

Multiple Linear Regression Assignment

Problem Statement

A Chinese automobile company Geely Auto aspires to enter the US market by setting up their manufacturing unit there and producing cars locally to give competition to their US and European counterparts.

They have contracted an automobile consulting company to understand the factors on which the pricing of cars depends. Specifically, they want to understand the factors affecting the pricing of cars in the American market, since those may be very different from the Chinese market. The company wants to know:

Which variables are significant in predicting the price of a car
How well those variables describe the price of a car
Based on various market surveys, the consulting firm has gathered a large dataset of different types of cars across the Americal market.

Business Goal

You are required to model the price of cars with the available independent variables. It will be used by the management to understand how exactly the prices vary with the independent variables. They can accordingly manipulate the design of the cars, the business strategy etc. to meet certain price levels. Further, the model will be a good way for management to understand the pricing dynamics of a new market.

Step 1: lets now load data and understand the data

```
In [1]: # Supress Warnings
```

```
import warnings
warnings.filterwarnings('ignore')
```

```
In [2]: import numpy as np
import pandas as pd
```

```
In [3]: GeelyAuto = pd.read_csv('CarPrice_Assignment.csv')
```

```
In [4]: # Lets check the head of the dataset
GeelyAuto.head()
```

Out[4]:

	car_ID	symboling	CarName	fueltype	aspiration	doornumber	carbody	drivewheel	engine
0	1	3	alfa-romero giulia	gas	std	two	convertible	rwd	
1	2	3	alfa-romero stelvio	gas	std	two	convertible	rwd	
2	3	1	alfa-romero Quadrifoglio	gas	std	two	hatchback	rwd	
3	4	2	audi 100 ls	gas	std	four	sedan	fwd	
4	5	2	audi 100ls	gas	std	four	sedan	4wd	

5 rows × 26 columns

```
In [5]: # Lets check the data description and info
GeelyAuto.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 205 entries, 0 to 204
Data columns (total 26 columns):
car_ID          205 non-null int64
symboling       205 non-null int64
CarName         205 non-null object
fueltype        205 non-null object
aspiration      205 non-null object
doornumber      205 non-null object
carbody         205 non-null object
drivewheel      205 non-null object
enginelocation  205 non-null object
wheelbase       205 non-null float64
carlength       205 non-null float64
carwidth        205 non-null float64
carheight       205 non-null float64
curbweight      205 non-null int64
enginetype      205 non-null object
cylindernumber  205 non-null object
enginesize      205 non-null int64
fuelsystem      205 non-null object
boreratio       205 non-null float64
stroke          205 non-null float64
compressionratio 205 non-null float64
horsepower      205 non-null int64
peakrpm         205 non-null int64
citympg         205 non-null int64
highwaympg      205 non-null int64
price           205 non-null float64
dtypes: float64(8), int64(8), object(10)
memory usage: 41.7+ KB
```

```
In [6]: GeelyAuto.describe()
```

```
Out[6]:
```

	car_ID	symboling	wheelbase	carlength	carwidth	carheight	curbweight	engir
count	205.000000	205.000000	205.000000	205.000000	205.000000	205.000000	205.000000	205.0
mean	103.000000	0.834146	98.756585	174.049268	65.907805	53.724878	2555.565854	126.9
std	59.322565	1.245307	6.021776	12.337289	2.145204	2.443522	520.680204	41.6
min	1.000000	-2.000000	86.600000	141.100000	60.300000	47.800000	1488.000000	61.0
25%	52.000000	0.000000	94.500000	166.300000	64.100000	52.000000	2145.000000	97.0
50%	103.000000	1.000000	97.000000	173.200000	65.500000	54.100000	2414.000000	120.0
75%	154.000000	2.000000	102.400000	183.100000	66.900000	55.500000	2935.000000	141.0
max	205.000000	3.000000	120.900000	208.100000	72.300000	59.800000	4066.000000	326.0

Step 2: Data Preparation

You can see that your dataset has many columns with categorical data.

But in order to fit a regression line, we would need numerical values and not string. Hence, we need to convert them to 1s and 0s.

```
In [7]: GeelyAuto.set_index('car_ID', inplace=True)
GeelyAuto.head()
```

```
Out[7]:
```

	symboling	CarName	fueltype	aspiration	doornumber	carbody	drivewheel	engineloc:
car_ID								
1	3	alfa-romero giulia	gas	std	two	convertible	rwd	
2	3	alfa-romero stelvio	gas	std	two	convertible	rwd	
3	1	alfa-romero Quadrifoglio	gas	std	two	hatchback	rwd	
4	2	audi 100 ls	gas	std	four	sedan	fwd	
5	2	audi 100ls	gas	std	four	sedan	4wd	

5 rows × 25 columns

```
In [8]: # Lets now split carname and consider only car company name
GeelyAuto['CarName']=GeelyAuto['CarName'].apply(lambda x: x.split(' ')[0])
```

```
In [9]: GeelyAuto['CarName'].unique()
```

```
Out[9]: array(['alfa-romero', 'audi', 'bmw', 'chevrolet', 'dodge', 'honda',
              'isuzu', 'jaguar', 'maxda', 'mazda', 'buick', 'mercury',
              'mitsubishi', 'Nissan', 'nissan', 'peugeot', 'plymouth', 'porsche',
              'porcshce', 'renault', 'saab', 'subaru', 'toyota', 'toyouta',
              'vokswagen', 'volkswagen', 'vw', 'volvo'], dtype=object)
```

```
In [10]: # We see some names from GeelyAuto CarName column having redudant or some name is.
GeelyAuto['CarName']=GeelyAuto['CarName'].apply(lambda x: 'volkswagen' if x=='vok
GeelyAuto['CarName']=GeelyAuto['CarName'].apply(lambda x: 'mazda' if x=='maxda' e
GeelyAuto['CarName']=GeelyAuto['CarName'].apply(lambda x: 'nissan' if x=='Nissan'
GeelyAuto['CarName']=GeelyAuto['CarName'].apply(lambda x: 'porsche' if x=='porcsh
GeelyAuto['CarName']=GeelyAuto['CarName'].apply(lambda x: 'toyota' if x=='toyouta
GeelyAuto['CarName']=GeelyAuto['CarName'].apply(lambda x: 'volkswagen' if x=='vw'
```

```
In [11]: GeelyAuto['CarName'].unique()
```

```
Out[11]: array(['alfa-romero', 'audi', 'bmw', 'chevrolet', 'dodge', 'honda',
               'isuzu', 'jaguar', 'mazda', 'buick', 'mercury', 'mitsubishi',
               'nissan', 'peugeot', 'plymouth', 'porsche', 'renault', 'saab',
               'subaru', 'toyota', 'volkswagen', 'volvo'], dtype=object)
```

```
In [12]: #Now Lets check for other columns
GeelyAuto.nunique()
```

```
Out[12]: symboling          6
CarName          22
fueltype         2
aspiration       2
doornumber       2
carbody         5
drivewheel       3
engineloation    2
wheelbase       53
carlength       75
carwidth        44
carheight       49
curbweight      171
enginetype       7
cylindernumber   7
enginesize      44
fuelsystem       8
boreratio       38
stroke          37
compressionratio 32
horsepower      59
peakrpm         23
citympg         29
highwaympg      30
price          189
dtype: int64
```

```
In [13]: #Now to change categorical data for this lets check unique data  
GeelyAuto.nunique()
```

```
Out[13]: symboling          6  
CarName          22  
fueltype         2  
aspiration       2  
doornumber       2  
carbody         5  
drivewheel       3  
enginelocation   2  
wheelbase       53  
carlength       75  
carwidth        44  
carheight       49  
curbweight      171  
enginetype       7  
cylindernumber   7  
enginesize      44  
fuelsystem       8  
boreratio       38  
stroke          37  
compressionratio 32  
horsepower      59  
peakrpm         23  
citympg         29  
highwaympg      30  
price          189  
dtype: int64
```

```
In [14]: #Now lets set all categorycall data with 2 possiblities with 1 or 0  
#array(['gas', 'diesel'], dtype=object) for fueltype  
def decodeFuelType(fuelType):  
    if fuelType=='gas':  
        return 1  
    else:  
        return 0  
GeelyAuto['fueltype']=GeelyAuto['fueltype'].apply(decodeFuelType)
```

```
In [15]: GeelyAuto['enginetype'].unique()
```

```
Out[15]: array(['dohc', 'ohcv', 'ohc', 'l', 'rotor', 'ohcf', 'dohcv'], dtype=object)
```

```
In [16]: # array(['std', 'turbo'], dtype=object) for aspiration
def decodeType(x):
    if x=='std':
        return 1
    else:
        return 0
GeelyAuto['aspiration']=GeelyAuto['aspiration'].apply(decodeType)

# array(['two', 'four'], dtype=object) for doornumber
def decodeType(x):
    if x=='two':
        return 1
    else:
        return 0
GeelyAuto['doornumber']=GeelyAuto['doornumber'].apply(decodeType)

# array(['front', 'rear'], dtype=object) for enginelocation
def decodeType(x):
    if x=='front':
        return 1
    else:
        return 0
GeelyAuto['enginelocation']=GeelyAuto['enginelocation'].apply(decodeType)
```

Dummy Variables

```
In [17]: #now to create some dummy variables for CarName category
CarName=pd.get_dummies(GeelyAuto['CarName'],drop_first = True,prefix='CarName',pr
#CarName.drop('Nissan', inplace=True,axis=1)
CarName.head()
```

Out[17]:

	CarName_audi	CarName_bmw	CarName_buick	CarName_chevrolet	CarName_dodge	CarName_nissan
car_ID						
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	0	0	0	0	0	0
4	1	0	0	0	0	0
5	1	0	0	0	0	0

5 rows × 7 columns

```
In [18]: GeelyAuto = pd.concat([GeelyAuto,CarName],axis=1)
```

```

In [19]: #now to create some dummy variables for carbody category
carbody=pd.get_dummies(GeelyAuto['carbody'],drop_first = True,prefix='carbody',pr
#carbody.head()
GeelyAuto = pd.concat([GeelyAuto,carbody],axis=1)

#now to create some dummy variables for drivewheel category
drivewheel=pd.get_dummies(GeelyAuto['drivewheel'],drop_first = True,prefix='drive
#drivewheel.head()
GeelyAuto = pd.concat([GeelyAuto,drivewheel],axis=1)
#now to create some dummy variables for enginetype category
enginetype=pd.get_dummies(GeelyAuto['enginetype'],drop_first = True,prefix='engin
#enginetype.head()
GeelyAuto = pd.concat([GeelyAuto,enginetype],axis=1)

#now to create some dummy variables for cylindernumber category
cylindernumber=pd.get_dummies(GeelyAuto['cylindernumber'],drop_first = True,prefi
#cylindernumber.head()
GeelyAuto = pd.concat([GeelyAuto,cylindernumber],axis=1)

#now to create some dummy variables for fuelsystem category
fuelsystem=pd.get_dummies(GeelyAuto['fuelsystem'],drop_first = True,prefix='fuels
#fuelsystem.head()
GeelyAuto = pd.concat([GeelyAuto,fuelsystem],axis=1)

```

```

In [20]: GeelyAuto.describe()

```

Out[20]:

	symboling	fueltype	aspiration	doornumber	enginelocation	wheelbase	carlength	
count	205.000000	205.000000	205.000000	205.000000	205.000000	205.000000	205.000000	205.000000
mean	0.834146	0.902439	0.819512	0.439024	0.985366	98.756585	174.049268	174.049268
std	1.245307	0.297446	0.385535	0.497483	0.120377	6.021776	12.337289	12.337289
min	-2.000000	0.000000	0.000000	0.000000	0.000000	86.600000	141.100000	141.100000
25%	0.000000	1.000000	1.000000	0.000000	1.000000	94.500000	166.300000	166.300000
50%	1.000000	1.000000	1.000000	0.000000	1.000000	97.000000	173.200000	173.200000
75%	2.000000	1.000000	1.000000	1.000000	1.000000	102.400000	183.100000	183.100000
max	3.000000	1.000000	1.000000	1.000000	1.000000	120.900000	208.100000	208.100000

8 rows × 65 columns

```

In [21]: GeelyAuto.drop(['CarName','carbody','drivewheel','enginetype','cylindernumber','f

```

```
In [22]: GeelyAuto.head()
```

Out[22]:

	symboling	fueltype	aspiration	doornumber	enginelocation	wheelbase	carlength	carwidth
car_ID								
1	3	1	1	1	1	88.6	168.8	64.
2	3	1	1	1	1	88.6	168.8	64.
3	1	1	1	1	1	94.5	171.2	65.
4	2	1	1	0	1	99.8	176.6	66.
5	2	1	1	0	1	99.4	176.6	66.

5 rows × 65 columns

In [23]: `GeelyAuto.nunique()`

```
Out[23]: symboling          6
fueltype          2
aspiration        2
doornumber        2
enginelocation    2
wheelbase         53
carlength         75
carwidth          44
carheight         49
curbweight        171
enginesize        44
boreratio         38
stroke           37
compressionratio  32
horsepower        59
peakrpm          23
citympg           29
highwaympg        30
price            189
CarName_audi      2
CarName_bmw       2
CarName_buick     2
CarName_chevrolet 2
CarName_dodge     2
CarName_honda     2
CarName_isuzu     2
CarName_jaguar    2
CarName_mazda     2
CarName_mercury   2
CarName_mitsubishi 2
...
CarName_saab      2
CarName_subaru    2
CarName_toyota    2
CarName_volkswagen 2
CarName_volvo     2
carbody_hardtop   2
carbody_hatchback 2
carbody_sedan     2
carbody_wagon     2
drivewheel_fwd    2
drivewheel_rwd    2
enginetype_dohcv  2
enginetype_l      2
enginetype_ohc    2
enginetype_ohcf   2
enginetype_ohcv   2
enginetype_rotor  2
cylindernumber_five 2
cylindernumber_four 2
cylindernumber_six 2
cylindernumber_three 2
cylindernumber_twelve 2
cylindernumber_two 2
```

```
fuelsystem_2bbl      2
fuelsystem_4bbl      2
fuelsystem_idi       2
fuelsystem_mfi       2
fuelsystem_mphi      2
fuelsystem_spdi      2
fuelsystem_spfi      2
Length: 65, dtype: int64
```

Step 3: Splitting the Data into Training and Testing Sets

As you know, the first basic step for regression is performing a train-test split.

```
In [24]: from sklearn.model_selection import train_test_split

# We specify this so that the train and test data set always have the same rows,
np.random.seed(0)
df_train, df_test = train_test_split(GeelyAuto, train_size = 0.7, test_size = 0.3)
```

Rescaling the Features

As you saw in the in our lectures of Simple Linear Regression, scaling doesn't impact our model. Here we can see that except for curbweight and peek rpm, all the columns have small integer values or categorical data. So it is extremely important to rescale the variables so that they have a comparable scale. If we don't have comparable scales, then some of the coefficients as obtained by fitting the regression model might be very large or very small as compared to the other coefficients. This might become very annoying at the time of model evaluation. So it is advised to use standardization or normalization so that the units of the coefficients obtained are all on the same scale. As you know, there are two common ways of rescaling:

1. Min-Max scaling
2. Standardisation (mean-0, sigma-1)

Option chosen will use MinMax scaling.

```
In [25]: from sklearn.preprocessing import MinMaxScaler
```

```
In [26]: scaler = MinMaxScaler()
```

```
In [27]: num_vars = ['symboling', 'wheelbase', 'carlength', 'carwidth', 'carheight', 'curbweight', 'horsepower', 'peakrpm', 'displacement']
df_train[num_vars] = scaler.fit_transform(df_train[num_vars])
```

```
df_train.head()
```

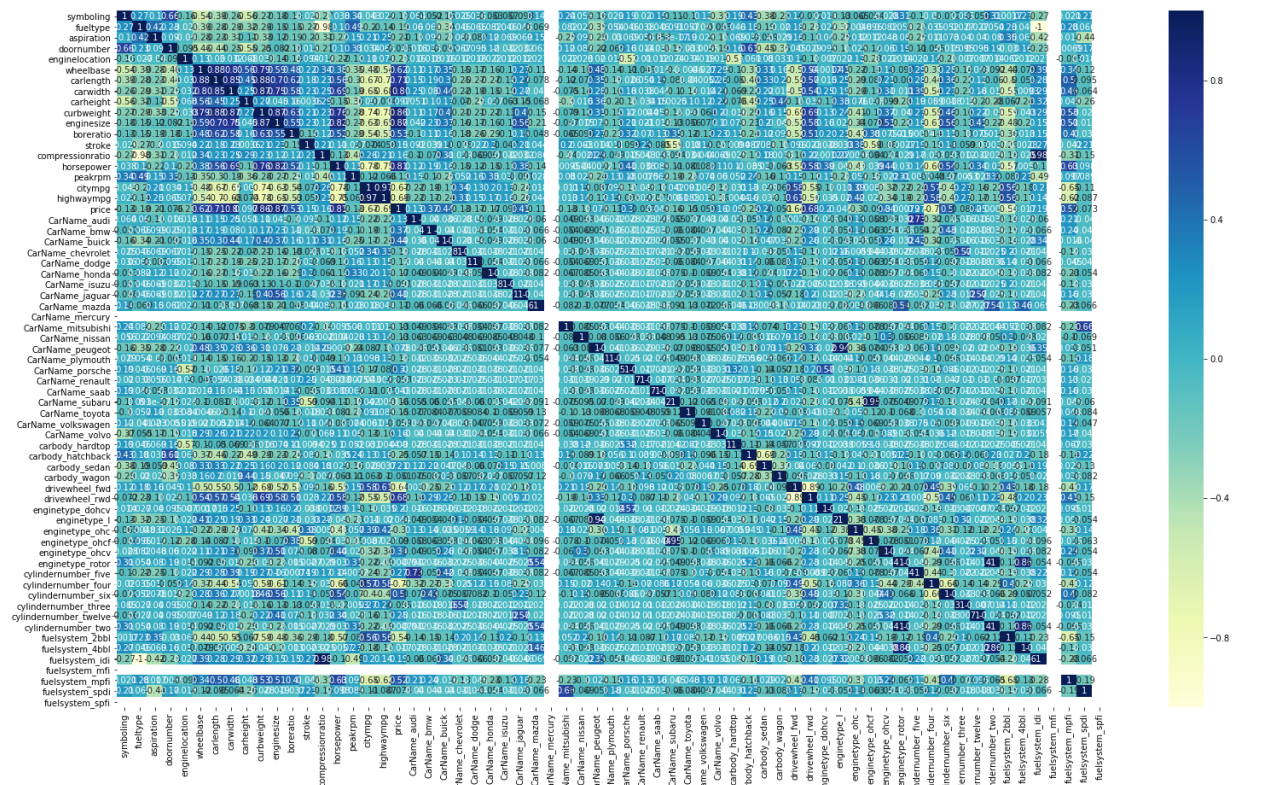
	symboling	fueltype	aspiration	doornumber	enginelocation	wheelbase	carlength	carwidth
car_ID								
123	0.6	1	1	0	1	0.244828	0.426016	0.29166
126	1.0	1	1	1	1	0.272414	0.452033	0.666666
167	0.6	1	1	1	1	0.272414	0.448780	0.30833
2	1.0	1	1	1	1	0.068966	0.450407	0.31666
200	0.2	1	0	0	1	0.610345	0.775610	0.57500

5 rows × 65 columns

```
import seaborn as sns
import matplotlib.pyplot as plt
```

Let's check the correlation coefficients to see which variables are highly correlated

```
plt.figure(figsize = (26, 15))
sns.heatmap(df_train.corr(), annot = True, cmap="YlGnBu")
plt.show()
```



```
In [31]: # With so many variables in our data set its very difficult to infer anything. Let's
col = ['symboling', 'wheelbase', 'carlength', 'carwidth', 'carheight', 'curbweight', 'engine
plt.figure(figsize = (26, 15))
sns.heatmap(df_train[col].corr(), annot = True, cmap="YlGnBu")
plt.show()
```



```
In [32]: df_train.corr()['price']
```

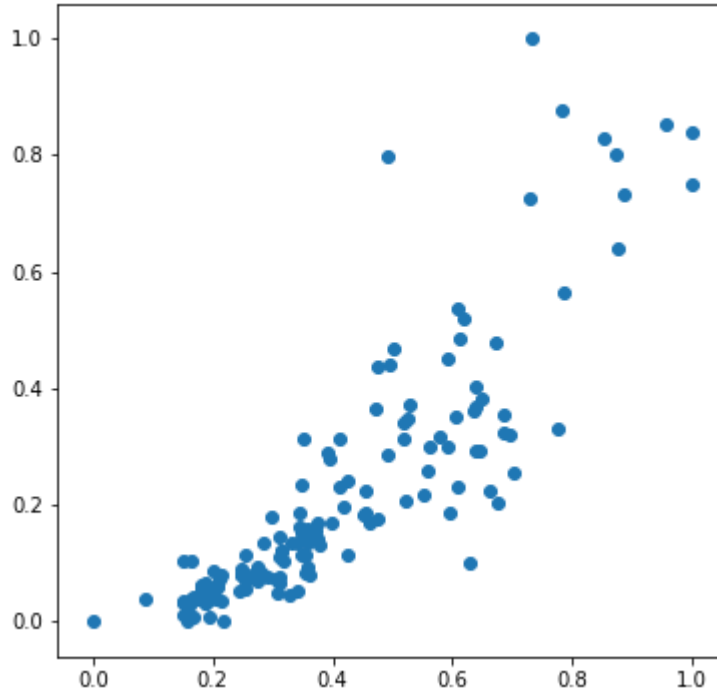
```
Out[32]: symboling          -0.129859
fueltype          -0.191150
aspiration        -0.206540
doornumber        -0.075936
engineloation     -0.226217
wheelbase         0.622591
carlength         0.713749
carwidth          0.799380
carheight         0.096631
curbweight        0.861860
enginesize        0.867915
boreatio          0.533591
stroke           0.152820
compressionratio  0.160847
horsepower        0.806183
peakrpm          -0.127431
citympg          -0.674290
highwaympg       -0.688389
price            1.000000
CarName_audi      0.131449
CarName_bmw       0.371790
CarName_buick     0.437268
CarName_chevrolet -0.132643
CarName_dodge     -0.165673
CarName_honda     -0.172485
CarName_isuzu     -0.091266
CarName_jaguar    0.405372
CarName_mazda     -0.105680
CarName_mercury   NaN
CarName_mitsubishi -0.134087
...
CarName_saab      0.042719
CarName_subaru    -0.164373
CarName_toyota    -0.148968
CarName_volkswagen -0.059238
CarName_volvo     0.161924
carbody_hardtop   0.089735
carbody_hatchback -0.252484
carbody_sedan     0.205018
carbody_wagon     -0.051173
drivewheel_fwd    -0.635202
drivewheel_rwd    0.677169
enginetype_dohcv  0.197875
enginetype_l      0.044246
enginetype_ohc    -0.297108
enginetype_ohcf   -0.089985
enginetype_ohcv   0.339468
enginetype_rotor  -0.000793
cylindernumber_five 0.271430
cylindernumber_four -0.695256
cylindernumber_six 0.500613
cylindernumber_three -0.085274
cylindernumber_twelve 0.247489
cylindernumber_two -0.000793
```

```
fuelsystem_2bbl      -0.537919
fuelsystem_4bbl      -0.017148
fuelsystem_idi       0.191150
fuelsystem_mfi       NaN
fuelsystem_mphi      0.519993
fuelsystem_spdi      -0.073240
fuelsystem_spfi      NaN
Name: price, Length: 65, dtype: float64
```

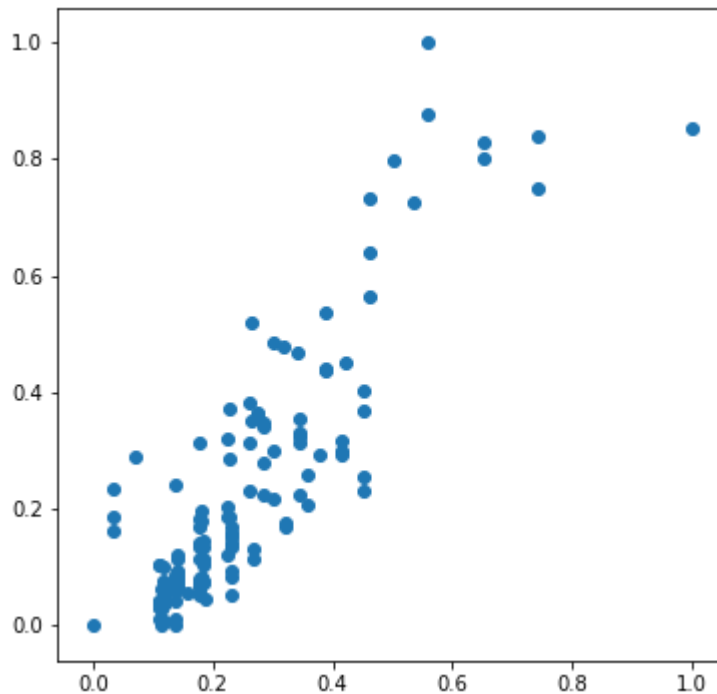
```
In [33]: #We see some relation between columns and Price so to narrow down the correlation
mask = df_train.corr()['price']>=0.3
df_train.corr()[mask]['price']
```

```
Out[33]: wheelbase      0.622591
carlength      0.713749
carwidth       0.799380
curbweight     0.861860
enginesize     0.867915
boretostratio  0.533591
horsepower     0.806183
price          1.000000
CarName_bmw    0.371790
CarName_buick  0.437268
CarName_jaguar 0.405372
CarName_porsche 0.302801
drivewheel_rwd 0.677169
enginetype_ohc 0.339468
cylindernumber_six 0.500613
fuelsystem_mphi 0.519993
Name: price, dtype: float64
```

```
In [34]: plt.figure(figsize=[6,6])  
plt.scatter(df_train.curbweight, df_train.price)  
plt.show()
```



```
In [35]: plt.figure(figsize=[6,6])  
plt.scatter(df_train.enginesize, df_train.price)  
plt.show()
```



if we see the top few scatter plot we see some linear

relation with the price now lets go further and analyse some data.

Dividing into X and Y sets for the model building

```
In [36]: y_train = df_train.pop('price')
X_train = df_train
```

Step 4: Building a linear model

Fit a regression line through the training data using statsmodels. Remember that in statsmodels, you need to explicitly fit a constant using `sm.add_constant(X)` because if we don't perform this step, statsmodels fits a regression line passing through the origin, by default.

```
In [37]: import statsmodels.api as sm

# Add a constant
X_train_lm = sm.add_constant(X_train[['curbweight']])

# Create a first fitted model with curbweight and price
lr = sm.OLS(y_train, X_train_lm).fit()
```

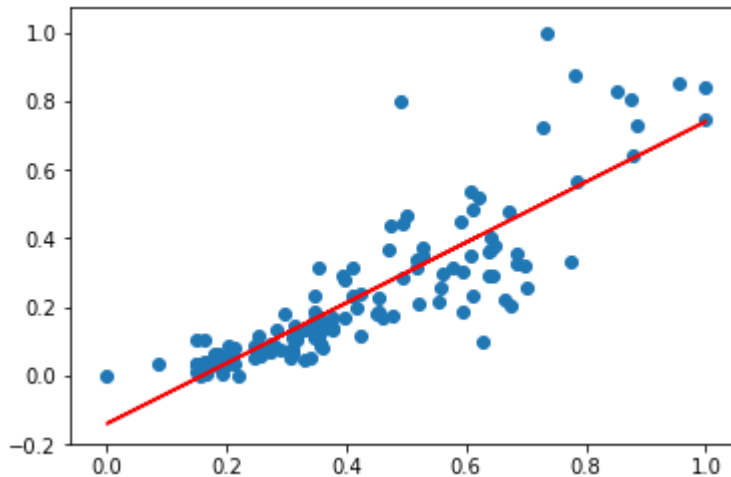
```
In [38]: # Now lets check the parameters obtained

lr.params
```

```
Out[38]: const          -0.139568
curbweight         0.879863
dtype: float64
```



```
In [39]: # Let's visualise the data with a scatter plot and the fitted regression line
plt.scatter(X_train_lm.iloc[:, 1], y_train)
plt.plot(X_train_lm.iloc[:, 1], -0.139568 + 0.879863*X_train_lm.iloc[:, 1], 'r')
plt.show()
```



```
In [40]: # Print a summary of the linear regression model obtained considering curbweight
print(lr.summary())
```

OLS Regression Results

```
=====
Dep. Variable:          price    R-squared:                0.743
Model:                  OLS      Adj. R-squared:            0.741
Method:                 Least Squares    F-statistic:            407.2
Date:                  Sun, 11 Nov 2018    Prob (F-statistic):      2.06e-43
Time:                  07:35:48    Log-Likelihood:          114.04
No. Observations:        143    AIC:                    -224.1
Df Residuals:           141    BIC:                    -218.2
Df Model:                1
Covariance Type:        nonrobust
=====
```

	coef	std err	t	P> t	[0.025	0.975]
const	-0.1396	0.020	-6.974	0.000	-0.179	-0.100
curbweight	0.8799	0.044	20.180	0.000	0.794	0.966

```
=====
Omnibus:                51.679    Durbin-Watson:           1.690
Prob(Omnibus):           0.000    Jarque-Bera (JB):        220.291
Skew:                    1.232    Prob(JB):                 1.46e-48
Kurtosis:                8.559    Cond. No.                 5.57
=====
```

Warnings:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

Add another variable

```
In [41]: # Add a constant
X_train_lm = sm.add_constant(X_train[['curbweight', 'enginesize']])

# Create a first fitted model with curbweight and price
lr = sm.OLS(y_train, X_train_lm).fit()
```

```
In [42]: lr.params
```

```
Out[42]: const          -0.128113
         curbweight      0.450496
         enginesize      0.678162
         dtype: float64
```

```
In [43]: print(lr.summary())
```

```

=====
                        OLS Regression Results
=====
Dep. Variable:          price      R-squared:                0.802
Model:                  OLS       Adj. R-squared:           0.799
Method:                 Least Squares   F-statistic:          284.0
Date:                  Sun, 11 Nov 2018   Prob (F-statistic):    5.31e-50
Time:                  07:35:48     Log-Likelihood:        132.84
No. Observations:      143          AIC:                  -259.7
Df Residuals:          140          BIC:                  -250.8
Df Model:               2
Covariance Type:       nonrobust
=====

```

	coef	std err	t	P> t	[0.025	0.975]
const	-0.1281	0.018	-7.239	0.000	-0.163	-0.093
curbweight	0.4505	0.076	5.890	0.000	0.299	0.602
enginesize	0.6782	0.105	6.489	0.000	0.472	0.885

```

=====
Omnibus:                 36.002    Durbin-Watson:           1.830
Prob(Omnibus):            0.000    Jarque-Bera (JB):         85.998
Skew:                     1.023    Prob(JB):                 2.12e-19
Kurtosis:                 6.202    Cond. No.                  17.3
=====

```

Warnings:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

We see some increase on the adjusted R square so this is a value addition.

Let move forward to use RFE for feature selection and see if we can create a better model

In [44]: *#Build a linear model considering all the variables to analyse the impact*

```
import statsmodels.api as sm
X_train_lm = sm.add_constant(X_train)

lr_1 = sm.OLS(y_train, X_train_lm).fit()

lr_1.params
```

```
Out[44]: const                2.306669e-02
symboling                 -9.879522e-03
fueltype                  -1.260443e-01
aspiration                -8.113676e-02
doornumber                -1.271982e-02
enginelocation            -1.903334e-01
wheelbase                 2.367996e-01
carlength                 -1.752292e-01
carwidth                  2.597715e-01
carheight                 -1.762854e-01
curbweight                3.291759e-01
enginesize                2.010441e+00
boreratio                 -6.214539e-01
stroke                   -1.812934e-01
compressionratio          -3.611938e-01
horsepower                -2.005293e-01
peakrpm                   1.878108e-01
citympg                   -5.156732e-02
highwaympg                1.273365e-01
CarName_audi              6.967675e-02
CarName_bmw               3.396067e-01
CarName_buick             4.930883e-02
CarName_chevrolet         -5.223870e-02
CarName_dodge             -1.089892e-01
CarName_honda             -6.885945e-02
CarName_isuzu             2.295335e-03
CarName_jaguar            -1.331677e-01
CarName_mazda             2.447154e-02
CarName_mercury           -5.682073e-15
CarName_mitsubishi        -1.273068e-01
...
CarName_saab              2.259475e-01
CarName_subaru            -4.025895e-02
CarName_toyota            1.648532e-02
CarName_volkswagen        2.285186e-02
CarName_volvo             1.069129e-01
carbody_hardtop           -9.714369e-02
carbody_hatchback         -1.058254e-01
carbody_sedan             -8.663686e-02
carbody_wagon             -6.762364e-02
drivewheel_fwd            -3.432830e-03
drivewheel_rwd            2.624883e-02
enginetype_dohcv          2.378634e-01
enginetype_l              2.070738e-01
enginetype_ohc            1.040682e-03
enginetype_ohcf           1.731411e-01
enginetype_ohcv          -2.752914e-02
```

enginetype_rotor	3.960833e-01
cylindernumber_five	2.665612e-01
cylindernumber_four	4.385606e-01
cylindernumber_six	1.188905e-01
cylindernumber_three	4.954953e-01
cylindernumber_twelve	-3.562871e-01
cylindernumber_two	3.960833e-01
fuelsystem_2bbl	2.322706e-02
fuelsystem_4bbl	-4.880078e-02
fuelsystem_idi	1.491109e-01
fuelsystem_mfi	0.000000e+00
fuelsystem_mphi	-1.610708e-02
fuelsystem_spdi	-2.618883e-02
fuelsystem_spfi	0.000000e+00

Length: 65, dtype: float64

```
In [45]: print(lr_1.summary())
```

```

OLS Regression Results

=====
Dep. Variable:          price    R-squared:                0.975
Model:                  OLS      Adj. R-squared:            0.958
Method:                 Least Squares    F-statistic:            57.59
Date:                  Sun, 11 Nov 2018    Prob (F-statistic):      1.40e-49
Time:                  07:35:48    Log-Likelihood:          280.02
No. Observations:      143    AIC:                    -444.0
Df Residuals:           85    BIC:                    -272.2
Df Model:               57
Covariance Type:        nonrobust
=====
=====

```

	coef	std err	t	P> t	[0.025
const	0.0231	0.142	0.162	0.872	-0.260
symboling	-0.0099	0.037	-0.265	0.792	-0.084
fueltype	-0.1260	0.100	-1.265	0.209	-0.324
aspiration	-0.0811	0.029	-2.778	0.007	-0.139
doornumber	-0.0127	0.015	-0.841	0.403	-0.043
enginelocation	-0.1903	0.058	-3.285	0.001	-0.306
wheelbase	0.2368	0.093	2.542	0.013	0.052
carlength	-0.1752	0.100	-1.747	0.084	-0.375
carwidth	0.2598	0.101	2.581	0.012	0.060
carheight	-0.1763	0.053	-3.341	0.001	-0.281
curbweight	0.3292	0.131	2.521	0.014	0.070
enginesize	2.0104	0.464	4.333	0.000	1.088
boreratio	-0.6215	0.170	-3.660	0.000	-0.959
stroke	-0.1813	0.083	-2.174	0.032	-0.347
compressionratio	-0.3612	0.271	-1.334	0.186	-0.900
horsepower	-0.2005	0.209	-0.958	0.341	-0.617
peakrpm	0.1878	0.051	3.688	0.000	0.087
citympg	-0.0516	0.154	-0.335	0.738	-0.357
highwaympg	0.1273	0.139	0.918	0.361	-0.148

0.403					
CarName_audi	0.0697	0.086	0.812	0.419	-0.101
0.240					
CarName_bmw	0.3396	0.093	3.664	0.000	0.155
0.524					
CarName_buick	0.0493	0.089	0.554	0.581	-0.128
0.226					
CarName_chevrolet	-0.0522	0.079	-0.665	0.508	-0.208
0.104					
CarName_dodge	-0.1090	0.066	-1.655	0.102	-0.240
0.022					
CarName_honda	-0.0689	0.082	-0.836	0.406	-0.233
0.095					
CarName_isuzu	0.0023	0.074	0.031	0.975	-0.145
0.149					
CarName_jaguar	-0.1332	0.089	-1.492	0.139	-0.311
0.044					
CarName_mazda	0.0245	0.066	0.370	0.712	-0.107
0.156					
CarName_mercury	-5.682e-15	1.99e-15	-2.850	0.005	-9.65e-15
-1.72e-15					
CarName_mitsubishi	-0.1273	0.067	-1.899	0.061	-0.261
0.006					
CarName_nissan	0.0268	0.068	0.396	0.693	-0.108
0.162					
CarName_peugeot	-0.2884	0.061	-4.719	0.000	-0.410
-0.167					
CarName_plymouth	-0.1125	0.064	-1.758	0.082	-0.240
0.015					
CarName_porsche	0.2637	0.111	2.367	0.020	0.042
0.485					
CarName_renault	-0.0109	0.079	-0.138	0.891	-0.168
0.146					
CarName_saab	0.2259	0.087	2.608	0.011	0.054
0.398					
CarName_subaru	-0.0403	0.108	-0.373	0.710	-0.255
0.175					
CarName_toyota	0.0165	0.063	0.263	0.794	-0.108
0.141					
CarName_volkswagen	0.0229	0.064	0.356	0.723	-0.105
0.151					
CarName_volvo	0.1069	0.092	1.164	0.247	-0.076
0.289					
carbody_hardtop	-0.0971	0.057	-1.704	0.092	-0.210
0.016					
carbody_hatchback	-0.1058	0.044	-2.386	0.019	-0.194
-0.018					
carbody_sedan	-0.0866	0.047	-1.838	0.070	-0.180
0.007					
carbody_wagon	-0.0676	0.050	-1.343	0.183	-0.168
0.032					
drivewheel_fwd	-0.0034	0.025	-0.140	0.889	-0.052
0.045					
drivewheel_rwd	0.0262	0.035	0.757	0.451	-0.043
0.095					
enginetype_dohcv	0.2379	0.168	1.415	0.161	-0.096
0.572					

enginetype_l 0.370	0.2071	0.082	2.528	0.013	0.044
enginetype_ohc 0.090	0.0010	0.045	0.023	0.981	-0.088
enginetype_ohcf 0.277	0.1731	0.052	3.324	0.001	0.070
enginetype_ohcv 0.050	-0.0275	0.039	-0.702	0.485	-0.106
enginetype_rotor 0.625	0.3961	0.115	3.438	0.001	0.167
cylindernumber_five 0.548	0.2666	0.141	1.885	0.063	-0.015
cylindernumber_four 0.803	0.4386	0.183	2.391	0.019	0.074
cylindernumber_six 0.324	0.1189	0.103	1.154	0.252	-0.086
cylindernumber_three 0.742	0.4955	0.124	3.992	0.000	0.249
cylindernumber_twelve -0.001	-0.3563	0.179	-1.992	0.050	-0.712
cylindernumber_two 0.625	0.3961	0.115	3.438	0.001	0.167
fuelsystem_2bbl 0.134	0.0232	0.056	0.416	0.679	-0.088
fuelsystem_4bbl 0.109	-0.0488	0.079	-0.616	0.540	-0.206
fuelsystem_idi 0.473	0.1491	0.163	0.915	0.363	-0.175
fuelsystem_mfi 0	0	0	nan	nan	0
fuelsystem_mpf 0.106	-0.0161	0.062	-0.262	0.794	-0.138
fuelsystem_spdi 0.104	-0.0262	0.066	-0.399	0.691	-0.157
fuelsystem_spfi 0	0	0	nan	nan	0

```

=====
Omnibus:                 34.574    Durbin-Watson:                 1.863
Prob(Omnibus):            0.000    Jarque-Bera (JB):             162.755
Skew:                     0.707    Prob(JB):                     4.55e-36
Kurtosis:                 8.032    Cond. No.                     1.07e+16
=====

```

Warnings:

- [1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
- [2] The smallest eigenvalue is 1.07e-29. This might indicate that there are strong multicollinearity problems or that the design matrix is singular.

As the warnings suggest there is a strong multicollinearity problem.

Checking VIF

Variance Inflation Factor or VIF, gives a basic quantitative idea about how much the feature variables are correlated with each other. It is an extremely important parameter to test our linear model. The formula for calculating VIF is:

$$VIF_i = \frac{1}{1-R_i^2}$$

```
In [46]: # Check for the VIF values of the feature variables.  
from statsmodels.stats.outliers_influence import variance_inflation_factor
```



```
In [47]: # Create a dataframe that will contain the names of all the feature variables and
vif = pd.DataFrame()
vif['Features'] = X_train.columns
vif['VIF'] = [variance_inflation_factor(X_train.values, i) for i in range(X_train
vif['VIF'] = round(vif['VIF'], 2)
vif = vif.sort_values(by = "VIF", ascending = False)
print(vif)
```

	Features	VIF
54	cylindernumber_three	inf
59	fuelsystem_idi	inf
35	CarName_subaru	inf
1	fueltype	inf
30	CarName_peugeot	inf
46	enginetype_l	inf
48	enginetype_ohcf	inf
56	cylindernumber_two	inf
50	enginetype_rotor	inf
4	enginelocation	inf
52	cylindernumber_four	470.310000
10	enginesize	372.550000
13	compressionratio	298.700000
11	boreratio	89.540000
53	cylindernumber_six	89.140000
14	horsepower	86.860000
51	cylindernumber_five	86.020000
61	fuelsystem_mphi	68.000000
16	citympg	59.320000
9	curbweight	55.090000
57	fuelsystem_2bbl	51.750000
17	highwaympg	49.470000
41	carbody_sedan	40.420000
36	CarName_toyota	34.580000
40	carbody_hatchback	32.310000
47	enginetype_ohc	31.320000
6	carlength	30.550000
29	CarName_nissan	29.560000
23	CarName_honda	29.190000
5	wheelbase	26.560000
..
42	carbody_wagon	21.290000
44	drivewheel_rwd	19.930000
20	CarName_buick	19.500000
28	CarName_mitsubishi	19.320000
32	CarName_porsche	18.580000
18	CarName_audi	18.110000
55	cylindernumber_twelve	16.190000
45	enginetype_dohcv	14.300000
37	CarName_volkswagen	14.020000
22	CarName_dodge	12.710000
62	fuelsystem_spdi	12.640000
12	stroke	12.540000
25	CarName_jaguar	11.920000
34	CarName_saab	11.240000
43	drivewheel_fwd	10.580000
58	fuelsystem_4bbl	9.410000

```

8          carheight  9.350000
2          aspiration  9.250000
21         CarName_chevrolet  9.240000
24         CarName_isuzu  8.190000
31         CarName_plymouth  8.110000
15          peakrpm  7.240000
49         enginetype_ohcv  6.610000
33         CarName_renault  6.250000
0          symboling  5.760000
39         carbody_hardtop  4.870000
3          doornumber  4.110000
27         CarName_mercury      NaN
60         fuelsystem_mfi      NaN
63         fuelsystem_spfi      NaN

```

[64 rows x 2 columns]

Dropping the Variable and Updating the Model

As you can notice some of the variable have high VIF values some with 'inf'. Such variables are insignificant and should be dropped.

As you might have noticed, the variable `fueltype` has high VIF inf and a high p-value (0.206) as well. Hence, this variable isn't of much use and should be dropped.

```
In [48]: X = X_train.drop('fueltype', 1,)
```

```
In [49]: # Build a second fitted model
X_train_lm = sm.add_constant(X)

lr_2 = sm.OLS(y_train, X_train_lm).fit()
# Print the summary of the model
print(lr_2.summary())
```

```

49
Time:                                07:35:48    Log-Likelihood:                280.
02
No. Observations:                    143    AIC:                            -44
4.0
Df Residuals:                        85    BIC:                            -27
2.2
Df Model:                            57

```

Covariance Type: nonrobust

```

=====
=====

```

		coef	std err	t	P> t	[0.02
5	0.975]					

const		-0.0715	0.169	-0.423	0.673	-0.40
7	0.264					
symboling		-0.0099	0.037	-0.265	0.792	-0.08

```
In [50]: # Calculate the VIFs again for the new model

vif = pd.DataFrame()
vif['Features'] = X.columns
vif['VIF'] = [variance_inflation_factor(X.values, i) for i in range(X.shape[1])]
vif['VIF'] = round(vif['VIF'], 2)
vif = vif.sort_values(by = "VIF", ascending = False)
vif
```

Out[50]:

	Features	VIF
49	enginetype_rotor	inf
45	enginetype_l	inf
29	CarName_peugeot	inf
53	cylindernumber_three	inf
55	cylindernumber_two	inf
3	enginelocation	2631.800000
51	cylindernumber_four	470.310000
34	CarName_subaru	447.870000
9	enginesize	372.550000
58	fuelsystem_idi	317.620000
12	compressionratio	298.700000
47	enginetype_ohcf	287.690000
10	boreratio	89.540000
52	cylindernumber_six	89.140000
13	horsepower	86.860000
50	cylindernumber_five	86.020000
60	fuelsystem_mphi	68.000000
15	citympg	59.320000
8	curbweight	55.090000
56	fuelsystem_2bbl	51.750000
16	highwaympg	49.470000
40	carbody_sedan	40.420000
35	CarName_toyota	34.580000
39	carbody_hatchback	32.310000
46	enginetype_ohc	31.320000
5	carlength	30.550000
28	CarName_nissan	29.560000
22	CarName_honda	29.190000
4	wheelbase	26.560000

	Features	VIF
25	CarName_mazda	26.350000
...
41	carbody_wagon	21.290000
43	drivewheel_rwd	19.930000
19	CarName_buick	19.500000
27	CarName_mitsubishi	19.320000
31	CarName_porsche	18.580000
17	CarName_audi	18.110000
54	cylindernumber_twelve	16.190000
44	enginetype_dohcv	14.300000
36	CarName_volkswagen	14.020000
21	CarName_dodge	12.710000
61	fuelsystem_spdi	12.640000
11	stroke	12.540000
24	CarName_jaguar	11.920000
33	CarName_saab	11.240000
42	drivewheel_fwd	10.580000
57	fuelsystem_4bbl	9.410000
7	carheight	9.350000
1	aspiration	9.250000
20	CarName_chevrolet	9.240000
23	CarName_isuzu	8.190000
30	CarName_plymouth	8.110000
14	peakrpm	7.240000
48	enginetype_ohcv	6.610000
32	CarName_renault	6.250000
0	symboling	5.760000
38	carbody_hardtop	4.870000
2	doornumber	4.110000
26	CarName_mercury	NaN
59	fuelsystem_mfi	NaN
62	fuelsystem_spfi	NaN

63 rows × 2 columns

In [51]: *# Calculate the VIFs again for the new model*

```
X1 = X.drop('enginetype_rotor', 1,)
vif = pd.DataFrame()
vif['Features'] = X1.columns
vif['VIF'] = [variance_inflation_factor(X1.values, i) for i in range(X1.shape[1])]
vif['VIF'] = round(vif['VIF'], 2)
vif = vif.sort_values(by = "VIF", ascending = False)
vif
```

Out[51]:

	Features	VIF
52	cylindernumber_three	inf
29	CarName_peugeot	inf
45	enginetype_l	inf
3	enginelocation	2631.800000
50	cylindernumber_four	470.310000
34	CarName_subaru	447.870000
9	enginesize	372.550000
57	fuelsystem_idi	317.620000
12	compressionratio	298.700000
47	enginetype_ohcf	287.690000
54	cylindernumber_two	105.250000

In [52]: *# Calculate the VIFs again for the new model*

```
X2 = X1.drop('cylindernumber_three', 1,)
vif = pd.DataFrame()
vif['Features'] = X2.columns
vif['VIF'] = [variance_inflation_factor(X2.values, i) for i in range(X2.shape[1])]
vif['VIF'] = round(vif['VIF'], 2)
vif = vif.sort_values(by = "VIF", ascending = False)
vif
```

Out[52]:

	Features	VIF
3	enginelocation	2631.80
50	cylindernumber_four	470.31
34	CarName_subaru	447.87
9	enginesize	372.55
56	fuelsystem_idi	317.62
12	compressionratio	298.70
47	enginetype_ohcf	287.69
45	enginetype_l	174.12
29	CarName_peugeot	121.59
53	cylindernumber_two	105.25
10	boreratio	89.54

```
In [53]: # Build a second fitted model
X_train_lm = sm.add_constant(X)

lr_2 = sm.OLS(y_train, X_train_lm).fit()
# Print the summary of the model
print(lr_2.summary())
```

OLS Regression Results

```
=====
Dep. Variable:          price      R-squared:                0.975
Model:                  OLS       Adj. R-squared:            0.958
Method:                 Least Squares   F-statistic:           57.59
Date:                  Sun, 11 Nov 2018   Prob (F-statistic):    1.40e-49
Time:                  07:35:50    Log-Likelihood:        280.02
No. Observations:      143          AIC:                  -444.0
Df Residuals:          85           BIC:                  -272.2
Df Model:              57
Covariance Type:       nonrobust
=====
```

```
=====
                                coef      std err          t      P>|t|      [0.025
0.975]
-----
const                -0.0715      0.169      -0.423      0.673      -0.407
0.264
symboling            -0.0099      0.037      -0.265      0.792      -0.084
0.064
aspiration           -0.0811      0.029      -2.778      0.007      -0.139
-0.023
doornumber           -0.0127      0.015      -0.841      0.403      -0.043
0.017
engineloation        -0.2218      0.056      -3.988      0.000      -0.332
-0.111
wheelbase            0.2368      0.093       2.542      0.013       0.052
0.422
carlength            -0.1752      0.100      -1.747      0.084      -0.375
0.024
carwidth             0.2598      0.101       2.581      0.012       0.060
0.460
carheight            -0.1763      0.053      -3.341      0.001      -0.281
-0.071
curbweight           0.3292      0.131       2.521      0.014       0.070
0.589
enginesize           2.0104      0.464       4.333      0.000       1.088
2.933
boreratio            -0.6215      0.170      -3.660      0.000      -0.959
-0.284
stroke              -0.1813      0.083      -2.174      0.032      -0.347
-0.016
compressionratio     -0.3612      0.271      -1.334      0.186      -0.900
0.177
horsepower           -0.2005      0.209      -0.958      0.341      -0.617
0.216
peakrpm              0.1878      0.051       3.688      0.000       0.087
0.289
=====
```

citympg	-0.0516	0.154	-0.335	0.738	-0.357
0.254					
highwaympg	0.1273	0.139	0.918	0.361	-0.148
0.403					
CarName_audi	0.0697	0.086	0.812	0.419	-0.101
0.240					
CarName_bmw	0.3396	0.093	3.664	0.000	0.155
0.524					
CarName_buick	0.0493	0.089	0.554	0.581	-0.128
0.226					
CarName_chevrolet	-0.0522	0.079	-0.665	0.508	-0.208
0.104					
CarName_dodge	-0.1090	0.066	-1.655	0.102	-0.240
0.022					
CarName_honda	-0.0689	0.082	-0.836	0.406	-0.233
0.095					
CarName_isuzu	0.0023	0.074	0.031	0.975	-0.145
0.149					
CarName_jaguar	-0.1332	0.089	-1.492	0.139	-0.311
0.044					
CarName_mazda	0.0245	0.066	0.370	0.712	-0.107
0.156					
CarName_mercury	3.465e-15	1.05e-15	3.299	0.001	1.38e-15
5.55e-15					
CarName_mitsubishi	-0.1273	0.067	-1.899	0.061	-0.261
0.006					
CarName_nissan	0.0268	0.068	0.396	0.693	-0.108
0.162					
CarName_peugeot	-0.2884	0.061	-4.719	0.000	-0.410
-0.167					
CarName_plymouth	-0.1125	0.064	-1.758	0.082	-0.240
0.015					
CarName_porsche	0.2637	0.111	2.367	0.020	0.042
0.485					
CarName_renault	-0.0109	0.079	-0.138	0.891	-0.168
0.146					
CarName_saab	0.2259	0.087	2.608	0.011	0.054
0.398					
CarName_subaru	-0.0087	0.118	-0.074	0.941	-0.243
0.226					
CarName_toyota	0.0165	0.063	0.263	0.794	-0.108
0.141					
CarName_volkswagen	0.0229	0.064	0.356	0.723	-0.105
0.151					
CarName_volvo	0.1069	0.092	1.164	0.247	-0.076
0.289					
carbody_hardtop	-0.0971	0.057	-1.704	0.092	-0.210
0.016					
carbody_hatchback	-0.1058	0.044	-2.386	0.019	-0.194
-0.018					
carbody_sedan	-0.0866	0.047	-1.838	0.070	-0.180
0.007					
carbody_wagon	-0.0676	0.050	-1.343	0.183	-0.168
0.032					
drivewheel_fwd	-0.0034	0.025	-0.140	0.889	-0.052
0.045					
drivewheel_rwd	0.0262	0.035	0.757	0.451	-0.043

0.095					
enginetype_dohcv	0.2379	0.168	1.415	0.161	-0.096
0.572					
enginetype_l	0.2071	0.082	2.528	0.013	0.044
0.370					
enginetype_ohc	0.0010	0.045	0.023	0.981	-0.088
0.090					
enginetype_ohcf	0.1416	0.059	2.393	0.019	0.024
0.259					
enginetype_ohcv	-0.0275	0.039	-0.702	0.485	-0.106
0.050					
enginetype_rotor	0.3961	0.115	3.438	0.001	0.167
0.625					
cylindernumber_five	0.2666	0.141	1.885	0.063	-0.015
0.548					
cylindernumber_four	0.4386	0.183	2.391	0.019	0.074
0.803					
cylindernumber_six	0.1189	0.103	1.154	0.252	-0.086
0.324					
cylindernumber_three	0.4955	0.124	3.992	0.000	0.249
0.742					
cylindernumber_twelve	-0.3563	0.179	-1.992	0.050	-0.712
-0.001					
cylindernumber_two	0.3961	0.115	3.438	0.001	0.167
0.625					
fuelsystem_2bbl	0.0232	0.056	0.416	0.679	-0.088
0.134					
fuelsystem_4bbl	-0.0488	0.079	-0.616	0.540	-0.206
0.109					
fuelsystem_idi	0.2752	0.230	1.198	0.234	-0.181
0.732					
fuelsystem_mfi	0	0	nan	nan	0
0					
fuelsystem_mphi	-0.0161	0.062	-0.262	0.794	-0.138
0.106					
fuelsystem_spdi	-0.0262	0.066	-0.399	0.691	-0.157
0.104					
fuelsystem_spfi	0	0	nan	nan	0
0					

```

=====
Omnibus:                34.574    Durbin-Watson:                1.863
Prob(Omnibus):           0.000    Jarque-Bera (JB):            162.755
Skew:                    0.707    Prob(JB):                     4.55e-36
Kurtosis:                8.032    Cond. No.                     1.08e+16
=====

```

Warnings:

- [1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
- [2] The smallest eigenvalue is 9.47e-30. This might indicate that there are strong multicollinearity problems or that the design matrix is singular.

After dropping two features we see VIF to be reduced and p value change as well, But the manual feature list selection will not not help due to the number of features lets move to

RFE

```
In [96]: from sklearn.model_selection import train_test_split

# We specify this so that the train and test data set always have the same rows,
np.random.seed(0)
df_train, df_test = train_test_split(GeelyAuto, train_size = 0.7, test_size = 0.3)
```

```
In [97]: from sklearn.preprocessing import MinMaxScaler
scaler = MinMaxScaler()
# Apply scaler() to all the columns except the 'yes-no' and 'dummy' variables
num_vars = ['symboling', 'wheelbase', 'carlength', 'carwidth', 'carheight', 'curbweight']
df_train[num_vars] = scaler.fit_transform(df_train[num_vars])
```

```
In [98]: df_train.head()
```

```
Out[98]:
```

	symboling	fueltype	aspiration	doornumber	enginelocation	wheelbase	carlength	carwidth
car_ID								
123	0.6	1	1	0	1	0.244828	0.426016	0.29166
126	1.0	1	1	1	1	0.272414	0.452033	0.66666
167	0.6	1	1	1	1	0.272414	0.448780	0.30833
2	1.0	1	1	1	1	0.068966	0.450407	0.31666
200	0.2	1	0	0	1	0.610345	0.775610	0.57500

5 rows × 65 columns

```
In [99]: df_train.describe()
```

```
Out[99]:
```

	symboling	fueltype	aspiration	doornumber	enginelocation	wheelbase	carlength
count	143.000000	143.000000	143.000000	143.000000	143.000000	143.000000	143.000000
mean	0.559441	0.909091	0.818182	0.440559	0.993007	0.411141	0.525476
std	0.239200	0.288490	0.387050	0.498199	0.083624	0.205581	0.204848
min	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
25%	0.400000	1.000000	1.000000	0.000000	1.000000	0.272414	0.399187
50%	0.600000	1.000000	1.000000	0.000000	1.000000	0.341379	0.502439
75%	0.600000	1.000000	1.000000	1.000000	1.000000	0.503448	0.669919
max	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000

8 rows × 65 columns

```
In [100]: from sklearn.feature_selection import RFE
          from sklearn.linear_model import LinearRegression
```

```
In [59]: # Running RFE with the output number of the variable as 10
          lm = LinearRegression()
          lm.fit(X_train,y_train)

          rfe = RFE(lm,10)
          rfe = rfe.fit(X_train,y_train)
```

```
In [60]: list(zip(X_train.columns,rfe.support_,rfe.ranking_))
```

```
('bore ratio', True, 1),
('stroke', False, 2),
('compressionratio', False, 25),
('horsepower', False, 46),
('peakrpm', False, 19),
('citympg', False, 38),
('highwaympg', False, 27),
('CarName_audi', False, 37),
('CarName_bmw', True, 1),
('CarName_buick', False, 36),
('CarName_chevrolet', False, 22),
('CarName_dodge', False, 18),
('CarName_honda', False, 21),
('CarName_isuzu', False, 53),
('CarName_jaguar', False, 32),
('CarName_mazda', False, 44),
('CarName_mercury', False, 48),
('CarName_mitsubishi', False, 13),
('CarName_nissan', False, 43),
('CarName_peugeot', False, 6),
```

```
In [61]: col = X_train.columns[rfe.support_]
          col
```

```
Out[61]: Index(['engine location', 'carwidth', 'curbweight', 'engine size', 'bore ratio',
               'CarName_bmw', 'CarName_porsche', 'cylindernumber_three',
               'cylindernumber_twelve', 'cylindernumber_two'],
              dtype='object')
```

```
In [62]: X_train.columns[~rfe.support_]
```

```
Out[62]: Index(['symboling', 'fueltype', 'aspiration', 'doornumber', 'wheelbase',  
               'carlength', 'carheight', 'stroke', 'compressionratio', 'horsepower',  
               'peakrpm', 'citympg', 'highwaympg', 'CarName_audi', 'CarName_buick',  
               'CarName_chevrolet', 'CarName_dodge', 'CarName_honda', 'CarName_isuzu',  
               'CarName_jaguar', 'CarName_mazda', 'CarName_mercury',  
               'CarName_mitsubishi', 'CarName_nissan', 'CarName_peugeot',  
               'CarName_plymouth', 'CarName_renault', 'CarName_saab', 'CarName_subaru',  
               'CarName_toyota', 'CarName_volkswagen', 'CarName_volvo',  
               'carbody_hardtop', 'carbody_hatchback', 'carbody_sedan',  
               'carbody_wagon', 'drivewheel_fwd', 'drivewheel_rwd', 'enginetype_dohcv',  
               'enginetype_l', 'enginetype_ohc', 'enginetype_ohcf', 'enginetype_ohcv',  
               'enginetype_rotor', 'cylindernumber_five', 'cylindernumber_four',  
               'cylindernumber_six', 'fuelsystem_2bbl', 'fuelsystem_4bbl',  
               'fuelsystem_idi', 'fuelsystem_mfi', 'fuelsystem_mphi',  
               'fuelsystem_spdi', 'fuelsystem_spfi'],  
              dtype='object')
```

```
In [63]: x_train_rfe=X_train[col]
```

```
In [64]: import statsmodels.api as sm
```

```
In [65]: x_train_rfe = sm.add_constant(x_train_rfe)
lm = sm.OLS(y_train,x_train_rfe).fit()
print(lm.summary())
```

OLS Regression Results

```
=====
Dep. Variable:          price      R-squared:                0.912
Model:                  OLS       Adj. R-squared:            0.906
Method:                 Least Squares   F-statistic:           137.1
Date:                  Sun, 11 Nov 2018   Prob (F-statistic):    1.21e-64
Time:                  07:35:52     Log-Likelihood:        190.87
No. Observations:        143       AIC:                   -359.7
Df Residuals:           132       BIC:                   -327.1
Df Model:                10
Covariance Type:        nonrobust
=====
```

```
=====
               coef      std err          t      P>|t|      [0.025
0.975]
-----
const               0.1704      0.084      2.022      0.045      0.004
0.337
enginelocation     -0.3298      0.086     -3.844      0.000     -0.500
-0.160
carwidth            0.3283      0.067      4.910      0.000      0.196
0.461
curbweight          0.2989      0.080      3.729      0.000      0.140
0.457
enginesize          0.5732      0.097      5.926      0.000      0.382
0.764
boreratio          -0.1088      0.037     -2.976      0.003     -0.181
-0.036
CarName_bmw         0.2453      0.029      8.327      0.000      0.187
0.304
CarName_porsche     0.1565      0.052      3.000      0.003      0.053
0.260
cylindernumber_three 0.1802      0.068      2.638      0.009      0.045
0.315
cylindernumber_twelve -0.0536      0.081     -0.659      0.511     -0.214
0.107
cylindernumber_two   0.1542      0.037      4.123      0.000      0.080
0.228
=====
```

```
Omnibus:           10.995   Durbin-Watson:           1.961
Prob(Omnibus):      0.004   Jarque-Bera (JB):         15.683
Skew:               0.423   Prob(JB):                  0.000393
Kurtosis:           4.385   Cond. No.                  38.7
=====
```

Warnings:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

Calculate VIF1

```
In [66]: from statsmodels.stats.outliers_influence import variance_inflation_factor
```

```
In [67]: vif = pd.DataFrame()  
x = x_train_rfe  
vif['features']=x.columns  
vif['vif']=[variance_inflation_factor(x.values, i ) for i in range(x.shape[1])]  
vif
```

Out[67]:

	features	vif
0	const	230.922520
1	enginelocation	1.663007
2	carwidth	4.920153
3	curbweight	9.263420
4	enginesize	7.225349
5	boreratio	1.852666
6	CarName_bmw	1.134883
7	CarName_porsche	1.818873
8	cylindernumber_three	1.054182
9	cylindernumber_twelve	1.494303
10	cylindernumber_two	1.237541

```
In [68]: # Running RFE with the output number of the variable as 12  
lm = LinearRegression()  
lm.fit(X_train,y_train)  
  
rfe = RFE(lm,12)  
rfe = rfe.fit(X_train,y_train)
```

```
In [69]: list(zip(X_train.columns,rfe.support_,rfe.ranking_))
```

```
Out[69]: [('symboling', False, 39),
 ('fueltype', False, 21),
 ('aspiration', False, 12),
 ('doornumber', False, 49),
 ('enginelocation', True, 1),
 ('wheelbase', False, 13),
 ('carlength', False, 18),
 ('carwidth', True, 1),
 ('carheight', False, 14),
 ('curbweight', True, 1),
 ('enginesize', True, 1),
 ('boreratio', True, 1),
 ('stroke', True, 1),
 ('compressionratio', False, 23),
 ('horsepower', False, 44),
 ('peakrpm', False, 17),
 ('citympg', False, 36),
 ('highwaympg', False, 25),
 ('CarName_audi', False, 35),
 ('CarName_bmw', True, 1),
 ('CarName_buick', False, 34),
 ('CarName_chevrolet', False, 20),
 ('CarName_dodge', False, 16),
 ('CarName_honda', False, 19),
 ('CarName_isuzu', False, 51),
 ('CarName_jaguar', False, 30),
 ('CarName_mazda', False, 42),
 ('CarName_mercury', False, 46),
 ('CarName_mitsubishi', False, 11),
 ('CarName_nissan', False, 41),
 ('CarName_peugeot', False, 4),
 ('CarName_plymouth', False, 15),
 ('CarName_porsche', True, 1),
 ('CarName_renault', False, 45),
 ('CarName_saab', False, 7),
 ('CarName_subaru', False, 9),
 ('CarName_toyota', False, 43),
 ('CarName_volkswagen', False, 40),
 ('CarName_volvo', False, 6),
 ('carbody_hardtop', False, 26),
 ('carbody_hatchback', False, 24),
 ('carbody_sedan', False, 27),
 ('carbody_wagon', False, 28),
 ('drivewheel_fwd', False, 48),
 ('drivewheel_rwd', False, 38),
 ('enginetype_dohcv', False, 10),
 ('enginetype_l', False, 5),
 ('enginetype_ohc', False, 50),
 ('enginetype_ohcf', False, 8),
 ('enginetype_ohcv', False, 37),
 ('enginetype_rotor', True, 1),
 ('cylindernumber_five', False, 3),
 ('cylindernumber_four', False, 2),
 ('cylindernumber_six', False, 33),
```

```
('cylindernumber_three', True, 1),  
( 'cylindernumber_twelve', True, 1),  
( 'cylindernumber_two', True, 1),  
( 'fuelsystem_2bbl', False, 47),  
( 'fuelsystem_4bbl', False, 31),  
( 'fuelsystem_idi', False, 22),  
( 'fuelsystem_mfi', False, 52),  
( 'fuelsystem_mpfi', False, 32),  
( 'fuelsystem_spdi', False, 29),  
( 'fuelsystem_spfi', False, 53)]
```



```
In [70]: col = X_train.columns[rfe.support_]
print(col)
x_train_rfe=X_train[col]
x_train_rfe = sm.add_constant(x_train_rfe)
lm = sm.OLS(y_train,x_train_rfe).fit()
print(lm.summary())
vif = pd.DataFrame()
x = x_train_rfe
vif['features']=x.columns
vif['vif']=[variance_inflation_factor(x.values, i ) for i in range(x.shape[1])]
print(vif)
```

```
Index(['enginolocation', 'carwidth', 'curbweight', 'enginesize', 'boreratio',
      'stroke', 'CarName_bmw', 'CarName_porsche', 'enginetype_rotor',
      'cylindernumber_three', 'cylindernumber_twelve', 'cylindernumber_two'],
      dtype='object')
```

OLS Regression Results

```
=====
Dep. Variable:          price      R-squared:                0.916
Model:                  OLS      Adj. R-squared:            0.909
Method:                 Least Squares      F-statistic:        130.2
Date:                  Sun, 11 Nov 2018      Prob (F-statistic):    6.86e-65
Time:                  07:35:53      Log-Likelihood:       194.22
No. Observations:      143      AIC:                  -364.4
Df Residuals:          131      BIC:                  -328.9
Df Model:              11
Covariance Type:       nonrobust
=====
```

```
=====
                                coef      std err          t      P>|t|      [0.025
0.975]
-----
const                0.2085        0.084        2.483      0.014      0.042
0.375
enginolocation       -0.3126        0.084       -3.704      0.000     -0.480
-0.146
carwidth             0.3477        0.066        5.266      0.000      0.217
0.478
curbweight           0.2791        0.079        3.534      0.001      0.123
0.435
enginesize           0.6626        0.101        6.541      0.000      0.462
0.863
boreratio            -0.1483        0.039       -3.788      0.000     -0.226
-0.071
stroke              -0.1067        0.043       -2.510      0.013     -0.191
-0.023
CarName_bmw          0.2323        0.029        7.917      0.000      0.174
0.290
CarName_porsche      0.1472        0.051        2.869      0.005      0.046
0.249
enginetype_rotor     0.0855        0.019        4.589      0.000      0.049
0.122
cylindernumber_three 0.1773        0.067        2.647      0.009      0.045
0.310
cylindernumber_twelve -0.1384        0.087       -1.598      0.112     -0.310
=====
```

```

0.033
cylindernumber_two      0.0855      0.019      4.589      0.000      0.049
0.122
=====
Omnibus:                13.877      Durbin-Watson:          2.071
Prob(Omnibus):          0.001      Jarque-Bera (JB):      24.547
Skew:                   0.444      Prob(JB):              4.67e-06
Kurtosis:               4.825      Cond. No.              9.27e+16
=====

```

Warnings:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

[2] The smallest eigenvalue is 4.99e-32. This might indicate that there are strong multicollinearity problems or that the design matrix is singular.

```

          features      vif
0          const 238.746750
1    engine_location  1.674027
2      carwidth  4.988429
3    curbweight  9.356507
4    enginesize  8.244360
5    boreratio  2.210802
6        stroke  1.512427
7   CarName_bmw  1.171284
8   CarName_porsche  1.828551
9   enginetype_rotor    inf
10 cylindernumber_three  1.054490
11 cylindernumber_twelve  1.762497
12 cylindernumber_two    inf

```

```

In [71]: # Running RFE with the output number of the variable as 8
lm = LinearRegression()
lm.fit(X_train,y_train)

rfe = RFE(lm,6)
rfe = rfe.fit(X_train,y_train)

```

```
In [72]: print(list(zip(X_train.columns,rfe.support_,rfe.ranking_)))
col = X_train.columns[rfe.support_]
print(col)
x_train_rfe=X_train[col]
x_train_rfe = sm.add_constant(x_train_rfe)
lm = sm.OLS(y_train,x_train_rfe).fit()
print(lm.summary())
vif = pd.DataFrame()
x = x_train_rfe
vif['features']=x.columns
vif['vif']=[variance_inflation_factor(x.values, i ) for i in range(x.shape[1])]
print(vif)
```

```
[('symboling', False, 45), ('fueltype', False, 27), ('aspiration', False, 18),
('doornumber', False, 55), ('enginelocation', True, 1), ('wheelbase', False, 1
9), ('carlength', False, 24), ('carwidth', True, 1), ('carheight', False, 20),
('curbweight', True, 1), ('enginesize', True, 1), ('boreratio', False, 4), ('st
roke', False, 6), ('compressionratio', False, 29), ('horsepower', False, 50),
('peakrpm', False, 23), ('citympg', False, 42), ('highwaympg', False, 31), ('Ca
rName_audi', False, 41), ('CarName_bmw', True, 1), ('CarName_buick', False, 4
0), ('CarName_chevrolet', False, 26), ('CarName_dodge', False, 22), ('CarName_h
onda', False, 25), ('CarName_isuzu', False, 57), ('CarName_jaguar', False, 36),
('CarName_mazda', False, 48), ('CarName_mercury', False, 52), ('CarName_mitsubi
shi', False, 17), ('CarName_nissan', False, 47), ('CarName_peugeot', False, 1
0), ('CarName_plymouth', False, 21), ('CarName_porsche', False, 3), ('CarName_r
enault', False, 51), ('CarName_saab', False, 13), ('CarName_subaru', False, 1
5), ('CarName_toyota', False, 49), ('CarName_volkswagen', False, 46), ('CarName
_volvo', False, 12), ('carbody_hardtop', False, 32), ('carbody_hatchback', Fals
e, 30), ('carbody_sedan', False, 33), ('carbody_wagon', False, 34), ('drivewhee
l_fwd', False, 54), ('drivewheel_rwd', False, 44), ('enginetype_dohcv', False,
16), ('enginetype_l', False, 11), ('enginetype_ohc', False, 56), ('enginetype_o
hcf', False, 14), ('enginetype_ohcv', False, 43), ('enginetype_rotor', False,
7), ('cylindernumber_five', False, 9), ('cylindernumber_four', False, 8), ('cyl
indernumber_six', False, 39), ('cylindernumber_three', True, 1), ('cylindernumb
er_twelve', False, 5), ('cylindernumber_two', False, 2), ('fuelsystem_2bbl', Fa
lse, 53), ('fuelsystem_4bbl', False, 37), ('fuelsystem_idi', False, 28), ('fuel
system_mfi', False, 58), ('fuelsystem_mphi', False, 38), ('fuelsystem_spdi', Fa
lse, 35), ('fuelsystem_spfi', False, 59)]
Index(['enginelocation', 'carwidth', 'curbweight', 'enginesize', 'CarName_bmw',
       'cylindernumber_three'],
      dtype='object')
```

OLS Regression Results

```
=====
Dep. Variable:          price      R-squared:            0.892
Model:                  OLS        Adj. R-squared:       0.887
Method:                 Least Squares    F-statistic:      186.8
Date:                   Sun, 11 Nov 2018    Prob (F-statistic): 4.11e-63
Time:                   07:35:53      Log-Likelihood:    175.94
No. Observations:       143          AIC:               -337.9
Df Residuals:           136          BIC:               -317.1
Df Model:                6
Covariance Type:        nonrobust
=====
```

	coef	std err	t	P> t	[0.025	0.975]

```
-----
-----
const                0.2907    0.076    3.841    0.000    0.141
    0.440
enginolocation       -0.4824    0.076   -6.379    0.000   -0.632
    -0.333
carwidth             0.3866    0.070    5.532    0.000    0.248
    0.525
curbweight           0.2679    0.078    3.451    0.001    0.114
    0.421
enginesize           0.4480    0.082    5.460    0.000    0.286
    0.610
CarName_bmw          0.2497    0.032    7.917    0.000    0.187
    0.312
cylindernumber_three 0.1926    0.075    2.584    0.011    0.045
    0.340
=====
Omnibus:                4.900    Durbin-Watson:                2.031
Prob(Omnibus):          0.086    Jarque-Bera (JB):            4.619
Skew:                   0.321    Prob(JB):                     0.0993
Kurtosis:               3.603    Cond. No.                     29.2
=====
```

Warnings:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

```

           features      vif
0           const 155.842927
1  enginolocation   1.080674
2           carwidth   4.490868
3           curbweight   7.265199
4           enginesize   4.348564
5           CarName_bmw   1.088127
6  cylindernumber_three   1.049848
```

Step 5: Residual Analysis of the train data

So, now to check if the error terms are also normally distributed (which is infact, one of the major assumptions of linear regression), let us plot the histogram of the error terms and see what it looks like.

```
In [78]: # Build a fourth fitted model
x_train_rfe=X_train[col]
X_train_lm = sm.add_constant(x_train_rfe)

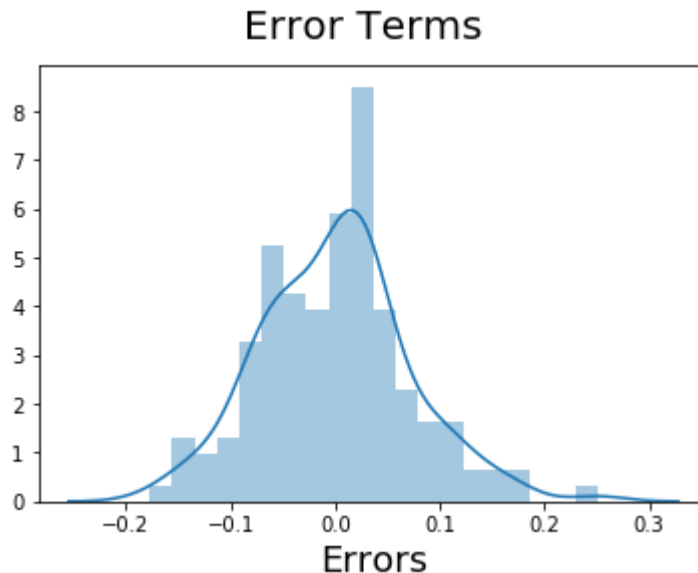
lr_4 = sm.OLS(y_train, X_train_lm).fit()
y_train_price = lr_4.predict(X_train_lm)
```

```
In [79]: # Plot the histogram of the error terms
fig = plt.figure()
sns.distplot(y_train - y_train_price, bins = 20)
fig.suptitle('Error Terms', fontsize = 20)
plt.xlabel('Errors', fontsize = 18)
# Plot heading
# X-Label
```

C:\ProgramData\Anaconda3\lib\site-packages\matplotlib\axes_axes.py:6462: UserWarning: The 'normed' kwarg is deprecated, and has been replaced by the 'density' kwarg.

warnings.warn("The 'normed' kwarg is deprecated, and has been "

Out[79]: Text(0.5,0,'Errors')



Step 6: Making Predictions Using the Final Model

Now that we have fitted the model and checked the normality of error terms, it's time to go ahead and make predictions using the final, i.e. fourth model.

Applying the scaling on the test sets

```
In [101]: #x_train_rfe=X_train[col]
from sklearn.model_selection import train_test_split

# We specify this so that the train and test data set always have the same rows,
np.random.seed(0)
df_train, df_test = train_test_split(GeelyAuto, train_size = 0.7, test_size = 0.3)
from sklearn.preprocessing import MinMaxScaler
scaler = MinMaxScaler()
# Apply scaler() to all the columns except the 'yes-no' and 'dummy' variables
num_vars = ['symboling', 'wheelbase', 'carlength', 'carwidth', 'carheight', 'curbweight']
df_train[num_vars] = scaler.fit_transform(df_train[num_vars])
num_vars = ['symboling', 'wheelbase', 'carlength', 'carwidth', 'carheight', 'curbweight']
df_test[num_vars] = scaler.transform(df_test[num_vars])
df_test.describe()
```

Out[101]:

	symboling	fueltype	aspiration	doornumber	enginelocation	wheelbase	carlength	carw
count	62.000000	62.000000	62.000000	62.000000	62.000000	62.000000	62.000000	62.000
mean	0.583871	0.887097	0.822581	0.435484	0.967742	0.437764	0.559481	0.480
std	0.271724	0.319058	0.385142	0.499868	0.178127	0.212861	0.189947	0.165
min	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.056911	0.183
25%	0.400000	1.000000	1.000000	0.000000	1.000000	0.313793	0.459350	0.358
50%	0.600000	1.000000	1.000000	0.000000	1.000000	0.387931	0.547967	0.441
75%	0.800000	1.000000	1.000000	1.000000	1.000000	0.570690	0.719919	0.516
max	1.000000	1.000000	1.000000	1.000000	1.000000	1.182759	1.089431	0.975

8 rows × 65 columns

```
In [104]: col = ['enginelocation', 'carwidth', 'curbweight', 'enginesize', 'CarName_bmw',
                'cylindernumber_three']
y_test = df_test.pop('price')
X_test = df_test[col]
y_train = df_train.pop('price')
X_train = df_train[col]
```

```
In [105]: # Adding constant variable to test dataframe
X_test_final = sm.add_constant(X_test)
X_train_final = sm.add_constant(X_train)
```

```
In [106]: lr_2 = sm.OLS(y_train, X_train_final).fit()
```

```
In [110]: lr_2.params
```

```
Out[110]: const                0.290687
enginelocation             -0.482417
carwidth                   0.386564
curbweight                 0.267873
enginesize                 0.447988
CarName_bmw               0.249720
cylindernumber_three      0.192642
dtype: float64
```

```
In [111]: # Print the summary of the model
print(lr_2.summary())
```

```

                                OLS Regression Results
=====
Dep. Variable:                  price    R-squared:                0.892
Model:                            OLS    Adj. R-squared:           0.887
Method:                 Least Squares    F-statistic:              186.8
Date:                  Sun, 11 Nov 2018    Prob (F-statistic):       4.11e-63
Time:                  10:30:01    Log-Likelihood:           175.94
No. Observations:                  143    AIC:                     -337.9
Df Residuals:                      136    BIC:                     -317.1
Df Model:                           6
Covariance Type:                  nonrobust
=====

```

	coef	std err	t	P> t	[0.025
const	0.2907	0.076	3.841	0.000	0.141
enginelocation	-0.4824	0.076	-6.379	0.000	-0.632
carwidth	0.3866	0.070	5.532	0.000	0.248
curbweight	0.2679	0.078	3.451	0.001	0.114
enginesize	0.4480	0.082	5.460	0.000	0.286
CarName_bmw	0.2497	0.032	7.917	0.000	0.187
cylindernumber_three	0.1926	0.075	2.584	0.011	0.045

```

=====
Omnibus:                  4.900    Durbin-Watson:           2.031
Prob(Omnibus):            0.086    Jarque-Bera (JB):        4.619
Skew:                     0.321    Prob(JB):                 0.0993
Kurtosis:                 3.603    Cond. No.                  29.2
=====

```

Warnings:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

```
In [108]: # Making predictions using the fourth model

y_pred_train_final = lr_2.predict(X_test_final)
y_pred_test_final = lr_2.predict(X_test_final)
```

```
In [113]: # Check the parameters obtained

lr_2.params
```

```
Out[113]: const                0.290687
engine.location              -0.482417
car.width                   0.386564
curb.weight                 0.267873
engine.size                 0.447988
CarName.bmw                 0.249720
cylinder.number.three      0.192642
dtype: float64
```

We can see that the equation of our best fitted line is: ¶

$$\text{price} = -0.4824 \times \text{engine.location} + 0.3865 \times \text{car.width} + 0.2678 \times \text{curb.weight} + 0.4479 \times \text{engine.size} + 0.2497 \times \text{CarName.bmw} + 0.192642 \times \text{cylinder.number.three}$$

```
In [ ]:
```