

Homework 01

一、 实验目标：

试下多项式曲线拟合

二、 实验过程：

拟合函数采用多项式：

$$y(x, w) = w_0 + w_1x + w_2x^2 + \dots + w_Mx^M$$

拟合方法采用最小二乘法，使误差函数最小：

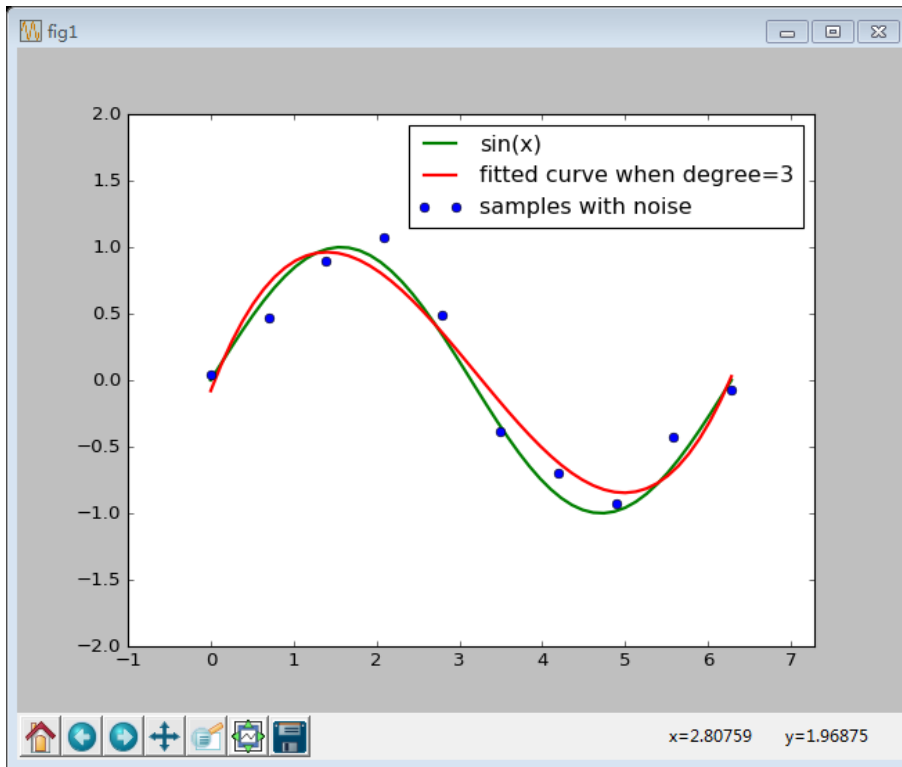
$$E(w) = \sum_{i=1}^n (y(x_i, w) - t(x_i))^2$$

第一步，生成采样点。`x=np.linspace()`生成固定步长的点，`y=np.sin(x)`，生成其 `sin` 值。采用 `numpy.random.normal(0, 0.1)`模拟噪声，生成模拟采样点

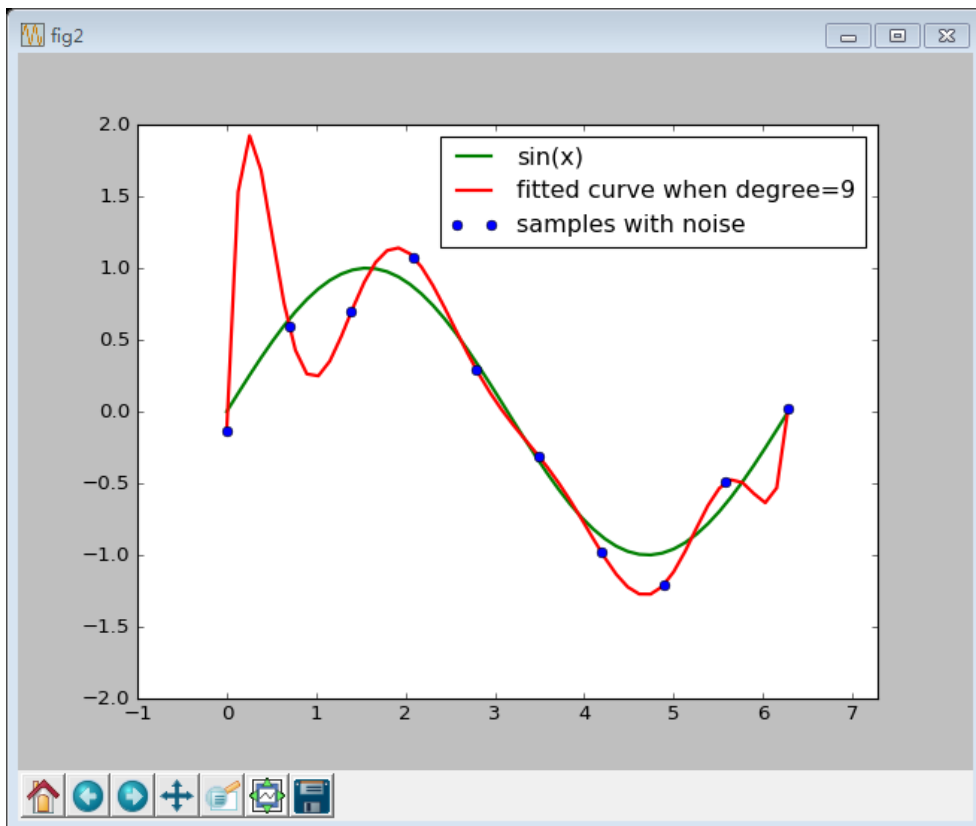
第二步，通过 `np.poly1d(p)`生成多项式函数，构造 `residual(p,x,y)`函数，即实际值与预期值的误差。通过 `leastsq` 函数，使用最小二乘法拟合曲线。并得到最终的多项式参数，绘制图像。

三、 实验结果：

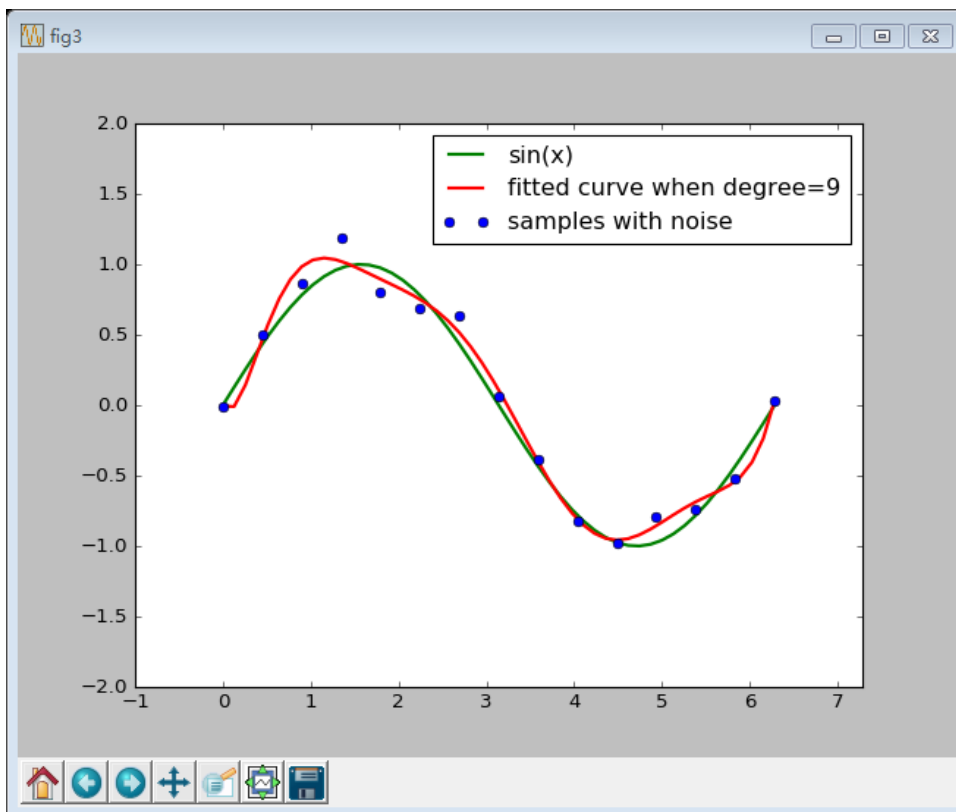
1. 当采样点个数为 10，`degree=3` 时，拟合曲线



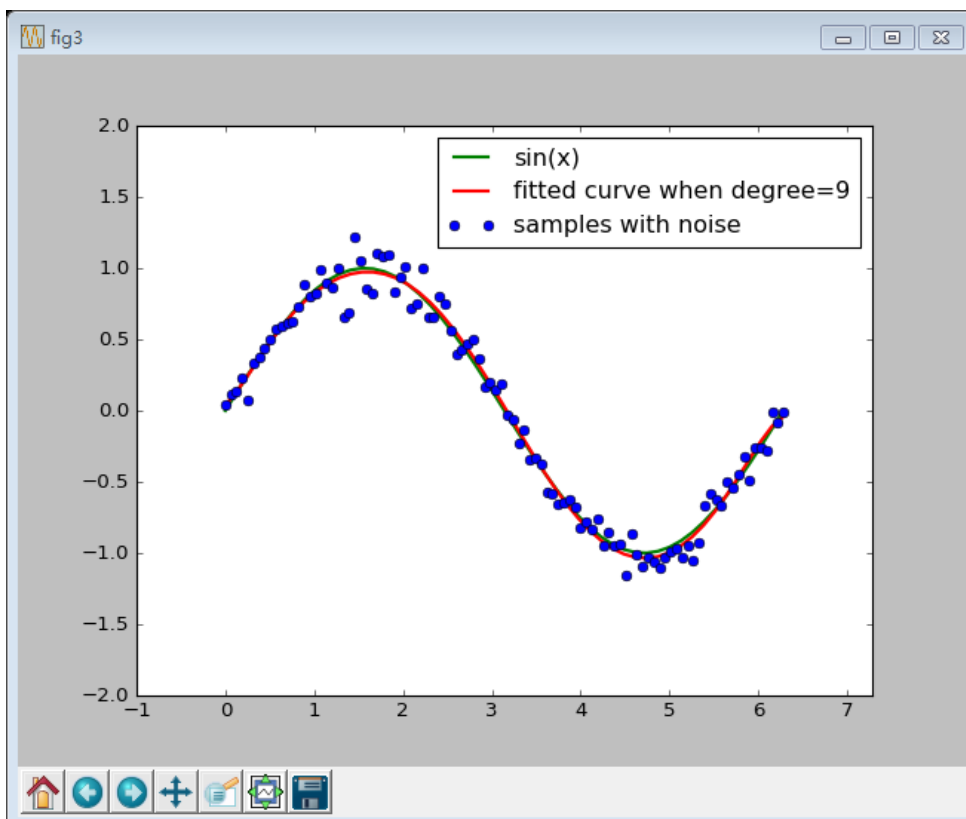
2. 当采样点个数为 10, degree=9 时, 拟合曲线



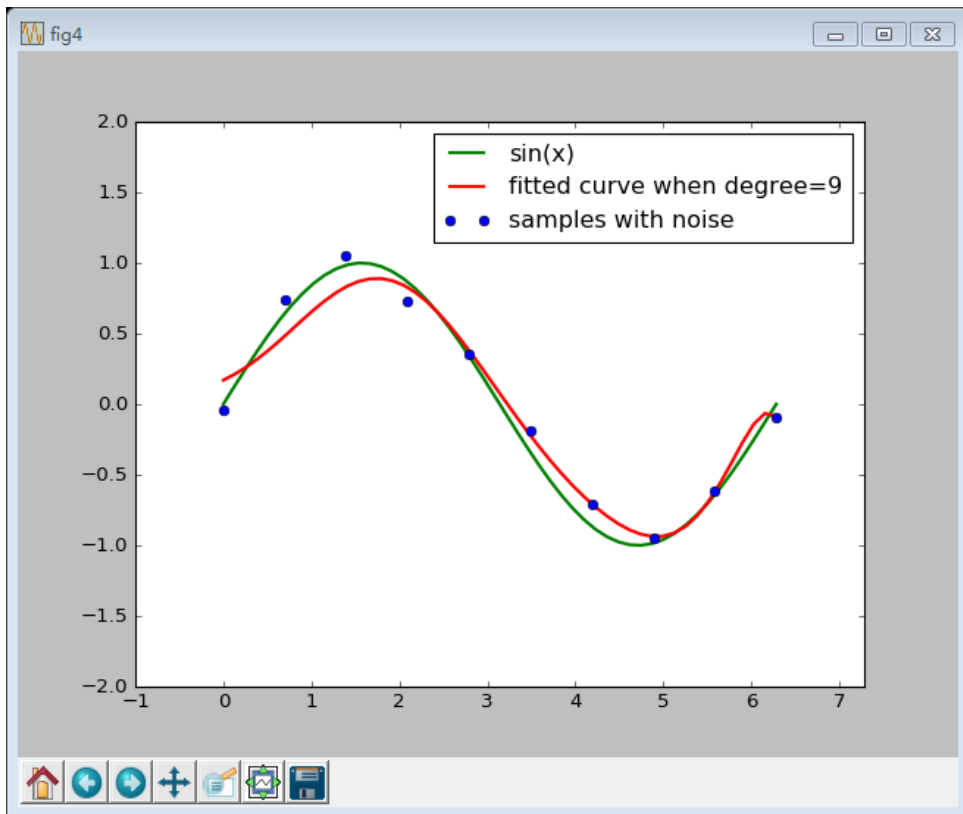
3. 当采样点个数为 15, degree=9 时, 拟合曲线



4. 当采样点个数为 100, $\text{degree}=9$ 时, 拟合曲线



5. 当采样点个数为 10, $\text{degree}=9$ 时, 加入正则项时, 拟合曲线



四、 实验代码及注释：

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.optimize import leastsq
import math

#Generate polynomial function
def fitting_func(p, x):
    f = np.poly1d(p)
    return f(x)

#Generate residual between polynomial function and real value
def residuals(p, x, y):
    return y - fitting_func(p, x)

def residuals_norm(p, x, y):
    ret = y - fitting_func(p, x)
    ret = np.append(ret, np.sqrt(regularization)*p)
    return ret

m0 = 3
m1 = 9
m2 = 15
```

```
regularization = 1
```

#Generate samples

```
x_10 = np.linspace(0, 2*math.pi, 10)
x_15 = np.linspace(0, 2*math.pi, 15)
x_100 = np.linspace(0, 2*math.pi, 100)
x_display = np.linspace(0, 2*math.pi)
y_10 = np.sin(x_10)
y_15 = np.sin(x_15)
y_100 = np.sin(x_100)
y_display = np.sin(x_display)
y_noise_10 = [np.random.normal(0, 0.1) + i for i in y_10]
y_noise_15 = [np.random.normal(0, 0.1) + i for i in y_15]
y_noise_100 = [np.random.normal(0, 0.1) + i for i in y_100]
```

#Generate random polynomial parameters

```
p0 = np.random.randn(m0+1)
p1 = np.random.randn(m1+1)
```

#fitting the curve with leastsq()

```
plsq0 = leastsq(residuals, p0, args=(x_10, y_noise_10))
plsq1 = leastsq(residuals, p1, args=(x_10, y_noise_10))
plsq2 = leastsq(residuals, p1, args=(x_15, y_noise_15))
plsq3 = leastsq(residuals, p1, args=(x_100, y_noise_100))
plsq4 = leastsq(residuals_norm, p1, args=(x_10, y_noise_10))

fig1 = plt.figure("fig1")
plt.ylim(-2, 2)
plt.xlim(-1, 2*math.pi+1)
plt.plot(x_display, y_display, color="green", linewidth="2", label="sin(x)")
plt.plot(x_display, fitting_func(plsq0[0], x_display), color="red", linewidth="2", label="fitted curve when degree=3")
plt.plot(x_10, y_noise_10, "o", color="blue", label="samples with noise")
plt.legend()
plt.show()
```

```
fig2 = plt.figure("fig2")
plt.ylim(-2, 2)
plt.xlim(-1, 2*math.pi+1)
plt.plot(x_display, y_display, color="green", linewidth="2", label="sin(x)")
plt.plot(x_display, fitting_func(plsq1[0], x_display), color="red", linewidth="2", label="fitted curve when degree=9")
```

```
plt.plot(x_10, y_noise_10, "o", color="blue", label="samples with noise")
plt.legend()
plt.show()

fig3 = plt.figure("fig3")
plt.ylim(-2, 2)
plt.xlim(-1, 2*math.pi+1)
plt.plot(x_display, y_display, color="green", linewidth="2", label="sin(x)")
plt.plot(x_display, fitting_func(plsq2[0], x_display), color="red", linewidth="2", label="fitted
curve when degree=9")
plt.plot(x_15, y_noise_15, "o", color="blue", label="samples with noise")
plt.legend()
plt.show()

fig3 = plt.figure("fig3")
plt.ylim(-2, 2)
plt.xlim(-1, 2*math.pi+1)
plt.plot(x_display, y_display, color="green", linewidth="2", label="sin(x)")
plt.plot(x_display, fitting_func(plsq3[0], x_display), color="red", linewidth="2", label="fitted
curve when degree=9")
plt.plot(x_100, y_noise_100, "o", color="blue", label="samples with noise")
plt.legend()
plt.show()

fig4 = plt.figure("fig4")
plt.ylim(-2, 2)
plt.xlim(-1, 2*math.pi+1)
plt.plot(x_display, y_display, color="green", linewidth="2", label="sin(x)")
plt.plot(x_display, fitting_func(plsq4[0], x_display), color="red", linewidth="2", label="fitted
curve when degree=9")
plt.plot(x_10, y_noise_10, "o", color="blue", label="samples with noise")
plt.legend()
plt.show()
```