Expectation Maximization Algorithm

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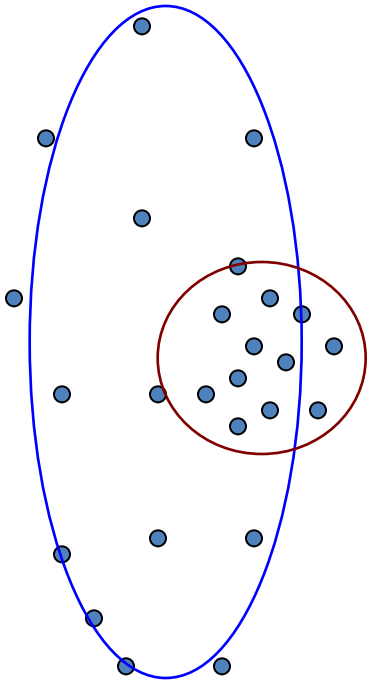
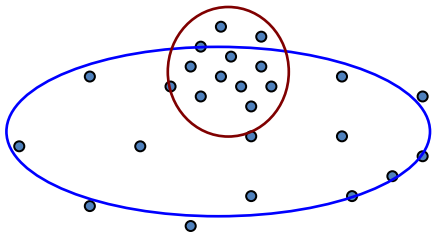
Slides adapted from Carlos Guestrin, Dan Klein, Luke Zettlemoyer and Dan Weld

The Evils of "**Hard Assignments**"?



- Clusters may overlap
- Some clusters may be "wider" than others
- Distances can be deceiving!

Probabilistic Clustering



- Try a probabilistic model!
  - allows overlaps, clusters of different size, etc.
- Can tell a *generative story* for data
  - P(X|Y) P(Y)
- Challenge: we need to estimate model parameters without labeled Ys

Y	X_1	X_2
??	0.1	2.1
??	0.5	-1.1
??	0.0	3.0
??	-0.1	-2.0
??	0.2	1.5
…	…	…

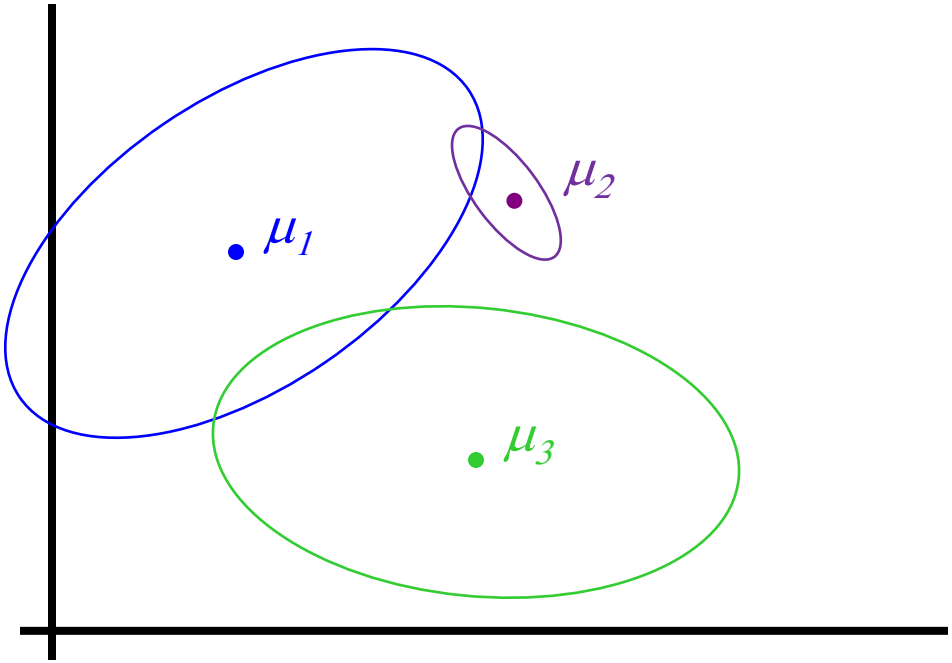
The General GMM assumption

- P(Y): There are k components
- P(X|Y): Each component generates data from a **multivariate Gaussian** with mean μ_i and covariance matrix Σ_i

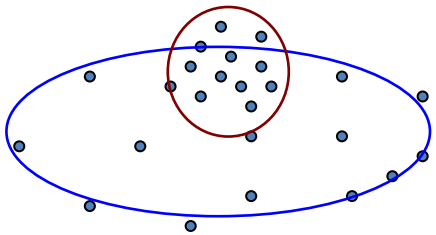
Each data point is sampled from a ***generative process*:**

1. Choose component i with probability *P(y=i)*
2. Generate datapoint ~ N(m_i, Σ_i)

Gaussian mixture model (GMM)



What Model Should We Use?



- Depends on X!
- Here, maybe Gaussian Naïve Bayes?
  - Multinomial over clusters Y
  - Gaussian over each X_i given Y

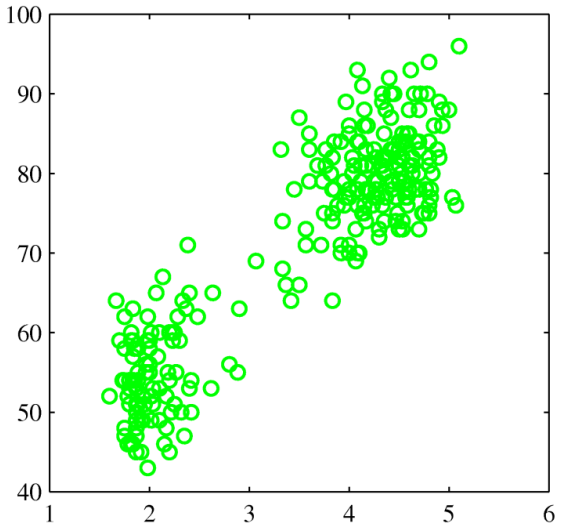
Y	X_1	X_2
??	0.1	2.1
??	0.5	-1.1
??	0.0	3.0
??	-0.1	-2.0
??	0.2	1.5
	…	…

$$p(Y_i = y_k) = \theta_k$$

$$P(X_i = x \mid Y = y_k) = \frac{1}{\sigma_{ik}\sqrt{2\pi}} \; e^{\frac{-(x-\mu_{ik})^2}{2\sigma_{ik}^2}}$$

Could we make fewer assumptions?

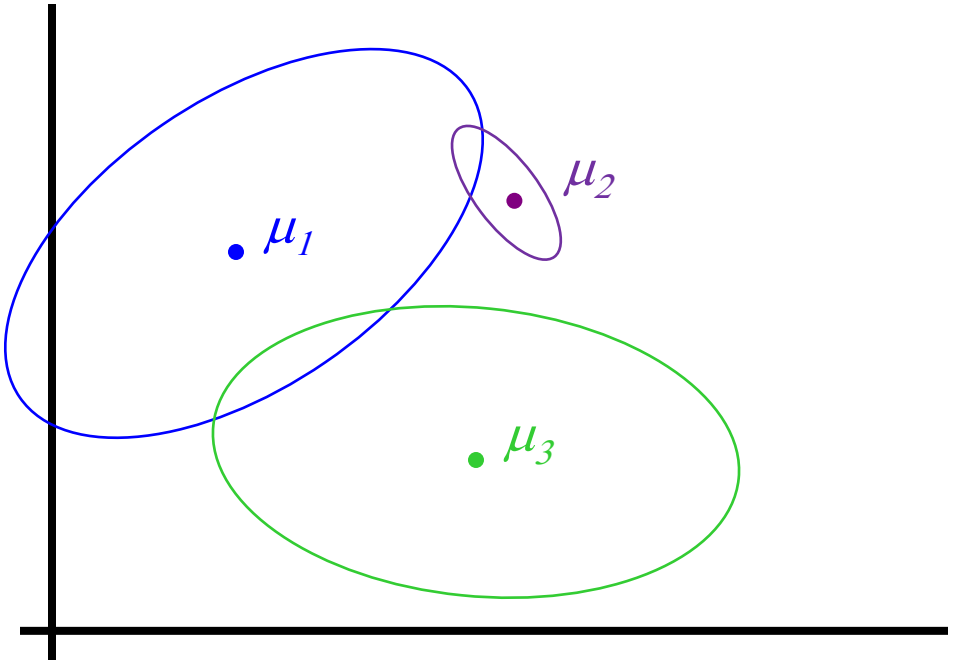
- What if the X_i co-vary?
- What if there are multiple peaks?
- Gaussian Mixture Models!
  - P(Y) still multinomial
  - P(**X**|Y) is a multivariate Gaussian dist'n



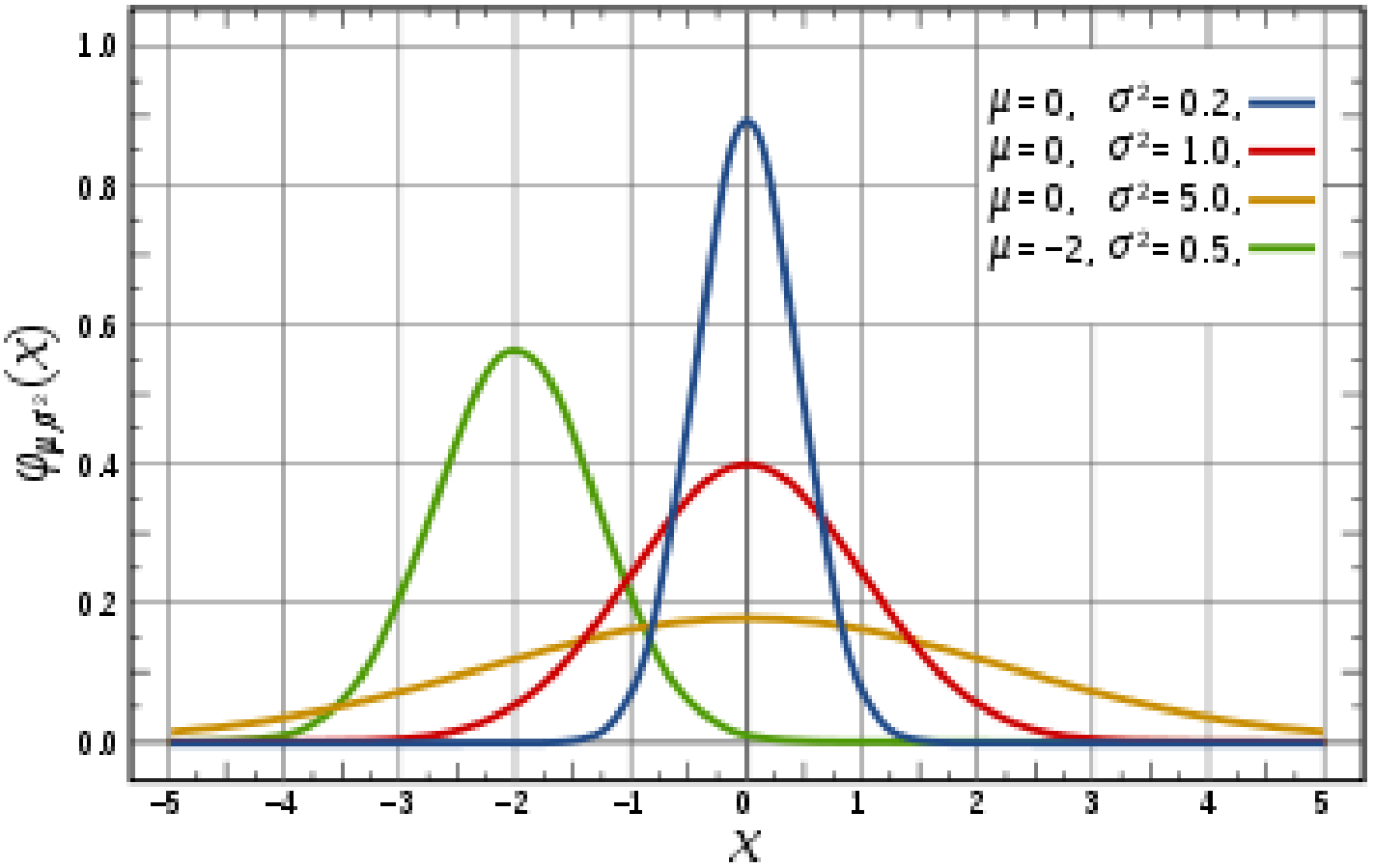
$$P(X = \mathbf{x}_j \mid Y = i) = \frac{1}{(2p)^{m/2} \| \Sigma_i \|^{1/2}} \exp\left[-\frac{1}{2} (\mathbf{x}_j - \mu_i)^T \Sigma_i^{-1} (\mathbf{x}_j - \mu_i) \right]$$

The General GMM assumption

1. What's a ***Multivariate*** Gaussian?
2. What's a ***Mixture Model***?



Review: Gaussians



$$P(x \mid \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

Learning Gaussian Parameters (given fully-observable data)

$$\hat{\mu}_{MLE} = \frac{1}{N}\sum_{i=1}^{N} x_i$$

$$\hat{\sigma}^2_{MLE} = \frac{1}{N}\sum_{i=1}^{N} (x_i - \hat{\mu})^2$$

Geometry of the Multivariate Gaussian

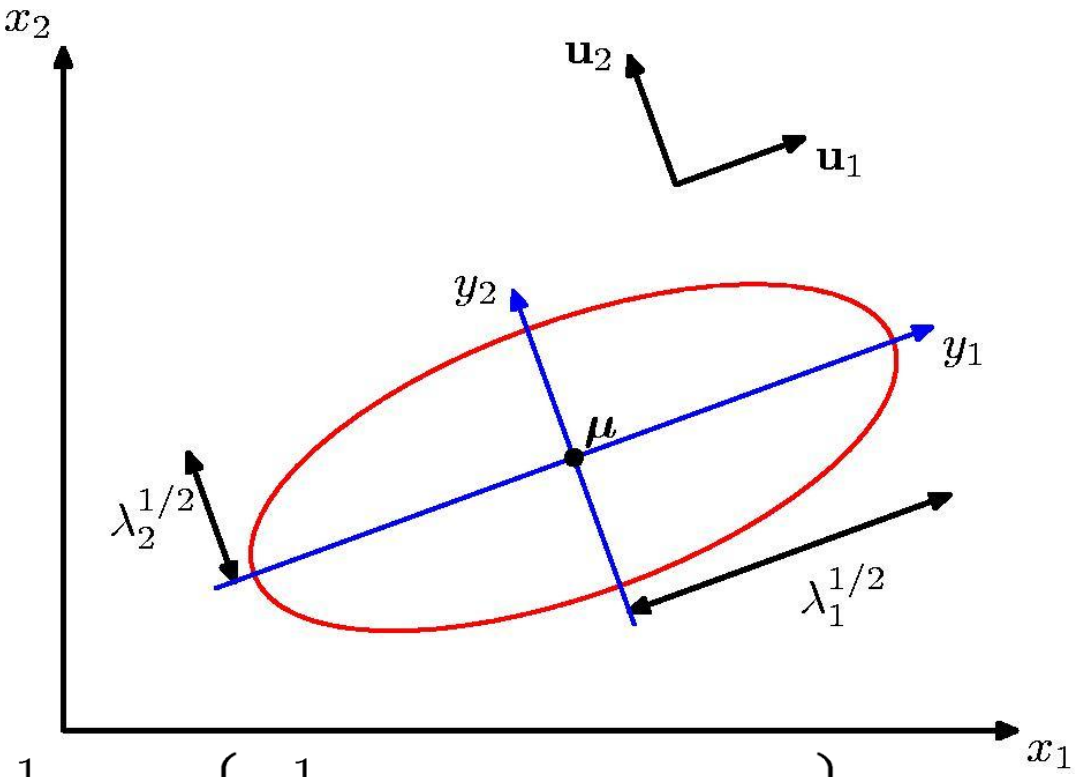
$$\Delta^2 = (\mathbf{x}-\boldsymbol{\mu})^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})$$

$$\boldsymbol{\Sigma}^{-1} = \sum_{i=1}^{D}\frac{1}{\lambda_i}\mathbf{u}_i\mathbf{u}_i^{\mathrm{T}}$$

$$\Delta^2 = \sum_{i=1}^{D}\frac{y_i^2}{\lambda_i}$$

$$y_i = \mathbf{u}_i^{\mathrm{T}}(\mathbf{x}-\boldsymbol{\mu})$$

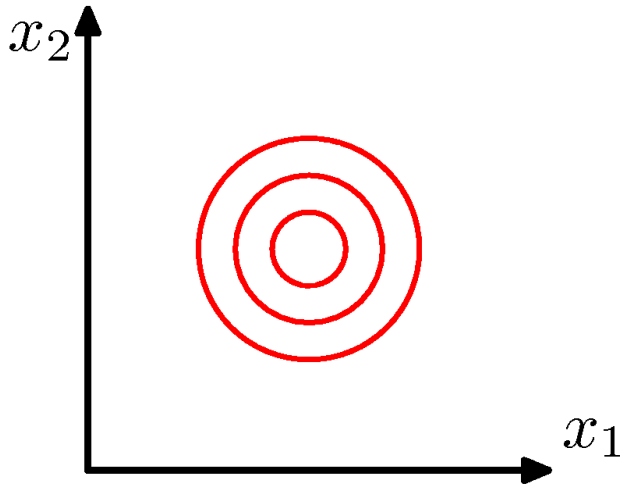
Eigenvalue, λ
Eigenvector $\mathbf{u}_i$



Covariance matrix, Σ, = degree to which x_i vary together

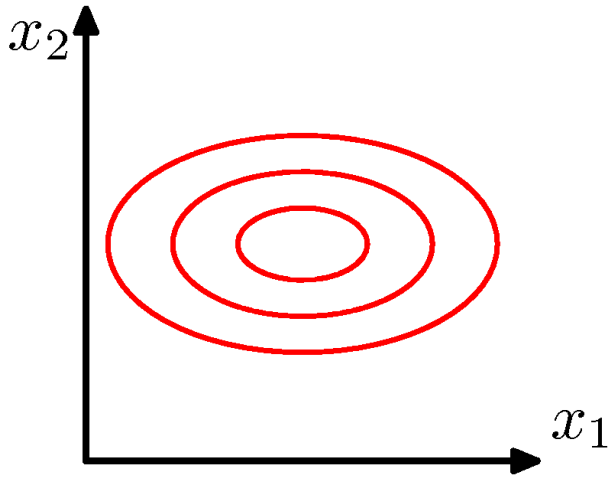
$$\mathcal{N}(\mathbf{x}|\boldsymbol{\mu},\boldsymbol{\Sigma}) = \frac{1}{(2\pi)^{D/2}}\frac{1}{|\boldsymbol{\Sigma}|^{1/2}}\exp\left\{-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})\right\}$$

Multivariate Gaussians



$\Sigma \propto$ identity matrix

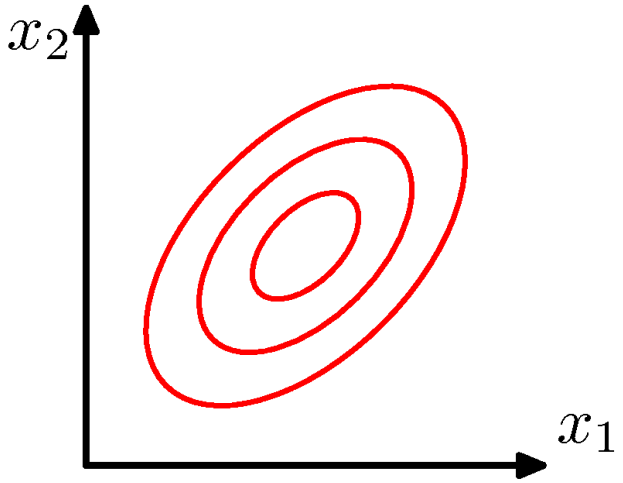
Multivariate Gaussians



Σ = diagonal matrix

X_i are independent *ala* Gaussian NB

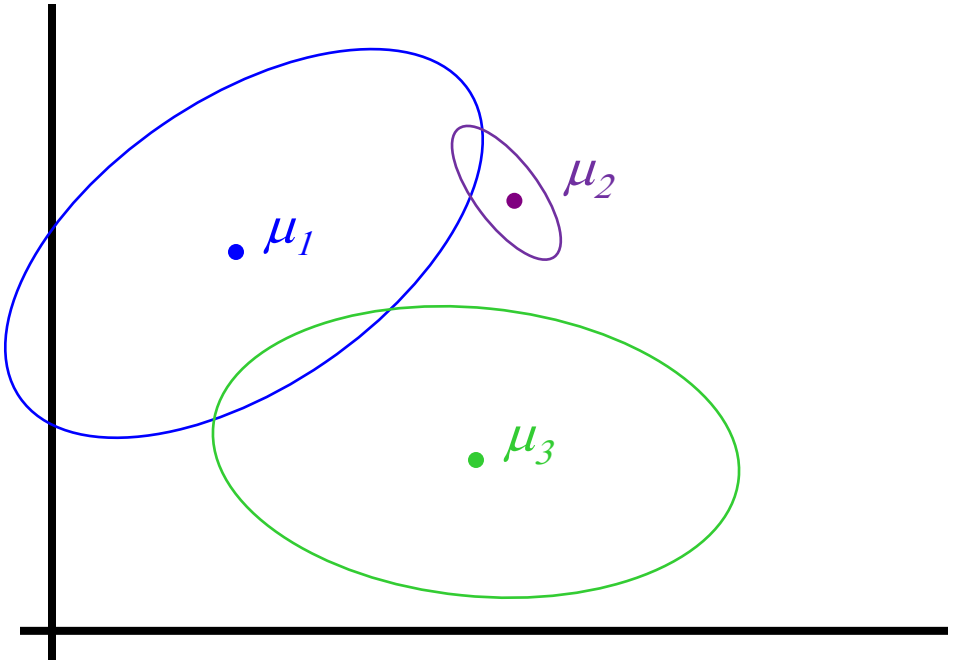
Multivariate Gaussians



Σ = arbitrary (semidefinite) matrix
specifies rotation (change of basis)
eigenvalues specify relative elongation

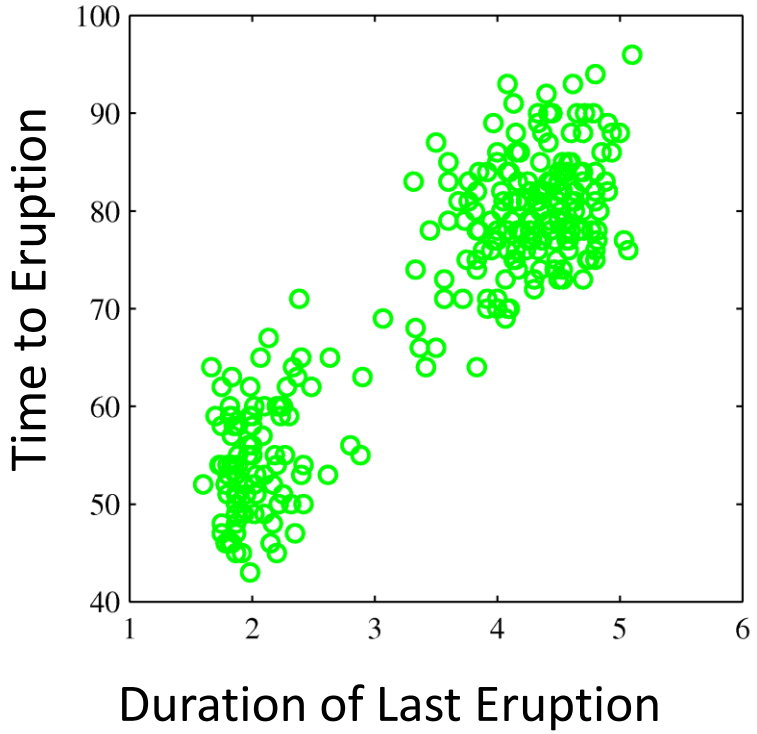
The General GMM assumption

1. What's a Multivariate Gaussian?
2. What's a ***Mixture Model***?



Mixtures of Gaussians (1)

Old Faithful Data Set



Mixtures of Gaussians (1)

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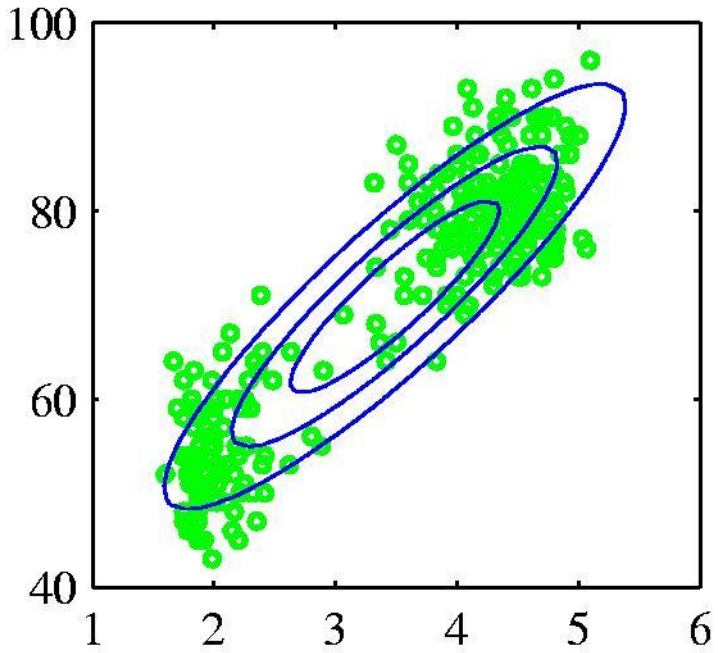
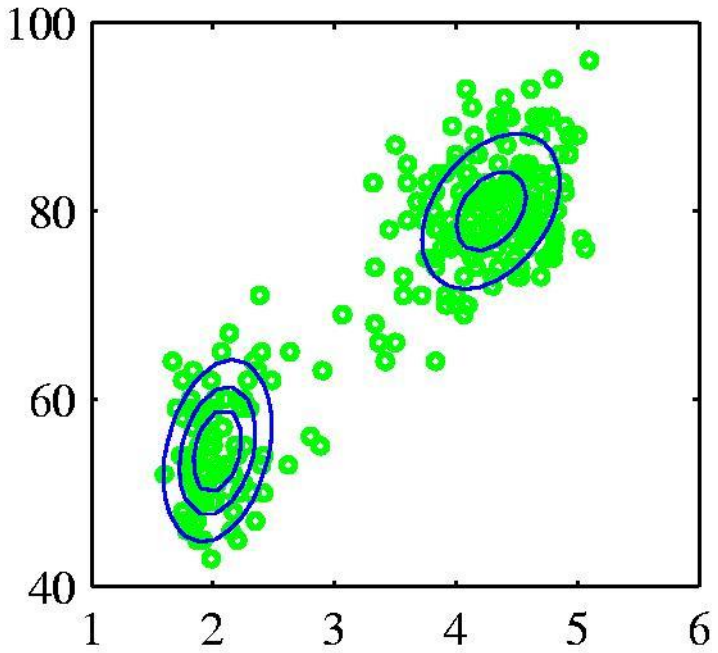


Single Gaussian



Mixture of two Gaussians

Mixtures of Gaussians (2)

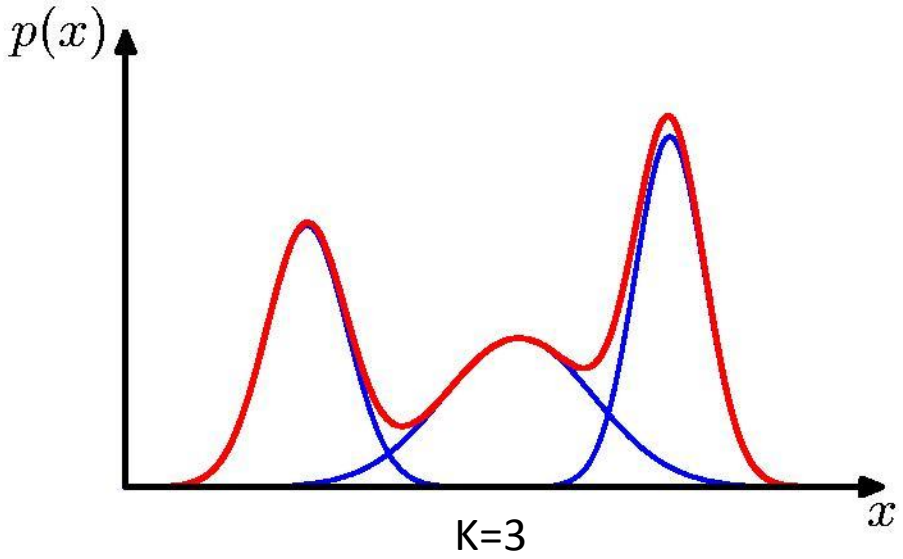
Combine simple models into a complex model:

$$p(\mathbf{x}) = \sum_{k=1}^{K} \pi_k \mathcal{N}(\mathbf{x}|\boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k)$$

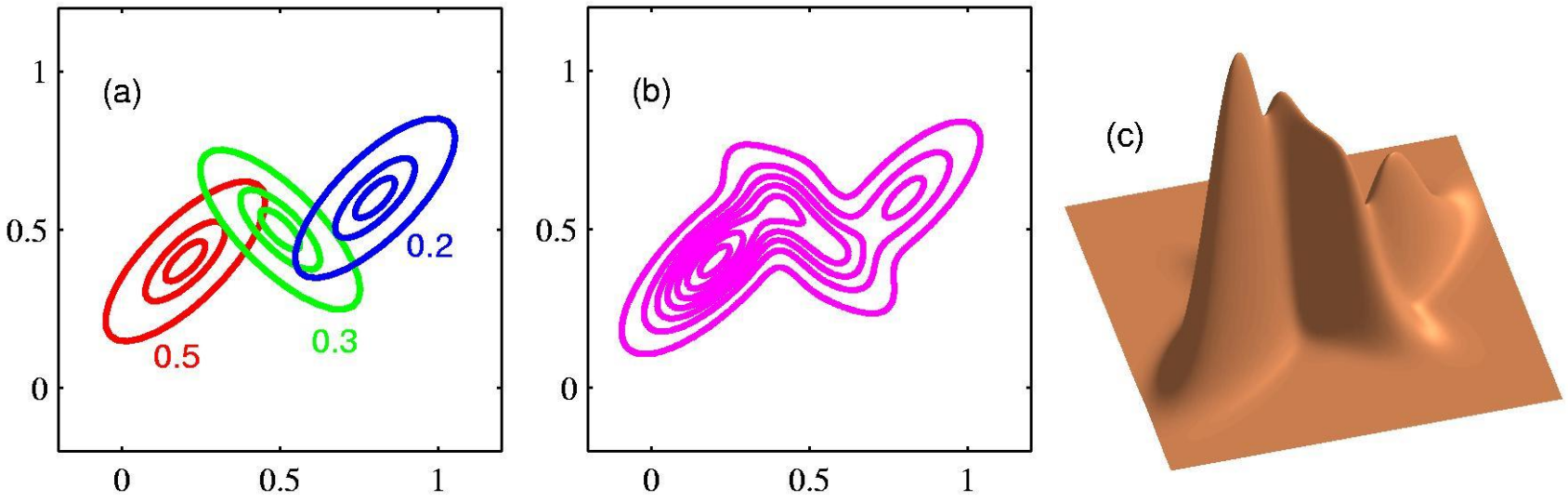
Component

Mixing coefficient

$$\forall k : \pi_k \geqslant 0 \qquad \sum_{k=1}^{K} \pi_k = 1$$

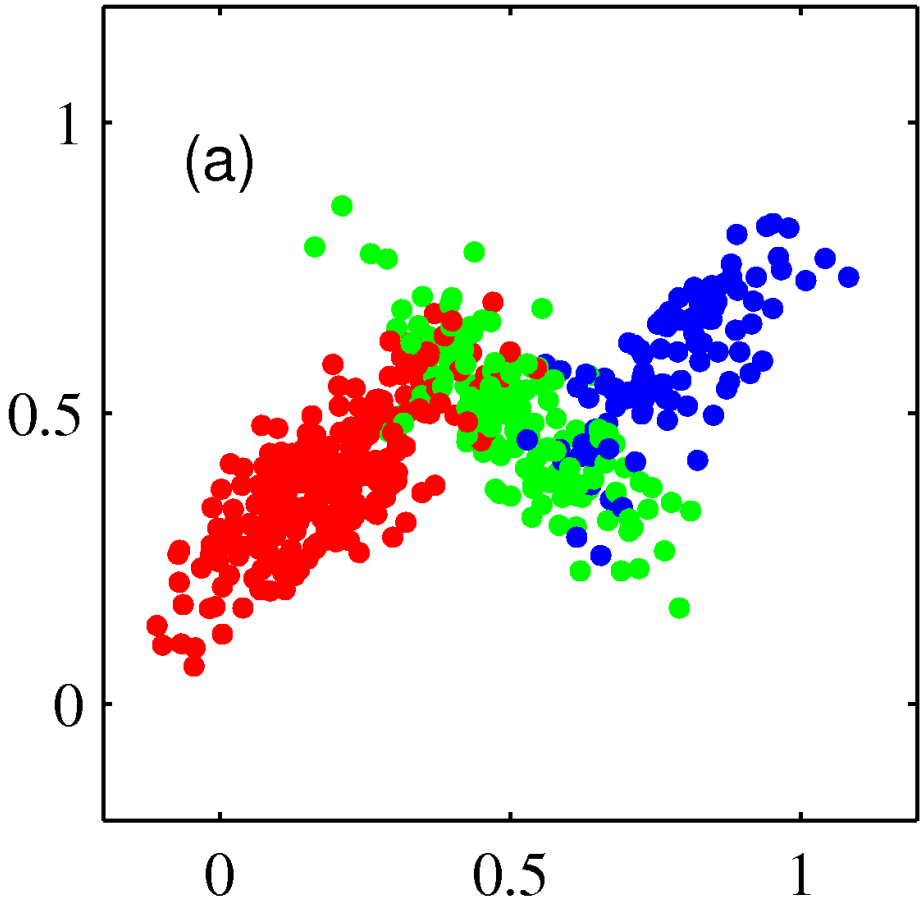


Mixtures of Gaussians (3)



Eliminating Hard Assignments to Clusters

Model data as mixture of multivariate Gaussians



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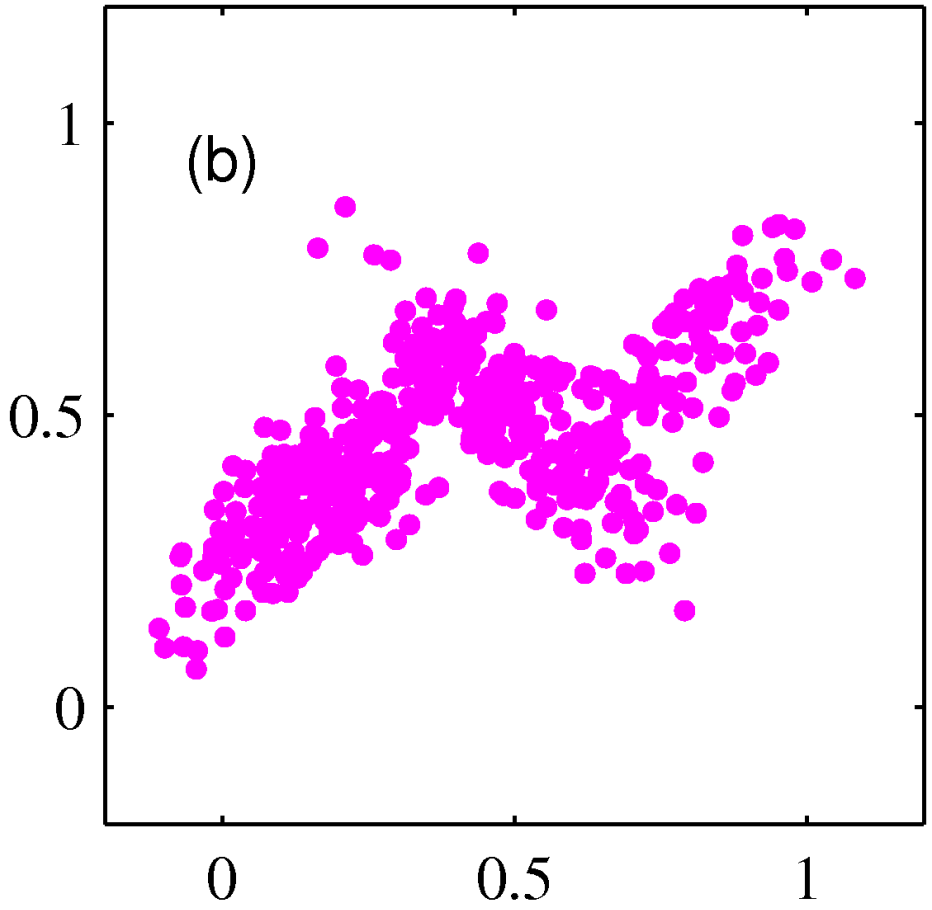

Eliminating Hard Assignments to Clusters

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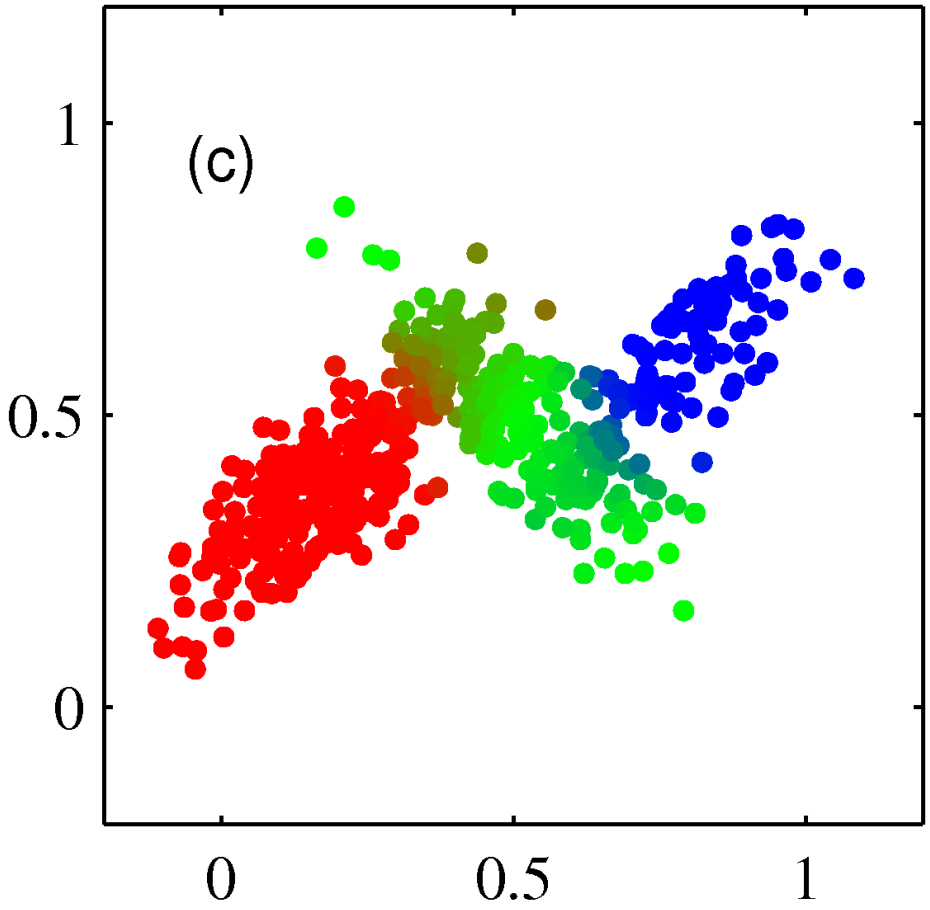


π_i = probability point was generated from i^{th} Gaussian

Detour/Review: Supervised MLE for GMM

- How do we estimate parameters for Gaussian Mixtures with fully supervised data?
- Have to define objective and solve optimization problem.

$$P(y=i,\mathbf{x}_j)=\frac{1}{(2p)^{m/2}\,\|\Sigma_i\|^{1/2}}\exp\left[-\frac{1}{2}\left(\mathbf{x}_j-\mu_i\right)^T\Sigma_i^{-1}\left(\mathbf{x}_j-\mu_i\right)\right]P(y=i)$$

- For example, MLE estimate has closed form solution:

$$\mu_{ML}=\frac{1}{n}\sum_{j=1}^{n}x_n \qquad \Sigma_{ML}=\frac{1}{n}\sum_{j=1}^{n}\left(\mathbf{x}_j-\mu_{ML}\right)\left(\mathbf{x}_j-\mu_{ML}\right)^T$$

Compare

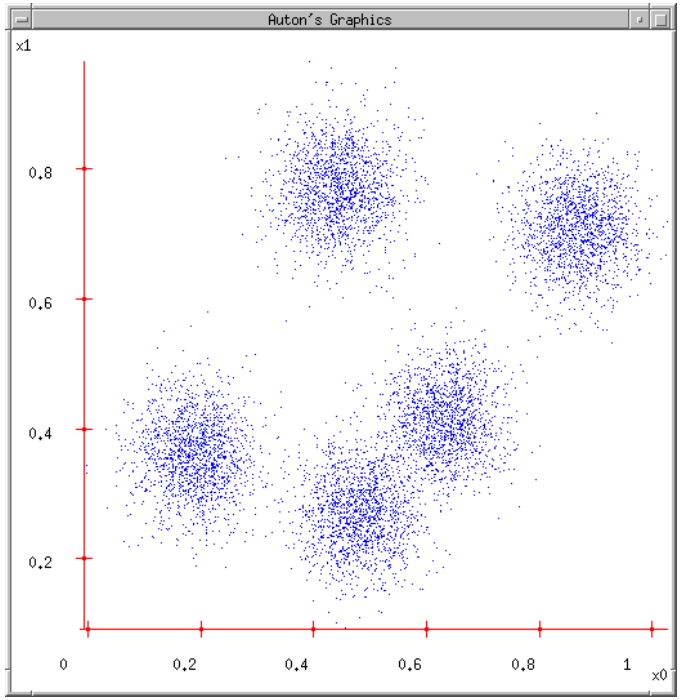
- Univariate Gaussian

$$\mu_{MLE} = \frac{1}{N}\sum_{i=1}^{N} x_i \qquad \sigma^2_{MLE} = \frac{1}{N}\sum_{i=1}^{N} (x_i - \hat{\mu})^2$$

- ***Mixture*** of ***Multi***variate Gaussians

$$m_{ML} = \frac{1}{n}\sum_{j=1}^{n} x_n \qquad S_{ML} = \frac{1}{n}\sum_{j=1}^{n} (\mathbf{x}_j - m_{ML})(\mathbf{x}_j - m_{ML})^T$$

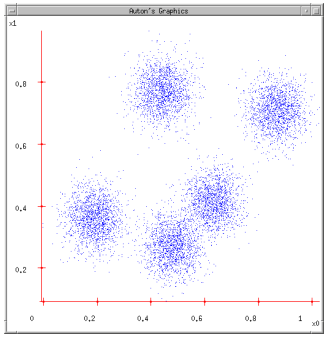
That was easy!
But what if ***unobserved data***?



- MLE:
  - $\text{argmax}_\theta \prod_j P(y_j, x_j)$
  - θ: all model parameters
    - eg, class probs, means, and variance for naïve Bayes
- But we don't know y_j's!!!
- Maximize ***marginal likelihood***:
  - $\text{argmax}_\theta \prod_j P(x_j) = \text{argmax} \prod_j \sum_{i=1}^{k} P(y_j=i, x_j)$

How do we optimize? Closed Form?

- Maximize ***marginal likelihood***:
  - $\text{argmax}_\theta \prod_j P(x_j) = \text{argmax} \prod_j \sum_{i=1}^{k} P(y_j=i,x_j)$
- Almost always a hard problem!
  - Usually no closed form solution
  - Even when P(X,Y) is convex, P(X) generally isn't…
  - For all but the simplest P(X), we will have to do gradient ascent, in a big messy space with lots of local optimum…



Learning general mixtures of Gaussian

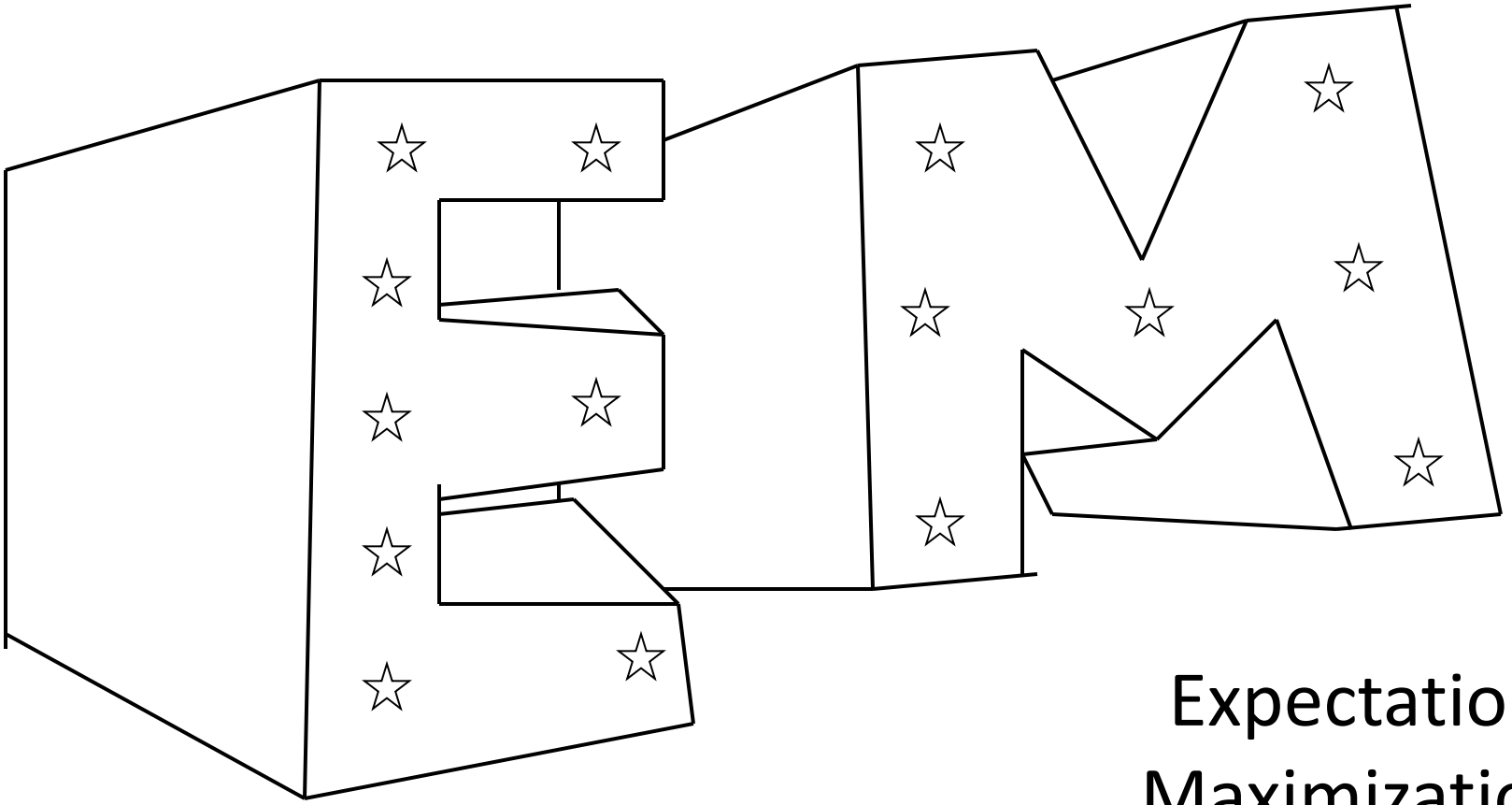
$$P(y=i \mid \mathbf{x}_j) \propto \frac{1}{(2\pi)^{m/2} \| \Sigma_i \|^{1/2}} \exp\left[-\frac{1}{2} \left(\mathbf{x}_j - \mu_i\right)^T \Sigma_i^{-1} \left(\mathbf{x}_j - \mu_i\right) \right] P(y=i)$$

- Marginal likelihood:

$$\prod_{j=1}^{m} P(\mathbf{x}_j) = \prod_{j=1}^{m} \sum_{i=1}^{k} P(\mathbf{x}_j, y=i)$$

$$= \prod_{j=1}^{m} \sum_{i=1}^{k} \frac{1}{(2\pi)^{m/2} \| \Sigma_i \|^{1/2}} \exp\left[-\frac{1}{2} \left(\mathbf{x}_j - \mu_i\right)^T \Sigma_i^{-1} \left(\mathbf{x}_j - \mu_i\right) \right] P(y=i)$$

- Need to differentiate and solve for μ_i, Σ_i, and P(Y=i) for i=1..k
- There will be no closed form solution, gradient is complex, lots of local optimum
- ***Wouldn't it be nice if there was a better way!?!***



Expectation
Maximization

The EM Algorithm

- A clever method for maximizing marginal likelihood:
  - $\text{argmax}_\theta \prod_j P(x_j) = \text{argmax}_\theta \prod_j \sum_{i=1}^{k} P(y_j=i,x_j)$
  - A type of gradient ascent that can be easy to implement (eg, no line search, learning rates, etc.)
- Alternate between two steps:
  - Compute an expectation
  - Compute a maximization
- Not magic: ***still optimizing a non-convex function with lots of local optima***
  - The computations are just easier (often, significantly so!)

EM: Two Easy Steps

Objective: $\text{argmax}_\theta \prod_j \sum_{i=1}^{k} P(y_j=i, x_j|\theta) = \sum_j \log \sum_{i=1}^{k} P(y_j=i, x_j|\theta)$

Data: $\{x_j \mid j=1 .. n\}$

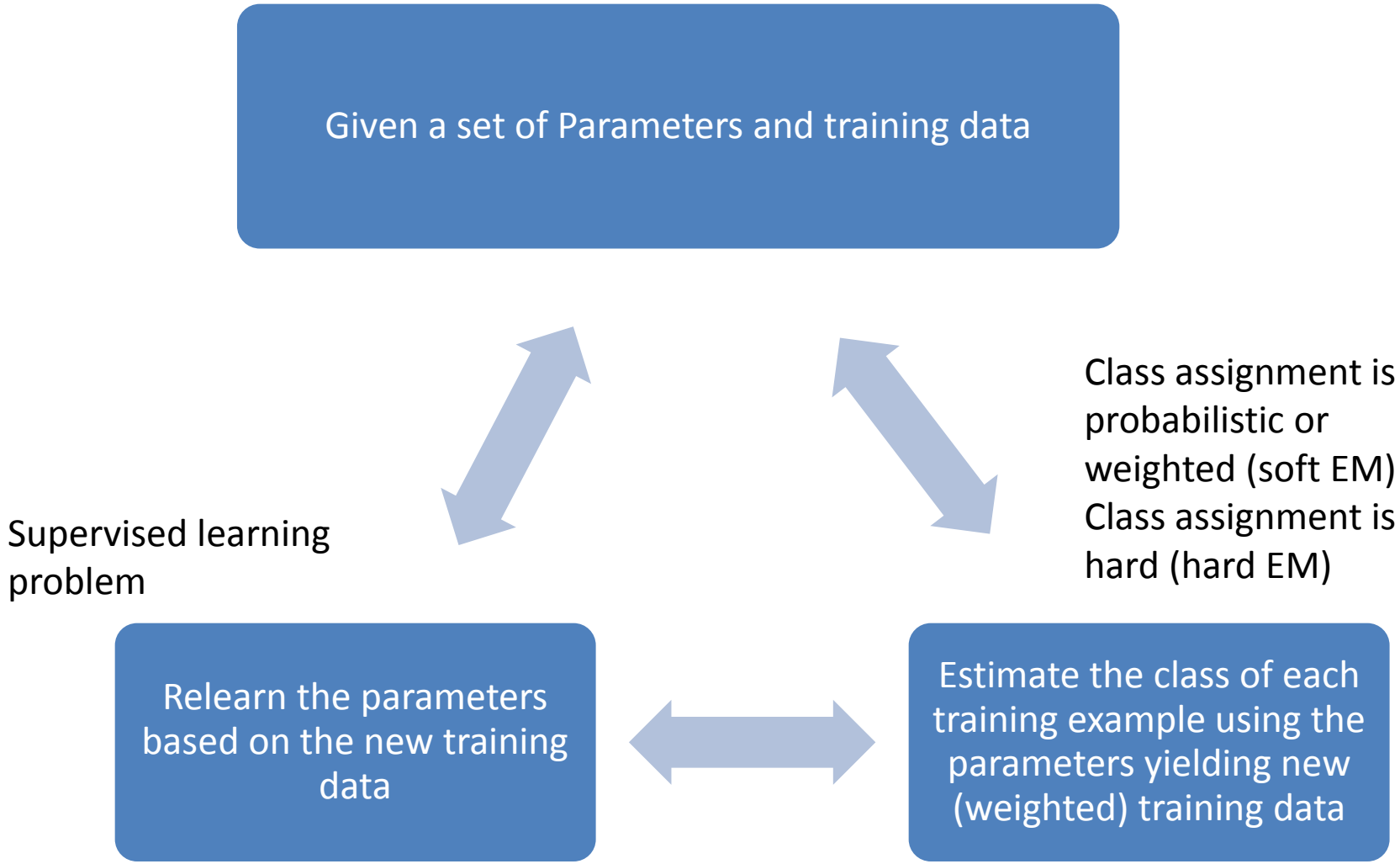
Notation a bit inconsistent
Parameters = $\theta=\lambda$

- **E-step**: Compute expectations to "fill in" missing y values according to current parameters, θ
  - For all examples j and values i for y, compute: $P(y_j=i \mid x_j, \theta)$

- **M-step**: Re-estimate the parameters with "weighted" MLE estimates
  - Set $\theta = \text{argmax}_\theta \sum_j \sum_i P(y_j=i \mid x_j, \theta) \log P(y_j=i, x_j|\theta)$

Especially useful when the E and M steps have closed form solutions!!!

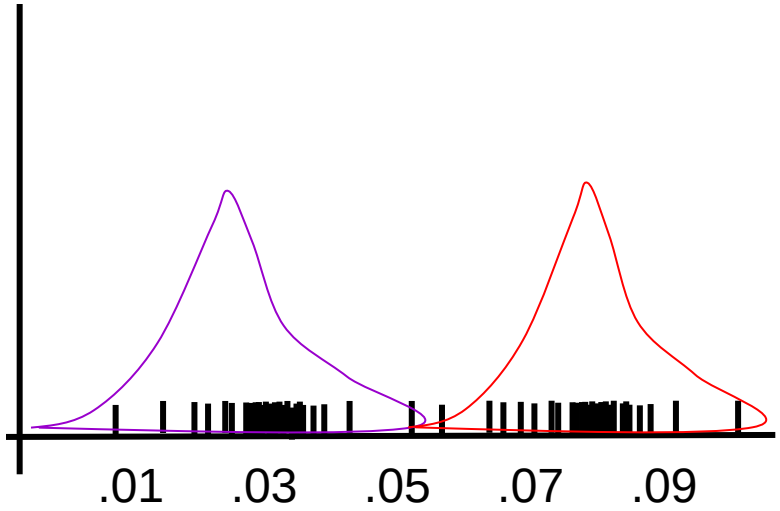
EM algorithm: Pictorial View



Simple example: learn means only!

Consider:

- 1D data
- Mixture of k=2 Gaussians
- Variances fixed to σ=1
- Dist'n over classes is uniform
- Just need to estimate μ_1 and μ_2



$$\prod_{j=1}^{m} \sum_{i=1}^{k} P(x, y=i) \propto \prod_{j=1}^{m} \sum_{i=1}^{k} \exp\left[-\frac{1}{2\sigma^2} \left\| x - \mu_i \right\|^2 \right] P(y=i)$$

EM for GMMs: only learning means

Iterate: On the *t*'th iteration let our estimates be

$$\lambda_t = \{ \mu_1^{(t)}, \mu_2^{(t)} \ldots \mu_k^{(t)} \}$$

E-step

Compute "expected" classes of all datapoints

$$p(y = i | x_j, \mu_1 \ldots \mu_k) \propto \exp\left(-\frac{1}{2\sigma^2} \| x_j - \mu_i \|^2 \right) P(y = i)$$

M-step

Compute most likely new **μ**s given class expectations

$$\mu_i = \frac{\sum_{j=1}^{m} P(y = i | x_j) x_j}{\sum_{j=1}^{m} P(y = i | x_j)}$$

E.M. for General GMMs

Iterate: On the *t*'th iteration let our estimates be

$$\lambda_t = \{ \mu_1^{(t)}, \mu_2^{(t)} \dots \mu_k^{(t)}, \Sigma_1^{(t)}, \Sigma_2^{(t)} \dots \Sigma_k^{(t)}, p_1^{(t)}, p_2^{(t)} \dots p_k^{(t)} \}$$

$p_i^{(t)}$ is shorthand for estimate of $P(y=i)$ on t'th iteration

E-step

Compute "expected" classes of all datapoints for each class

$$P(y = i | x_j, \lambda_t) \propto p_i^{(t)} p(x_j | \mu_i^{(t)}, \Sigma_i^{(t)})$$

Just evaluate a Gaussian at x_j

M-step

Compute weighted MLE for **μ** given expected classes above

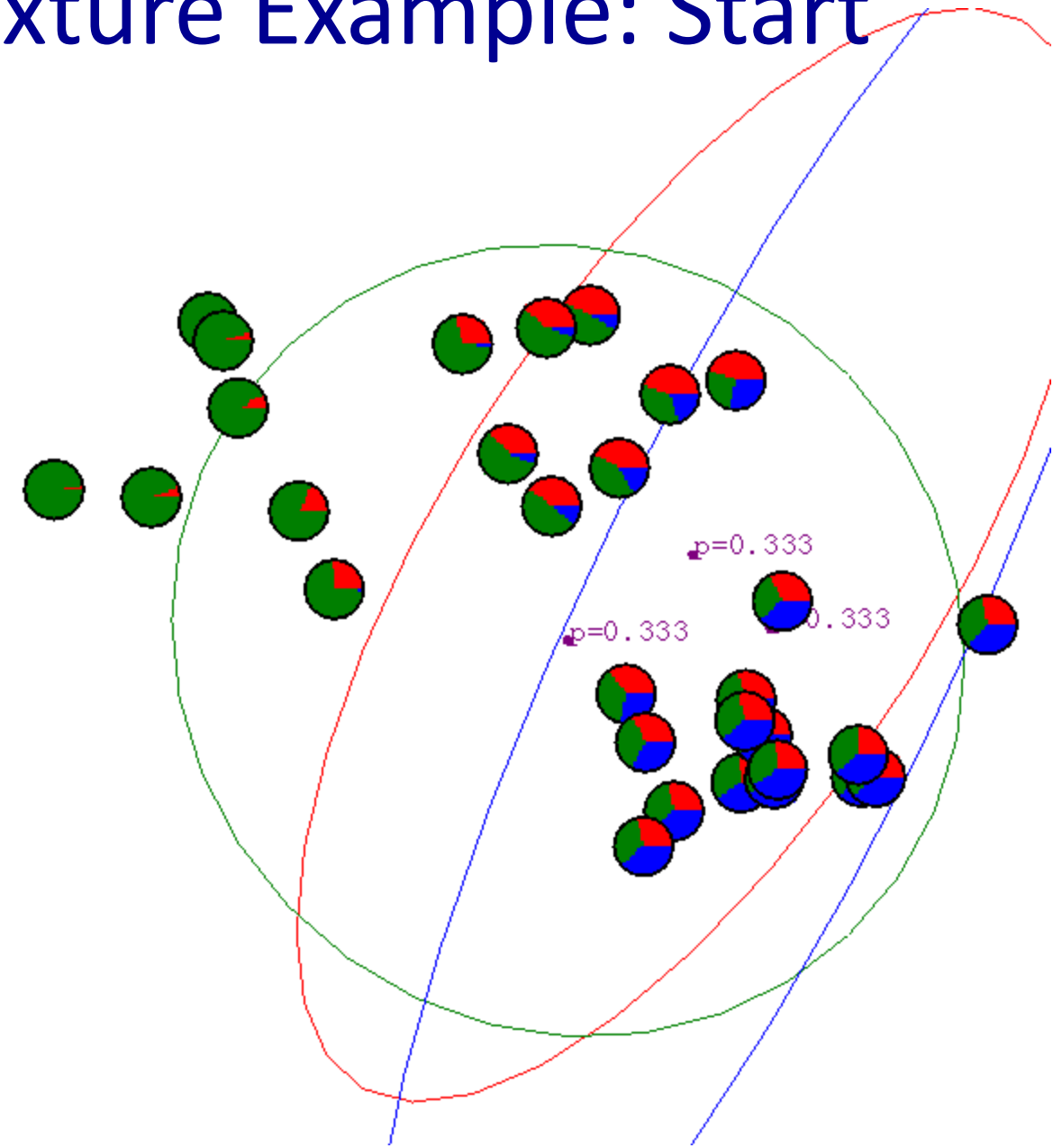
$$\mu_i^{(t+1)} = \frac{\sum_j P(y = i | x_j, \lambda_t) x_j}{\sum_j P(y = i | x_j, \lambda_t)}$$

$$\Sigma_i^{(t+1)} = \frac{\sum_j P(y = i | x_j, \lambda_t) [x_j - \mu_i^{(t+1)}][x_j - \mu_i^{(t+1)}]^T}{\sum_j P(y = i | x_j, \lambda_t)}$$

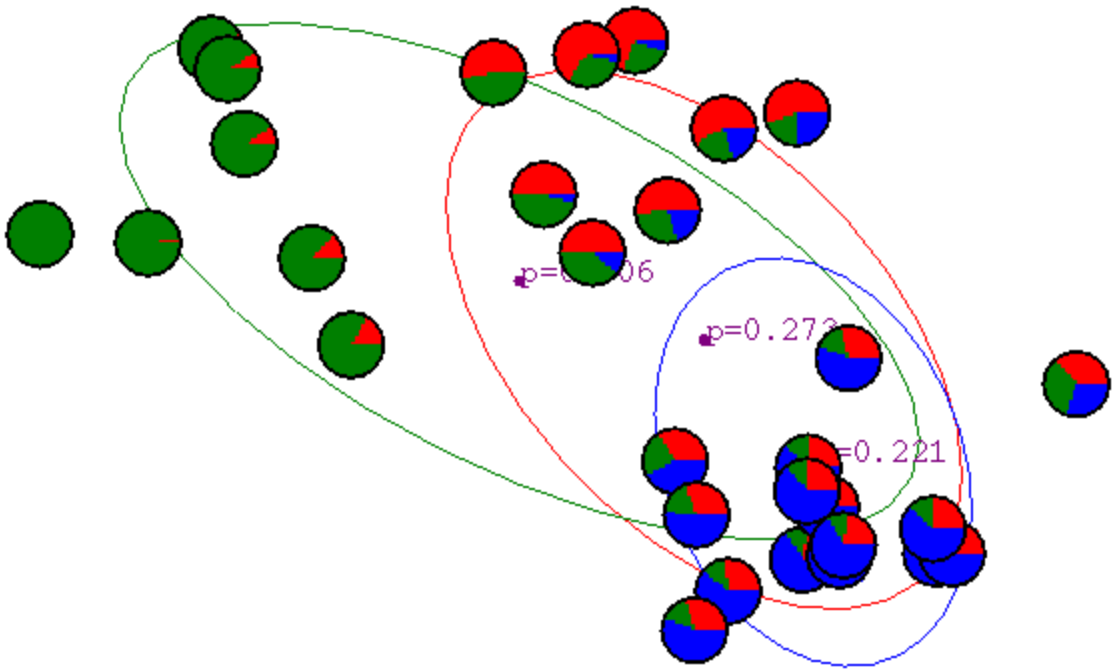
$$p_i^{(t+1)} = \frac{\sum_j P(y = i | x_j, \lambda_t)}{m}$$

m = #training examples

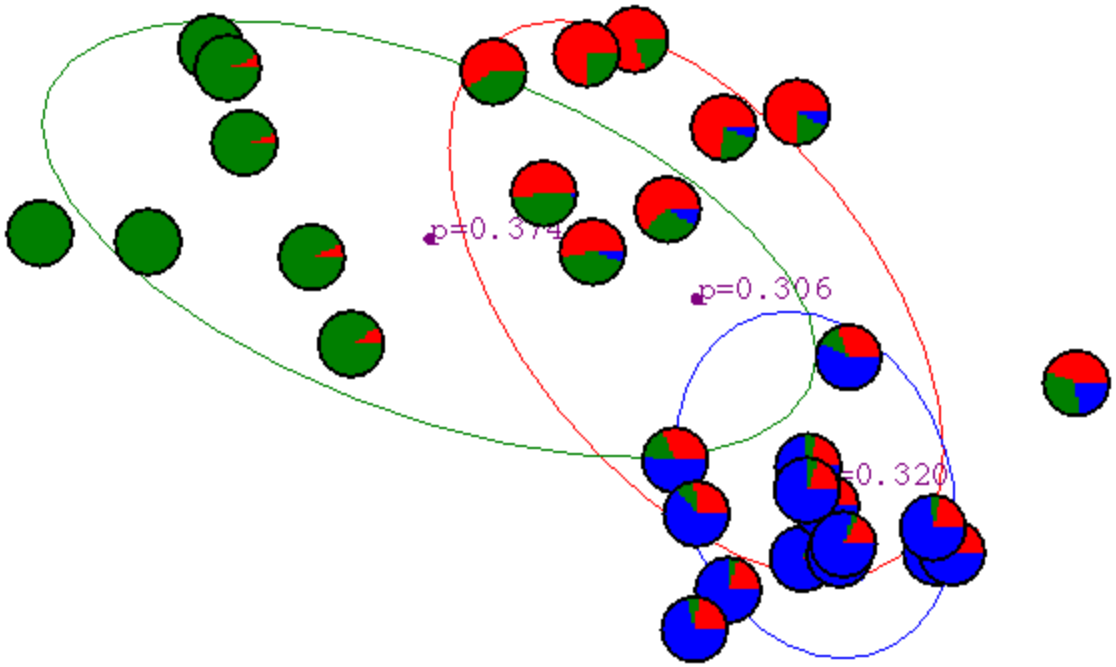
Gaussian Mixture Example: Start



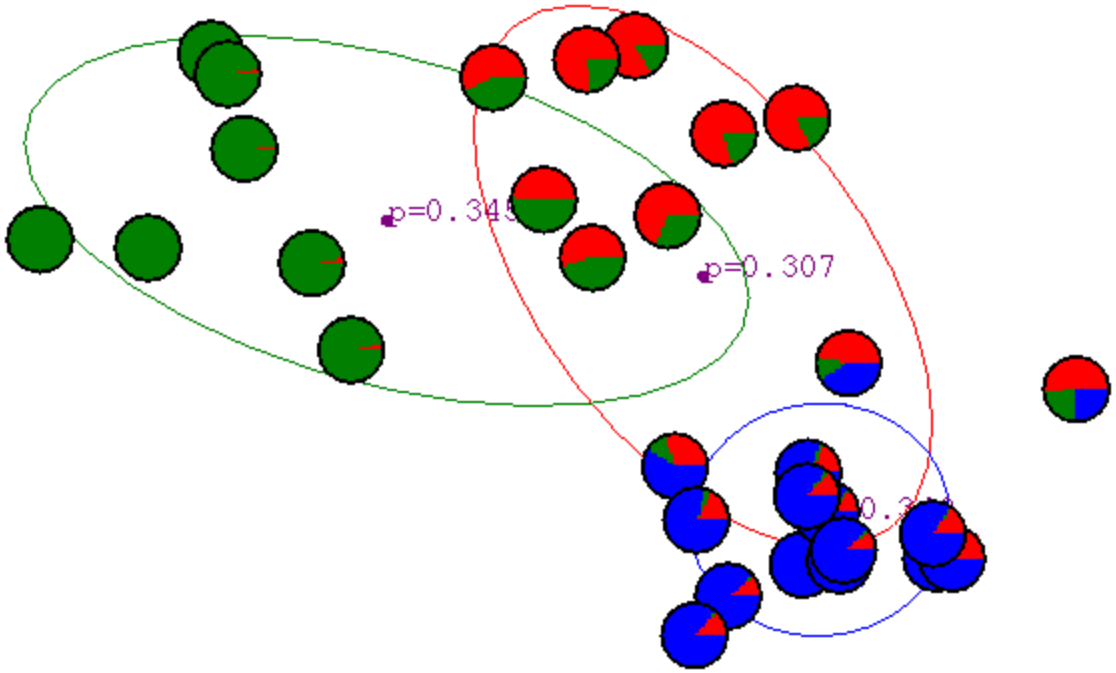
After first iteration



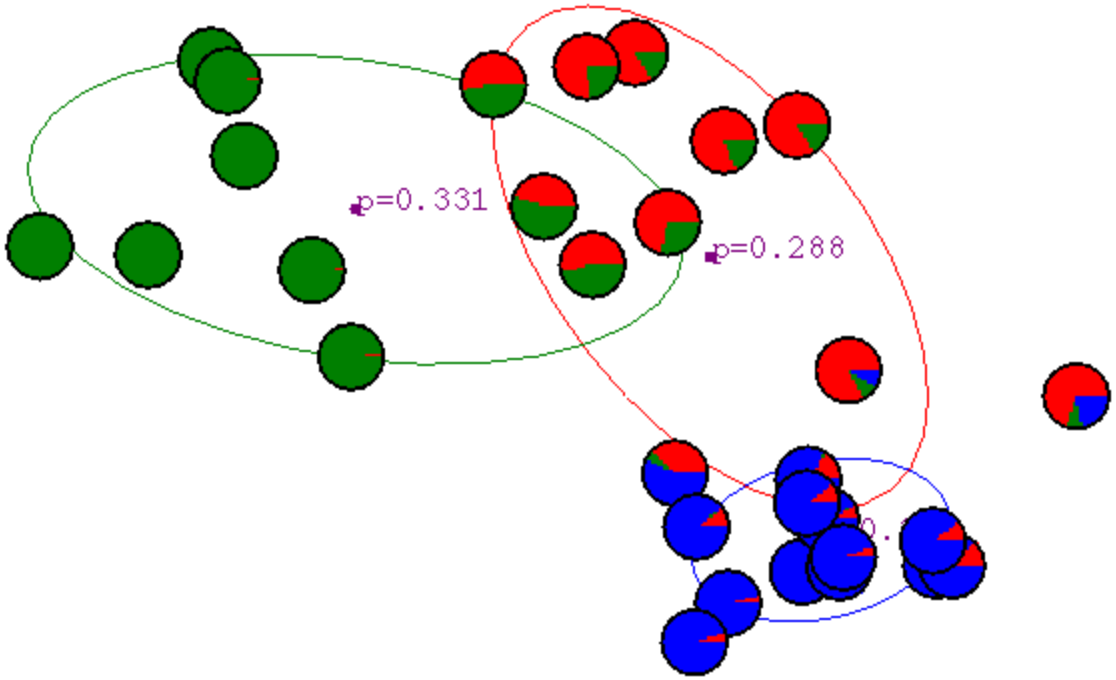
After 2nd iteration



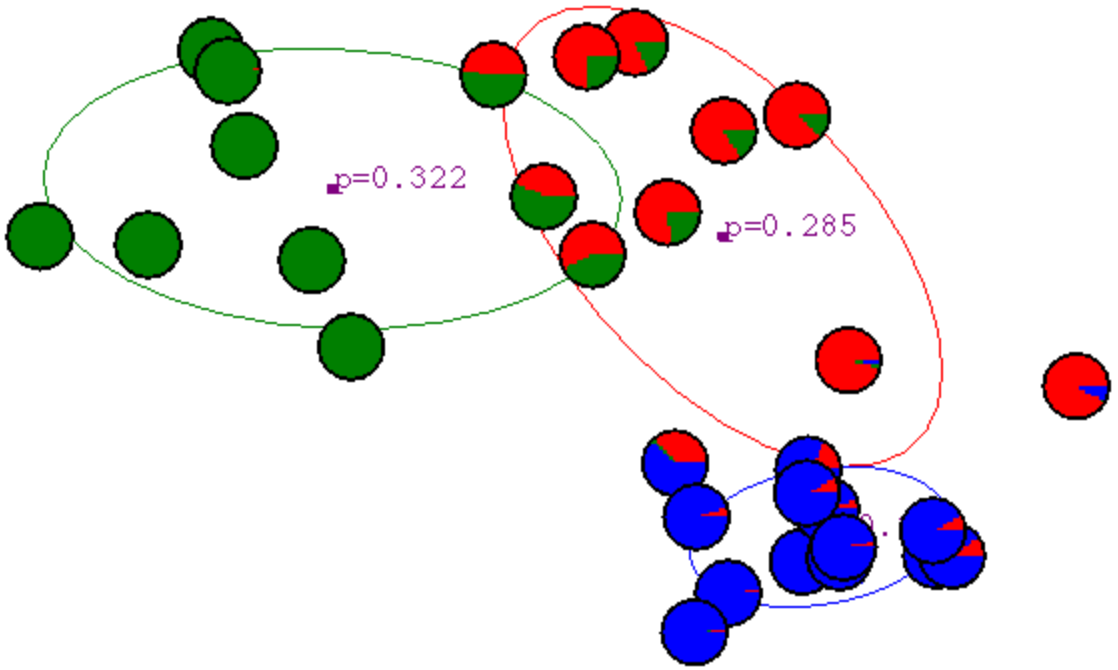
After 3rd iteration



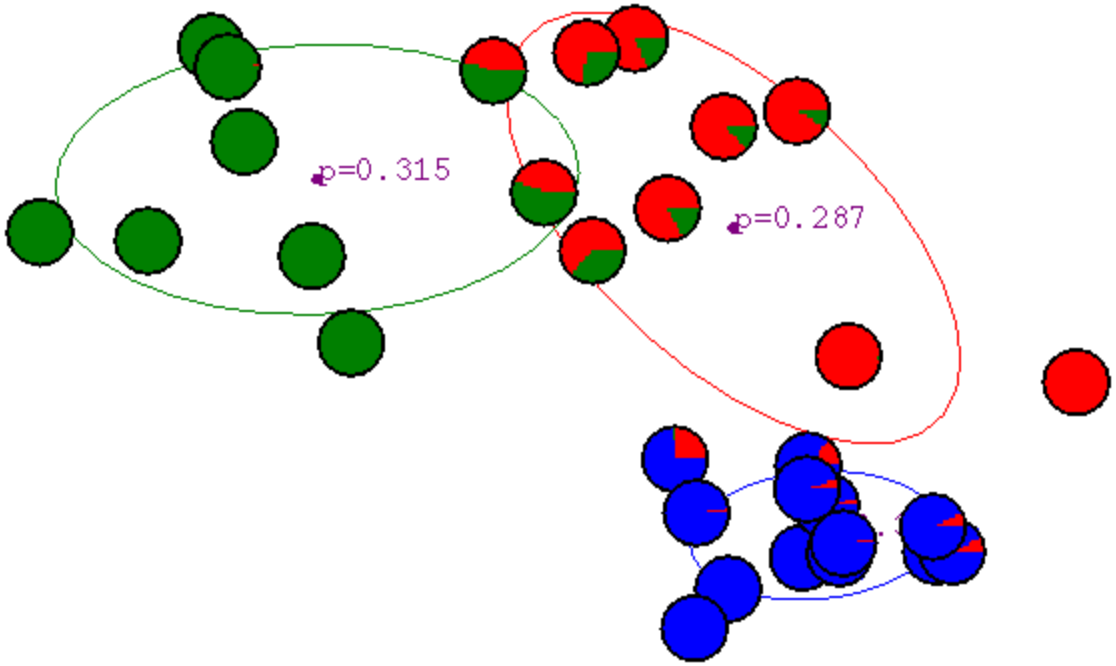
After 4th iteration



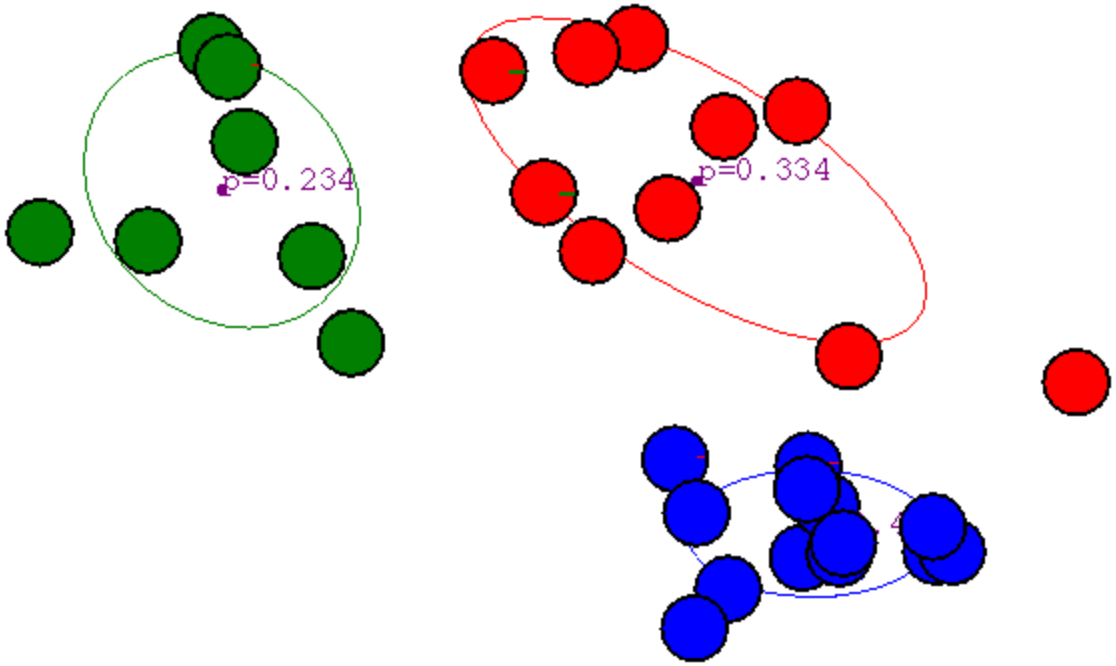
After 5th iteration



After 6th iteration



After 20th iteration



What if we do hard assignments?

Iterate: On the *t*'th iteration let our estimates be

$$\theta_t = \{ \mu_1^{(t)}, \mu_2^{(t)} \ldots \mu_k^{(t)} \}$$

E-step

Compute "expected" classes of all datapoints

$$p(y = i | x_j, \mu_1 \ldots \mu_k) \propto \exp\left(-\frac{1}{2\sigma^2} \| x_j - \mu_i \|^2 \right) P(y = i)$$

M-step

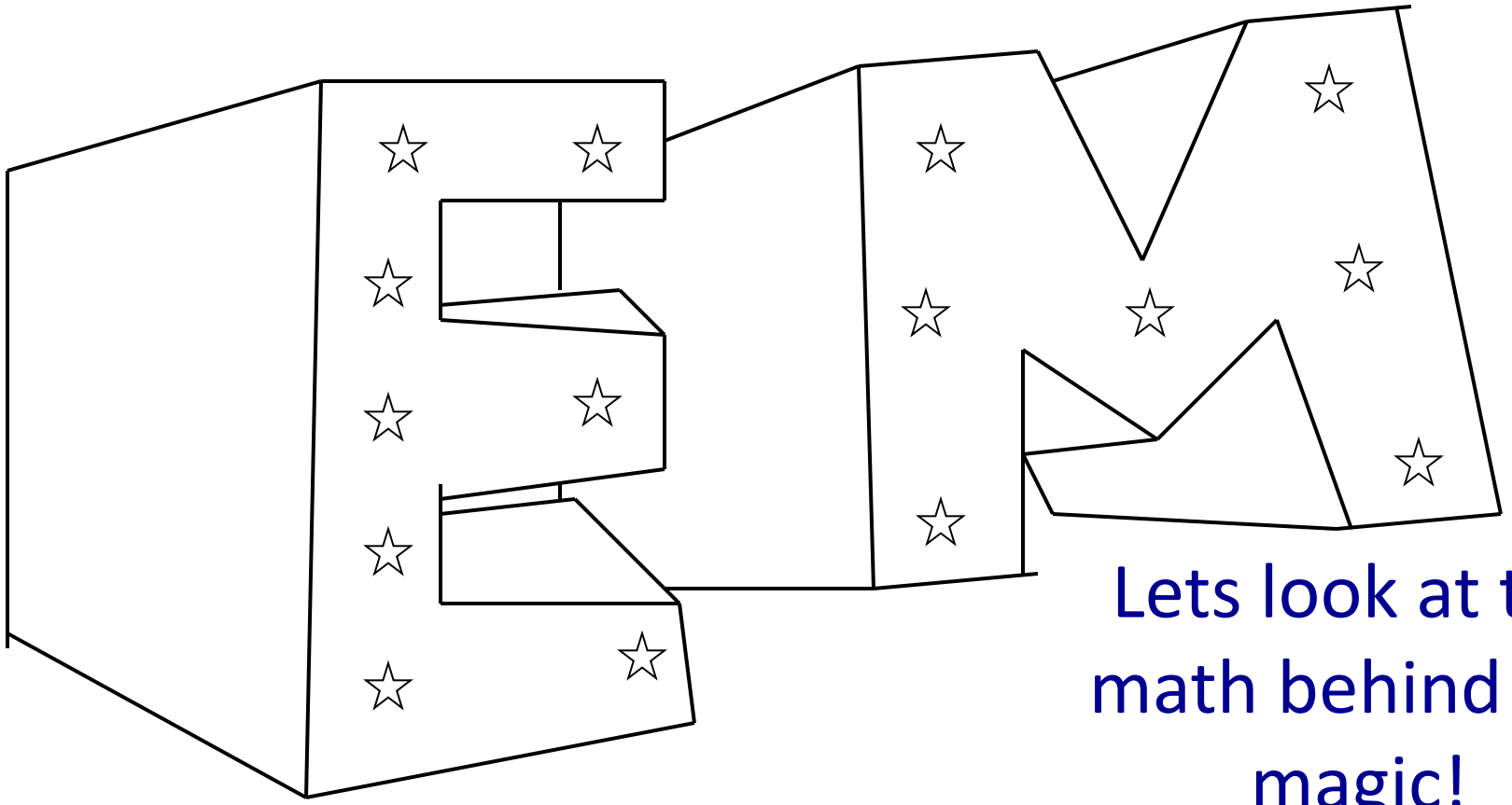
Compute most likely new μs given class expectations

$$\mu_i = \frac{\sum_{j=1}^{m} P(y = i | x_j) x_j}{\sum_{j=1}^{m} P(y = i | x_j)}$$

$$\mu_i = \frac{\sum_{j=1}^{m} \delta(y = i, x_j) x_j}{\sum_{j=1}^{m} \delta(y = i, x_j)}$$

δ represents hard assignment to "most likely" or nearest cluster

Equivalent to k-means clustering algorithm!!!



Lets look at the math behind the magic!

We will argue that EM:

- Optimizes a bound on the likelihood
- Is a type of coordinate ascent
- Is guaranteed to converge to a (often local) optima

The general learning problem with missing data

- Marginal likelihood: **x** is observed,

  z (eg class labels, **y**) is missing:

$$\begin{aligned}\ell(\theta : \mathcal{D}) &= \log \prod_{j=1}^{m} P(\mathbf{x}_j \mid \theta) \\ &= \sum_{j=1}^{m} \log P(\mathbf{x}_j \mid \theta) \\ &= \sum_{j=1}^{m} \log \sum_{\mathbf{z}} P(\mathbf{x}_j, \mathbf{z} \mid \theta)\end{aligned}$$

- Objective: Find argmax_θ l(θ:Data)

A Key Computation: E-step

- **x** is observed, **z** is missing
- Compute probability of missing data given current choice of θ
  - Q(**z**|$\mathbf{x}_j$) for each $\mathbf{x}_j$
    - e.g., probability computed during classification step
    - corresponds to "classification step" in K-means

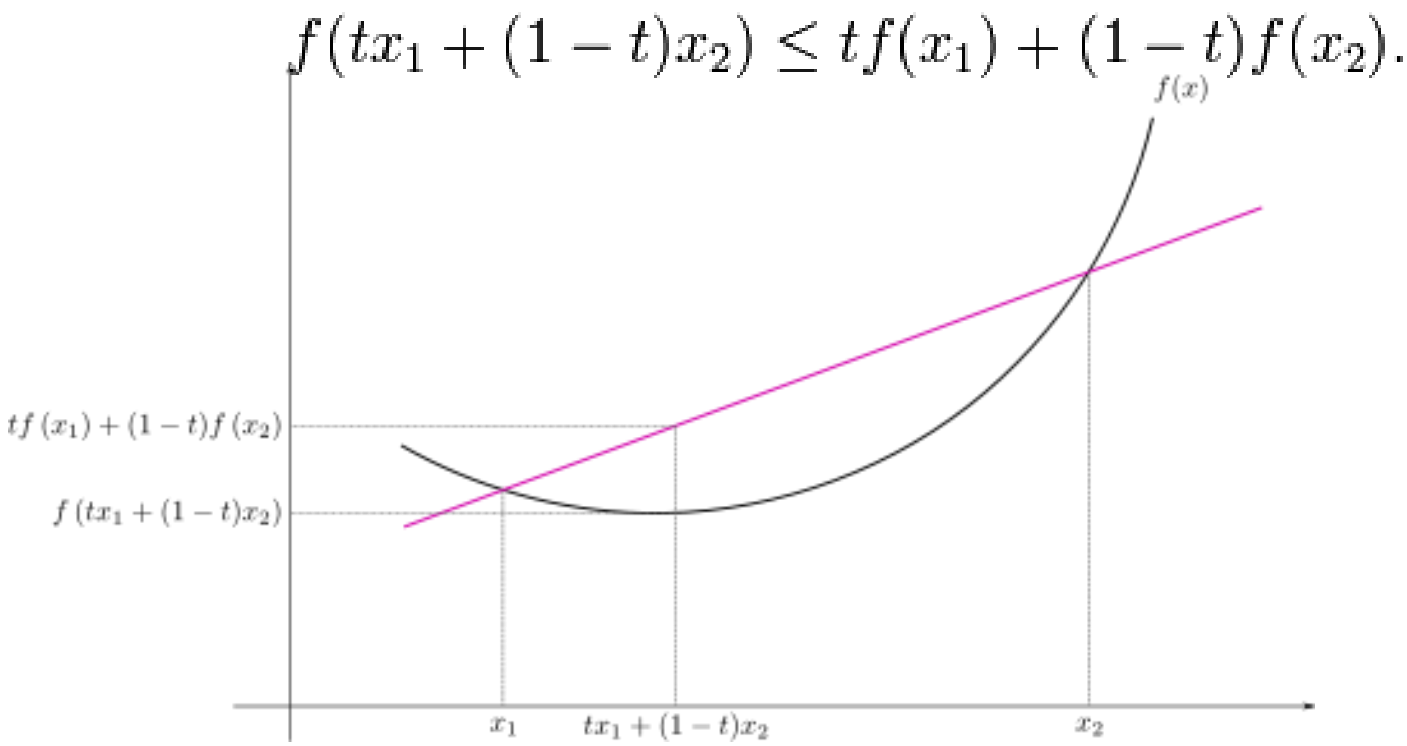
$$Q^{(t+1)}(\mathbf{z} \mid \mathbf{x}_j) \;=\; P(\mathbf{z} \mid \mathbf{x}_j, \theta^{(t)})$$

Properties of EM

- We will prove that
  - EM converges to a local minima
  - Each iteration improves the log-likelihood
- How? (Same as k-means)
  - E-step can never decrease likelihood
  - M-step can never decrease likelihood

Jensen's inequality

- **Theorem**: log $\sum_z$ P(**z**) f(**z**) $\geq$ $\sum_z$ P(**z**) log f(**z**)
  - **e.g., Binary case for convex function f:**



Applying Jensen's inequality

- Use: $\log \sum_{\mathbf{z}} P(\mathbf{z})\, f(\mathbf{z}) \geq \sum_{\mathbf{z}} P(\mathbf{z}) \log f(\mathbf{z})$

$$\ell(\theta^{(t)} : \mathcal{D}) = \sum_{j=1}^{m} \log \sum_{\mathbf{z}} Q^{(t+1)}(\mathbf{z} \mid \mathbf{x}_j) \frac{P(\mathbf{z}, \mathbf{x}_j \mid \theta^{(t)})}{Q^{(t+1)}(\mathbf{z} \mid \mathbf{x}_j)}$$

$$\geq \sum_{j=1}^{m} \sum_{z} Q^{(t+1)}(z \mid x_j) \log \left(\frac{p(z, x_j \mid \theta^{(t)})}{Q^{(t+1)}(z \mid x_j)} \right)$$

$$= \sum_{j=1}^{m} \sum_{z} Q^{(t+1)}(z \mid x_j) \log \left(p(z, x_j \mid \theta^{(t)}) \right) - \sum_{j=1}^{m} \sum_{z} Q^{(t+1)}(z \mid x_j) \log \left(Q^{(t+1)}(z \mid x_j) \right)$$

$$\ell(\theta^{(t)} : \mathcal{D}) \geq \sum_{j=1}^{m} \sum_{\mathbf{z}} Q^{(t+1)}(\mathbf{z} \mid \mathbf{x}_j) \log P(\mathbf{z}, \mathbf{x}_j \mid \theta^{(t)}) + m.H(Q^{(t+1)})$$

The M-step

Lower bound:

$$\ell(\theta^{(t)} : \mathcal{D}) \geq \sum_{j=1}^{m} \sum_{\mathbf{z}} Q^{(t+1)}(\mathbf{z} \mid \mathbf{x}_j) \log P(\mathbf{z}, \mathbf{x}_j \mid \theta^{(t)}) + m.H(Q^{(t+1)})$$

- Maximization step:

$$\theta^{(t+1)} \leftarrow \arg\max_{\theta} \sum_{j=1}^{m} \sum_{\mathbf{z}} Q^{(t+1)}(\mathbf{z} \mid \mathbf{x}_j) \log P(\mathbf{z}, \mathbf{x}_j \mid \theta)$$

- We are optimizing a lower bound!
- Use expected counts to do weighted learning:
  - If learning requires Count(**x**,**z**)
  - Use $E_{Q(t+1)}$[Count(**x**,**z**)]
  - *Looks a bit like boosting!!!*

Convergence of EM

- Define: potential function F(θ,Q):

$$\ell(\theta : \mathcal{D}) \;\geq\; F(\theta, Q) = \sum_{j=1}^{m} \sum_{\mathbf{z}} Q(\mathbf{z} \mid \mathbf{x}_j) \log \frac{P(\mathbf{z}, \mathbf{x}_j \mid \theta)}{Q(\mathbf{z} \mid \mathbf{x}_j)}$$

  – lower bound from Jensen's inequality

- EM is coordinate ascent on F!
  – Thus, maximizes lower bound on marginal log likelihood

M-step can't decrease F(θ,Q): by definition!

$$\theta^{(t+1)} \leftarrow \arg\max_{\theta} \sum_{j=1}^{m} \sum_{\mathbf{z}} Q^{(t+1)}(\mathbf{z} \mid \mathbf{x}_j) \log P(\mathbf{z}, \mathbf{x}_j \mid \theta)$$

- We are maximizing F directly, by ignoring a constant!

$$F(\theta, Q^{(t+1)}) = \sum_{j=1}^{m} \sum_{\mathbf{z}} Q^{(t+1)}(\mathbf{z} \mid \mathbf{x}_j) \log P(\mathbf{z}, \mathbf{x}_j \mid \theta) + m.H(Q^{(t+1)})$$

E-step: more work to show that F(θ,Q) doesn't decrease

- KL-divergence: measures distance between distributions

$$KL(Q||P) = \sum_z Q(z) \log \frac{Q(z)}{P(z)}$$

- KL=zero if and only if Q=P

E-step also doesn't decrease F: Step 1

- Fix θ to $\theta^{(t)}$, take a max over Q:

$$\ell(\theta^{(t)} : \mathcal{D}) \;\geq\; F(\theta^{(t)}, Q) = \sum_{j=1}^{m} \sum_{\mathbf{z}} Q(\mathbf{z} \mid \mathbf{x}_j) \log \frac{P(\mathbf{z}, \mathbf{x}_j \mid \theta^{(t)})}{Q(\mathbf{z} \mid \mathbf{x}_j)}$$

$$= \sum_{j=1}^{m} \sum_{z} Q(z \mid x_j) \log \left(\frac{P(z \mid x_j, \theta^{(t)}) P(x_j \mid \theta^{(t)})}{Q(z \mid x_j)} \right)$$

$$= \sum_{j=1}^{m} \sum_{z} Q(z \mid x_j) \log \left(P(x_j \mid \theta^{(t)}) \right) - \sum_{j=1}^{m} \sum_{z} Q(z \mid x_j) \log \left(\frac{Q(z \mid x_j)}{P(z \mid x_j, \theta^{(t)})} \right)$$

$$= \ell(\theta^{(t)} : \mathcal{D}) - \sum_{j=1}^{m} KL \left(Q(\mathbf{z} \mid \mathbf{x}_j) || P(\mathbf{z} \mid \mathbf{x}_j, \theta^{(t)}) \right)$$

E-step also doesn't decrease F: Step 2

- Fixing θ to $\theta^{(t)}$:

$$\ell(\theta^{(t)} : \mathcal{D}) \geq F(\theta^{(t)}, Q) = \ell(\theta^{(t)} : \mathcal{D}) + \sum_{j=1}^{m} \sum_{\mathbf{z}} Q(\mathbf{z} \mid \mathbf{x}_j) \log \frac{P(\mathbf{z} \mid \mathbf{x}_j, \theta^{(t)})}{Q(\mathbf{z} \mid \mathbf{x}_j)}$$

$$= \ell(\theta^{(t)} : \mathcal{D}) - \sum_{j=1}^{m} KL\left(Q(\mathbf{z} \mid \mathbf{x}_j) || P(\mathbf{z} \mid \mathbf{x}_j, \theta^{(t)})\right)$$

- Now, the max over Q yields:
  - $Q(z|x_j) \leftarrow P(z|x_j, \theta^{(t)})$
  - Why? The likelihood term is a constant; the KL term is zero iff the arguments are the same distribution!!
  - So, the E-step is actually a maximization / tightening of the bound. It ensures that:

$$F(\theta^{(t)}, Q^{(t+1)}) = \ell(\theta^{(t)} : \mathcal{D})$$

EM is coordinate ascent

$$\ell(\theta : \mathcal{D}) \;\geq\; F(\theta, Q) = \sum_{j=1}^{m} \sum_{\mathbf{z}} Q(\mathbf{z} \mid \mathbf{x}_j) \log \frac{P(\mathbf{z}, \mathbf{x}_j \mid \theta)}{Q(\mathbf{z} \mid \mathbf{x}_j)}$$

- **M-step**: Fix Q, maximize F over θ (a lower bound on $\ell(\theta : \mathcal{D})$):

$$\ell(\theta : \mathcal{D}) \;\geq\; F(\theta, Q^{(t)}) = \sum_{j=1}^{m} \sum_{\mathbf{z}} Q^{(t)}(\mathbf{z} \mid \mathbf{x}_j) \log P(\mathbf{z}, \mathbf{x}_j \mid \theta) + m.H(Q^{(t)})$$

- **E-step**: Fix θ, maximize F over Q:

$$\ell(\theta^{(t)} : \mathcal{D}) \geq F(\theta^{(t)}, Q) \;=\; \ell(\theta^{(t)} : \mathcal{D}) - m \sum_{j=1}^{m} KL\left(Q(\mathbf{z} \mid \mathbf{x}_j) || P(\mathbf{z} \mid \mathbf{x}_j, \theta^{(t)})\right)$$

  - "Realigns" F with likelihood:

$$F(\theta^{(t)}, Q^{(t+1)}) \;=\; \ell(\theta^{(t)} : \mathcal{D})$$

What you should know

- K-means for clustering:
  - algorithm
  - converges because it's coordinate ascent
- Know what agglomerative clustering is
- EM for mixture of Gaussians:
  - Also coordinate ascent
  - How to "learn" maximum likelihood parameters (locally max. like.) in the case of unlabeled data
  - Relation to K-means
    - Hard / soft clustering
    - Probabilistic model
- Remember, E.M. can get stuck in local minima,
  - And empirically it ***DOES***

Acknowledgements

- K-means & Gaussian mixture models presentation contains material from excellent tutorial by Andrew Moore:
  - http://www.autonlab.org/tutorials/
- K-means Applet:
  - http://www.elet.polimi.it/upload/matteucc/Clustering/tutorial_html/AppletKM.html
- Gaussian mixture models Applet:
  - http://www.neurosci.aist.go.jp/%7Eakaho/MixtureEM.html