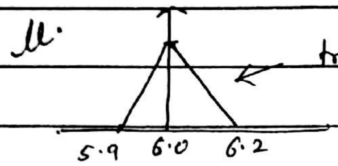
Soft Computing Sonal Mam P.T.I

24/03/14.

Fuzzy logic by Timothy Ross.

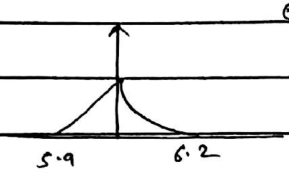
fuzzy → not clear / ambiguous.
impreeise.

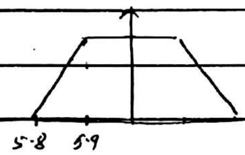
fuzzy no.
voltage around 6V.



triangular membership f^n.

exponential





trapezoidal.

fuzzy variable –
ex. hot, tall, small, short ...
introduced by Zadeh – father of fuzzy.

fuzzy sets — crisp sets.

Q.1) differentiate bet^n crisp logic set & fuzzy logic set.

x, $\mu_A(x) \rightarrow$ 0 to 1.

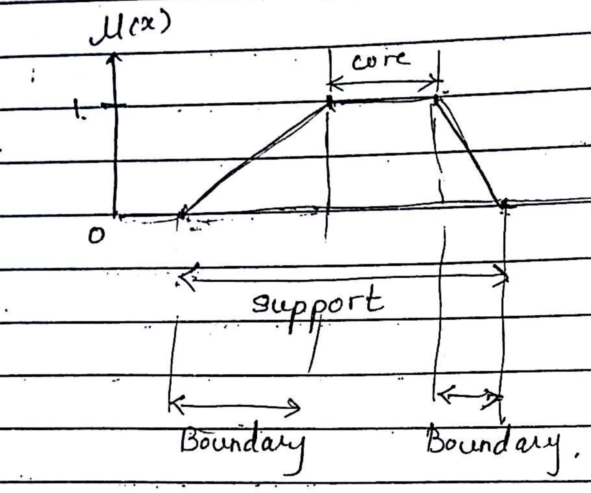
Q.2) what are diff. features of membership function.

define

core – comprises those elements x of the universe
s.t. $\mu_a(x) = 1$

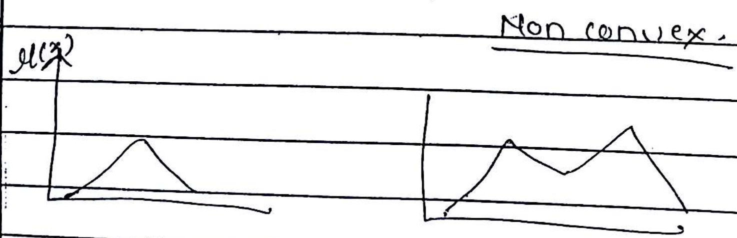
<u>support</u> – region of the universe that is characterised by non zero membership.

<u>Boundary</u> – boundaries comprise those elements x of the universe s.t. $0 < \mu_a(x) < 1$.



Normal fuzzy set –
at least one element x in the universe whose membership

define <u>convex fuzzy set</u> – membership values are strictly monotonically '↑' or '↓'.



<u>cross over points</u> – $\mu_a(x) = 0.5$

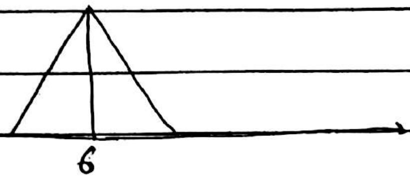
<u>Height</u> – defined as max $\{\mu_a(x)\}$.

Q.3) prob. –

Define a fuzzy set of integers close to 6.

(8M).

triangular.



Bell shape m.f. $\mu_A(x) = \dfrac{1}{1+(x-6)^2}$.

$A = \{ x, \mu_A(x) \mid x \in X \}$.

$= \{ (3, 0.1), (4, 0.2), (5, 0.5) (6, 1)$
$(7, 0.5) (8, 0.2) (9, 0.1) \}$

Q.4) enlist different membership function.

- triangular
- trapezoidal

graph

equations.

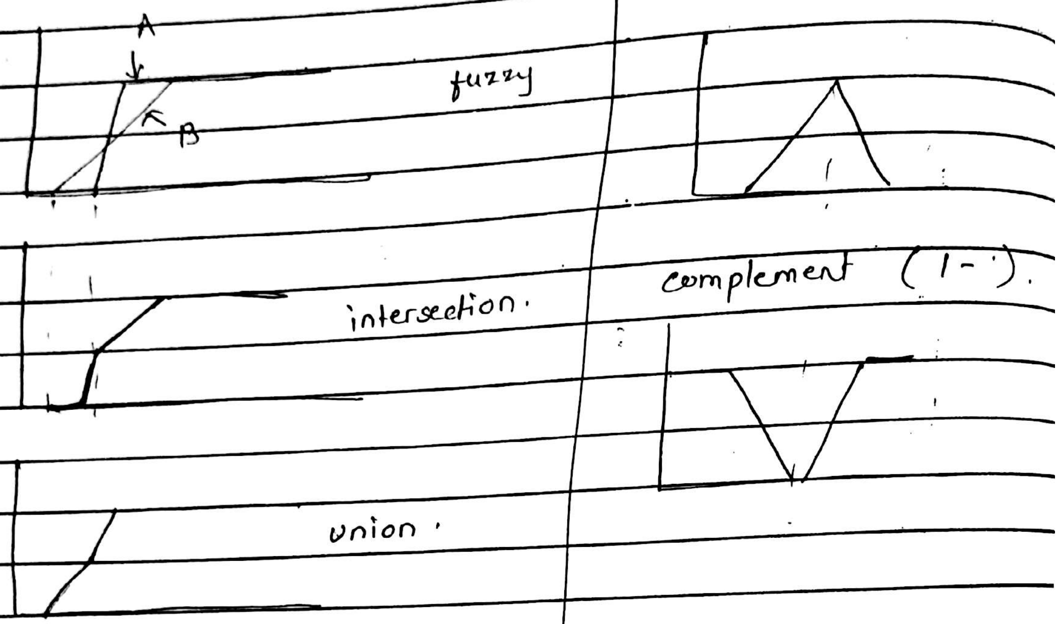
Q.5) operations on fuzzy sets.

intersection

complement

Union

ex.



imp.

** Fuzzy composition

$I_a \propto R_{se}$, speed $\propto I_a$.

what is relatⁿ betⁿ R_{se} & speed.

- max . min
- max . product.

* extension principle.
* α-cut & fuzzy set
* fuzzy linguistic variable
* projections of fuzzy set.

- Defuzzification methods.

3 problems — 1 for 20 M.

1) washing machine
2) train approaching station
3) shower.

Fuzzy Sets

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Introduction -

A classical set is a set with a crisp boundary
for example -
a classical set A of real numbers greater than 10
can be expressed as,

$$A = \{x / x > 10\}$$

Here there is a clear, unambiguous boundary 10 such that if x is greater than this number then x belongs to the set.

Although classical sets are suitable for various applications and have proven to be an important tool for mathematics and computer science, they do not reflect the nature of human concepts and thoughts, which tend to be abstract and imprecise.

As an illustration - mathematically we can express the set of tall persons as a ~~grp~~/collection of persons having ~~his~~ height more than 6 ft.
as,

$$A = \{x / x > 6\}$$

where,
$A \rightarrow$ is 'tall person'
$x \rightarrow$ is 'height'

Yet this is an unnatural and inadequate way of representing our usual concept of a 'tall person'.

The dichotomous nature of the classical set would classify a person 6.00 ft tall as a 'tall person', but not a person 5.99 ft. tall. This distinction is intuitively unreasonable. The flaws comes from the sharp transition between inclusion & exclusion in a set.

In contrast to classical set, a fuzzy set, as a name implies is a set without a crisp boundary. i.e. the transition from "belong to a set" to "not belong to a set" is gradual. It is a smooth transition characterised by membership functions that give fuzzy sets flexibility in modeling commonly used linguistic expressions such as "the water is hot" or "the temp is high".

→ Fuzziness does not come from the randomness of the constituent members of the sets, but from the uncertain and imprecise nature of abstract thoughts and concepts.

Basic Definitions and Terminology

Let X be the space of objects and x be the generic element of X.
A classical set A, $A \subseteq X$, is defined as a collection of elements or objects $x \in X$, such that each x can either belong or not belong to the set A.
By defining the characteristics function for each element x in X, we can represent a classical set A by a set of ordered pair $(x, 0)$ or $(x, 1)$ which indicates $x \notin A$ or $x \in A$ resp.

A fuzzy set expresses the degree to which an element belongs to a set.
Thus, the characteristic function of a fuzzy set is allowed to have values between 0 & 1. that denotes the degree of membership of an element in a given set.

Def.ⁿ - Fuzzy sets and membership functions.

If X is a collection of objects denoted by x, then fuzzy set A in X is defined as a set of ordered pairs -

$$A = \{ (x, \mu_A(x)) / x \in X \}$$

$\mu_A(x) \rightarrow$ membership of object x for fuzzy set A.

The MF (membership function) maps each element of X to a membership grade (or membership value) between 0 & 1.

→ Thus, the defn. of fuzzy set is a simple extension of the defn. of classical set in which the characteristic function is permitted to have any value between 0 and 1.

→ If values of $\mu_A(x)$ are restricted to 0 or 1 then A reduces to a classical set and $\mu_A(x)$ is the characteristic function of A.

→ X : referred to as universe of discourse or simply universe.
It consists of discrete objects (ordered / non-ordered) or continuous space.

ex. 1 fuzzy set with a discrete non ordered universe.

Let X = {Mumbai, Pune, Nagpur} be the set of cities one may choose to live in.

The fuzzy set C = "desirable city to live in" may be described as,

C = {(Mumbai, 0.9), (Pune, 0.8), (Nagpur, 0.6)}

Here, the universe of discourse X is discrete and it contains non-ordered objects - in this case - three cities in the state of Maharashtra.

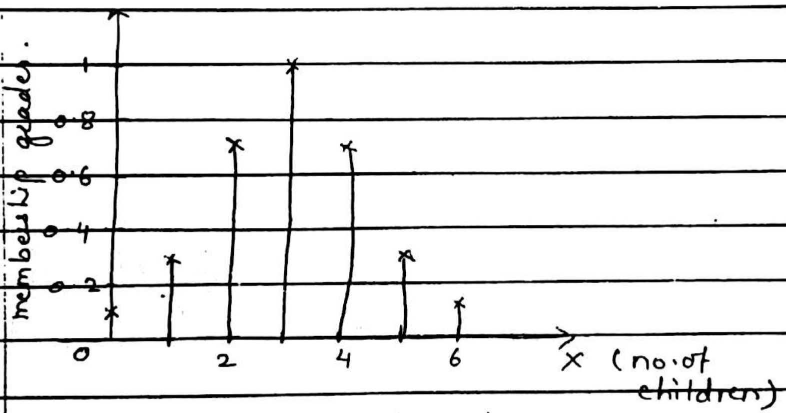
The membership grades listed above are quite subjective, you can come up with three different but legitimate values to reflect his/her preference.

ex 2 : fuzzy set with discrete ordered universe.

Let $X = \{0, 1, 2, 3, 4, 5, 6\}$ be the set of numbers of children, a family may choose to have.
Then,
the fuzzy set A = "sensible number of children in a family".
may be described as

$$A = \{(0, 0.1), (1, 0.3), (2, 0.7), (3, 1), (4, 0.7), (5, 0.3), (6, 0.1)\}.$$

Here universe X is discrete ordered.



MF on discrete universe.

Again here the membership grades are subjective measures.

Ex. 3 : Fuzzy sets with a continuous universe.

Let $X = R^+$ be the set of possible ages of human beings.

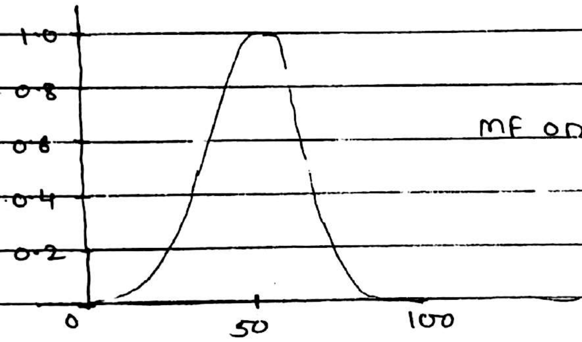
Then the fuzzy set

$$B = \text{"about 50 yrs old"}$$

may be expressed as

$$B = \{ (x, \mu_B(x)) / x \in X \}$$

where $\mu_B(x) = \dfrac{1}{1 + \left(\dfrac{x-50}{10}\right)^4}$



MF on a continuous universe.

From the preceding examples, it is clear that the construction of the set depends on two things - identification of a suitable universe of discource

- specification of an appropriate MF. membership function.

→ The specification of MF is subjective, that means, the MF specified for the same concept by different persons may vary considerably.
(ex. - "sensible no. of children in a family")

→ This subjectivity comes from the individual differences in perceiving or expressing abstract concepts & has little to do with randomness.

∴ the subjectivity and non randomness of fuzzy sets is the primary difference between the study of fuzzy sets and probability theory, which deals with objective treatment of random phenomena.

→ For simplicity of notation, we now introduce an alternative way of denoting a fuzzy set.

A fuzzy set A ~~is~~ can be denoted as,

$$A = \begin{cases} \sum_{x_i \in X} \mu_A(x_i)/x_i & \text{if } X \text{ is a collection of discrete objects.} \\ \int_X \mu_A(x)/x & \text{if } X \text{ is a continuous space (usually the real line } R\text{).} \end{cases}$$

The Σ & $\int$ sign in above equation stand for union of $(x, \mu_A(x))$ pairs.
they do not indicate summation or integration.
Also, "/" sign is only a marker and does not imply division.

∴ alternative expressions →

~~ex~~ $C = 0.9/\text{mumbai} + 0.8/\text{pune} + 0.6/\text{Nagpur}$.

$A = 0.1/0 + 0.3/1 + 0.7/2 + 1.0/3 + 0.7/4 + 0.3/5 + 0.1/6$.

$$B = \int_{R^+} \frac{1}{1+\left(\frac{x-50}{10}\right)^4} \Big/ x$$

In practice, when the universe of discourse X is a continuous space (the real line R or its subset), we usually partition X into several fuzzy sets whose MF's cover X in a more or less uniform manner.

These fuzzy sets, which usually carry names that conform to adjectives appearing in our daily linguistic usage such as "large", "medium" or "small" are called linguistic values or linguistic labels.
Thus the universe of discourse X is often called the linguistic variable.

ex: linguistic variables and linguistic values.

Let X = "age".
then we define fuzzy sets "young", "middle aged" & "old" that are characterised by MF, $\mu_{old}(x)$, $\mu_{middleaged}(x)$ and $\mu_{old}(x)$. respectively.

A fuzzy set is uniquely specified by its membership function. To describe membership functions more specifically, we shall define the nomenclature used in the literature.

Def^n: **Support** –

The support of a fuzzy set A is the set of all points x in X such that $\mu_A(x) > 0$:

$$\text{Support}(A) = \{ x \mid \mu_A(x) > 0 \}$$

Def^n – **Core** –

The core of a fuzzy set A is the set of all points x in X such that $\mu_A(x) = 1$

$$\text{Core}(A) = \{ x \mid \mu_A(x) = 1 \}$$

Def^n – **Normality** –

A fuzzy set A is normal if its core is non-empty.
In other words, we can always find a point $x \in X$ such that $\mu_A(x) = 1$.

Def^n – **cross over points** –

A cross over point of a fuzzy set A is a point $x \in X$ at which $\mu_A(x) = 0.5$

$$\text{crossover}(A) = \{ x \mid \mu_A(x) = 0.5 \}.$$

Defⁿ- fuzzy singleton -

A fuzzy set whose support is a single point in X which with $\mu_A(x) = 1$ is called a fuzzy singleton.

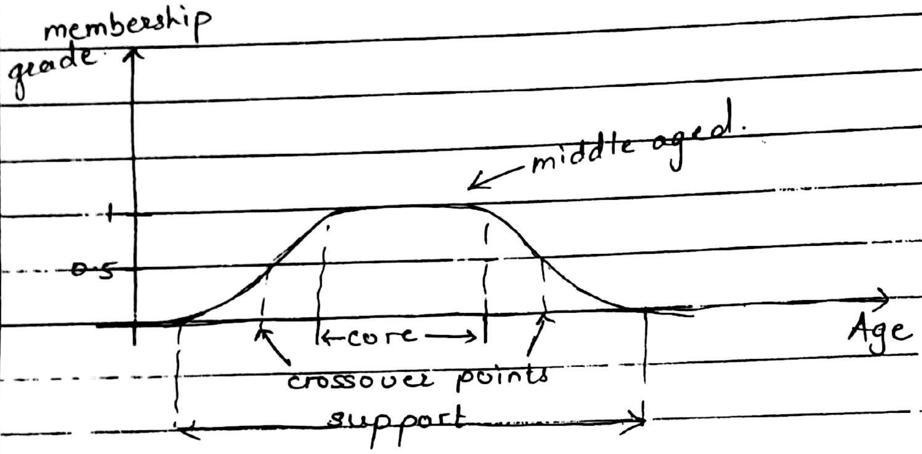


fig. :- fuzzy set "middle aged".

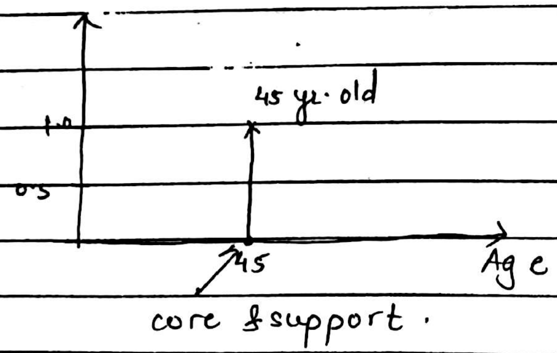


fig. :- Fuzzy singleton "45 yr old".

Def^n - α-cut, strong α-cut

The α-cut or α-level set of a fuzzy set A is a crisp set defined by

$$A_\alpha = \{ x \mid \mu_A(x) \geq \alpha \}$$

strong α-cut or strong α-level set are defined similarly -

$$A_\alpha' = \{ x \mid \mu_A(x) > \alpha \}$$

using the notation for a level set, we can express the support and core of a fuzzy set A as,

$$\text{support}(A) = A_0'$$

and

$$\text{core}(A) = A_1 \quad \text{respectively.}$$

Def^n - Convexity -

A fuzzy set A is convex iff for any $x_1, x_2 \in X$ and any $\lambda \in [0,1]$

$$\mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \min\{\mu_A(x_1), \mu_A(x_2)\}$$

Alternatively, A is convex if all its α-level sets are convex.

Hebbian learning problem.

c̄ binary and continuous activation function.

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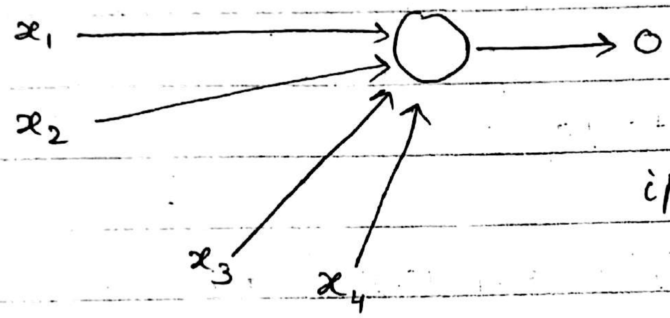
Assume the n/w shown below.



$$\text{i/p } x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

initial weight vector

$$w' = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0.5 \end{bmatrix}$$

needs to be trained using set of 3 i/p vectors as,

$$x_1 = \begin{bmatrix} 1 \\ -2 \\ 1.5 \\ 0 \end{bmatrix} \quad x_2 = \begin{bmatrix} 1 \\ -0.5 \\ -2 \\ -1.5 \end{bmatrix} \quad x_3 = \begin{bmatrix} 0 \\ 1 \\ -1 \\ 1.5 \end{bmatrix}$$

for an arbitrary choice of learning constant $c = 1$. Find the final wt. after one iteration of training using

a) Bipolar binary activation function.

b) Bipolar continuous activation function.

Step 1 :-

$$net^1 = w^{1t} \cdot x_1 = [1 \ -1 \ 0 \ 0.5] \begin{bmatrix} 1 \\ -2 \\ 1.5 \\ 0 \end{bmatrix}$$

$$= 1 + 2 + 0 + 0$$

$$= 3$$

$$o^1 = f(net^1) = sgn(3) = +1$$

$$\therefore \Delta w^1 = c \cdot r \cdot x_1 \qquad \because r = o^1$$

$$= c \cdot o^1 x_1$$

$$= 1 \cdot 1 \cdot \begin{bmatrix} 1 \\ -2 \\ 1.5 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 1.5 \\ 0 \end{bmatrix}$$

$$\therefore w^2 = w^1 + \Delta w^1$$

$$= \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0.5 \end{bmatrix} + \begin{bmatrix} 1 \\ -2 \\ 1.5 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ 1.5 \\ 0.5 \end{bmatrix}$$

Step 2 :-

$$net^2 = w^{2t} \cdot x_2 = [2 \ -3 \ 1.5 \ 0.5] \begin{bmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \end{bmatrix}$$

$$= 2 + 1.5 - 3 - 0.75$$

$$= -0.25$$

$$\therefore o^2 = f(net^2) = sgn(-0.25) = -1$$

Scanned by CamScanner

$$\therefore \Delta w^2 = c \cdot r \cdot x_2$$
$$= 1 \cdot (-1) \cdot \begin{bmatrix} 1 \\ -0.5 \\ -2 \\ -1.5 \end{bmatrix} = \begin{bmatrix} -1 \\ 0.5 \\ 2 \\ 1.5 \end{bmatrix}$$

$$\therefore w^3 = w^2 + \Delta w^2$$
$$= \begin{bmatrix} 2 \\ -3 \\ 1.5 \\ 0.5 \end{bmatrix} + \begin{bmatrix} -1 \\ 0.5 \\ 2 \\ 1.5 \end{bmatrix} = \begin{bmatrix} 1 \\ -2.5 \\ 3.5 \\ 2.0 \end{bmatrix}$$

step3 :-

$$net^3 = w^{3t} \cdot x_3 = \begin{bmatrix} 1 & -2.5 & 3.5 & 2.0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ -1 \\ 1.5 \end{bmatrix}$$
$$= -2.5 - 3.5 + 3$$
$$= -3$$

$$\therefore o^3 = f(net^3) = sgn(-3) = -1$$

$$\therefore \Delta w^3 = c \cdot o^3 x_3$$
$$= 1 \cdot (-1) \cdot \begin{bmatrix} 0 \\ +1 \\ -1 \\ 1.5 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 1 \\ -1.5 \end{bmatrix}$$

$$\therefore w^4 = w^3 + \Delta w^3$$
$$= \begin{bmatrix} 1 \\ -2.5 \\ 3.5 \\ 2 \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \\ 1 \\ -1.5 \end{bmatrix} = \begin{bmatrix} 1 \\ -3.5 \\ 4.5 \\ 0.5 \end{bmatrix}$$

// final weights.

$$o = \frac{2}{1+e^{-\lambda net}} - 1$$

Let $\lambda = 1$

Step 1:-

$$net^1 = w^{1t} x_1 = \begin{bmatrix} 1 & -1 & 0 & 0.5 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 1.5 \\ 0 \end{bmatrix}$$

$$= 3$$

$$\therefore o^1 = \frac{2}{1+e^{-\lambda net^1}} - 1$$

$$= \frac{2}{1+e^{-3}} - 1$$

$$= 0.905$$

$$\therefore \Delta w^1 = c \cdot r \cdot x = c \cdot o^1 \cdot x_1$$

$$= 1 \cdot 0.905 \begin{bmatrix} 1 \\ -2 \\ 1.5 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0.905 \\ -1.81 \\ 1.3575 \\ 0 \end{bmatrix}$$

$$\therefore w^2 = w^1 + \Delta w^1$$

$$= \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0.5 \end{bmatrix} + \begin{bmatrix} 0.905 \\ -1.81 \\ 1.3575 \\ 0 \end{bmatrix} = \begin{bmatrix} 1.905 \\ -2.81 \\ 1.3575 \\ 0.5 \end{bmatrix}$$

ep 2:- $net^2 = w^{2t} \cdot x_2 = [1.905 \quad -2.81 \quad 1.3575 \quad 0.5] \begin{bmatrix} 1 \\ -0.5 \\ -2 \\ -1.5 \end{bmatrix}$

$= -0.154$

$$o^2 = \frac{2}{1+e^{0.154}} - 1$$

$= \cancel{-0.077} \quad -0.0768$

$\therefore \Delta w^2 = c \cdot o^2 \cdot x_2 = (1)(-0.0768) \cdot \begin{bmatrix} 1 \\ -0.5 \\ -2 \\ -1.5 \end{bmatrix}$

$$= \begin{bmatrix} -0.0768 \\ 0.0384 \\ 0.1536 \\ 0.1152 \end{bmatrix}$$

$$\therefore w^3 = w^2 + \Delta w^2 = \begin{bmatrix} 1.905 \\ -2.81 \\ 1.3575 \\ 0.5 \end{bmatrix} + \begin{bmatrix} -0.0768 \\ 0.0384 \\ 0.1536 \\ 0.1152 \end{bmatrix} = \begin{bmatrix} 1.8282 \\ -2.7716 \\ 1.5111 \\ 0.6152 \end{bmatrix}$$

p3:-

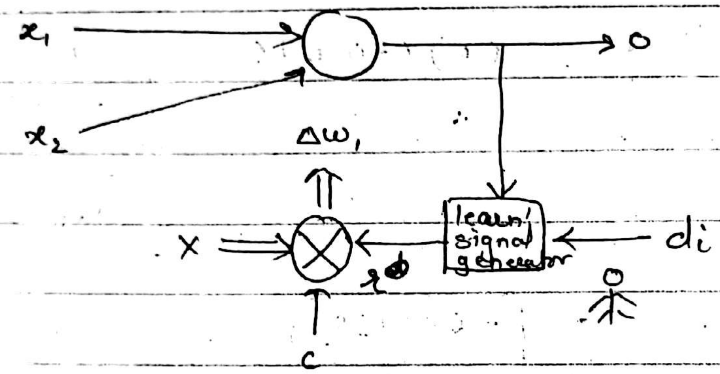
$net^3 = w^{3t} x_3 = [1.8282 \quad -2.7716 \quad 1.5111 \quad 0.6152] \cdot \begin{bmatrix} 0 \\ 1 \\ -1 \\ 1.5 \end{bmatrix}$

$= -3.3599$

$$o^3 = \frac{2}{1+e^{3.3599}} - 1 = -0.9328$$

$\therefore \Delta w^3 = c \cdot o^3 \cdot x_2 = (1) \cdot (-0.9328) \cdot [0 \quad\quad [0$

neuron network as shown below have been im[...]
starting $\bar{c}$ $w^1 = [1 \ \ -1]^t$ for $c = 1$.
using inputs -
$x_1 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ $x_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $x_3 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$, $x_4 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$



sol^n :- A) weights for bipolar binary f(net)

step I

i) $net^1 = w^{1t} \cdot x_1 = [1 \ \ -1] \cdot \begin{bmatrix} 1 \\ -2 \end{bmatrix} = 3$

$o^1 = f(net^1) = sgn(3) = +1$

$\Delta w^1 = c \cdot r \cdot x = c \cdot o^1 x_1$

$= (1) \cdot (1) \cdot \begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

$\therefore w^2 = w^1 + \Delta w^1$

$= \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$

) $net^2 = w^{2t} \cdot x_2 = \begin{bmatrix} 2 & -3 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = -3$

$\therefore o^2 = sgn(-3) = -1$

$\therefore \Delta w^2 = c \cdot r \cdot x = c \cdot o^2 \cdot x_2$

$= (1) \cdot (-1) \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$

$\therefore w^3 = w^2 + \Delta w^2$

$= \begin{bmatrix} 2 \\ -3 \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \\ -4 \end{bmatrix}$

i) $net^3 = w^{3t} \cdot x_3 = \begin{bmatrix} 2 & -4 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 3 \end{bmatrix} = 4 - 12 = -8$

$\therefore o^3 = -1$

$\therefore \Delta w^3 = c \cdot o^3 \cdot x_3 = (1) \cdot (-1) \cdot \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} -2 \\ -3 \end{bmatrix}$

$\therefore w^4 = w^3 + \Delta w^3$

$= \begin{bmatrix} 2 \\ -4 \end{bmatrix} + \begin{bmatrix} -2 \\ -3 \end{bmatrix} = \begin{bmatrix} 0 \\ -7 \end{bmatrix}$

v) $net^4 = w^{4t} \cdot x_4 = \begin{bmatrix} 0 & -7 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -1 \end{bmatrix} = 7$

$\therefore o^4 = +1$

$\therefore \Delta w^4 = c \cdot o^4 \cdot x_4 = (1) \cdot (1) \cdot \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$\therefore w^5 = w^4 + \Delta w^4$

$= \begin{bmatrix} 0 \\ -7 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ -8 \end{bmatrix}$

$\begin{bmatrix} -2 \end{bmatrix}$

$\therefore O^5 = +1$

$\Delta w^5 = c \cdot O^5 \cdot x_1 = (1) \cdot (1) \cdot \begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

$\therefore w^6 = w^5 + \Delta w^5$

$= \begin{bmatrix} 1 \\ -8 \end{bmatrix} + \begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 2 \\ -10 \end{bmatrix}$

ii) $net^6 = w^{6t} \cdot x_2 = \begin{bmatrix} 2 & -10 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = -10$

$\therefore O^6 = -1$

$\therefore \Delta w^6 = c \cdot O^6 \cdot x_2 = (1)(-1) \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$

$\therefore w^7 = w^6 + \Delta w^6$

$= \begin{bmatrix} 2 \\ -10 \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \\ -11 \end{bmatrix}$

iii) $net^7 = w^{7t} \cdot x_3 = \begin{bmatrix} 2 & -11 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 3 \end{bmatrix} = 4 - 33 =$

$\therefore O^7 = -1$

$\therefore \Delta w^7 = c \cdot O^7 \cdot x_3 = (1)(-1) \cdot \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} -2 \\ -3 \end{bmatrix}$

$\therefore w^8 = w^7 + \Delta w^7 = \begin{bmatrix} 2 \\ -11 \end{bmatrix} + \begin{bmatrix} -2 \\ -3 \end{bmatrix} = \begin{bmatrix} 0 \\ -14 \end{bmatrix}$

iv) $net^8 = w^{8t} \cdot x_4 = \begin{bmatrix} 0 & -14 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -1 \end{bmatrix} = 0 + 14 = 14$

$O^8 = +1$

$\therefore \Delta w^8 = c \cdot O^8 \cdot x_4 = (1)(1) \cdot \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$\therefore w^9 = w^8 + \Delta w^8 = \begin{bmatrix} 1 \\ -15 \end{bmatrix}$.

III

i) $net^9 = w^{9t} \cdot x_1 = \begin{bmatrix} 1 & -15 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -2 \end{bmatrix} = 1 + 30 = 31$

$\therefore o^9 = +1 \quad \therefore \Delta w^9 = (1) \cdot (1) \cdot \begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

$\therefore w^{10} = \begin{bmatrix} 1 \\ -15 \end{bmatrix} + \begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 2 \\ -17 \end{bmatrix}$

ii) $net^{10} = w^{10t} \cdot x_2 = \begin{bmatrix} 2 & -17 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = 0 - 17 = -17$

$\therefore o^{10} = -1 \quad \therefore \Delta w^{10} = (1) \cdot (-1) \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$

$\therefore w^{11} = \begin{bmatrix} 2 \\ -17 \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \\ -18 \end{bmatrix}$

iii) $net^{11} = w^{11t} \cdot x_3 = \begin{bmatrix} 2 & -18 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 3 \end{bmatrix} = 4 - 54 = -50$

$\therefore o^{11} = -1 \quad \therefore \Delta w^{11} = (1)(-1) \cdot \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} -2 \\ -3 \end{bmatrix}$

$\therefore w^{12} = \begin{bmatrix} 0 \\ -21 \end{bmatrix}$

iv) $net^{12} = w^{12t} \cdot x_4 = \begin{bmatrix} 0 & -21 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -1 \end{bmatrix} = 21$

$\therefore o^{12} = +1 \quad \therefore \Delta w^{12} = (1)(1) \cdot \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$\therefore w^{13} = \begin{bmatrix} 1 \\ -22 \end{bmatrix}$

step I.

i) $net' = \frac{17}{1+e^{.7}}$

i) $net^1 = w^{1t} \cdot x_1 = [1 \ \ -1] \cdot \begin{bmatrix} 1 \\ -2 \end{bmatrix} = 3$

$O^1 = \frac{2}{1+e^{-3}} - 1 = 0.905$

$\Delta w^1 = c \cdot O^1 \cdot x_1 = (1) \cdot (0.905) \cdot \begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 0.905 \\ -1.81 \end{bmatrix}$

$\therefore w^2 = w^1 + \Delta w^1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \begin{bmatrix} 0.905 \\ -1.81 \end{bmatrix} = \begin{bmatrix} 1.905 \\ -2.81 \end{bmatrix}$

ii) $net^2 = w^{2t} \cdot x_2$
$= [1.905 \ \ -2.81] \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = -2.81$

$\therefore O^2 = \frac{2}{1+e^{2.81}} - 1 = -0.8864$

$\therefore \Delta w^2 = c \cdot O^2 \cdot x_2 = (1) \cdot (-0.8864) \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -0.8864 \end{bmatrix}$

$\therefore w^3 = \begin{bmatrix} 1.905 \\ -2.81 \end{bmatrix} + \begin{bmatrix} 0 \\ -0.8864 \end{bmatrix} = \begin{bmatrix} 1.905 \\ -3.6964 \end{bmatrix}$

iii) $net^3 = w^{3t} \cdot x_3$
$= [1.905 \ \ -3.6964] \cdot \begin{bmatrix} 2 \\ 3 \end{bmatrix} = -7.2792$

$\therefore O^3 = \frac{2}{1+e^{7.2792}} - 1 = -0.9986$

$\therefore \Delta w^3 = (1) \cdot (-0.9986) \cdot \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} -1.9972 \\ -2.9958 \end{bmatrix}$

$\therefore w^4 = \begin{bmatrix} -0.0922 \\ -6.6922 \end{bmatrix}$

iv) $net^4 = w^{4t} \cdot x_4 = [-0.0922 \quad -6.6922] \cdot \begin{bmatrix} 1 \\ -1 \end{bmatrix} = 6.6$

$\therefore o^4 = \dfrac{2}{1+e^{-6.6}} - 1 = 0.9973$

$\therefore \Delta w^4 = (1) \cdot (0.9973) \cdot \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0.9973 \\ -0.9973 \end{bmatrix}$

$\therefore w^5 = \begin{bmatrix} 0.9051 \\ -7.6895 \end{bmatrix}$

p II i) $net^5 = w^{5t} \cdot x_1 = [0.9051 \quad -7.6895] \begin{bmatrix} 1 \\ -2 \end{bmatrix} = 16.2841$

$\therefore o^5 = \dfrac{2}{1+e^{-16.2841}} - 1 = 0.9999 \approx 1$

$\Delta w^5 = c \cdot o^5 \cdot x_1 = (1) \cdot (0.9999) \begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

$\therefore w^6 = w^5 + \Delta w^5 = \begin{bmatrix} 0.9051 \\ -7.6895 \end{bmatrix} + \begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 1.9051 \\ -9.6895 \end{bmatrix}$

ii) $net^6 = w^{6t} \cdot x_2 = [1.9051 \quad -9.6895] \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = -9.6895$

$\therefore o^6 = \dfrac{2}{1+e^{9.6895}} - 1 = -0.9998 \approx -1$

$\Delta w^6 = c \cdot o^6 \cdot x_2 = (1)(-0.9998) \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$

iii) $net^7 = w^{7t} \cdot x_3 = [$

$\therefore w^7 = w^6 + \Delta w^6 = \begin{bmatrix} 1.9051 \\ -9.6895 \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 1.9051 \\ -10.6895 \end{bmatrix}$

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$$= -28.2583$$

$$\therefore o^7 = \frac{2}{1+e^{28.2583}} - 1 = -1$$

$$\therefore \Delta w^7 = c \cdot o^7 \cdot x_3 = (1)(-1) \cdot \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} -2 \\ -3 \end{bmatrix}$$

$$\therefore w^8 = w^7 + \Delta w^7 = \begin{bmatrix} 1.9051 \\ -10.6895 \end{bmatrix} + \begin{bmatrix} -2 \\ -3 \end{bmatrix} = \begin{bmatrix} -0. \\ -13. \end{bmatrix}$$

iv) $$net^8 = w^{8t} \cdot x_4 = [-0.0949 \;\; -13.6895] \cdot \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$= 13.5946$$

$$\therefore o^8 = \frac{2}{1+e^{-13.5946}} - 1 = 0.9999 \approx 1$$

$$\therefore \Delta w^8 = c \cdot o^8 \cdot x_4 = (1)(1) \cdot \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\therefore w^9 = w^8 + \Delta w^8 = \begin{bmatrix} -0.0949 \\ -13.6895 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0. \\ -14. \end{bmatrix}$$

Q. Perceptron learning P. 2.17 Zurada pg: 87.

Implement the peceptron rule training of the n/w from fig. 2.23 using f(net) = sgn(net), c = 1: and the following data specifying the initial weights w^1 and the two training pairs.

$$w^1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \left(x_1 = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}, d_1 = -1 \right),$$

$$\left(x_2 = \begin{bmatrix} 0 \\ -1 \\ -1 \end{bmatrix}, d_2 = 1 \right)$$ ∴ Repeat the training sequence $(x_1, d_1), (x_2, d_2)$ until two correct responses in a row are achieved.

List net^k values obtained during training.

Sol^n:- i) $net^1 = w^{1t} \cdot x_1 = [0 \ 1 \ 0] \cdot \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} = 1$

$\therefore o^1 = sgn(net^1) = +1$

$$\therefore \Delta w^1 = c \cdot r \cdot x$$
$$= c \cdot (d_1 - o^1) \cdot x_1$$
$$= -2c x_1$$
$$= -2 \cdot (1) \cdot \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -4 \\ -2 \\ 2 \end{bmatrix}$$

$\because d_1 = -1$ & $o^1 = +1$
∴ update weights.

$$\therefore w^2 = \begin{bmatrix} \cancel{2}\ 0 \\ \cancel{1}\ 1 \\ \cancel{-1}\ 0 \end{bmatrix} + \begin{bmatrix} -4 \\ -2 \\ 2 \end{bmatrix} = \begin{bmatrix} -4 \ \cancel{-2} \\ -1 \ \cancel{1} \\ 2 \ \cancel{1} \end{bmatrix}$$

ii) $net^2 = w^{2t} \cdot x_2 = [-4 \ -1 \ 2] \cdot \begin{bmatrix} 0 \\ -1 \\ -1 \end{bmatrix} = \cancel{0}\ 1 - 2 = -1$

$$\begin{bmatrix} -1 \\ 2 \end{bmatrix} + \begin{bmatrix} -2 \\ -2 \end{bmatrix} = \begin{bmatrix} -3 \\ 40 \end{bmatrix}$$

$$net^3 = w^{3t} \cdot x_1 = [-4 \quad -3 \quad 0] \cdot \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} = -8 - 3$$
$$= -11$$

$\therefore o^3 = sgn(-11) = -1$

and $d_1 = -1$

They are same

$\therefore$ No need to update weights.

$w^4 =$ $\therefore \Delta w^3 = [0 \quad 0 \quad 0]^t$

$\therefore w^4 = w^3 = [-4 \quad -3 \quad 0]^t$.

List of net^k values obtained during training -

$net^1 = 1$

$net^2 = -1$

$net^3 = -11$.

$net^4 = +3$.

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-3+0

-11

Q. Solve the following classification problem using perceptron rule. Apply each input vector in order, for as many repeatations as it takes to ensure that the problem is solved.

$$\left(x_1 = \begin{bmatrix} 2 \\ 2 \end{bmatrix}, d_1 = 0 \right), \left(x_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}, d_2 = 1 \right)$$

$$\left(x_3 = \begin{bmatrix} -2 \\ 2 \end{bmatrix}, d_3 = 0 \right), \left(x_4 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, d_4 = 1 \right)$$

$b^0 = 0$, $c = 1$.

ng

Sol^n:-

as values of d are 0 or 1

∴ activation function is unipolar function.

i) $net^1 = w^{0t} \cdot x_1 = [0 \ \ 0] \cdot \begin{bmatrix} 2 \\ 2 \end{bmatrix} = 0$

$$o' = sgn(net' + b^0)$$
$$= sgn(0 + 0)$$
$$= +1$$

compare o' $\bar{c}$ d_1 . They are different

∴ update weights

error $e = d_1 - o'$

$= 0 - 1 = -1$

$$\therefore \Delta w^0 = c \cdot (d_1 - o') \cdot x_i$$
$$= (1) \cdot (-1) \cdot \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} -2 \\ -2 \end{bmatrix}$$

$$o^2 = sgn(net^2 + b^1)$$ $b^1 = b^0 +$

$= 0 +$

$$= sgn(2 - 1)$$
$$= +1$$

compare o^2 and d_2 They are same
∴ No change in weights

$$\therefore w^2 = w^1 = \begin{bmatrix} -2 \\ -2 \end{bmatrix}$$

$e = d_2 - o^2 = 0$ ∴ $b^2 = b^1 + e$
$= -1$

iii $net^3 = w^{2t} \cdot x_3 = \begin{bmatrix} -2 & -2 \end{bmatrix} \begin{bmatrix} -2 \\ 2 \end{bmatrix} = 4 - 4$

$$o^3 = sgn(net^3 + b^2)$$
$$= sgn(0 - 1) = \cancel{+1}$$ $\cancel{e = d_3 - o^3}$
$$= sgn(-1) = 0$$

compare o^3 c̄ d_3 They are ~~diff~~ sa
~~∴ update weights~~

$$\cancel{\Delta w^3 = \alpha \cdot c \cdot (d_3 - o^3) \cdot x_3}$$
$$\cancel{= (1) \cdot (1) \cdot \begin{bmatrix} -2 \\ 2 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}}$$

∴ $w^4 =$

∴ No updates.

$$\therefore w^3 = w^2 = \begin{bmatrix} -2 \\ -2 \end{bmatrix}$$ $b^3 = b^2 = -1$

iv) $net^4 = \omega^{3t} \cdot x_4 = \begin{bmatrix} -2 & -2 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = 2 - 2 = 0$

$\therefore O^4 = sgn(net^4 + b^3)$

$= sgn(0 + -1) = sgn(-1) = 0.$

$d_4 = 1$

$\therefore$ d_4 & O^4 are different

$\therefore$ update weights.

error $\cdot$ $e = d_4 - O^4 = 1 - 0 = 1$

$\Delta\omega^3 = c \cdot (d_4 - O^4) \cdot x_4$

$= (1)(1) \cdot \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

$\omega^4 = \omega^3 + \Delta\omega^3 = \begin{bmatrix} -2 \\ -2 \end{bmatrix} + \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ -1 \end{bmatrix}$

$b^4 = b^3 + e = -1 + 1 = 0$

v) $net^5 = \omega^{4t} \cdot x_1 = \begin{bmatrix} -3 & -1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 2 \end{bmatrix} = -6 - 2 = \cancel{-4}\ -8$

$O^5 = sgn(net^5 + b^4) =$

$= sgn(-8 + 0)$

$= 0$

compare $O^5 = 0$ $\bar{c}$ $d_1 = 0$

they are same

$\therefore$ No updates

$o^6 = sgn(net^6 + b^5)$
$= sgn(-1+0)$
$= 0$

compare $o^6 = 0$ $\bar{c}$ $d_2 = 1$
they are diff
$\therefore$ update weights
error $e = d_2 - o^6 = 1 - 0 = 1$

$\therefore \Delta w^5 = c \cdot e \cdot x_2$
$= (1)(1) \cdot \begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

$\therefore w^6 = w^5 + \Delta w^5 = \begin{bmatrix} -3 \\ -1 \end{bmatrix} + \begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} -2 \\ -3 \end{bmatrix}$

$b^6 = b^5 + e = 0 + 1 = 1$

vii) $net^7 = w^{6t} \cdot x_3 = [-2 \quad -3] \begin{bmatrix} -2 \\ 2 \end{bmatrix} = 4$
$= $

$o^7 = sgn(net^7 + b^6)$
$= sgn(-2+1)$
$= 0$

compare $o^7 = 0$ $\bar{c}$ $d_3 = 0$
they are same
$\therefore$ No updates

$\therefore w^7 = w^6 = \begin{bmatrix} -2 \\ -3 \end{bmatrix}$

$b^7 = b^6 = 1$

iii) net^8

$o^8 =$
$=$
$=$
compa
they
$\therefore$

Δ

Δw^7

$\therefore w^8 =$

$b^8 =$

viii) $net^8 = w^{7t} x_4 = \begin{bmatrix} -2 & -3 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = 2 - 3 = -1$

$o^8 = sgn(net^8 + b^7)$
$= sgn(-1 + 1)$
$= 0$

compare $o^8 = 0$ $\bar{c}$ $d_4 = 1$
they are diff.
$\therefore$ update weights.

$\therefore$ error $e = d_4 - o^8 = 1 - 0 = 1$

$\Delta w^7 = c \cdot e \cdot x_4$
$= (1) \cdot (1) \cdot \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

$\therefore w^8 = w^7 + \Delta w^7 = \begin{bmatrix} -2 \\ -3 \end{bmatrix} + \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ -2 \end{bmatrix}$

$b^8 = b^7 + e = 1 + 1 = 2$

- $\Delta w_{ij} = c \cdot (d_i - O_i) f'(net_i) x_j$

$j = 1, 2, \cdots n.$

- initial weights - Any
- supervised learning
- Neuron char - continuous
- feedforward.

→ The delta learning rule is valid only f[or] continuous activation function and in supervi[sed] training mode.

→ The learning signal for this rule is called 'delta' and is defined as follows -

$$r = [d_i - f(w_i^t x)] \cdot f'(w_i^t x) \quad \text{---(1)}$$

the term $f'(w_i^t x)$ is → derivative of the activation functio[n] $f(net)$.

$\because net = w_i^t x.$

→ This learning rule can be easily deriv[ed] from condition of least squared error between O_i and d_i

$$E = \frac{1}{2}(d_i - O_i)^2 \quad \text{----(2)}$$

OR $$E = \frac{1}{2}[d_i - f(w_i^t x)]^2 \quad \text{---(}$$

→ calculating the gradient vector wrt w_i of the squared error :

$$\nabla E = -(d_i - O_i)\, f'(w_i^t x)\, x \qquad \text{----(4)}$$

→ Since the minimization of the error requires the weight changes to be in the negative gradient direction, we take

$$\Delta w_i = -\eta \nabla E \qquad \text{-----(5)}$$

where,

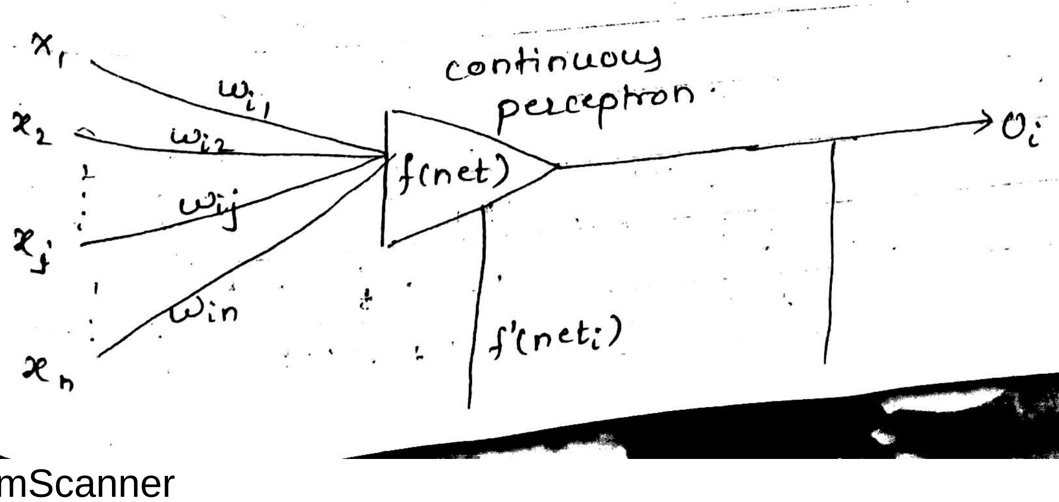
$\eta \rightarrow$ +ve const

from (4) & (5)

$$\Delta w_i = \eta\,(d_i - O_i)\, f'(net_i)\, x \qquad \text{---(6)}$$

→ considering the use of general learning rule $\Delta w = c \cdot r \cdot x$ and using the delta learning signal in (1), weight adjustment becomes,

$$\Delta w_i = c \cdot (d_i - O_i)\, f'(net_i)\, x \qquad \text{---(7)}$$



$\overline{2}$ function

b) $f'(net) = o(1-o)$ for unipolar continuous

Sol^n: —

a) To prove $f'(net) = \frac{1}{2}(1-o^2)$.

$R.H.S. = \frac{1}{2}(1-o^2)$

for bipolar continuous function,

$$o = \frac{2}{1+e^{-\lambda net}} - 1$$

assuming $\lambda = 1$

$$RHS = \frac{1}{2}(1-o^2)$$

$$= \frac{1}{2}\left[1-\left(\frac{2}{1+e^{-net}}-1\right)^2\right]$$

$$= \frac{1}{2}\left[1-\left(\frac{4}{(1+e^{-net})^2} - 2\cdot\frac{2}{1+e^{-net}} + 1\right)\right]$$

$$= \frac{1}{2}\cdot\frac{4}{1+e^{-net}}\left(\frac{-1}{1+e^{-net}} + 1\right)$$

$$= \frac{2}{1+e^{-net}}\left(\frac{-1+1+e^{-net}}{1+e^{-net}}\right)$$

$$= \frac{2}{1+e^{-net}}\cdot\frac{+e^{-net}}{1+e^{-net}}$$

$$= \frac{2e^{-net}}{(1+e^{-net})^2}$$

Now

$f(ne$

$f'(n$

Now LHS = $f'(net)$

$$f(net) = \frac{2}{1+e^{-net}} - 1$$

$$f'(net) = \frac{d}{dnet}\left(\frac{2}{1+e^{-net}} - 1\right)$$

$$= \frac{d}{dnet}\left(\frac{2}{1+e^{-net}}\right) - 0$$

$$= \frac{1+e^{-net} \cdot \frac{d}{dnet}(2) - 2 \cdot \frac{d}{dnet}(1+e^{-net})}{(1+e^{-net})^2}$$

$$= \frac{-2 \cdot -e^{-net}}{(1+e^{-net})^2}$$

$$= \frac{2e^{-net}}{(1+e^{-net})^2}$$

Sol^n — RHS = $O(1-O)$

for unipolar continuous f^n.

$$O = \frac{1}{1+e^{-\lambda net}}, \quad \text{assume } \lambda = 1$$

$$\therefore RHS = O(1-O)$$

$$= \frac{1}{1+e^{-net}}\left(1 - \frac{1}{1+e^{-net}}\right)$$

$$= \frac{1}{1+e^{-net}} - \frac{1}{(1+e^{-net})^2}$$

$$= \frac{1+e^{-net}-1}{(1+e^{-net})^2}$$

$$= \frac{e^{-net}}{(1+e^{-net})^2}$$

$$L.H.S. = f'(net)$$

$$= \frac{d}{dnet}\left(\frac{1}{1+e^{-net}}\right)$$

$$= \frac{1+e^{-net} \cdot \frac{d}{dnet}(1) - 1 \cdot \frac{d}{dnet}(1+}{(1+e^{-net})^2}$$

$$= \frac{-(-e^{-net})}{(1+e^{-net})^2}$$

$$= \frac{e^{-net}}{(1+e^{-net})^2}$$

$\lambda = 1$

SONAL.

Q. Perform two training steps of the network using the delta learning rule for $\lambda = 1$ and $c = 0.25$. Train the network using the following data pairs.

$$\left(x_1 = \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}, d_1 = -1\right), \left(x_2 = \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix}, d_2 = 1\right)$$

The initial weights are $w' = [1 \; 0 \; 1]^t$

Hint: $f'(net) = \frac{1}{2}(1 - O^2)$.

Sol^n:-

$$\Delta w = c \cdot (d_i - O_i) \cdot f'(net) \cdot x.$$

i) $$net' = w'^t \cdot x_1 = [1 \; 0 \; 1]^t \cdot \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix} = 2 - 1 = 1$$

$$\therefore O' = f(net') = \frac{2}{1 + e^{-\lambda net'}} - 1$$

$$= \frac{2}{1 + e^{-1}} - 1$$

$$= 0.462$$

$1 + e^{-net}$

$$f'(net') = \frac{1}{2}(1 - O^2)$$

$$= 0.5(1 - 0.462^2)$$

$$= 0.393$$

Now,

$$\Delta w' = c \cdot (d_1 - O_1) f'(net') \cdot x_1$$

$$= 0.25(-1 - 0.462)(0.393) \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$$

$=$

$$= \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} -0.288 \\ 0 \\ 0.144 \end{bmatrix} = \begin{bmatrix} 0.712 \\ 0 \\ 1.144 \end{bmatrix}$$

ii) $net^2 = w^{2t} \cdot x_2 = [0.712 \quad 0 \quad 1.144] \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix}$

$$= -0.432$$

$$O^2 = \frac{2}{1 + e^{+0.432}} - 1$$

$$= -0.2127$$

$$f'(net^2) = \frac{1}{2}(1 - O^2)$$

$$= 0.5(1 - (-0.212)^2)$$

$$= 0.477$$

Now,

$$\Delta w^2 = c \cdot (d_2 - O_2)\, f'(net^2) \cdot x_2$$

$$= 0.25(1 + 0.212) \cdot 0.477 \cdot \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix}$$

$$= 0.144 \cdot \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix} = \begin{bmatrix} 0.144 \\ -0.288 \\ -0.144 \end{bmatrix}$$

$$\therefore w^3 = w^2 + \Delta w^2 = \begin{bmatrix} 0.712 \\ 0 \\ 1.144 \end{bmatrix} + \begin{bmatrix} 0.144 \\ -0.288 \\ -0.144 \end{bmatrix}$$

$$= \begin{bmatrix} \overset{0.856}{\cancel{0.568}} \\ -0.288 \\ 1 \end{bmatrix}$$

Q: Find final weights after 1st iteration

$X_1 = \begin{bmatrix} 1 \\ -2 \\ 0 \\ -1 \end{bmatrix}$ $X_2 = \begin{bmatrix} 0 \\ 1.5 \\ -0.5 \\ -1 \end{bmatrix}$, $X_3 = \begin{bmatrix} -1 \\ 1 \\ 0.5 \\ -1 \end{bmatrix}$ $w^1 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0.5 \end{bmatrix}$

$d_1 = -1$, $d_2 = -1$, $d_3 = 1$, $c = 0.1$, $\lambda = 1$

use bipolar continuous activation function.

Sol^n :-

$\Delta w = c \cdot (d - o) \cdot f'(net) \cdot x$

and $f'(net) = \frac{1}{2} \cdot (1 - o^2)$

Fuzzy set theory

SONAL

Lotfi Zadeh in 1965

fuzziness ⟷ vagueness.

entire real world is complex.
this complexity is arises from uncertainty.
uncertainty based on random processes → probability theory.

uncertainty is characterised by non-random processes
- uncertainty due to partial info.
- due to info that is not fully reliable
- due to inherent imprecision in the language
- due to receipt of info from multiple sources about a problem c is conflicting.

→ fuzzy set theory provides effective problem solving approach.

Fuzzy logic is a form of multivalued logic to deal with reasoning that is approximate rather than precise.

crisp logic — fuzzy logic.

crisp logic	fuzzy logic	
Is water colorless?	Is ram honest?	
	extremly honest	(1)
y/n.	extremly dishonest	(0)
	very honest	(0.85)
	honest at times	(0.4)
↓	↓	
crisp set	fuzzy set.	

— The closer one looks at a real world problem, the fuzzier becomes its solution.

Fuzzy logic — paradigm of computing c̄ words

— a tech. to deal c̄ imprecision & info. granul

— provides a mechanism for representing linguistic constructs such as 'high', low, tall, many, few.

— based on notion of relative graded members ∴ functions of cognitive processes.

Imprecise & vague data → Fuzzy logic system → decisions.

FLS accepting imprecise & vague data & providing a decision.

The utility of fuzzy sets lies in their ability to model uncertain or ambiguous data and to provide suitable decisions.

Fuzzy set operations

1) Union ($\tilde{A} \cup \tilde{B}$)

$$\mu_{\tilde{A} \cup \tilde{B}}(x) = \max \left[\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x) \right] = \mu_{\tilde{A}}(x) \vee \mu_{\tilde{B}}(x) \quad , \forall x \in U$$

$\vee$ ↑ max operator

2) Intersection ($\tilde{A} \cap \tilde{B}$)

$$\mu_{\tilde{A} \cap \tilde{B}}(x) = \min \left[\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x) \right] = \mu_{\tilde{A}}(x) \wedge \mu_{\tilde{B}}(x) \quad , \forall x \in U$$

$\wedge$ ↑ min operator

3) Complement ($\bar{\tilde{A}}$)

$$\mu_{\bar{\tilde{A}}}(x) = 1 - \mu_{\tilde{A}}(x) \quad \forall x \in U.$$

4) Algebraic sum ($\tilde{A} + \tilde{B}$)

$$\mu_{\tilde{A}+\tilde{B}}(x) = \mu_{\tilde{A}}(x) + \mu_{\tilde{B}}(x) - \mu_{\tilde{A}}(x) \cdot \mu_{\tilde{B}}(x)$$

5) Algebraic product ($\tilde{A} \cdot \tilde{B}$)

$$\mu_{\tilde{A} \cdot \tilde{B}}(x) = \mu_{\tilde{A}}(x) \cdot \mu_{\tilde{B}}(x).$$

6) Bounded sum ($\tilde{A} \oplus \tilde{B}$)

$$\mu_{\tilde{A} \oplus \tilde{B}}(x) = \min \left[1, \mu_{\tilde{A}}(x) + \mu_{\tilde{B}}(x) \right]$$

7) Bounded difference ($\tilde{A} \odot \tilde{B}$)

$$\mu_{\tilde{A} \odot \tilde{B}} = \max \left[0, \mu_{\tilde{A}}(x) - \mu_{\tilde{B}}(x) \right]$$

Consider two fuzzy sets

$$\underset{\sim}{A} = \left\{ \frac{1}{2} + \frac{0.3}{4} + \frac{0.5}{6} + \frac{0.2}{8} \right\}$$

$$\underset{\sim}{B} = \left\{ \frac{0.5}{2} + \frac{0.4}{4} + \frac{0.1}{6} + \frac{1}{8} \right\}$$

perform union, intersection, complement & diff. over fuzzy sets A & B.

1) Union :- $\underset{\sim}{A} \cup \underset{\sim}{B} = \max\left[\mu_{\underset{\sim}{A}}(x), \mu_{\underset{\sim}{B}}(x) \right]$

$$= \left\{ \frac{1}{2} + \frac{0.4}{4} + \frac{0.5}{6} + \frac{0.1}{8} \right\}$$

2) Intersection $\underset{\sim}{A} \cap \underset{\sim}{B} = \min\left[\mu_{\underset{\sim}{A}}(x), \mu_{\underset{\sim}{B}}(x) \right]$

$$= \left\{ \frac{0.5}{2} + \frac{0.3}{4} + \frac{0.1}{6} + \frac{0.2}{8} \right\}$$

3) Complement $\bar{\underset{\sim}{A}} = 1 - \mu_{\underset{\sim}{A}}(x)$

$$= \left\{ \frac{0}{2} + \frac{0.7}{4} , \frac{0.5}{6} + \frac{0.8}{8} \right\}$$

$$\bar{\underset{\sim}{B}} = 1 - \mu_{\underset{\sim}{B}}(x)$$

$$= \left\{ \frac{0.5}{2} + \frac{0.6}{4} + \frac{0.9}{6} + \frac{0}{8} \right\}$$

4) Difference

$$\underset{\sim}{A} | \underset{\sim}{B} = \underset{\sim}{A} \cap \bar{\underset{\sim}{B}} = \min\left[\mu_{\underset{\sim}{A}}(x), \mu_{\bar{\underset{\sim}{B}}}(x) \right]$$

$$= \left\{ \frac{0.5}{2} + \frac{0.3}{4} + \frac{0.5}{6} + \frac{0}{8} \right\}$$

$$\underset{\sim}{B} | \underset{\sim}{A} = B \cap \bar{\underset{\sim}{A}} = \min\left[\mu_{\underset{\sim}{B}}(x), \mu_{\bar{\underset{\sim}{A}}}(x) \right]$$

$$= \left\{ \frac{0}{2} + \frac{0.4}{4} + \frac{0.1}{6} + \frac{0.8}{8} \right\}$$

Given two fuzzy sets

$$\underset{\sim}{B_1} = \left\{ \frac{1}{1.0} + \frac{0.75}{1.5} + \frac{0.3}{2.0} + \frac{0.15}{2.5} + \frac{0}{3.0} \right\}$$

$$\underset{\sim}{B_2} = \left\{ \frac{1}{1.0} + \frac{0.6}{1.5} + \frac{0.2}{2.0} + \frac{0.1}{2.5} + \frac{0}{3.0} \right\}$$

find

$$\underset{\sim}{B_1} \cup \underset{\sim}{B_2} = \max\left[\mu_{\underset{\sim}{B_1}}(x), \mu_{\underset{\sim}{B_2}}(x) \right]$$

$$= \left\{ \frac{1}{1.0} + \frac{0.75}{1.5} + \frac{0.3}{2.0} + \frac{0.15}{2.5} + \frac{0}{3.0} \right\}$$

$$\underset{\sim}{B_1} \cap \underset{\sim}{B_2} = \min\left[\mu_{\underset{\sim}{B_1}}(x), \mu_{\underset{\sim}{B_2}}(x) \right]$$

$$= \left\{ \frac{1}{1.0} + \frac{0.6}{1.5} + \frac{0.2}{2.0} + \frac{0.1}{2.5} + \frac{0}{3.0} \right\}$$

$$\overline{\underset{\sim}{B_1}} = 1 - \mu_{\underset{\sim}{B_1}}(x)$$

$$= \left\{ \frac{0}{1.0} + \frac{0.25}{1.5} + \frac{0.7}{2.0} + \frac{0.85}{2.5} + \frac{1}{3.0} \right\}$$

$$\overline{\underset{\sim}{B_2}} = 1 - \mu_{\underset{\sim}{B_2}}(x)$$

$$= \left\{ \frac{0}{1.0} + \frac{0.4}{1.5} + \frac{0.8}{2.0} + \frac{0.9}{2.5} + \frac{1}{3.0} \right\}$$

$$\underset{\sim}{B_1} \mid \underset{\sim}{B_2} = B_1 \cap \overline{B_2}$$

$$= \min\left[\mu_{\underset{\sim}{B_1}}(x), \mu_{\overline{\underset{\sim}{B_2}}}(x) \right]$$

$$= \left\{ \frac{0}{1.0} + \frac{0.4}{1.5} + \frac{0.3}{2.0} + \frac{0.15}{2.5} + \frac{0}{3.0} \right\}$$

$$\overline{\underset{\sim}{B_1} \cup \underset{\sim}{B_2}} = 1 - (\underset{\sim}{B_1} \cup \underset{\sim}{B_2})$$

$$= \left\{ \frac{0}{1.0} + \frac{0.25}{1.5} + \frac{0.7}{2.0} + \frac{0.85}{2.5} + \frac{1}{3.0} \right\}$$

$$\overline{\underset{\sim}{B_1} \cap \underset{\sim}{B_2}} = 1 - (\underset{\sim}{B_1} \cap \underset{\sim}{B_2})$$

$$= \left\{ \frac{0}{1} + \frac{0.4}{} + \frac{0.8}{} + \frac{0.9}{} + \frac{1}{} \right\}$$

$$= \left\{ \frac{1}{1.0} + \frac{0.75}{1.5} + \frac{0.7}{2.0} + \frac{0.85}{2.5} + \frac{1}{3.0} \right\}$$

$$\underset{\sim}{B_2} \cap \bar{\underset{\sim}{B_2}} = \min\left[\mu_{\underset{\sim}{B_2}}(x), \mu_{\bar{\underset{\sim}{B_2}}}(x) \right]$$

$$= \left\{ \frac{0}{1.0} + \frac{0.4}{1.5} + \frac{0.2}{2.0} + \frac{0.1}{2.5} + \frac{1}{3.0} \right\}$$

$$\underset{\sim}{B_2} \cup \bar{\underset{\sim}{B_2}} = \max\left[\mu_{\underset{\sim}{B_2}}(x), \mu_{\bar{\underset{\sim}{B_2}}}(x) \right]$$

$$= \left\{ \frac{1}{1.0} + \frac{0.6}{1.5} + \frac{0.8}{2.0} + \frac{0.9}{2.5} + \frac{1}{3.0} \right\}$$

Consider two fuzzy sets

$$\underset{\sim}{A} = \left\{ \frac{0.2}{1} + \frac{0.3}{2} + \frac{0.4}{3} + \frac{0.5}{4} \right\}$$

$$\underset{\sim}{B} = \left\{ \frac{0.1}{1} + \frac{0.2}{2} + \frac{0.2}{3} + \frac{1}{4} \right\}$$

Algebraic sum $\underset{\sim}{A} + \underset{\sim}{B}$

$$\mu_{\underset{\sim}{A}+\underset{\sim}{B}} = \mu_{\underset{\sim}{A}}(x) + \mu_{\underset{\sim}{B}}(x) - \mu_{\underset{\sim}{A}}(x) \cdot \mu_{\underset{\sim}{B}}(x)$$

$$= \left\{ \frac{0.3 - 0.02}{1} + \frac{0.5 - 0.06}{2} + \frac{0.6 - 0.08}{3} + \frac{1.5 - 0.5}{4} \right\}$$

$$= \left\{ \frac{0.28}{1} + \frac{0.44}{2} + \frac{0.52}{3} + \frac{0.5}{4} \right\}$$

Algebraic product $\underset{\sim}{A} \cdot \underset{\sim}{B}$

$$\mu_{\underset{\sim}{A}\cdot\underset{\sim}{B}} = \mu_{\underset{\sim}{A}}(x) \cdot \mu_{\underset{\sim}{B}}(x)$$

$$= \left\{ \frac{0.02}{1} + \frac{0.06}{2} + \frac{0.08}{3} + \frac{0.5}{4} \right\}$$

Bounded sum $\underset{\sim}{A} \oplus \underset{\sim}{B}$

$$\mu_{\underset{\sim}{A}\oplus\underset{\sim}{B}} = \min\left[1, \mu_{\underset{\sim}{A}}(x) + \mu_{\underset{\sim}{B}}(x) \right]$$

$$= \min\left\{ 1, \left[\frac{0.3}{1} + \frac{0.5}{2} + \frac{0.6}{3} + \frac{\cancel{0.5}\ 1.5}{4} \right] \right\}$$

$$= \left\{ \frac{0.3}{1} + \frac{0.5}{2} + \frac{0.6}{3} + \frac{\cancel{0.5}\ 1.0}{4} \right\}$$

i) Bounded difference $\mu_{\tilde{A} \ominus \tilde{B}}(x)$

$$= \max\left[0, \mu_{\tilde{A}}(x) - \mu_{\tilde{B}}(x)\right]$$

$$= \max\left\{0, \left[\frac{0.1}{1} + \frac{0.1}{2} + \frac{0.2}{3} + \frac{0.5}{4}\right]\right\}$$

$$= \left\{\frac{0.1}{1} + \frac{0.1}{2} + \frac{0.2}{3} + \frac{0.5}{4}\right\}.$$

) The discretised membership functions for a transistor and a register are

$$\mu_{\tilde{T}} = \left\{\frac{0}{0} + \frac{0.2}{1} + \frac{0.7}{2} + \frac{0.8}{3} + \frac{0.9}{4} + \frac{1}{5}\right\}$$

$$\mu_{\tilde{R}} = \left\{\frac{0}{0} + \frac{0.1}{1} + \frac{0.3}{2} + \frac{0.2}{3} + \frac{0.4}{4} + \frac{0.5}{5}\right\}$$

i) Algebraic sum $\tilde{A} + \tilde{B}$

$\cancel{\mu_{\tilde{T}} + \mu_{\tilde{R}} z =}$ $\mu_{\tilde{T}+\tilde{R}}(x) = \mu_{\tilde{T}}(x) + \mu_{\tilde{R}}(x) - \mu_{\tilde{T}}(x) \cdot \mu_{\tilde{R}}(x)$

$$= \left[\frac{0}{0} + \frac{0.3}{1} + \frac{1.0}{2} + \frac{1.0}{3} + \frac{1.3}{4} + \frac{1.5}{5}\right\}$$

$$- \left[\frac{0}{0} + \frac{0.02}{1} + \frac{0.21}{2} + \frac{0.16}{3} + \frac{0.36}{4} + \frac{0.5}{5}\right]$$

$$= \left\{\frac{0}{0} + \frac{0.28}{1} + \frac{0.79}{2} + \frac{0.84}{3} + \frac{0.94}{4} + \frac{1.0}{5}\right\}$$

Algebraic product $\tilde{A} \cdot \tilde{B}$

$$\mu_{\tilde{T} \cdot \tilde{R}}(x) = \mu_{\tilde{T}}(x) \cdot \mu_{\tilde{R}}(x)$$

$$= \left\{\frac{0}{0} + \frac{0.02}{1} + \frac{0.21}{2} + \frac{0.16}{3} + \frac{0.36}{4} + \frac{0.5}{5}\right\}$$

Bounded sum $\tilde{T} \oplus \tilde{R}$

$$\mu_{\underset{\sim}{T} \odot \underset{\sim}{R}}(x) = \max\left[0,\ \mu_{\underset{\sim}{T}}(x) - \mu_{\underset{\sim}{R}}(x)\right]$$

$$= \max\left\{0, \left[\frac{0}{0} + \frac{0.1}{1} + \frac{0.4}{2} + \frac{0.6}{3} + \frac{0.5}{4} + \right.\right.$$

$$= \left\{\frac{0}{0} + \frac{0.1}{1} + \frac{0.4}{2} + \frac{0.6}{3} + \frac{0.5}{4} + \frac{0.5}{5}\right\}$$

Fuzzy relations.

1) consider the following two fuzzy sets — SONAL

$A = \left\{ \frac{0.3}{x_1} + \frac{0.7}{x_2} + \frac{1}{x_3} \right\}$

and $B = \left\{ \frac{0.4}{y_1} + \frac{0.9}{y_2} \right\}$

The cartesian product performed over fuzzy sets $\underset{\sim}{A}$ and $\underset{\sim}{B}$ results in relation $\underset{\sim}{R}$

$$\underset{\sim}{R} = \underset{\sim}{A} \times \underset{\sim}{B} = \begin{array}{c|cc} & y_1 & y_2 \\ \hline x_1 & 0.3 & 0.3 \\ x_2 & 0.4 & 0.7 \\ x_3 & 0.4 & 0.9 \end{array}$$

$\mu_R(x_1, y_1) = \min(\mu_A(x_1), \mu_B(y_1))$

2) Two fuzzy relations are given by

$$\underset{\sim}{R} = \begin{array}{c|cc} & y_1 & y_2 \\ \hline x_1 & 0.6 & 0.3 \\ x_2 & 0.2 & 0.9 \end{array} \quad \text{and} \quad \underset{\sim}{S} = \begin{array}{c|ccc} & z_1 & z_2 & z_3 \\ \hline y_1 & 1 & 0.5 & 0.3 \\ y_2 & 0.8 & 0.4 & 0.7 \end{array}$$

Obtain fuzzy relation T as a composition between the fuzzy relations.

The composition between two given fuzzy relations is performed in two ways

a) max-min composition

b) max-product composition.

max-min composition of $R(X,Y)$ and $S(Y,Z)$ is defined as,

$\therefore T = R \circ S =$ $x_2 \begin{bmatrix} 0.8 & 0.4 & 0.7 \end{bmatrix}$

$$\mu_T(x_1, z_1) = \max\{\min[\mu_R(x_1, y_1), \mu_S(y_1, z_1)]; \min[\mu_R(x_1, y_2), \mu_S(y_2, z_1)]$$

$$= \max\{0.6, 0.3\}$$

$$= 0.6$$

$$\mu_T(x_1, z_2) = \max\{\min[\mu_R(x_1, y_1), \mu_S(y_1, z_2)] \min[\mu_R(x_1, y_2), \mu_S(y_2, z_2)]\}$$

$$= \max\{0.5, 0.3\}$$

$$= 0.5$$

$$\mu_T(x_1, z_3) = \max\{\min[\mu_R(x_1, y_1), \mu_S(y_1, z_3)], \min[\mu_R(x_1, y_2), \mu_S(y_2, z_3)]\}$$

$$= \max\{0.3, 0.3\}$$

$$= 0.3$$

$$\mu_T(x_2, z_1) = \max\{\min[\mu_R(x_2, y_1), \mu_S(y_1, z_1)], \min[\mu_R(x_2, y_2), \mu_S(y_2, z_1)]\}$$

$$= \max\{0.2, 0.8\}$$

$$= 0.8$$

$$\mu_T(x_2, z_2) = \max\{\min[\mu_R(x_2, y_1), \mu_S(y_1, z_2)], \min[\mu_R(x_2, y_2), \mu_S(y_2, z_2)]\}$$

$$= \max\{0.2, 0.4\}$$

$$= 0.4$$

$$\mu_T(x_2, z_3) = \max\{\min[\mu_R(x_2, y_1), \mu_S(y_1, z_3)]; \min[\mu_R(x_2, y_2), \mu_S(y_2, z_3)]\}$$

$$= \max\{0.2, 0.7\}$$

$$= 0.7$$

max-product composition

max-product composition of $R(x,y)$ and $S(y,z)$ is defined as

$T(x,z) = R \bullet S$

$$R(x,y) \bullet S(y,z) = \max_{y \in Y} \left[\mu_R(x,y) \cdot \mu_S(y,z) \right]$$

$$= \bigvee_{y \in Y} \left[\mu_{\tilde{R}}(x,y) \cdot \mu_{\tilde{S}}(y,z) \right]$$

$\forall x \in X,\ z \in Z.$

$$\therefore T = \begin{array}{c c} & \begin{array}{ccc} z_1 & z_2 & z_3 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \end{array} & \begin{bmatrix} 0.6 & 0.3 & 0.21 \\ 0.72 & 0.36 & 0.63 \end{bmatrix} \end{array}$$

$$\mu_{\tilde{T}}(x_1, z_1) = \max\{[\mu_{\tilde{R}}(x_1, y_1) \cdot \mu_{\tilde{S}}(y_1, z_1)], [\mu_R(x_1, y_2) \cdot \mu_{\tilde{S}}(y_2, z_1)]\}$$

$$= \max\{0.6,\ 0.24\}$$

$$= 0.6$$

$\mu_{\tilde{T}}(x_1, z_2) = \max\{0.3,\ 0.12\} = 0.3$

$\mu_{\tilde{T}}(x_1, z_3) = \max\{0.18,\ 0.21\} = 0.21$

$\mu_{\tilde{T}}(x_2, z_1) = \max\{0.2,\ 0.72\} = 0.72$

$\mu_{\tilde{T}}(x_2, z_2) = \max\{0.10,\ 0.36\} = 0.36$

$\mu_{\tilde{T}}(x_2, z_3) = \max\{0.06,\ 0.63\} = 0.63$

04/03/14.
Fuzzy

fuzzy

fuzzy

fuzzy
ex.
int

fuzzy

Q.1) differe

x,

Q.2) what
define
core

Soft computing Sonal Mam P.T.1

24/03/14.

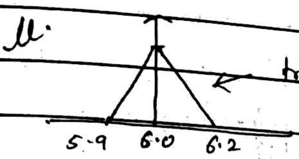
Fuzzy logic by Timothy Ross.

fuzzy → not clear / ambiguous.
"imprecise.

fuzzy no.

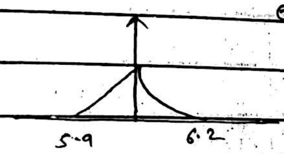
voltage around 6V.

μ.

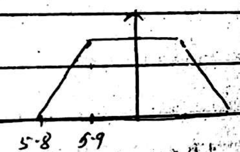


← triangular membership f^n.

exponential



trapezoidal.



fuzzy variable –
ex. hot, tall, smaller, short
introduced by Zadeh – father of fuzzy.

fuzzy sets — crisp sets.

Q.1) differentiate bet^n crisp logic & fuzzy logic.
set set.

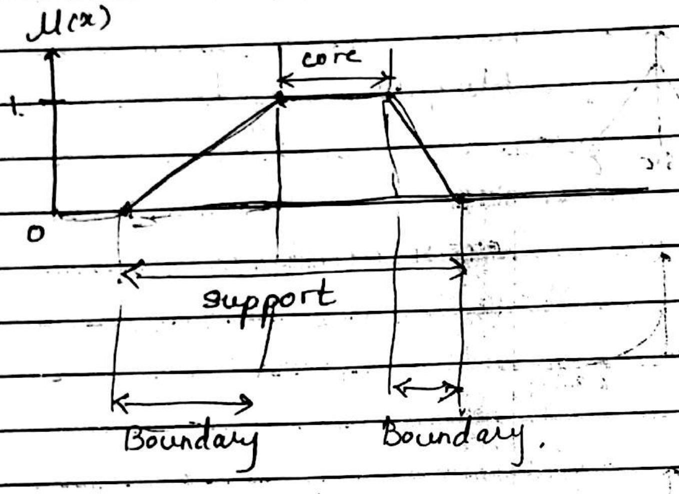
$x, \mu_A(x) \rightarrow 0$ to 1.

membership function.

Q.2) what are diff. features of membership function.
define
core – comprises those elements x of the universe
s.t. $\mu_A(x) = 1$

support - region of the universe that is characterised by non zero membership.

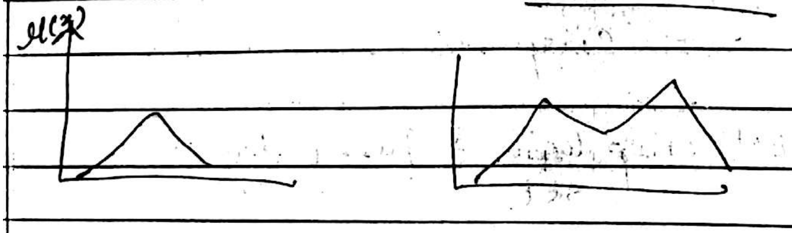
Boundary - boundaries comprise those elements x of the universe s.t. $0 < \mu_a(x) < 1$.



Normal fuzzy set -
at least one element x in the universe whose membership

convex fuzzy set - membership values are strictly monotonically ↑ or ↓.

Non convex.



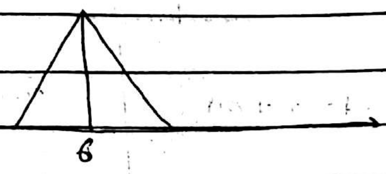
cross over points - $\mu_a(x) = 0.5$

Height - defined as $\max \{ \mu_a(x) \}$.

Q.3) prob. –

Define a fuzzy set of integers close to 6. (8M).

triangular –



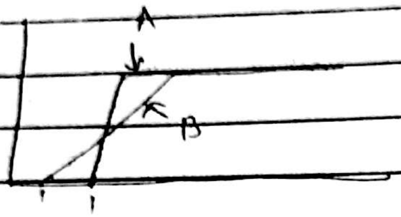
Bellshape m.f. $\mu_A(x) = \frac{1}{1+(x-6)^2}$.

$A = \{ x, \mu_A(x) \mid x \in X \}$.

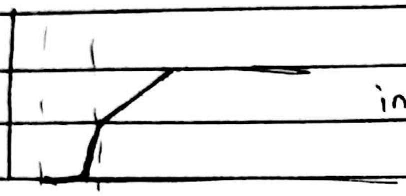
$= \{ (3, 0.1), (4, 0.2), (5, 0.5) (6, 1)$
$(7, 0.5) (8, 0.2) (9, 0.1) \}$

4) enlist different membership function.
- triangular
- trapezoidal   graph
  equations.

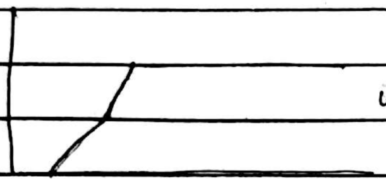
Q.5) operations on fuzzy sets.
intersection
complement
Union



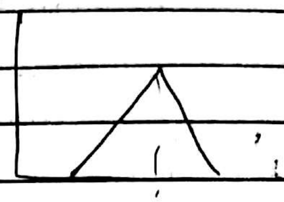
fuzzy



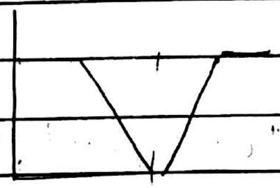
intersection.



union



complement (1-).



Fuzzy composition

$I_a \propto R_{se}$, speed $\propto I_a$.

what is relatⁿ betⁿ R_{se} & speed.

- max . min
- max . product

extension principle.

α-cut & fuzzy set

fuzzy linguistic variable

projections of fuzzy set

Defuzzification methods.

problems — 1 for 20M.

1) washing machine
2) train approaching station
3) shower.

Fuzzy Sets

SONAL.

Introduction -

A classical set is a set with a crisp boundary for example -
a classical set A of real numbers greater than 10 can be expressed as,

$$A = \{x / x > 10\}$$

Here there is a clear, unambiguous boundary 10 such that if x is greater than this number then x belongs to the set.

Although classical sets are suitable for various applications and have proven to be an important tool for mathematics and computer science, they do not reflect the nature of human concepts and thoughts, which tend to be abstract and imprecise.

As an illustration - mathematically we can express the set of tall persons as a ~~gr~~/collection of persons having ~~his~~ height more than 6 ft. as,

$$A = \{x / x > 6\}$$

where,
A → is 'tall person'
x → is 'height'.
Yet this is an unnatural and inadequate way of representing our usual concept of a 'tall person'.

The dichotomous nature of the classical set would classify a person 6.00 ft tall as a 'tall person', but not a person 5.99 ft tall. This distinction is intuitively unreasonable. The flaws comes from the sharp transition between inclusion & exclusion in a set.

In contrast to classical set, a fuzzy set, as a name implies is a set without a crisp boundary. i.e. the transition from "belong to a set" to "not belong to a set" is gradual. It is a smooth transition characterised by membership functions that give fuzzy sets flexibility in modeling commonly used linguistic expressions such as "the water is hot" or "the temp is high".

> Fuzziness does not come from the randomness of the constituent members of the sets, but from the uncertain and imprecise nature of abstract thoughts and concepts.

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Basic Definitions and Terminology

Let X be the space of objects and x be the generic element of X.
A classical set A, $A \subseteq X$, is defined as a collection of elements or objects $x \in X$, such that each x can either belong or not belong to the set A.
By defining the characteristics function for each element x in X, we can represent a classical set A by a set of ordered pair $(x, 0)$ or $(x, 1)$ which indicates $x \notin A$ or $x \in A$ resp.

A fuzzy set expresses the degree to which an element belongs to a set.
Thus, the characteristic function of a fuzzy set is allowed to have values between 0 & 1. that denotes the degree of membership of an element in a given set.

- Fuzzy sets and membership functions.

If X is a collection of objects denoted by x, then fuzzy set A in X is defined as a set of ordered pairs -

$$A = \{(x, \mu_A(x)) / x \in X\}$$

$\mu_A(x) \rightarrow$ membership of object x for fuzzy set A.

The MF (membership Function) maps each element of X to a membership grade (or membership value) between 0 & 1.

→ Thus, the def^n of fuzzy set is a simple extension of the def^n of classical set in which the characteristic function is permitted to have any value between 0 and 1.

→ If values of $\mu_A(x)$ are restricted to 0 or 1 then A reduces to a classical set and $\mu_A(x)$ is the characteristic function of A.

→ X : referred to as universe of discourse or simply universe.
It consists of discrete objects (ordered / non-ordered) or continuous space.

ex. 1 fuzzy set with a discrete non ordered universe.

Let X = { Mumbai, Pune, Nagpur } be the set of cities one may choose to live in.

The fuzzy set C = " desirable city to live in" may be described as,

$$C = \{ (\text{Mumbai}, 0.9), (\text{Pune}, 0.8), (\text{Nagpur}, 0.6) \}$$

Here, the universe of discourse X is discrete and it contains non-ordered objects - in this case - three cities in the state of maharashtra.

The membership grades listed above are quite subjective, you can come up with three different but legitimate values to reflect his/her preference.

ex 2 : fuzzy set with discrete ordered universe. ③

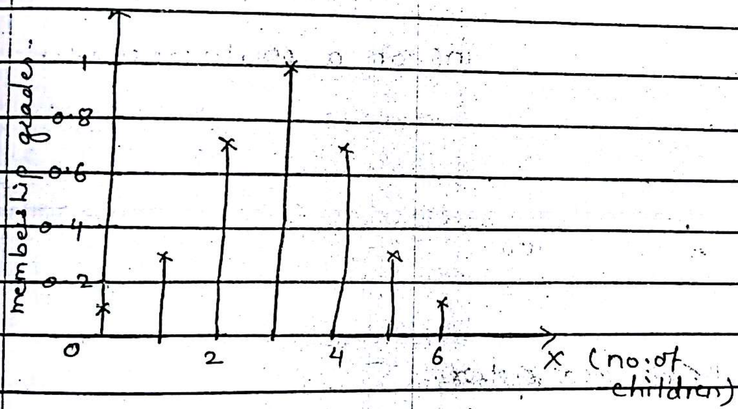
Let $X = \{0, 1, 2, 3, 4, 5, 6\}$ be the set of numbers of children, a family may choose to have. Then,

the fuzzy set A = " sensible number of children in a family ".

may be described as

$$A = \{(0, 0.1), (1, 0.3), (2, 0.7), (3, 1), (4, 0.7), (5, 0.3), (6, 0.1)\}.$$

Here universe X is discrete ordered.



... MF on discrete universe.

Again here the membership grades are subjective measures.

3 : Fuzzy sets with a continuous universe.

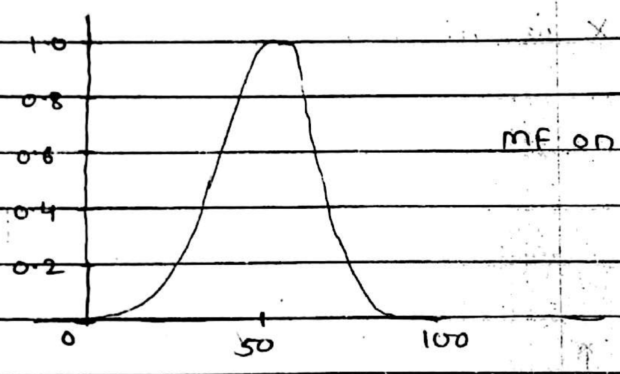
Let $X = R^+$ be the set of possible ages of human beings.
Then the fuzzy set

$$B = \text{"about 50 yrs old"}$$

may be expressed as

$$B = \{ (x, \mu_B(x)) / x \in X \}$$

where $\mu_B(x) = \dfrac{1}{1 + \left(\dfrac{x-50}{10}\right)^4}$



MF on a continuous universe.

From the preceding examples, it is clear that the construction of the set depends on two things – identification of a suitable universe of discourse
– specification of an appropriate MF. membership function.

The specification of MF is subjective, that means, the MF specified for the same concept by different persons may vary considerably.
(ex. – "sensible no. of children in a family").

→ This subjectivity comes from the individual differences in perceiving or expressing abstract concepts & has little to do with randomness.

∴ the subjectivity and non randomness of fuzzy sets is the primary difference between the study of fuzzy sets and probability theory, which deals with objective treatment of random phenomena.

→ For simplicity of notation, we now introduce an alternative way of denoting a fuzzy set.

A fuzzy set A ~~is~~ can be denoted as,

$$A = \begin{cases} \sum_{x_i \in X} \mu_A(x_i)/x_i & \text{if } X \text{ is a collection of discrete objects.} \\ \int_X \mu_A(x)/x & \text{if } X \text{ is a continuous space (usually the real line } R). \end{cases}$$

The Σ & $\int$ sign in above equation stand for union of $(x, \mu_A(x))$ pairs.

they do not indicate summation or integration.

Also, "/" sign is only a marker and does not imply division.

∴ alternative expressions →

$$C = 0.9/\text{mumbai} + 0.8/\text{pune} + 0.6/\text{Nagpur}.$$

$$A = 0.1/0 + 0.3/1 + 0.7/2 + 1.0/3 + 0.7/4 + 0.3/5 + 0.1/6.$$

$$B = \int_{R^+} \frac{1}{1+\left(\frac{x-50}{10}\right)^4} \Big/ x$$

In practice, when the universe of discourse X is a continuous space (the real line R or its subset), we usually partition X into several fuzzy sets whose MF's cover X in a more or less uniform manner.

These fuzzy sets, which usually carry names that conform to adjectives appearing in our daily linguistic usage such as "large", "medium" or "small" are called linguistic values or linguistic labels.
Thus the universe of discourse X is often called the linguistic variable.

- linguistic variables and linguistic values.

Let X = "age".
then we define fuzzy sets "young", "middle aged" & "old" that are characterised by MF, $\mu_{old}(x)$, $\mu_{middleaged}(x)$ and $\mu_{old}(x)$. respectively.

A fuzzy set is uniquely specified by its membership function. To describe membership functions more specifically, we shall define the nomenclature used in the literature.

Def^n:- **Support** –

The support of a fuzzy set A is the set of all points x in X such that $\mu_A(x) > 0$.

$$\text{Support}(A) = \{ x \mid \mu_A(x) > 0 \}$$

Def^n:- **Core** –

The core of a fuzzy set A is the set of all points x in X such that $\mu_A(x) = 1$.

$$\text{Core}(A) = \{ x \mid \mu_A(x) = 1 \}$$

Def^n:- **Normality** –

A fuzzy set A is normal if its core is non-empty.
In other words, we can always find a point $x \in X$ such that $\mu_A(x) = 1$.

Def^n:- **cross over points** –

A cross over point of a fuzzy set A is a point $x \in X$ at which $\mu_A(x) = 0.5$

$$\text{crossover}(A) = \{ x \mid \mu_A(x) = 0.5 \}.$$

Fuzzy singleton -

A fuzzy set whose support is a single point in X which with $\mu_A(x) = 1$ is called a fuzzy singleton.

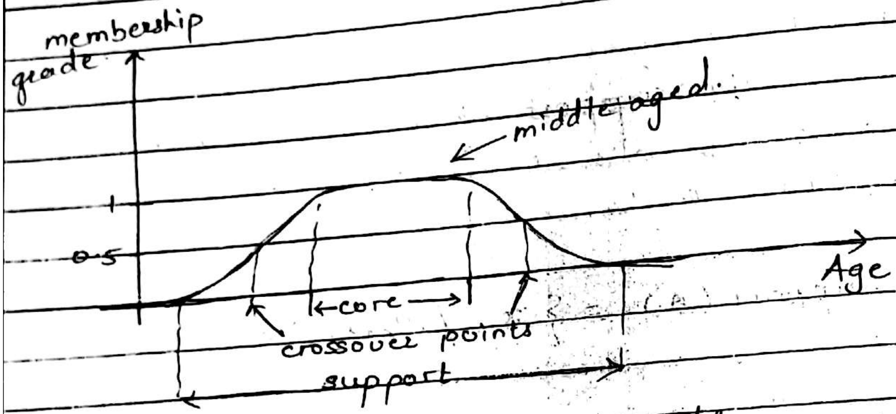


fig. :- fuzzy set "middle aged".

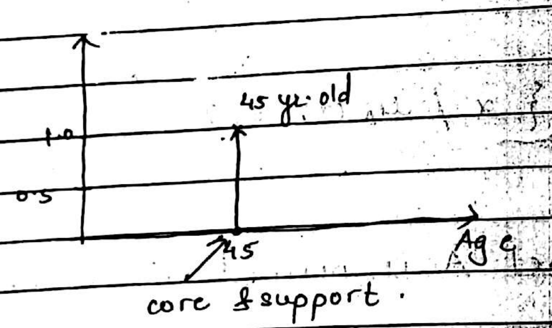


fig. :- Fuzzy singleton "45 yr old".

Defn - α-cut, strong α-cut

The α-cut or α-level set of a fuzzy set A is a crisp set defined by

$$A_\alpha = \{ x \mid \mu_A(x) \geq \alpha \}$$

strong α-cut or strong α-level set are defined similarly -

$$A_\alpha' = \{ x \mid \mu_A(x) > \alpha \}$$

using the notation for a level set, we can express the support and core of a fuzzy set A as,

$$\text{support}(A) = A_0'$$

and

$$\text{core}(A) = A_1 \quad \text{respectively.}$$

Defn - Convexity -

A fuzzy set A is convex iff for any $x_1, x_2 \in$ and any $\lambda \in [0,1]$

$$\mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \min\{\mu_A(x_1), \mu_A(x_2)\}$$

Alternatively, A is convex if all its α-level sets are convex.