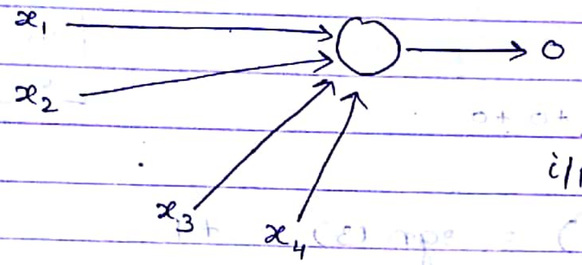
Hebbian learning problem.
$\bar{c}$ binary and continuous activation function.

SONAL

Assume the n/w shown below.



i/p $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$

initial weight vector

$$w' = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0.5 \end{bmatrix}$$

needs to be trained using set of 3 i/p vectors as,

$$x_1 = \begin{bmatrix} 1 \\ -2 \\ 1.5 \\ 0 \end{bmatrix} \quad x_2 = \begin{bmatrix} 1 \\ -0.5 \\ -2 \\ -1.5 \end{bmatrix} \quad x_3 = \begin{bmatrix} 0 \\ 1 \\ -1 \\ 1.5 \end{bmatrix}$$

for an arbitrary choice of learning constant $c=1$.
Find the final wt. after one iteration of training using
a) Bipolar binary activation function.
b) Bipolar continuous activation function.

Soln :-

a) Bipolar Binary activation function -

Step 1 :-

$$net^1 = w^{1t} \cdot x_1 = [1 \ -1 \ 0 \ 0.5] \begin{bmatrix} 1 \\ -2 \\ 1.5 \\ 0 \end{bmatrix}$$

$$= 1 + 2 + 0 + 0$$

$$= 3$$

$$o^1 = f(net^1) = sgn(3) = +1$$

$$\therefore \Delta w^1 = c \cdot r \cdot x_1 \qquad \because r = o^1$$

$$= c \cdot o^1 x_1$$

$$= 1 \cdot 1 \cdot \begin{bmatrix} 1 \\ -2 \\ 1.5 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 1.5 \\ 0 \end{bmatrix}$$

$$\therefore w^2 = w^1 + \Delta w^1$$

$$= \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0.5 \end{bmatrix} + \begin{bmatrix} 1 \\ -2 \\ 1.5 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ 1.5 \\ 0.5 \end{bmatrix}$$

Step 2 :- $net^2 = w^{2t} \cdot x_2 = [2 \ -3 \ 1.5 \ 0.5] \begin{bmatrix} 1 \\ -0.5 \\ -2 \\ -1.5 \end{bmatrix}$

$$= 2 + 1.5 - 3 - 0.75$$

$$= -0.25$$

$$\therefore o^2 = f(net^2) = sgn(-0.25) = -1$$

$$\therefore \Delta w^2 = c \cdot r \cdot x_2$$

$$= 1 \cdot (-1) \cdot \begin{bmatrix} 1 \\ -0.5 \\ 2 \\ -1.5 \end{bmatrix} = \begin{bmatrix} -1 \\ 0.5 \\ 2 \\ 1.5 \end{bmatrix}$$

$$\therefore w^3 = w^2 + \Delta w^2$$

$$= \begin{bmatrix} 2 \\ -3 \\ 1.5 \\ 0.5 \end{bmatrix} + \begin{bmatrix} -1 \\ 0.5 \\ 2 \\ 1.5 \end{bmatrix} = \begin{bmatrix} 1 \\ -2.5 \\ 3.5 \\ 2.0 \end{bmatrix}$$

<u>step 3 :-</u>

$$net^3 = w^{3t} \cdot x_3 = \begin{bmatrix} 1 & -2.5 & 3.5 & 2.0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ -1 \\ 1.5 \end{bmatrix}$$

$$= -2.5 - 3.5 + 3$$

$$= -3$$

$$\therefore o^3 = f(net^3) = sgn(-3) = -1$$

$$\therefore \Delta w^3 = c \cdot o^3 x_3$$

$$= 1 \cdot (-1) \cdot \begin{bmatrix} 0 \\ +1 \\ -1 \\ 1.5 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 1 \\ -1.5 \end{bmatrix}$$

$$\therefore w^4 = w^3 + \Delta w^3$$

$$= \begin{bmatrix} 1 \\ -2.5 \\ 3.5 \\ 2 \end{bmatrix} + \begin{bmatrix} 0 \\ +1 \\ 1 \\ -1.5 \end{bmatrix} = \begin{bmatrix} 1 \\ -3.5 \\ 4.5 \\ 0.5 \end{bmatrix}$$

// final weights.

b) Bipolar continuous activation function:

$$o = \frac{2}{1+e^{-\lambda net}} - 1$$

Let $\lambda = 1$

Step 1:-

$$net^1 = w^{1t} x_1 = \begin{bmatrix} 1 & -1 & 0 & 0.5 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 1.5 \\ 0 \end{bmatrix}$$

$$= 3$$

$$\therefore o^1 = \frac{2}{1+e^{-\lambda net^1}} - 1$$

$$= \frac{2}{1+e^{-3}} - 1$$

$$= 0.905$$

$$\therefore \Delta w^1 = c \cdot r \cdot x = c \cdot o^1 \cdot x_1$$

$$= 1 \cdot 0.905 \cdot \begin{bmatrix} 1 \\ -2 \\ 1.5 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0.905 \\ -1.81 \\ 1.3575 \\ 0 \end{bmatrix}$$

$$\therefore w^2 = w^1 + \Delta w^1$$

$$= \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0.5 \end{bmatrix} + \begin{bmatrix} 0.905 \\ -1.81 \\ 1.3575 \\ 0 \end{bmatrix} = \begin{bmatrix} 1.905 \\ -2.81 \\ 1.3575 \\ 0.5 \end{bmatrix}$$

step 2 :- $net^2 = w^{2t} \cdot x_2 = [1.905 \quad -2.81 \quad 1.3575 \quad 0.5] \begin{bmatrix} 1 \\ -0.5 \\ -2 \\ -1.5 \end{bmatrix}$

$= -0.154$

$o^2 = \dfrac{2}{1+e^{0.154}} - 1$

$= \cancel{-0.077} \quad -0.0768$

$\therefore \Delta w^2 = c \cdot o^2 \cdot x_2 = (1)(-0.0768) \cdot \begin{bmatrix} 1 \\ -0.5 \\ -2 \\ -1.5 \end{bmatrix}$

$= \begin{bmatrix} -0.0768 \\ 0.0384 \\ 0.1536 \\ 0.1152 \end{bmatrix}$

$\therefore w^3 = w^2 + \Delta w^2 = \begin{bmatrix} 1.905 \\ -2.81 \\ 1.3575 \\ 0.5 \end{bmatrix} + \begin{bmatrix} -0.0768 \\ 0.0384 \\ 0.1536 \\ 0.1152 \end{bmatrix} = \begin{bmatrix} 1.8282 \\ -2.7716 \\ 1.5111 \\ 0.6152 \end{bmatrix}$

step 3 :-

$net^3 = w^{3t} x_3 = [1.8282 \quad -2.7716 \quad 1.5111 \quad 0.6152] \cdot \begin{bmatrix} 0 \\ 1 \\ -1 \\ 1.5 \end{bmatrix}$

$= -3.3599$

$o^3 = \dfrac{2}{1+e^{3.3599}} - 1 = -0.9328$

$\therefore \Delta w^3 = c \cdot o^3 \cdot x_3 = (1) \cdot (-0.9328) \cdot \begin{bmatrix} 0 \\ 1 \\ -1 \\ 1.5 \end{bmatrix} = \begin{bmatrix} 0 \\ -0.9328 \\ 0.9328 \\ -1.3992 \end{bmatrix}$

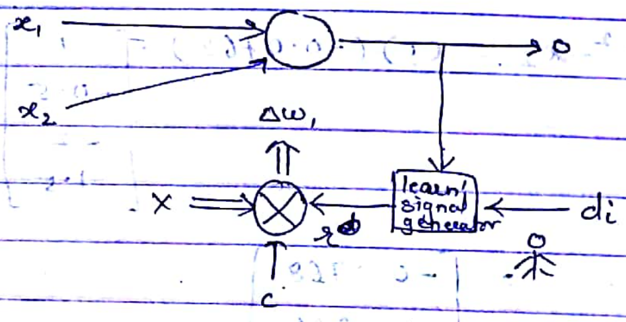
$\therefore w^4 = w^3 + \Delta w^3 = \begin{bmatrix} 1.8282 \\ -2.7716 \\ 1.5111 \\ 0.6152 \end{bmatrix} + \begin{bmatrix} 0 \\ -0.9328 \\ 0.9328 \\ -1.3992 \end{bmatrix} = \begin{bmatrix} 1.8282 \\ -3.7044 \\ 2.4439 \\ -0.784 \end{bmatrix}$

Q. Zurada P2.16

Four steps of Hebbian learning of a single neuron network as shown below have been implement starting ε $w^1 = [1 \ \ -1]^t$ for $c = 1$.

using inputs -

$$x_1 = \begin{bmatrix} 1 \\ -2 \end{bmatrix} \quad x_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad x_3 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \quad x_4 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$



Sol^n:- A) weights for bipolar binary f(net)

Step I

i) $net^1 = w^{1t} \cdot x_1 = [1 \ \ -1] \cdot \begin{bmatrix} 1 \\ -2 \end{bmatrix} = 3$

$o^1 = f(net^1) = sgn(3) = +1$

$\Delta w^1 = c \cdot o^1 x_1$

$= (1) \cdot (1) \cdot \begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

$\therefore w^2 = w^1 + \Delta w^1$

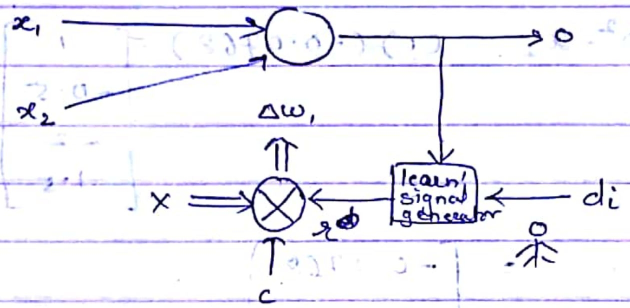
$= \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$

Q. Zurada P2.16

Four steps of Hebbian learning of a single neuron network as shown below have been implemented starting $\bar{c}$ $w^1 = [1 \ \ -1]^t$ for $c = 1$.

using inputs -

$$x_1 = \begin{bmatrix} 1 \\ -2 \end{bmatrix} \quad x_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad x_3 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \quad x_4 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$



Sol^n:- A) weights for bipolar binary f(net)

Step I

i) $net^1 = w^{1t} \cdot x_1 = [1 \ \ -1] \cdot \begin{bmatrix} 1 \\ -2 \end{bmatrix} = 3$

$o^1 = f(net^1) = sgn(3) = +1$

$\Delta w^1 = c \cdot r \cdot x = c \cdot o^1 x_1$

$= (1) \cdot (1) \cdot \begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

$\therefore w^2 = w^1 + \Delta w^1$

$= \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$

ii) $net^2 = w^{2t} \cdot x_2 = [2 \quad -3] \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = -3$

$\therefore o^2 = sgn(-3) = -1$

$\therefore \Delta w^2 = c \cdot r \cdot x = c \cdot o^2 \cdot x_2$

$= (1) \cdot (-1) \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$

$\therefore w^3 = w^2 + \Delta w^2$

$= \begin{bmatrix} 2 \\ -3 \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \\ -4 \end{bmatrix}$

iii) $net^3 = w^{3t} \cdot x_3 = [2 \quad -4] \cdot \begin{bmatrix} 2 \\ 3 \end{bmatrix} = 4 - 12 = -8$

$\therefore o^3 = -1$

$\therefore \Delta w^3 = c \cdot o^3 \cdot x_3 = (1) \cdot (-1) \cdot \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} -2 \\ -3 \end{bmatrix}$

$\therefore w^4 = w^3 + \Delta w^3$

$= \begin{bmatrix} 2 \\ -4 \end{bmatrix} + \begin{bmatrix} -2 \\ -3 \end{bmatrix} = \begin{bmatrix} 0 \\ -7 \end{bmatrix}$

iv) $net^4 = w^{4t} \cdot x_4 = [0 \quad -7] \cdot \begin{bmatrix} 1 \\ -1 \end{bmatrix} = 7$

$\therefore o^4 = +1$

$\therefore \Delta w^4 = c \cdot o^4 \cdot x_4 = (1) \cdot (1) \cdot \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$\therefore w^5 = w^4 + \Delta w^4$

$= \begin{bmatrix} 0 \\ -7 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ -8 \end{bmatrix}$

Step II

i) $net^5 = w^{5t} \cdot x_1 = [1 \quad -8] \cdot \begin{bmatrix} 1 \\ -2 \end{bmatrix} = 1 + 16 = 17$

$\therefore o^5 = +1$

$\Delta w^5 = c \cdot o^5 \cdot x_1 = (1) \cdot (1) \cdot \begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

$\therefore w^6 = w^5 + \Delta w^5$

$= \begin{bmatrix} 1 \\ -8 \end{bmatrix} + \begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 2 \\ -10 \end{bmatrix}$

ii) $net^6 = w^{6t} \cdot x_2 = [2 \quad -10] \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = -10$

$\therefore o^6 = -1$

$\therefore \Delta w^6 = c \cdot o^6 \cdot x_2 = (1)(-1) \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$

$\therefore w^7 = w^6 + \Delta w^6$

$= \begin{bmatrix} 2 \\ -10 \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \\ -11 \end{bmatrix}$

iii) $net^7 = w^{7t} \cdot x_3 = [2 \quad -11] \cdot \begin{bmatrix} 2 \\ 3 \end{bmatrix} = 4 - 33 = -29$

$\therefore o^7 = -1$

$\therefore \Delta w^7 = c \cdot o^7 \cdot x_3 = (1)(-1) \cdot \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} -2 \\ -3 \end{bmatrix}$

$\therefore w^8 = w^7 + \Delta w^7 = \begin{bmatrix} 2 \\ -11 \end{bmatrix} + \begin{bmatrix} -2 \\ -3 \end{bmatrix} = \begin{bmatrix} 0 \\ -14 \end{bmatrix}$

iv) $net^8 = w^{8t} \cdot x_4 = [0 \quad -14] \cdot \begin{bmatrix} 1 \\ -1 \end{bmatrix} = 0 + 14 = 14$

$o^8 = +1$

$\therefore \Delta w^8 = c \cdot o^8 \cdot x_4 = (1)(1) \cdot \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$\therefore w^9 = w^8 + \Delta w^8 = \begin{bmatrix} 1 \\ -15 \end{bmatrix}$

Step III

i) $net^9 = w^{9t} \cdot x_1 = [1 \quad -15] \cdot \begin{bmatrix} 1 \\ -2 \end{bmatrix} = 1 + 30 = 31$

$\therefore o^9 = +1 \quad \therefore \Delta w^9 = (1) \cdot (1) \cdot \begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

$\therefore w^{10} = \begin{bmatrix} 1 \\ -15 \end{bmatrix} + \begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 2 \\ -17 \end{bmatrix}$

ii) $net^{10} = w^{10t} \cdot x_2 = [2 \quad -17] \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = 0 - 17 = -17$

$\therefore o^{10} = -1 \quad \therefore \Delta w^{10} = (1) \cdot (-1) \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$

$\therefore w^{11} = \begin{bmatrix} 2 \\ -17 \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \\ -18 \end{bmatrix}$

iii) $net^{11} = w^{11t} \cdot x_3 = [2 \quad -18] \cdot \begin{bmatrix} 2 \\ 3 \end{bmatrix} = 4 - 54 = -50$

$\therefore o^{11} = -1 \quad \therefore \Delta w^{11} = (1)(-1) \cdot \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} -2 \\ -3 \end{bmatrix}$

$\therefore w^{12} = \begin{bmatrix} 0 \\ -21 \end{bmatrix}$

iv) $net^{12} = w^{12t} \cdot x_4 = [0 \quad -21] \cdot \begin{bmatrix} 1 \\ -1 \end{bmatrix} = 21$

$\therefore o^{12} = +1 \quad \therefore \Delta w^{12} = (1)(1) \cdot \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$\therefore w^{13} = \begin{bmatrix} 1 \\ -22 \end{bmatrix}$.

B) weights for bipolar ~~binary~~ continuous f(net), $\lambda = 1$

step I:

i) ~~net~~ = $\frac{1?}{1+e^{?}}$

i) $net^1 = w^{1t} \cdot x_1 = [1 \quad -1] \cdot \begin{bmatrix} 1 \\ -2 \end{bmatrix} = 3$

$O^1 = \frac{2}{1+e^{-3}} - 1 = 0.905$

$\Delta w^1 = c \cdot O^1 \cdot x_1 = (1) \cdot (0.905) \cdot \begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 0.905 \\ -1.81 \end{bmatrix}$

$\therefore w^2 = w^1 + \Delta w^1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \begin{bmatrix} 0.905 \\ -1.81 \end{bmatrix} = \begin{bmatrix} 1.905 \\ -2.81 \end{bmatrix}$

ii) $net^2 = w^{2t} \cdot x_2$

$= [1.905 \quad -2.81] \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = -2.81$

$\therefore O^2 = \frac{2}{1+e^{2.81}} - 1 = -0.8864$

$\therefore \Delta w^2 = c \cdot O^2 \cdot x_2 = (1) \cdot (-0.8864) \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -0.8864 \end{bmatrix}$

$\therefore w^3 = \begin{bmatrix} 1.905 \\ -2.81 \end{bmatrix} + \begin{bmatrix} 0 \\ -0.8864 \end{bmatrix} = \begin{bmatrix} 1.905 \\ -3.6964 \end{bmatrix}$

iii) $net^3 = w^{3t} \cdot x_3$

$= [1.905 \quad -3.6964] \cdot \begin{bmatrix} 2 \\ 3 \end{bmatrix} = -7.2792$

$\therefore O^3 = \frac{2}{1+e^{7.2792}} - 1 = -0.9986$

$\therefore \Delta w^3 = (1) \cdot (-0.9986) \cdot \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} -1.9972 \\ -2.9958 \end{bmatrix}$

$\therefore w^4 = \begin{bmatrix} -0.0922 \\ -6.6922 \end{bmatrix}$.

iv) $net^4 = w^{4t} \cdot x_4 = [-0.0922 \quad -6.6922] \begin{bmatrix} 1 \\ -1 \end{bmatrix} = 6.6$

$\therefore o^4 = \dfrac{2}{1+e^{-6.6}} - 1 = 0.9973$

$\therefore \Delta w^4 = (1) \cdot (0.9973) \cdot \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0.9973 \\ -0.9973 \end{bmatrix}$

$\therefore w^5 = \begin{bmatrix} 0.9051 \\ -7.6895 \end{bmatrix}$

step II i) $net^5 = w^{5t} \cdot x_1 = [0.9051 \quad -7.6895] \begin{bmatrix} 1 \\ -2 \end{bmatrix} = 16.2841$

$\therefore o^5 = \dfrac{2}{1+e^{-16.2841}} - 1 = 0.9999 \approx 1$

$\Delta w^5 = c \cdot o^5 \cdot x_1 = (1) \cdot (0.9999) \begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

$\therefore w^6 = w^5 + \Delta w^5 = \begin{bmatrix} 0.9051 \\ -7.6895 \end{bmatrix} + \begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 1.9051 \\ -9.6895 \end{bmatrix}$

ii) $net^6 = w^{6t} \cdot x_2 = [1.9051 \quad -9.6895] \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = -9.6895$

$\therefore o^6 = \dfrac{2}{1+e^{9.6895}} - 1 = -0.9998 \approx -1$

$\Delta w^6 = c \cdot o^6 \cdot x_2 = (1) \cdot (-0.9998) \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$

iii) $net^7 = w^{7t} \cdot x_3 =$

$\therefore w^7 = w^6 + \Delta w^6 = \begin{bmatrix} 1.9051 \\ -9.6895 \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 1.9051 \\ -10.6895 \end{bmatrix}$

iii) $net^7 = w^{7t} \cdot x_3 = [1.9051 \;\; -10.6895] \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

$= -28.2583$

$\therefore o^7 = \dfrac{2}{1+e^{28.2583}} - 1 = -1$

$\therefore \Delta w^7 = c \cdot o^7 \cdot x_3 = (1)(-1) \cdot \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} -2 \\ -3 \end{bmatrix}$

$\therefore w^8 = w^7 + \Delta w^7 = \begin{bmatrix} 1.9051 \\ -10.6895 \end{bmatrix} + \begin{bmatrix} -2 \\ -3 \end{bmatrix} = \begin{bmatrix} -0.0949 \\ -13.6895 \end{bmatrix}$

iv) $net^8 = w^{8t} \cdot x_4 = [-0.0949 \;\; -13.6895] \cdot \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$= 13.5946$

$\therefore o^8 = \dfrac{2}{1+e^{-13.5946}} - 1 = 0.9999 \approx 1$

$\therefore \Delta w^8 = c \cdot o^8 \cdot x_4 = (1)(1) \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$\therefore w^9 = w^8 + \Delta w^8 = \begin{bmatrix} -0.0949 \\ -13.6895 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0.9051 \\ -14.6895 \end{bmatrix}$

Q. Perceptron learning
Implement the
from fig. 2.23
and the follo
w^1 and the

$w^1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $(x_1$

$(x_2 = \begin{bmatrix} 0 \\ -1 \\ -1 \end{bmatrix}$

sequence $(x_i, d_i$
responses in a
List net^k valu

Sol^n:- i) $net^1 = w^{1t} \cdot x$

$\therefore o^1 = sgn(net^1$

$\therefore \Delta w^1 = c \cdot r \cdot$
$= c \cdot (c$
$= -2c$
$= -2 \cdot$

$\therefore w^2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} +$

ii) $net^2 = w^{2t} \cdot x_2$

$\therefore o^2 = sgn(-1)$

$\therefore \Delta w^2 =$
$= +2($

Q. Perceptron learning P. 2.17 Zurada pg. 87.

Implement the perceptron rule training of the n/w from fig. 2.23 using $f(net) = sgn(net)$, $c = 1$, and the following data specifying the initial weights w^1 and the two training pairs

$$w^1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \left(x_1 = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}, d_1 = -1 \right),$$

$$\left(x_2 = \begin{bmatrix} 0 \\ -1 \\ -1 \end{bmatrix}, d_2 = 1 \right)$$ Repeat the training

sequence (x_1, d_1); (x_2, d_2), until two correct responses in a row are achieved.

List net^k values obtained during training.

Sol^n:-

i) $net^1 = w^{1t} \cdot x_1 = [0 \;\; 1 \;\; 0] \cdot \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} = 1$

$\therefore o^1 = sgn(net^1) = +1$

$\therefore \Delta w^1 = c \cdot r \cdot x$ | $\because d_1 = -1$ & $o^1 = +1$

$= c \cdot (d_1 - o^1) \cdot x_1$ | $\therefore$ update weights.

$= -2 c x_1$

$= -2 \cdot (1) \cdot \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -4 \\ -2 \\ 2 \end{bmatrix}$

$$\therefore w^2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} -4 \\ -2 \\ 2 \end{bmatrix} = \begin{bmatrix} -4 \\ -1 \\ 2 \end{bmatrix}$$

ii) $net^2 = w^{2t} \cdot x_2 = [-4 \;\; -1 \;\; 2] \cdot \begin{bmatrix} 0 \\ -1 \\ -1 \end{bmatrix} = 1 - 2 = -1$

$\therefore o^2 = sgn(-1) = -1$ and $d_2 = 1$ $\quad \therefore$ update weights

$\therefore \Delta w^2 = +2 c x_2$

$= +2(1) \cdot \begin{bmatrix} 0 \\ -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \\ -2 \end{bmatrix}$

4

$\therefore \omega^3 = \omega^2 + \Delta\omega^2$

$$= \begin{bmatrix} -4 \\ -1 \\ 2 \end{bmatrix} + \begin{bmatrix} 0 \\ -2 \\ -2 \end{bmatrix} = \begin{bmatrix} -4 \\ -3 \\ 0 \end{bmatrix}$$

$$net^3 = \omega^{3t} \cdot x_1 = [-4 \quad -3 \quad 0] \cdot \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} = -8 - 3 + 0 = -11$$

$\therefore o^3 = \operatorname{sgn}(-11) = -1$

and $d_1 = -1$

They are same

$\therefore$ No need to update weights.

$\omega^4 = \therefore \Delta\omega^3 = [0 \quad 0 \quad 0]^t$

$\therefore \omega^4 = \omega^3 = [-4 \quad -3 \quad 0]^t$.

→ perform 1 more.

List of net^k values obtained during training -

$net^1 = 1$

$net^2 = -1$

$net^3 = -11$

$net^4 = +3$

Q. Solve the following classification problem using perceptron rule. Apply each input vector in order, for as many repeatations as it takes to ensure that the problem is solved.

$$\left(x_1 = \begin{bmatrix} 2 \\ 2 \end{bmatrix}, d_1 = 0\right), \left(x_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}, d_2 = 1\right)$$

$$\left(x_3 = \begin{bmatrix} -2 \\ 2 \end{bmatrix}, d_3 = 0\right), \left(x_4 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, d_4 = 1\right)$$

$b^0 = 0$, $c = 1$.

Sol^n:-

as values of d are 0 or 1,
$\therefore$ activation function is unipolar function.

i) $net^1 = w^{0t} \cdot x_1 = [0 \;\; 0] \cdot \begin{bmatrix} 2 \\ 2 \end{bmatrix} = 0$

$o^1 = sgn(net^1 + b^0)$
$\quad = sgn(0 + 0)$
$\quad = +1$

compare o^1 & d_1 They are different
update weights.
error $e = d_1 - o^1$
$\quad = 0 - 1 = -1$

$\therefore \Delta w^0 = c \cdot (d_1 - o^1) \cdot x_1$
$\quad = (1) \cdot (-1) \cdot \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} -2 \\ -2 \end{bmatrix}$

$\therefore w^1 = w^0 + \Delta w^0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -2 \\ -2 \end{bmatrix} = \begin{bmatrix} -2 \\ -2 \end{bmatrix}$.

ii) $net^2 = w^{1t} \cdot x_2 = \begin{bmatrix} -2 & -2 \end{bmatrix} \begin{bmatrix} -1 \\ -2 \end{bmatrix} = -2+4 = 2$

$o^2 = sgn(net^2 + b^1)$ $\quad b^1 = b^0 + (d_1 - o^1) = 0 + -1 = -1$

$= sgn(2 - 1)$
$= +1$

compare o^2 and d_2 They are same
∴ No change in weights.

$\therefore w^2 = w^1 = \begin{bmatrix} -2 \\ -2 \end{bmatrix}$

$e = d_2 - o^2 = 0 \quad \therefore b^2 = b^1 + e$

iii $net^3 = w^{2t} \cdot x_3 = \begin{bmatrix} -2 & -2 \end{bmatrix} \begin{bmatrix} -2 \\ 2 \end{bmatrix} = 4 - 4 = 0$

$o^3 = sgn(net^3 + b^2)$
$= sgn(0 - 1) = \cancel{1}$ $\quad \cancel{e = d_3 - o^3 = 0 - (-1)} = 1$
$= sgn(-1) = 0.$

compare o^3 c̄ d_3 They are ~~diff.~~ same
~~∴ update weights~~

~~$\Delta w^3 = \cancel{w^3} + c \cdot (d_3 - o^3) \cdot x_3$~~
~~$= (1) \cdot (1) \cdot \begin{bmatrix} -2 \\ 2 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$~~

$\therefore \cancel{w^4} =$

∴ No updates.

$\therefore w^3 = w^2 = \begin{bmatrix} -2 \\ -2 \end{bmatrix}$ $\quad b^3 = b^2 = -1$

iv) $net^4 = w^{3t} \cdot x_4 = [-2 \ \ -2] \begin{bmatrix} -1 \\ 1 \end{bmatrix} = 2 - 2 = 0$

$\therefore o^4 = sgn(net^4 + b^3)$

$= sgn(0 + -1) = sgn(-1) = 0.$

$d_4 = 1$

$\therefore d_4$ & o^4 are different

$\therefore$ update weights.

error $e = d_4 - o^4 = 1 - 0 = 1$

$\Delta w^3 = c \cdot (d_4 - o^4) \cdot x_4$

$= (1)(1) \cdot \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

$w^4 = w^3 + \Delta w^3 = \begin{bmatrix} -2 \\ -2 \end{bmatrix} + \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ -1 \end{bmatrix}$

$b^4 = b^3 + e = -1 + 1 = 0$

v) $net^5 = w^{4t} \cdot x_1 = [-3 \ \ -1] \cdot \begin{bmatrix} 2 \\ 2 \end{bmatrix} = -6 - 2 = \cancel{-4} \ -8$

$o^5 = sgn(net^5 + b^4)$

$= sgn(-8 + 0)$

$= 0$

compare $o^5 = 0$ c̄ $d_1 = 0$

they are same

$\therefore$ No updates

$w^5 = w^4 = \begin{bmatrix} -3 \\ -1 \end{bmatrix}$

$b^5 = b^4 = 0$

vi) $net^6 = w^{5t} \cdot x_2 = [-3 \quad -1] \begin{bmatrix} 1 \\ -2 \end{bmatrix} = -3+2$
$= -1$

$o^6 = sgn(net^6 + b^5)$
$= sgn(-1+0)$
$= 0$

compare $o^6 = 0$ & $d_2 = 1$
they are diff.
∴ update weights
error $e = d_2 - o^6 = 1 - 0 = 1$

∴ $\Delta w^5 = c \cdot e \cdot x_2$
$= (1)(1) \cdot \begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

∴ $w^6 = w^5 + \Delta w^5 = \begin{bmatrix} -3 \\ -1 \end{bmatrix} + \begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} -2 \\ -3 \end{bmatrix}$

$b^6 = b^5 + e = 0 + 1 = 1$

vii) $net^7 = w^{6t} \cdot x_3 = [-2 \quad -3] \begin{bmatrix} -2 \\ 2 \end{bmatrix} = 4 - 6$
$= -2$

$o^7 = sgn(net^7 + b^6)$
$= sgn(-2+1)$
$= 0$

compare $o^7 = 0$ & $d_3 = 0$
they are same
∴ No updates.

∴ $w^7 = w^6 = \begin{bmatrix} -2 \\ -3 \end{bmatrix}$

$b^7 = b^6 = 1$

viii) $net^8 = w^{7t} x_4 = [-2 \; -3] \begin{bmatrix} -1 \\ 1 \end{bmatrix} = 2 - 3 = -1$

$o^8 = sgn(net^8 + b^7)$

$= sgn(-1+1)$

$= 0$

compare $o^8 = 0$ & $d_4 = 1$

they are diff.

$\therefore$ update weights.

$\therefore$ error $e = d_4 - o^8 = 1 - 0 = 1$

$\Delta w^7 = c \cdot e \cdot x_4$

$= (1) \cdot (1) \cdot \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

$\therefore w^8 = w^7 + \Delta w^7 = \begin{bmatrix} -2 \\ -3 \end{bmatrix} + \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ -2 \end{bmatrix}$

$b^8 = b^7 + e = 1 + 1 = 2$

→ For this method of training, the weights are initialised at any values.

→ also called continuous perceptron training rule.

⇒ Delta learning rule.

- $\Delta w_{ij} = c \cdot (d_i - o_i) f'(net_i) x_j$

$j = 1, 2, \ldots n.$

- initial weights – Any
- Supervised learning
- Neuron char – continuous.
- feed forward.

→ The delta learning rule is valid only for continuous activation function and in supervised training mode.

→ The learning signal for this rule is called 'delta' and is defined as follows -

$$r = [d_i - f(w_i^t x)] \cdot f'(w_i^t x) \quad \text{----(1)}$$

the term $f'(w_i^t x)$ is → derivative of the activation function $f(net)$.

$\because net = w_i^t x.$

→ This learning rule can be easily derived from condition of least squared error between o_i and d_i

$$E = \frac{1}{2}(d_i - o_i)^2 \quad \text{----(2)}$$

OR $$E = \frac{1}{2}[d_i - f(w_i^t x)]^2 \quad \text{---(3)}$$

→ Calculating the gradient vector wrt w_i of the squared error.

$$\nabla E = -(d_i - O_i) f'(w_i^t x) x \quad \cdots (4)$$

→ Since the minimization of the error requires the weight changes to be in the negative gradient direction, we take

$$\Delta w_i = -\eta \nabla E \quad \cdots (5)$$

where,

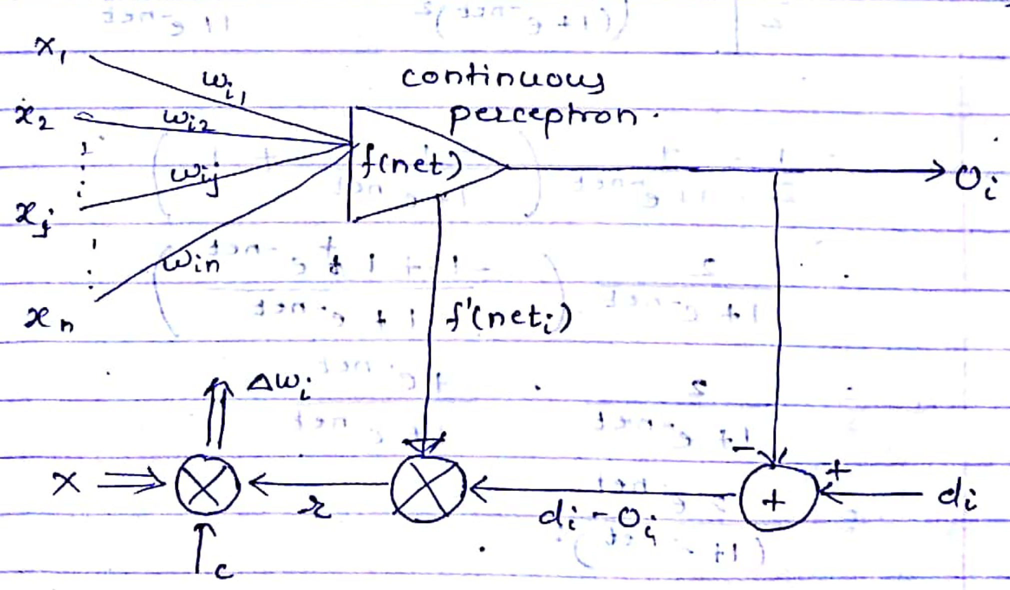
$\eta \rightarrow$ +ve const

from (4) & (5)

$$\Delta w_i = \eta (d_i - O_i) f'(net_i) x \quad \cdots (6)$$

→ Considering the use of general learning rule $\Delta w = c \cdot r \cdot x$ and using the delta learning signal in (1), weight adjustment becomes,

$$\Delta w_i = c (d_i - O_i) f'(net_i) x \quad \cdots (7)$$



Prove that

a) $f'(net) = \frac{1}{2}(1-o^2)$ for bipolar continuous function

b) $f'(net) = o(1-o)$ for unipolar continuous.

Soln: —

a) To prove $f'(net) = \frac{1}{2}(1-o^2)$.

$$R.H.S. = \frac{1}{2}(1-o^2)$$

for bipolar continuous function,

$$O = \frac{2}{1+e^{-\lambda net}} - 1$$

assuming $\lambda = 1$

$$RHS = \frac{1}{2}(1-o^2)$$

$$= \frac{1}{2}\left[1 - \left(\frac{2}{1+e^{-net}} - 1\right)^2\right]$$

$$= \frac{1}{2}\left[1 - \left(\frac{4}{(1+e^{-net})^2} - 2 \cdot \frac{2}{1+e^{-net}} + 1\right)\right]$$

$$= \frac{1}{2} \cdot \frac{4}{1+e^{-net}}\left(\frac{-1}{1+e^{-net}} + 1\right)$$

$$= \frac{2}{1+e^{-net}}\left(\frac{-1+1+e^{-net}}{1+e^{-net}}\right)$$

$$= \frac{2}{1+e^{-net}} \cdot \frac{+e^{-net}}{1+e^{-net}}$$

$$= \frac{2e^{-net}}{(1+e^{-net})^2}.$$

Now: LHS = $f'(net)$

$$f(net) = \frac{2}{1+e^{-net}} - 1$$

$$f'(net) = \frac{d}{dnet}\left(\frac{2}{1+e^{-net}} - 1\right)$$

$$= \frac{d}{dnet}\left(\frac{2}{1+e^{-net}}\right) - 0$$

$$= \frac{1+e^{-net} \cdot \frac{d}{dnet}(2) - 2 \cdot \frac{d}{dnet}(1+e^{-net})}{(1+e^{-net})^2}$$

$$= \frac{-2 \cdot -e^{-net}}{(1+e^{-net})^2}$$

$$= \frac{2e^{-net}}{(1+e^{-net})^2}$$

b) To prove $f'(net) = O(1-O)$ for unipolar continuous function.

Sol^n — RHS $= O(1-O)$

for unipolar continuous f^n.

$$O = \frac{1}{1+e^{-\lambda net}}, \quad \text{assume } \lambda = 1$$

$\therefore$ RHS $= O(1-O)$

$$= \frac{1}{1+e^{-net}}\left(1 - \frac{1}{1+e^{-net}}\right)$$

$$= \frac{1}{1+e^{-net}} - \frac{1}{(1+e^{-net})^2}$$

$$= \frac{1+e^{-net}-1}{(1+e^{-net})^2}$$

$$= \frac{e^{-net}}{(1+e^{-net})^2}$$

L.H.S. $= f'(net)$

$$= \frac{d}{dnet}\left(\frac{1}{1+e^{-net}}\right)$$

$$= \frac{1+e^{-net} \cdot \frac{d}{dnet}(1) - 1 \cdot \frac{d}{dnet}(1+e^{-net})}{(1+e^{-net})^2}$$

$$= \frac{-(-e^{-net})}{(1+e^{-net})^2}$$

$$= \frac{e^{-net}}{(1+e^{-net})^2}$$

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SONAL.

Q. Perform two training steps of the network using the delta learning rule for $\lambda = 1$ and $c = ~~0.0~~\ 0.25$. Train the network using the following data pairs.

Zurada

$$\left(x_1 = \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}, d_1 = -1 \right), \left(x_2 = \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix}, d_2 = 1 \right)$$

The initial weights are $w' = [1\ 0\ 1]^t$

Hint: $f'(net) = \frac{1}{2}(1 - o^2)$.

Sol^n:– $\Delta w = c \cdot (d_i - O_i) \cdot f'(net) \cdot x$.

i) $net' = w'^t \cdot x_1 = [1\ 0\ 1]^t \cdot \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix} = 2 - 1 = 1$

$\therefore O' = f(net') = \frac{2}{1 + e^{-\lambda net'}} - 1$

$= \frac{2}{1 + e^{-1}} - 1$

$= 0.462$

$f'(net') = \frac{1}{2}(1 - O^2)$

$= 0.5(1 - 0.462^2)$

$= 0.393$

Now,

$\Delta w' = c \cdot (d_1 - O_1) f'(net') \cdot x_1$

$= 0.25(-1 - 0.462)(0.393) \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$

$= -0.144 \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$

$= \begin{bmatrix} -0.288 \\ 0 \\ +0.144 \end{bmatrix}$

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$$\omega^2 = \omega^1 + \Delta\omega^1$$

$$= \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} -0.288 \\ 0 \\ 0.144 \end{bmatrix} = \begin{bmatrix} 0.712 \\ 0 \\ 1.144 \end{bmatrix}$$

ii) $net^2 = \omega^{2t} \cdot x_2 = \begin{bmatrix} 0.712 & 0 & 1.144 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix}$

$$= -0.432$$

$$O^2 = \frac{2}{1 + e^{+0.432}} - 1$$

$$= -0.2127$$

$$f'(net^2) = \frac{1}{2}(1 - O^2)$$

$$= 0.5(1 - (-0.212)^2)$$

$$= 0.477$$

Now,

$$\Delta\omega^2 = c \cdot (d_2 - O_2)\, f'(net^2) \cdot x_2$$

$$= 0.25(1 + 0.212) \cdot 0.477 \cdot \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix}$$

$$= 0.144 \cdot \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix} = \begin{bmatrix} 0.144 \\ -0.288 \\ -0.144 \end{bmatrix}$$

$$\therefore \omega^3 = \omega^2 + \Delta\omega^2 = \begin{bmatrix} 0.712 \\ 0 \\ 1.144 \end{bmatrix} + \begin{bmatrix} 0.144 \\ -0.288 \\ -0.144 \end{bmatrix}$$

$$= \begin{bmatrix} \overset{0.856}{\cancel{0.568}} \\ -0.288 \\ 1 \end{bmatrix}$$

Q. Find final weights after 1^{st} iteration

$$x_1 = \begin{bmatrix} 1 \\ -2 \\ 0 \\ -1 \end{bmatrix}, \quad x_2 = \begin{bmatrix} 0 \\ 1.5 \\ -0.5 \\ -1 \end{bmatrix}, \quad x_3 = \begin{bmatrix} -1 \\ 1 \\ 0.5 \\ -1 \end{bmatrix} \quad w' = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0.5 \end{bmatrix}$$

$d_1 = -1$, $d_2 = -1$, $d_3 = 1$, $c = 0.1$, $\lambda = 1$

use bipolar continuous activation function.

Sol^n :- $\Delta w = c \cdot (d - o) \cdot f'(net) \cdot x$

and $f'(net) = \frac{1}{2}(1 - o^2)$