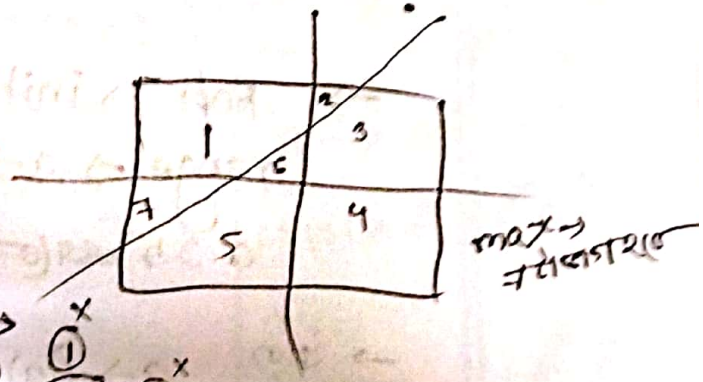
Mathematical Analysis (1st class) Monday (1st week) → 26/09/22
Time - 10.45 AM

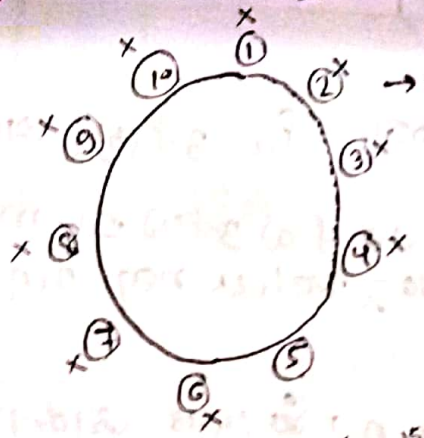
4 set / 3.5 set ← Concrete mathematics

3 set / 3.5 set ← Introduction to probability model - 12th edition

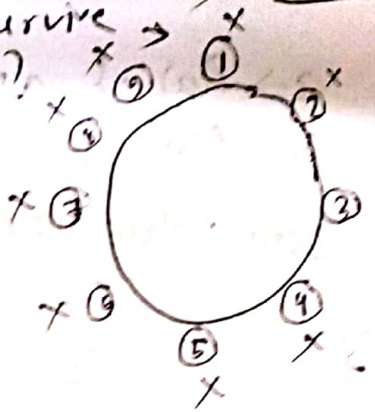
ct → 2 ct → average count 2 ct

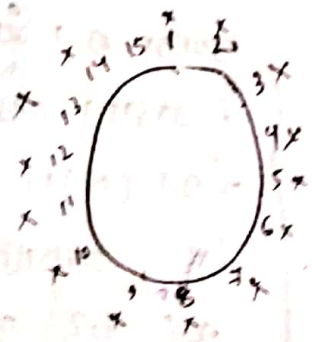
→ Recurrence problem





→ কয়টি survive করে?





Topic Name : Mathematical analysis (2nd class)

Day : Wednesday (3rd day)

Time : 10.45 am Date : 28 / 9 / 22

Recurrent problem

Tower of Hanoi →

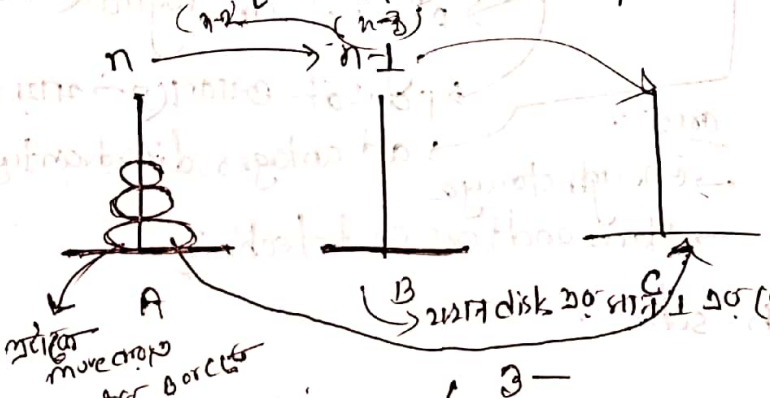
H.W

1.1, 1.2, 1.3 → concrete mathematics.

~~Numerical~~ Mathmetical analysis (3rd class) Monday ~~Tuesday~~ (5th day) 10.00am | 10.10.22

chapter-1:

→ derive the tower of hanoi problem? How can we derive the equation of hanoi problem?



smaller disk

→ Recursion / Recursive function / How can execute recursive function

if (n>1)
then n-1=B.
A=C.
n-1=C.

if n=1
then A=C.

recursive function

base

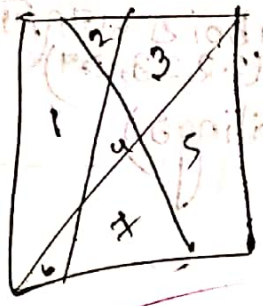
H.W → 1.1, 1.2, 1.3

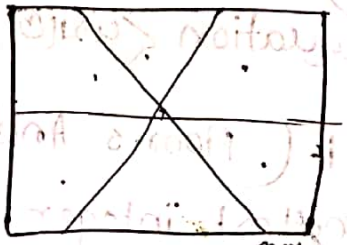
Topic Name: Mathematical analysis (3rd class) Day: Monday (5th day)

Time: 10am Date: 10/10/22

Lines in the plane → (condition)

↳ maximum no. of crossing কতটা হবে



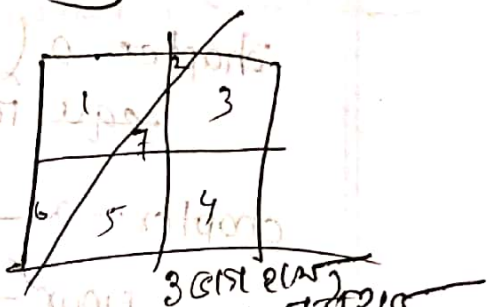


$L_0 = 1$

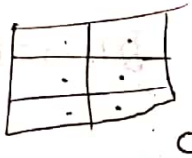
$L_1 = 2$ (max)

$L_2 = 4$ (max)

$L_3 = 7$



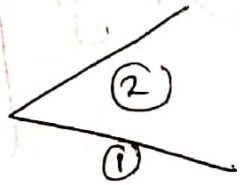
3 টার্গেট থাকলে max 7টা হবে

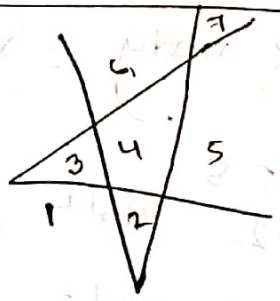


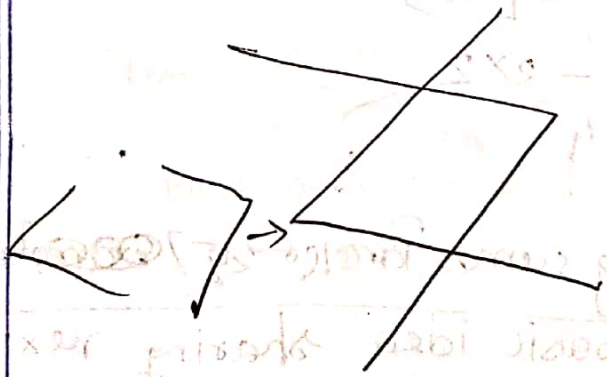
এটা যেভাবে draw করলে 4টা হবে (minimum)

Mathematical analysis (4th class) Tuesday (6th day)

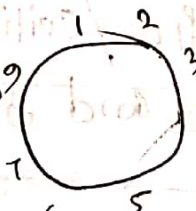
9.40am / 11.10.22



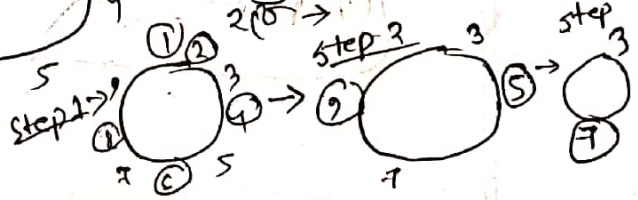




→ এটা কে survive করাবে ?



→ আমরা ques টা কিভাবে step by step draw করছি



Topic Name: Mathematical analysis (4th class) Day: Tuesday (6th day)
Time: 10.30am Date: 11 / 10 / 22

Practice problem- সব ই করতে হবে,
sir যেগুলো দিছেন → 6, 7, 8, 11, 13, 15.

chapter-2 (sums)
~~equ~~ কিছু equation দেখাতে হবে। boi এ দেখানো (2.2, 2.3, ...)

chapter-3 → 3.1 (Floors And Ceilings)
Floor → greatest integer
Ceiling → least integer

3.4 → ভালোভাবে পড়তে বলছেন (MOD: The binary operation)

Important:

$\lfloor n/m \rfloor$

$n = m\lfloor n/m \rfloor + n \bmod m$

$x \bmod y = x - y\lfloor x/y \rfloor$, for $y \neq 0$ } V.V.I

$x = 5$
$y = 2$ হলে,

$5 \bmod 2 = 5 - 2\lfloor 5/2 \rfloor$
$= 5 - 2 \times 2$
$= 5 - 4$
$= 1$

3.5 → floor/Ceiling sums দিতে হবে? ~~example~~

chapter 2 and 3 basic idea sharing next class

4.1 → Divisibility

$m \backslash n \Leftrightarrow m > 0$ and $n = mk$ for some integer k.

→ greatest common divisor

$$\gcd(m,n) = \max\{k \mid k \backslash m \text{ and } k \backslash n\}.$$

↳ k হলো এমন একটি number যার দিয়ে m কেও ভাগ করা যায়, n কেও ভাগ করা যায়।

Ex: $\gcd(8,16) = \max\{1,2,4,8\}$
$= 8$

$\gcd(12,18) = \max(1,2,3,6)$
$= 6$

$\gcd(5,7) = \max(1)$
$= 1$

→ if $n > 0$, we have $\gcd(0,n) = n$.

Guys: কোন number যেখানে আছে 0, এই process এ যেখানেও বসাতে পারো এমন eqn এর implementation আসতে পারে

$\gcd(0,n) = n$

$\gcd(m,n) = \max\{k \mid k \backslash m \text{ and } k \backslash n\}$

$\gcd(0,n) = \max_k\{1/0 \text{ and } 1 \backslash n\}$

$= \max\{1, \ldots n \backslash 0 \text{ and } n \backslash n$

$= \max\{1, \ldots n\}$ [কেননা $n \backslash 0$ ভাগ হয় $n \backslash n$]

$= n.$

Topic Name: Mathematical analysis (5th class) Day: Monday (10th day)
Time: 10am Date: 17/10/22

lcm: least common multiple

$lcm(m,n) = \min\{k \mid k \geq 0, m \mid k \text{ and } n \mid k\}$;

- Explain the eudid's algorithm from gcd.
- Calculate the eudids, gcd (20, 15)

$gcd(0,n) = n$

$gcd(m,n) = gcd(n \bmod m, m)$; for $m > 0$

$gcd(12,18) = gcd(18 \bmod 12, 12)$

calculate this using Euclids algorithm

$= gcd(6,12)$

$= gcd(12 \bmod 6, 6)$

$= gcd(0,6)$

$= 6$. [$\because gcd(0,n) = n$; $n = 6$]

- $lcm(m,n) \times gcd(m,n) = m \times n$
- ques: lcm

Topic Name: Mathematical analysis(5thclass) Day: Monday (10th day)

Time: 10am Date: 19 / 11 / 22

4.2 → primes

→ what is prime number?

→ prime no. equation

→ prime and relatively prime

→ what is relatively prime?

6 and 25 → composite but relatively prime

composite no. relatively prime

Concrete mathematics (6th class) Wednesday (11th day)
10.45 am / 19.11.22

Number theory

Euclid Algorithm

GCD(0,n) = n ; n > 0

Gcd(m,n) = Gcd(n % m, m)

GCD(6,12) ?

GCD(6,12) = GCD(12 % 6, 6)

= GCD(0, 6)

= 6

int sum;

int

int

int

i++ n

if

Topic Name: Concrete mathematics (7th class) Day: Tuesday (14th day)
Time: 9.40am Date: 25/10/22

• What is prime number?

4.2

Any positive integer n can be written as a product of primes —

n (Any number) $= p_1 \cdots p_m = \prod_{k=1}^{m} p_k, \quad p_1 \le \cdots \le p_m$.

p_n (All prime numbers) $= p_1 = 2$ [because p_1 = first prime number]

Prime numbers: $2, 3, 5, 7, 11, \ldots$
$p_1, p_2, p_3, p_4, p_5, \ldots$

• If $n=5$, then equation এমন হবে —

$n=5$
$p_3 = 5$; $n = p_3$

• If $n=6$, then, $n = p_1 \cdot p_2$

• If $n=35$, then, $n = p_2 \cdot p_3$

} sequence of the prime number

$\Rightarrow$ $n=16$; $p_1^4 = 2^4$

• $n = \prod_p p^{n_p}$.

$\Rightarrow 12 = 2 \cdot 2 \cdot 3 = 2^2 \cdot 3 - - - = (2, 1, 0, 0 \cdots)$
$= p_1 \cdot p_2$.

2nd place এ 2 এর কত power
3rd place এ 3 এর কত power
1st place এ 2 এর কত power

$\Rightarrow 25 = 5 \cdot 5 = 5^2 = (0, 0, 2, 0 \cdots)$

$= 18 = 2 \cdot 3^2 = (1, 2, 0, 0)$.

$n = \prod_p p^{n_p}$

• Using the prime method calculate gcd / lcm এমন ques আসতে পারে।

gcd:
$K_p = \min(m_p, n_p)$

lcm:
$K_p = \max(m_p, n_p)$

• gcd (12, 18)

$12 = 2^2 \cdot 3^1$

$18 = 2^1 \cdot 3^2$

we know,

$gcd = K_p = \min(m_p, n_p)$

$= ~~\min(2~~ \; 2^{\min(2,1)} \cdot 3^{\min(1,2)}$

$= 2^1 \cdot 3^1$

$= 6.$

• lcm (12, 18)

$lcm = K_p = \max(m_p, n_p)$

$= 2^{\max(2,1)} \cdot 3^{\max(1,2)}$

$= 2^2 \cdot 3^2 = 36.$

4.3 → ভাল করে পড়তে বলেছেন

Topic Name: Mathematical analysis (7th class) Day: Tuesday (14th day)
Time: 9.40am Date: 25/10/22

4.4 Factorial Factors:

$$n! = 1 \cdot 2 \cdots n = \prod_{K=1}^{n} K$$

113 page → calculation (এখান থেকে ২০ → (slide-127page)

4.5 Relative Primality

→ দুইটা number কে 1 ছাড়া prime না হলে তারা Relatively prime হয়ে যায়।

- Define relative prime.
- ex দিয়ে কাকে বলে relatively prime বোঝাতে।
- gcd বের করা
- Calculate the number using prime factor.

4.6 Modulus কি?

$$a \equiv b \pmod{m} \Leftrightarrow a \bmod m = b \bmod m$$

Ex: $9 \equiv 16 \pmod 5$

$9 \bmod 5 = 4$ [5|9|1
5
4

$(-16) \bmod 5 = -16 - 5\left[\frac{-16}{5}\right]$

$= -16 - 5(-1)\left[\frac{16}{5}\right]$

$= -16 + 5 \cdot 4$

$= 4$

5) −16 (−4
−20
4

Topic Name: Mathematical analysis (7th class) Day: Tuesday (14th day)

Time: 9.40am Date: 25 / 10 / 22

- congruent eq. কি?

$a \equiv b \pmod{m}$ ~~→ a i~~

যখন m দিয়ে a ও b কে modules করলে একই থাকে

a is congruent to b modulo m

- congruent এর কিছু ques আসবে, math solve

42 — Phi and mu সম্পর্কে কি?

Mathmetical analysis (8th class) Monday (18th day) 9.40 am / 31/10/22

- Eulerc function

prove পড়তে হবে।

Ex:

18 ---

} Antor পড়াইছে।

Chapterc-5

- Binomial co-efficient
- Pascal triangle

} নিজেরা পড়বা।

• Binomial co-efficients কি?

Chapter-6

• Special number কি?

6.1

• Stirling numbers কি?

Types — 2 types

$\left\{ {n \atop k} \right\}$ 2nd kind

$\left\{ {n \atop k} \right\}$ → n number of total things into k nonempty subset

$\left[{n \atop k} \right]$ 1st kind

$\left[{n \atop k} \right]$ → n objects into k cycles instead of subsets

Table 258

Stirling triangle for subsets:

৪টা item

$\left\{ {4 \atop 2} \right\} = 7$

subset হবে ২টো (subset must nonempty)

suppose, $n = \{1, 2, 3, 4\}$
এর থেকে ২টা subset বানাতে হবে but subset must be non empty.

$\{1\} \cup \{2, 3, 4\}$, red → subset 2 (K)
$\{2\} \cup \{1, 3, 4\}$ — 1, 2, 3, 4 → n (item)
$\{3\} \cup \{1, 2, 4\}$
$\{4\} \cup \{1, 2, 3\}$
$\{1, 2\} \cup \{3, 4\}$
$\{1, 3\} \cup \{2, 4\}$
$\{1, 4\} \cup \{2, 3\}$

$\Rightarrow 2^{n-1} = 2^{4-1}$
$= 2^3 = 8$ (৪ হবে কারণ ২ যখন ভাগ set গুলো থেকে আলাদা যেটা বানাই empty set নিয়ে যেমন {1,2,3,4},{})

How we can derive this equation?
হল 2^{n-1}.

Topic Name: Mathematical analysis (9th class) Day: Tuesday (9th day)
Time: 10 am Date: 1 / 11 / 22

- $\left\{ {4 \atop 3} \right\} = 6$

$n = \{1,2,3,4\}$

~~$\{1\} \cup \{2,3,4\}$~~

$+ \{1\} \cup \{2\} \cup \{3,4\}$

$\{1\} \cup \{3\} \cup \{2,4\}$

$\{1\} \cup \{4\} \cup \{2,3\}$

$\{2\} \cup \{3\} \cup \{1,4\}$

$\{2\} \cup \{4\} \cup \{1,3\}$

$\{3\} \cup \{4\} \cup \{1,2\}$

- $\left\{ {n \atop k} \right\} = K \left\{ {n-1 \atop k} \right\} + \left\{ {n-1 \atop k-1} \right\}$; integer $n \neq 0$

$\left\{ {4 \atop 3} \right\} = 3 \left\{ {3 \atop 3} \right\} + \left\{ {3 \atop 2} \right\}$

$= 3 \times 1 + 2 \left\{ {2 \atop 2} \right\} + \left\{ {2 \atop 1} \right\}$

$\left\{ {2 \atop 2} \right\}$ এবং $\left\{ {2 \atop 1} \right\}$ → k এবারার 1 সমান always 1

$= 3 + 2 + 1$

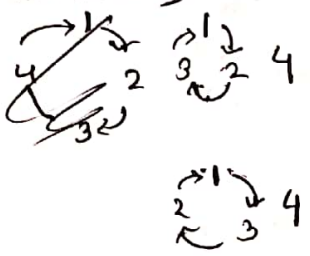
$= 6$

Topic Name: Mathematical analysis (9th class) Day: Tuesday (19th day)
Time: 10am Date: 1 11 '22

• 1st kind $\begin{bmatrix} n \\ k \end{bmatrix}$

• $\begin{bmatrix} 4 \\ 2 \end{bmatrix}$ = no of different cycle (disjoint) =11

[1,2,3][4], [1,3,4][2], [1,2,4][3], [2,3,4][1],
[1,3,2][4], [1,4,3][2], [1,4,2][3], [2,4,3][1],
[1,2][3,4], [1,3][2,4], [1,4][2,3],



Mathematical analysis (10th class) Wednesday (20th day)
10.45 am / 2111/22

$$\begin{bmatrix} n \\ k \end{bmatrix} = (n-1)\begin{bmatrix} n-1 \\ k \end{bmatrix} + \begin{bmatrix} n-1 \\ k-1 \end{bmatrix} \quad \text{integer } n \geq 0$$

$$\begin{bmatrix} 6 \\ 2 \end{bmatrix} = (6-1)\begin{bmatrix} 5 \\ 2 \end{bmatrix} + \begin{bmatrix} 5 \\ 1 \end{bmatrix}$$

$$= 5\left\{(5-1)\begin{bmatrix} 4 \\ 2 \end{bmatrix} + \begin{bmatrix} 4 \\ 1 \end{bmatrix}\right) + (5-1)\begin{bmatrix} 4 \\ 1 \end{bmatrix} + \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$

$$+4\left(3\begin{bmatrix} 3 \\ 1 \end{bmatrix} + \begin{bmatrix} 3 \\ 0 \end{bmatrix}\right) + 0$$

6.2 Eulerian numbers → Total no. of elements

$$\left\langle\!\left\langle \begin{matrix} n \\ k \end{matrix} \right\rangle\!\right\rangle$$ → no of ascents

$$= \quad k_i < k_{i+1}$$

12<3 — 2 ascent
1<3>2 — 1 "
2>1<3 — 1 "
2<3>1 — 1 "
3>1<2 — 1 "
3>2>1 — 0 "

Topic Name: ____ Day: ____
Time: Date: / /

$\left\langle {3 \atop 0} \right\rangle = 1$ (আগের page এ 0 no. of ascents সেট আছে তাই 1)

$\left\langle {3 \atop 1} \right\rangle = 4$ (আগের page এ 1 no. of 〃 4টা 〃 〃 4)

$\left\langle {3 \atop 2} \right\rangle = 1.$ (〃 〃 〃 2 〃 and 〃 〃 1)

$\{1, 2, 3, 4\}$

$1<3>2<4 - 2$

$1<4>2<3 - 2$

$1<2<4>3 - 2$

$1<3<4>2 - 2$

$2<3>1<4 - 2$

$2<4>1<3 - 2$

$2<3<4>1 - 2$

$2>1<3<4 - 2$

$3<4>1<2 - 2$

$3>1<2<4 - 2$

$4>1<2<3 - 2.$

$\left\langle {4 \atop 2} \right\rangle = \underline{11}$

Second order Eulerian triangle

$$\left\langle\!\left\langle {2 \atop 1} \right\rangle\!\right\rangle = 2$$

11 22

$11 < 22$ ⎤
$2 > 1 = 1 < 2$ ⎦ → (2)

6.3 → Harmonic number → পড়া

6.4 কয়েকটা summation

6.5 - Bernoulli numbers

- simple definition + example (short ques)
- short note type আসবে explain লাগবে
- Eulerian triangle থেকে কোনো কিছু আসে
- Equation derive করতে আসে / prove করতে আসে
- Derive the second order Eulerian triangle
- Mathematical problem আসতে আসে – বড় $\left\langle {4 \atop 2} \right\rangle$ →
- এই chapter থেকে মাত্র একটা ques আসবে

6.3

· What is the harmonic number?

$$H_n = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} = \sum_{k=1}^{n} \frac{1}{k}$$

n	0	1	2	3	4	5	6	7	8	9	10
	0	1	$\frac{3}{2}$	$\frac{11}{6}$	$\frac{25}{12}$	$\frac{137}{60}$	$\frac{49}{20}$	$\frac{363}{140}$	$\frac{761}{280}$	$\frac{7129}{2520}$	$\frac{7381}{2520}$

$1 + \frac{1}{2}$

6.5 Bernoulli numbers

Special number → ~~stir~~
1st kind / 2nd kind (stirling)

Binomial pascal; triangle

- Explain the stirling no. ~~of~~ of 1st kind / 2nd kind
- Derive " " eqn.
- Derive করতে দিতে পারে এবং একটা কাজ। নাও হতে পারে

Ans হবে।

- Direct value দিবে। আবার Math solve
- 4, 3 হলে তাহ stirling no. কত?

- Eulerian no.
- Harmonic
- Bernoulli

$1 + \frac{1}{2} = \frac{2+1}{2} = \frac{3}{2}$

$\frac{3}{2} + \frac{1}{3} = \frac{3 \times 3 + 2}{6}$

8 week off chilo

Topic Name : Mathematical analysis (11th class)

Day: Tuesday (9th week)

Time: 9.40am Date: 22/11/22

Probablity statics বইয়ের chapter-1,2 পড়া

Topic Name: Mathematical analysis (12th class) Day: Wednesday (9th week)
Time: 10.45 am Date: 30/11/22

Introduction to probability theory

1.2

→ Sample space and events কি?

↓ যতগুলো result আসতে পারে

↳ যে ঘটনা ঘটবে (coin মারলে H আসবে H হচ্ছে event)

$S = \{H, T\}$ → (coin মারলে H অথবা T আসতে পারে সেই sample space)

↳ sample space

→ some examples → বহুত হবে

→ Two coins →

$S = \{(H,H), (H,T), (T,H), (T,T)\}$

↳ sample space (Two coins outcome)

5. If the experiment consists of measuring the lifetime of a car, then the sample space consists of all non-negative real numbers. (মানুষ মারা যাবে life time. যেহেতু time ঋণাত্মক হবে না এবং infinity দিয়া হইতে পারে) That is —

→ $S = [0, \infty]$

Event: sub set of sample space

→ Two dice flips. make event — where the value of First dice is greater than 3 and some of the two dice is greater than 6.

$S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6),$
$(2,1), (2,2), (2,3), (2,4), (2,5), (2,6),$
$(3,1), (3,2), (3,3), (3,4), (3,5), (3,6),$
$(4,1), (4,2), (4,3), (4,4), (4,5), (4,6),$
$(5,1), (5,2), (5,3), (5,4), (5,5), (5,6),$
$(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$ = 15 event

→ Union
→ Intersection } (বুঝতে হবে)

1.3. Probabilities defined on events

- conditions →

i) $0 \leq P(E) \leq 1$ [1 এর সমান বা ছোট হবে]

ii) $P(S) = 1$ [Full sample space probability 1]

iii) For any sequence of events $E_1, E_2 \dots$ that are mutually exclusive; that is, events for which $E_n E_m = \phi$ where $n \neq m$, then

(mutually exclusive ↳ common থাকবে না)

$$P\left(\bigcup_{n=1}^{\infty} E_n\right) = \sum_{n=1}^{\infty} P(E_n)$$

$$S = \{(H,H), (H,T), (T,H), (T,T)\}$$
(E_1, E_2, E_3, E_4)

$E_1 = \frac{1}{4}$ (probability)

$E_2 = \frac{1}{4}$

$E_3 = \frac{1}{4}$

$E_4 = \frac{1}{4}$

যদি E_1 ও E_2 নেওয়া হয়,

তবে, $E_1 . E_2 = \frac{1}{2}$

$E_1 . E_2 = \frac{1}{4} + \frac{1}{4}$

$= \frac{2}{4} = \frac{1}{2}$

Ex. 1.1 করতে হবে

Ex. 1.2, Ex. 1.3 } অনুরূপ গুলো হবে

→ Remark - page-13 (slide) → সব equation

1.4 Conditional probabilities

$P(E|F)$

$$\Rightarrow P(E|F) = \frac{P(EF)}{P(F)}$$

Topic Name : Mathematical analysis (12th class)

Day : Wednesday (9th week)

Time : 11.30pm Date : 30 / 11 / 22

Ex – 1.4, 1.5, 1.6, 1.7.

chapter – (অধ্যায়)

Topic Name: Mathematical analysis (18th class) Day: Wednesday (10th week)
Time: 11 am Date: 7 / 12 / 22

Chapter-1
Page-15

Exercise: 1
Replace করলে এটা হবে

Ans: $S = \{RR, RG, RB, GG, GR, GB, BB, BR, BG\}$

replace না করলে

$S = \{RG, RB, GR, GB, BR, BG\}$

Exercise: 2

$\frac{1}{9}$.

Chapter-2 (Random variable).

Example → 2.1, 2.3

Topic Name: Mathematical analysis (14th class) Day: Monday (11th week)

Time: 10am Date: 12/12/22

chapter-2 (probability models)

Page-26 → cdf equation

" -27 → 2.2 (Discrete random variables)

important: what is probability mass function?

2.2.1 → Bernoulli random just definition (Ex-2.6, 2.7).

2.2.3 → Geometric - definition

2.2.4 → poisson → "

2.3 → continuous → "

2.3.1 → Uniform → " (Ex-2.13, 2.14)

2.3.2 → Exponential random → definition

2.3.4 → Normal random → "

2.4 → Expectation

2.4.1 → The discrete case (Ex 2.15, 2.16)

2.4.2 → The continuous case

2.4.3 → Example-2.23, Expectation of a random

2.8 → Stochastic processes

Maths → Exercise → 1, 2,

Topic Name : Mathematical analysis (15th class) Day: Tuesday (11th week)

Time: 10am Date: 13 / 12 / 22

chapter-3

3.1 → [illegible]

3.2 → The discrete case (Ex- 3.1, 3.2)

3.3 → The continuous case (Ex- 3.5 (slide) / 3.6 (বই), দুইটা same.

3.4 → Reading task

3.4.1 → computing variances

3.5 → computing probability

3.6.1 → A list model

3.6.2 → A random graph

Exercise → 3.1

chapter-4 Markov chain [illegible]

Topic Name : Mathematical analysis (16th class)

Day : wednesday (12th week)

Time : 11 am Date : 21 / 12 / 22

Chapter-4

Markov chains

→ Introduction - 4.1

→ Ex-4.1, 4.4

Ex-4.3, 4.8, 4.9

→ Classification of states - 4.3

Mathematical analysis (17th class)

Monday (13th week)
9 40am / 26/12/22

Chap - 5

5.1, 5.2, Exp. Dis for properties → only main ones (5.2.2)

5.3, 5.3.1 (From to definition → property ग्रहण)

5.34 (properties), software reliability

5.4.2 (compound poisson with example)

Chapter - 6

6.1, Birth & Death model details (6.3**) with Ex.

6.2 with example, 6.4