \frac{v}{\lambda} = \frac{5700}{3.2 \times 10^{-3}} = 1781250\ Hz = 1.8\ MHz$$

Hence, the fundamental frequency of the crystal is 1.8 MHz.

4. **The ultrasonic pulse echo method is employed to detect possible defects in a steel bar of thickness 45 cm if pulse arrival times are 30 and 80 microseconds, find the distance of the defect from the end of the bar at which the ultrasonic pulse enters the bar.**

Solution

Here,

Thickness of the steel bar $= 45\ cm = 0.45\ m$

Echo times $(t_1) = 30\ \mu s = 30 \times 10^{-6}\ s$

$(t_2) = 80\ \mu s = 80 \times 10^{-6}\ s$

Assume that $x\ m$ is the distance of the possible defect in steel bar from the end of the bar at which ultrasonic pulse enters the bar. The pulse covers a distance of $2x$ in arriving back to the end after being reflected from the defect. Thus, we can write,

$$t_1 = \frac{2x}{v}$$

i.e., $30 \times 10^{-6}\ s = \frac{2x}{v}$ ... (i)

The second pulse will arrive after being reflected from far end of the bar. Therefore, it covers a distance $2 \times 0.45\ m = 0.90\ m$ in $80 \times 10^{-6}\ s$. Thus,

$$80 \times 10^{-6}\ s = \frac{2 \times 0.45\ m}{v} \quad \text{... (ii)}$$

Dividing equation (i) by equation (ii), we obtain,

$$\frac{3}{8} = \frac{x}{0.45}$$

$\therefore\ x = 0.17\ m$

The distance of the flaw from nearest end is 0.17 m.

5. **A quartz crystal with a thickness of 0.4 mm and a density of 2650 kgm^{-3} vibrates longitudinally producing ultrasonic waves. Find the fundamental frequency of vibration if the Young's modulus of quartz is $7.5 \times 10^{10} Nm^{-2}$.**

Solution:

Here,

Thickness of the crystal $(t) = 0.4\ mm = 4 \times 10^{-4}\ m$

Density of the crystal $(\rho) = 2650\ kgm^{-3}$

Young's modulus of elasticity $(Y) = 7.5 \times 10^{10} Nm^{-2}$

We have,

Fundamental frequency of vibration $(f) = \frac{k}{2t}\sqrt{\frac{Y}{\rho}}$

$k = 1$ for the fundamental mode of vibration. Therefore,

$$f = \frac{1}{2\times4\times10^{-4} m}\sqrt{\frac{7.5\times10^{10} Nm^{-2}}{2650\ kgm^{-3}}}$$

$$= 6.65 \times 10^{6} Hz = 6.65\ MHz$$

Hence, the fundamental frequency of the vibration is 6.65 MHz.

6. Calculate the fundamental frequency of quartz crystal of 3 mm thickness. Given that the density of the crystal is 2650 kgm^{-3} and Young's modulus of elasticity is $7.5 \times 10^{10} Nm^{-2}$.

Solution: Proceed as solution of Q. No. 5 on page no. 67

7. An ultrasonic generator has a quartz crystal vibrating at its fundamental frequency. The thickness of the crystal is 2.5 mm, density 2650 kgm^{-3} and Young modulus is $7.5 \times 10^{10} Nm^{-2}$. Calculate the fundamental frequency of vibration and second overtone.

Solution: Proceed as solution of Q. No. 5 on page no. 67

[for the second overtone, $k = 3$]

Chapter 4

LENSES

4.1 SIGN CONVENTION

The distance of the object and image from the refracting surface is a vector quantity and these distances must be represented with proper signs.

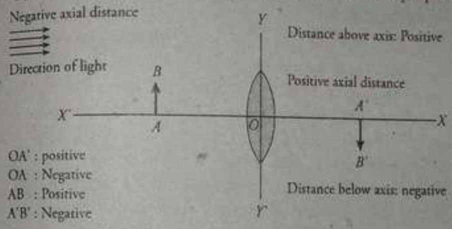

Figure: Sign convention

The above figure depicts the conventions of signs used in accordance with the convention of coordinate geometry. In general, the figures are drawn with the incident light travelling from left right. Considering the centre of the refracting surface at origin O and its axis along XX', distances measured to the left of O are taken as negative. Distances measured upward and normal to the X–axis are taken as positive whereas downward distances are taken as negative.

If we consider AB as an object in above figure, its distance OA is negative. If $A'B'$ represents an image of AB, its distance OA' is positive. The size of the object AB is positive while size of the image $A'B'$ is negative.

4.2 LENS TERMINOLOGY

A *lens* is a portion of a transparent medium bounded by two spherical surfaces or by one spherical surface and a plane surface. It is usually made of glass. The centres of two spherical surfaces which form the lens are known as *centres of curvature*. The distance of the refracting surface from the respective centre of curvature is called *radius of curvature*. The middle point of the lens from either side is said to be *optical centre*. The incident ray and emergent ray through the optical centre of lens are parallel to each other, *i.e.*, rays don't refract while passing through the optical centre. The imaginary line passing through the centres of curvature is said to be *principal axis*. A point on the principal axis, where rays of a parallel beam of light, also parallel to the principal axis actually converge or from where it appears to come from is said to be *principal focus*. A linear distance between optical centre to principal focus is known as *focal length*.

4.3 CARDINAL POINTS

Whenever refraction takes place through a thin lens, position and size of an image formed is determined by neglecting the thickness of less, as it is very small compared to distances of object and image from it. In case of thick lens or coaxial system of lenses, we cannot proceed with this assumption. The method of finding position and size of final image by considering refraction at each surface of a lens successively is extremely tedious and complex process. To overcome this difficulty, Gauss (1941) proved that the positions of certain specific points are known. These specific points are known as cardinal points. The pair of the points forms six cardinal points of an optical system: two focal points, two principal points and two nodal points. All six points are situated on the optical axis of system and are conjugate to each other.

4.4 REFRACTION THROUGH A LENS

When light form the object O falls on the first reflecting surface of the lens, the image I' is formed at the distance v' from the reflecting surface.

Thus,

$$\frac{\mu_2}{v'} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R_1}$$

where, R_1 is the radius of curvature of refracting surface

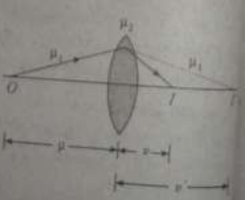

Figure: Refraction through a thin lens

The image I' acts as virtual object in medium of refracting index μ_2 for second refracting surface and final image I at distance v from the second refracting surface with radius of curvature R_2 such that;

$$\frac{\mu_1}{v} - \frac{\mu_2}{v'} = \frac{\mu_1 - \mu_2}{R_2}$$

On simplifying, we obtain,

$$\frac{1}{v} - \frac{1}{u} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

where, $\mu = \frac{\mu_2}{\mu_1}$ is the refractive index of material of lens

4.5 POWER OF LENS

Power of a lens is measure of its ability to produce convergence or divergence of a parallel beam of light. It is defined as the reciprocal of focal length expressed in meters. Thus,

$$\text{Power of a lens } (P) = \frac{1}{\text{Focal length in meters}}$$

The S.I. unit of power of lens is Diopter (D).

If two lenses of focal lengths f_1 and f_2 are in contact, then,

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$P = P_1 + P_2$$

where, P_1 and P_2 are power of two lenses and P is their equivalent power. If two lenses are coaxially separated by a distance d, their equivalent power is:

$$P = P_1 + P_2 - dP_1P_2$$

4.6 CHROMATIC ABERRATION

The equations connecting object distance, image distance, focal length, refractive index, etc. are based on the assumption that angles made by the rays of light with axis are small. In practice, lenses are used to form images of points situated off axis as well. In general non paraxial rays of light from an object point do not meet at a single point after refraction through lens. The refractive index and hence the focal length of a lens are different for different wavelengths of light. Therefore, a number of colored images are formed by the lens for non–monochromatic light. These images even though formed by paraxial rays are at different position and are of different sizes.

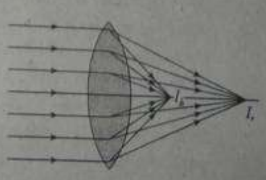
Figure: Chromatic aberration produced by a convex lens

The deviations from the actual size, shape and position of an image as calculated by simple lens equations are called *aberrations* produced by a lens.

The aberrations produced by a variation of refractive index with wavelength of light are called *chromatic aberrations*. In other words, a single lens produces colored images of an object illuminated by white light; such defect is called chromatic aberration. Elimination of the defect in a system of lenses is called *achromatism*.

4.7 SOLVED EXAM QUESTIONS

1. **What are common defects in the image produced by a single lens? Find the condition for achromatism of two lenses separated by distance.** [T.U. 2061 Ashwin]

Solution:

The common defects in the image produced by a single lens are:

i) Chromatic Aberrations
ii) Monochromatic Aberrations
  a) Spherical Aberrations
  b) Coma
  c) Astigmatism
  d) Curvature of a field
  e) Distortion

Conditions for Achromatism of two lenses

An elimination of chromatic aberration in a system of lenses is called *achromatism*. The achromatic combination is made by placing two lenses of different materials and suitable focal length in contact or separated by a finite distance.

Consider two lenses of focal lengths f_1 and f_2 separated by a distance d. The equivalent focal length F for the combination of two lenses is;

$$\frac{1}{F} = \frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{f_1 f_2}$$

On differentiating, we obtain,

$$-\frac{dF}{F^2} = -\frac{df_1}{f_1^2}-\frac{df_2}{f_2^2}-d\left(\frac{df_1}{f_1^2 f_2}+\frac{df_2}{f_1 f_2^2}\right)$$

For achromatism,

$$dF = 0$$

For lenses of same material,

$$-\frac{df_1}{f_1} = -\frac{df_2}{f_2} = \omega$$

where, ω is dispersive power of the lens material. Thus,

$$\frac{\omega}{f_1}+\frac{\omega}{f_2}-d\left(\frac{\omega}{f_1 f_2}+\frac{\omega}{f_1 f_2}\right) = 0$$

or, $$\frac{1}{f_1}+\frac{1}{f_2} = \frac{2d}{f_1 f_2}$$

$$\therefore \quad d = \frac{f_1+f_2}{2}$$

Hence, two lenses must be separated by the distance equal to mean focal length of the two lenses for achromatic combination.

2. **Two thin converging lenses of focal lengths 50 cm and 40 cm are placed co-axially 30 cm apart in air. Determine the positions of cardinal points.** [T.U. 2062 Baishakh]

Solution:

Here,

$f_1 = 50\ cm$

$f_2 = 40\ cm$

$d = 30\ cm$

We have,

$$f = \frac{f_1 f_2}{f_1+f_2-d} = \frac{50\ cm \times 40\ cm}{50\ cm + 40\ cm - 30\ cm} = \frac{10}{3}\ cm$$

$$\alpha = +d\frac{f}{f_2} = 30 \times \frac{10}{3\times 40} = 2.5\ cm$$

Thus the first principal point P_1 is at a distance of 2.5 cm to the right of the first lens.

$$\beta = -d\frac{f}{f_1} = 30 \times \frac{10}{3\times 50} = 2\ cm$$

The second principal point P_2 is at distance 2 cm to the left of the second lens.

The first focal point F_1 is at a distance;

$$f_1 - f = \left(50-\frac{10}{3}\right) cm = 46.67\ cm \text{ from the first lens}$$

The second focal point F_2 is at a distance;

$$f - f_2 = \left(\frac{10}{3}-40\right)\ cm = -36.67\ cm \text{ from the second lens}$$

As the medium on the two sides of the lens system is the same, the nodal points N_1 and N_2 coincide with P_1 and P_2.

3. What are cardinal points? Explain their significance with reference to a co-axial lens system using ray diagram. [T.U. 2063 Baishakh]

Solution:

Whenever refraction takes place through a thin lens, position and size of an image formed is determined by neglecting the thickness of less, as it is very small compared to distances of object and image from it. In case of thick lens or coaxial system of lenses, we cannot proceed with this assumption. The method of finding position and size of final image by considering refraction at each surface of a lens successively is extremely tedious and complex process. To overcome this difficulty, Gauss (1941) proved that the positions of certain specific points are known. These specific points are known as cardinal points. The pair of the point from six cardinal point of an optical system: two focal points, two principal points and two nodal points. All six points are situated on the optical axis of system and are conjugate to each other.

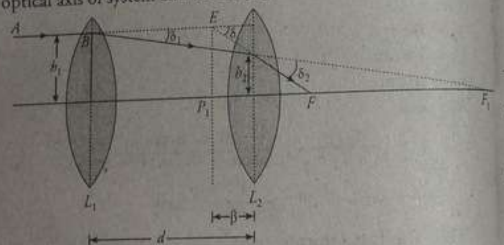

Figure: Co-axial lens system

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4. Two thin lenses of focal lengths f_1 and f_2 separated by a distance d have an equivalent focal length 50 *cm*. The combination satisfies the conditions for no chromatic aberration and minimum spherical aberration. Find the values of f_1, f_2 and d. Assume that both the lenses are of same material. [T.U. 2063 Baishakh]

Solution:

Here,

Equivalent focal length $(F) = 50\ cm = 0.5\ m$

Two thin lenses of focal lengths f_1 and f_2 separated by a distance d. For no chromatic aberration, we write,

$$d = f_1 - f_2$$

For minimum spherical aberration, we can write,

$$d = \frac{f_1 + f_2}{2}$$

From these two equations, we obtain,

$$f_1 = 3f_2$$

$$d = 3f_2 - f_2 = 2f_2$$

The equivalent focal length for combination of two lenses is,

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

$$\text{or, } \frac{1}{0.5} = \frac{1}{3f_2} + \frac{1}{f_2} - \frac{2f_2}{3f_2 f_2}$$

$$\text{or, } 2 = \frac{2}{3f_2}$$

$$\therefore f_2 = \frac{1}{3}\ m = 0.33\ m$$

Substituting f_2, we obtain,

$$f_1 = 3 \times 0.33\ m = 0.99\ m$$

$$d = 2 \times 0.33\ m = 0.66\ m$$

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5. Two lenses of focal lengths 8 cm and 4 cm are placed at a distance apart. Calculate the position of the principal points to form an achromatic combination. [T.U. 2064 Poush]

Solution:

Here,

$f_1 = 8\ cm$

$f_2 = 4\ cm$

$$d = \frac{f_1 + f_2}{2} = \frac{8\ cm + 4\ cm}{2} = 6\ cm$$

$$f = \frac{f_1 f_2}{f_1 + f_2 - d} = \frac{8\ cm \times 4\ cm}{8\ cm + 4\ cm - 6\ cm}$$

$$= \frac{16}{3}\ cm$$

$$\alpha = +d\frac{f}{f_2} = 6 \times \frac{16}{3 \times 4} = 8\ cm$$

$$\beta = -d\frac{f}{f_1} = -6 \times \frac{16}{3 \times 8} = -4\ cm$$

The principle points are 8 *cm* and −4 *cm* to form an achromatic combination.

6. **What are cardinal points in an optical system? Use a suitable diagram to explain the principal points.** **[T.U. 2064 Poush]**

Solution:

Cardinal Points

See the solution of Q. No. 3 on page no. 74

Consider a thick lens system or coaxial lens system. The system has two principal foci F_1 and F_2. The parallel ray of incident light after refraction intersect the principal axis at F_2. When incident ray and final emergent ray are produced, they intersect at a point H_2. A plane passing through H_2 and perpendicular to principal axis intersects the principal axis at P_2 is referred as second principal point. Similarly ray from F_1, after refraction passes parallel to the principal axis. The incident ray from F_1 and final emergent ray, if produced meet at H_1. A plane containing H_1 and perpendicular to principal axis is termed as first principal plane. The point P_1 in the principal axis is referred as first principal point.

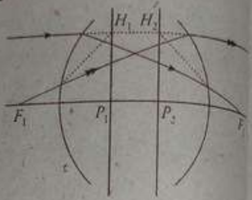

Figure: Principal points

7. **What are chromatic and monochromatic aberrations? List out different kinds of monochromatic aberrations. What is spherical aberration and discuss how it can be reduced to a minimum?** **[T.U. 2065 Shrawan]**

Solution:

The refractive index of the material of a lens is different for different wavelengths of light. Hence the focal length of a lens is different for different wavelengths. Further, as the magnification of image is dependent of focal length of la lens, the size of the image is different for different wavelengths. The variation of the image distance from lens with refractive index measure axial or longitudinal chromatic aberration and variation in the size of the image measures lateral chromatic aberration.

The different kinds of monochromatic aberrations are;

a) Spherical Aberrations
b) Coma

c) Astigmatism
d) Curvature of a field
e) Distortion

The failure of rays to pass through a single point after reflection from a curved spherical surface is called spherical aberration. Spherical aberrations can be minimized in following ways.

i) Spherical aberrations can be minimized by using stops, which reduce the effective lens aperture. The stop used can be such as to permit either the axial rays of light or the marginal rays of light. However, the amount of light passing through the lens is reduced and image appears less bright.
ii) Plano–convex lenses are used in optical instruments so as to reduce the spherical aberration. When the curved surface of the lens faces the incident or emergent light whichever is more parallel to the axis, the spherical aberration is minimum.
iii) Spherical aberration can be minimized by using two plano–convex lenses separated by a distance equal to the difference in their focal length. In this arrangement, the total deviation is equally shared by two lenses and spherical aberration is minimum.
iv) Spherical aberration for a convex lens is positive and that for a concave lens is negative. Thus, by a suitable combination of convex lens and concave lens, it can be made minimized.

8. **What are cardinal points of an optical system? Determine the equivalent focal length of a combination of two thin lenses separated by a finite distance. Hence find the position of two principal points.** **[T.U. 2065 Chaitra]**

Solution:

Cardinal Points

See the solution of Q. No. 3 on page no. 74

Consider two thin lenses L_1 and L_2 having focal lengths f_1 and f_2 are placed coaxially. They are separated by a finite distance d. Consider a monochromatic ray AB parallel to principal axis is incident on lens L_1 at a height h_1. This ray is refracted towards BF_1 in absence of lens L_2. The F_1 is principal focus of lens L_1. The deviation produced by this lens is,

$$\delta_1 = \frac{h_1}{f_1}$$

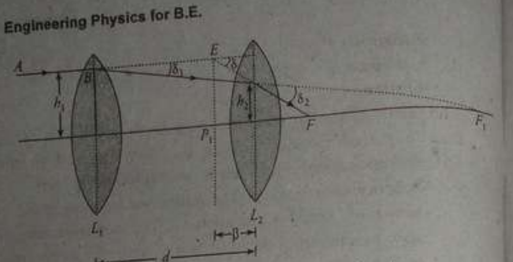

Figure: Refraction through the combination of two lenses

The ray BF_1 meets lens L_2 at point C, at the height h_2 from principal axis and gets emerged from this lens towards CF, where, F is the principal focus of the combination of two lenses L_1 and L_2. The deviation due to second lens is,

$$\delta_2 = \frac{h_2}{f_2}$$

where, f_2 is the focal length of the lens L_2

An incident ray AB is produced and final emergent ray FC will intersect at point. Thus, a thin lens placed at E, will produce same deviation as the lenses L_1 and L_2 separated at the distance d. An imaginary such lens of focal length, is called equivalent lens. The deviation produced by equivalent lens is:

$$\delta = \frac{h_1}{f}$$

From geometry, we write,

$$\delta = \delta_1 + \delta_2$$

$$\frac{h_1}{f} = \frac{h_1}{f_1} + \frac{h_1}{f_2}$$

Since ΔBL_1F_1 and ΔBL_2F_1 are similar. This provides the sides of these triangele have proportionate relation.

$$i.e., \frac{BL_1}{L_1F_1} = \frac{CL_2}{L_2F_1}$$

$$\frac{h_1}{f_1} = \frac{h_2}{f_1 - d} \qquad (\because L_1L_2 = d)$$

$$\therefore h_2 = \frac{h_1(f_1 - d)}{f_1}$$

Thus,

$$\frac{h_1}{f} = \frac{h_1}{f_1} + \frac{h_1(f_1 - d)}{f_1 \cdot f_2}$$

$$\therefore \frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

This provides the equivalent focal length of two thin lenses separated by the distance d.

$$f = \frac{f_1 f_2}{f_1 + f_2 - d} = -\frac{f_1 f_2}{\Delta}$$

where, $\Delta = -(f_1 + f_2 - d)$ and is known as optical separation or optical interval between the two lenses

It is numerically equal to the distance between the second principal focus of the first lens and first principal focus of the second lens.

9. **Two thin lenses of focal lengths f_1 and f_2 separated by a distance d have an equivalent focal length 50 cm. The combination satisfies the conditions for no chromatic aberration and minimum spherical aberration. Assuming both the lenses are of same material, find the values of f_1, f_2 and d. [T.U. 2065 Chaitra]**

Solution: See the solution of Q. No. 4 on page no. 74

10. **Two thin lenses having focal lengths 10 cm and 4 cm are coaxially separated by a distance of 5 cm. Find the equivalent focal length of the combination. Determine also the positions of the principal points. [T.U. 2068 Shrawan]**

Solution:

Here,

$f_1 = 10\ cm$

$f_2 = 4\ cm$

$d = 5\ cm$

$$f = \frac{f_1 f_2}{f_1 + f_2 - d} = \frac{10\ cm \times 4\ cm}{10\ cm + 4\ cm - 6\ cm} = 5\ cm$$

$$\alpha = +d\frac{f}{f_2} = 5 \times \frac{5}{4} = 6.25\ cm$$

$$\beta = -d\frac{f}{f_1} = -5 \times \frac{5}{10} = -2.5\ cm$$

The equivalent focal length of combination is 5 cm. The principal points are at 6.25 cm from the right of the first lens and second lens is at the distance of 2.5 cm left from second lens.

11. **Two identical thin convex lenses of focal length 8 *cm* each are coaxial and 4 *cm* apart. Find the equivalent focal length and the positions of the principal points. Also find the position of the object for which image is formed at infinity.** **[P.U. 2002]**

Solution:

Here,

$f_1 = 8\ cm$

$f_2 = 4\ cm$

$$d = \frac{f_1+f_2}{2} = \frac{8\ cm + 4\ cm}{2} = 6\ cm$$

$$f = \frac{f_1 f_2}{f_1+f_2-d} = \frac{8\ cm \times 4\ cm}{8\ cm + 4\ cm - 6\ cm} = \frac{16}{3}\ cm$$

$$\alpha = +d\frac{f}{f_2} = 6 \times \frac{16}{3\times 4} = 8\ cm$$

$$\beta = -d\frac{f}{f_1} = -6 \times \frac{16}{3\times 8} = -4\ cm$$

The equivalent focal length of combination is $\frac{16}{3}\ cm$. The principal points are at 8 *cm* from the right of the first lens and second lens is at the distance of 4 *cm* left from second lens.

For the final image to be formed at infinity,

$V = \infty$

$f = \frac{16}{3}\ cm$

Therefore,

$$\frac{1}{V} - \frac{1}{U} = \frac{1}{f}$$

$$\therefore\ U = -f = -\frac{16}{3}\ cm$$

But,

$$u = U + \alpha = -\frac{16}{3} + 8 = \frac{8}{3}\ cm$$

Hence, the object is at a distance of $\frac{8}{3}\ cm$ to the left of the first lens.

12. **Define cardinal points. Derive the expression for combined focal length of two thin lenses placed at certain distance apart. [P.U. 2003]**

Solution: See the solution of Q. No. 8 on page no. 77

13. **What are cardinal points? Explain spherical and chromatic aberration in optical image. Obtain condition of achromatize for two thin lenses placed co-axially in contact.** **[P.U. 2004]**

Solution:

Cardinal Points

See the solution of Q. No. 3 on page no. 74

Spherical and Chromatic aberrations

The failure of rays to pass through a single point after reflection from a curved spherical surface is called spherical aberration. Spherical aberrations can be minimized by using stops, which reduce the effective lens aperture. The stop used can be such as to permit either the axial rays of light or the marginal rays of light. However, the amount of light passing through the lens is reduced and image appears less bright. Spherical aberration can be minimized by using two plano–convex lenses separated by a distance equal to the difference in their focal length. In this arrangement, the total deviation is equally shared by two lenses and spherical aberration is minimum.

The refractive index of the material of a lens is different for different wavelengths of light. Hence the focal length of a lens is different for different wavelengths. Further, as the magnification of image is dependent of focal length of la lens, the size of the image is different for different wavelengths. The variation of the image distance from lens with refractive index measure axial or longitudinal chromatic aberration and variation in the size of the image measures lateral chromatic aberration.

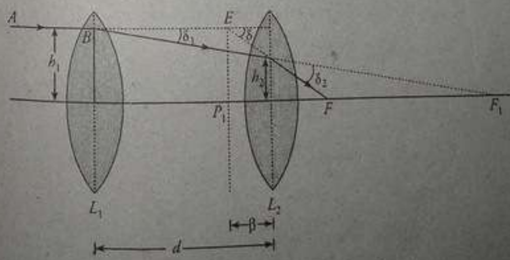

Figure: Refraction through the combination of two lenses

Consider two thin lenses L_1 and L_2 having focal lengths f_1 and f_2 are placed coaxially. They are separated by a finite distance d. Consider a monochromatic ray AB parallel to principal axis is incident on lens L_1 at a height h_1. This ray is refracted towards BF_1 in absence of lens L_2. The F_1 is principal focus of lens L_1. The deviation produced by this lens is,

$$\delta_1 = \frac{h_1}{f_1}$$

The ray BF_1 meets lens L_2 at point C, at the height h_2 from principal axis and gets emerged from this lens towards CF, where F is the principal focusof the combination of two lenses L_1 and L_2. The deviation due to second lens is,

$$\delta_2 = \frac{h_2}{f_2}$$

where, f_2 is the focal length of the lens L_2

An incident ray AB is produced and final emergent ray FC will intersect at point. Thus, a thin lens placed at E, will produce same deviation as the lenses L_1 and L_2 separated at the distance d. An imaginary such lens of focal length, is called equivalent lens. The deviation produced by equivalent lens is;

$$\delta = \frac{h_1}{f}$$

From geometry, we write,

$$\delta = \delta_1 + \delta_2$$

$$\text{or, } \frac{h_1}{f} = \frac{h_1}{f_1} + \frac{h_1}{f_2}$$

Since ΔBL_1F_1 and ΔBL_2F_1 are similar. This provides the sides of these triangles have proportionate relation.

$$\text{i.e., } \frac{BL_1}{L_1F_1} = \frac{CL_2}{L_2F_1}$$

$$\text{or, } \frac{h_1}{f_1} = \frac{h_2}{f_1 - d} \qquad (\because L_1L_2 = d)$$

$$\therefore h_2 = \frac{h_1(f_1 - d)}{f_1}$$

Thus,

$$\frac{h_1}{f} = \frac{h_1}{f_1} + \frac{h_1(f_1 - d)}{f_1 \cdot f_2}$$

$$\therefore \frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

This provides the equivalent focal length of two thin lenses separated by the distance d.

$$f = \frac{f_1 f_2}{f_1 + f_2 - d} = -\frac{f_1 f_2}{\Delta}$$

where, $\Delta = -(f_1 + f_2 - d)$ and is known as optical separation or optical interval between the two lenses.

14. Two thin convex lenses of focal lengths 20 *cm* and 40 *cm* are placed co-axially 20 *cm* apart. Find the position of principal points of the lens system. **[P.U. 2004]**

Solution:

Here,

$f_1 = 20\ cm$

$f_2 = 40\ cm$

$$d = \frac{f_1 + f_2}{2} = \frac{20\ cm + 40\ cm}{2} = 30\ cm$$

$$f = \frac{f_1 f_2}{f_1 + f_2 - d}$$

$$= \frac{20\ cm \times 40\ cm}{20\ cm + 40\ cm - 30\ cm} = \frac{8}{3}\ cm$$

$$\alpha = +d\frac{f}{f_2} = 30 \times \frac{8}{3 \times 40} = 2\ cm$$

$$\beta = -d\frac{f}{f_1} = -30 \times \frac{8}{3 \times 20} = -4\ cm$$

The equivalent focal length of combination is $\frac{8}{3}$ *cm*. The principal points are at 2 *cm* from the right of the first lens and second lens is at the distance of 4 *cm* left from second lens.

15. Two thin lenses of focal lengths f_1 and f_2 seperated by a distance d have an equivalent focal length 50 *cm*. The combination satisfies the conditions for no chromatic aberration and minimum spherical aberration. Find the values of f_1, f_2 and d. Assume that both the lenses are of same material. **[P.U. 2005]**

Solution: See the solution of Q. No. 4 on page no. 74

16. Define the cardinal points of a system of co-axial lines. Calculate the equivalent focal length and principal points of two thin co-axial lenses separated by a finite distance d. **[P.U. 2007]**

Solution: See the solution of Q. No. 8 on page no. 77

17. What do you mean by the optical separation? Two lenses of focal lengths f_1 and f_2 are placed coaxially at a certain distance d apart in air. Derive an expression for the equivalent focal length of the combination and find the position of two principal planes.
[P.U. 2008]

Solution:

Optical separation is defined as the distance between the second principal focus of the first lens and first principal focus of the second lens.

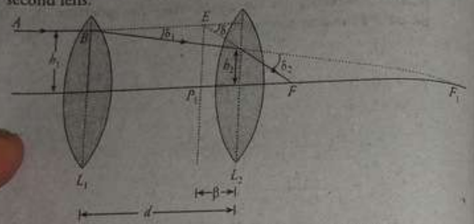

Figure: Refraction through the combination of two lenses

Consider two thin lenses L_1 and L_2 having focal lengths f_1 and f_2 are placed coaxially. They are separated by a finite distance d. Consider a monochromatic ray AB parallel to principal axis is incident on lens L_1 at a height h_1. This ray is refracted towards BF_1 in absence of lens L_2. The F_1 is principal focus of lens L_1. The deviation produced by this lens is,

$$\delta_1 = \frac{h_1}{f_1}$$

The ray BF_1 meets lens L_2 at point C, at the height h_2 from principal axis and gets emerged from this lens towards CF, where, F is the principal focus of the combination of two lenses L_1 and L_2. The deviation due to second lens is,

$$\delta_2 = \frac{h_2}{f_2}$$

where, f_2 is the focal length of the lens L_2. An incident ray AB is produced and final emergent ray FC will intersect at point. Thus, a thin lens placed at E, will produce same deviation as the lenses L_1 and L_2 separated at the distance d. An imaginary such lens of focal length, is called equivalent lens. The deviation produced by equivalent lens is;

$$\delta = \frac{h_1}{f}$$

From geometry, we write,

$$\delta = \delta_1 + \delta_2$$

$$\text{or, } \frac{h_1}{f} = \frac{h_1}{f_1} + \frac{h_1}{f_2}$$

Since ΔBL_1F_1 and ΔBL_2F_1 are similar. This provides the sides of these triangles have proportionate relation.

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$$\frac{h_1}{f_1} = \frac{h_2}{f_1 - d} \qquad (\because L_1L_2 = d)$$

$$\therefore h_2 = \frac{h_1(f_1 - d)}{f_1}$$

Thus,

$$\frac{h_1}{f} = \frac{h_1}{f_1} + \frac{h_1(f_1 - d)}{f_1 \cdot f_2}$$

$$\therefore \frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

This provides the equivalent focal length of two thin lenses separated by the distance d.

$$f = \frac{f_1 f_2}{f_1 + f_2 - d} = -\frac{f_1 f_2}{\Delta}$$

where, $\Delta = -(f_1 + f_2 - d)$ and is known as optical separation or optical interval between the two lenses.

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18. Two lenses of focal lengths f_1 and f_2 are placed coaxially at a certain distance d apart in air. Prove that $F = \frac{f_1 f_2}{\Delta}$, where, the symbols have their usual meaning. **[P.U. 2010]**

Solution: See the solution of Q. No. 8 on page no. 77

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19. What are cardinal points? Derive the expression for combined focal length of two thin lenses placed at certain distance apart. [P.U. 2011]

Solution: See the solution of Q. No. 8 on page no. 77

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20. Two thin lenses (same material) of focal lengths f_1 and f_2 separated by a distance d have an equivalent focal length 50 *cm*. The combination satisfies the conditions for no chromatic aberration and minimum spherical aberration. Find the values of f_1, f_2 and d. **[P.U. 2011]**

Solution: See the solution of Q. No. 4 on page no. 74

Chapter 5

FIBRE OPTICS

5.1 FIBRE OPTICS AND OPTICAL FIBRE

Fibre optics is branch of physics that deals with the propagation of ligh waves via optical fibre. In optical fibres, the light waves lunched in on end of the fibre and it is passed through the other end without any lo of signals. The light waves passes through the optical fibre due to th total internal reflection of light. The light waves undergo total intern reflection more than hundred thousand times within the length of meter of the optical fibre.

An optical fibre is a dielectric wave guide that transmits light signals fro one place to another place. It consists of central core within whic propagation of electromagnetic field is confirmed and which surrounded by a cladding layer. The refractive index of the core is alwa greater than that of cladding layer. The cladding layer is surrounded by thin layer of buffer coating or jacket. The core and cladding layer made up of pure silica glass, whereas buffer coating is made of plastics cross sectional view of typical optical fibre is shown in figure below.

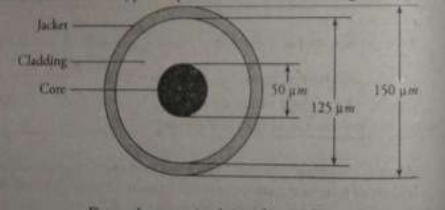

Figure: A cross sectional view of optical fibre

5.2 BASIC PRINCIPLE OF FIBRE OPTICS

The basic principle behind the propagation via optical fibre is tot internal reflection. The refractive index is one of the most importa optical parameters of a medium. It is defined as the optical density material with respect to vacuum. Mathematically,

$$\text{Refractive index of the medium } (\mu) = \frac{\text{Velocity of light in vaccum } (c)}{\text{Velocity of light in a medium } (v)}$$

The refractive index of vacuum is 1, that of air is 1.0002, that of water is 1.333, that of fused silica is 1.452 and that of crown glass is 1.517.

Whenever a light ray travels from an optically denser medium to optically rarer medium, it bends away from normal, *i.e.*, $i > r$. An angle of incidence in a denser medium is said to be *critical angle* C, if angle of refraction in a rarer medium is right angle. Whenever angle of incidence is greater than critical angle, incident ray does not refract. It reflects back in same medium. This phenomenon is called *total internal reflection*. At critical angle,

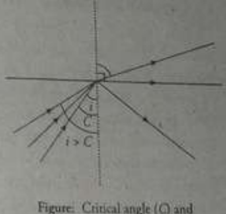

Figure: Critical angle (C) and total internal reflection

$$\mu_1 \sin C = \mu_2 \sin 90^\circ$$

where, μ_1 and μ_2, $(\mu_1 > \mu_2)$ are refractive indices of denser and rarer media

$$\text{or,} \quad \sin C = \frac{\mu_2}{\mu_1}$$

$$\therefore \quad C = \sin^{-1}\left(\frac{\mu_2}{\mu_1}\right)$$

This equation gives the value of critical angle.

5.3 CHARACTERISTICS OF OPTICAL FIBRES

The optical fibres have typical characteristics that make them highly attractive as a transmission medium. They offer the following characteristics;

i) **Potential bandwidth**

An optical fibre has enormous potential bandwidth, resulting from the use of optical carrier frequencies around $2 \times 10^{14}\ Hz$. With such a high frequency and bandwidth roughly equal to 10% of the carrier frequency.

ii) **Transmission loss**

Optical fibres have low transmission losses, as low as $0.1\ dB\ km^{-1}$.

iii) **Small size and weight**

The diameter of optical fibre is not greater than human hair.

iv) **Ruggedness and flexibility**

Optical fibre has a very high tensile strengths and possibility of being bent or twisted without damage.

v) Immunity to electromagnetic interference

It is an inherent characteristic of an optical fibre, viewed as a dielectric waveguide.

5.4 CLASSIFICATION OF OPTICAL FIBERS

Optical fibers are constructed to fulfill the requirements of particular purposes. The major requirement is in the area of bandwidth, *i.e.*, low medium, high medium or ultra high medium. In addition, a fibre may be constructed for its extremely low attenuation per kilometer. In some cases, tensile strength is the most desired characteristic. The optical fibres are classified into three major categories on the basis of *material, number of modes and refractive index profile.*

Based on the material used for preparation of optical fibres, they are classified into glass fibres and plastic fibres.

Glass fibres

In case of glass fibres, the core and cladding of optical fibres are prepared using glass. Silica is the basic raw material for the preparation of glass fibres. Silica has the refractive index of 1.458 at 850 *nm*. In order to increase or decrease or decrease or decrease the refractive index of Silica, doping materials such as germanium dioxide, phosphorus pentoxide, boron oxide, etc. are added. The addition of germanium dioxide or phosphorus pentoxide with silica increases the refractive index of silica whereas the addition of boron oxide decreases the refractive index of it.

Examples of fiber compositions are as follows:

i) Germanium dioxide + Silica (Core); Silica (Cladding)
ii) Phosphorus pentoxide + Silica (Core); Silica (Cladding)
iii) Silica (Core); Phosphorus pentoxide + Silica (Cladding)

Plastic fibres

The plastic fibres are typically made up of plastics and they are of low cost. Although they exhibit considerably greater signal attenuation than glass fibers, the plastic fibres can be handled without special care due to its toughness and durability. Due to its high refractive index differences between core and cladding materials, plastic fibers yield high numerical aperture of 0.16 and large angle of acceptance up to 70°. A polystyrene core ($\mu_1 = 1.60$) and a polymethylmethacrylate cladding ($\mu_2 = 1.49$), a polymethylmethacrylate core ($\mu_1 = 1.49$) and a cladding made of its co-polymer ($\mu_2 = 1.40$), etc. are examples of plastic fibres.

Based on the refractive index profile, optical fibres are classified into step index optical fibres and graded index optical fibres.

Step index optical fibres

The optical fibre in which refractive indices of core and cladding is constant. The refractive index of core μ_1 is slightly greater than that of cladding. Such optical fibre is called step index optical fibre. Therefore, there is noticeable boundary between the core and cladding. There is high transmission loss due to splitting of light singnals.

A step index fibre can be prepared single mode or multi–mode. Only one signal is passed through a single mode step index fibre, whereas more than one signal can be passed through multimode step index fibres.

In single mode step index fibre, the transmission is quicker as compared to other fibers. There is minimum dispersion. In this fibre, the bandwidth of information transmission is maximum and accuracy in reproducing pulses at the receiving end is high. However, the cost of fabrication is high in single mode step index fibre. The fabrication process is cumbersome as well. The source to fibre aperture is smallest as compared to other fibres. Due to the very small size of the central core, it is difficult to couple light in and out of such fibre. A highly directed light source such as a laser can couple light on the single mode step index fibres.

In multi–mode step index fibre, it is easy to couple light into and out of fibre due to large central core. The source to fibre aperture is relatively large. These fibres are economical. It is simple to fabricate. However, the transmission of information is slowest as compared to other fibres. In this fibre, the bandwidth of information transmission is minimum and distortion of pulse of light is maximum.

Graded Index optical fibres

In a graded index optical fibres, the refractive indices of the core vary with radial distances. The refractive index μ_1 is maximum at the centre of the core and it it minimum at the core–cladding interface μ_2. The refractive index decreases parabolically in proportion to the distance away from the centre of fibre to a constant value at the cladding. As compared to single mode fibre, the light can be easily coupled into and out of the fibre. However, as compared to multi–mode step index fibre, it is difficult to couple light into and out of the fibre. They are easier to construct as compared to single mode optical fibre. However, the construction is more difficult as compared to multi–mode step index optical fibre. The distortion of light pulse is more as compared to single mode optical fibre. However, the distortion is less as compared to multi–mode index optical fibre.

Based on the number of modes of propagation, optical fibres are classified into single mode optical fibres and multi–mode optical fibres.
The number of modes of propagation through an optical fibre is;

$$N = \frac{d \times (NA)^2}{2\lambda}$$

where, d is the diameter of the optical fibre core, NA is the numerical aperture and λ is the wavelength of the light

Single mode optical fibres

If only one mode is passed through a fibre at a particular time, then it is said to be single mode optical fibre. The core diameter of the single mode optical fibre is nearly 8 to 9 μm. The core, cladding and sheath or jacket specification for a single mode optical fibre is $8.5/125/250\ \mu m$. The difference between the refractive indices of core and cladding is made very low. Due to this low difference between the refractive indices the critical angle at core cladding interface is very large. Therefore, the light rays that make very larger value of the angle of incidence at the core–cladding interface will pass through the fibre. So only one ray passes through the fibre, *i.e.*, the fundamental mode alone travels through the fibre. The single mode optical fibres are generally operated at 1300 nm and 1550 nm. The attenuation is low for single mode optical fibre in 1550 nm wavelength operation.

Multi–mode optical fibres

If more than one mode is passed through the optical fibre, it is said to be multi–mode optical fibre. A multi–mode optical fibres have core diameters of 50 μm or greater. A large number of signals are passed through the multimode fibres because of its large diameter. The multimode fibres are available in three different sizes. They are;

i) Core/Cladding/Jacket diameters ⇒ $50/125/250\ \mu m$
   Core/Cladding/Jacket diameters ⇒ $50/125/900\ \mu m$
ii) Core/Cladding/Jacket diameters ⇒ $62.5/125/250\ \mu m$
   Core/Cladding/Jacket diameters ⇒ $62.5/125/900\ \mu m$
iii) Core/Cladding/Jacket diameters ⇒ $100/140/250\ \mu m$

5.5 APPLICATIONS OF OPTICAL FIBRES

Fibre optics essentially deals with communication including voice signals, video signals and digital data. This is done by transmission of light through optical fibres. An optical fibre system consists of three basic parts: a light source, an optical fibre and a light detector. A source may be light emitting diodes [LED's]. An optical fibre transmits light waves. The detector can be either an avalanche photo diode [APD] or positive intrinsic negative [PIN] diode. Basically, a fibre optic system converts an electrical signal to an infrared light signal. This signal is transmitted through an optical fibre. At the end of optical fibre, it is reconverted into an electrical signal.

The optical fibres are used in fabrication of fiberscope in endoscopy in medical sciences. Such fiberscopes are used in visualization of internal organs of human body. Optical fibres in combination of laser probe help to visualization of internal portion of human body and cauterization of tissues. Optical fibres are useful in industry as well. It can be used to examine welds, nozzles and combustion chambers inside aircraft engines which are inaccessible for observations in other available procedures.

5.6 ADVANTAGES OF OPTICAL FIBRES

The optical fibres have a lot of advantages over wireless or radio systems. That is why communication industries have introduced fibre optic systems significantly. The followings are the main advantages of optical fibres.

i) **Low attenuation**

Attenuation in an optical fibre is markedly lower than that of co–axial cable or twisted pair and is constant over a very wide range. Therefore, transmission within the wide range of distance is possible without repeaters, etc.

ii) **Smaller size and light weight**

Optical fibres are considerable thinner than co–axial cable or bundled twisted pair cable. Therefore, they occupy much less space.

iii) **Electromagnetic isolation**

Electromagnetic waves generated from electrical disturbances or electrical noise does not interfere with light signals. As a result, the system is not vulnerable to interference impulse noise or cross talk.

iv) **Physical connections**

No physical electrical connection is required between the senders and receives.

v) **Reliability**

Optical fibres are much more reliable because they can better withstand environmental conditions such as pollution radiation and salts produce no corrosion. Moreover, it is nominally affected by nuclear radiation. Its life is longer comparison to copper wire.

vi) **Security and privacy**

There is no interference in optical fibres and hence transmission is more secure and private because it is very difficult to tap into an optical fibre.

vii) **Greater bandwidth**

Bandwidth of optical fibres is higher than that of an equivalent wire transmission line.

viii) **Isolation coating**

As optical fibres are very good dielectrics, isolation coating is not required.

ix) **Higher data rate**

Data rate is much higher in an optical fibre and hence much more information can be carried out by each fibre in comparison to equivalent copper cables.

x) **Lower cost**

The cost per channel is lower than that of an equivalent wire cable system.

5.7 SOLVED EXAM QUESTIONS

1. **What are the uses of optical fibres?** **[T.U. 2061 Baishakh]**

Solution:

Uses of optical fibres

The use and demand for optical fiber has grown tremendously and optical–fibre applications are numerous. Telecommunication applications are widespread, ranging from global networks to desktop computers. These involve the transmission of voice, data or video over distances of less than a meter to hundreds of kilometers, using one of a few standard fibre designs in one of several cable designs.

Carriers use optical fiber to carry plain old telephone service across their nationwide networks. Local exchange carriers use fibre to carry this same service between central office switches at local levels, and sometimes as far as the neighborhood or individual home.

Optical fibre is also used extensively for transmission of data. Multinational firms need secure reliable systems to transfer data and financial information between buildings to the desktop terminals or computers and to transfer data around the world. Cable television companies also use fibre for delivery of digital video and data services. The high bandwidth provided by fibre makes it the perfect choice for transmitting broadband signals such as high–definition television (HDTV) telecasts.

Intelligent transportation systems, such as smart highways with intelligent traffic lights, automated tollbooths, and changeable message signs, also use fiber–optic–based telemetry systems.

Another important application for optical fibre is the biomedical industry. Fiber–optic systems are used in most modern telemedicine devices for transmission of digital diagnostic images. Other applications for optical fiber include space, military, automotive, and the industrial sector.

2. **Give the reasons for attenuation and distortions of light through the optical fibres.** **[T.U. 2061 Ashwin]**

Solution:

Attenuation

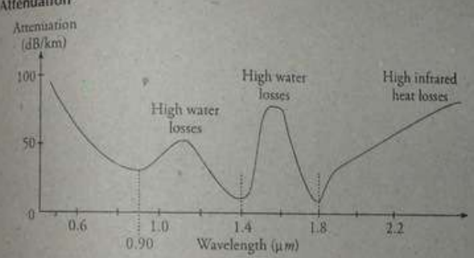

Figure: Typical attenuation versus wavelength curve of optical fibre

It is a decrease of magnitude of power of the light beam. It is represented in dB per unit length. The attenuation of optical signal with wavelength is shown in figure. Figure shows that there is low attenuation at 0.9 μm, 1.4 μm and 1.8 μm. These three wavelengths are called optical windows. The loss of signal at 0.9 μm is higher than losses at 1.4 μm and 1.8 μm. Thus, the devices have been designed to operate and exploit the low attenuation of 1.4 μm and 1.8 μm wavelengths. The attenuation in optical fibre may be due to absorption losses, scattering losses and radiation losses.

Distortion

In the transmission of optical signal, a pulse output is sometimes wider than input pulse; *i.e.*, pulse gets distorted as it moves through the fibre. The distortion of pulse is due to the dispersion effect which is measured in terms of nanoseconds per kilometer.

3. **Trace the diagram of step index and graded index optical fibres.** [T.U. 2062 Baishakh]

Solution:

The diagrams of step index optical fibre and graded index optical fibre are:

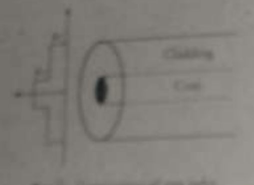

Figure: Construction of step index optical fibre

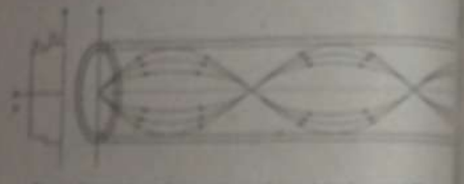

Figure: Construction and propagation signals in graded index optical fibre

4. **What is the principle behind functioning of an optical fibre? Describe the various types of optical fibres.** [T.U. 2063 Baishakh]

Solution:

Principle of fibre optics

The basic principle behind the propagation via optical fibre is total internal reflection. The refractive index is one of the most important optical parameters of a medium. It is defined as the optical density of material with respect to vacuum.

Mathematically,

Refractive index of the medium,

$$(\mu) = \frac{\text{velocity of light in vacuum (c)}}{\text{velocity of light in a medium (v)}}$$

The refractive index of vacuum is 1, that of air is 1.00029, that of water is 1.333, that of fused silica is 1.452 and that of crown glass is 1.52

Whenever a light ray travels from an optically denser medium to optically rarer medium, it bends away from normal, *i.e.*, $i > r$. An angle of incidence in a denser medium is said to be *critical angle C*, if angle of refraction in a rarer medium is right angle. Whenever angle of incidence is greater than critical angle, incident ray does not refract. It reflects back in same medium. This phenomenon is called *total internal reflection*.

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Based on the number of modes of propagation, optical fibres are classified into single mode optical fibres and multi–mode optical fibres. The number of modes of propagation through an optical fibre is;

$$N = \frac{d \times (NA)^2}{2\lambda}$$

where, d is the diameter of the optical fibre core, NA is the numerical aperture and λ is the wavelength of the light

Single mode optical fibres

If only one mode is passed through a fibre at a particular time, then it is said to be single mode optical fibre. The core diameter of the single mode optical fibre is nearly 8 to 9 μm. The core, cladding and sheath or jacket specification for a single mode optical fibre is 8.5/125/250 μm. The difference between the refractive indices of core and cladding is made very low. Due to this low difference between the refractive indices, the critical angle at core cladding interface is very large. Therefore, the light rays that make very larger value of the angle of incidence at the core–cladding interface will pass through the fibre. So only one ray passes through the fibre, *i.e.*, the fundamental mode alone travels through the fibre. The single mode optical fibres are generally operated at 1300 nm and 1550 nm. The attenuation is low for single mode optical fibre at 1550 nm wavelength operation.

Multi–mode optical fibres

If more than one mode is passed through the optical fibre, it is said to be multi–mode optical fibre. A multi–mode optical fibres have core diameters of 50 μm or greater. A large number of signals are passed through the multimode fibres because of its large diameter. The multimode fibres are available in three different sizes. They are;

i) Core/Cladding/Jacket diameters ⇒ 50/125/ 250 μm
   Core/Cladding/Jacket diameters ⇒ 50/125/ 900 μm
ii) Core/Cladding/Jacket diameters ⇒ 62.5/125/ 250 μm
   Core/Cladding/Jacket diameters ⇒ 62.5/125/ 900 μm
iii) Core/Cladding/Jacket diameters ⇒ 100/140/ 250 μm

5. **The use of optical fibre is increasing day by day. What is the physics behind it? Discuss the advantages of optical fibres in modern communication technology.** **[T.U. 2064 Po...**

Solution:

The basic principle behind the propagation via optical fibre is ... internal reflection. The refractive index is one of the ... important optical parameters of a medium.

The optical fibres have typical characteristics that make ... highly attractive as a transmission medium. An optical fibre ... enormous potential bandwidth, resulting from the use of op... carrier frequencies around $2 \times 10^{14}\,Hz$. With such a ... frequency and bandwidth roughly equal to 10% of the ca... frequency. Optical fibres have low transmission losses, as lo... $0.1\,dB\,km^{-1}$. The diameter of optical fibre is not greater ... human hair. Optical fibre has a very high tensile strength ... possibility of being bent or twisted without damage.

Advantages of optical fibres

The optical fibres have a lot of advantages over wireless or ... systems. That is why communication industries have introd... fibre optic systems significantly. The followings are the ... advantages of optical fibres.

i) Low attenuation

Attenuation in an optical fibre is markedly lower than that of ... axial cable or twisted pair and is constant over a very wide ... Therefore, transmission within the wide range of distan... possible without repeaters, etc.

ii) Smaller size and light weight

Optical fibres are considerable thinner than co–axial cab... bundled twisted pair cable. Therefore, they occupy much less ...

iii) Electromagnetic isolation

Electromagnetic waves generated from electrical disturban... electrical noise does not interfere with light signals. As a resu... system is not vulnerable to interference impulse noise or cros...

iv) Physical connections

No physical electrical connection is required between the s... and receives.

v) Reliability

Optical fibres are much more reliable because they can ... withstand environmental conditions such as pollution ... and salts produce no corrosion. Moreover, it is nominally ... *by nuclear radiation*. Its life is longer comparison to copp...

vi) Security and privacy

There is no interference in optical fibres and hence transmission is more secure and private because it is very difficult to tap into an optical fibre.

vii) Greater bandwidth

Bandwidth of optical fibres is higher than that of an equivalent wire transmission line.

viii) Isolation coating

As optical fibres are very good dielectrics, isolation coating is not required.

ix) Higher data rate

Data rate is much higher in an optical fibre and hence much more information can be carried out by each fibre in comparison to equivalent copper cables.

x) Lower cost

The cost per channel is lower than that of an equivalent wire cable system.

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6. **What are mono-mode and multimode optical fibres? Differentiate between step index and graded index optical fibre. Also write down the applications of optical fibre in communication system.** **[T.U. 2065 Shrawan]**

Solution:

Mono–mode and multimode optical fibres

See the solution of Q. No. 4 on page no. 94

Differences between step index and graded index optical fibres

Step index optical fibres

See the solution of Q. No. 4 on page no. 94

Graded Index optical fibres

See the solution of Q. No. 4 on page no. 94

Applications of optical fibres

See the solution of Q. No. 1 on page no. 92

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7. **What is an optical fibre? How is it made? Write down the main differences between step index and graded index multimode optical fibres.** **[T.U. 2065 Chaitra]**

Solution:

Optical fibre

An optical fibre is a dielectric wave guide that transmits light signals from one place to another place. It consists of central core within which propagation of electromagnetic field is confirmed and which is surrounded by a cladding layer. The refractive index of the core is always greater than that of cladding layer. The cladding layer is surrounded by a thin layer of buffer coating or jacket. The core and cladding layer are made up of pure silica glass, whereas buffer coating is made of plastics. A cross sectional view of typical optical fibre is shown in figure.

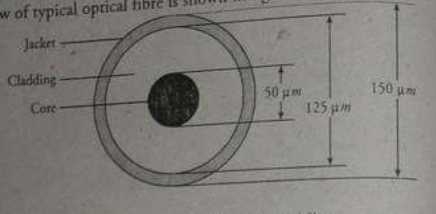

Figure: A cross sectional view of optical fibre

Differences between step index and graded index optical fibres
See the solution of Q. No. 4 on page no. 94

8. What is an optical fibre? Explain the graded index optical fibre and also write the application of optical fibre in communication system as well as medical science. [T.U. 2065 Kartik]

Solution:

Optical fibre
See the solution of Q. No. 7 on page no. 99

Graded Index optical fibres
See the solution of Q. No. 4 on page no. 94

Application in communications

Fibre optics essentially deals with communication including voice signals, video signals and digital data. This is done by transmission of light via optical fibres. An optical fibre system consists of three basic parts: a light source, an optical fibre and a light detector. A source may be light emitting diodes [LED's]. An optical fibre transmits light waves. The detector can be either an avalanche photo diode [APD] or positive intrinsic negative [PIN] diode. Basically, a fibre optic system converts an electrical signal to an infrared light signal. This signal is transmitted through an optical fibre. At the end of optical fibre, it is reconverted into an electrical signal.

Medical applications

The optical fibres are used in fabrication of fiberscope in endoscopy in medical sciences. Such fiberscopes are used in

visualization of internal organs of human body. Optical fibres in combination of laser probe help to visualization of internal portion of human body and cauterization of tissues.

9. Define acceptance angle of an optical fibre. Derive the relation for Numerical Aperture (NA) of the optical fibre. Also write down its significance. [T.U. 2067 Ashadh]

Solution:

Acceptance angle of an optical fibre

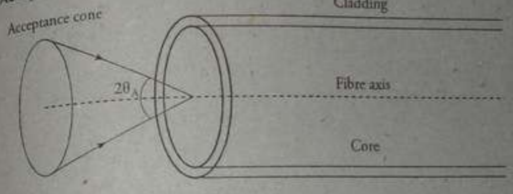

Figure: Acceptance angle and acceptance cone

The light signal rays that enter optical fibre within the angle $2\theta_A$ will be accepted by the fibre. The angle $2\theta_A$ is called acceptance angle. In 3D, it is an acceptance cone with semi-vertical angle θ_A. An accepatace cone is illustrated in figure.Light signals aimed at the optical fibre within this cone will be accepted and propagated to the far end. The larger the acceptance cone, easier the launching of signals into the optical fibre. It may be noted that the numerical aperture and acceptance angle are independent to optical fibre dimensions.

Numerical aperture (NA) of the optical fibre

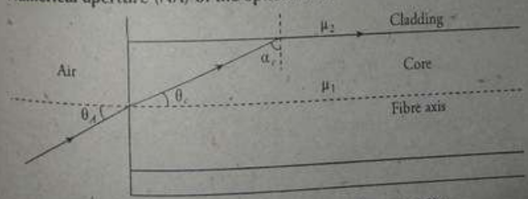

Figure: Propagation of a signal in a step index optical fibre

Consider a light signal that enters the fibre at angle θ_A with respect to fibre axis and strikes on core–cladding interface at angle θ_c as shown in the figure. According to Snell's law, we have,

$\sin\alpha_c = \frac{\mu_2}{\mu_1}$

Since,

$\sin\alpha_c = \cos(90^\circ - \alpha_c) = \cos\theta_c$

We obtain,

$\cos\theta_c = \frac{\mu_2}{\mu_1}$

or, $\sin\theta_c = \sqrt{1-\cos^2\theta_c} = \sqrt{1-\left(\frac{\mu_2}{\mu_1}\right)^2}$

If signal is lunched from air and μ_0 is the refractive index of air, then

$\frac{\sin\theta_A}{\sin\theta_c} = \frac{\mu_1}{\mu_0}$

Therefore,

$$\mu_0 \sin\theta_A = \mu_1 \sin\theta_c = \mu_1\sqrt{1-\left(\frac{\mu_2}{\mu_1}\right)^2} = \sqrt{\mu_1^2-\mu_2^2}$$

The term $\mu_0 \sin\theta_A$ is defined as numerical aperture (NA) of optical fibre. As $\mu_0 \approx 1$,

$NA = \sin\theta_A = \sqrt{\mu_1^2-\mu_2^2}$

Numerical aperture is a dimension less quantity. Its value ranges from 0.14 to 0.50. It is a measure of light signal gathering power of the fibre.

Furthermore, light signal propagation in an optical fibre associated with the parameter *"relative refractive index difference"* well. It is defined by;

Relative refractive index difference,

$$\Delta = \frac{\mu_1-\mu_2}{\mu_1} = \frac{\mu_1^2-\mu_2^2}{\mu_1(\mu_1+\mu_2)} \cong \frac{(NA)^2}{2\mu_1^2} = \frac{(NA)^2}{2\mu_{core}^2}.$$

$(\because \mu_1+\mu_2 \cong 2$

Therefore,

$NA = \mu_{core}\sqrt{2\Delta}$

Numerical aperture is a measure of light signal gathering power of the fibre.

---

10. What is an optical fibre? Discuss its types. Derive the relation numerical aperture (NA) in an optical fibre. [T.U. 2067 Mangsir]

Solution:

Optical fibre

See the solution of Q. No. 7 on page no. 99

Types of optical fibre

See the solution of Q. No. 4 on page no. 94

Numerical aperture (NA) of the optical fibre

See the solution of Q. No. 9 on page no. 101

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11. Write down the principle of optical fibre and show that numerical aperture $(NA) = \mu_{core}\sqrt{2\Delta}$, where symbols have their own meaning. [T.U. 2068 Shrawan]

Solution:

Principle of optical fibre

See the solution of Q. No. 4 on page no. 94

At critical angle,

$\mu_1 \sin C = \mu_2 \sin 90^\circ$

where, μ_1 and μ_2, $(\mu_1 > \mu_2)$ are refractive indices of denser and rarer media.

or, $\sin C = \frac{\mu_2}{\mu_1}$

$\therefore\ C = \sin^{-1}\left(\frac{\mu_2}{\mu_1}\right)$

This equation gives the value of critical angle.

Numerical aperture (NA) of the optical fibre

See the solution of Q. No. 9 on page no. 101

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12. What is an optical fibre? Explain the type of physics behind its functioning. What are their types and explain the application of optical fibre in communications. [P.U. 2005]

Solution:

Optical fibre

See the solution of Q. No. 7 on page no. 99

Principle of optical fibre

See the solution of Q. No. 4 on page no. 94

- **Physics of its functioning**

  Optical fibers are constructed to fulfill the requirements of particular purposes. The major requirement is in the area of bandwidth, *i.e.,* low medium, high medium or ultra high medium. In addition, a fibre may be constructed for its extremely low attenuation per kilometer. In some cases, tensile strength is the most desired characteristic. The optical fibres are classified into three major categories on the basis of *material, number of modes and refractive index profile.*

Types of optical fibre
See the solution of Q. No. 4 on page no. 94

Application in communications
See the solution of Q. No. 8 on page no. 100

13. What is optical fibre? Discuss its working principle. [P.U. 2011]

Solution: See the solution of Q. No. 1 and 4 on page no. 92 and 94

5.8 ADDITIONAL SOLVED PROBLEMS

1. A step index optical fibre has core index 1.43 and cladding index 1.4. Calculate critical angle, critical propagation angle and numerical aperture of the optical fibre.

Solution:

Here,
Refractive index of core, $(\mu_1) = 1.43$
Refractive index of cladding, $(\mu_2) = 1.4$
Now,
Critical angle,

$$C = \sin^{-1}\left(\frac{\mu_2}{\mu_1}\right) = \sin^{-1}\left(\frac{1.4}{1.43}\right) = 78.24°$$

Critical propagation angle,

$$\theta_c = \cos^{-1}\left(\frac{\mu_2}{\mu_1}\right) = \cos^{-1}\left(\frac{1.4}{1.43}\right) = 11.76$$

Numerical aperture,

$$NA = \mu_0 \sin\theta_A = \sqrt{\mu_1^2 - \mu_2^2} = \sqrt{(1.43)^2 - (1.4)^2} = 0.29$$

2. A step index fibre has a core of refractive index 1.55 and a cladding of refractive index 1.53 if the signal is lunched from a medium of refractive index 1.3. Find the numerical aperture and acceptance angle.

Solution:

Here,
Refractive index of core, $(\mu_1) = 1.43$
Refractive index of cladding, $(\mu_2) = 1.4$
Now,
Numerical aperture,

$$NA = \mu_0 \sin\theta_A = 1.3\sin\theta_A = \sqrt{\mu_1^2 - \mu_2^2}$$
$$= \sqrt{(1.55)^2 - (1.53)^2} = 0.25$$
$$\sin\theta_A = \frac{0.25}{1.3} = 0.19$$

$$\therefore\ \theta_A = \sin^{-1}(0.19) = 10.95°$$

Hence,
Acceptance angle, $2\theta_A = 21.90$

3. The refractive of indices of core and cladding of an optical fibre are 1.45 and 1.4 respectively. Calculate the numerical aperture, acceptance angle and Δ.

Solution:

Numerical aperture,

$$NA = \mu_0 \sin\theta_A = \sqrt{\mu_1^2 - \mu_2^2} = \sqrt{(1.45)^2 - (1.4)^2}$$
$$= 0.38$$

or, $1 \times \sin\theta_A = 0.38$

$\therefore\ \theta_A = 22.33°$

Acceptance angle,

$2\theta_A = 44.66°$

$$\Delta = \frac{\mu_1 - \mu_2}{\mu_1} = \frac{1.45 - 1.40}{1.45} = 0.034$$

4. Compute the numerical aperture and the acceptance angle of an optical fibre form the following data:
μ_1 (core) = 1.55 and μ_2 (cladding) = 1.5

Solution:

We have,
Numerical aperture,

$$NA = \mu_0 \sin\theta_A$$
$$= \sqrt{\mu_1^2 - \mu_2^2}$$
$$= \sqrt{(1.55)^2 - (1.50)^2}$$
$$= 0.39$$
$$\therefore\ \theta_A = \sin^{-1}(0.39)$$
$$= 22.36°$$

Hence,
Acceptance angle,

$$2\theta_A = 2 \times 22.36°$$
$$= 45.92°$$

5. Compute the NA, acceptance angle and critical angle of optical fibre having refractive index of core $\mu_1 = 1.5$ and refractive index of cladding $\mu_1 = 1.45$.

Solution: Proceed as solution of Q. No. 4 on page no. 105

Chapter 6

INTERFERENCE

6.1 PRINCIPAL OF SUPERPOSITION

The principal of superposition states that the resultant displacement at any point and any instant may be found by adding the instantaneous displacements that would be produced at the point by the individual wave trains if each were present alone. In the case of light wave, by displacement we mean the magnitude of electric field or magnetic field intensity.

6.2 SUPERPOSITION OF WAVES

Superposition of waves of equal phase and frequency

Let us assume that two sinusoidal waves of the same frequency are travelling together in a medium. The waves have the same phase, without any phase angle difference between them. Then the crest of one wave falls exactly on the crest of the other wave and so do the troughs. The resultant amplitude is got by adding the amplitudes of each wave point by point. The resultant amplitude is the sum of the individual amplitudes.

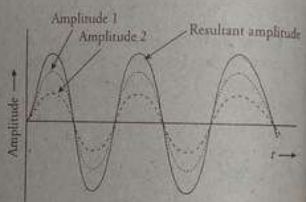

Figure: Superposition of waves of equal phase and frequency

i.e., $A = A_2 + A_2 + \dots\dots$

The resultant intensity is the square of the sum of the amplitudes

$$I = (A_1 + A_2 + A_3 + \dots\dots)^2 \qquad 6.1$$

Superposition of waves of constant phase difference

Let us consider two waves that have the same frequency but have certain constant phase angle difference between them. The two waves have a certain differential phase angle ϕ. In this case the crest of one wave does not exactly coincide with the crest of the other wave. The resultant amplitude and intensity can be obtained by trigonometry.

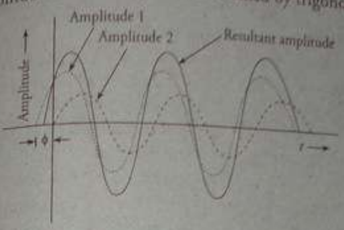

Figure: Superposition of two sine waves of constant phase difference

The two waves having the same frequency ($\omega = 2\pi f$) and a constant phase difference (ϕ) can be represented by the equations.

$$Y_1 = a \sin \omega t$$
$$Y_2 = b \sin(\omega t + \phi) \qquad 6.2$$

where, ϕ is the constant phase difference, a, b are the amplitudes and ω is the angular frequency of the waves

The resultant amplitude Y is given by;

$$\begin{aligned} Y &= Y_1 + Y_2 = a \sin \omega t + b \sin(\omega t + \phi) \\ &= a \sin \omega t + b(\sin \omega t \cos \phi + \cos \omega t \sin \phi) \\ &= a \sin \omega t + b \sin \omega t \cos \phi + b \cos \omega t \sin \phi \\ &= (a + b \cos \phi) \sin \omega t + b \cos \omega t \sin \phi \end{aligned} \qquad 6.3$$

If R is the amplitude of the resultant wave and θ is the phase angle then,

$$\begin{aligned} Y &= R \sin(\omega t + \phi) \\ &= R(\sin \omega t \cos \theta + \cos \omega t \sin \theta) \\ &= R \cos \theta \sin \omega t + R \sin \theta \cos \omega t \end{aligned} \qquad 6.4$$

Comparing equations (1.3) and (1.4); we obtain,

$$R \cos \theta = a + b \cos \phi$$
$$R \sin \theta = b \sin \phi$$

or, $$R^2 = a^2 + b^2 + 2ab \cos \phi$$

$$\theta = \tan^{-1} \frac{b \sin \phi}{a + b \cos \phi} \qquad 6.5$$

Clearly, R is maximum when $\phi = 2n\pi$

and is minimum when $\phi = (2n + 1)\pi$

where, $n = 0, 1, 2, 3, \dots\dots$

When, ϕ is an even multiple of π we say that waves are in phase and when ϕ is an odd multiple of π, the waves are out of phase.

When the amplitude of waves are equal to a say, then,

$$I = 2a^2(1 + \cos\phi) = 4a^2 \cos^2\frac{\phi}{2} \quad 6.6$$

A plot of I versus ϕ is shown in figure. Clearly, this reveals that the light distribution from the superposition of waves will consist of alternately bright and dark bands called interference fringes. Such fringes can be observed visually if projected on a screen or recorded photo–electrically. In the above discussion we have not considered travelling waves (*i.e.*, waves in which displacement is also a function of distance). If λ is the wavelength, then the change of phase that occurs over a distance λ is 2π. Thus, if the difference in phase between two waves arriving at a point is 2π, then difference in the path travelled by these waves is λ. Let the phase difference of two waves arriving at a point be δ and the corresponding path difference be x. For a path difference of λ, the phase difference $= 2\pi$. Therefore, for a path difference of x.

$$\text{Phase difference} = \delta = \frac{2\pi}{\lambda} \times x$$

$$= \frac{2\pi}{\lambda} \times \text{path difference}$$

and

$$\text{Path difference} = x$$

$$= \frac{\lambda}{2\pi} \times \text{phase difference}$$

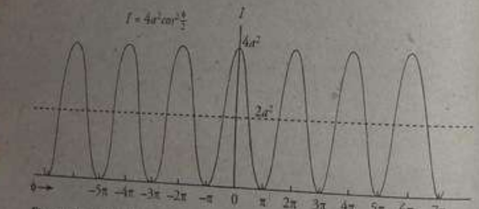

Figure: Intensity distribution for the interference fringes from two waves of same frequency and amplitude

6.3 SUPERPOSITION OF WAVES OF DIFFERENT FREQUENCIES

So far we have assumed that the waves have the same frequency. But light is never truly monochromatic. Many light sources emit quasimonochromatic light *i.e.*, light emitted will be predominantly of one frequency but will still contain other ranges of frequencies. When waves of different frequencies are superimposed, the result is more complicated.

6.4 SUPERPOSITION OF WAVES OF RANDOM PHASE DIFFERENCES

When waves having random phase differences between them superimpose, no discernible interference pattern is produced. The resultant intensity is got by adding the square of the individual amplitudes.

$$I = \sum_{i=1}^{N} A_i^2 = A_1^2 + A_2^2 + A_3^2 + \ldots\ldots\ldots \quad 6.7$$

6.5 YOUNG'S DOUBLE SLIT EXPERIMENT

We have seen in the previous section that two waves with a constant phase difference will produce an interference pattern. Let us see how it can be realized in practice. Let us used two conventional light sources (like two sodium lamps) illuminating two pin holes as shown in figure. Then we will find that no interference pattern is observed on the screen. This can be understood from the following reasoning. In a conventional light source, light comes from a large number of independent atoms each atom emitting light for about 10^{-9} seconds *i.e.*, light emitted by an atom, is essentially a pulse lasting for only 10^{-9} seconds. Even if the atoms were emitting under similar conditions, waves from different atoms would differ in their initial phases. Consequently, light coming out from the holes S_1 and S_2 will have a fixed phase relationship for a period of about 10^{-9} seconds. Hence, the interference pattern will keep on changing every billionth of a second. The human eye can notice intensity changes which last at least for a tenth of a second and hence we will observe a uniform intensity over the screen. However, if we have a camera whose time of shutter can be made less than 10^{-9} sec, then the film will record an interference pattern. We can summarize the above argument by noting that light beams from two independent sources do not have a fixed phase relationship over a prolonged time period and hence, do not produce any stationary interference pattern.

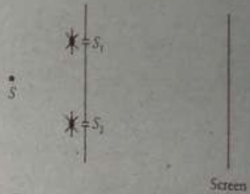

Figure: If two sodium lamps illuminate two pin holes S_1 and S_2 no interference pattern is observed on the screen

Thomas Young in 1802 devised an ingenious but simple method to lock the phase relationship between two sources. The trick lies in the division of a wave front into two. These two split wave fronts act as if they emanated from two sources having a fixed phase relationship and therefore, when these two waves were allowed to interfere, a stationary interference pattern was produced. In the actual experiment a light source illuminated a tiny pin hole S.

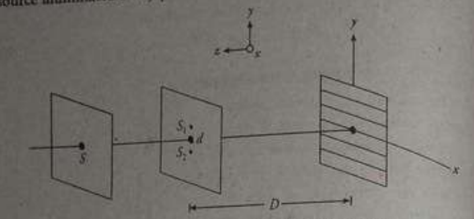

Figure: Young's arrangement to produce interference pattern

Light diverging from this pin hole fell on a barrier containing two rectangular apertures S_1 and S_2 which were very close to each other and were located equidistant from S. Spherical waves traveling from S_1 and S_2 were coherent and on the screen beautiful interference fringes could be obtained. In the centre screen, where the light waves from two slits have travelled through equal distances and where the path difference is zero, we have zeroth-order maximum. But maxima will occur whenever the path difference is one wavelength λ or an integral multiple of wavelength $n\lambda$. The integer n is called the order of interference.

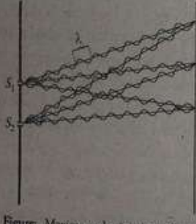

Figure: Maxima and minima in Young's double slit experiment

When the path difference is a multiple of $\left(n+\frac{1}{2}\right)\lambda$, we observe a dark fringe. In order to calculate the position of the maxima, we proceed as follows. Let d be the distance between the slits and D be the distance of the screen from the slits.

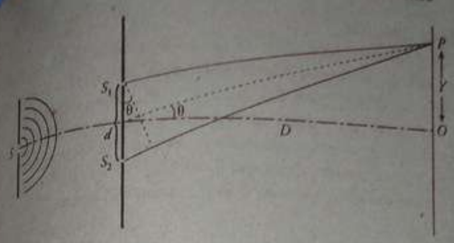

Figure: Path difference in Young's double slit experiment

Let P be the position of the maximum, then, the path difference between the two waves reaching P is:

$$d\sin\theta = n\lambda$$

$$\sin\theta = \frac{n\lambda}{d};\ (n = 1, 2, 3, \dots\dots\dots)$$

where, λ is the wavelength of light used and θ is the angle as shown in the figure. If Y is the distance of the point P from O, the centre of the screen, then we have,

$$Y = D\tan\theta$$

For small angles of θ,

$$Y = D\tan\theta = D\sin\theta$$

$$Y = \frac{Dn\lambda}{d} = D\sin\theta$$

$$\lambda = \frac{DY}{Dn} \qquad 6.8$$

Clearly, fringe width $= Y_{n+1} - Y_n = \beta = \frac{D\lambda}{d}$ 6.9

Hence, by measuring the distance between slits, the distance to the screen and the distance from the central fringe to some fringe on either side, the wavelength of light producing the interference pattern may be determined.

6.5 COHERENCE

An important concept associated with the idea of interference is coherence. Coherence means that two or more electromagnetic waves are in a fixed and predictable phase relationship to each other. In general the phase between two electromagnetic waves can vary from point to point (in space) or change from instant to instant (in time). There are thus two independent concepts of coherence namely temporal coherence and spatial coherence.

Temporal coherence

This type of coherence refers to the correlation between the field a[illegible] point and the field at the same point at a later time *i.e.*, the rela[illegible] between $E(x, y, z, t_1)$ and $E(x, y, z, t_2)$. If the phase difference betw[illegible] the two fields is constant during the period normally covered [illegible] observations, the wave is said to have temporal coherence. If the pha[illegible] difference change many times and in an irregular way during the short[illegible] period of observation, the wave is said to be non coherent.

Spatial coherence

The waves at different points in space are said to be space coherent [illegible] they preserve a constant phase difference over any time t. This is possib[illegible] even when two beams are individually time incoherent, as long as a[illegible] phase change in one of the beams is accompanied by a simultaneo[illegible] equal phase change in the other beam (this is what happens in Young[illegible] double slit experiment). With the ordinary light sources, this is possib[illegible] only if the two beams have been produced in the same part of the sourc[illegible] Time coherence is a characteristic of a single beam of light whereas spa[illegible] coherence concerns the relationship between two separate beams of ligh[illegible] Interference is a manifestation of coherence.

Light waves come in the form of wave trains because light is produc[illegible] during deexcitation of electrons in atoms. These wave trains are of fin[illegible] length. Each wave train contains only a limited number of waves. Th[illegible] length of the wave train Δs is called the *coherence length*. It is the produ[illegible] of the number of waves N contained in wave train and their wavelength[illegible] *i.e.*, $Ns = N\lambda$. Since velocity is defined as the distance travelled per un[illegible] of time, it takes a wave train of length Δs, as a certain length of time 'Δt' to pass a given point.

$$\Delta t = \frac{\Delta s}{c}$$

where, c is the velocity of light

The length of time Δt is called the *coherence time*. The degree of tempor[illegible] coherence can be measured using a Michelson's interferometer.

It is clear from the above discussion that the important condition f[illegible] observing interference is that the two sources should be coherent. Th[illegible] observations of interference are facilitated by reducing the separati[illegible] between the sources of light producing interference. Further, in [illegible] Young's double slit experiment the distance between two sources and th[illegible] screen should be large. The contrast between the bright and dark fring[illegible]

[illegible]ved by making equal the amplitudes of the light sources [illegible] interference. Further, the sources must be narrow and [illegible]hromatic. The concept of coherence is discussed in greater detail [illegible] chapter on lasers.

TYPES OF INTERFERENCE

[illegible] phenomenon of interference is divided into two classes depending [illegible] mode of production of interference. They are:

- Interference produced by the division of wavefront and
- Interference produced by the division of amplitude

[illegible] first case the incident wavefront is divided into two parts by [illegible] use of the phenomenon of reflection, refraction or diffraction. [illegible] two parts of the wavefront travel unequal distances and reunite to [illegible] interference fringes. Young's double slit experiment is a classic [illegible] for this. In Young's double slit experiment one uses two narrow [illegible] isolate beams from separate portions of the primary wavefront. In [illegible] second case the amplitude of the incident light is divided into two [illegible] either by parallel reflection or refraction. These light waves with [illegible] amplitude reinforce after travelling different distances and [illegible] interference. Newton's rings are an example for this type.

INTERFERENCE IN THIN FILMS

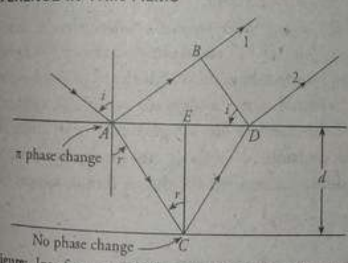

Figure: Interference in plane parallel films due to reflected light

[illegible] colours of thin films, soap bubbles and oil slicks can be explained as [illegible] the phenomena of interference. In all these examples, the [illegible] of interference pattern is by the division of amplitude. For [illegible] if a plane wave falls on a thin film then the wave reflected from [illegible] upper surface interferes with the wave reflected from the lower [illegible]. Such studies have many practical applications as provided by the [illegible] of production of non–reflecting coatings.

Interference in plane parallel films due to reflected light

Let us consider a plane parallel film as shown in the figure. Let light is incident at A. Part of the light is reflected toward B and the other part is refracted into the film towards C. This second part is reflected at C and emerges at D, and is parallel to the first part. At normal incidence, the path difference between rays 1 and 2 is twice the optical thickness of the film.

Path difference $= 2\mu d$

At oblique incidence the path difference is given by;

$$\text{Path difference} = \mu(AC + AD) - AB$$

$$= \frac{2\mu d}{\cos r} - AB$$

$$= \frac{2\mu d}{\cos r} - 2\mu d \tan r \cdot \sin r$$

Since $AB = AD \sin i = 2AE \cdot \sin i = 2d \tan r \cdot \sin i = 2d \tan r \cdot \mu \cdot \sin r$

i.e., $$\text{Path difference} = 2\mu d \left\{\frac{1}{\cos r} - \tan r \cdot \sin r\right\}$$

$$= 2\mu d \left\{\frac{1 - \sin^2 r}{\cos r}\right\} = 2\mu d \cdot \cos r$$

where, μ is the refractive index of the medium between the surfaces

Since for air; $\mu = 1$, the path difference between rays 1 and 2 is given by

Path difference $= 2d \cos r$

While calculating the path difference, the phase change that might occur during reflection has to be taken into account. Whenever light is reflected from an interface beyond which the medium has lower index of refraction, the reflected wave undergoes no phase change. When the medium beyond the interface has a higher refractive index there is phase change of π. The transmitted waves do not experience any phase change. Hence, the condition for maxima for the air film to appear bright is;

$$2\mu d \cos r + \frac{\lambda}{2} = n\lambda$$

or, $$2\mu d \cos r = n\lambda - \frac{\lambda}{2} = (2n - 1)\frac{\lambda}{2}$$

where, $n = 1, 2, 3, \ldots\ldots$

The film will appear dark in the reflected light when

$$2\mu d \cos r + \frac{\lambda}{2} = (2n - 1)n\lambda$$

or, $$2\mu d \cos r = n\lambda$$

where, $n = 1, 2, 3, \ldots\ldots$

Interference in plane parallel films due to transmitted light

Figure illustrates the geometry for observing interference in plane parallel films due to transmitted light. We have two transmitted rays CT and EU which are derived from the same point source and hence, are in a position to interfere. The effective path difference between these two rays is given by;

$$\text{Path difference} = \mu(CD + DE) - CP$$

But $$\mu = \frac{\sin i}{\sin r} = \frac{\frac{CP}{DE}}{\frac{QE}{CE}} = \frac{CP}{QE} \qquad [\because CP = \mu(QE)]$$

or, $$\text{Path difference} = \mu(CD + DQ + QE) - \mu(QE) = \mu(CD + DQ)$$

$$= \mu(ID + DQ) = \mu(QI) = 2\mu d \cos r$$

In this case it should be noted that, no phase change occurs when the rays are refracted unlike in the case of reflection. Hence, the condition for maxima is $2\mu d \cos r = n\lambda$ and the condition for minima is $2\mu d \cos r = (2n - 1)n\lambda$.

Thus, the conditions of maxima and minima in transmitted light are just the reverse of the condition for reflected light.

Interference in wedge shaped film

Let us consider two plane surfaces GH and G_1H_1 inclined at an angle α and enclosing a wedge shaped film. The thickness of the film increases from G to H as shown in the figure. Let μ be the refractive index of the material of the film. When this film is illuminated there is interference between two systems of rays, one reflected from the front surface and the other obtained by internal reflection at the back surface.

The path difference is given by;

$$\text{Path difference} = \mu(BC + CD) - BF$$

$$\text{Path difference} = \mu(BE + EC + CD) - \mu BE$$

$$\left[\because \sin i = \frac{BF}{BD};\ \sin r = \frac{BE}{BD};\ \mu = \frac{\sin i}{\sin r} \Rightarrow \mu = \frac{BF}{BE}\right]$$

$$\text{Path difference} = \mu(EC + CD) = \mu(EC + CP) = \mu EP$$

$$= 2\mu d \cos(r + \alpha)$$

Due to reflection an additional phase difference of $\frac{\lambda}{2}$ is introduced.

Hence,

$$\text{Path difference} = 2\mu d \cos(r + \alpha) + \frac{\lambda}{2}$$

For constructive interference

$$2\mu d \cos(r + \alpha) + \frac{\lambda}{2} = n\lambda$$

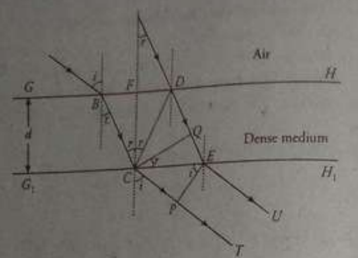

Figure: Interference in plane parallel films due to transmitted light

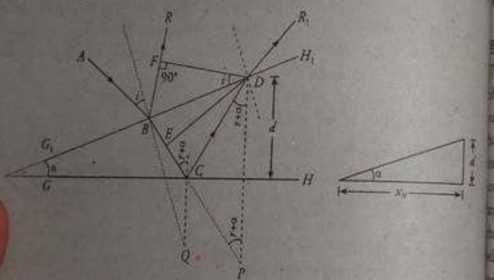

Figure: Interference in a wedge shaped film

or, $\quad 2\mu d\cos(r+\alpha) \quad = (2n-1)\frac{\lambda}{2}$

where, $n = 1, 2, 3, \ldots\ldots\ldots$

For destructive interference

$$2\mu d\cos(r+\alpha)+\frac{\lambda}{2} = (2n+1)n\lambda$$

or, $\quad 2\mu d\cos(r+\alpha) \quad = n\lambda$

where, $n = 0, 1, 2, 3, \ldots\ldots\ldots$

Spacing between two consecutive bright bands is obtained as follows.

For n^{th} maxima;

$$2\mu d\cos(r+\alpha) = (2n-1)\frac{\lambda}{2}$$

Let this band be obtained at a distance X_n from thin edge as shown in figure. For near normal incidence, $r = 0$. Assuming, $\mu = 1$,

From the figure,

$$d = X_n \tan\alpha$$

$$2X_n \tan\alpha \cos\alpha = (2n-1)\frac{\lambda}{2}$$

$$2X_n \sin\alpha = (2n-1)$$

For $(n+1)^{th}$ maxima;

$$2X_{n+1}\sin\alpha = (2n+1)\frac{\lambda}{2}$$

$$2(X_{n+1}-X_n)\sin\alpha = \lambda$$

Fringe spacing;

$$\beta = X_{n+1}-X_n = \frac{\lambda}{2\sin\alpha} = \frac{\lambda}{2\alpha}$$

where, α is small and measured in radians

9 COLOURS OF THIN FILMS

The discussion of the interference due to parallel film and at a wedge should now enable us to understand as to why films appear coloured. To summarize, the incident light is split up by reflection at the top and bottom of the film. The split rays are in a position to interfere and interference of these rays is responsible for colours. Since the interference condition is a function of thickness of the film, the wavelength and the angle of refraction, different colours are observed at different positions of the eye. The colours for which the condition of maxima will be satisfied will be seen and others will be absent. It should be noted here that the conditions for maxima and minima in transmitted light are opposite to that of reflected light. Hence, the colours that are absent in reflected light will be present in transmitted light. The colours observed in transmitted and reflected light are complimentary.

10 NEWTON'S RINGS

When a plano–convex lens with its convex surface is placed on a plane glass plate, an air film of gradually increasing thickness is formed between the two. If monochromatic light is allowed to fall normally and viewed as shown in the figure, then alternate dark and bright circular fringes are observed. The fringes are circular because the air film has a circular symmetry. Newton's rings are formed because of the interference between the waves reflected from the top and bottom surfaces of the air film formed between the plates as shown in the figure.

The path difference between these rays (*i.e.*, rays 1 and 2) is;

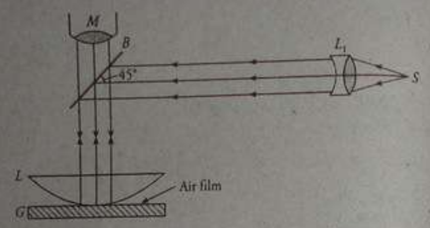

Figure: Experimental set up for viewing Newton's rings

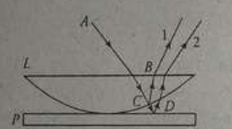

Figure: Interference in Newton's rings setup

$$2\mu d \cos r + \frac{\lambda}{2}$$

Since $r = 0, \mu = 1$

i.e., Path difference $= 2d + \frac{\lambda}{2}$

At the point of contact $d = 0$, the path difference is $\frac{\lambda}{2}$. Hence, the cen spot is dark. The condition for bright fringe is;

$$2d + \frac{\lambda}{2} = n\lambda$$

or, $2d = \frac{(2n-1)\lambda}{2}$

where, $n = 1, 2, 3, \dots\dots\dots$

and the condition for dark fringe is;

$$2d + \frac{\lambda}{2} = (2n + 1)\frac{\lambda}{2}$$

or, $2d = n\lambda$

where, $n = 0, 1, 2, 3, \dots\dots\dots$

Now, let us calculate the diameters of these fringes. Let LOL' be the le placed on the glass plate AB. The curved surface LOL' is part of spherical surface with the centre at C. Let R be the radius of curva and r be the radius of Newton's ring corresponding to constant thickness d.

From the property of the circle,

i.e., $NP \times NQ = NO \times ND$

i.e., $r \times r = d(2R - d)$

$= 2Rd - d^2$

$= 2Rd$

i.e., $r^2 = 2Rd$

or, $d = \frac{r^2}{2R}$

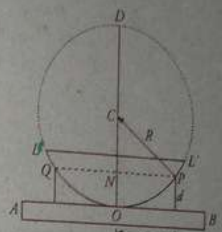

Figure: Calculation of diameter of Newton's ring

Thus, for a bright fringe

$$\frac{2r^2}{2R} = \frac{(2n-1)\lambda}{2}$$

or, $r^2 = \frac{(2n-1)\lambda R}{2}$

Replacing r by $\frac{D}{2}$ where D is the diameter, we get,

$$D_n = \sqrt{2\lambda R}\sqrt{2n-1}$$

Similarly, for a dark fringe,

$$\frac{2r^2}{2R} = n\lambda$$

or, $r^2 = n\lambda R$

or, $D_n^2 = 4n\lambda R$

or, $D_n = 2\sqrt{n\lambda R}$

Thus, the diameters of the rings are proportional to the square roots of the natural numbers.

By measuring the diameter of the Newton's rings, it is possible to calculate the wavelength of light as follows. We have for the diameter of the n^{th} dark fringe.

$$D_n^2 = 4n\lambda R$$

Similarly diameter for the $(n+p)^{th}$ dark fringe;

$$D^2 = 4(n+p)\lambda R$$

or, $D_{n+p}^2 - D_n^2 = 4pR\lambda$

$$\lambda = \frac{D_{n+p}^2 - D_n^2}{4pR}$$

λ can be calculated using this formula.

Newton's rings set up could also be used to determine the refractive index of a liquid. First the experiment is performed when there is air film between the lens and the glass plate. The diameters of the n^{th} and $(n+p)^{th}$ fringes are determined. Then, we have,

$$D_{n+p}^2 - D_n^2 = 4p\lambda R$$

Now, the liquid whose refractive index is to be determined is poured into the container without disturbing the entire arrangement. Again the diameter of the n^{th} and $(n+p)^{th}$ dark fringes are determined. Again, we have,

$$D'^2_{n+p} - D'^2_n = \frac{4p\lambda R}{\mu}$$

From the above equations;

$$\mu = \frac{D^2_{n+p} - D^2_n}{D'^2_{n+p} - D'^2_n}$$

6.11 SOLVED EXAM QUESTIONS

1. **What is the cause of colored image produce by the reflected light form thin films? Does it have anything to do with the colorful pattern of rainbow?** **[T.U. 2061 Baishakh]**

Solution:

When white light is incident on a thin film, the light which comes from any point from it will not include the colour, whose wavelength satisfies the relation,

$2\mu t\cos r = n\lambda$ in the reflected system.

Therefore, film will appear colored and color will depend upon the thickness and the angle of inclination. If r and t are constant, color will be uniform. In the case of oil on water, different colors are seen because r and t vary.

The colorful pattern of rainbow is not different from the above phenomenon. It's a dispersion of white light from the water droplets on the atmospheric air.

---

2. **A soap film of refractive index 1.33 is viewed at an angle of 35° to normal. It has thickness 5×10^{-7} cm. For what wavelength will the film be non-reflecting?** **[T.U. 2061 Baishakh]**

Solution:

Here,

Refractive index of soap film $(\mu) = 1.33$

Thickness of the film $(t) = 5 \times 10^{-7}\ cm$

Let, i be an angle of incidence and r be an angle of refraction, then,

$$\mu = \frac{\sin i}{\sin r}$$

$$\text{or,}\quad \sin r = \frac{\sin i}{\mu} = \frac{\sin 35^\circ}{1.33} = 0.43$$

$$\therefore\quad \cos r = \sqrt{1-\sin^2 r} = \sqrt{1-(0.43)^2} = 0.90$$

Applying the relation;

$$2\mu t\cos r = n\lambda$$

For first order, $n = 1$;

$$\lambda_1 = 2 \times 1.33 \times 5 \times 10^{-7} \times 0.90 = 12 \times 10^{-7}\ m$$

This lies in the infrared (invisible) region.

For second order, $n = 2$;

$$\lambda_2 = 1.33 \times 5 \times 10^{-7} \times 0.90 = 6 \times 10^{-7}\ m$$

This lies in the visible region.

For third order, $n = 3$;

$$\lambda_3 = 2 \times 1.33 \times 5 \times 10^{-7} \times 0.30 = 4 \times 10^{-7}\ m$$

This lies in the visible region.

For fourth order, $n = 4$;

$$\lambda_4 = \frac{1}{2} \times 1.33 \times 5 \times 10^{-7} \times 0.90 = 3 \times 10^{-7}\ m$$

This lies in the ultraviolet (invisible) region.

Thus, for the wavelengths $6 \times 10^{-7}\ m$ and $4 \times 10^{-7}\ m$, soap film is non reflecting.

---

3. **In the Newton's rings experiment the diameter of the tenth ring changes from 1.4 cm to 1.27 cm when a liquid is introduced between the lens and the plate. Calculate the refractive index of the liquid.** **[T.U. 2061 Ashwin]**

Solution:

We have,

For liquid medium, $(D_1^2) = \frac{4n\lambda R}{\mu}$

For air medium, $(D_2^2) = 4n\lambda R$

Dividing, we obtain,

$$\mu = \left(\frac{D_2}{D_1}\right)^2 = \left(\frac{1.4\ cm}{1.27\ cm}\right)^2 = 1.215$$

The refractive index of the liquid is 1.215.

---

4. **Derive the conditions of constructive and destructive interference using the mathematical theory for superposition of two waves of same frequency.** **[T.U. 2061 Ashwin]**

Solution:

Consider a narrow monochromatic source S and two pinholes A and B equidistant from S. A and B act as two coherent sources separated by a distance d. Let a screen be placed at a distance D from the coherent sources. The point C on the screen is equidistant from A and B. Therefore, the path difference between the two waves is zero. Thus, the point C has maximum intensity.

Consider a point P at a distance x from C. The waves reach at the point P from A and B.

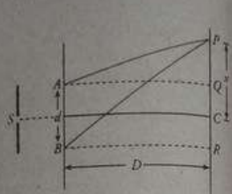

Figure: Interference fringes

Here,

$PQ \quad = x - \frac{d}{2}$

$PR \quad = x + \frac{d}{2}$

Thus,

$$(BP)^2 - (AP)^2 = \left[D^2 - \left(x + \frac{d}{2}\right)^2\right] - \left[D^2 - \left(x - \frac{d}{2}\right)^2\right]$$

$$= 2xd$$

$$\therefore \quad BP - AP = \frac{2xd}{BP + AP}$$

But,

$BP \approx AP \approx D$

Path difference $= BP - AP$

$= \frac{xd}{D}$

Phase difference $= \frac{2\pi}{\lambda}\left(\frac{xd}{D}\right)$

If the path difference is a whole number multiple of wavelength of λ, the point P is bright, *i.e.*,

Bright fringes appear. Therefore,

$\frac{xd}{D} = n\lambda$

where, $n = 0,1,2,3,\ldots\ldots\ldots$

or, $x = \frac{n\lambda D}{d}$ ... (i)

This relation gives the distance of the bright fringes form the point C. At C, the path difference is zero and bright is fringe is formed.

When $n = 1$; $x_1 = \frac{\lambda D}{d}$

When $n = 2$; $x_2 = \frac{2\lambda D}{d}$

When $n = 3$; $x_3 = \frac{3\lambda D}{d}$

....

....

....

When $n = n$; $x_n = \frac{n\lambda D}{d}$

Thus, the distance between any two consecutive bright fringes is:

$$x_2 - x_1 = \frac{2\lambda D}{d} - \frac{\lambda D}{d}$$

$$= \frac{\lambda D}{d} \quad \ldots \text{(ii)}$$

If the path difference is an odd multiple of half wavelength, the point P is dark, *i.e.*, dark fringes appear.

$\frac{xd}{D} = (2n + 1)\frac{\lambda}{2}$

where, $n = 0,1,2,3,\ldots\ldots\ldots$

or, $x = \frac{(2n+1)\lambda D}{d}$ ... (iii)

This relation gives the distances of dark fringes from the point C.

When $n = 1$; $x_1 = \frac{\lambda D}{2d}$

When $n = 2$; $x_2 = \frac{3\lambda D}{2d}$

When $n = 3$; $x_3 = \frac{5\lambda D}{2d}$

....

....

....

When $n = n$; $x_n = \frac{(2n+1)\lambda D}{2d}$

Thus, the distance between any two consecutive bright fringes is:

$$x_2 - x_1 = \frac{3\lambda D}{2d} - \frac{\lambda D}{2d}$$

$$= \frac{\lambda D}{d} \quad \ldots \text{(iv)}$$

The distance between any two consecutive bright or dark fringes is known as fringe width. Therefore, alternately bright and dark parallel fringes are formed. The fringes are formed on both sides of C. All the fringes are of equal in width and are independent of the order of the fringe. However, the breadth of a bright or dark fringe is half of the fringe width.

5. **What is interference of light? Derive the expression for the radii of dark and bright rings in Newton's ring experiment for reflected light.** **[T.U. 2062 Baishakh]**

Solution:

Interference of light

The phenomenon of interference is defined as the modification of the distribution of light energy obtained by the superposition of two or more light waves.

Newton's rings

Whenever a convex surface of a plano-convex lens is placed on a plane glass plate, a thin film is produced between two surfaces. The thickness of the air film so produced increases as we go outwards from the point of contact. The loci of all the points corresponding to same thickness are circles concentric with the point of contact. The circular fringes so produced are known as Newton's rings. The fringes produced with monochromatic light are circular. The fringes are concentric circles with the point of contact as the centre. When fringes are viewed with white light we observe colored fringes. With monochromatic light, bright and dark circular fringes are produced in the air film. Newton's rings can be formed due to reflected as well as transmitted light.

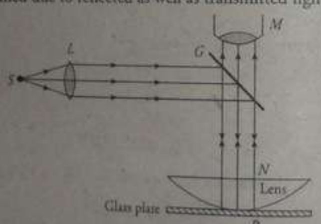

Figure: Experimental arrangement of Newton's ring experiment

Theory

A monochromatic light ray AB falls normally on air film at the point B after passing through lens. Part of it is reflected from point B (rarer medium) on glass-air boundary and goes upwards along BF without phase reversal and partially refracts into air film along BC.

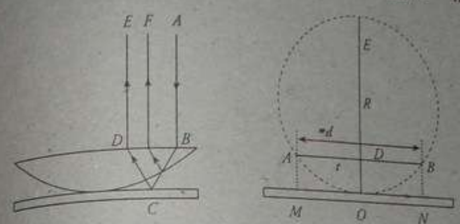

Figure: Newton's ring formation in reflected system and mathematical determination of Newton's ring diameter

At point C again reflection takes place on glass plate (denser medium). The reflected light goes along CDE with a phase difference of π or path difference $\frac{\lambda}{2}$. The two rays BF and CDE produce interference fringes depending upon their path difference. To obtain an expression for the diameter of a ring, consider the vertical section of AOB of plano-convex lens through the centre of curvature.

$OD = t$

$AB = d$

The radius of curvature of the lens is R. The lens is in contact with glass plate MON at O such that points A and B are equidistant from O. Complete the circle $AOBE$ and draw diameter. Draw BN and AM perpendicular to plane MN. The thickness of the air film will be zero at O, around O circular fringes will be obtained. The path differences between two rays are reflected from B and other reflected from C is $2\mu t \cos\gamma$. Since incidence is normal so angle of refraction is zero and refractive index of air, $\mu = 1$. Therefore,

Path difference $= 2t$

As one of the rays CDE suffers reflection on denser medium, an additional path difference $\frac{\lambda}{2}$ is introduced.

$\therefore$ Path difference $= 2t + \frac{\lambda}{2}$

The points A and B being equidistant form O will lie on bright ring of diameter AB if path difference is $n\lambda$.

i.e., $2t + \frac{\lambda}{2} = n\lambda$

$$2t = \frac{(2n-1)}{2}\lambda; \qquad n = 0, 1, 2, 3, \ldots\ldots$$

For dark rings,

$2t = n\lambda;$ $\qquad n = 0, 1, 2, 3, \ldots\ldots$

From figure,

$AD \times DB = OD(2R - OD)$

Since,

$AD = DB = r$

$OD = BM = AM = t$

Thus,

$r^2 = t(2R - t)$

or, $r^2 = 2Rt$ $\qquad (\because 2R - t \approx 2R)$

$\therefore t = \dfrac{r^2}{2R}$

On substituting value of t for bright and dark rings, we obtain,

For bright rings,

$r^2 = \dfrac{(2n-1)\lambda R}{2}$

$\therefore r = \sqrt{\dfrac{(2n-1)\lambda R}{2}}$

For dark rings,

$r = \sqrt{n\lambda R}$

These relations are expressions for radii of bright and dark rings respectively.

---

6. **An air wedge of angle 0.01 radians is illuminated by monochromatic light of wave length 600 *nm* falling normally on it. At what distance from the edge of the wedge, will the twentieth fringe be observed by reflected light.** [T.U. 2063 Baishakh]

Solution:

Here,

Air wedge angle, $(\theta) = 0.01$ radians

Wavelength of monochromatic light, $(\lambda) = 600\ nm = 600 \times 10^{-9}\ m$

$(n) = 12$

We have,

$2t = n\lambda$

and $\theta = \dfrac{t}{x}$

or, $2\theta x = n\lambda$

$\therefore x = \dfrac{n\lambda}{2\theta} = \dfrac{12 \times 600 \times 10^{-9}}{2 \times 0.01} = 3 \times 10^{-3}\ m$

The 12^{th} fringe will be observed at the distance $3 \times 10^{-3}\ m$ form the edge of the wedge.

Newton's rings formed by sodium light between a flat glass plate and a convex lens are viewed normally. What will be the order of the dark ring which will have double the diameter of 40^{th} ring? [T.U. 2063 Baishakh]

Solution:

For dark rings, diameter of the ring is;

$D_n^2 = 4n\lambda R = (4 \times 40)\lambda R$ $\qquad \ldots$ (i)

When the diameter of ring is doubled,

$D_2^2 = 4m\lambda R$

or, $(2D_n)^2 = 4m\lambda R$ $\qquad \ldots$ (ii)

Dividing equation (i) by (ii), we obtain,

$\dfrac{1}{4} = \dfrac{40}{m}$

$\therefore m = 160$

The order of the required dark ring is 160.

How can we form the Newton's rings in reflected light? Show that the diameter of the dark rings are proportional to the square root of natural numbers and that of bright rings are proportional to square root of odd numbers in the reflected light. [T.U. 2064 Poush]

Solution:

Newton's rings

See the solution of Q. No. 5 on page no. 124

On substituting value of t for bright and dark rings, we obtain,

For bright rings,

$r^2 = \dfrac{(2n-1)\lambda R}{2}$

$\therefore r = \sqrt{\dfrac{(2n-1)\lambda R}{2}}$

or, $D = 2\sqrt{\dfrac{(2n-1)\lambda R}{2}}$ $\qquad \ldots$ (i)

For dark rings,

$r = \sqrt{n\lambda R}$

or, $D = 2\sqrt{n\lambda R}$ $\qquad \ldots$ (ii)

Equations (i) and (ii) give the diameters of bright and dark rings respectively. The diameters of the dark rings are proportional to the square root of natural numbers and that of bright rings are proportional to square root of odd numbers in the reflected light.

9. A screen containing two slits 0.1 mm apart is 1.20 m from the viewing screen. Light of wavelength 500 nm, falls on the slits from a distant source. Approximately how far apart will the interference fringes be on the screen? [T.U. 2064 Pous]

Solution:

Here,

Slit separation, $(d) = 0.1\ mm = 0.1 \times 10^{-3} m$

Distance between slit and screen, $(D) = 1.20\ m$

Wavelength of light, $(\lambda) = 500\ nm = 500 \times 10^{-9} m$

Fringe width, $(\beta) = ?$

We have,

$$\beta = \frac{\lambda D}{d} = \frac{500 \times 10^{-9} \times 1.20}{0.1 \times 10^{-3}} = 6 \times 10^{-3}\ m$$

The interference fringes are $6 \times 10^{-3}\ m$ apart from each other.

---

10. What are coherent sources? Show that the separation of two successive maxima or minima depends on the wavelength of light used, distance between the slit and the screen and the slit width. [T.U. 2065 Shrawan]

Solution:

Coherent sources

Two sources, which emit waves of the same frequency, wavelength, same amplitude and have zero phase difference or constant phase difference at all the times are called *coherent sources*.

For the remaining part

See the solution of Q. No. 4 on page no. 121

The equations (iii) and (iv) shows that separation of two successive maxima or minima depends on the wavelength of light used, distance between the slit and the screen and the slit width.

---

11. What are Newton's rings? Determine the wavelength of light using Newton's rings method. Also explain the use of monochromatic ray of light in this method. [T.U. 2065 Shrawan]

Solution:

Newton's rings

See the solution of Q. No. 5 on page no. 124

The figure depicts the experimental arrangement for determination of the wavelength of monochromatic light using Newton's rings. A monochromatic source of light S emits

rays that fall on lens L. The parallel beam of light from this lens is reflected by the glass plate G inclined at an angle of 45° to the horizontal. A plano-convex lens is placed just below the glass plate on the surface of another glass plate. Newton's rings are viewed through travelling microscope M focused on the air film. Circular bright and dark rings are seen with the central dark. With the help of a travelling microscope, measure the diameter of the n^{th} dark ring. Suppose the diameter of the n^{th} ring be D_n. Thus,

$$(D_n)^2 = 4n\lambda R \quad \ldots (i)$$

Measure the diameter of the $(n+m)^{th}$ dark ring. Its diameter is;

$$(D_{n+m})^2 = 4(n+m)\lambda R \quad \ldots (ii)$$

Subtracting these equations we obtain,

$$(D_{n+m})^2 - (D_n)^2 = 4m\lambda R$$

$$\text{or, } \lambda = \frac{(D_{m+n})^2 - (D_n)^2}{4mR} \quad \ldots (iii)$$

This relation determines the wavelength of monochromatic light. The radius of curvature of the lower surface of the lens is determined with the help of a spherometer but more accurately it can be determined by Boy's method. Hence the wavelength of a given monochromatic source of light can be determined.

Monochromatic light gives alternative bright and dark rings are formed. Diameter of such rings depends on the wavelength of light. No overlapping of alternative dark and bright rings takes place so that diameter can be accurately measured using travelling microscope. When white light is used, the diameter of the rings of the different colors will differ and colored rings are observed. Only the first few rings are clear and after that due to overlapping of the rings of different colors, rings cannot be viewed.

A soap film of refractive index $\frac{4}{3}$ and the thickness 20 μm is illuminated by white light incident at an angle of 49.6°. The light reflected by it is examined by a spectroscope in which is found dark band corresponding to a wavelength 500 nm. Calculate the order of interference of the dark band. [T.U. 2065 Shrawan]

Here,

Refractive index of a soap film, $(\mu) = \frac{4}{3}$

Thickness of a soap film, $(t) = 20\ \mu m = 20 \times 10^{-6}\ m$

Angle of incidence, $(i) = 49.6°$

Wavelength of light, $(\lambda) = 500\ nm = 500 \times 10^{-9}\ m$

We have,

$$\mu = \frac{\sin i}{\sin r}$$

$$\text{or,}\quad \sin r = \frac{\sin i}{\mu} = \frac{\sin 49.6°}{\frac{4}{3}} = 0.57$$

$$\therefore\quad \cos r = \sqrt{1-\sin^2 r} = \sqrt{1-(0.57)^2} = 0.82$$

Thus,

$$2\mu t \cos r = n\lambda$$

$$\text{or,}\quad n = \frac{2\mu t \cos r}{\lambda} = \frac{2 \times 4 \times 20 \times 10^{-6} \times 0.82}{3 \times 500 \times 10^{-9}} \approx 87$$

The order of interference of dark band is 87.

---

13. **Newton's rings are formed by reflected light of wavelength 5895 Å with a liquid between the plane and curved surfaces. If the diameter of the sixth bright ring is 3 *mm* and the radius of the curved surface is 100 *cm*, calculate the refractive index of the liquid.** **[T.U. 2065 Chaitra]**

Solution:

Here,

Wavelength of light, $(\lambda) = 5895\ Å = 5895 \times 10^{-10}\ m$

Diameter of 6^{th} bright ring, $(D_6) = 3\ mm = 3 \times 10^{-3}\ m$

Radius of curvature, $(R) = 100cm = 1\ m$

We have,

$$D_n = 2\sqrt{\frac{(2n-1)\lambda R}{2\mu}}$$

$$\text{or,}\quad D_6 = 2\sqrt{\frac{(2 \times 6 - 1)\lambda R}{2\mu}}$$

$$\text{or,}\quad 3 \times 10^{-3} m = 2\sqrt{\frac{(2 \times 6 - 1)5895 \times 10^{-10} \times 1}{2\mu}}$$

$$\therefore\quad \mu = 1.20$$

The refractive index of a liquid is 1.20.

14. **In Newton's rings experiment the radius of the fourth and twelfth rings are 0.26 *cm* and 0.37 *cm* respectively. Find the diameter of the 24th dark ring.** **[T.U. 2065 Kartik]**

Solution:

Here,

Diameter of 4^{th} ring, $(D_4) = 0.26\ cm = 2.6 \times 10^{-3}\ m$

Diameter of 12^{th} ring, $(D_{12}) = 0.37\ cm = 3.7 \times 10^{-3}\ m$

$m = 12 - 4 = 8$

We have,

$$\lambda = \frac{(D_{m+n})^2 - (D_n)^2}{4mR} = \frac{(D_{12})^2 - (D_4)^2}{4 \times 8R}$$

$$\text{or,}\quad \lambda R = \frac{(3.7 \times 10^{-3})^2 - (2.6 \times 10^{-3})^2}{32}$$

$$\therefore\quad \lambda R = 2.17 \times 10^{-7} \quad \ldots (i)$$

Now,

$$D_{24} = 2\sqrt{24\lambda R} = 2\sqrt{24 \times 2.17 \times 10^{-7}} = 4.56 \times 10^{-3}\ m$$

The diameter of 24^{th} dark ring is $4.56 \times 10^{-3}\ m$.

---

15. **Explain how interference fringes are formed by a thin wedge shaped film, when examined by normally reflected light. How will you estimate the difference of film thickness between two points?** **[T.U. 2067 Ashadh]**

Solution:

Consider two planes surfaces OM and ON inclined at an angle θ and enclosing a wedge shaped air film. The thickness of the air film increases from O to. When the air film is viewed with reflected monochromatic light, a system of equidistant interference fringes is observed which are parallel to the line of intersection of the two surfaces.

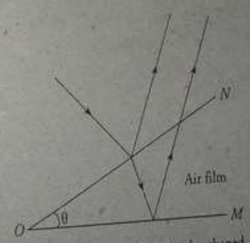

Figure: Interference fringe wedge shaped air film

The interfering rays do not enter the eyes parallel to each other but they appear to diverge from a point near the film. The effect is best observed when the angle of incidence is small.

We consider n^{th} bright fringe occurs at P_n. The thickness of the air film at P_n is P_nQ_n. As the angel of incidence is small,

$\cos r = 1$

Applying the relation for a bright fringe,

$$2\mu t \cos r = (2n+1)\frac{\lambda}{2}$$

For air medium,

$\mu = 1$

and $P_nQ_n = t$

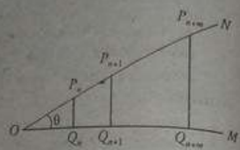

Figure: Probable position of fringes in interference of wedge film

Thus,

$$2P_nQ_n = (2n+1)\frac{\lambda}{2} \quad \text{... (i)}$$

The next bright ring occurs at P_{n+1} position such that

$$2P_{n+1}Q_{n+1} = [2(n+1)+1]\frac{\lambda}{2}$$

$$= (2n+3)\frac{\lambda}{2} \quad \text{... (ii)}$$

Subtracting, we obtain,

$$2P_{n+1}Q_{n+1} - 2P_nQ_n = \lambda$$

$$\therefore \quad P_{n+1}Q_{n+1} - P_nQ_n = \frac{\lambda}{2}$$

Thus the next bright fringe will occur at the point where the thickness of the air film increase by $\frac{\lambda}{2}$. Suppose the $(n+m)^{th}$ bright ring is at P_{n+m}. There will be m bright fringes between P_n and P_{n+m} such that:

$$P_{n+m}Q_{n+m} - P_nQ_n = \frac{m\lambda}{2}$$

If the distance;

$$Q_nQ_{n+m} = x$$

$$\text{or,} \quad \theta = \frac{P_{n+m}Q_{n+m} - P_nQ_n}{Q_nQ_{n+m}}$$

$$= \frac{m\lambda}{2x}$$

$$\text{or,} \quad x = \frac{m\lambda}{2\theta}$$

Thus, the angle of inclination between OM and ON can be known. As x is the distance corresponding to m fringes, the fringe width is;

$$\beta = \frac{x}{m} = \frac{\lambda}{2\theta}$$

The interference fringe due to wedge shaped film is the basis for Newton's rings and Haidinger fringes. The result is applicable to determine the wavelength of incident light.

In Newton's ring arrangement of a source emitting two wavelengths 6×10^{-7} m and 5.9×10^{-7} m is used. It is found that n^{th} dark ring due to one wavelength coincides with $(n+1)^{th}$ dark ring due to other. Find the diameter of n^{th} dark ring if radius of curvature of lens is 0.9 m. **[T.U. 2067 Ashadh]**

Solution:

Here,

Radius of curvature of lens, $(R) = 0.9\ m$

Wavelengths of a light source, $(\lambda_1) = 6 \times 10^{-7}\ m$

$(\lambda_2) = 5.9 \times 10^{-7}\ m$

Diameter of n^{th} dark ring for λ_1 = Diameter of $(n+1)^{th}$ dark ring for λ_2

$$\text{or,} \quad (D_n)^2 = (D_{n+1})^2$$

$$\text{or,} \quad 4n\lambda_1 R = 4(n+1)\lambda_2 R$$

$$\text{or,} \quad 4n \times 6 \times 10^{-7}\ m = 4(n+1) \times 5.9 \times 10^{-7}\ m$$

$$\text{or,} \quad 24\,n = 23.6\,n + 23.6$$

$$\therefore \quad n = 59$$

The diameter of n^{th} dark ring,

$$(D_n)^2 = 4n\lambda_1 R$$

$$= 4 \times 59 \times 6 \times 10^{-7} \times 0.9$$

$$\therefore \quad D_n = 1.13 \times 10^{-2}\ m$$

The diameter of n^{th} dark ring is $1.13 \times 10^{-2}\ m$.

A plano-convex lens of radius 300 cm is placed on an optically flat glass plate and is illuminated by monochromatic light. The diameter of the 8^{th} dark ring in the transmitted system is 0.72 cm. Calculate the wavelength of light used. **[T.U. 2067 Mangsir]**

Solution:

Here,

Radius of curvature of a lens, $R = 300\ cm = 3\ m$

Diameter of the 8^{th} dark ring, $D_8 = 0.72\ cm = 7.2 \times 10^{-3}\ m$

For a transmitted light, we have,

$$D_n = 2\sqrt{\frac{(2n-1)\lambda R}{2}}$$

$$\text{or,} \quad \lambda = \frac{(D_8)^2}{2(2 \times 8 - 1)R} = \frac{(7.2 \times 10^{-3})^2}{2(2 \times 8 - 1) \times 3}$$

$$\therefore \quad \lambda = 3756.52 \times 10^{-10}\ m$$

$$= 3756.52\ \text{Å}$$

The wavelength of monochromatic light = 3756.52 Å

18. **Why are Newton's rings circular? Discuss and derive the necessary theory of Newton's ring experiment for transmitted light.** [T.U. 2068 Shrawan]

Solution:

The Newton's rings are circular because the air film has a circular symmetry. In case of transmitted light, the interference fringes are produced such that fro bright rings,

$$2\mu t\cos\gamma = n\lambda$$

and for dark rings,

$$2\mu t\cos\gamma = (2n-1)\frac{\lambda}{2}$$

Figure: Newton's ring for transmitted light

For air film trapped between glass plate and plano-convex lens,

$$\mu = 1$$

Since angle γ is small, $\cos\gamma = 1$. Thus,

For bright rings,

$$2t = n\lambda$$

For dark rings,

$$2t = (2n-1)\frac{\lambda}{2}$$

From figure,

$$AD \times DB = OD(2R - OD)$$

Since,

$$AD = DB = r$$

$$OD = BM = AM = t$$

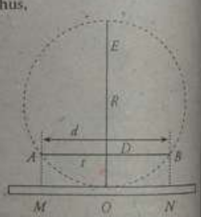

Figure: Mathematical determination of Newton's ring diameter

Thus,

$$r^2 = t(2R - t)$$

or, $r^2 = 2Rt$ $\quad (\because 2R - t \approx 2R)$

$\therefore$ $t = \dfrac{r^2}{2R}$

On substituting value of t for bright and dark rings, we obtain,

For bright rings,

$$r^2 = n\lambda R$$

For dark rings,

$$r^2 = \frac{(2n-1)\lambda R}{2}$$

When $n = 0$, for bright rings; $r = 0$. Therefore, central Newton's ring for transmitted light is bright, *i.e.*, just opposite to the ring pattern due to reflected light.

What is interference of light? Explain with necessary theory the Newton's ring method of measuring the wavelength of light. [P.U. 2003]

Solution:

Interference of light

See the solution of Q. No. 5 on page no. 124

For the remaining part

See the solution of Q. No. 11 on page no. 128

A soap film of refractive index $\frac{4}{3}$ and the thickness 1.5×10^{-4} cm is illuminated by white light incident at an angle of 60°. The light reflected by it is examined by a spectroscope in which is found dark band corresponding to a wavelength 5×10^{-5} cm. Calculate the order of interference of the dark band. [P.U. 2003]

Solution:

Here,

Refractive index of a soap film, $(\mu) = \frac{4}{3}$

Thickness of a soap film, $(t) = 1.5 \times 10^{-4}\ cm = 1.5 \times 10^{-6}\ m$

Angle of incidence, $(i) = 60°$

Wavelength of light, $(\lambda) = 5 \times 10^{-5}\ cm = 5 \times 10^{-7}\ m$

We have,

$$\mu = \frac{\sin i}{\sin r}$$

or, $\sin r = \dfrac{\sin i}{\mu} = \dfrac{\sin 60°}{\frac{4}{3}} = 0.65$

$\therefore$ $\cos r = \sqrt{1 - \sin^2 r} = \sqrt{1 - (0.65)^2} = 0.76$

Thus,

$$2\mu t\cos r = n\lambda$$

or, $n = \dfrac{2\mu t\cos r}{\lambda}$

$$= \frac{2 \times 4 \times 1.5 \times 10^{-6} \times 0.76}{3 \times 5 \times 10^{-7}} \approx 54$$

The order of interference of dark band is 54.

21. Explain how Newton's rings are formed and describe with necessary theory, the method for the determination of wavelength of monochromatic light. Explain the blooming of lenses. [P.U. 2004]

Solution: See the solution of Q. No. 5 on page no. 124

Blooming of lenses

The process of coating a film on the lens is called *blooming*. A very thin coating on the lens surface can reduce reflections of light considerably. The amount of reflection of light at a boundary depends on the difference in refraction index between the two materials.

Ideally, the coating material should have a refractive index so that the amount of reflection at each surface is nearly equal. The destructive interference can occur nearly completely for one particular wavelength. The thickness of the film is chosen so that light reflecting from the front and rare surface of the film destructively interferes.

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22. Light of wavelength 6.0×10^{-5} cm falls normally on the thin wedge shape film of refractive index 1.4, forming fringes that are 2 *mm* apart. Find the angle of the wedge. [P.U. 2004]

Solution:

Here,

Wavelength of light, $(\lambda) = 6.0 \times 10^{-5}\ cm = 6.0 \times 10^{-7}\ m$

Refractive index, $(\mu) = 1.4$

Fringe width, $(\beta) = 2\ mm = 2 \times 10^{-3} m$

We have,

$$\beta = \frac{\lambda}{2\theta\mu}$$

$$\text{or, } \theta = \frac{\lambda}{2\mu\beta}$$

$$= \frac{6.0 \times 10^{-7}}{2 \times 2 \times 10^{-3} \times 1.4}$$

$$= 1.07 \times 10^{-4}\ m$$

The angle of wedge is $1.07 \times 10^{-4}\ m$.

---

23. Explain the phenomenon of interference of light. Give the theory of the Newton's rings. How fringes can be used to find the wavelength of light? [P.U. 2005]

Solution: See the solution of Q. No. 5 and 11 on page no. 124 and 128

24. A soap film of refractive index 1.33 and the thickness $1.5 \times 10^{-4}\ cm$ is illuminated by white light incident at an angle of 45°. The light reflected by it is examined by a spectroscope in which is found dark band corresponding to a wavelength 5.63×10^{-5} cm. Calculate the order of interference of the dark band. [P.U. 2005]

Solution: Proceed as solution of Q. No. 20 on page no. 135

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25. What are coherent sources? Explain the phenomenon of interference in thin films. [P.U. 2007]

Solution:

Coherent sources

Two sources are said to be coherent if they emit light waves of the same frequency, nearly of same amplitude and are always in phase with each other. It means that the two sources must emit radiation of the same wavelength. In actual practice, it is not possible to have two independent sources which are coherent. For experimental purposes, two virtual sources formed from a single source can act as coherent sources.

Interference in thin films

A familiar example of interference in thin film is beautiful colors produced by thin film of oil on the surface of water or thin film of soap bubble. Newton and Hooke observed and developed the interference phenomenon due to multiple reflections of thin transparent materials. Hooke observed such phenomenon in thin films of mica and similar thin transparent plates. Newton was able to show the interference rings to explain phenomenon on the basis of interference between light reflected from the top and the bottom surface of an air film.

Consider a transparent film of thickness t and refractive index μ. An incident ray SA falls on the upper surface of the film is partially reflected along AT and partially refracted along AB. At point B, part of it is reflected along BC and finally emerges out along CQ. The differences in path between two rays AT and CQ can be determined. The normals CN and AM are drawn on AT and BC respectively. The angle of incidence is i and angle of refraction is r. CB is produced to meet AE produced at point P.

The optical path difference,

$$x = \mu(AB + BC) - AN$$

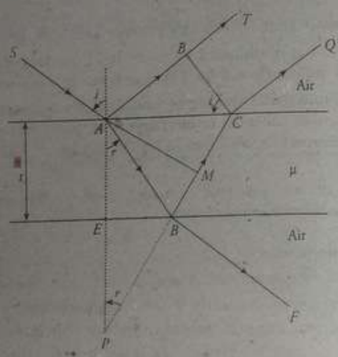

Figure: Interference in thin films due to reflected light

$$\mu = \frac{\sin i}{\sin r} = \frac{AN}{CM}$$

Thus,

$$x = \mu(AB + BC) - \mu CM = \mu(PC - CM) = \mu PM$$

In ΔAPM,

$$\cos r = \frac{PM}{AP}$$

or, $PM = AP \cos r = (AE + EP)\cos r = 2t\cos r$

$\therefore\ x = 2\mu t \cos r$

This expression in the case of reflected light does not represent the actual path difference and only the apparent path difference. It has been established on the basis of electromagnetic theory that when light is reflected from the surface of an optically denser medium a phase change π equivalent to a path difference $\frac{\lambda}{2}$ occurs. Thus the actual path difference in this case is;

$$x = 2\mu t \cos r - \frac{\lambda}{2}$$

For a constructive interference, $x = n\lambda$; film appears bright.

$$n\lambda = 2\mu t \cos r - \frac{\lambda}{2}; \quad n = 0,1,2,3, \ldots\ldots\ldots$$

For a destructive interference, $(2n+1)\frac{\lambda}{2}$; film appears dark.

$$(2n+1)\frac{\lambda}{2} = 2\mu t \cos r - \frac{\lambda}{2}$$

or, $2\mu t \cos r = (n+1)\lambda$

Since n is an integer only, therefore $(n+1)$ can be taken as n.

$$2\mu t \cos r = n\lambda; \quad n = 0,1,2,3, \ldots\ldots\ldots$$

The interference pattern will not be perfect because the intensities of the rays will not be same and their amplitudes are different. The amplitudes will depend on the amount of light reflected and transmitted through films.

A wedge shaped air film having an angle 45' 30" is illustrated by monochromatic light and fringes are observed normally. If the fringe width is 0.12 *cm* calculate the wavelength of light used.

[P.U. 2007]

Solution:

Here,

Angle of wedge shaped air film, $(\theta) = 45'30'' = 45.5 \times \frac{\pi}{2}$ radians

Fringe width, $(\beta) = 0.12\ cm = 1.2 \times 10^{-3}\ m$

We have,

$$\beta = \frac{\lambda_0}{2\theta}$$

or, $\lambda = 2\theta\beta = 2 \times \left(45.5 \times \frac{\pi}{2}\right) \times 1.2 \times 10^{-3} = 0.171\ m$

The wavelength of monochromatic source used is $0.171\ m$. This wavelength is extremely small compared to practical monochromatic lights. The information given in question has significant errors.

Explain the interference due to thin film and shows that the reflected and transmitted systems are complementary to each other.

[P.U. 2008]

Solution:

Interference in thin films

See the solution of Q. No. 25 on page no. 137

Consider a transparent film of thickness t and refractive index μ. An incident ray SA falls on the upper surface of the film is refracted along AB. At point B, part of it is reflected along BC and and partly refracted along BR . The light ray BC after reflection at C, finally emerges along DQ. Since the reflection takes place in rarer medium, no phase change occurs. The normals BM and DN are drawn on CD and BR respectively. The angle of incidence is i and angle of refraction is r. DC is produced to meet BP produced at point P. The optical path difference is,

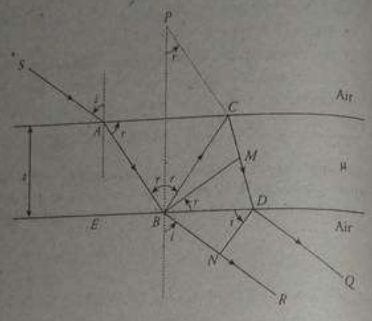

Figure: Interference due to transmitted light in thin films

$$x = \mu(BC + CD) - BN$$

$$\mu = \frac{\sin i}{\sin r} = \frac{BN}{MD}$$

Thus,

$$x = \mu(AB + BC) - \mu CM = \mu(PC - CM) = \mu PM$$

In ΔBPM,

$$\cos r = \frac{PM}{BP}$$

or, $PM = BP\cos r = 2t\cos r$

$\therefore\ x = 2\mu t\cos r$

This relation gives path difference for transmitted light in th film interference.

For bright fringes, the path difference $x = n\lambda$

$$\therefore\ 2\mu t\cos r = n\lambda;\ n = 0, 1, 2, 3, \ldots\ldots \qquad \ldots \text{(iii)}$$

For dark fringes, the path difference $x = (2n+1)\frac{\lambda}{2}$

$$2\mu t\cos r = (2n+1)\frac{\lambda}{2};\ n = 0, 1, 2, 3, \ldots\ldots \qquad \ldots \text{(iv)}$$

The interference fringes obtained are less distinct in transmit light because of unequal amplitude of transmitted light. Howe when angle of incidence is nearly 45°, the fringes are more distin The relation (i), (ii), (iii) and (iv) shows that the interference d to thin film and shows that the reflected and transmitted syste are complementary to each other.

What do you mean by coherent sources of light? Write down the analytical treatment of interference and show that the distance between two consecutive dark and bright fringes is equal.[P.U. 2010]

Solution: See the solution of Q. No. 4 and 10 on page no. 124 and 128

A soap film $5 \times 10^{-5}\ cm$ thick is viewed at an angle 35° to the normal. Find the wavelengths of light in the visible spectrum which will be absent from the reflected light ($\mu = 1.33$). [P.U. 2010]

Solution:

Here,

Thickness of a soap film, $(t) = 5 \times 10^{-5}\ cm = 5 \times 10^{-7}\ m$

Angle of incidence, $(i) = 35°$

Refractive index, $(\mu) = 1.33$

$$\mu = \frac{\sin i}{\sin r}$$

or, $\sin r = \frac{\sin i}{\mu} = \frac{\sin 35°}{1.33} = 0.43$

or, $\cos r = \sqrt{1 - \sin^2 r} = \sqrt{1 - (0.43)^2} = 0.90$

We have,

$$2\mu t\cos r = n\lambda$$

For the first order, $n = 1$

$$\lambda_1 = 2 \times 1.33 \times 5 \times 10^{-7} \times 0.90 = 11.97 \times 10^{-7}\ m$$

which lies in the infra-red region

For the second order, $n = 2$

$$\lambda_2 = 2 \times 1.33 \times 5 \times 10^{-7} \times \frac{0.90}{2} = 5.98 \times 10^{-7} m$$

which lies in visible region

For the third order, $n = 3$

$$\lambda_3 = 2 \times 1.33 \times 5 \times 10^{-7} \times \frac{0.90}{3}$$

$$= 3.99 \times 10^{-7} m$$

which lies in visible region

For the fourth order, $n = 4$

$$\lambda_2 = 2 \times 1.33 \times 5 \times 10^{-7} \times \frac{0.90}{4}$$

$$= 5.98 \times 10^{-7} m$$

which lies in ultra-violet region

Hence, the absent wavelengths in the reflected light are $5.98 \times 10^{-7} m$ and $3.99 \times 10^{-7} m$.

30. What is interference of light? Give the theory of Newton's rings in the case of reflected system and describe how the refractive index of liquid is determined by Newton's ring method.[P.U. 2011]

Solution: See the solution of Q. No. 5 on page no. 124

Suppose the diameter of the n^{th} ring be D_n. Thus,

$$(D_n)^2 = 4n\lambda R \qquad \dots \text{(i)}$$

Measure the diameter of the $(n+m)^{th}$ dark ring. Its diameter is,

$$(D_{n+m})^2 = 4(n+m)\lambda R \qquad \dots \text{(ii)}$$

Subtracting these equations we obtain,

$$(D_{n+m})^2 - (D_n)^2 = 4m\lambda R \qquad \dots \text{(iii)}$$

The liquid is poured in the container C without disturbing the arrangement. The air film between the lower surface of the lens and the upper surface of the plate is replaced by the liquid. The diameters of the n^{th} ring and $(n+m)^{th}$ ring are determined. Let μ be the refractive index of liquid. If D_n' and D_{n+m}' be the diameters of n^{th} ring and $(n+m)^{th}$ ring, then,

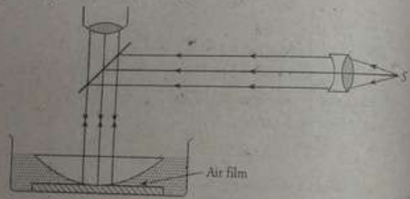

Figure: Determination of refractive index of a liquid using Newton's rings

$$(D_n')^2 = \frac{4n\lambda R}{\mu}$$

$$(D_{n+m}')^2 = \frac{4(n+m)\lambda R}{\mu}$$

Thus,

$$(D_{n+m}')^2 - (D_n')^2 = \frac{4m\lambda R}{2\mu}$$

$$\therefore \mu = \frac{4m\lambda R}{(D_{m+n}')^2 - (D_n')^2} \qquad \dots \text{(iv)}$$

Substituting equation (iii) into equation (iv), we obtain,

$$\mu = \frac{(D_{m+n})^2 - (D_n)^2}{(D_{n+m}')^2 - (D_n')^2} \qquad \dots \text{(v)}$$

This relation determines the refractive index of a liquid.

In Young's two slit experiment, the separation of four bright fringes is 2.4 mm when the wavelength of light is 6×10^{-7} m. The distance from the slit to the screen is 1.0 m. Calculate the separation of the slits. [P.U. 2011]

Solution:

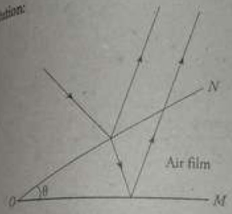

Figure: Interference fringe wedge shaped air film

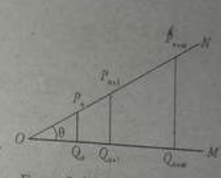

Figure: Probable position of fringes in interference of wedge film

Here,

The separation of four bright fringes, $(x_4) = 2.4\ mm = 2.4 \times 10^{-3} m$

Wavelength of light, $(\lambda) = 6 \times 10^{-7}\ m$

Distance from slit to screen, $(D) = 1.0\ m$

We have,

$$d = \frac{4\lambda D}{x_4} = \frac{4 \times 6 \times 10^{-7} \times 1.0}{2.4 \times 10^{-3}} = 10^{-3}\ m$$

The separation of the slits is $10^{-3}\ m$.

What is meant by blooming of lenses? Derive an expression for the interference in the thin films and wedge shape. [P.U. 2002]

Solution:

Blooming of lenses

See the solution of Q. No. 21 on page no. 136

Interference in thin films

See the solution of Q. No. 27 on page no. 139

Interference in the wedge shape

See the solution of Q. No. 15 on page no. 131

6.12 Additional Solved Problems

1. **Two narrow and parallel slits 0.08 *cm* apart are illuminated by light of frequency 8×10^{-11} *KHz*. It is desired to have a fringe width of 6 $\times 10^{-4}$ *m*. Where should the screen be placed from the slits?**

Solution:

$d = 0.08\ cm = 0.08 \times 10^{-2}\ m$

$\beta = 6 \times 10^{-4}\ m$

Frequency $(v) = 8 \times 10^{11}\ KHz$

i.e., $\lambda = \frac{c}{v} = \frac{3 \times 10^{8}}{8 \times 10^{11} \times 10^{3}}\ m$

$\lambda = ?$

From $\beta = \frac{\lambda D}{d}$

We have,

$$\therefore \quad D = \frac{\beta D}{\lambda} = \frac{6 \times 10^{-4} \times 0.08 \times 10^{-2} \times 8 \times 10^{14}}{3 \times 10^{8}} = 1.28\ m$$

2. **In Young's double slit experiment, a source of light of wavelength 4200 Å is used to obtain interference fringes of width 0.64×10^{-2} *m*. What should be the wavelength of the light source to obtain fringes 0.46×10^{-2} *m* wide, if the distance between screen and the slits is reduced to half the initial value?**

Solution:

In the first case;

$\lambda = 4200\ Å = 4200 \times 10^{-10}\ m$

$\beta = 0.64 \times 10^{-2}\ m$

$$\therefore \quad 0.64 \times 10^{-2} = \frac{4200 \times 10^{-10} \times D}{d} \quad \dots (i)$$

In the second case;

$\beta = 0.46 \times 10^{-2}\ m$

$\lambda = ?$

$$\therefore \quad 0.46 \times 10^{-2} = \frac{\lambda \times \frac{D}{2}}{d} = \frac{2d}{\lambda d} \quad \dots (ii)$$

Dividing equation (i) by (ii); we get,

$$\frac{0.64 \times 10^{-2}}{0.46 \times 10^{-2}} = \frac{4200 \times 10^{-10} \times D}{d} \times \frac{2d}{\lambda d}$$

$$\therefore \quad \lambda = \frac{4200 \times 10^{-10} \times 2 \times 0.46}{0.64}$$

$$= 6037.5\ Å$$

3. **In Young's double slit experiment, the distance between the slits is 1 *mm*. The distance between the slit and the screen is 1 metre. The wavelength used in 5893 Å. Compare the intensity at a point distance 1 *mm* from the centre to that at its centre. Also find the minimum distance from the centre of a point where the intensity is half of that at the centre.**

Solution:

Path difference at a point on the screen distance y from the central point is;

$= \frac{Y \cdot d}{D}$

Here,

$Y = 1\ mm = 1 \times 10^{-3}\ m$

$D = 1\ m$

$d = 1\ mm = 1 \times 10^{-3}\ m$

$$\therefore \quad \text{Path difference} = \frac{1 \times 10^{-3} \times 1 \times 10^{-3}}{1} = 1 \times 10^{-6}\ m = \Delta$$

$$\text{Phase difference} = \frac{2\pi}{\lambda}\Delta = \frac{10^{-6} \times 2\pi}{5893 \times 10^{-10}} = 3.394\pi \text{ radians}$$

$\therefore$ Ratio of intensity with the central maximum is;

$$\cos^2\frac{\delta}{2} = \cos^2(1.697\pi) = 0.3372$$

When the intensity is half of the maximum, if δ is the phase difference, we have,

$$\cos^2\frac{\delta}{2} = 0.5$$

$$\text{or,} \quad \frac{\delta}{2} = 45°$$

$$\text{or,} \quad \delta = \frac{\pi}{2}$$

$$\text{Path difference} = \Delta = \delta\frac{\lambda}{2\pi}$$

$$= \frac{\pi}{2} \times \frac{\lambda}{2\pi} = \frac{\lambda}{4}$$

Distance of the point on the screen from the centre;

$$(Y) = \Delta\frac{D}{d}$$

$$= \frac{\lambda}{4} \times \frac{1}{1 \times 10^{-3}}$$

$$= \frac{5893 \times 10^{-10}}{1 \times 10^{-3}}$$

$$= 1.473 \times 10^{-4}\ m$$

4. In a double slit experiment, fringes are produced using light of wavelength 4800 Å. One slit is covered by a thin plate of glass refractive index 1.4 and the other slit by another plate of glass of the same thickness but of refractive index 1.7. On doing so the central bright fringe shifts to the position originally occupied by the fifth bright fringe from the centre. Find the thickness of the glass plate.

Solution:

We have,

$$n\lambda = (\mu - \mu')t$$

Here,

$$n = 5$$
$$\mu - \mu' = 0.3$$
$$\lambda = 4800 \times 10^{-10}\ m$$
$$\therefore\ t = \frac{5 \times 4800 \times 10^{-10}}{0.3} = 8.0 \times 10^{-8}\ m$$

---

5. A drop of oil of volume 0.2 c.c. is dropped on a surface of tank of water of area 1 m^2. The film spreads uniformly over the whole surface. White light which is incident normally is observed through a spectrometer. The spectrum is seen to contain one dark band whose centre has a wavelength $5.5 \times 10^{-5}\ cm$ in air. Find the refractive index of oil.

Solution:

The thickness of the film $(d) = \frac{0.2\ cm}{100 \times 100} = 2 \times 10^{-5}\ cm$

The film appears dark by reflected light for a wavelength λ given by the relation;

$$2\mu d \cos r = n\lambda$$

For normal incidence $r = 0$

$$\cos r = 1$$

Further $n = 1$ and $\lambda = 5.5 \times 10^{-5}\ cm$

$$\mu = \frac{n\lambda}{2t\cos r} = \frac{1 \times 5.5 \times 10^{-5}}{2 \times 2 \times 10^{-5} \times 1} = 1.375$$

---

6. A soap film $5 \times 10^{-5}\ cm$ thick is viewed at an angle of 35° to the normal. Find the wavelengths of light in the visible spectrum which will be absent from the reflected light ($\mu = 1.33$).

Solution:

Let, i be the angle of incidence and r be the angle of refraction. Then,

$$\mu = \frac{\sin i}{\sin r}$$

or, $1.33 = \frac{\sin 35°}{\sin r}$

or, $r = 25.55°$

or, $\cos r = 0.90$

Applying the relation,

$$2\mu d \cos r = n\lambda$$

where, $d = 5.5 \times 10^{-5}\ cm$

i) For the first order, $n = 1$

$\lambda_1 = 2 \times 1.33 \times 5 \times 10^{-5} \times 0.90 = 12.0 \times 10^{-5}\ cm$

which lies in the infrared (invisible) region.

ii) For the first order, $n = 2$

$\lambda_2 = 1.33 \times 5 \times 10^{-5} \times 0.90 = 6.0 \times 10^{-5}\ cm$

which lies in the visible region.

iii) Similarly, taking $n = 3$

$\lambda_3 = 4.0 \times 10^{-5}\ cm$

which also lies in the visible region.

iv) If $n = 1$

$\lambda_4 = 3.0 \times 10^{-5}\ cm$

which lies in the ultraviolet (invisible) region.

Hence, absent wavelengths in the reflected light are $6.0 \times 10^{-5}\ cm$ and $4.0 \times 10^{-5}\ cm$.

---

7. Two glass plates enclose a wedge shaped air film, touching at one edge and separated by a wire of 0.05 mm diameter at a distance 15 cm from that edge. Calculate the fringe width. Monochromatic light of $\lambda = 6000$ Å from a broad source falls normally on the film.

Solution:

Here,

Fringe width $(\beta) = \frac{\lambda}{2\alpha}$

Clearly, $(\alpha) = \frac{0.05\ mm}{15\ cm} = \frac{0.005}{15}$ radians

$$(\beta) = \frac{\lambda}{2\alpha}$$
$$= \frac{6000 \times 10^{-5} \times 15}{2 \times 0.005} = 0.09\ cm$$

8. An air wedge of angle 0.01 radians is illuminated by monochromatic light of 6000 Å falling normally on it. At what distance from the edge of the wedge, will the 10th fringe be observed by reflected light.

Solution:

Here,

$\alpha = 0.01$ radians

$n = 10$

$\lambda = 6000 \times 10^{-10}\ m$

$2d = \frac{(2n-1)\lambda}{2}$

where, d is the thickness of wedge

But $\alpha = \frac{d}{x}$

or, $d = \alpha x$

$\therefore\ 2\alpha x = \frac{(2n-1)\lambda}{2}$

Here, $n = 10$

$$x = \frac{(2n-1)\lambda}{4\alpha}$$

$$= \frac{19 \times 6000 \times 10^{-10}}{4 \times 0.01}\ m$$

$$= 2.85 \times 10^{-4}\ m$$

9. A thin equiconvex lens of focal length 4 meters and refractive index 1.50 rests on and is in contact with an optical flat. Using light of wavelength 5460 Å, Newton's rings are viewed normally by reflection. What is the diameter of the 5th bright ring?

Solution:

Here,

The diameter of the n^{th} bright ring is given by;

$$D = \sqrt{2(2n-1)\lambda R}$$

Here,

$n = 5$

$\lambda = 5460 \times 10^{-6}\ cm$

$f = 400\ cm$

$\mu = 1.50$

We have,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

Here,

$R_1 = R_2 = R$

$$\therefore\ \frac{1}{f} = (\mu - 1)\frac{2}{R}$$

i.e. $\frac{1}{400} = (1.50 - 1)\frac{2}{R}$

or, $R = 400\ cm$

In Newton's ring experiment, the diameter of the 4th and 12th dark rings are 0.400 cm and 0.700 cm respectively. Find the diameter of the 20th dark ring.

Solution:

We have,

$$D^2_{n+p} - D^2_n = 4p\lambda R$$

Here,

$(n+p) = 12$

$n = 4$

$p = 12 - 4 = 8$

$\therefore\ D^2_{12} - D^2_4 = 4 \times 3 \times \lambda R$ ……… ... (i)

$D^2_{20} - D^2_4 = 4 \times 16 \times \lambda R$ ……… ... (ii)

Dividing equation (ii) by (i); we get,

$$\frac{D^2_{20} - D^2_4}{D^2_{12} - D^2_4} = \frac{4 \times 16 \times \lambda R}{4 \times 8 \times \lambda R} = 2$$

or, $\frac{D^2_{20} - (0.4)^2}{(0.7)^2 - (0.4)^2} = 2$

or, $D_{20} = 0.906\ cm$

In a Newton's ring experiment the diameter of the 10th ring changes from 1.40 to 1.27 cm when a liquid is introduced between the lens and the plate. Calculate the refractive index of the liquid.

Solution:

When the liquid is used the diameter of the 10th ring is given by;

$$(D'_{10})^2 = \frac{4 \times 10 \times \lambda R}{\mu} \quad \text{... (i)}$$

For air medium

$$(D_{10})^2 = 4 \times 10 \times \lambda R \quad \text{... (ii)}$$

Dividing equation (i) by (ii); we get,

$$\mu = \frac{D^2_{10}}{D'^2_{10}} = 1.215$$

12. In a Newton's ring experiment the diameter of the 5th dark ring was 0.3 cm and the diameter of 25h ring was 0.8 cm, if the radius of curvature of the plano-convex lens is 100 cm, find the wavelength of the light used.

Solution:

$$\lambda = \frac{D_{n+p}^2 - D_n^2}{4pR}$$

Here,

$$D_{25} = 0.8\,cm$$
$$D_5 = 0.3\,cm$$
$$p = 25 - 5 = 20$$
and $$R = 100\,cm$$

$$\therefore \lambda = \frac{(0.8)^2 - (0.3)^2}{4 \times 20 \times 100} = 4.87 \times 10^{-5}\,cm$$

Chapter 7

DIFFRACTION

INTRODUCTION

Whenever light passes through a small opening or aperture by the side of a small obstacle, it bends to some extent into the region of geometrical shadow and its intensity falls off rapidly. This deviation is extremely small if the wavelength is small in comparison to the dimension of the obstacle. But the deviation becomes much more pronounced when the dimensions of the aperture or opaque disc are comparable with wavelength of light.

The phenomenon of bending of light around corners of an obstacle and their spreading into the geometrical shadow of an object is called diffraction.

According to the rectilinear propagation of light, light always travels in a straight line. But, whenever light passes through a small opening or aperture by the side of a small obstacle, it bends to some extent into the region of geometrical shadow, thus violating the rectilinear propagation of light.

FRESNEL DIFFRACTION AND FRAUNHOFFER DIFFRACTION

Fresnel diffraction occurs when a source and screen are placed at a finite distance from an obstacle with a sharp edge. In this case, no lenses are used to make parallel rays. Thus, the incident wave front is either spherical or cylindrical in this diffraction.

Fraunhoffer diffraction occurs when source and screen placed at infinite distance. In this diffraction, incident wave front is a plane wave front. The incident rays are made parallel by some means.

TRANSMISSION GRATING AND REFLECTION GRATING

A diffraction grating consists of a large number of narrow slits side by side. The slits are separated by opaque spaces. When a wavefront is incident on grating surface, light is transmitted through the slits and obstructed by the opaque portions. Such grating is called transmission grating.

The space between any two lines is transparent to the light and the lined portion is opaque to light. Such surfaces act as a transmission grating. If the lines are drawn on a silvered surface then light is reflected from the portions of the mirror in between any two lines and such surfaces act as reflection grating.

7.4 SOLVED EXAM QUESTIONS

1. **Define diffraction. How does the phenomenon of diffraction violate the rectilinear propagation of light? What are the advantages and disadvantages of spectrum produced by grating over the spectrum produced by a prism? [T.U. 2061 Baishakh]**

Solution:

Whenever light passes through a small opening or aperture by the side of a small obstacle, it bends to some extent into the region of geometrical shadow and its intensity falls off rapidly. The deviation is extremely small when wavelength is small in comparison to the dimension of the obstacle. But the deviation becomes much more pronounced when the dimensions of the aperture or opaque disc are comparable with wavelength of light. The phenomenon of bending of light around corners of an obstacle and their spreading into the geometrical shadow of an object is called *diffraction*.

According to the rectilinear propagation of light, light always travels in a straight line. But, whenever light passes through a small opening or aperture by the side of a small obstacle, it bends to some extent into the region of geometrical shadow, thus violating the rectilinear propagation of light.

Advantages and disadvantages of grating spectrum and prism spectrum

With a grating, a number of spectra of different orders can be obtained on the two sides of the central maximum whereas with a prism only one spectrum can be obtained.

The spectra obtained with a grating are comparatively pure than those with a prism.

- Knowing the grating element $(a + b)$ and measuring the diffraction ... single spectrum, the wavelength of any spectral line can be measured accurately. But in the case of a prism the angles of deviation are not directly related to the wavelength of the spectral line. The angles of deviation are dependent on the refractive index of the material of the prism, which depends on the wavelength of light.

With a grating, the diffraction angle for violet end of the spectrum is less than that for red. With a prism, the angle of deviation for the violet ray of light is more than that for red rays of light.

The intensities of the spectral lines with a grating are much less than with a prism. In a grating spectrum, most of the incident light energy is associated with the undispersed central bright maximum and the rest of the energy is distributed in the different order spectra on the two sides of the central maximum. But in a prism, most of the incident light energy is distributed in a single spectrum and hence brighter spectral lines are obtained.

The dispersive power of a grating is;

$$\frac{d\theta}{d\lambda} = \frac{nN}{\cos\theta}$$

This is constant for particular order. Thus, the spectral lines are evenly distributed and spectrum obtained with a grating is said to be rational. The refractive index of the material of prism changes more rapidly at the violet end than at the red end of the spectrum.

The dispersive power of a prism is $\frac{d\mu}{(\mu-1)}$. It has higher value in the violet region of the spectrum than in the red region. Hence, there will be more spreading of the spectral lines towards the violet and the spectrum obtained with a prism is said to be irrational.

The resolving power of a grating is much higher than that of a prism. Hence the same two nearby spectral lines appear better resolved with a grating than a prism.

The spectra obtained with different gratings are identical because the dispersive power and resolving power of a grating do not depend on the nature of the material of the grating. But the spectra obtained with different prisms are never identical because both dispersive power and resolving power of a prism depend on the nature of the material of the prism.

Define diffraction. Distinguish between Fresnel and Fraunhoffer diffraction. [T.U. 2062 Baishakh]

Solution:

Diffraction

The phenomenon of bending of light around corners of an obstacle and their spreading into the geometrical shadow of an object is called diffraction.

Fresnel diffraction occurs when a source and screen are placed at a finite distance from an obstacle with a sharp edge. In this case, no lenses are used to make parallel rays. Thus, the incident wave front is either spherical or cylindrical in this diffraction.

Fraunhoffer diffraction occurs when source and screen placed at infinite distance. In this diffraction, incident wave front is a plane wave front. The incident rays are made parallel by some means.

3. What angular separation is produced between the spectral by a grating of 2000 lines per cm in the second order diffraction pattern when it is illuminated normally by a light that contains wavelengths 6000 Å and 6010 Å? [T.U. 2062 Baishakh]

Solution:

Here,

$$a+b=\frac{1}{N}=\frac{1}{2000}\ cm$$

For the second order diffraction with $\lambda_1 = 6000$ Å

$$(a+b)\sin\theta_1 = n\lambda_1$$

or, $\sin\theta_1 = \frac{n\lambda_1}{a+b} = 2\times 6000\ \text{Å}\times 2000\ cm^{-1}$

$= 2\times 6000\times 10^{-8} cm \times 2000\ cm^{-1} = 0.24$

$\therefore\ \theta_1 = \sin^{-1}(0.24) = 13.89°$

For the second order diffraction with $\lambda_2 = 6010$ Å

$$(a+b)\sin\theta_2 = n\lambda_2$$

or, $\sin\theta_2 = \frac{n\lambda_2}{a+b} = 2\times 6010\ \text{Å}\times 2000\ cm^{-1}$

$= 2\times 6010\times 10^{-8} cm\times 2000\ cm^{-1} = 0.2404$

$\therefore\ \theta_2 = \sin^{-1}(0.2404) = 13.91°$

Angular separation, $(\theta_2-\theta_1) = 13.91° - 13.89° = 0.02° = 1.2'$

Angular separation is produced between the spectral by a grating of 2000 lines per cm in the second order diffraction pattern when it is illuminated normally by a light that contains wavelengths 6000 Å and 6010 Å is 1.2'.

---

4. Derive an expression for the intensity distribution due to Fraunhoffer diffraction at a single slit and show that the intensity of the first subsidiary maxima is 4.5% of that of principal maxima. [T.U. 2063 Baishakh]

Solution:

Consider a collimated beam of monochromatic light of wavelength λ, produced by a point source S placed at the focus of a spherical lens L_1, incident normally on a narrow slit AB of width a. The incident plane wavefront on the slit AB can be imagined to be divided into a large number of infinitesimally small strips. The path difference between the secondary waves emanating from the extreme points A and B is $a\sin\theta$ where a is width of the slit and $\angle ABL = \theta$. For a parallel beam of incident light, the amplitude of vibration of the waves from each strip can

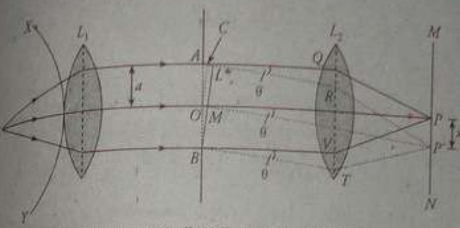

Figure: Fraunhoffer diffraction at a single slit

be taken to be the same. As one consider the secondary waves in a direction inclined at an angle θ from the point B upwards, the path difference changes and hence the phase difference also increases. Let α be the phase difference between the secondary waves from the points B and A of the slit. As the wavefront is divided into a large number of strips can be obtained by vector polygon method. Since the amplitudes are small and the phase difference increases infinitesimally small amounts from strip to strip. Thus, the vibration polygon coincides with the circular arc OP gives the direction of the initial vector and NM the direction of the final vector due to the secondary waves from A. K is the centre of the circular arc.

$$\angle MNP = 2\alpha$$
$$\angle OKM = 2\alpha$$

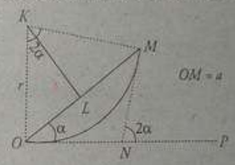

Figure: Intensity distribution for a circular wavefront

In the ΔOKL,

$$OL = r\sin\alpha$$

where, r is the radius of the circular arc

Chord $OM = 2OL = 2r\sin\alpha$

The length of the arc OM is proportional to the width of the slit.

Therefore, length of the arc $OM = ka$

where, k is a constant and a is the width of the slit

and $2\alpha = \frac{arc\ OM}{radius} = \frac{ka}{r}$

i.e., $2r = \frac{ka}{\alpha}$

Thus,

Chord $OM = \frac{ka}{\alpha}\sin\alpha$

or, $A \quad = \frac{\sin\alpha}{\alpha} ka$

$\therefore A \quad = A_0 \frac{\sin\alpha}{\alpha}$

Thus, the resultant amplitude of vibration at appoint on the screen is $A_0 \frac{\sin\alpha}{\alpha}$ and intensity I at a point is;

$$I = A^2 = A_0^2 \left(\frac{\sin\alpha}{\alpha}\right)^2 = I_0 \left(\frac{\sin\alpha}{\alpha}\right)^2$$

i.e., the intensity at point on the screen is proportional to $\left(\frac{\sin\alpha}{\alpha}\right)^2$.

For the principal maxima, $\alpha \to 0$

$$\frac{\sin\alpha}{\alpha} = 1$$

Hence the intensity is maximum, the maximum intensity is;

$$I = I_0 \left(\frac{\sin\alpha}{\alpha}\right)^2 = I_0$$

For the first secondary maxima, $\alpha = \frac{3\pi}{2}$

Thus,

$$I = I_0 \left(\frac{\sin\alpha}{\alpha}\right)^2 = I_0 \left(\frac{-1}{\frac{3\pi}{2}}\right)^2 = 0.045\, I_0$$

i.e., the intensity of the first subsidiary maxima is 4.5 % of that of principal maxima.

---

5. What do you understand by diffraction of light? Explain the distribution of intensity with a special reference of light in single slit. **[T.U. 2064 Poush]**

Solution:

Diffraction of light

See the solution of Q. No. 2 on page no. 153

For the remaining part

See the solution of Q. No. 4 on page no. 154

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6. A parallel beam of monochromatic light is allowed to be incident normally on a plane transmission grating having 5000 lines per cm and the third order spectral line is found to be diffracted through 35°. Calculate the wavelength of light. [T.U. 2065 Shrawan]

Solution:

Here,

$$a + b = \frac{1}{N} = \frac{1}{5000}\, cm$$

For the third order diffraction, $n = 3$

Thus,

$$(a + b)\sin\theta = n\lambda$$

or, $\lambda \quad = (a + b)\sin\theta$

$$= \frac{1}{3} \times \frac{1}{5000}\, cm \times \sin 35° = 3.824 \times 10^{-5}\, cm$$

$\therefore \lambda \quad = 3824 Å$

The wavelength of monochromatic light, $\lambda = 3824$ Å.

Distinguish between Franhoffer and Fresnel diffraction. **[T.U. 2065 Chaitra]**

Solution: See the solution of Q. No. 2 on page no. 153

What is the highest order spectrum which may be seen with monochromatic light of wavelength 559 *nm*, by means of diffraction grating with 15000 lines per inch? [T.U. 2065 Kartik]

Solution:

For the highest order spectrum,

$$\sin\theta_n = 1$$

Thus,

$$(a + b) = n\lambda$$

or, $n \quad = \frac{(a + b)}{\lambda} = \frac{1}{N\lambda} = \frac{1}{15000}\, inch \times \frac{1}{559\, nm}$

$$= \frac{1}{15000} \times 2.54\, cm \times \frac{1}{559 \times 10^{-9} \times 100\, cm}$$

$\therefore n \quad = 3$

The highest order spectrum which may be seen with monochromatic light of wavelength 559 *nm* is 3.

Show that the intensity of second order maxima of Fraunhoffer's single slit diffraction is $\left(\frac{2}{5\pi}\right)^2$ times the intensity of central maxima. **[T.U. 2066 Ashadh]**

Solution: See the solution of Q. No. 4 on page no. 154

For the first secondary maxima, $\alpha = \frac{5\pi}{2}$

Thus,

$$I = I_0 \left(\frac{\sin\alpha}{\alpha}\right)^2 = I_0 \left(\frac{1}{\frac{5\pi}{2}}\right)^2 = \left(\frac{2}{5\pi}\right)^2 I_0$$

i.e., the intensity of second order maxima of Fraunhoffer's single slit diffraction is $\left(\frac{2}{5\pi}\right)^2$ times the intensity of central maxima.