National Institute of Science Education And Research

FIRST SEMESTER, 2021-22
MidSemester Examination

P401

Classical Mechanics II

(Time allowed: TWO hours)

NOTE: Answer **All** questions. Examination is closed books and closed notes. Total number of points is 120. Be precise and to the point.

1. (a) For a rigid body composed of N particles, show that the number of degrees of freedom, in general, is 6. Give an example of a rigid body where this number is smaller than 6. Show that if the internal forces between the particles are central forces then internal forces don't do any work.

(b) For the case of symmetric top, derive the conjugate momenta, Hamiltonian and EOM using the Euler-Lagrange equations for the Lagrangian

$$L = \frac{I_1}{2}\left(\dot{\phi}^2 \sin^2\theta + \dot{\theta}^2\right) + \frac{I_3}{2}\left(\dot{\psi} + \dot{\phi}\cos\theta\right)^2 - Mgl\cos\theta \tag{1}$$

Consider a special case when $\dot{\theta} = 0$ always i.e. $\theta = \theta_0$. Find the conditions for the solution to exist in this case and interpret them. Distinguish between the cases $\theta_0 > \frac{\pi}{2}$ and $\theta_0 < \frac{\pi}{2}$

2. (a) Consider a particle with charge e moving with velocity $\vec{v}$ in a constant, uniform magnetic field $\vec{B}$ with interaction Lagrangian $L_i = e\vec{v}\cdot\vec{A}$. Using the gauge in which the vector potential $\vec{A}$ is given in terms of position vector $\vec{r}$ by $\vec{A} = -\frac{1}{2}\vec{r}\times\vec{B}$, show that in a rotating frame, the total Lagrangian can be reduced to that of a simple harmonic oscillator.

(b) Find the eigenvalues corresponding to 2×2 matrix A satisfying $AA^T = I$ and $DetA = 1$.

(c) Find the equation of motion for a scalar field ϕ with the Lagrangian density

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial_\nu\phi\eta^{\mu\nu} - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4 \tag{2}$$

Here m, λ are constants and $\eta_{\mu\nu}$ is the Minkwoski metric.

3. Please answer following questions.

(a) Consider the motion of a torque-free symmetric top from the space-fixed frame. Show that angular momentum vector $\vec{L}$, angular velocity vector $\vec{\omega}$ and the vector corresponding to the body z-axis (symmetry axis) $\hat{e_3}$ lie in a plane. Show that $\vec{\omega}$ and $\hat{e_3}$ precess around the angular momentum vector.

(b) If $A(\hat{n},\Phi)$ is a fixed-axis rotation matrix and if R is some other rotation, show that $RA(\hat{n},\Phi)R^T = A(R\hat{n},\Phi)$, where R^T is the transpose of rotation R. If needed, you can use the axis-angle representation $A = I + N\sin\Phi + N^2(1-\cos\Phi)$ where $N_{ij} = -\epsilon_{ijk}n_k$ is the dual of axis direction n_k.

(c) Find the moment of inertia for a solid sphere of radius R and mass M with centre at the origin.