28/8/2014 ①

Ihara Zeta Function

$$\prod_{\substack{\text{Cycles of length} \\ \text{prime } p}} \frac{1}{\left(1-q^{-s|p|}\right)} \quad q\text{-regular graph} = IZF$$

$$\prod_{s} \frac{1}{\left(1-\frac{1}{p^{s}}\right)} = RZF \left(1+\frac{1}{2^{s}}\right)\left(1-\frac{1}{2^{s}}\right)$$

$$Z_1(s)\left(\det\left(I-A_{1}2^{-s}+2^{1-2s}I\right)\right) = \left(1-2^{-2s}\right)^{|V|-|E|}$$

$$Z_2(s)\left(\det\left(I-A_{2}3^{-s}+3^{1-2s}I\right)\right) = \left(1-3^{-2s}\right)^{|V|-|E|}$$

$$Z_3(s)\left(\det\left(I-A_{3}5^{-s}+5^{1-2s}I\right)\right) = \left(1-5^{-2s}\right)^{|V|-|E|}$$

$$Z_4(s)\left(\det\left(I-A_{4}7^{-s}+7^{(1-2s)}I\right)\right) = \left(1-7^{-2s}\right)^{|V|-|E|}$$

$$Z_5(s)\left(\det\left(I-A_{5}11^{-s}+11^{(1-2s)}I\right)\right) = \left(1-11^{-2s}\right)^{|V|-|E|}$$

$\vdots$

$$Z_1 \times Z_2 \times \ldots (\det_1)(\det_2) \ldots \ldots (\ ) = (RZF)^{|V|-|E|}(RZF_+)^{|V|-|E|}$$

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2 regular 3-regular 5-regular

30/8/2014 ②

$$Z(s)\left(\det\left(I-A_{1}2^{-s}+2^{1-2s}I\right)\right) = \left(1-2^{-s}\right)^{|V|-|E|}\left(1+2^{-s}\right)^{|V|-|E|}$$

$$Z_2(s)\left(\det\left(I-A_{2}3^{-s}+3^{1-2s}I\right)\right) = \left(1-3^{-s}\right)^{|V|-|E|}\left(1+3^{-s}\right)^{|V|-|E|}$$

$\vdots$

$$= RZF$$

$$I2^{s} - A + 2^{1-s}I = 0$$

$$A - I2^{s} - 2^{1-s}I = 0$$

$$A - \left(2^{s}+2^{1-s}\right)I = 0.$$

$\vdots$

For Ramanujan graphs.

1) $Z_i(s) = 0$ as they satisfy RH & Zero of RHS should also be zero of $Z_i(s)$

2) For non-ramanujan graphs:

$$A - \left(q^{s}+q^{1-s}\right)I = 0.$$

for some q (prime).

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③

$\Rightarrow \lambda$ is an eigen value of Adjacency matrix of prime regularity.

$$\left(q^{s}+q^{1-s}\right) = \lambda$$

$$q^{a+ib} + q^{1-a-ib}$$

$$q^{a+ib} + \frac{q}{q^{a+ib}} = \frac{q^{a^2-b^2}+q}{q^{a+ib}} \left(\frac{q^{a-ib}}{q^{a-ib}}\right)$$

$$\frac{q^{2s}+q}{q^{s}} = \lambda$$

Adjacency matrix of n vertices and q-regular graph

$$n\times n \begin{bmatrix} 0 & 1 & 1 & 1 & \cdots & 1 & 0 \\ 1 & 0 & 1 & \cdots & \cdots & \cdots & 0 \end{bmatrix} \quad \begin{matrix} q\,1^{s} \\ q\,1^{s} \end{matrix}$$

$$q^{s}\,1 \quad q\,1^{s}$$

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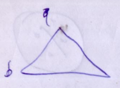
④



2 regular

$$\begin{array}{c|ccc} & a & b & c \\ \hline a & 0 & 1 & 1 \\ b & 1 & 0 & 1 \\ c & 1 & 1 & 0 \end{array} - \lambda \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$= \begin{vmatrix} -\lambda & 1 & 1 \\ 1 & -\lambda & 1 \\ 1 & 1 & -\lambda \end{vmatrix} = -\lambda\left(\lambda^2-1\right) - \left(-\lambda-1\right) + \left(1+\lambda\right).$$

$$-\lambda^3 + \lambda + \lambda + 1 + \lambda + 1 = 0.$$

$$-\lambda^3 + 3\lambda + 2 = 0$$

$$\lambda^3 - 3\lambda - 2 = 0 \qquad 2 = 2^{s} + 2^{1-s}$$

$$\lambda^3 - \lambda - 2\lambda - 2 = 0.$$

$$\lambda^2(\lambda-1) - 2(\lambda+1) = 0.$$

$$\lambda = 2 \qquad 8-6-2 = 0$$

$$8-8 = 0$$

$$(\lambda-2)(\lambda^2-$$

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$$2 = 2^{s} + 2^{1-s}$$

$$2 = \frac{2^{2s} + 2}{2^{s}}$$

$$2^{s+1} = 2^{2s} + 2$$

$$2^{s+1} = 2\left(2^{2s-1} + 1\right).$$

$$2^{s} = 2^{2s-1} + 1$$

$$2^{a+ib} = 2^{2a+2ib-1} + 1.$$

$$2^{a}\,2^{ib} = \frac{2^{a}\,2^{2ib}}{2} + 1$$

$$2^{a+1}\,2^{ib} = 2^{a}\,2^{2ib} + 2$$

$$2^{a+1}\,2^{ib} = 2^{a}\,4^{ib} + 2 \qquad \text{if } a = 1/2$$

$$= \sqrt{2}\,4^{ib} + 2$$

$$2\sqrt{2}\,2^{ib} = \sqrt{2}\left(4^{ib} + \sqrt{2}\right).$$

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6

$$2\,2^{ib} = 4^{ib} + \sqrt{2}$$

$$2^{ib} = \frac{2^{2ib} + \sqrt{2}}{2}$$

$$2^{ib} = 2^{2ib-1} + \frac{1}{\sqrt{2}}$$

$$2^{2ib-1} - 2^{ib} = \frac{-1}{\sqrt{2}} \qquad \text{if } 2^{ib} = k$$

$$\frac{k^{2}}{2} - k = \frac{-1}{\sqrt{2}}$$

$$k^{2} - 2k = -\sqrt{2}$$

$$k^{2} - 2k + \sqrt{2} = 0.$$

$$\frac{+2 \pm \sqrt{4 - 4\sqrt{2}}}{2}$$

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7

2 is an eigen value for 2-regular graph adjacency matrix.

$$\Rightarrow 2^{s} + 2^{1-s} = 2$$

if RH is true $s = \frac{1}{2} + ib$

$$2^{\frac{1}{2}+ib} + 2^{1-\frac{1}{2}-ib} = 2$$

$$\Rightarrow 2^{\frac{1}{2}+ib} + 2^{\frac{1}{2}-ib} = 2$$

$$\sqrt{2}\left(2^{ib} + 2^{-ib}\right) = 2$$

$$2^{ib} + 2^{-ib} = \sqrt{2}$$

if $2^{ib} = y$

$$y + \frac{1}{y} = \sqrt{2}$$

$$y^{2} - \sqrt{2}\,y + 1 = 0$$

$$y = \frac{\sqrt{2} \pm \sqrt{2 - 4}}{2}$$

$$= \frac{\sqrt{2} \pm i\sqrt{2}}{2}$$

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8

$$y = \frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}},\ \frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}$$

$$\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}} = 2^{ib}$$

$$= e^{ib\ln 2} =$$

$$\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}} = \cos b\ln 2 + i\sin b\ln 2$$

$$\tan b\ln 2 = \frac{1}{\sqrt{2}} \Big/ \frac{1}{\sqrt{2}} = 1$$

$$b\ln 2 = \frac{\pi}{4}$$

$$b = \frac{\pi}{4\ln 2} = \frac{3.14\ldots}{\ln 16}$$

Maximum eigen value of q-regular graph is q.

$$L = D - A$$

$$L = Id - A$$

$$= -(A - Id)$$

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Then $\quad q^s + q^{1-s} = q$ (9)

6 March Thursday 06-03-2014

$$q^{s-1} + q^{-s} = 1$$

$$q^{2s-1} + 1 = q^s$$

$$q^{2s-1} - q^s + 1 = 0.$$

$s = a + ib$

$$q^{2a-1+2ib} - q^{a+ib} + 1 = 0.$$

Case

(1) if $a \neq 1/2$

$q^s = y$

$$\frac{(y)^2}{q} - y + 1 = 0$$

$$y^2 - qy + q = 0$$

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(10) 07-03-2014 Kritikai ★ March Friday 7

$$y = \frac{q \pm \sqrt{q^2 - 4q}}{2}$$

$= q$

$$q^s = \frac{q \pm \sqrt{q^2 - 4q}}{2}$$

$$q^{a+ib} = \frac{q \pm \sqrt{q^2 - 4q}}{2}$$

q^{a+ib}

~~$(a+ib)\log q = \frac{q \pm \sqrt{q^2 - 4q}}{2}$~~

$q^{a+ib} =$

$$q^a q^{ib} = \frac{q \pm \sqrt{q^2 - 4q}}{2}$$

For $q = 2$

2 regular graph

$$\frac{2 \pm \sqrt{4-8}}{2}$$

$$\frac{2 \pm i2}{2}$$

$2^s = 1 \pm i$

$2^{\frac{1}{2}+ib} = 1 \pm i$

$\sqrt{2}\, 2^{ib} = 1 \pm i$

$2^{ib} = \frac{1}{\sqrt{2}} \pm \frac{i}{\sqrt{2}}$

$e^{ib\ln 2} = \frac{1}{\sqrt{2}} \pm \frac{i}{\sqrt{2}}$

$\cos b\ln 2 + i\sin b\ln 2$

$= \frac{1}{\sqrt{2}} \pm \frac{1}{\sqrt{2}}$

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$$q^a q^{ib} = \frac{q \pm \sqrt{q^2 - 4q}}{2}$$ (11)

8 March Saturday 08-03-2014

Case (2) if $a = 1/2$

$$\sqrt{q}\, q^{ib} = \frac{\sqrt{q}\times\sqrt{q} \pm \sqrt{q-4}\sqrt{q}}{2}$$

$$q^{ib} = \frac{\sqrt{q} \pm \sqrt{q-4}}{2}$$

~~$ib\log q = \sqrt{q} \pm \sqrt{\ }$~~

$$e^{ib\log q} = \frac{\sqrt{q} \pm \sqrt{q-4}}{2}$$

9 Sunday 09-03-2014

$$\cos b\log q + i\sin b\log q = \frac{\sqrt{q} \pm \sqrt{q-4}}{2}$$

$\Rightarrow$ $\quad \mathrm{Im}() = 0$

$b\log q = 0.$ but $q \neq 0$

$\Rightarrow a \neq 1/2 \qquad b \neq 0$

$\boxed{\text{Is RH false ?}}$

for some s $\quad a \neq 1/2$

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steps in previous proof (12) (if it proves)

10-03-2014 March Monday 10

(1) Construct infinitely many prime-regular graphs.

(2) Using Ihara Zeta functions for all these graphs equate to Riemann Zeta Function (pages 1 & 2)

(3) Zeros of RZF in RHS (page 2) should either be 3.1) zeros of $Z_i(s)$ - the Ihara Zeta functions of the prime regular graphs (or) 3.2) ~~roots of the~~ eigen values of the product of Ihara identities for all graphs.

(4) 3.1) implies graph is Ramanujan (because graph has to satisfy RH in its IZF to be Ramanujan)

3.2) is for non-ramanujan graphs and maximum eigen value of p-regular graph is p. Thus $q^s + q^{1-s} = q$ (eigenvalue = degree) ~~(5) Assuming~~

(5) Assuming $Re(s) = 1/2 = a$, there seems to be a contradiction as in page 12 in deriving $Im(s) = b$. Probably hints at RH being false for some s.

Ka Shrinivaasan

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To verify

(13)

11 March Tuesday 11-03-2014

1) Can a p-regular graph have eigenvalues other than p ?

2) Is it right to equate q^{ib} to

$$\frac{\sqrt{q} \pm \sqrt{q-4}}{2}$$

3) Can q^{ib} be real

if the eigenvalue is less than q for q regular graphs

$y^2 - qy + t \qquad t < q$

$$q^s = \frac{q \pm \sqrt{q^2-4t}}{2}$$

If $s = \frac{1}{2} + ib$

$$q^s = \sqrt{q}\, q^{ib} = \frac{q \pm \sqrt{q^2-4t}}{2}$$

or $$q^{ib} = \frac{q \pm \sqrt{q^2-4t}}{2\sqrt{q}}$$

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$$e^{ib\log q} = \cos b\log q + i\sin b\log q = q^{ib}$$ (14)

12-03-2014 March Wednesday 12

$$= \frac{q \pm \sqrt{q^2-4t}}{2\sqrt{q}}$$

$$\boxed{\cos b\log q = \frac{q \pm \sqrt{q^2-4t}}{2\sqrt{q}} \quad \text{and} \quad \sin b\log q = 0}$$

$\Rightarrow b = 0$ or $\log q = 0$ a contradiction

thus $Re(s)$ cannot be $\frac{1}{2}$ for all zeroes of RZF

Ka. Shriniwaasan
30/8/2014

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