Dr. Tanuja K. Sawde

Defuzzification Methods

Defuzzification is the process of conversion of a fuzzy quantity into a precise quantity.

The output of a fuzzy process may be union of two or more fuzzy membership functions defined on the universe of discourse of the o/p variables.

A fuzzy o/p process may involve many o/p parts & the membership function representing each part of the o/p can have any shape.

We have

$$\underset{\sim}{C}_n = \bigcup_{i=1}^{n} \underset{\sim}{C}_i = \underset{\sim}{C}$$

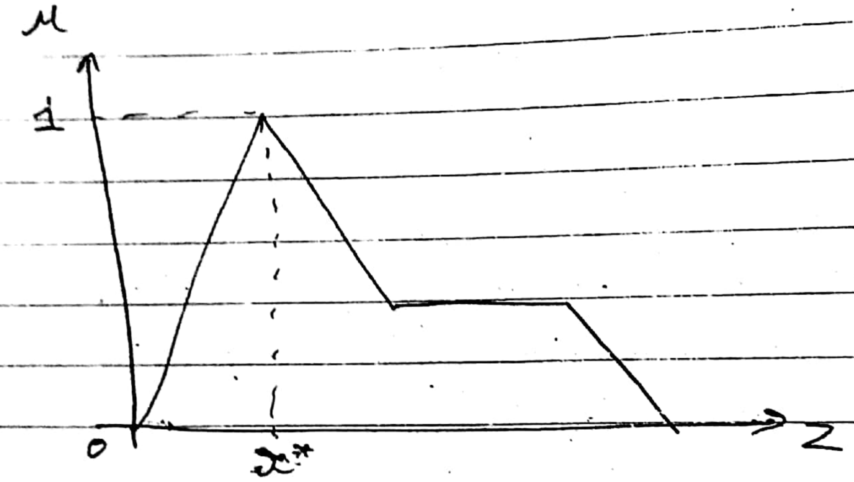
Defuzzification methods.

1.) Max-membership principle.
2.) Centroid method.
3.) Weighted average method.
4.) Mean-max membership.
5.) Center of sums.
6.) Center of largest area
7.) First of maxima, last of maxima.

1.) Max-Membership Principle.

This method is also known as height method & is limited to peak output functions

$$\mu_C(x^*) \geq \mu_{\underset{\sim}{C}}(x) \text{ for } x \in X.$$



max-membership defuzzification method.

2.) Centroid Method.

This method is also known as center of mass, center of area or center of gravity method. It is the most commonly used defuzzification method. The defuzzified o/p x^* is defined as.

$$x^* = \frac{\int \mu_C(x) \cdot x\,dx}{\int \mu_{\underset{\sim}{C}}(x)\,dx}.$$

when $\int$ denotes an algebraic integration.

3.) Weighted Average method

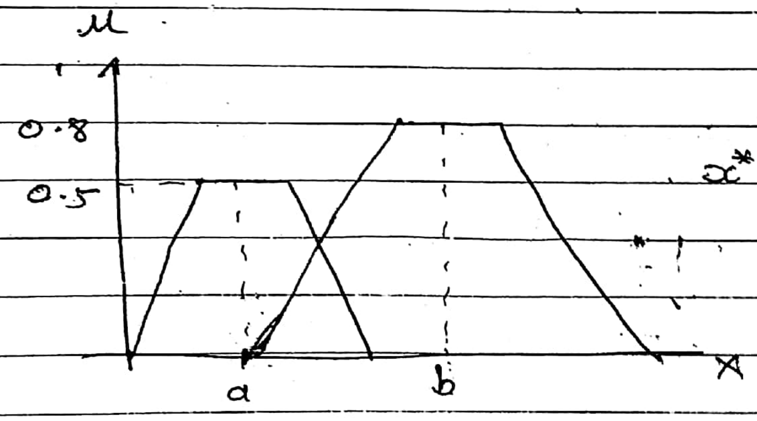
This method is valid for symmetrical o/p membership functions only.

Each membership function is weighted by its maximum membership value.

The output in this case is given by.

$$x^* = \frac{\sum \mu_C(\bar{x}_i) \cdot \bar{x}_i}{\sum \mu_C(\bar{x}_i)}$$

where Σ denotes algebraic sum & $\bar{x}_i$ is the maximum of the i^{th} membership function.



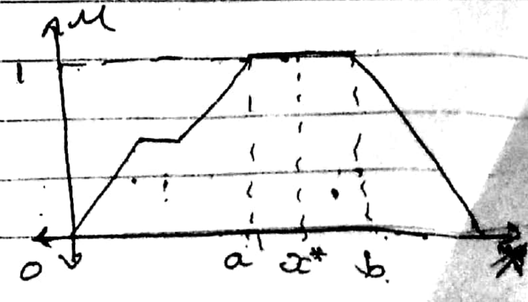
$$x^* = \frac{0.5a + 0.8b}{0.5 + 0.8}$$

4.) Mean-max Membership.

This method is also known as the middle of the maxima.

$$x^* = \frac{\sum_{i=1}^{n} \bar{x}_i}{n}$$

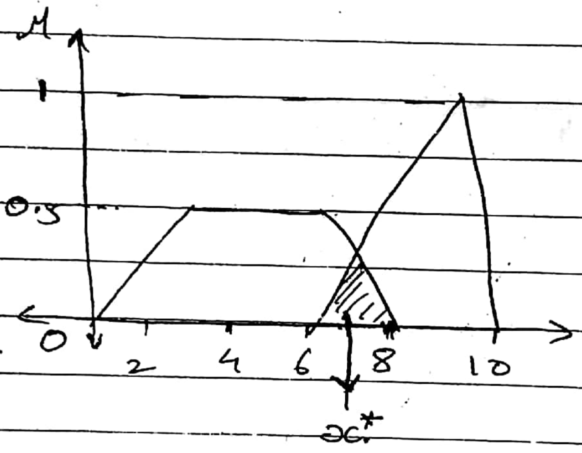
$$x^* = \frac{a+b}{2}$$



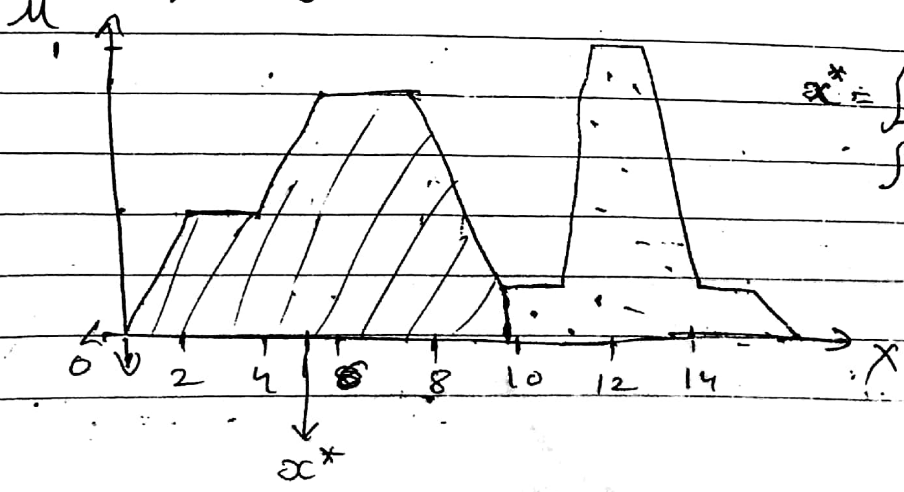
5.) Center of Sums

This method employs the algebraic sum of the individual fuzzy subsets instead of their union.
The calculations here are very fast, but the main drawback is that intersecting areas are added twice.

$$x^* = \frac{\int_x x \sum_{i=1}^{n} \mu_{C_i}(x)\,dx}{\int_x \sum_{i=1}^{n} \mu_{C_i}(x)\,dx}$$



6.) Center of largest area



$$x^* = \frac{\int \mu_{C_i}(x)\cdot x\,dx}{\int \mu_{C_i}(x)\,dx}$$

Meena

Lecture 6

Defuzzification Methods

- There may be situations where the output of a fuzzy process needs to be a single scalar quantity as opposed to a fuzzy set.

- Defuzzification is the conversion of a fuzzy quantity to a precise quantity, just as fuzzification is the conversion of a precise quantity to a fuzzy quantity.
- The output of a fuzzy process can be the logical union of two or more fuzzy membership functions defined on the universe of discourse of the output variable.

For example, suppose a fuzzy output is comprised of two parts: the first part, $\tilde{C}_1$, a trapezoidal shape and the second part, $\tilde{C}_2$, a triangular membership shape. The union of these two membership functions i.e., $\tilde{C} = \tilde{C}_1 \cup \tilde{C}_2$, involves the max operator, which graphically is the outer envelope of the two shapes shown in figures below. Of course, a general fuzzy output process can involve many output parts (more than two) and the membership function representing each part of the output can

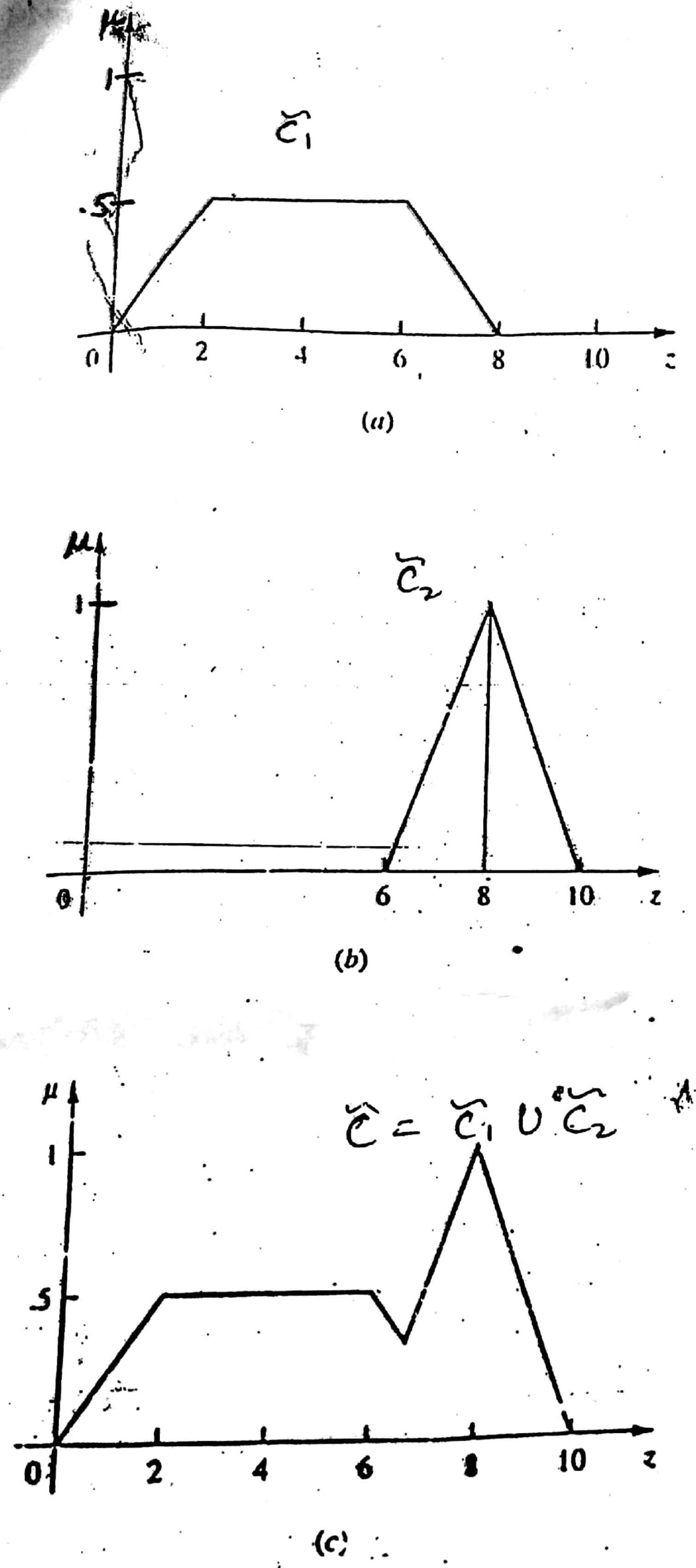


FIGURE 5.4
Typical fuzzy process output: (*a*) first part of fuzzy output; (*b*) second part of fuzzy output; (*c*) union of both parts.

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have shapes other than triangles and trapezoids.

There are seven methods of defuzzification that have been proposed by investigators in recent years.

1. Centroid Method or Center of gravity (COG) center of area (COA)

2. Weighted average method

3. Center of sums (COS)

4. Mean of maxima (MOM) / Min - max

5. Max membership principle / Height method

6. Center of largest area

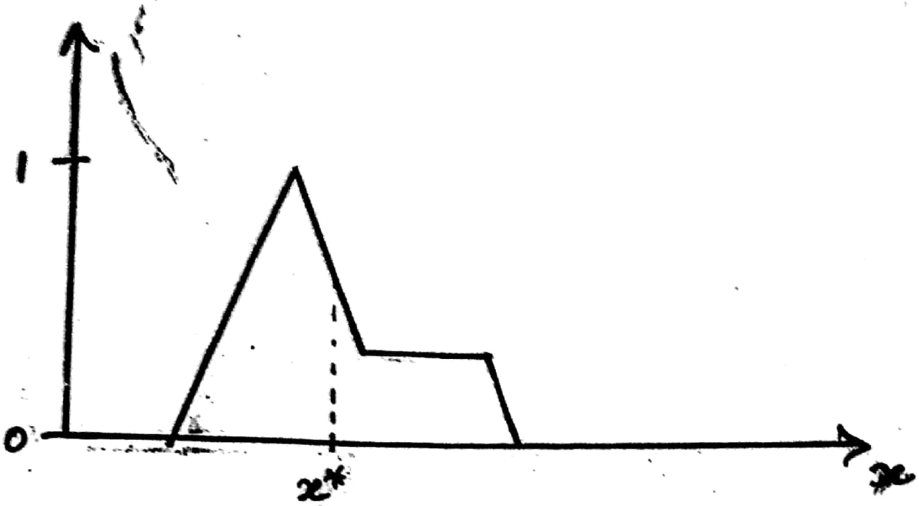
First or last of maxima

1. Centroid Method - It obtains the center of area (x^*) occupied by the fuzzy set. It is given by the algebraic expression

$$x^* = \frac{\int \mu(x)\, x\, dx}{\int \mu(x)\, dx}$$

$\int$ denotes algebraic integration for a continuous membership function.

This method works for unsymmetrical membership functions also.



2. <u>Weighted average method</u> is only valid for symmetrical output membership functions. It is given by the algebraic expression

$$x^* = \frac{\sum_{i=1}^{n} x_i \mu(x_i)}{\sum_{i=1}^{n} \mu(x_i)}$$

where Σ denotes an algebraic sum for a discrete membership function.
n = no. of elements in the sample
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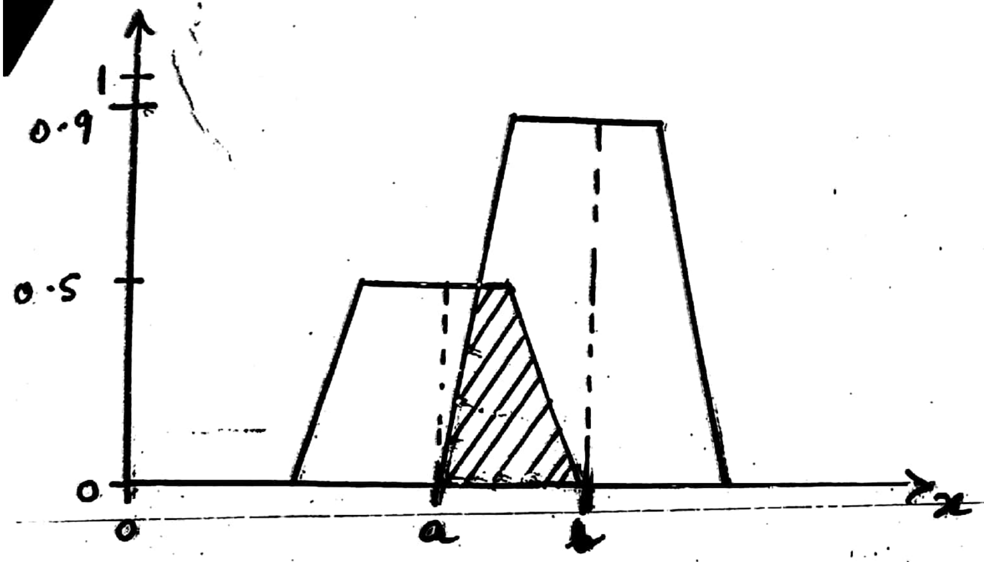
$\mu(x_i)$ = membership function
This method determines the center of area under the combined membership function (union of the fuzzy sets). The overlapping area is considered only once.
The weighted average is formed by weighting each membership function in the output by its respective maximum

membership value. The weights are max. individual membership values.



The defuzzified value is :-

$$x^* = \frac{a(0.5) + b(0.9)}{0.5 + 0.9}$$

Since the method is limited to symmetrical membership functions, the values a and b are the means of their respective shapes. This method is slow and computationally complex.

3. Center of Sums - This is faster than many defuzzification methods that are presently in use. This process involves the algebraic sum of individual output fuzzy sets instead of their union. One drawback is that the intersecting area is added twice. The defuzzified value is given by :-

$$x^* = \frac{\sum_{i=1}^{N} x_i \sum_{k=1}^{n} \mu_{\tilde{A}_k}(x_i)}{\sum_{i=1}^{N} \sum_{k=1}^{n} \mu_{\tilde{A}_k}(x_i)}$$

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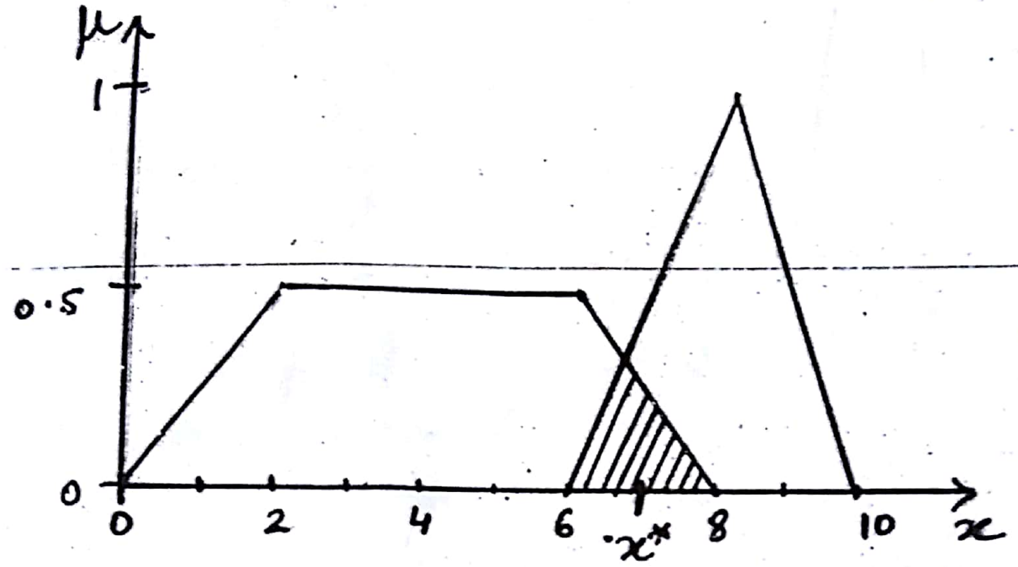
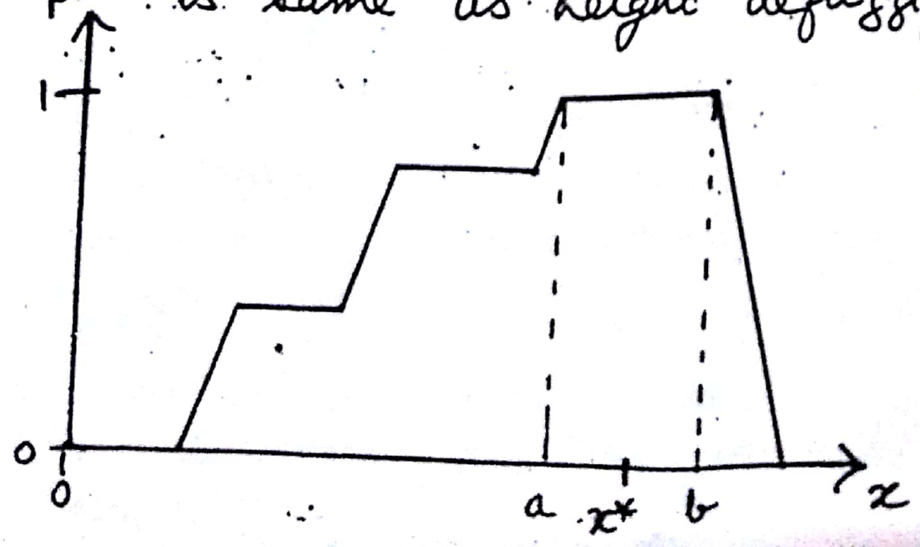


Mean of Maxima (MOM) / Min-Max / Middle of maxima

One simple way of defuzzifying the output is to take the crisp value with the highest degree of membership. In cases with more than one element having the max. value, the mean value of the maxima is taken. This is same as height defuzzification method for peaked fuzzy sets.

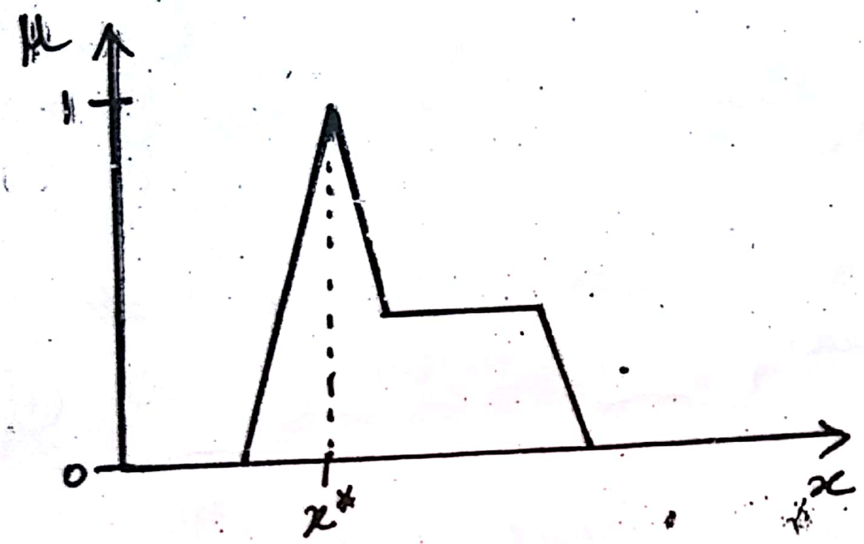


$$x^* = \frac{a+b}{2}$$

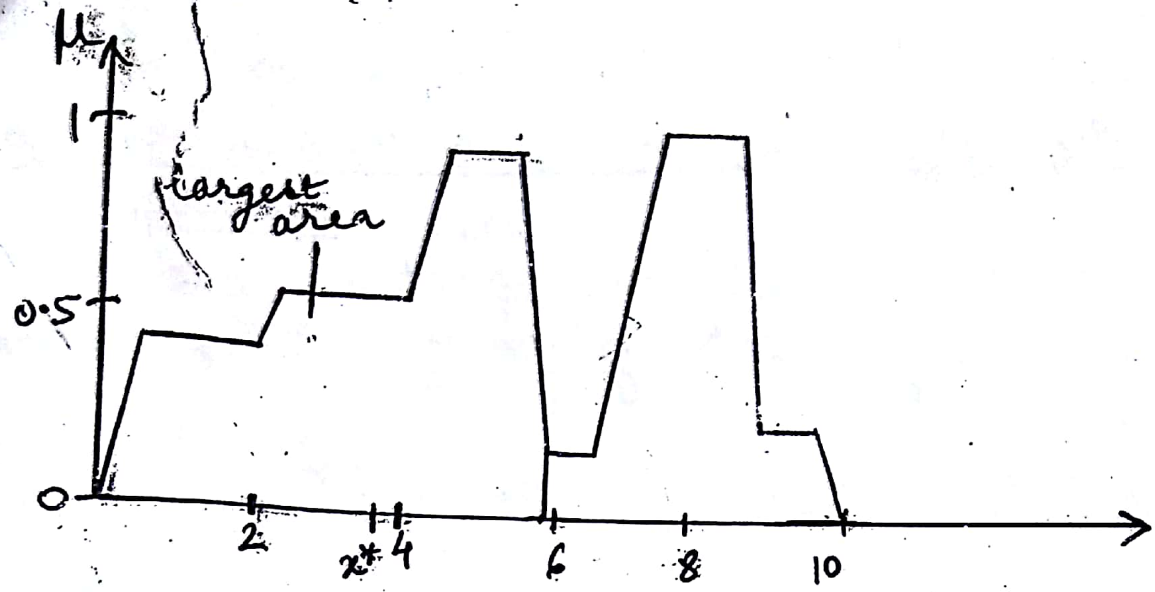
Height defuzzification method/ max. membership principle

This method is used for peaked output fuzzy sets only. It is given by the algebraic expression

$$\mu(x^*) \geq \mu(x) \text{ for all } x \in X$$



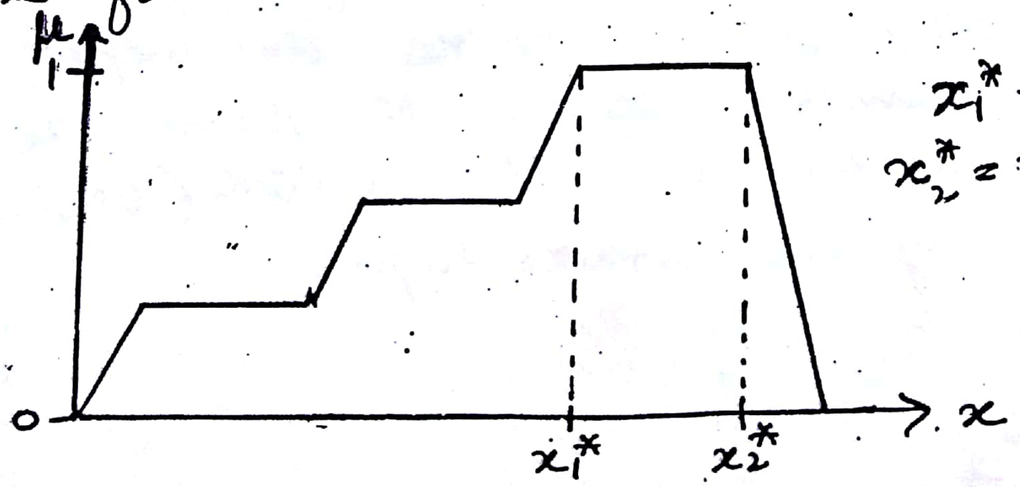
Center of largest area — If the output fuzzy set has at least two convex subregions (i.e. it is nonconvex) then the largest convex fuzzy subregion with the largest area is used to calculate the defuzzified value x^* of the output. When the output fuzzy set is convex, x^* is the same as that calculated by the centroid method (because then there is only one convex region)



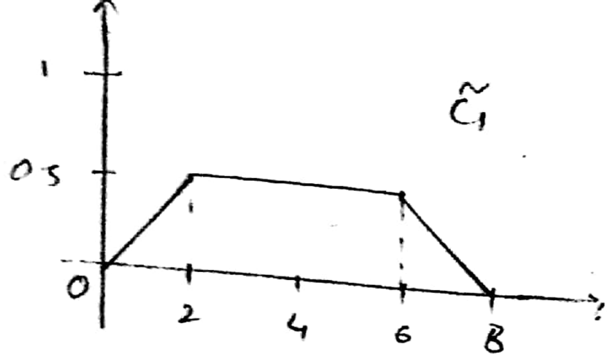
$$x^* = \frac{\int \mu_{\tilde{A}_m}(x)\, x\, dx}{\int \mu_{\tilde{A}_m}(x)\, dx}$$

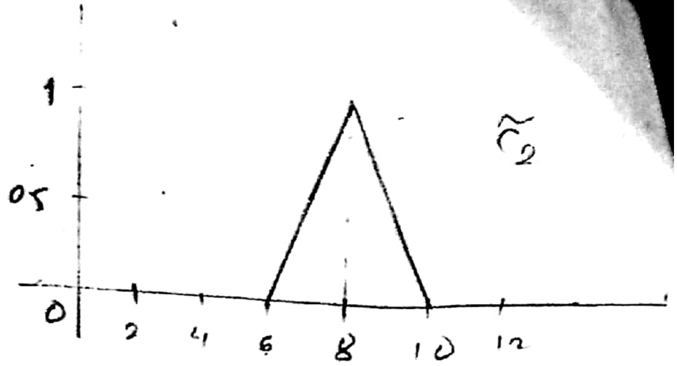
where $\tilde{A}_m$ is the convex subregion that has the largest area.

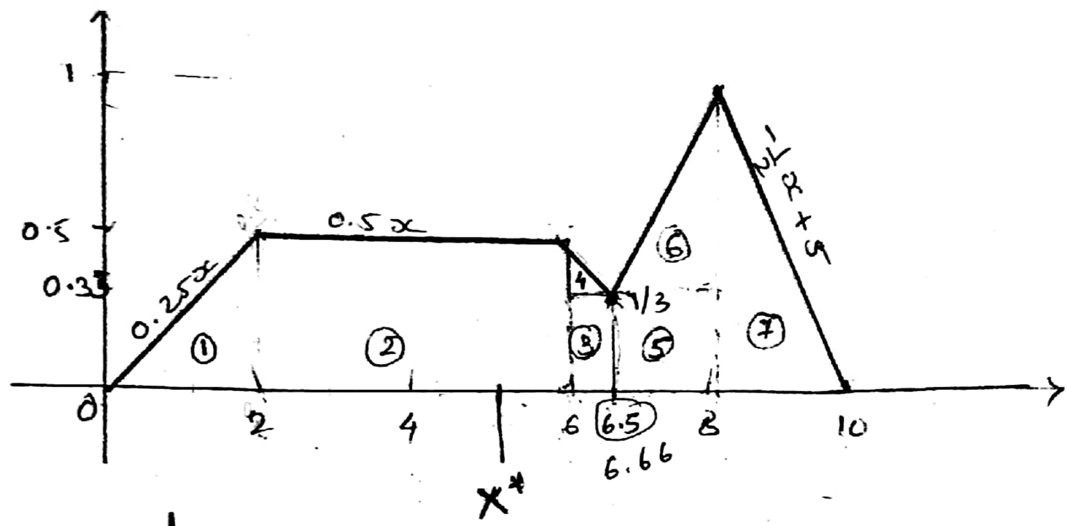
7. <u>First (or last) of maxima</u>:- This method uses the union of all individual output fuzzy sets to find out the smallest/largest value of the domain with maximized membership degree. First, the largest height in the union is determined. Then the first of the maxima is found or the last of the maxima is found.



x_1^* = First of maxima
x_2^* = Last of maxima







① Centroid method

$2 \times 0.5 = 1$.

sub-region	Area (A)	$\bar{x}$	$A\bar{x}$
1	$0.5 = \frac{2\times0.5}{2}$	1.33 $\frac{0+2+2}{3}$	0.665
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3	$0.15 = 0.3\times0.5$	6.25 $\frac{6+6.5}{2}$	0.937
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6	0.525	7.5 $\frac{6.5+8+8}{3}$	3.937
7	1	8.66 $\frac{8+8+10}{3}$	8.66
	4.675		25.769

$$x^* = \frac{\sum A\bar{x}}{\sum A}$$

① $\bar{x} = \frac{0+2+2}{3} = 1.33$

$$x^* = \frac{25.769}{4.675} = 5.512$$

(2) Weighted average method:

find the center of each fuzzy set.

Center of $\tilde{C_1} = 4$, center of $\tilde{C_2} = 8$

membership of $\tilde{C_1}$ at 4 = 0.5

" " $\tilde{C_2}$ at 8 = 1

$$\therefore x^* = \frac{(4 \times 0.5) + (8 \times 1)}{1 + 0.5} = \underline{\underline{6.66}}$$

(3) Center of sum:

Find area of individual curve.

Area of $\tilde{C_1} = (8+4) \times 0.5/2 = 3$

Area of $\tilde{C_2} = (4 \times 1) \times 1/2 = 2$

Center of $\tilde{C_1} = 4$,

center of $\tilde{C_2} = 8$

$$\therefore x^* = \frac{(4 \times 3) + (8 \times 2)}{3+2} = \underline{\underline{5.6}}$$

(4) Height defuzzification:

$$x^* = \underline{\underline{8}}$$

(5) Mean of Maxima:

$$x^* = 8$$

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Meena

Shalini Bhatia   Lecture 5

Q- Design a fuzzy logic controller for a train approaching or leaving a station. The inputs are distance from the station and speed of the train. The output is the amount of brake power used. Use four descriptors for each variable and appropriate defuzzification method.

Solution

The rule base is summarized as follows.

Distance \ Speed	Very Slow	Slow	Fast	Very Fast
Very Close	Very light 1	Heavy 2	Heavy 3	very heavy 4
Close	Light 5	Light 6	Heavy 7	Heavy 8
Far	Light 9	Very Light 10	Light 11	Heavy 12
Very far	Very light 13	Very Light 14	Light 15	Light 16

Now let us consider the control action if the distance is 100 m and the speed is 24.6 km/hr.

A speed of 24.6 km/hr. has a membership of 0.58 in slow and 0.21 in fast i.e.

$\mu_{slow} = 0.58$

$\mu_{fast} = 0.21$

A distance of 100 m has a membership of 0.29 in close and 0.88 in far i.e.

$\mu_{close} = 0.29$

$\mu_{far} = 0.88$

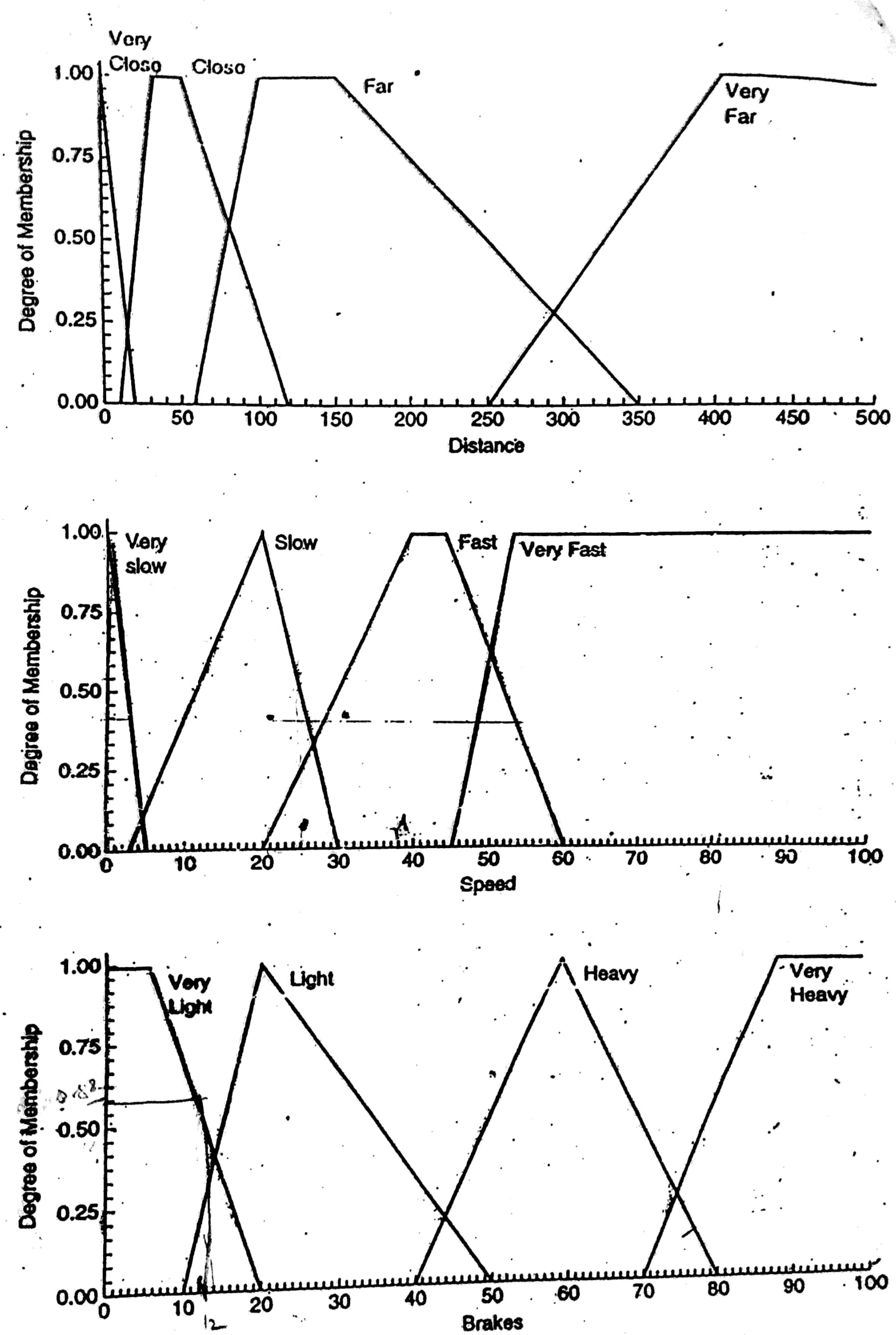

Figure 5.6 Definition of membership functions for a) distance, b) speed, c) brakes.

(2)

-ethod:

Defuzzification (min - max) The min operation

μ slow AND μ close = min (0.58, 0.29) = 0.29

μ slow AND μ far = min (0.58, 0.88) = 0.58.

μ fast AND μ close = min (0.21, 0.29) = 0.21

μ fast AND μ far = min (0.21, 0.88) = 0.21

The rules that fire are 6, 7, 10 and 11.
the max. operation gives :-

0.29 OR 0.58 OR 0.21 OR 0.21 = 0.58

0.58 corresponds to rule 10 or very light break power.

x-axis value.

Hence we get Brake power = 12% from the membership function of Brake power.

1 — 0.58

x

100

$\frac{x}{0.58}$ $\frac{100 \times 0.58}{1}$ 12

16 10

100 x

$\frac{x}{100}$ = $\frac{100 \times 10}{16}$

Q-1 Design a fuzzy controller to regulate the temperature of a domestic shower. Assume that:-

(a) Temperature is adjusted by a single mixer tap.
(b) Flow of water is constant.
(c) Control variable is the ratio of hot water to cold water input.

Design should clearly mention the descriptors used and the control variable, set of rules to generate control action and defuzzification.

Q-2 Design a fuzzy controller for maintaining the temperature of water in a tank at a fixed level. Input variables are cold water flow into the tank and steam flow into the tank. For cooling cold water flow is regulated and for raising the temperature steam flow is regulated. Design a fuzzification scheme for input variables. Formulate the control problem in terms of fuzzy inference rules incorporating the degree of relevance of each rule. Design a scheme which shall regulate water and steam flows properly.

(4)

Q-3 Design a fuzzy controller to control the feed amount of the coagulant for the water purification plant. Raw water is purified by injecting chemicals at rates related to water quality. Aluminium sulphate or PAC (polymerized aluminium chloride) is used as coagulant. Aluminium sulphate is less expensive than PAC but is not effective in low temperature water. Assume inputs as water temperature (cold, normal, hot) and grade of water (low, medium, high), output variable as amount of coagulant (small, medium, large). Derive the rules for control action and defuzzification. The design should be supported by figures wherever necessary. Clearly indicate that if water temperature is low and grade of water quality is low then PAC is used in large amount.

Meena

Lecture 6

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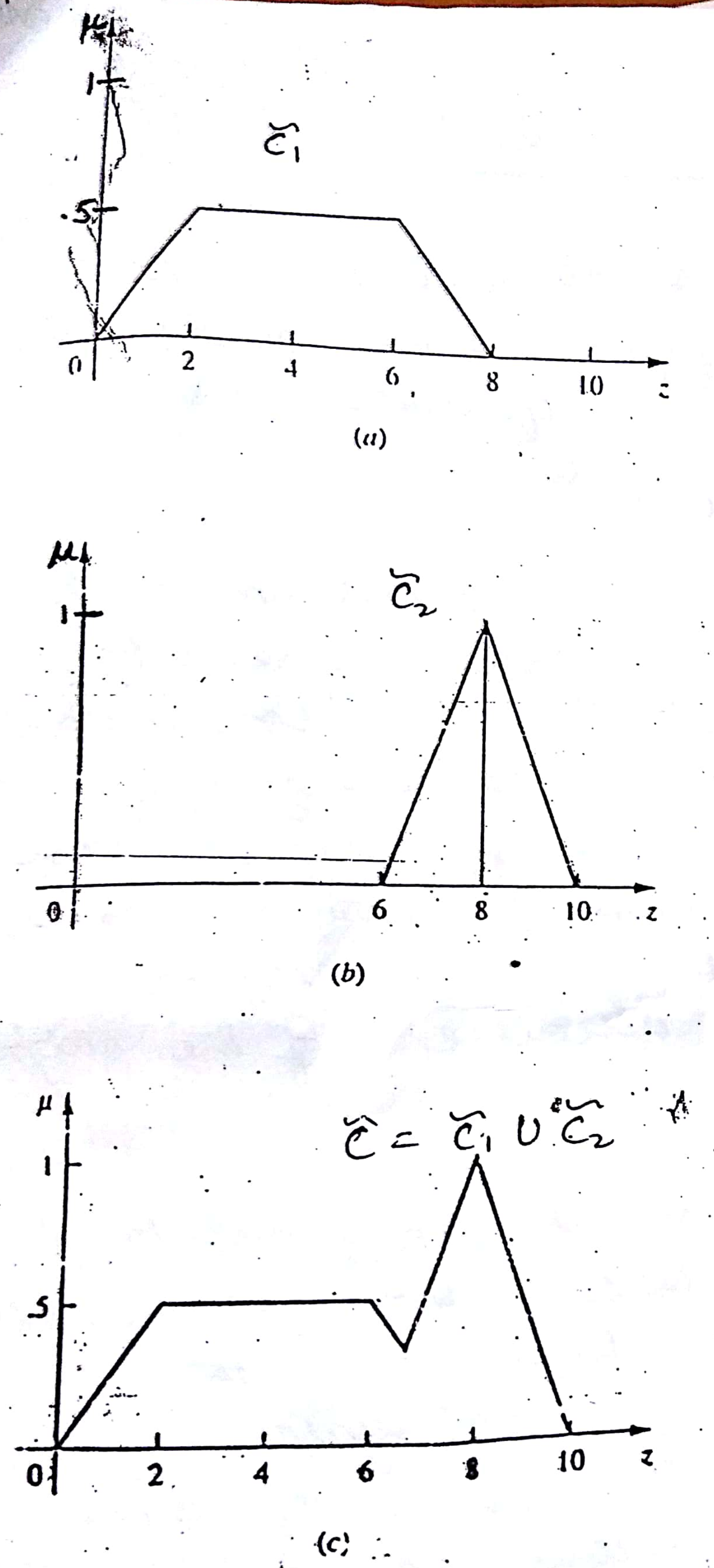

FIGURE 5.4
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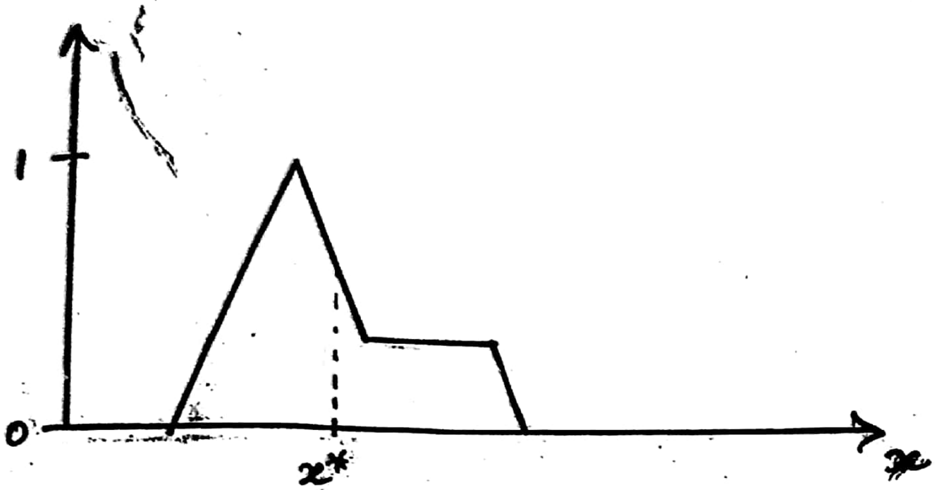

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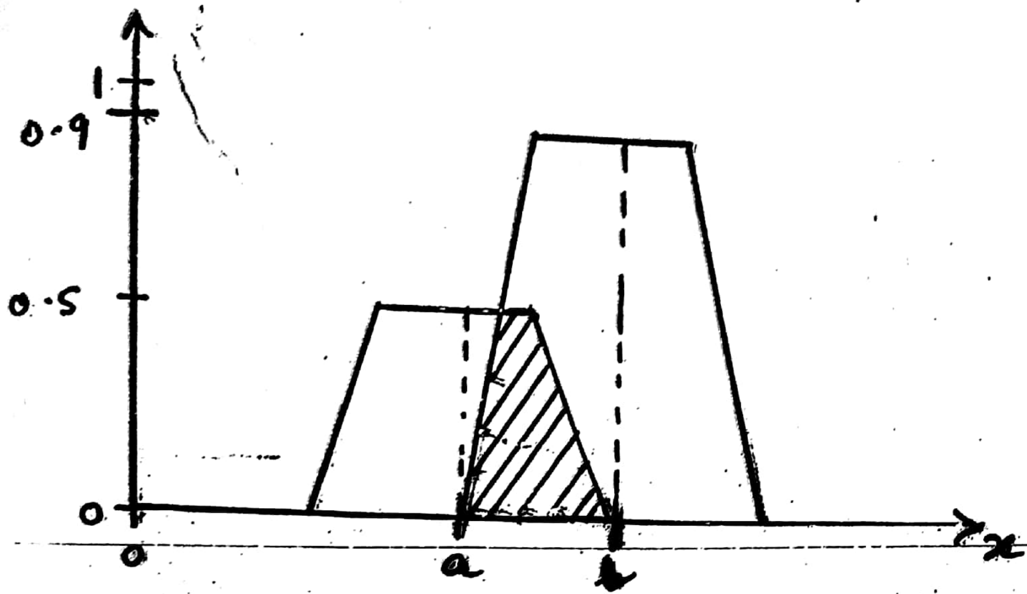

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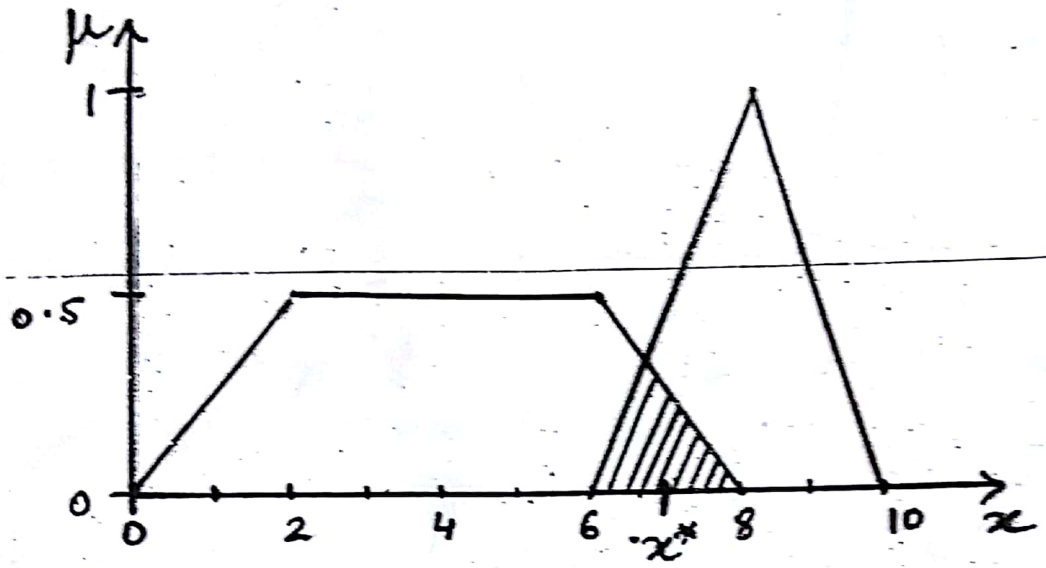

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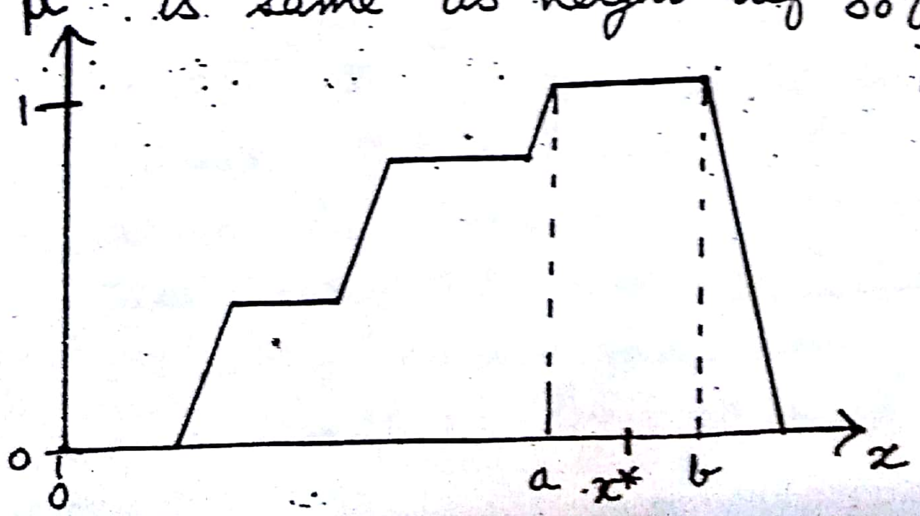

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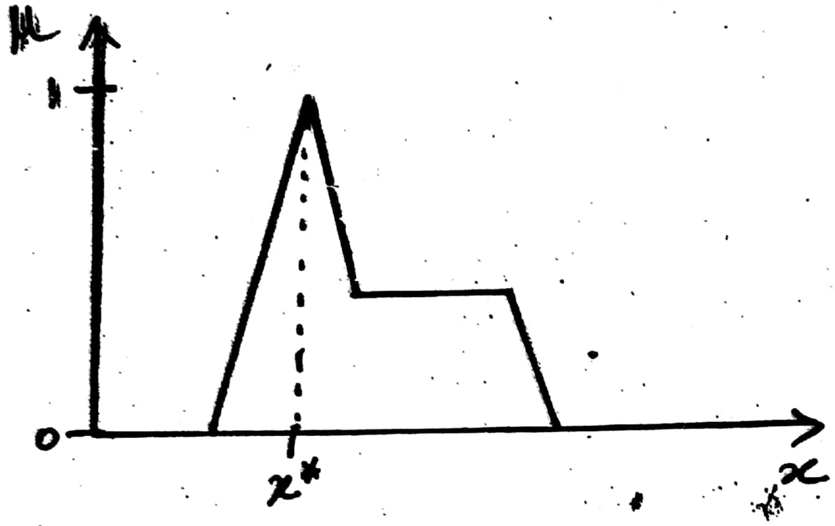

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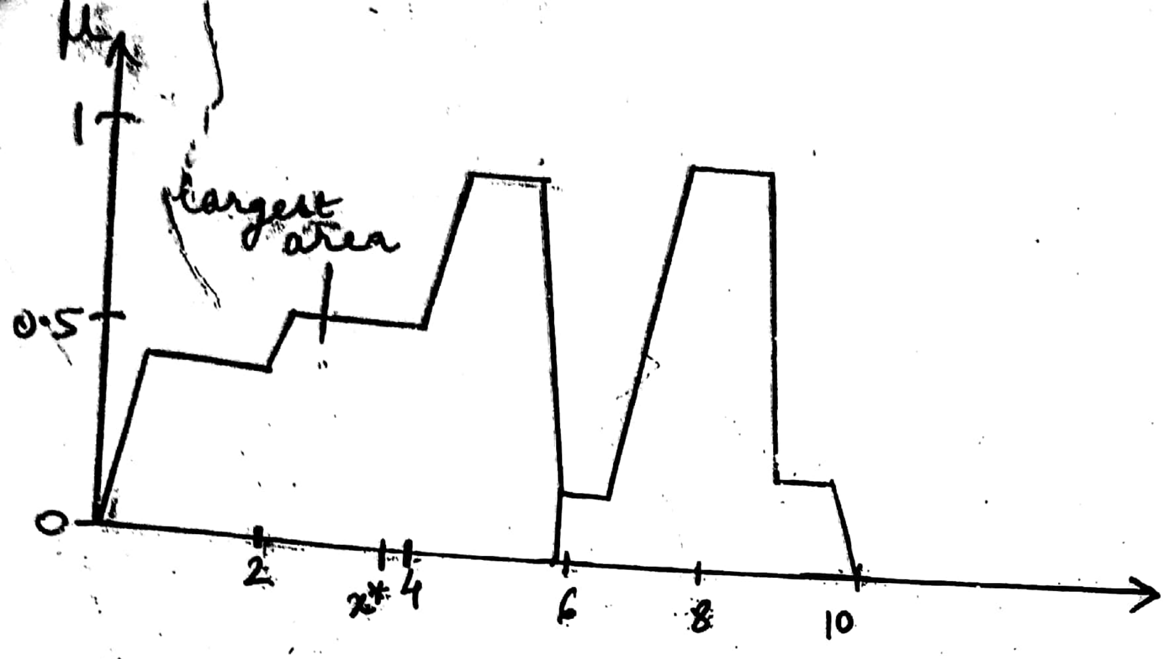

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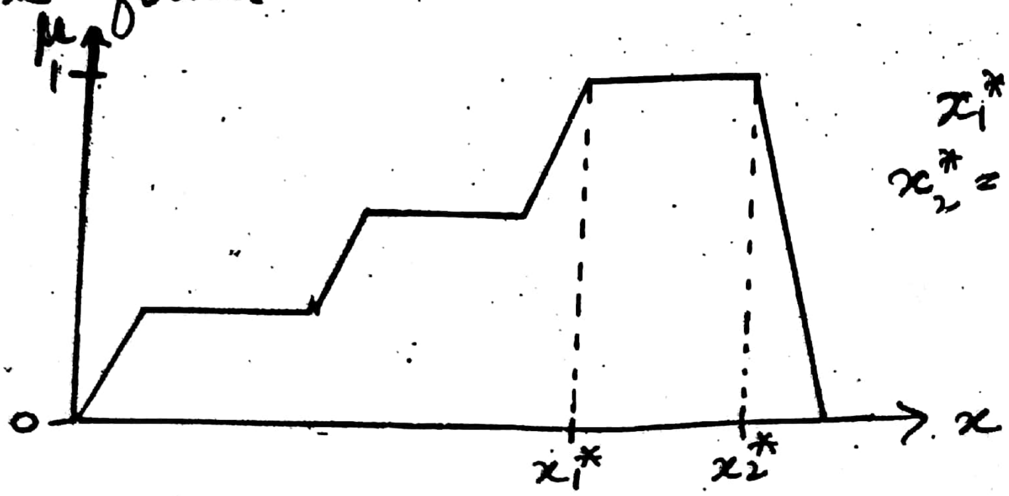
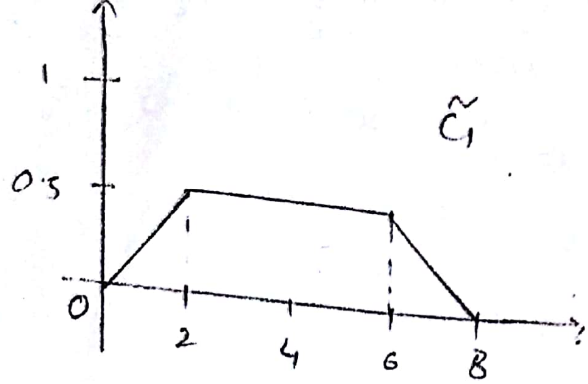
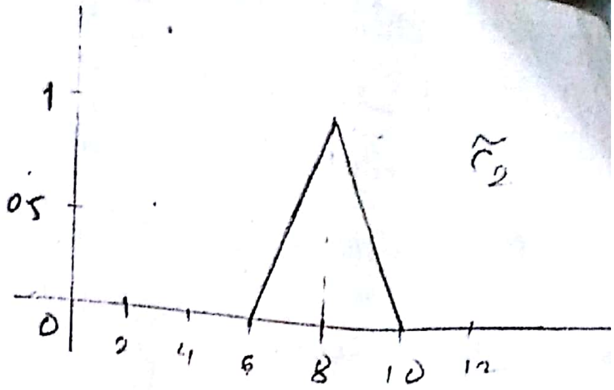
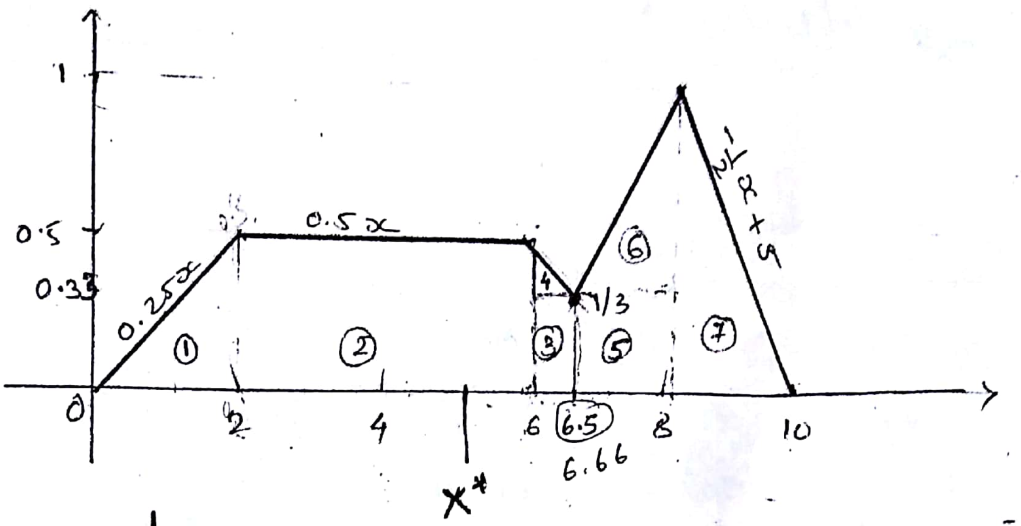


x_1^* = First of maxima
x_2^* = Last of maxima







① Centroid method   2×0.5 = 1.

Sub-region	Area (A)	$\bar{x}$	$A\bar{x}$
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$$x^* = \frac{\Sigma A\bar{x}}{\Sigma A}$$

① $\bar{x} = \frac{0+2+2}{3} = 1.33$

$$x^* = \frac{25.769}{4.675} = 5.512$$

0.3×0.5
0.15

0.2×0.5
2
$\frac{0.10}{2}$ 0.05

② Weighted average method:

find the center of each fuzzy set.

Center of $\tilde{C}_1 = 4$, center of $\tilde{C}_2 = 8$

membership of $\tilde{C}_1$ at 4 = 0.5

" " $\tilde{C}_2$ at 8 = 1

$$\therefore x^* = \frac{(4 \times 0.5) + (8 \times 1)}{1 + 0.5} = \underline{\underline{6.66}}$$

③ Center of sum:

Find area of individual curve.

Area of $\tilde{C}_1 = (8+4) \times 0.5/2 = 3$

Area of $\tilde{C}_2 = (4 \times 1) \times 1/2 = 2$

Center of $\tilde{C}_1 = 4$,

center of $\tilde{C}_2 = 8$

$$\therefore x^* = \frac{(4 \times 3) + (8 \times 2)}{3+2} = \underline{\underline{5.6}}$$

④ Height defuzzification:

$$x^* = \underline{\underline{8}}$$

⑤ Mean of maxima:

$$x^* = 8$$