Unit - IV

Graph Theory

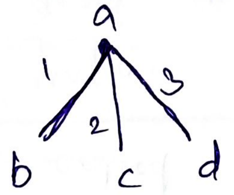
A graph $G = \{V, E\}$ consists of a non-empty set V called the set of vertices (nodes, points) and a set E of ordered (or) unordered pairs of elements of V called the set of edges, such that there is a mapping from the set E to the set of ordered (or) unordered pairs of elements of V.

Incident and adjacent vertices:-

when vertex v_i is an end vertex of some edge v_j, v_i, v_j are said to be incident with each other.

If two distinct vertex x & y are connect by edge then they are called adjacent vertex. Generally the graph G is denoted by $G = \{V, E\}$.

eg:-



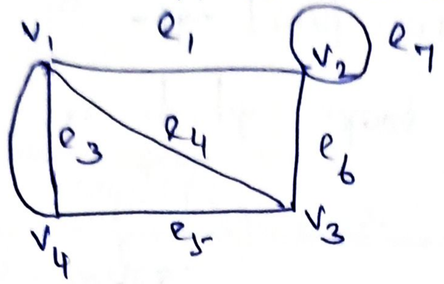
Here (a,b), (a,c) & (a,d) are adjacent vertex, and the edges 1,2,3 are incident edges.

The given graph $G = \{4, 3\}$, Since 4 vertex and 3 edges.

Parallel edges:-

The graph does not allow any line joining to the point (or) vertex to itself, such a graph is called loop.

The graph has an pair vertex has the more than edges then it is called parallel edges.



Since $G = \{4, 7\}$.

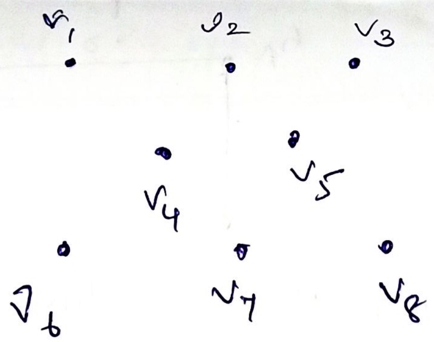
Let e_2 & e_3 are called parallel edges

e_7 are called loop (or) self loop.

Parallel graph :-

A graph contain contain only the vertex set and there is no edge set then the graph is called null graph :-

eg :-

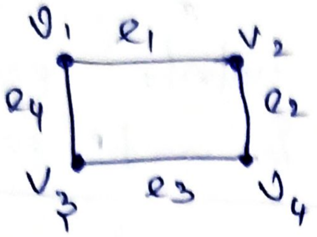


Directed and Undirected graph :-

If the graph $G = \{V, E\}$ each edge $e \in E$ is associated with an ordered pair of vertices then, G is called a directed graph (or) digraph.

If each edge is associated with an unordered pair of vertices then G is called an undirected graph.

eg :-



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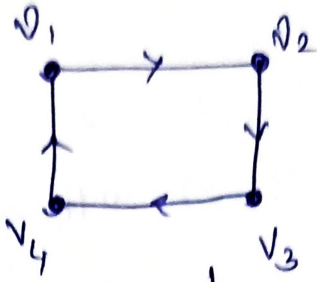
undirected graph



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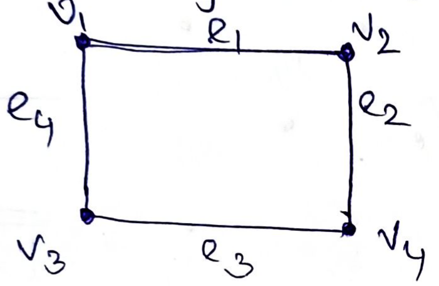
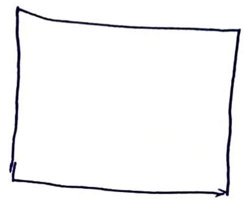
Directed graph.

Labelled & UnLabelled graph:-

A graph G is called Labelled if its vertices and edges are denoted one other by name, Otherwise they are unlabelled graph.



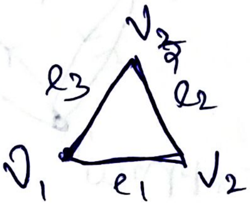


Labelled graph.

unlabelled graph.

Finite & Infinite graph:-

A graph with finite no of vertices as well as finite no of edges are called finite graph otherwise, it is an infinite graph.



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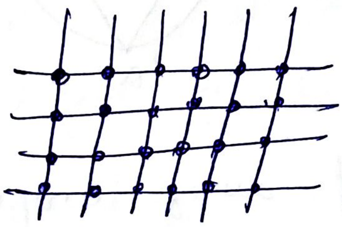
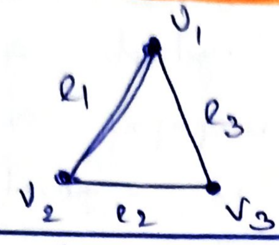
finite graph



Infinite graph.

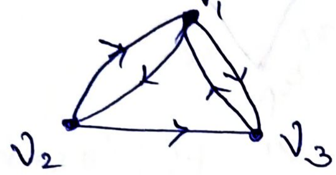
Simple Graph:- A graph in which there is only one edge between a pair of vertices is called a simple graph.



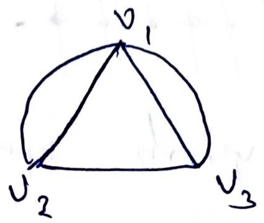
$\rightarrow$ Simple graph.

Multi-graph :-

A graph which contain same parallel edges is called multi-graph.



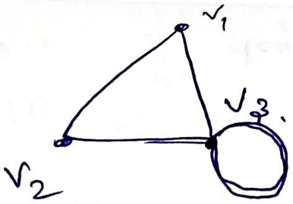
Directed Multi-graph



Undirected Multigraph.

Pseudograph :-

A graph in which loops and parallel edges are allowed is called a pseudo graph.

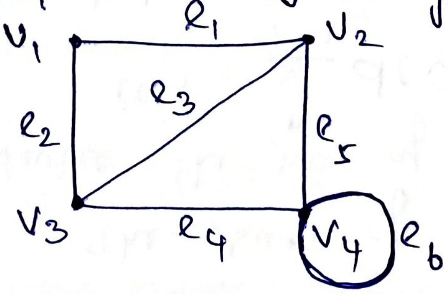


$\rightarrow$ pseudograph.

Degree of vertex :-

The degree of a vertex in an undirected graph is the no of edges incident with it, with the exception that a loop at a vertex contributes twice to the degree of that vertex.

It is denoted by deg (v). (or) deg(v_i) (or) d(v_i)



Here deg(v_1) = 2,

deg(v_2) = 3 = deg(v_3).

d(v_4) = 4.

Handshaking Theorem:-

The sum of degree of a vertex of a graph G twice the no of edges.

i.e., $\sum_{i=1}^{n} d(v_i) = 2e.$

Proof:- Let us consider a graph 'G' with 'e' edges and 'n' vertices $v_1, v_2 \ldots v_n$.

Since, every edges is incident with exactly two vertices, every edge contributes 2 degrees.

$\therefore$ All the 'e' edges contribute ($2e$) to the sum of the degree of the vertices.

$\therefore$ The sum of the degree's of all the vertices in G is the twice the no of edges in G.

i.e., $\sum_{i=1}^{\infty} deg(v_i) = 2e.$

Theorem:- The number of vertices of odd degree in an undirected graph is even.

Proof:- Let $G = \{V, E\}$ be the undirected graph.

Let V_1, V_2 be the set of vertices of G of even and odd degree's resply.

Then, by Previous theorem,

$$2e = \sum_{v_i \in V_1} deg(v_i) + \sum_{v_j \in V_2} deg(v_j) \longrightarrow (1).$$

Since, each $deg(v_i)$ is even, $\sum_{v_i \in V_1} (deg(v_i))$ is even.

As the LHS of (1) is is even, we get,

$$\sum_{v_j \in V_2} (deg(v_j)) \text{ is even.}$$

Since each $\deg(v_j)$ is odd, the no of terms contained in $\sum_{v_j \in V_2} \deg(v_j)$ or in V_2 is even.

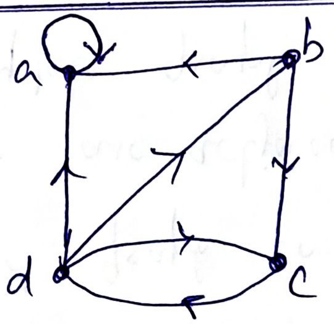
$\therefore$ The no of vertices of odd degree is even.

In degree & Out degree:-

In a directed graph, the no of edges with v as their terminal vertex (the no of edges that converge at v) is called the in-degree of v and is denoted as $\deg^-(v)$.

The no of edges with v as their initial vertex, (the no of edges that emanate from v) is called the out-degree of v and is denoted as $\deg^+(v)$.

Note:- A vertex with zero in degree is called Source and a vertex with zero out-degree is called a Sink.



$\deg^-(a) = 3$ | $\deg^+(a) = 1$

$\deg^-(b) = 1$ | $\deg^+(b) = 2$

$\deg^-(c) = 2$ | $\deg^+(c) = 1$

$\deg^-(d) = 1$ | $\deg^+(d) = 3$

Here $\sum \deg^-(v) = \sum \deg^+(v) =$ the no edges $= 7$.

also, loop is contribute one degree for indegree and one degree for out degree.

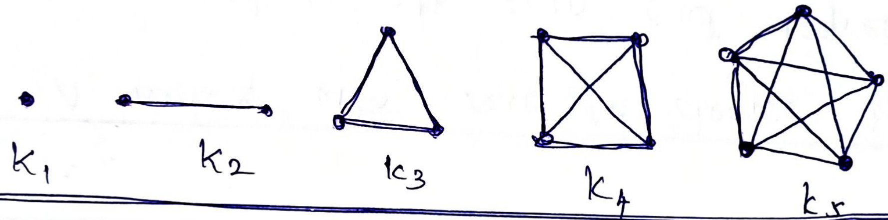
Some special graphs:-

Complete graph:-

A simple graph in which there is exactly one edge between each pair of distinct vertices is called a complete graph.

A graph in which two distinct vertices (or) vertex are adjacent is called a complete graph. The complete graph with n vertices and it's denoted by k_n.

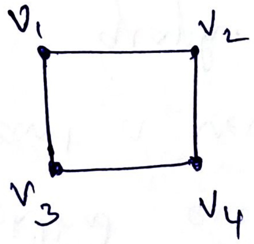
eg:-



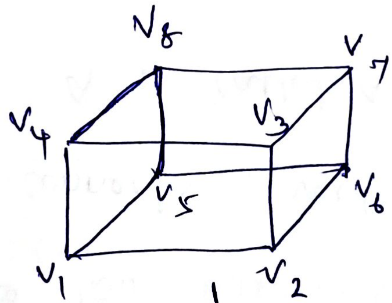
Regular graph:-

If every graph of a simple graph has the same degree, then the graph is called a regular graph. If every vertex in a regular graph has degree n, then the graph is n-regular.

eg:-



→ 2 regular



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4 regular.

→ 3 regular.

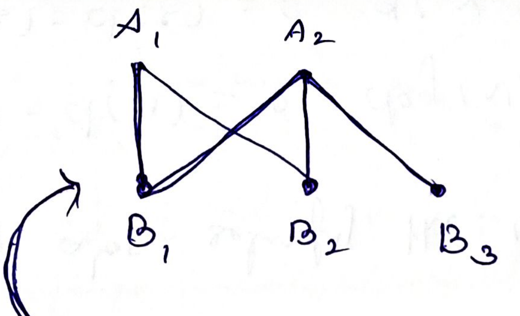
Bipartite graph:-

If the vertex set V of a simple graph $G = \{V, E\}$ can be partitioned into two subsets V_1 and V_2 such that every edge of G connects a vertex in V_1 and a vertex in V_2 then G is called a bipartite graph.

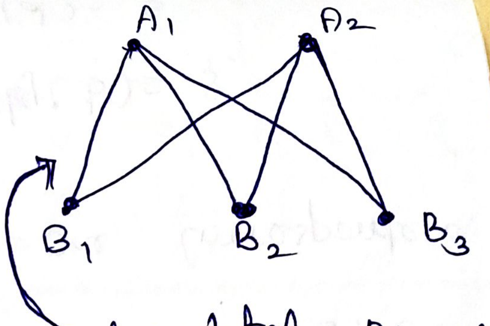
If each vertex of V_1 is connected with every vertex of V_2 by an edge, then G is called a completely bipartite graph.

If V_1 contains 'm' vertices and V_2 contains 'n' vertices, the completely bipartite graph is denoted by $K_{m,n}$.

eg:-



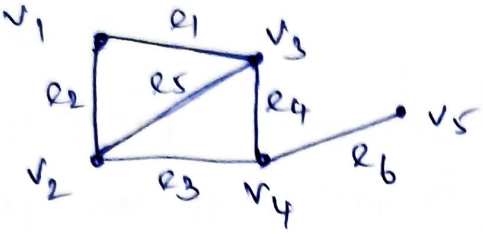
Bipartite graph

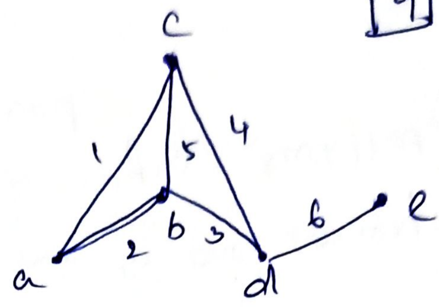


Completgly Bipartite

Isomorphic Graphs:-

Two graphs G_1 & G_2 are said to be isomorphic to each other if there is a one to one correspondence between their vertices and their edges such that incident relationship preserved.





(G_1)

Consider the graph G_1, G_2 are Isomorphic, because the vertices of $G_1 = \{v_1, v_2, v_3, v_4, v_5\}$ and $G_2 = \{a, b, c, d, e\}$.

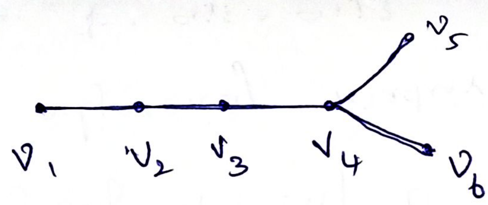
Their edges also satisfy the one-to-one correspondence.

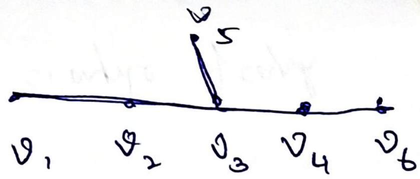
$d(v_1) = d(a) = 2$, $\deg(v_2) = \deg(b) = 3$,

$d(v_3) = d(c) = 3$, $d(v_4) = d(d) = 3$

$d(v_5) = d(e) = 1$

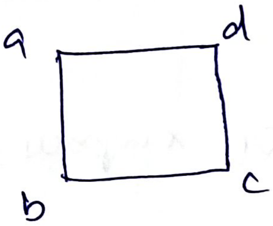
eg:- 2



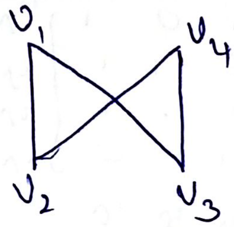


It is not isomorphic.

eg:- 3



(G_1)



(G_2)

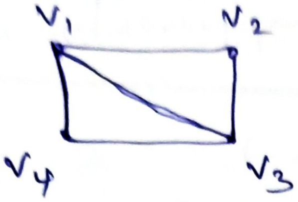
both G_1 and G_2 are Isomorphic graphs.

Matrix representation of a graph:-

(i) **adjacency Matrix:-** The adjacency matrix of a simple graph G with 'n' vertices and no parallel edges is an $n \times n$ matrix, A (or) $A_G = a_{ij}$

$$a_{ij} = \begin{cases} 1, & \text{if } v_i, v_j \text{ is an edges of } G \\ 0, & \text{otherwise} \end{cases}$$

eg:-



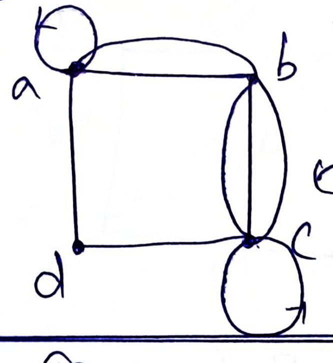
$\therefore$ Adjacency matrix is
$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array}\begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array}.$$

Note: ① Since the simple graph has no loop, each diagonal entry of $A = a_{ij} = 0$

Note ② The adjacency matrix of simple graph is symmetric. $\{\because a_{ji} = a_{ij}\}$.

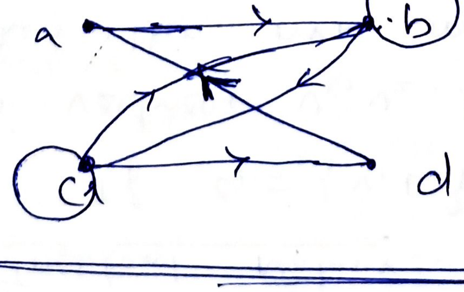
Note ③ $\deg(v_i)$ is equal to the number of 1's in the i^{th} row (or) i^{th} column.

Note ④ from the pseudograph following,



$$\begin{array}{c} \\ a \\ b \\ c \\ d \end{array}\begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 1 & 2 & 0 & 1 \\ 2 & 0 & 3 & 0 \\ 0 & 3 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

Note ⑤



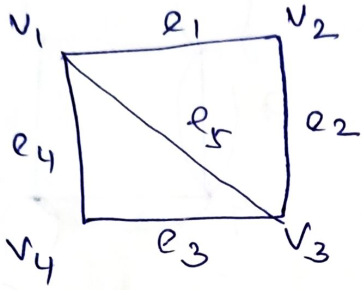
$$\begin{array}{c} \\ a \\ b \\ c \\ d \end{array}\begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Incident Matrix :-

If $G = \{V, E\}$ is an undirected graph with 'n' vertices $v_1, v_2 \ldots v_n$ and 'm' edges $e_1, e_2 \ldots e_m$ then then $n \times m$ matrix $B = \{b_{ij}\}$.

$$b_{ij} = \begin{cases} 1 & \text{when edge } e_j \text{ is incident on } v_i \\ 0, & \text{otherwise} \end{cases}$$

eg :-



$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{ccccc} e_1 & e_2 & e_3 & e_4 & e_5 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

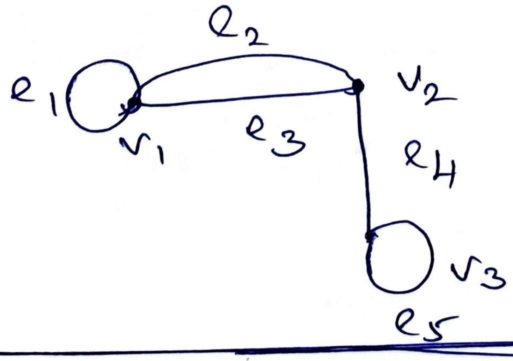
Note :- ① Each column of B contains exactly two units entries.

Note :- ② A row with all 0 entries, corresponds to an isolated vertex.

Note :- ③ A row with an single unit entry corresponds to a pendant vertex.

Note :- ④ $\deg(v_i)$ is equal to the number of 1's in the i^{th} row.

Note :- ⑤ Incidence Matrices can also be used to pseudographs.



$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \end{array} \begin{array}{c} \begin{array}{ccccc} e_1 & e_2 & e_3 & e_4 & e_5 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

Problems:-

① find the no of vertices, the graph with 10 vertices each of degree six?

Soln:- $\sum_{i=1}^{n} d(v_i) = 2e$

$60 = 2e$

$\therefore \boxed{e = 30}$

Here $n = 10$ & each degree $= 6$.

$\therefore \sum d(v_i) = 10 \times 6 = 60.$

② S.T the maxi no of edges in a simple graph with n vertices is $\frac{n(n-1)}{2}$.

Soln:- Use hand-shaking theorem,

$\sum_{i=1}^{n} d(v_i) = 2e$, where e is the no of edges with vertices in the graph G.

i.e., $d(v_1) + d(v_2) + \cdots d(v_n) = 2e$

W.K.T, the maxi no of degree of each vertex in the graph be $n-1$.

$\therefore$ ① $\Rightarrow$ $(n-1) + (n-1) + \cdots (n-1) = 2e.$

$n(n-1) = 2e$

$\therefore \boxed{e = \frac{n(n-1)}{2}}.$

3. P.T the no of edges in a bipartite graph with n vertices is atmost $n^2/2$.

Soln:- Let the vertex set be partitioned into the subsets V_1 and V_2. Let V_1 contain x vertices then V_2 contains $(n-x)$ vertices.

$\therefore$ The largest no of edges,

$f(x) = x(n-x)$ is a function of x.

To find x:-

If $f(x)$ is maximum,

$f'(x) = n - 2x, \quad f''(x) = -2.$

$\therefore f'(x) = 0 \Rightarrow n - 2x = 0 \quad \therefore \boxed{x = n/2}.$

$f''(x) = -2 \Rightarrow f''(n/2) < 0.$

$\therefore x = n/2$, $f(x)$ is maximum.

$\therefore$ maximum no of edges required $\Big\} = f(n/2) = nx - \cancel{x}^2$

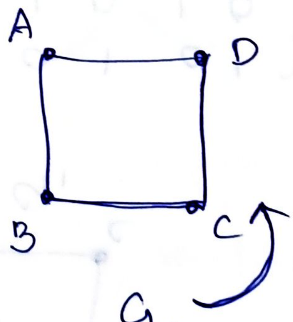
$= n \times n/2 - (n/2)^2$

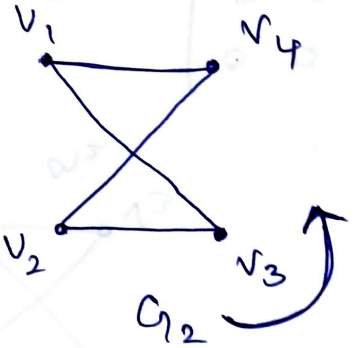
$= n^2/2 - n^2/4$

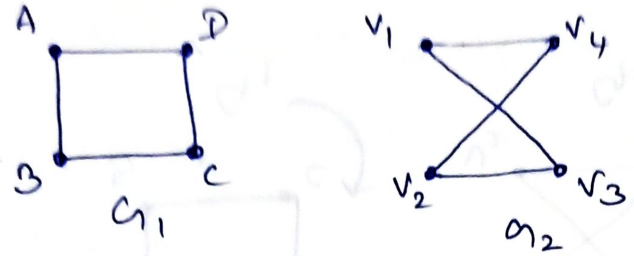
$= \dfrac{2n^2}{4} = n^2/2.$

4. The two labeled graphs G_1 and G_2 with adjacency vertices A_1 and A_2 respectively are isomorphic iff there exists permutation matrix P such that $PA_1P^T = A_2$.

eg:-







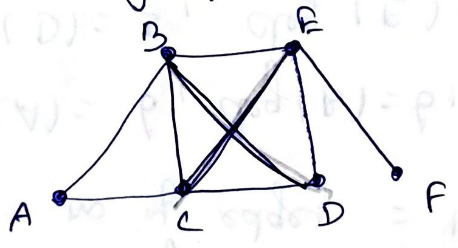
$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}.$$

we get $4!$ permutation $= 24$.

If we, assume $p = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.

$\therefore PA_1P^T = A_2$, Hence any two graphs G_1 and G_2 are isomorphic such that $A \to v_1$, $B \to v_3$, $C \to v_2$ and $D \to v_4$.

5. (i). find the no of vertices, the no of vertices, the no of edges and the degree of each vertex in the following undirected graphs. verify also the handshaking theorem in each case.



the no of vertices $= 6$

no of edges $= 9$

$deg(A) = 2$, $deg(B) = 4$,

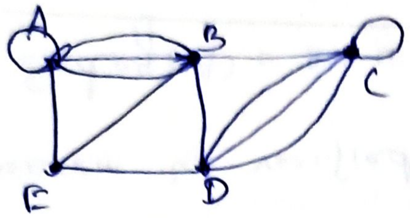
$deg(C) = 4$, $deg(D) = 3$

$deg(E) = 4$, $deg(F) = 1$

$\therefore \Sigma\, deg(A) = 2+4+4+3+4+1 = 18 = 2 \times 9 = 2 \times$ no of edges.

$\therefore$ theorem is verified.

(ii).



the no of vertices = 5.
no of edges = 13

$deg(A) = 6$, $deg(B) = 6$, $deg(C) = 6$
$deg(D) = 5$, $deg(E) = 3$.

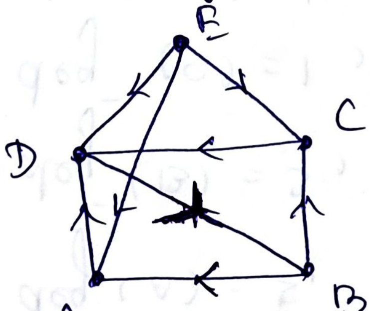
$\therefore \Sigma\, deg(A) = 2 \times$ no of edges.

$\therefore$ Hence, $6 + 6 + 6 + 5 + 3 = 2 \times 13$.

$\therefore 26 = 26$.

the theorem is verified

6. find the in-degree and out-degree of each of
(i). the following graphs. Also, verify that the sum of the in-degree's (or) out-degree's equals the no of edges.

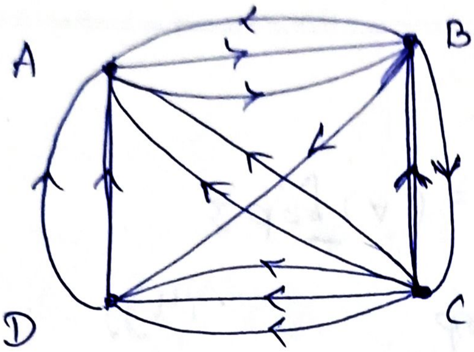


(G_1).

$deg^{-}(A) = 2$, $deg^{-}(B) = 1$, $deg^{-}(C) =$
$deg^{-}(D) = 3$, $deg^{-}(E) = 0$.
$deg^{+}(A) = 1$, $deg^{+}(B) = 2$, $deg^{+}(C) =$
$deg^{+}(D) = 1$, $deg^{+}(E) = 3$.

$\therefore \Sigma\, deg^{+}(A) = \Sigma\, deg^{+}(A) = 8$.
= the no edges of G_1.

(ii)



Graph of (G_2).

$\deg^-(A) = 5, \quad \deg^+(A) = 2$

$\deg^-(B) = 3, \quad \deg^+(B) = 3$

$\deg^-(C) = 1, \quad \deg^+(C) = 6$

$\deg^-(D) = 4, \quad \deg^+(D) = 2$

$\Sigma \deg^-(A) = \Sigma \deg^+(A) = 13$

= the no of edges of G_2.

paths, cycles and connectivity :-

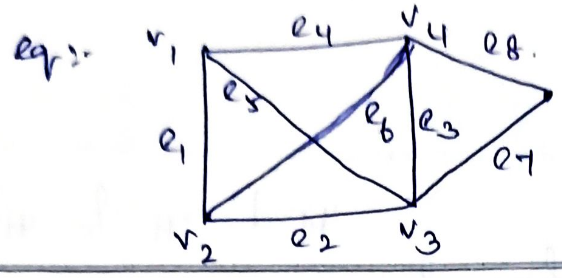
Definition :- (Path)

A path in a graph is a finite alternating sequence of vertices and edges, beginning and ending with vertices, such that each edge is incident on the vertices preceding and following it.

If the edges in a path are distinct, it is called a simple path.

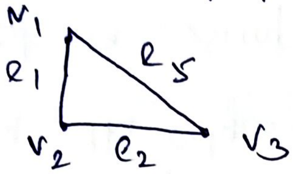
Note :- The number of edges in a path is called the length of the path.

eg:-



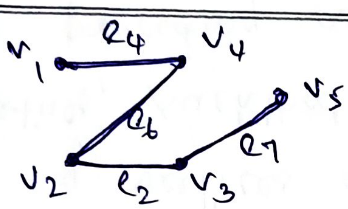
$\Rightarrow$ given graph.

from the above graph



$\Rightarrow$ { $v_1 e_1 v_2 e_2 v_3 e_5 v_1 e_1 v_2$ is a path. $\because$ e_1 contains twice. }.

Also,



$\Rightarrow$ is a simple path.

{$\because$ $v_1 e_4 v_4 e_6 v_2 e_2 v_3 e_7 v_5$ is a simple path. }.

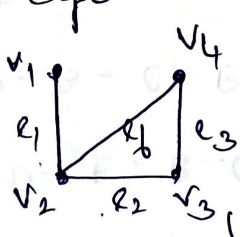
$\because$ no edges appear morethan once.

Note:- The number of edges in a path is called the length of the path.

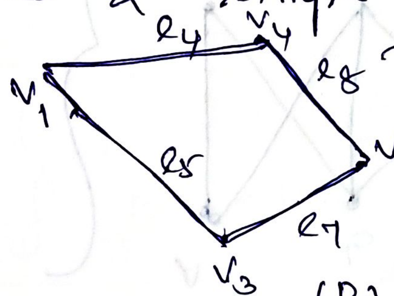
If the initial and final vertices of a path are the same (non-zero length), the path is called a circuit (or) cycle.

If the initial and final vertices of a simple path of non-zero length are the same, the simple path is called a simple circuit (or) a simple cycle.

eg:-



(A).



(B).

$\rightarrow$ is a simple circuit of length 4

from the above, $v_1 e_1 v_2 e_2 v_3 e_3 v_4 e_6 v_2 e_1 v_1$ is a circuit of length 5. from (A).

Also, $v_1 e_5 v_3 e_7 v_5 e_8 v_4 e_4 v_1$ is a simple circuit of length 4. from (B).

18 **example 7.1** Find which of the following vertex sequences are simple paths, paths, closed paths (circuits) and simple circuits with respect to the graph shown.

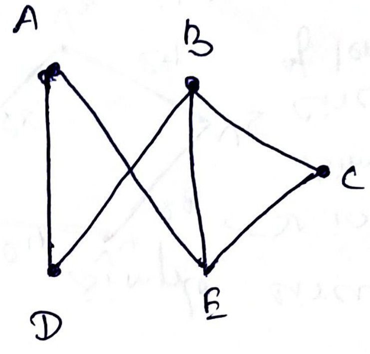
(a) A - D - E - B - C

(b) A - D - B - C - E

(c) A - E - C - B - E - A

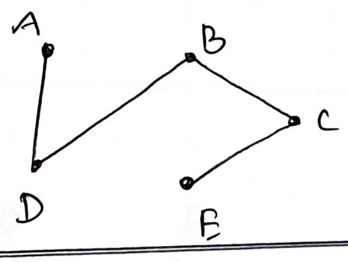
(d) A - D - B - E - C - B

(e) C - B - D - A - E - C



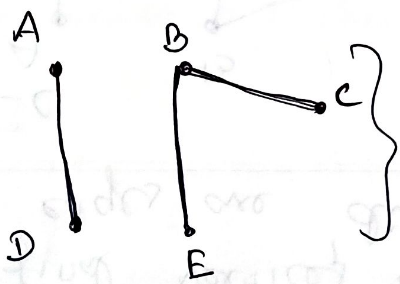
(a) **Solution:-** A - D - B - C - E is a simple path between the vertices A and E, Since the vertices and edges involved are distinct.

eg:- A - D - B - C - E



is a simple path.

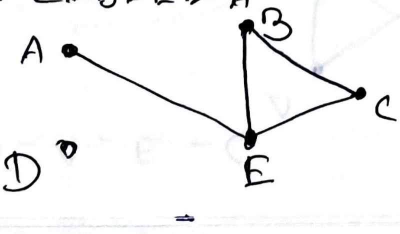
(b) A - D - E - B - C



is not a path.

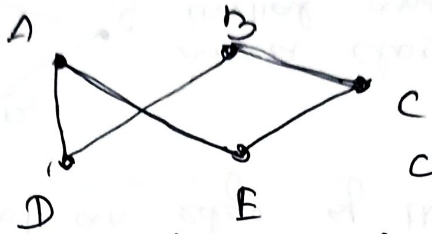
Since, DE is not an edge of the given graph.

(c). A - E - C - B - E - A



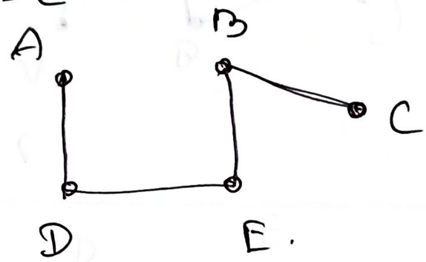
is a closed path, Since the initial and final vertices are the same and the vertex E, twice.

(d) C - B - D - A - E - C



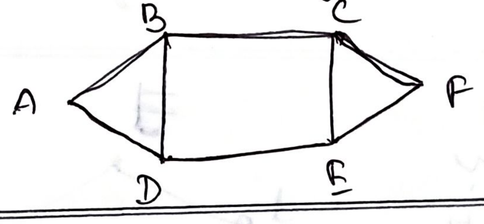
is a simple circuit, since the initial and final vertices are the same and the vertices and edges are distinct.

(e) A - D - E - B - C



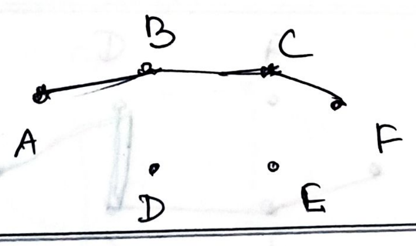
is a path (not a simple path) as the vertex B appears twice.

2. Find all the simple path from A to F and all the circuits in the given graph in the following.



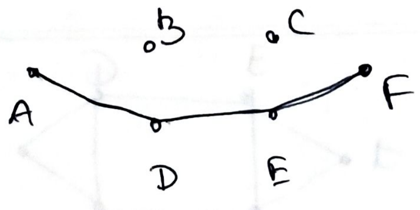
Soln:- The following are the simple path for A to F.

(1)



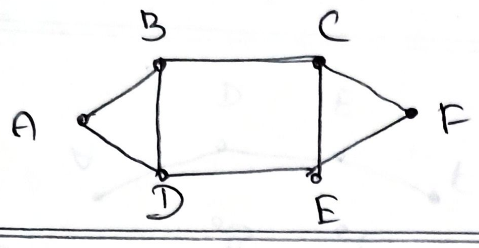
A - B - C - F is a simple path.

(2)

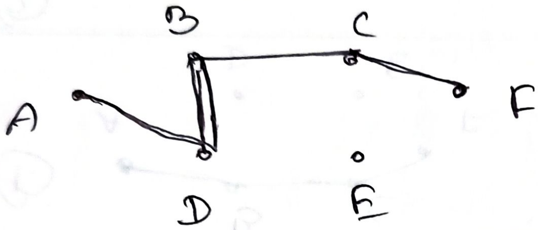


A - D - E - F is a simple path.

(3)

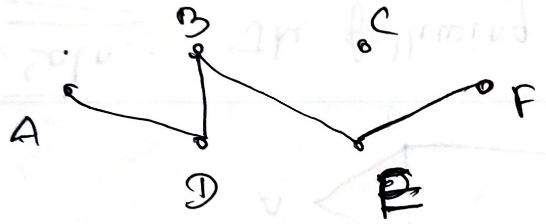


→ given graph.



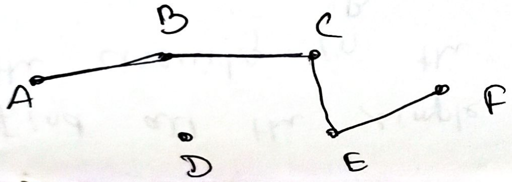
A – D – B – C – F.
is a simple path.

(4)



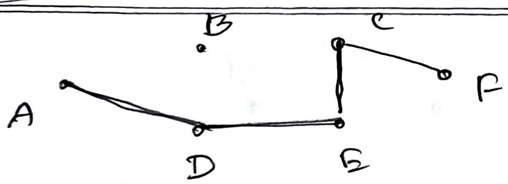
A – D – B – E – F
is a simple path.

(5)



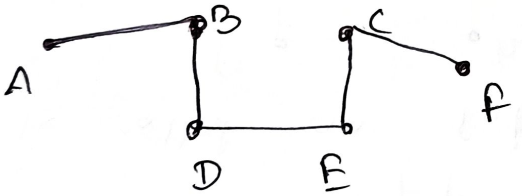
A – B – C – E – F is a simple path.

(6)



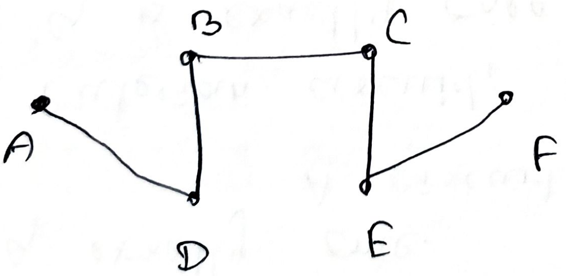
A – D – E – L – F is a simple path.

(7)



A – B – D – E – C – F is a simple path.

(8)



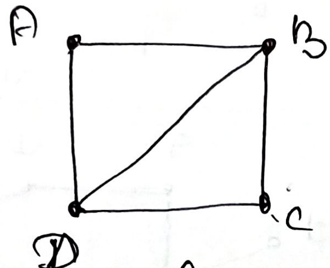
A – D – B – C – E – F is a simple path.

Eulerian and Hamiltanian graphs:-

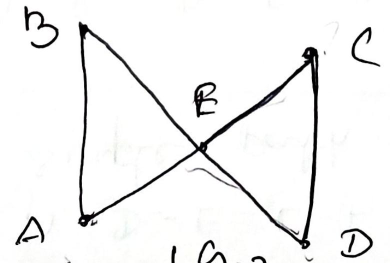
Definition:- A path of graph G is called an Eulerian path, if it includes each edge of G exactly once.

A circuit of a graph G is called an Eulerian circuit, if it includes each edge of G is exactly once.

A graph containing an Eulerian circuit is called an Eulerian graph.



(G_1)



(G_2).

The graph G_1 is an Eulerian path between B and D namely, B - D - C - B - A - D.

Since, it includes each of the edges exactly once.

The graph G_2 is an Eulerian circuit, namely A - E - C - D - E - B - A, Since, it includes each of the edges exactly once.

$\therefore$ G_2 is an Euler graph, as it contains an Eulerian circuit.

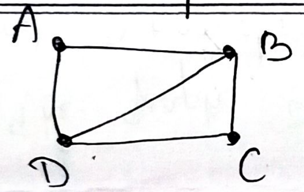
Theorem:- 1

The connected graph contains an Euler circuit, if and only if (iff) each of it's vertices is of even degree.

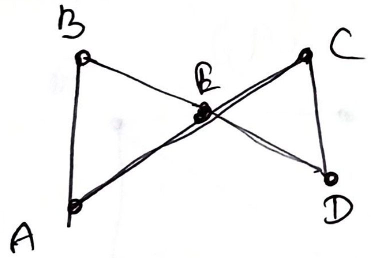
Theorem :- 2

A connected graph contains an Euler path, if and only if (iff) each of it has exactly two vertices of odd degree.

Note:- The Euler path will have the odd degree vertices as its ends points.



⟹ The vertices B and D are of degree 3 ⟹ it's an Euler path.



⟹ All the vertices are of even degree.

∴ It's an Euler Circuit.

Hamiltonian path:-

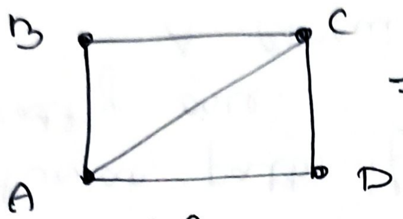
A path of a graph G is called a Hamiltonian path, if it includes each vertex of G exactly once.

A Circuit of a graph G is called a Hamiltonian circuit, if it includes each vertex of G exactly once, the starting and end vertices which appear twice.

23

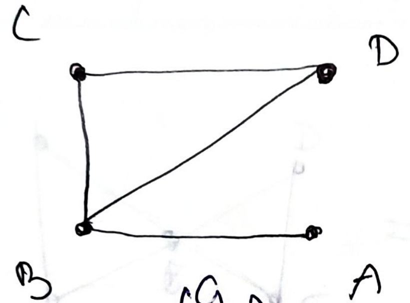
A graph containing a Hamiltonian circuit is called a Hamiltonian graph.

eg:-



(G_1)

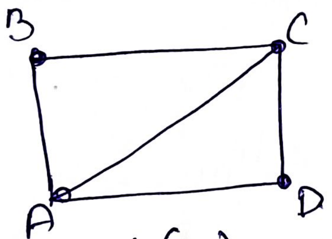
$\Rightarrow$ The graph G_1 has a Hamiltonian circuit namely, A-B-C-D-A. We note that, in this circuit all the vertices appear but, not all edges.



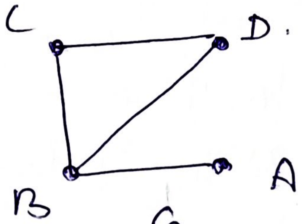
(G_2)

$\Rightarrow$ The graph G_2, has a Hamiltonian path, namely, A-B-C-D, but not a Hamiltonian Circuit.

properties of Hamiltonian path and Hamiltonian circuit:-



(G_1)



G_2

Prop: 1 :- From the above G_1 and G_2, it is clear that the path obtained by deleting any one edge from a Hamiltonian Circuit is a Hamiltonian path.

Prop: 2 :- Also, Hamiltonian circuits contains a Hamiltonian path. But it's converse is true.

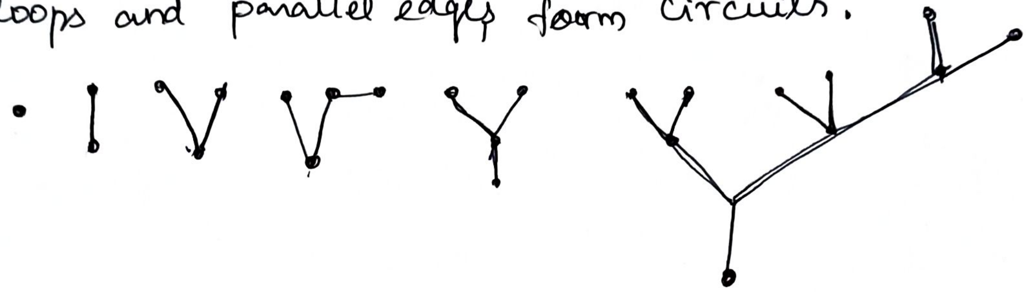
Prop :- 3 A complete graph K_n will always Hamiltonian circuit.

Prop: 4 :- The above graph may contain more than 1 circuit.

Trees :-

Definition :- A connected graph without any circuits is called a tree.

Obviously a tree has to be a simple graph, since loops and parallel edges form circuits.



Some properties of Trees :-

Property :- ①

An undirected graph is a tree, iff, there is a unique simple path between every pair of vertices.

Proof :- (i) Let the undirected graph T be a tree. Then, by definition of a tree, T is connected. Hence, there is a simple path between any pair of vertices, say v_i and v_j.

(ii) Let a unique path exist between every pair of vertices in T. Then, T is connected. If possible, let T contain a circuit. This means that, there is a pair of vertices v_i and v_j between which two distinct path exist, which is against the data.

Hence, T cannot have a circuit and so T is a tree.

Property : 2

A tree with 'n' vertices has $(n-1)$ edges.

Proof:- The property is true for $n=1,2,3$ as seen from the graph.

From the mathematical Induction property, the property be true for all tree with less than 'n' vertices.

Consider T with 'n' vertices. Let e_k be the edge connecting the vertices v_i and v_j of T.

If we delete the edge e_k from T, T becomes disconnected $(T-e_k)$ consists of exactly two components say T_1 and T_2 which are connected.

Since, T did not contain any circuit, T_1 and T_2 also will not have circuits.

Hence, both T_1 and T_2 are trees, each having less than 'n' vertices, say r and "$n-r$" resp'ly.

By the induction,

T_1 has $r-1$ edges and T_2 has $(n-r-1)$ edges.

$\therefore T$ has $(r-1)+(n-r-1)+1 = n-1$ edges.

$\therefore$ a tree has n vertices has $(n-1)$ edges.

Property : 3

Any connected graph with 'n' vertices and $(n-1)$ edges is a tree.

Property :- 4

Any circuitless graph with 'n' vertices and $(n-1)$ edges is a tree.

Spanning Trees :-

Definition :-

If the sub-graph T of a connected graph "G" is a tree containing all the vertices of G then, T is called a spanning tree of G.

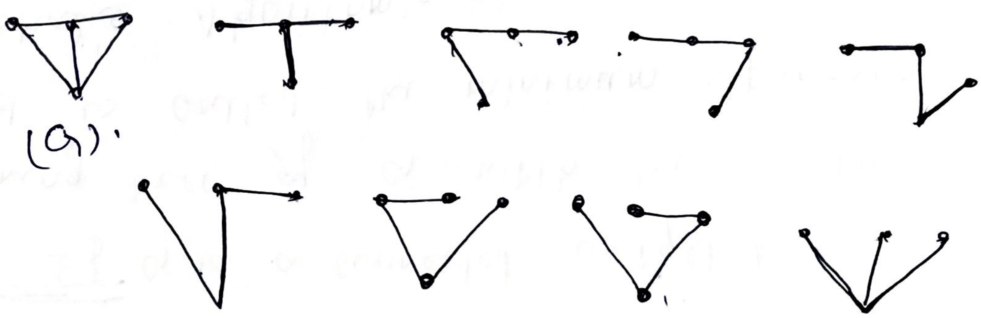
eg :-

Since, G has 4 vertices, any spanning tree of G will also have 4 vertices and hence, 3 edges.

Since, G has 5 edges, removal of 2 edges may result in spanning tree. This can be done in ${}^5C_2 = 10$ ways, but 2 of these 10 ways result in disconnected graphs.

All the possible spanning trees are following :-

eg :-



(G).

Minimum Spanning Tree :-

Definition :-

If G is a connected weighted graph, the spanning tree of G with the smallest total weight is called the minimum spanning tree of G.

Kruskal's Algorithm :-

Step :- 1

The edges of the given graph G are arranged in the order of increasing weights.

Step :- 2

An edge G with minimum weight is selected as an edge of the required spanning tree.

Step :- 3

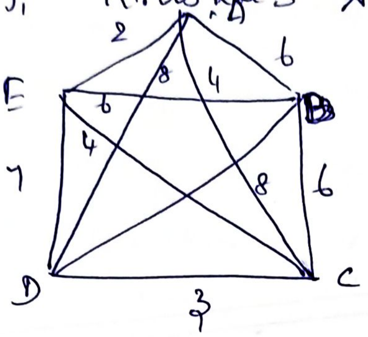
Edges with minimum weight that do not from a circuit with the already selected edges are succesively added.

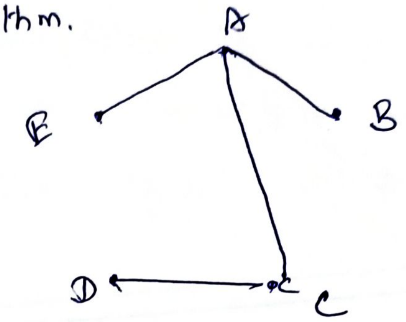
Step :- 4

The procedure is stopped after (n-1) edges have been selected.

12cm

(1) find the minimum spanning tree for the weighted graph using, kruskal's Algorithm.





we first arrang the edges in the increasing order of the edges and proceed as per kruskal's Algorithm.

Edge	weight	Included in the Spanning, tree or not.	If not included Circuit formed.
AE	2	yes	–
CD	3	yes	–
AC	4	yes	–
CE	4	NO	A – ~~E~~ – C – A
AB	6	yes	–
BC	6	NO	A – B – C – A
BE	6	NO	B – E – AB
DE	7	–	–
AD	8	–	–
BD	8	–	–

Since, there are 5 vertices in the graph, we should stop the procedure for finding the edges of the minimum spanning tree, where 4 edges have been found out.

The edges of the minimum spanning tree are AE, CD, AC, AB, whose total length is 15.

There are 5 other alternative minimum spanning trees of total length 15, whose edges are listed below

(1) AE, CD, AC, BC (ii) AE, CD, AC, BE

(3) AE, CD, CE, AB. (4) AE, CD, CE, BC

(5) AE, CD, CE, BE.

10(m).

Prefix and postfix [Expression Trees].

1. from the expression $((a-c)*d) / (a+(b-d))$ as a binary tree.

formula

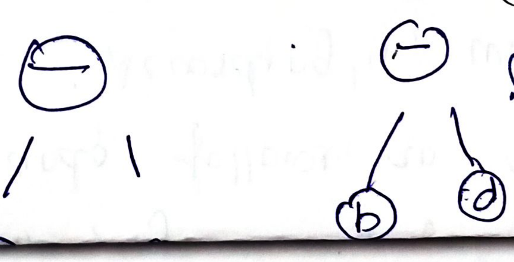
$-, /, *, +.$

(sign, lf, rt) → Prefix = −AB.

lf, rt, sign. Postfix = AB+,

left, Sign, Rt. Infix = A − B.

Soln:-



① − ②

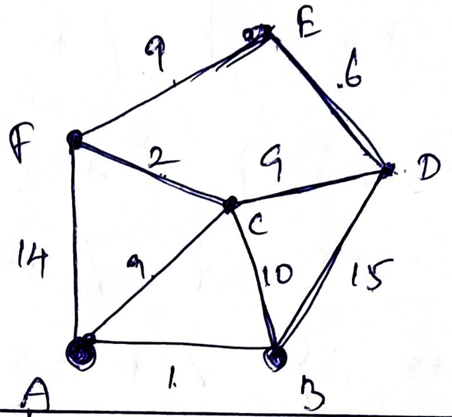
$= 8\ (2\ 5\ -)\ 1\ 3\ -\ *\ /$ $\{\because\ 2-5=-3\}$.

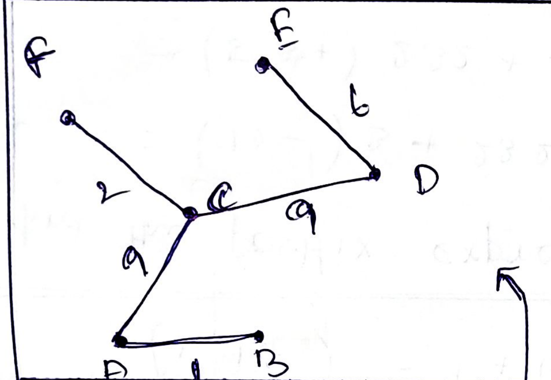
$= [8\ (-3)]\ [1\ 3\ -]\ *\ /$ $\{\because\ 1-3=-2\}$

$= 8\ [(-3)\ (-2)]\ *\ /$ $\{\because\ -2*-3=6\}$

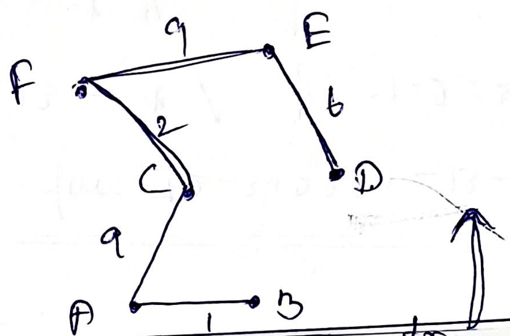
$= [8\ 6\ /] = 8/6 = 4/3$

2. Find the minimum spanning tree for the weighted graph shown in the figure by using Kruskal's Algorithm.





Spanning tree ①



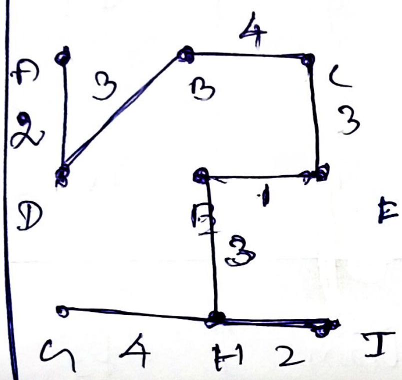
Spanning tree ②.

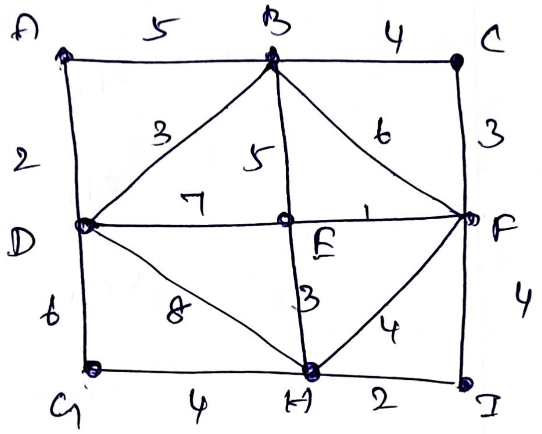
Edge	weights	Included spanning tree (or) not	If not circuit formed.
AB	1	Yes	–
FC	2	Yes	–
ED	6	Yes	–
AC	9	Yes	–
CD	9	Yes	–
AF	14	NO	A – C – F – A
BD	15	NO	B – D – C – B.

∴ The minimum spanning weights is 27.

Q3. Use Kruskal's Algorithm to find a minimum Spanning tree for the weighted graph shown in the below.

Spanning tree.





Given graph.

Conclusion:- The no of given vertices are 9 and edges are 8.
∴ Hence there are, length of the spanning tree = 22.

Edge.	weights	Included Spanning tree (or) not	If not Circuit formed.
EF	1	yes.	—
AD	2	yes	—
HI	2	yes	—
DB	3	yes	—
CF	3	yes	—
EF	3	yes	—
BC	4	yes	—
GH	4	yes	—
FH	4	NO	F-E-H-F.
FI	4	NO	E-F-I-H-E.
AB	5	NO	E-F-I-E.
BE	5	NO	A-D-B-A.
BF	6 7	NO	|
DG.	6 8	NO	etc.

∴ length = 1+2+2+3+3+3+4+4
= 22.