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Name: Ranjana Karmaker

Roll no: 2609 Reg. no: 689

1) Solve the system of linear equations us[ing] Cholesky's factorization method

$$2x-6y+8z=24$$
$$5x+4y-3z=2$$
$$3x+y+2z=16$$

Soln: The system can be written as,

$$AX=B$$

where $A=\begin{bmatrix}2 & -6 & 8\\5 & 4 & -3\\3 & 1 & 2\end{bmatrix}$, $X=\begin{bmatrix}x\\y\\z\end{bmatrix}$, $B=\begin{bmatrix}24\\2\\16\end{bmatrix}$

Let, $A=LU$

where,

$L=\begin{bmatrix}1 & 0 & 0\\l_{21} & 1 & 0\\l_{31} & l_{32} & 1\end{bmatrix}$ and $U=\begin{bmatrix}u_{11} & u_{12} & u_{13}\\0 & u_{22} & u_{23}\\0 & 0 & u_{33}\end{bmatrix}$

$$A=\begin{bmatrix}1 & 0 & 0\\l_{21} & 1 & 0\\l_{31} & l_{32} & 1\end{bmatrix}\cdot\begin{bmatrix}u_{11} & u_{12} & u_{13}\\0 & u_{22} & u_{23}\\0 & 0 & u_{33}\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ l_{21}u_{11} & l_{21}u_{12}+u_{22} & l_{21}u_{13}+u_{23} \\ l_{31}u_{11} & l_{31}u_{12}+l_{32}u_{22} & l_{31}u_{13}+l_{32}u_{23}+u_{33} \end{bmatrix} = \begin{bmatrix} 2 & -6 & 8 \\ 5 & 4 & -3 \\ 3 & 1 & 2 \end{bmatrix}$$

$\therefore u_{11} = 2 \qquad l_{21} = \frac{5}{2} \qquad u_{23} = -23$

$u_{12} = -6 \qquad u_{22} = 19 \qquad l_{32} = \frac{10}{19}$

$u_{13} = 8 \qquad l_{31} = \frac{3}{2} \qquad u_{33} = \frac{40}{19}$

$$\therefore U = \begin{bmatrix} 2 & -6 & 8 \\ 0 & 19 & -23 \\ 0 & 0 & \frac{40}{19} \end{bmatrix} \text{ and } L = \begin{bmatrix} 1 & 0 & 0 \\ \frac{5}{2} & 1 & 0 \\ \frac{3}{2} & \frac{10}{19} & 1 \end{bmatrix}$$

$AX = B$

$\Rightarrow LUX = B$

Let, $UX = Y$ where, $Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$

$\therefore LY = B$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ \frac{5}{2} & 1 & 0 \\ \frac{3}{2} & \frac{10}{19} & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 24 \\ 2 \\ 16 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} y_1 \\ \frac{5}{2}y_1 + y_2 \\ \frac{3}{2}y_1 + \frac{10}{19}y_2 + y_3 \end{bmatrix} = \begin{bmatrix} 24 \\ 2 \\ 16 \end{bmatrix}$$

$y_1 = 24$

$y_2 = -58$

$y_3 = \frac{200}{19}$

$$Y = \begin{bmatrix} 24 \\ -58 \\ \frac{200}{19} \end{bmatrix}$$

$\therefore Ux = Y$

$$\Rightarrow \begin{bmatrix} 2 & -6 & 8 \\ 0 & 19 & 23 \\ 0 & 0 & \frac{40}{19} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 24 \\ -58 \\ \frac{200}{19} \end{bmatrix}$$

$\therefore \; 2x - 6y + 8z = 24$

$19y - 23z = -58$

$\frac{40}{19} z = \frac{200}{19}$

Solving these,

$z = 5$

$y = 3$

$x = 1$

2. Solve the following equations of by Cholesky's factorisation method:

$$2x+y+4z=12$$
$$8x-3y+2z=20$$
$$4x-11y-z=33$$

Solnº: The system can be written as,

$AX=B$ where,

$$A=\begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix}, X=\begin{bmatrix} x \\ y \\ z \end{bmatrix}, B=\begin{bmatrix} 12 \\ 20 \\ 33 \end{bmatrix}$$

Let, $A=LU$ where,

$$L=\begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \quad U=\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$LU=A$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}=\begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ l_{21}U_{11} & l_{21}U_{12}+U_{22} & l_{21}U_{13}+U_{23} \\ l_{31}U_{11} & l_{31}U_{12}+l_{32}U_{22} & l_{31}U_{13}+l_{32}U_{23}+U_{33} \end{bmatrix} = \begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix}$$

$U_{11}=2 \quad l_{21}=4 \quad l_{31}=2$

$U_{12}=1 \quad U_{22}=-3-l_{21}U_{12}=-3-4=-7$

$U_{13}=4 \quad U_{23}=2-l_{21}U_{13}=2-16=-14$

$2\times 1 + l_{32}\times -7 = 11$

$\Rightarrow l_{32} = -\frac{9}{7}$

$8+18+U_{33}=-1$

$\Rightarrow U_{33}=-27$

$\therefore LUX=B$

$\Rightarrow LY=B$ [where $UX=Y$]

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 2 & -\frac{9}{7} & 1 \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 20 \\ 33 \end{bmatrix}$$

$\Rightarrow Y_1 = 12$

$4Y_1 + Y_2 = 20$

$\Rightarrow Y_2 = -28$

$2Y_1 - \frac{9Y_2}{7} + Y_3 = 33$

$\Rightarrow Y_3 = -27$

NOW, $UX = Y$

$$\Rightarrow \begin{bmatrix} 2 & 1 & 4 \\ 0 & -7 & -14 \\ 0 & 0 & -27 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ -28 \\ -27 \end{bmatrix}$$

$\therefore z = 1$

$-7y - 14z = -28$

$\Rightarrow -7y = -14$

$\therefore y = 2$

$2x + y + 4z = 12$

$\Rightarrow 2x + 2 + 4 = 12$

$\Rightarrow x = 3$

$\therefore x, y, z = 3, 2, 1.$