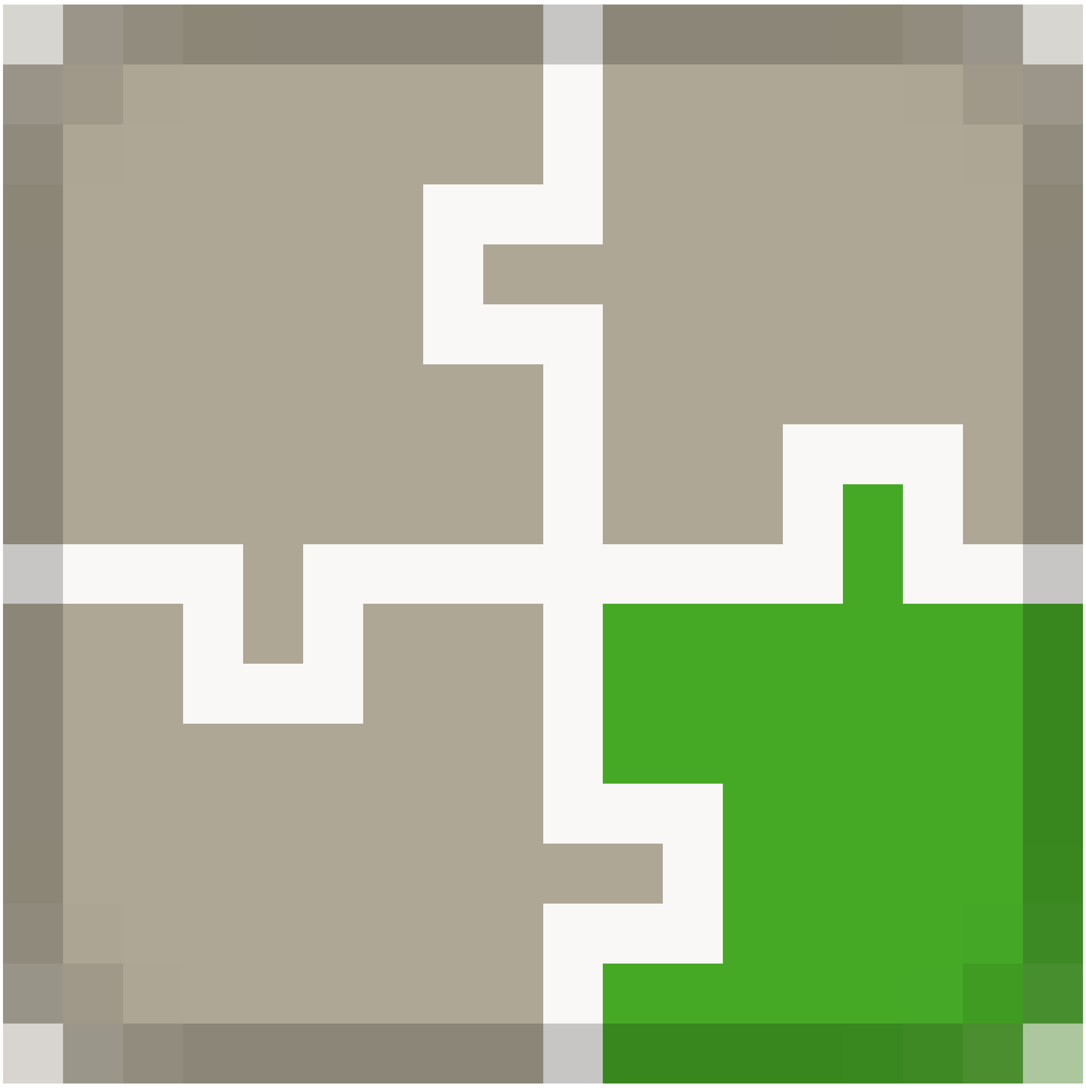

Object 1

Object 2

sci.math

✉

Complement of a function

ka_shrinivaasan

Sep 16, 2003, 2:58:04 PM9/16/03

to
Hi all,

Is there a way to find a complement of a function defined on
integers(Both domain and range are integers)?

What I mean is :
Suppose you have a set of positive integers = {1,2,3,4,5....}
Let us say we create a subset of even integers out of it =
{2,4,6,8,....}
This set of even numbers can be generated by using the function f(x) =
2x, x=1,2,3...
The complement of this set is set of odd numbers and they can be
generated using g(x) = 2x-1 , x=1,2,3,4,....
So I define g(x)=2x-1 to be the complement function of f(x)=2x over
the set of integers.

Is there a generic way to arrive at the complement function given
an arbitrary integer function? (I think we can do it using Taylor
series. But seems more mechanical)

thanks and regards,
K.Srinivasan

David C. Ullrich

Sep 16, 2003, 4:14:53 PM9/16/03

Reply to author
You do not have permission to reply to author in this group
Forward
Delete
You do not have permission to delete messages in this group
Link
Report message as abuse
Show original message
Either email addresses are anonymous for this group or you need the view member email addresses permission to view the original message
to
On 16 Sep 2003 02:28:04 -0700, ka_shri...@yahoo.com
(ka_shrinivaasan) wrote:

How do you do it using Taylor series?

>thanks and regards,
>K.Srinivasan

****

David C. Ullrich

Mitch Harris

Sep 16, 2003, 6:23:22 PM9/16/03

Reply to author
You do not have permission to reply to author in this group
Forward
Delete
You do not have permission to delete messages in this group
Link
Report message as abuse
Show original message
Either email addresses are anonymous for this group or you need the view member email addresses permission to view the original message
to
David C. Ullrich wrote:
> On 16 Sep 2003 02:28:04 -0700, ka_shri...@yahoo.com
>
>>Is there a way to find a complement of a function defined on
>>integers(Both domain and range are integers)?
>>

>>What I mean is :
>>Suppose you have a set of positive integers = {1,2,3,4,5....}
>>Let us say we create a subset of even integers out of it =
>>{2,4,6,8,....}
>>This set of even numbers can be generated by using the function f(x) =
>>2x, x=1,2,3...
>>The complement of this set is set of odd numbers and they can be
>>generated using g(x) = 2x-1 , x=1,2,3,4,....
>>So I define g(x)=2x-1 to be the complement function of f(x)=2x over
>>the set of integers.
>>
>>Is there a generic way to arrive at the complement function given
>>an arbitrary integer function? (I think we can do it using Taylor
>>series. But seems more mechanical)
>
> How do you do it using Taylor series?

generating functions? 1/(1-z) - f(z)
the function has to be strictly increasing and the gf is really a
characteristic function (all coeffs are 0 or 1)

e.g. 2n <=> 1/(1-x^2), not 2/(1-x)^2

Mitch

Will Twentyman

Sep 16, 2003, 6:38:45 PM9/16/03

Reply to author
You do not have permission to reply to author in this group
Forward
Delete
You do not have permission to delete messages in this group
Link
Report message as abuse
Show original message
Either email addresses are anonymous for this group or you need the view member email addresses permission to view the original message
to
ka_shrinivaasan wrote:

> Hi all,
>
>
> Is there a way to find a complement of a function defined on
> integers(Both domain and range are integers)?
>
> What I mean is :
> Suppose you have a set of positive integers = {1,2,3,4,5....}
> Let us say we create a subset of even integers out of it =
> {2,4,6,8,....}
> This set of even numbers can be generated by using the function f(x) =
> 2x, x=1,2,3...

> The complement of this set is set of odd numbers and they can be
> generated using g(x) = 2x-1 , x=1,2,3,4,....
> So I define g(x)=2x-1 to be the complement function of f(x)=2x over
> the set of integers.

Note: g(x) is *a* complement function. h(x)=2x+1 is also a complement
of f(x). Finding such a function may be non-trivial. Don't know if
that helps or not.

--
Will Twentyman
email: wtwentyman at copper dot net

David C. Ullrich

Sep 16, 2003, 7:50:12 PM9/16/03

Reply to author
You do not have permission to reply to author in this group
Forward
Delete
You do not have permission to delete messages in this group
Link
Report message as abuse
Show original message
Either email addresses are anonymous for this group or you need the view member email addresses permission to view the original message
to
On Tue, 16 Sep 2003 14:53:22 +0200, Mitch Harris
<har...@tcs.inf.tu-dresden.de> wrote:

>David C. Ullrich wrote:
>> On 16 Sep 2003 02:28:04 -0700, ka_shri...@yahoo.com
> >
>>>Is there a way to find a complement of a function defined on
>>>integers(Both domain and range are integers)?
>>>
>>>What I mean is :
>>>Suppose you have a set of positive integers = {1,2,3,4,5....}
>>>Let us say we create a subset of even integers out of it =
>>>{2,4,6,8,....}
>>>This set of even numbers can be generated by using the function f(x) =
>>>2x, x=1,2,3...
>>>The complement of this set is set of odd numbers and they can be
>>>generated using g(x) = 2x-1 , x=1,2,3,4,....
>>>So I define g(x)=2x-1 to be the complement function of f(x)=2x over
>>>the set of integers.
>>>
>>>Is there a generic way to arrive at the complement function given
>>>an arbitrary integer function? (I think we can do it using Taylor
>>>series. But seems more mechanical)
>>

>> How do you do it using Taylor series?
>
>generating functions? 1/(1-z) - f(z)

That's a way to use power series to obtain the complement of
a set. It doesn't give the "complement function". (Given a function
f with _range_ a subset of f we want another function g with
range equal to the complement of the range of f...)

>the function has to be strictly increasing and the gf is really a
>characteristic function (all coeffs are 0 or 1)
>
>e.g. 2n <=> 1/(1-x^2), not 2/(1-x)^2
>
>Mitch

****

David C. Ullrich

Mitch Harris

Sep 16, 2003, 8:17:58 PM9/16/03

Reply to author
You do not have permission to reply to author in this group
Forward
Delete
You do not have permission to delete messages in this group
Link
Report message as abuse
Show original message
Either email addresses are anonymous for this group or you need the view member email addresses permission to view the original message
to
David C. Ullrich wrote:
> On Tue, 16 Sep 2003 14:53:22 +0200, Mitch Harris
>>David C. Ullrich wrote:
>>>On 16 Sep 2003 02:28:04 -0700, ka_shri...@yahoo.com
>>>
>>>>Is there a way to find a complement of a function defined on
>>>>integers(Both domain and range are integers)?
>>>>
>>>>What I mean is :
>>>>Suppose you have a set of positive integers = {1,2,3,4,5....}
>>>>Let us say we create a subset of even integers out of it =
>>>>{2,4,6,8,....}
>>>>This set of even numbers can be generated by using the function f(x) =
>>>>2x, x=1,2,3...
>>>>The complement of this set is set of odd numbers and they can be
>>>>generated using g(x) = 2x-1 , x=1,2,3,4,....
>>>>So I define g(x)=2x-1 to be the complement function of f(x)=2x over
>>>>the set of integers.
>>>>

>>>>Is there a generic way to arrive at the complement function given
>>>>an arbitrary integer function? (I think we can do it using Taylor
>>>>series. But seems more mechanical)
>>>
>>>How do you do it using Taylor series?
>>
>>generating functions? 1/(1-z) - f(z)
>
> That's a way to use power series to obtain the complement of
> a set. It doesn't give the "complement function". (Given a function
> f with _range_ a subset of f we want another function g with
> range equal to the complement of the range of f...)

Hmmm... then just extract the function from this characteristic function?

2n <=> 1/(1-x^2)
<complement> 1/(1-x) - 1/(1-x^2)
= x/(1-x^2)
<=> 2n-1
(not really a generating function coeff extraction but easy to see anyway)

Yes, this last step is not particularly tractable for anything other
than combinations of linear functions. (I think quadratic functions have
a basis using theta functions)

Do you see another way for the original problem?

Mitch

Justin Davis

Sep 16, 2003, 10:08:06 PM9/16/03

Reply to author
You do not have permission to reply to author in this group
Forward
Delete
You do not have permission to delete messages in this group
Link
Report message as abuse
Show original message
Either email addresses are anonymous for this group or you need the view member email addresses permission to view the original message
to
ka_shri...@yahoo.com (ka_shrinivaasan) wrote in message news:<46b9342d.03091...@posting.google.com>...

The complement will not, in general, be uniquely determined. For
instance, if we use the whole set of integers and select the ones
generated by f(n) = 2n, every function of the form g(n) = 2n - (2m +
1) is going to be a complement to f(n) for all integers m. With more
complicated sets, even your generating function is not going to be
uniquely determined. The problem is that you're going to begin with a
set. Either you already know a generating function, or you'll be hard
pressed to find one, since you'll have to know all terms of the

sequence without knowing a generating function to give them to you.

The best you can do, if you want a unique answer, is to give the set
and its complement without speaking of functions.

>
> thanks and regards,
> K.Srinivasan

cdj

Sep 17, 2003, 12:36:50 AM9/17/03

Reply to author
You do not have permission to reply to author in this group
Forward
Delete
You do not have permission to delete messages in this group
Link
Report message as abuse
Show original message
Either email addresses are anonymous for this group or you need the view member email addresses permission to view the original message
to
Will Twentyman <wtwen...@read.my.sig> wrote in message news:<3f65b951$1...@newsfeed.slurp.net>...

Seems to me this is more of a "theory of computability" issue than
anything. For the given example, the OP essentially gave us a program
for churning out a particular set of integers/naturals, and then gave
us a program that churns out the complement of the first set. Of
course there will in general be many programs to churn out the
complement, if there are any at all.

Among other things, if the original set (eg of evens, as per the OP)
isn't recursive, then that's a good time to give up on a "complement
program"....

Generally speaking, the notions of "recursive set", "recursively
enumerable set", and the like seem quite relevant to the OP's
question...

cdj

John Coleman

Sep 17, 2003, 1:17:20 AM9/17/03

Reply to author
You do not have permission to reply to author in this group
Forward

Delete
You do not have permission to delete messages in this group
Link
Report message as abuse
Show original message
Either email addresses are anonymous for this group or you need the view member email addresses permission to view the original message
to
ka_shri...@yahoo.com (ka_shrinivaasan) wrote in message news:<46b9342d.03091...@posting.google.com>...

As another poster has pointed out, g will not be unique in general. A more serious problem is that even if f is computable then it doesn't follow that any computable g would work. The range of a computable function from integers to integers is by definition recursively enumerable and it is a standard result that the complement of an r.e. set need not be r.e.

Hope this helps

-john coleman

Eric J. Wingler

Sep 17, 2003, 2:18:51 AM9/17/03

Reply to author
You do not have permission to reply to author in this group
Forward
Delete
You do not have permission to delete messages in this group
Link
Report message as abuse
Show original message
Either email addresses are anonymous for this group or you need the view member email addresses permission to view the original message
to
John Coleman wrote:

If we restrict ourselves to functions from the positive integers into the positive integers and we take g to be an increasing function, then g is unique. If I calculated correctly, g would satisfy the equation g(f(x)-x) = f(x)-1. Of course this may not be of much use to actually compute g.

Eric J. Wingler (win...@math.ysu.edu)
Dept. of Mathematics and Statistics
Youngstown State University
One University Plaza
Youngstown, OH 44555-0001
330-941-1817

Robert Israel

Sep 17, 2003, 3:25:26 AM9/17/03

Reply to author
You do not have permission to reply to author in this group
Forward
Delete
You do not have permission to delete messages in this group
Link
Report message as abuse
Show original message
Either email addresses are anonymous for this group or you need the view member email addresses permission to view the original message
to
In article <3F677733...@math.ysu.edu>,

Eric J. Wingler <win...@math.ysu.edu> wrote:
>John Coleman wrote:
>
>> ka_shri...@yahoo.com (ka_shrinivaasan) wrote in message
>news:<46b9342d.03091...@posting.google.com>...

>> > Is there a way to find a complement of a function defined on

>> > integers(Both domain and range are integers)?

>> > What I mean is :
>> > Suppose you have a set of positive integers = {1,2,3,4,5....}
>> > Let us say we create a subset of even integers out of it =
>> > {2,4,6,8,....}
>> > This set of even numbers can be generated by using the function f(x) =
>> > 2x, x=1,2,3...
>> > The complement of this set is set of odd numbers and they can be
>> > generated using g(x) = 2x-1 , x=1,2,3,4,....
>> > So I define g(x)=2x-1 to be the complement function of f(x)=2x over
>> > the set of integers.

>> > Is there a generic way to arrive at the complement function given
>> > an arbitrary integer function? (I think we can do it using Taylor
>> > series. But seems more mechanical)

I don't see what Taylor series could have to do with this.

>> As another poster has pointed out, g will not be unique in general. A
>> more serious problem is that even if f is computable then it doesn't
>> follow that any computable g would work. The range of a computable
>> function from integers to integers is by definition recursively
>> enumerable and it is a standard result that the complement of an r.e.
>> set need not be r.e.

>If we restrict ourselves to functions from the positive integers into

>the positive integers and we take g to be an
>increasing function, then g is unique. If I calculated correctly, g
>would satisfy the equation g(f(x)-x) = f(x)-1. Of

This is wrong: e.g. if f(1)=1, f(2)=3, f(3)=4 then g(f(3)-3)=g(1)=2 but
f(3)-1=3.

>course this may not be of much use to actually compute g.

I think you're also assuming f(x) is increasing. If we do that, then
g is computable, e.g. g(x) = min {w >= x: f(w+1-x) > w}. (Of course,
if g(x) doesn't exist, i.e. f misses fewer than x values, a search
for such w would never terminate, and there is no way in general
to predict this)

Robert Israel isr...@math.ubc.ca
Department of Mathematics http://www.math.ubc.ca/~israel
University of British Columbia
Vancouver, BC, Canada V6T 1Z2

David C. Ullrich

Sep 17, 2003, 3:58:42 AM9/17/03

"Extracting" a function whose _values_ are the indices where the
generating function has a non-zero coefficient is exactly equivalent
to the original problem.

>2n <=> 1/(1-x^2)
> <complement> 1/(1-x) - 1/(1-x^2)
> = x/(1-x^2)
> <=> 2n-1
>(not really a generating function coeff extraction but easy to see anyway)
>
>Yes, this last step is not particularly tractable for anything other
>than combinations of linear functions. (I think quadratic functions have
>a basis using theta functions)
>
>Do you see another way for the original problem?

No, it seems to me like the sort of problem that one should simply not expect a simple answer to.

Eric J. Wingler

Sep 18, 2003, 1:10:03 AM9/18/03

to
Robert Israel wrote:

> In article <3F677733...@math.ysu.edu>,
> Eric J. Wingler <win...@math.ysu.edu> wrote:
>
>
> >If we restrict ourselves to functions from the positive integers into
> >the positive integers and we take g to be an
> >increasing function, then g is unique. If I calculated correctly, g
> >would satisfy the equation g(f(x)-x) = f(x)-1. Of
>
> This is wrong: e.g. if f(1)=1, f(2)=3, f(3)=4 then g(f(3)-3)=g(1)=2 but
> f(3)-1=3.

Yes, I posted this a little too quickly. I had in mind a function f whose values at consecutive integers are not consecutive; that is, f(n) < f(n+1) - 1 for each n>0. For example, if f(x) = 3x, then we get g(2x) = 3x-1. Since x must be a positive integer, this equation doesn't tell us what g of an odd number is, but we can get an idea of how fast g grows.

> >course this may not be of much use to actually compute g.
>
> I think you're also assuming f(x) is increasing. If we do that, then
> g is computable, e.g. g(x) = min {w >= x: f(w+1-x) > w}. (Of course,
> if g(x) doesn't exist, i.e. f misses fewer than x values, a search
> for such w would never terminate, and there is no way in general
> to predict this)
>
> Robert Israel isr...@math.ubc.ca

Yes, I assumed that f is increasing. I interpreted the original poster's question as to how one would break the sequence of positive integers into two disjoint subsequences. One of these subsequences would be given by {f(n)}, the other by what he called the complement of f, {g(n)}.

Will Twentyman

Sep 18, 2003, 1:37:12 AM9/18/03

Reply to author
You do not have permission to reply to author in this group
Forward
Delete
You do not have permission to delete messages in this group
Link
Report message as abuse
Show original message
Either email addresses are anonymous for this group or you need the view member email addresses permission to view the original message
to
cdj wrote:

Good point. This leads to the issue of whether g can be constructed.
That would depend on f.

Mitch Harris

Sep 18, 2003, 2:16:03 PM9/18/03

Reply to author
You do not have permission to reply to author in this group
Forward
Delete
You do not have permission to delete messages in this group
Link
Report message as abuse
Show original message
Either email addresses are anonymous for this group or you need the view member email addresses permission to view the original message
to
David C. Ullrich wrote:
> On Tue, 16 Sep 2003 16:47:58 +0200, Mitch Harris
>>David C. Ullrich wrote:
>>>On Tue, 16 Sep 2003 14:53:22 +0200, Mitch Harris
>>>>David C. Ullrich wrote:
>>>>>On 16 Sep 2003 02:28:04 -0700, ka_shri...@yahoo.com
>>>>>
>>>>>>Is there a way to find a complement of a function defined on
>>>>>>integers(Both domain and range are integers)?
>>>>generating functions? 1/(1-z) - f(z)

>>>
>>>That's a way to use power series to obtain the complement of
>>>a set. It doesn't give the "complement function". (Given a function
>>>f with _range_ a subset of f we want another function g with
>>>range equal to the complement of the range of f...)
>>
>>Hmmm... then just extract the function from this characteristic function?
>
> "Extracting" a function whose _values_ are the indices where the
> generating function has a non-zero coefficient is exactly equivalent
> to the original problem.

I agree with "exactly", but I think differently than your intention. I think the generating function method puts the question on a basis in which objects can be manipulated formally. I don't think I've just reworded the problem. Some of the steps may be intractable (or even currently unknown how to implement), but

To give a better example (one that is tractable), suppose you have the function f(n) = 3n. It is easy to see intuitively that the the complement of the range is the set of numbers that are not congruent to 0 mod 3. But what is the -function- g(x) whose range is this set (i.e. g(0) = 1, g(1) = 2, g(2) = 4, g(3) = 5, g(4) = 7, etc...). How do you construct that function given just f? (Sure we have notation that we can use to immediately solve it intuitively:

"if x = 0 mod 2, g(x) = 3x/2 + 1,
if x = 1 mod 2, g(x) = 3(x-1)/2 + 2"

but by the method I gave eventually you compute for the complement the generating function (x+x^2)/(1-x^3) (from which of course one can formally extract the above)

the missing hard part is converting from a gf (a monotonically increasing one) to the gf of the characteristic function (and back).
I don't see how to do it for anything other than f having a range the union of linear functions.

Mitch

Reply all
Reply to author
Forward

0 new messages