Arrow's Theorem, Circuit For Democracy and Psuedorandom Choice and P Versus NP - Draft

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Abstract

In this draft article a circuit for majority voting that simulates the real-world electors is constructed and its relation to Arrow's theorem and implications for P Vs NP is described. This is a typeset version that unifies already existing handwritten notes and drafts.

1 Pseudorandom Choice and Circuit for Democracy

1. Conventional Majority circuit that outputs 1 if atleast half the inputs are one is chosen
2. Inputs to this majority circuit are SAT circuits for each voter in a democracy.
3. Wishlist of each of the voter is reduced to a CNF formula and a SAT circuit is constructed for it.
4. The variables and clauses in the voter SAT are expectations of each of the voter that is assigned 1 or 0 by the voter while deciding to vote for or against.
5. Thus above Majority circuit augmented with SAT oracles or circuits is in NP. This reduction from 3-SAT is rather trivial.
6. Above circuit makes democracy an NP-complete problem.
7. Using the arguments for 100% convergence case of P(good) series (mentioned in bibliography links below) there exists a Pseudorandom generator based Choice in P or NC that achieves the same objective thereby creating an intriguing anomaly.
8. In the P(good) series if both LHS(Pseudorandom choice) and RHS(Majority voting) are 1 (zero error), then there is a polytime algorithm (pseudorandom choice) to the NP problem of Majority voting which is a counterexample. One way functions exist when the series does not converge and do not exist when series converges to 1.Thus $P \neq NP$ if and only if there is no perfection in voting (and in general any entity or process in universe).

9. Moreover, finding even a single perfect voter who never makes an error itself looks to be intractable. Infact this circuit requires all voter SATs to be perfect for the series to converge to 1 thereby placing a tighter restriction than mere intractability.

2 Boolean function sensitivity, Condorcet Elections, Arrow's Theorem and P(Good) series

1. *https : //sites.google.com/site/kuja27/ImplicationRandomGraphConvexHullsAndPerfectVoterProblem* 01 − 11*.pdf* on decidability of existence of perfect voter and the probability series for a good choice of *https : //sites.google.com/site/kuja27/CircuitForComputingErrorProbabilityOfMajorityVoti* are related to already well studied problems in social choice theory but problem definition is completely different.

2. Arrow's theorem of social choice for an irrational outcome in condorcet election of more than 2 candidates and its complexity theory fourier analysis proof [GilKalai] are described in *www.cs.cmu.edu/ odonnell/papers/analysis − survey.pdf*.

3. Irrational outcome is a paradox where the society is "confused" or "ranks circularly" in choice of a candidate in multipartisan condorcet voting. Rational outcome converges to 91.2% with a possibility of 8.8% irrational outcome.

4. What is perplexing is the fact that this seems to contravene guarantee of unique restricted partitions described based on money changing problem and lattices in *https : //sites.google.com/site/kuja27/Sc* 04−17*.pdf* and *https : //sites.google.com/site/kuja27/IntegerPartitionAndHashFunctions*$_2$014*.pdf* which are also for elections in multipartisan setting (if condorcet election is done).

5. Probably a "rational outcome" is different from "good choice" wh ere rationality implies without any paradoxes in ranking alone without giving too much weightage to the "goodness" of a choice by the elector. Actual real-life elections are not condorcet elections where NAE tuples are generated.

6. It is not a conflict between Arrow's theorem as finding atleast 1 denumerant in multipartisan voting partition is NP-complete (as it looks) - which can be proved by ILP as in point 20 of *https : //sites.google.com/site/kuja27/IntegerPartitionAndHashFunctions*$_2$014*.pdf* - the assumption is that candidates are not ranked; they are voted independently in a secret ballot by electors and they can be more than 3. The elector just chooses a candidate and votes without giving any ordered ranking for the candidates which makes it non-condorcet.

7. Moreover Arrow's theorem for 3 candidate condorcet election implies a non-zero probability of error in voting which by itself prohibits a perfect voting system.

8. If generalized to any election and any number of candidates it could be an evidence in favour of P != NP by proving that perfect voter does not exist and using democracy Maj+SAT circuits and P(Good) probability series convergence (As described in handwritten notes and drafts:

   (a) *http : //sourceforge.net/projects/acadpdrafts/files/ImplicationGraphsPGoodEquationAndPNotE*
   (b) *https : //sites.google.com/site/kuja27/ImplicationRandomGraphConvexHullsAndPerfectVoterPr* 01 − 11*.pdf*,

(c) $https://sites.google.com/site/kuja27/LowerBoundsForMajorityVotingPseudorandomChoice.pdf$

(d) $https://sites.google.com/site/kuja27/CircuitForComputingErrorProbabilityOfMajorityVoting_2$

(e) $https://sites.google.com/site/kuja27/PhilosophicalAnalysisOfDemocracyCircuitAndPRGChoice$ $03-26.pdf$).

9. But it is not known that if Arrow's theorem can be generalized for infinite number of candidates as above and whether such an electoral system is decidable.

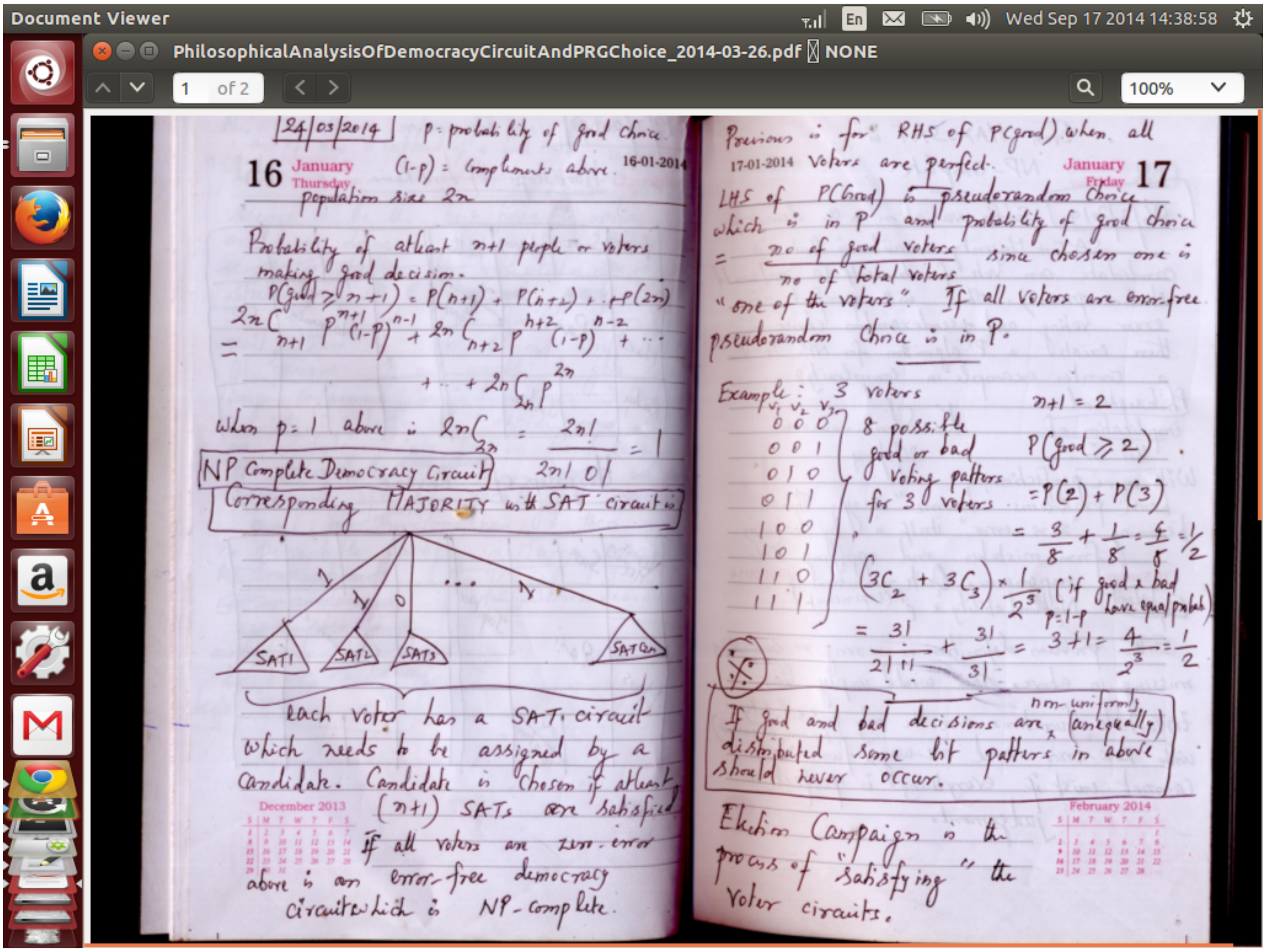
10. The possibility of a circular ranking in 3-condorcet election implies that there are some scenarios where voter can err though not exactly an error in making a "good choice" (or Perfect Voter Problem is decidable in case of 3 candidates condorcet election).

11. Error by a Voter SAT circuit implies that voter votes 0 instead of 1 and 1 instead of 0. This is nothing but the sensitivity of the voter boolean function i.e number of erroneous variable assignments by the voter that change the per-voter decision input to the Majority circuit.

12. Thus more the sensitivity or number of bits to be flipped to change the voter decision, less the probability of error by the voter. If sensitivity is denoted by s, 1/q is probability that a single bit is flipped and probability of error by the voter is p then $p = k/q^s$ for some constant k which is derived by the conditional probability that Pr[m bits are flipped] = Pr[m-th bit is flipped/(m-1) bits already flipped]*Pr[(m-1) bits are flipped] = $1/q$ *$1/q^{m-1}$ = $1/q^m$ (and m=s) .

13. This expression for p can be substituted in the Probability series defined in $https://sites.google.com/site/ku$ Probability of single bit is 1/q if the number of variables across all clauses is q and each variable is flipped independent of the other.

3 Majority Circuit with SAT input voters - Illustration with a P(good) series example



24/03/2014 p = probability of good choice

16 January Thursday (1-p) = complements above. 16-01-2014

population size 2n

Probability of atleast n+1 people or voters making good decision.

$P(\text{Good} \geq n+1) = P(n+1) + P(n+2) + \dots + P(2n)$

$= {}^{2n}C_{n+1}\, p^{n+1}(1-p)^{n-1} + {}^{2n}C_{n+2}\, p^{n+2}(1-p)^{n-2} + \dots + \dots + {}^{2n}C_{2n}\, p^{2n}$

when $p = 1$ above is ${}^{2n}C_{2n} = \frac{2n!}{2n!\,0!} = 1$

NP Complete Democracy Circuit

Corresponding MAJORITY with SAT circuit

SAT1 SAT2 SAT3 SATn

Each voter has a SAT_i circuit which needs to be assigned by a candidate. Candidate is chosen if atleast (n+1) SATs are satisfied. If all voters are zero-error above is an error-free democracy circuit which is NP-complete.

December 2013

Previous is for RHS of P(good) when all voters are perfect. 17-01-2014 January 17 Friday

LHS of P(Good) is pseudorandom choice which is in P and probability of good choice $= \frac{\text{no of good voters}}{\text{no of total voters}}$ since chosen one is "one of the voters". If all voters are error-free pseudorandom choice is in P.

Example: 3 voters $n+1 = 2$

v_1 v_2 v_3
0 0 0
0 0 1
0 1 0
0 1 1
1 0 0
1 0 1
1 1 0
1 1 1

8 possible good or bad voting patterns for 3 voters

$P(\text{Good} \geq 2) = P(2) + P(3) = \frac{3}{8} + \frac{1}{8} = \frac{4}{8} = \frac{1}{2}$

$({}^3C_2 + {}^3C_3) \times \frac{1}{2^3}$ (if good & bad have equal probab $p = 1-p$)

$= \frac{3!}{2!\,1!} + \frac{3!}{3!} = 3+1 = \frac{4}{2^3} = \frac{1}{2}$

If good and bad decisions are non-uniformly (unequally) distributed some bit patterns in above should never occur.

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Ekalvi Campaign is the process of "satisfying" the voter circuits.

4 Acknolwdgement

I dedicate this article to God.

5 Bibliography

References

[1] The series convergence is illustrated in *http://sourceforge.net/p/asfer/code/HEAD/tree/cpp−src/miscellaneous/MajorityVotingErrorProbabilityConvergence.JPG*

[2] Algorithms for Intrinsic Merit of a Document - Recursive Gloss Overlap Algorithm for wordnet subgraph - *http://arxiv.org/abs/1006.4458*

[3] Slides iluustrating WordNet subgraph or Definition Graph Construction using Recursive Gloss Overlap Algorithm - *https : //sites.google.com/site/kuja27/PresentationTAC2010.pdf?attredirects* = 0

[4] Document Summarization from WordNet subgraph obtained by Recursive Gloss Overlap - *https : //sites.google.com/site/kuja27/DocumentSummarization*$_u$*sing*$_S$*pectralGraphTheory*$_R$*GOGraph*$_2$0

[5] Primitive implementation of the above at: *http : //sourceforge.net/p/asfer/code/HEAD/tree/python−src/InterviewAlgorithm/*

[6] TAC 2010 - Update Summarization using Interview Algorithm - *http : //www.nist.gov/tac/publications/*2010*/participant.papers/CMI*$_I$*IT.proceedings.pdf*

[7] WordNet - *http : //wordnet.princeton.edu/*

[8] Equating Majority Voting and Pseudorandom Choice - *https : //sites.google.com/site/kuja27/LowerBoundsForMajorityVotingPseudorandomChoice.pdf?attredirects* 0

[9] Probability of good majority choice Series Derivation - *https : //sites.google.com/site/kuja27/CircuitForComputingErrorProbabilityOfMajorityVoting*$_2$014*.pdf*

[10] Axiom of Choice and Majority Voting *https : //sites.google.com/site/kuja27/IndepthAnalysisOfVariantO* 0

[11] A PRG for Pseudorandom Choice using Chaos Theory Attractors *https : //sites.google.com/site/kuja27/ChaoticPRG.pdf?attredirects* = 0

[12] Interview Algorithm and Interactive Proof *https : //sites.google.com/site/kuja27/InterviewAlgorithmInPS* 0

[13] Equivalence of Integer Partitions and Hash Functions (This also gives an alternative proof of NP-completeness of multipartisan democracy using reduction from restricted partition problem Money Changing Problem and Schur theorem for partitions)- *https : //sites.google.com/site/kuja27/IntegerPartitionAndHashFunctions*$_2$014*.pdf*

[14] Handwritten notes: Philosophical analysis of Democracy circuit and Pseudorandom choice *https : //sites.google.com/site/kuja27/PhilosophicalAnalysisOfDemocracyCircuitAndPRGChoice*$_2$0140 0

[15] Handwritten notes: on Schur Theorem, Restricted Partitions and Hash Functions, NP-Completeness of MultiPartisan Majority Voting *https : //sites.google.com/site/kuja27/SchurTheoremMCPAndDistinctPartitions*$_2$0140417*.pdf?attredirects* = 0

[16] Handwritten notes: An experimental theory of Convex Hull of the logical implication graph and Perfect Voter problem - *https : //sites.google.com/site/kuja27/ImplicationRandomGraphConvexHullsAndPerfectVoterProblem*$_2$01401

[17] Handwritten notes: Experimental Notes on Logical Implication Graphs: An experimental logical implication graph model for P and NP - *http : //sourceforge.net/projects/acadpdrafts/files/ImplicationGraphsPGoodEquationAndPNotEqualToNP*

[18] A gadget for randomized space filling to simulate some natural phenomena and LP for it - $https://sites.google.com/site/kuja27/Analysis\%20of\%20a\%20Randomized\%20Space\%20Filling\%20Alg$

[19] Social Choice Theory, Arrow's Theorem and Boolean functions - $http://www.ma.huji.ac.il/$ $kalai/CHAOS.pdf$