ROBOT SENSING & NAVIGATION –EECE554

LAB-3 –INERTIAL MEASUREMENT UNIT

STATIONARY DATA (10-15 MINS)

Data Capturing: The sensor was placed stationary in relatively calm surroundings and as far as possible away from magnetic disturbances from electronic devices and mechanical vibrations. The rosbag has been collected for 10 to 15 mins plotting linear acceleration, angular acceleration, magnetic field (converted from gauss to tesla units) and orientation from the $VNYMYR string. (converted from quaternions to Euler angles).

1. Linear Acceleration data –XYZ Axes

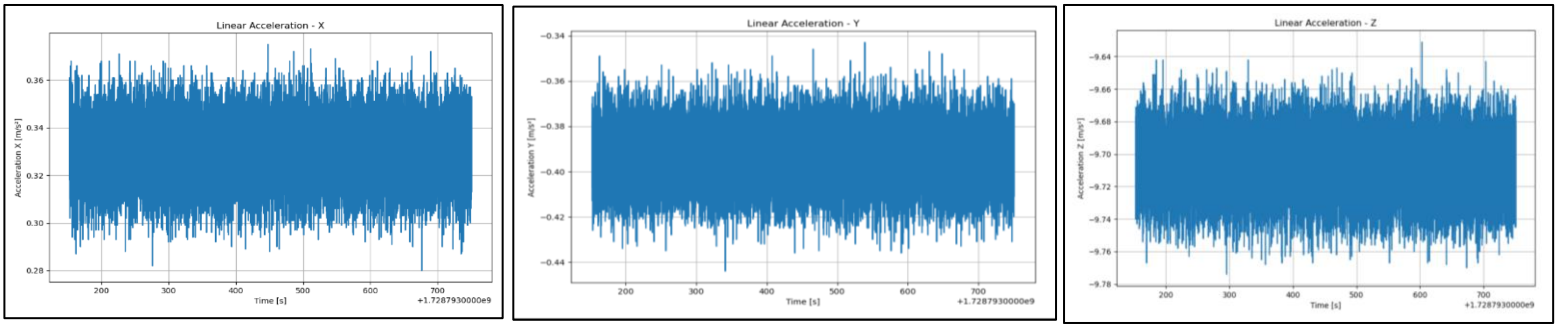


Figure-1

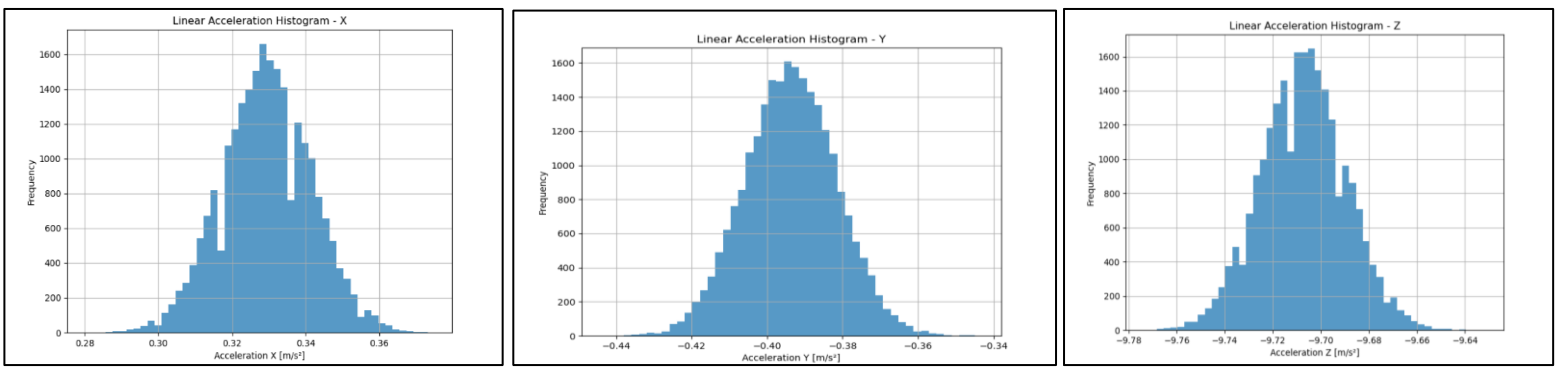


Figure-2

Observations: The following observations were made from figures 1 & 2.

Stability: In figure 1, the linear acceleration varies across time in 3 axes and indicates a stable data capture with random noise due to vibrations kept to minimal.

Mean: From figure 2, It can be observed that mean for X axes is centered around 0.33 m/s^2 which indicates a slight tilt of the sensor from the true horizontal plane and similarly in Y-axes, the mean is centered at -0.39 m/s^2 indicating tilt in negative y axis. The z-axes data is consistent and centered at -9.7 m/s^2 which the downward gravitational acceleration ~ 9.8 m/s^2.

Standard Deviation: The standard deviation of the plots in figure shows that the standard deviation ranges from 0.02 m/s^2 to 0.03 m/s^2 which is minimal and suggests stable sensor performance.

Distribution: The distribution of all 3 histograms is a normal distribution plot which is consistent with the sensor behavior.

Range: The range of the plot of X axis is 0.28 to 0.37 m/s^2 while for Y axis, it is -0.44 to -0.34 m/s^2 and for Z axis - -9.77 to -9.64 m/s^2 ~ 9.81 m/s^2-Accelaration due to Gravity.

Inference: The linear acceleration data confirms that the sensor was indeed stationary, with expected gravitational effects on the Z-axis and minor tilts affecting X and Y axes. The consistent noise levels across all axes suggest stable sensor performance.

2. Angular Velocity -XYZ Axes

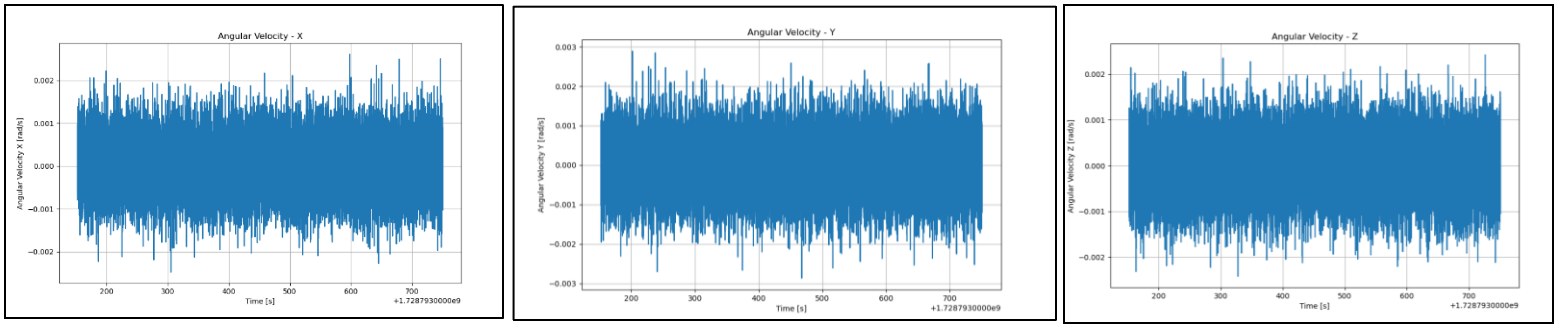


Figure-3

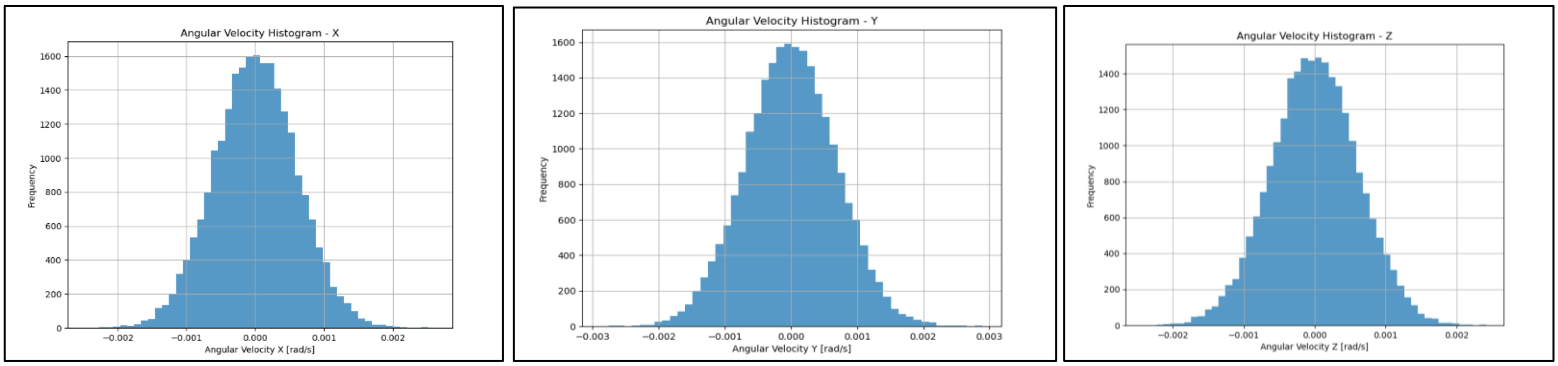


Figure-4

Observations: The following observations were made from figures 3 & 4.

Stability: In figure 3, the angular velocity varies across time in 3 axes and indicates a stable data capture with random noise due to vibrations kept to minimal.

Mean: From figure 4, It can be observed that mean for all the three axes is almost zero (0) rad/s indicating stationary position of the IMU sensor.

Standard Deviation: The standard deviation of the plots in figure 4 shows that the standard deviation is 0.001 rad/s for X and Z axes and 0.0015 rad/s for Y axis which minimal signifying the stationary state.

Distribution: The distribution of all 3 histograms is a normal distribution plot which is consistent with the sensor behavior.

Range: The range of all 3 histograms is from -0.002 rad/s to 0.002 rad/s indicating a stable plot.

Inference: The angular velocity data corroborates the stationary condition of the sensor, with only minimal noise present.

3. Magnetic Field – XYZ Axes

The magnetic field data has been collected from IMU sensor's magnetometers which indicates value in gauss which has later been converted into tesla units during the analysis part and the driver has been corrected accordingly.

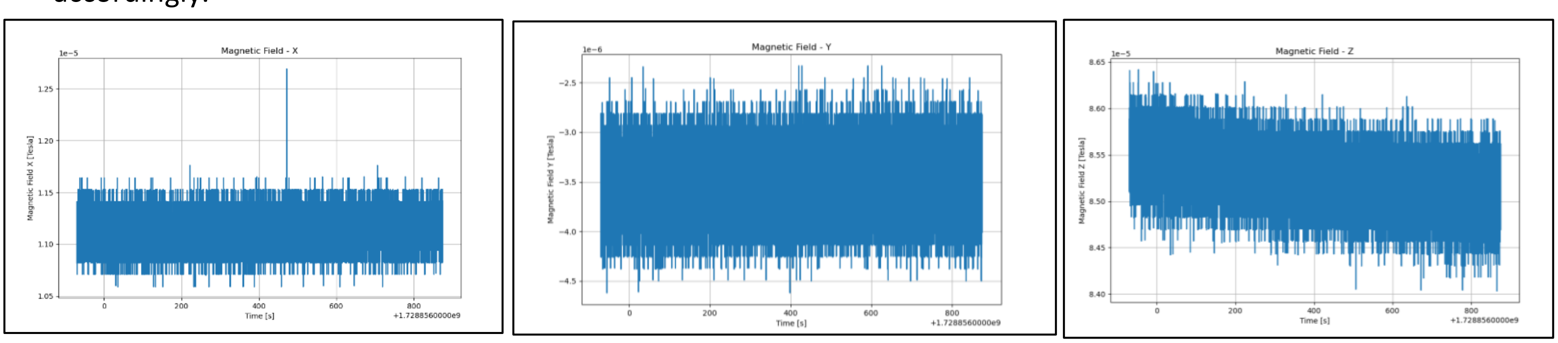


Figure-5

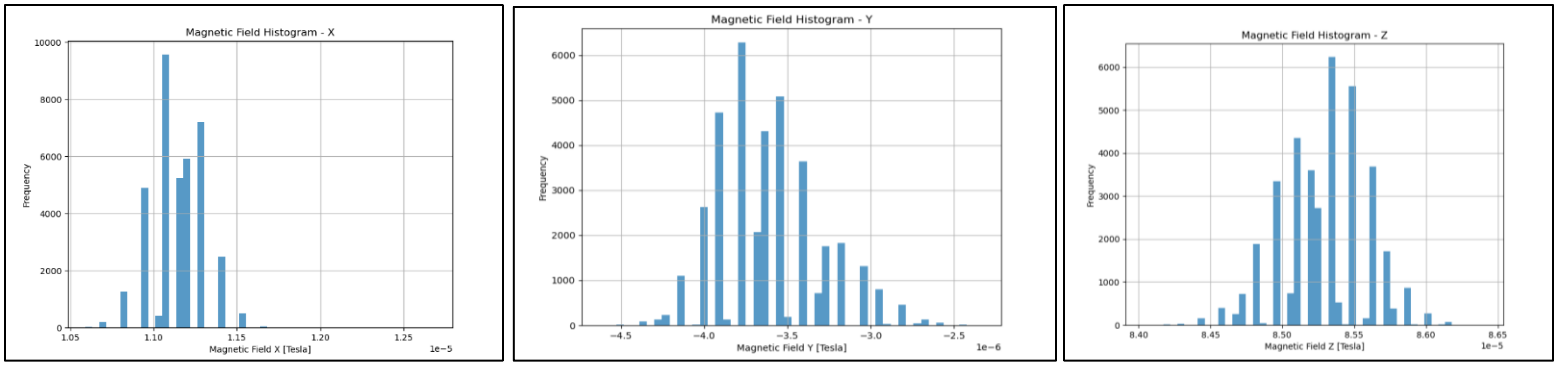


Figure-6

Observations: The following observations were made from figures 5 & 6.

Stability: In figure 5, the magnetic field varies across time in 3 axes with occasional spikes indicates a stable data capture with random noise due to low field magnetic disturbances.

Mean: From figure 6, It can be observed that mean for X axes is centered around 1.10 x 10^{-5} T which indicates a slight disturbance of the sensor due to external factors like presence of metal object near it and similarly in Y-axes, the mean is centered at -3.50 x 10^{-6} T indicating minor noise variations. The z-axes data is the most consistent and centered at 8.55 x 10^{-5} T.

Standard Deviation: The histogram plot of all the axes is slightly skewed indicating low field magnetic disturbances to the sensor during data collection. The standard deviation of the plot ranges from 0.5 x 10^{-6} T for Y axis to 0.01 x 10^{-5} T, 0.03 x 10^{-5} T in X and Z axis respectively.

Distribution: The distribution of all 3 histograms is slightly skewed normal distribution plot indicating slight disturbances.

Range: The range of the plot of X axis is between 1.075 x 10^{-5} T 1.175 x 10^{-5} T while for Y axis, it is -4.25 x 10^{-6} T to -2.5 x 10^{-6} T and for Z axis – 8.425 x 10^{-5} T to 8.625 x 10^{-5} T.

Inference: The magnetic field data shows stability with some variability, likely due to the sensor's sensitivity to local magnetic field fluctuations.

4. Orientation-Roll, Pitch, Yaw-Euler angles in radians

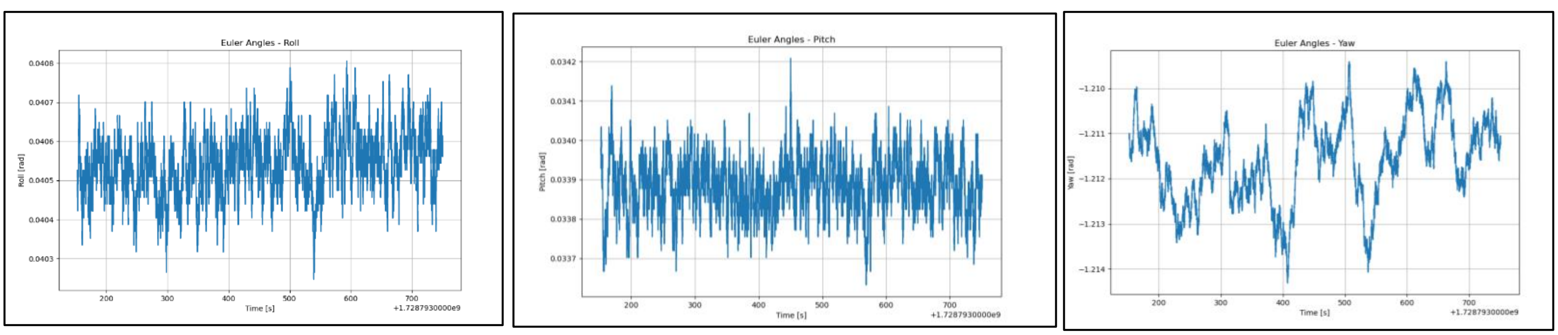


Figure-7

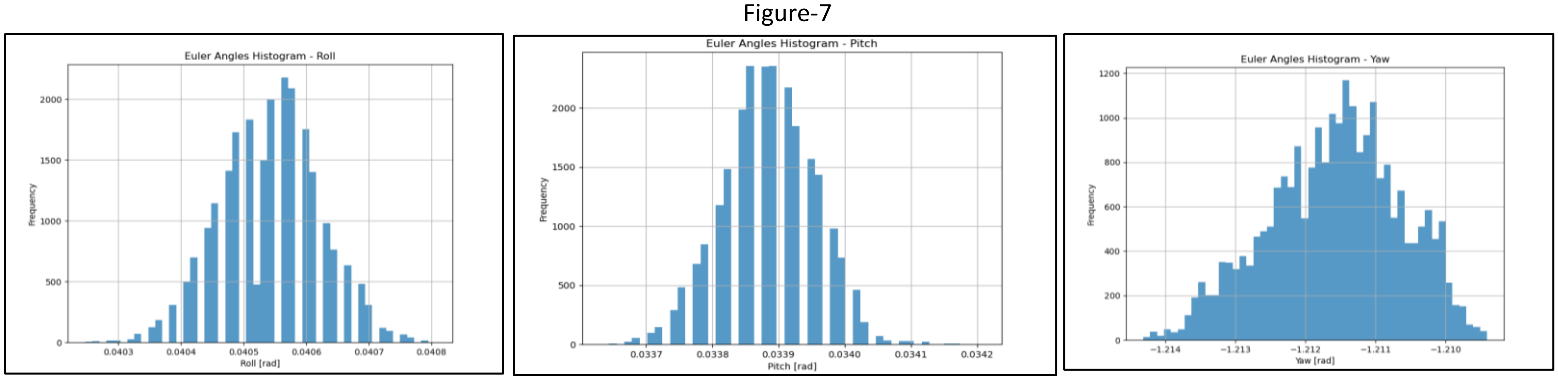


Figure-8

Observations: The following observations were made from figures 7 & 8.

Stability: In figure 7, the Euler angles-roll, pitch and yaw varies across time in 3 axes with noise disturbances.

Mean: From figure 8, It can be observed that mean for roll and pitch plots are almost near zero at 0.00405 rad and 0.0339 rad respectively but there is slightly negative mean of -1.211 rad for yaw plot indicating a minimal drift in yaw angle.

Standard Deviation: The standard deviation of the plots in figure 8 shows that the standard deviation is roughly 0.0001 rad for yaw, pitch and roll plots indicating a stable and consistent readings.

Distribution: The distribution of all 3 histograms is a normal distribution plot which is consistent with the sensor behavior.

Range: The ranges of roll and pitch are almost similar at 0.0403 rads to 0.408 rads for roll and 0.0337 rads to 0.0341 rads for pitch but there is slight disturbance in the yaw values due to a slight drift in yaw positioning with range at -1.215 rad to -1.205 rad.

Inference: The orientation data shows a stable position with minor tilts, which is consistent with the accelerometer data.

Conclusion:

- Sensor Stability: The VN-100 sensor has shown excellent stability over the observed duration with consistent readings across all parameters measured with slight noise creeping in.
- Accuracy: The accelerometer and gyroscope values align well with expected values which indicate the stationary position of the IMU sensor, thus signifying good calibration and accuracy.
- Environmental factors: Minor variations and disturbances detected in acceleration and orientation data suggest the sensor wasn't perfectly level positioned. This is normal and could be due to the mounting surface or fine-scale gravitational variations.
- Magnetic Sensitivity: The magnetometer showed more variability compared to acceleration measurements, which is expected due to its sensitivity to local magnetic field fluctuations.
- Noise Characteristics: All measurements exhibited normal distributions with expected levels of noise for a MEMS-based IMU, indicating proper functioning of the sensor.
- Data Quality: The consistency and stability of the data over the extended period suggests that the VectorNav VN-100 IMU is suitable for applications requiring high-precision motion sensing and orientation determination.

ALLAN VARIANCE OF IMU

Allan variance analysis has been performed on stationary data that has been captured for the duration of 5hrs period thereby a significant amount of data for in-depth analysis.

To compute the Allan variance for the IMU sensor, we need to consider internal sensors present in them, for which let us consider the 3 DOF gyroscope sensor present. In scope of this analysis, we can consider 1 axis of gyroscope sensor.

The five-hour dataset has been collected as a rosbag file which has been converted to. mat (MATLAB data) file for the purposes of analysis.

Five-hour dataset: https://drive.google.com/drive/folders/1_0skGjwHp16QEziWwAHnnYMjPLjWIxzZ

Sources of Noise in Inertial Sensors

1. **White Noise (Quantization Noise):**
    - **Description:** Random noise that affects the sensor readings at all frequencies. It is typically due to electronic noise in the sensor circuitry.
    - **Modeling:** Modeled as a Gaussian distribution with zero mean. In Allan variance plots, it appears as a slope of -0.5.
2. **Bias Instability:**
    - **Description:** A slow, random drift in the sensor output due to imperfections in the sensor's components and environmental factors.
    - **Modeling:** Represented as a constant offset that changes slowly over time. In Allan variance plots, it appears as a flat region.
3. **Random Walk:**
    - **Description:** A cumulative error that grows with time due to integration of white noise.
    - **Modeling:** Modeled as a random walk process with a slope of +0.5 in Allan variance plots.

a) Allan Variance Calculation

The Allan variance is calculated as follows:

Log *L* stationary gyroscope samples with a sample period τ_0. Let Ω be the logged samples.

$$\Omega(t) = \Omega_{Ideal}(t) + Bias_N(t) + Bias_B(t) + Bias_K(t)$$

For each sample, calculate the output angle θ:

$$\theta(t) = \int^t \Omega(t')dt'$$

For discrete samples, the cumulative sum multiplied by τ_0 can be used.

In our case, the sample period is defined to be 40hz and hence given in as input.

For calculating the Allan variance,

$$\sigma^2(\tau) = \frac{1}{2\tau^2} < (\theta_{k+2m} - 2\theta_{k+m} + \theta_k)^2 >$$

where $\tau = m\tau_0$ and $<>$ is the ensemble average.

The ensemble average can be expanded to:

$$\sigma^2(\tau) = \frac{1}{2\tau^2(L-2m)} \sum_{k=1}^{L-2m} (\theta_{k+2m} - 2\theta_{k+m} + \theta_k)^2$$

b) Allan deviation and Noise characteristics calculation.

To obtain the noise parameters for the gyroscope, use the following relationship between the Allan variance and the two-sided power spectral density (PSD) of the noise parameters in the original data set Ω. The relationship is:

$$\sigma^2(\tau) = 4\int_0^\infty S_\Omega(f)\frac{sin^4(\pi f\tau)}{(\pi f\tau)^2}df$$

Angle Random Walk(N)

The angle random walk is characterized by the white noise spectrum of the gyroscope output. The PSD is represented by: $S_\Omega(f) = N^2$ where *N* = angle random walk coefficient. The units of *N* are $(rad/s)/\sqrt{Hz}$

Substituting into the original PSD equation and performing integration yields:

$$\sigma^2(\tau) = \frac{N^2}{\tau}$$

Rate Random Walk

The rate random walk is characterized by the red noise (Brownian noise) spectrum of the gyroscope output. The PSD is represented by: $S_\Omega(f) = (\frac{K}{2\pi})^2\frac{1}{f^2}$

Where *K* = rate random walk coefficient, the units of *K* are $(rad/s)\sqrt{Hz}$

Substituting into the original PSD equation and performing integration yields:

$$\sigma^2(\tau) = \frac{K^2\tau}{3}$$

Bias Instability

The bias instability is characterized by the pink noise (flicker noise) spectrum of the gyroscope output. The PSD is represented by:

$$S_\Omega(f) = \begin{cases} (\frac{B^2}{2\pi})\frac{1}{f} & : f \le f_0 \\ 0 & : f > f_0 \end{cases}$$

Where *B* = bias instability coefficient, f_0 = cut-off frequency

Substituting into the original PSD equation and performing integration yields:

$$\sigma^2(\tau) = \frac{2B^2}{\pi}[\ln 2 + \; - \frac{sin^3 x}{2x^2}(sinx + 4xcosx) + Ci(2x) - Ci(4x)]$$

Where $x = \pi f_0 \tau$, *Ci* = cosine-integral function

When τ is much longer than the inverse of the cutoff frequency, the PSD equation is:

$$\sigma^2(\tau) = \frac{2B^2}{\pi}\ln 2$$

The above equation is a line with a slope of 0 when plotted on a log-log plot of $\sigma(\tau)$ versus τ.

The value of *B* can be read directly off this line with a scaling of $\sqrt{\frac{2\ln 2}{\pi}} \approx 0.664$. The units of *B* are rad/s.

These calculations have been performed using MATLAB for each of the 3 axes of gyroscope, accelerometer and magnetometer of which only gyroscope and accelerometer's values are compared with VN-100 datasheet.

Observations – Comparison with VN-100 datasheet- VectorNav-VN100

Gyroscope readings and comparison with data sheet of VN-100.

Noise parameters	Angular Velocity -X		Angular Velocity -Y		Angular Velocity -Z	
	Datasheet	Our Value	Datasheet	Our Value	Datasheet	Our Value
B ($\circ/hr.$)	<10	4.2665	<10	3.9126	<10	9.037
N ($\circ/s/\sqrt{Hz}$)	0.0035	0.0083	0.0035	0.0061	0.0035	0.0079

Table-1

Table-1 shows the Bias instability and Noise Density values of gyroscope sensor and its comparison with datasheet values of VN-100 sensor for all the 3 axes has been determined using the above-mentioned formulae for Allan variance and deviation.

Accelerometer readings and comparison with datasheet of VN-100

Noise parameters	Linear Acceleration -X		Linear Acceleration -Y		Linear Acceleration -Z	
	Datasheet	Our Value	Datasheet	Our Value	Datasheet	Our Value
B (mg)	<0.04	0.6005e-5	<0.04	0.117e-3	<0.04	0.117e-3
N (mg/$\sqrt{Hz}$)	0.14	0.1663e-3	0.14	0.1663e-3	0.14	0.2844e-4

Table-2

Table-2 shows the Bias instability and Noise Density values of accelerometer sensor for linear acceleration. and its comparison with datasheet values of VN-100 sensor for all the 3 axes has been determined using the above-mentioned formulae for Allan variance and deviation.

Inference

- From table-1, it can be inferred that the values of noise density N are more than the specification value of the datasheet for angular velocity, but the bias instability value B is within the given specification.
- Similarly, the data in table-2 shows us that, the values of bias instability B as well as the noise density values N are well within the specification given for the IMU sensor's accelerometer.

The Allan deviation plots for Linear acceleration-XYZ axis, Angular Velocity-XYZ axis and Magnetic field values in tesla after conversion -XYZ axis:

Gyroscope-XYZ Plots

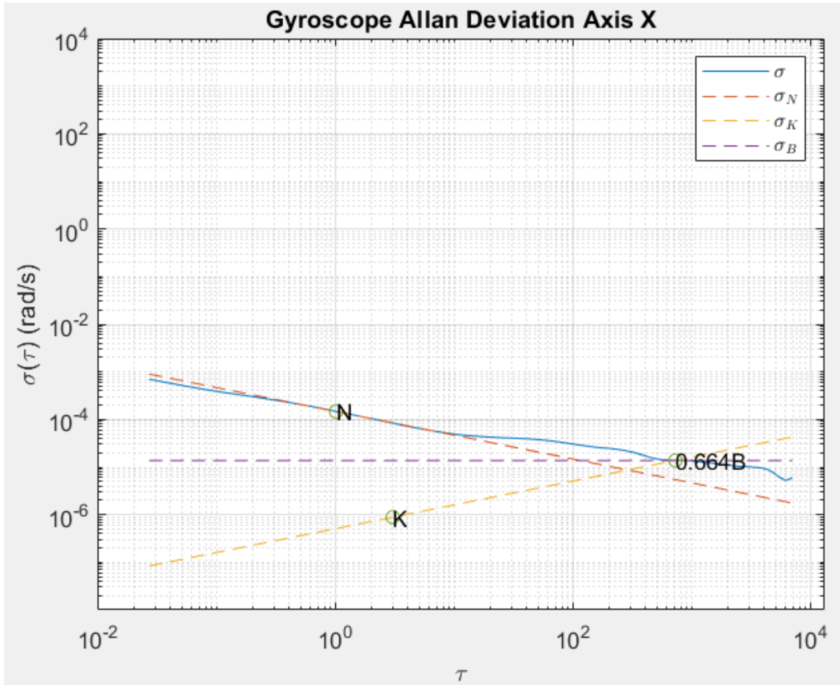


Figure-9

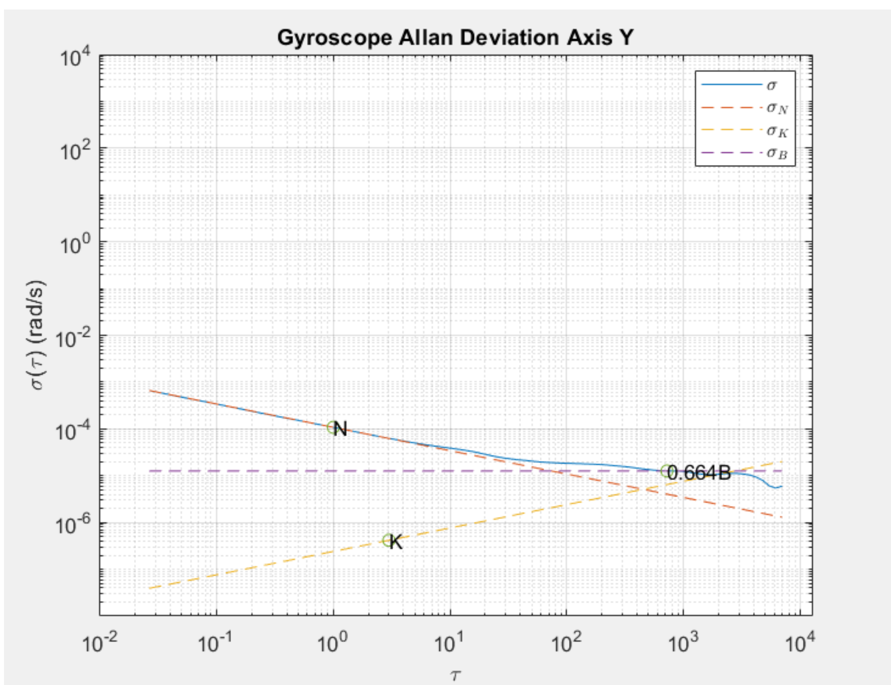


Figure-10

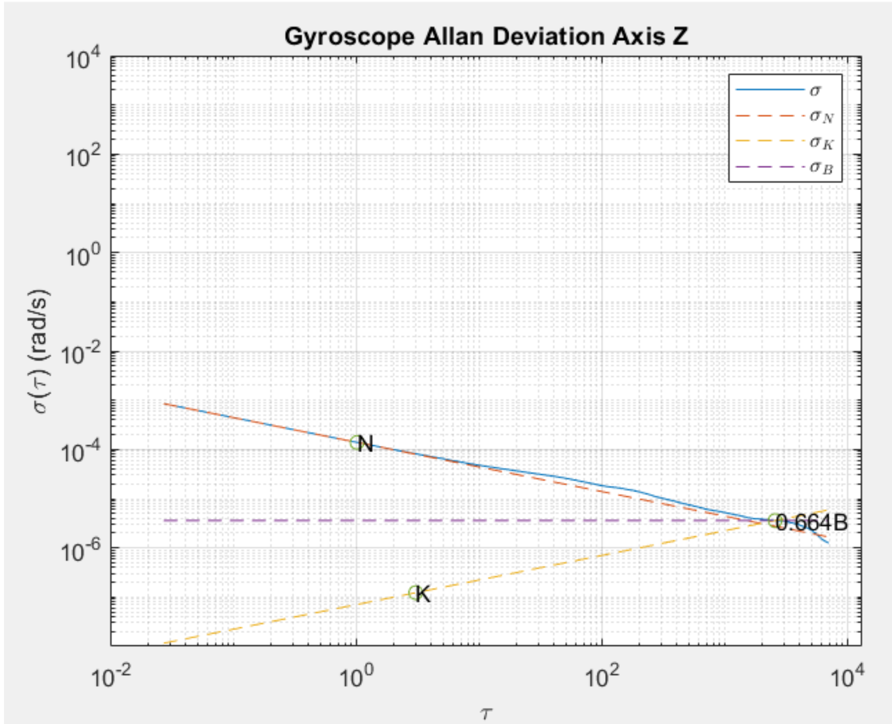


Figure-11

Accelerometer-XYZ Plots

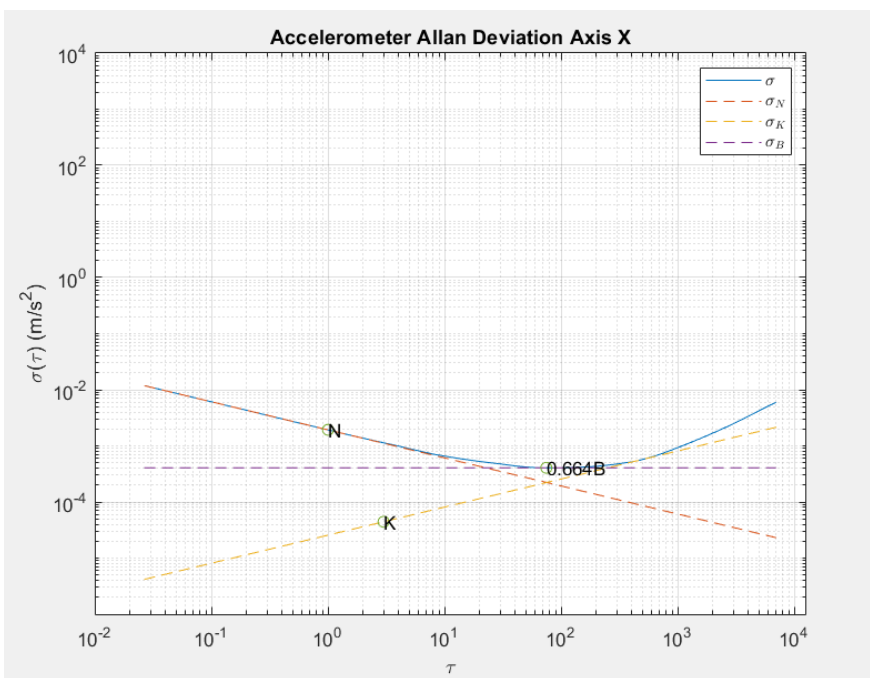


Figure-12

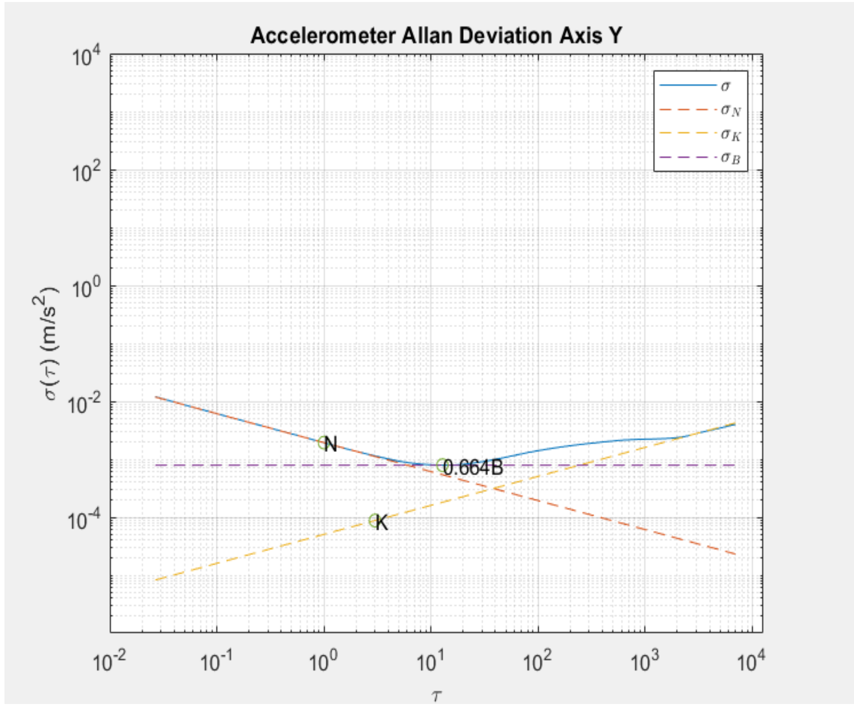


Figure-13

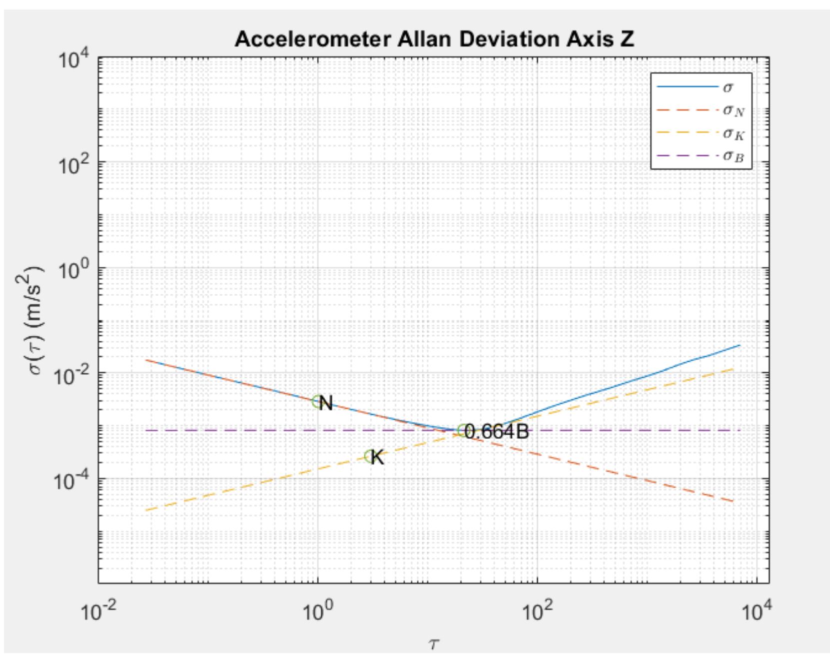


Figure-14

Magnetic Field -XYZ Axis

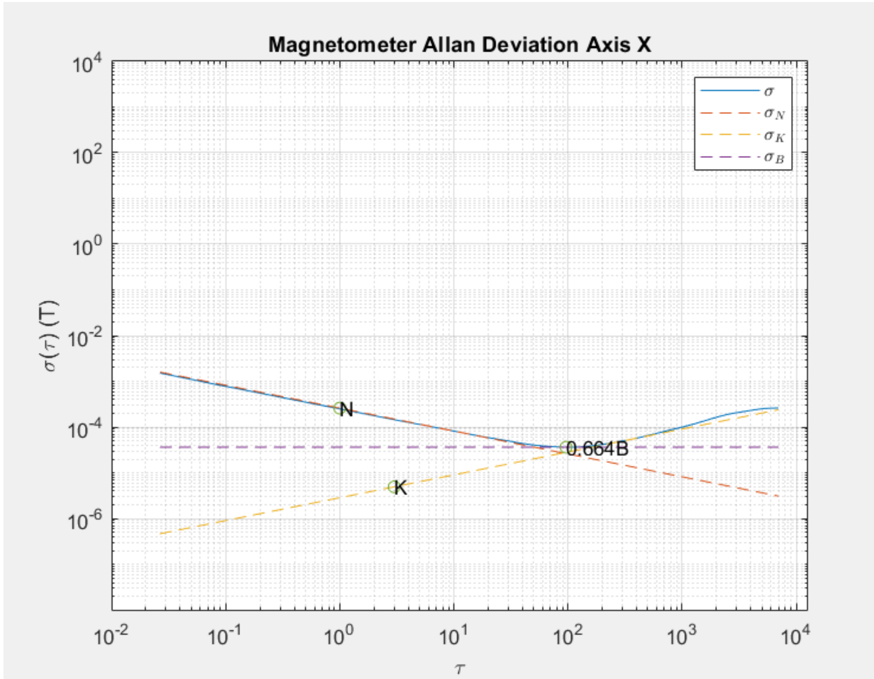


Figure-15

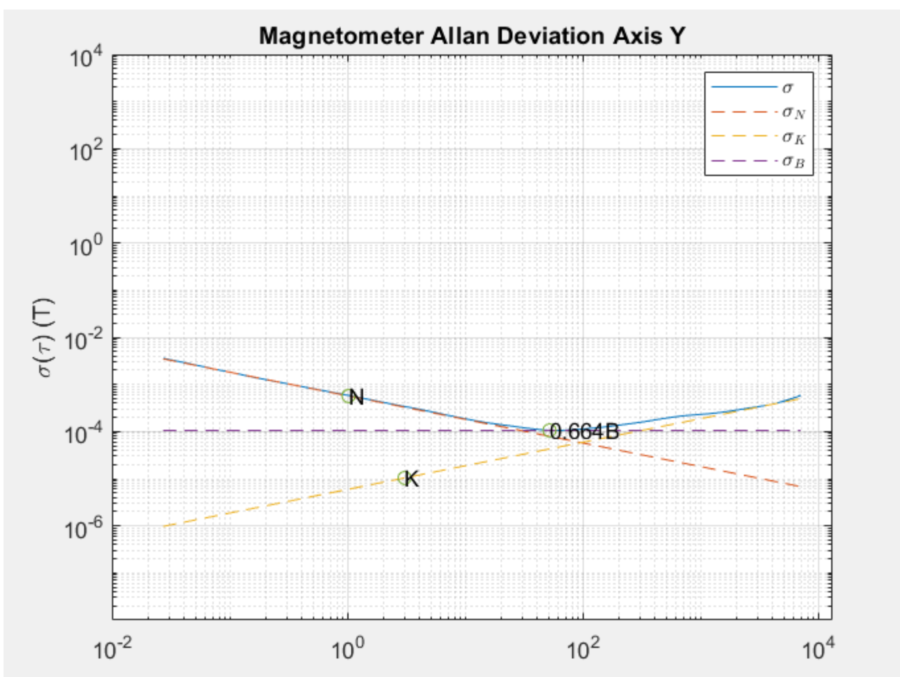


Figure-16

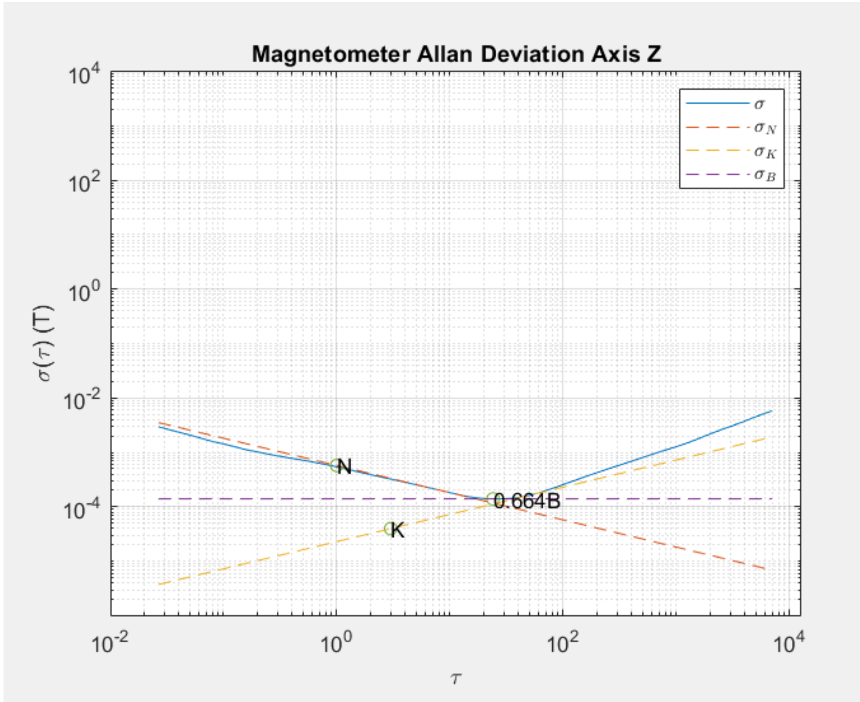
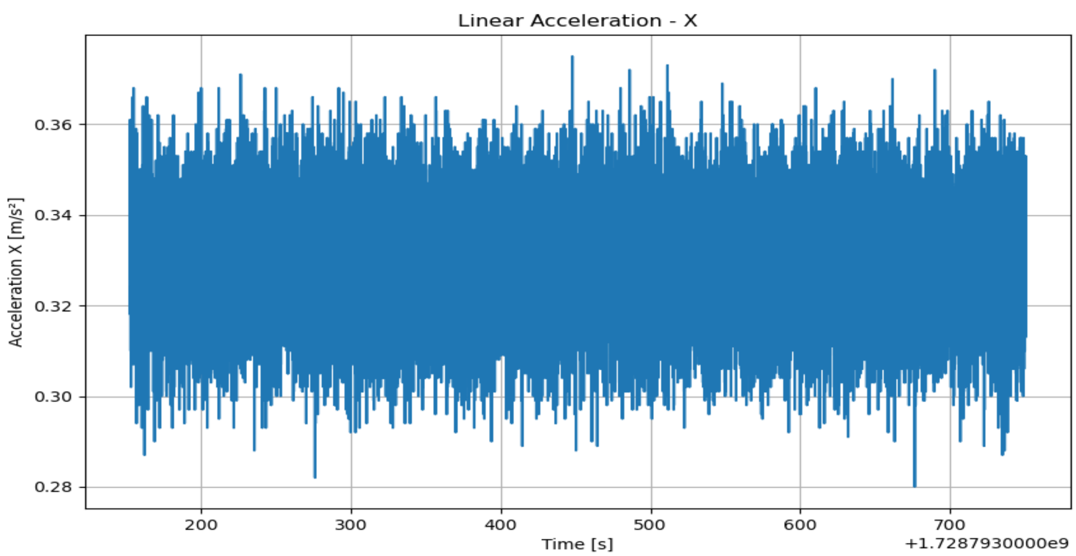
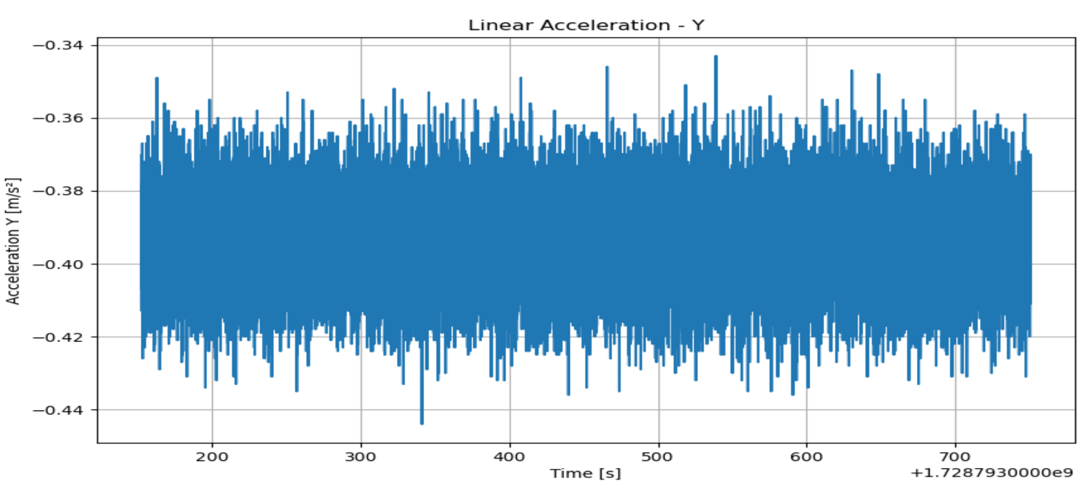


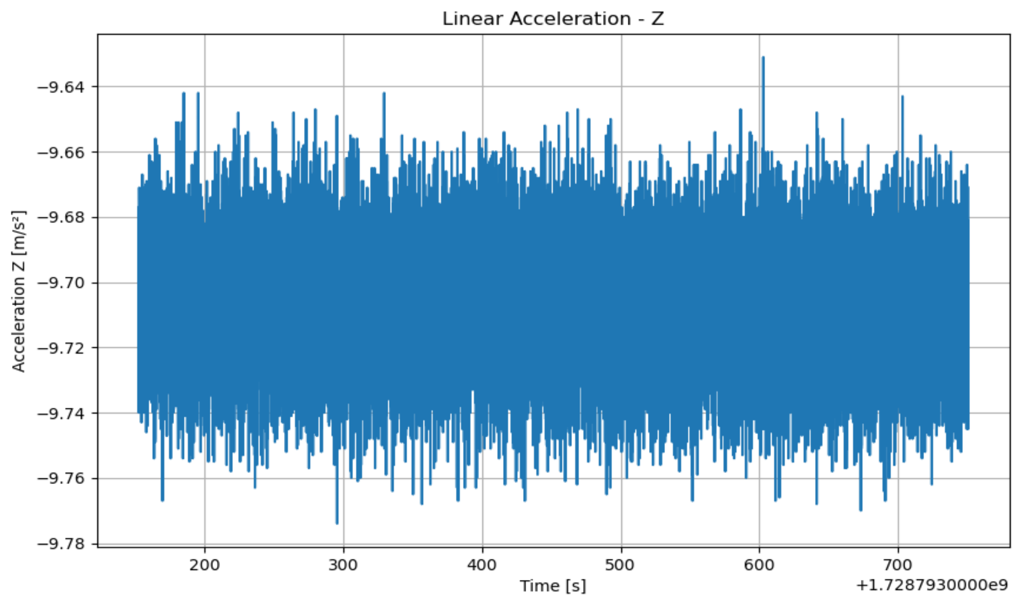
Figure-17

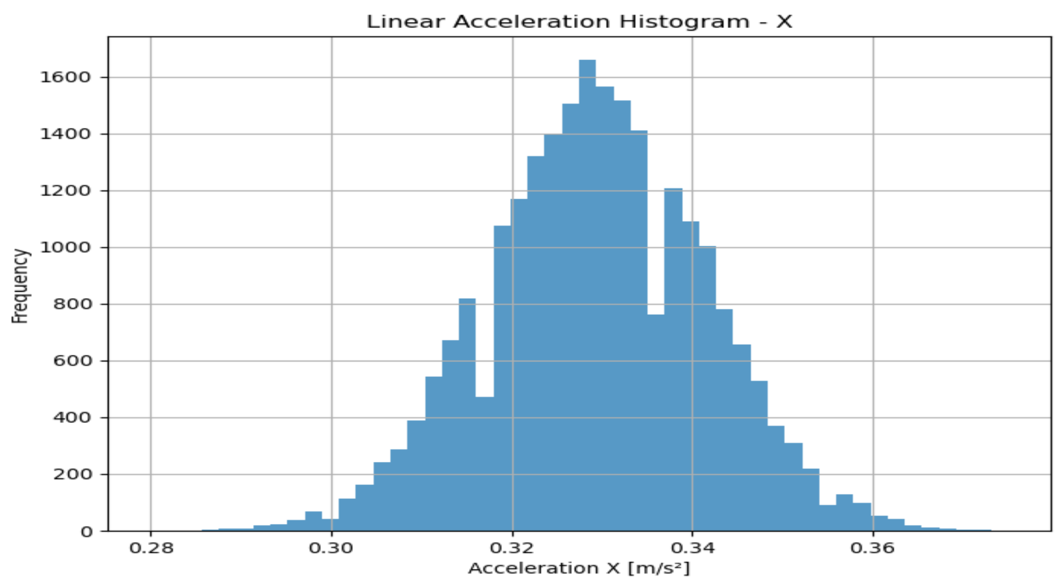
Appendix

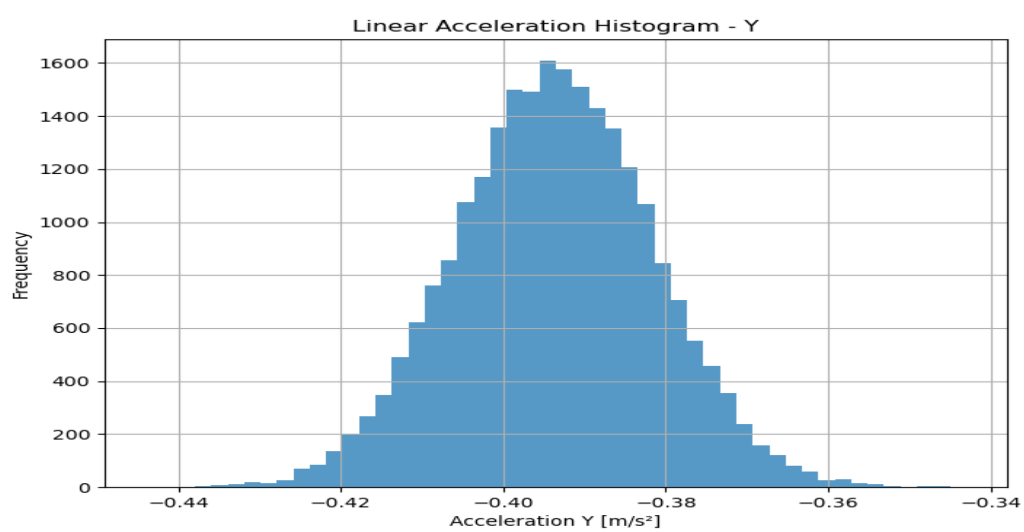
Please find attached all the time variance and histogram plots for reference.

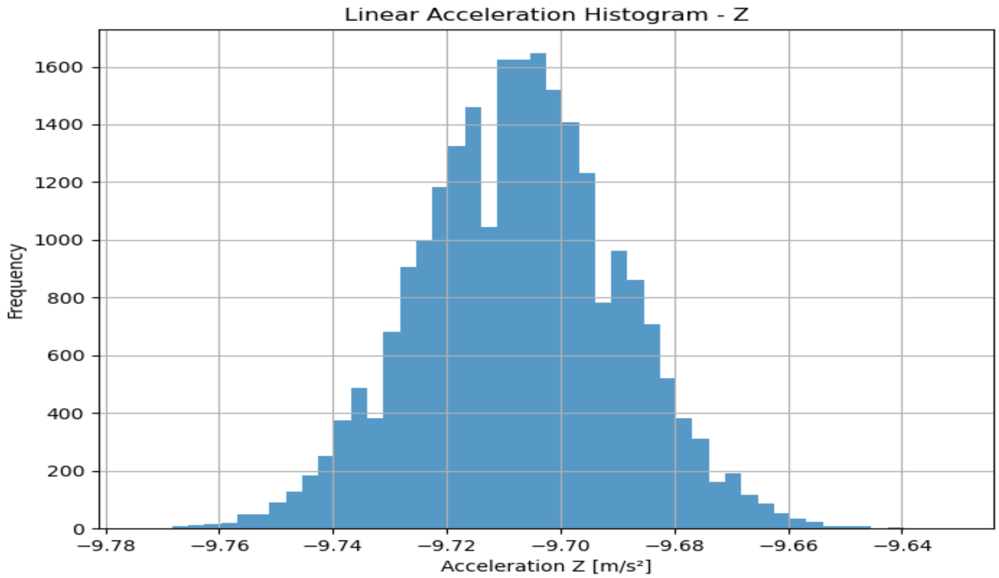


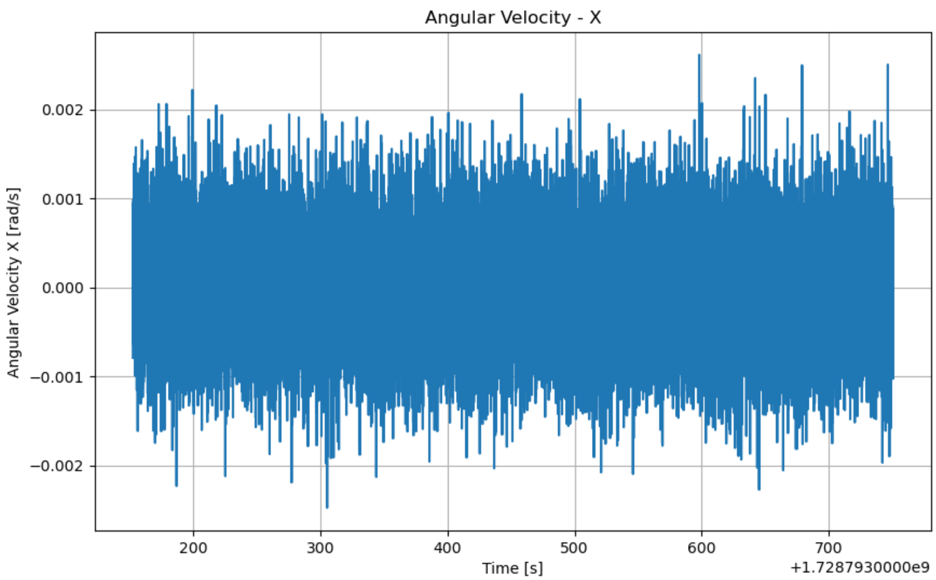


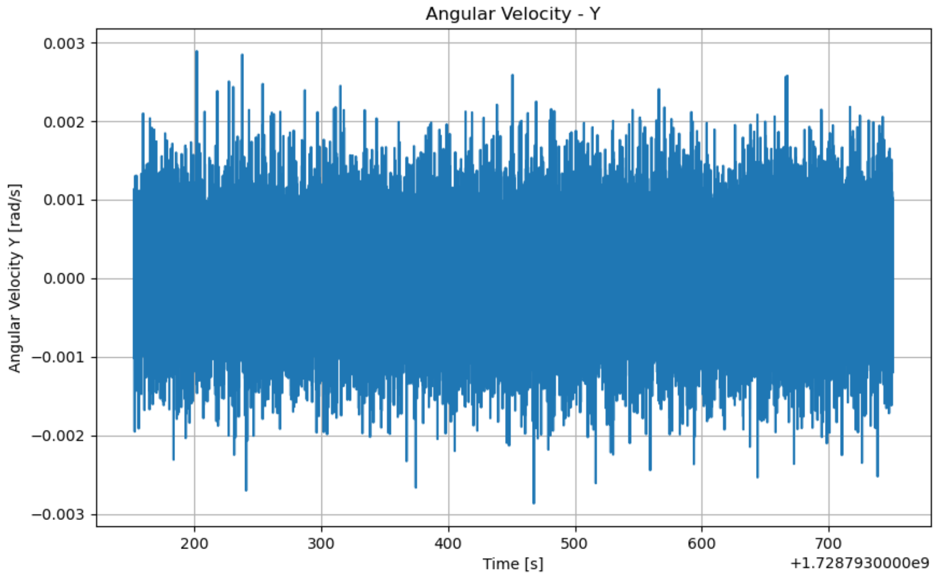

Linear Acceleration - Z
−9.64
−9.66
−9.68
−9.70
−9.72
−9.74
−9.76
−9.78
Acceleration Z [m/s²]
200
300
400
500
600
700
Time [s]
+1.7287930000e9


Linear Acceleration Histogram - X
0
200
400
600
800
1000
1200
1400
1600
Frequency
0.28
0.30
0.32
0.34
0.36
Acceleration X [m/s²]

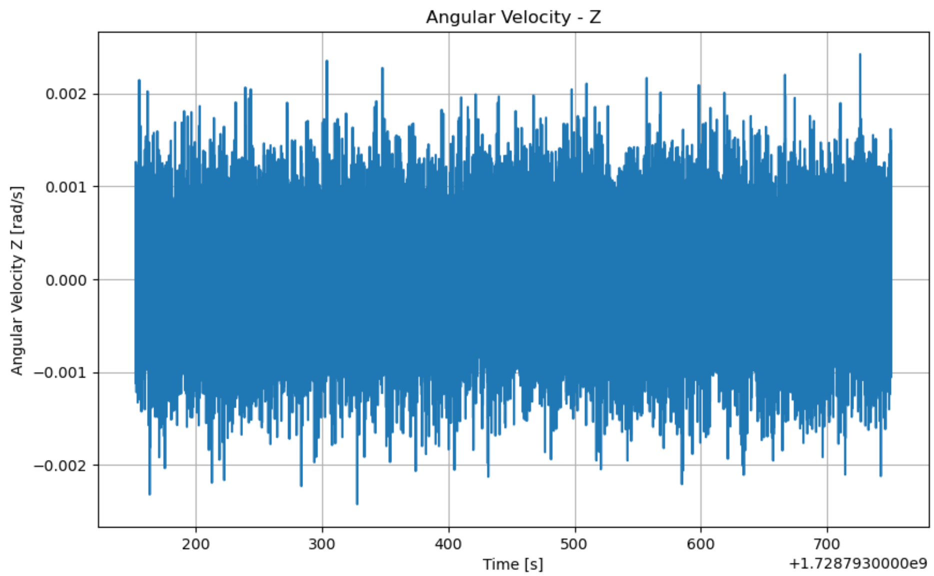

Linear Acceleration Histogram - Y
0
200
400
600
800
1000
1200
1400
1600
Frequency
−0.44
−0.42
−0.40
−0.38
−0.36
−0.34
Acceleration Y [m/s²]


Linear Acceleration Histogram - Z
Frequency
Acceleration Z [m/s²]
0
200
400
600
800
1000
1200
1400
1600
−9.78
−9.76
−9.74
−9.72
−9.70
−9.68
−9.66
−9.64

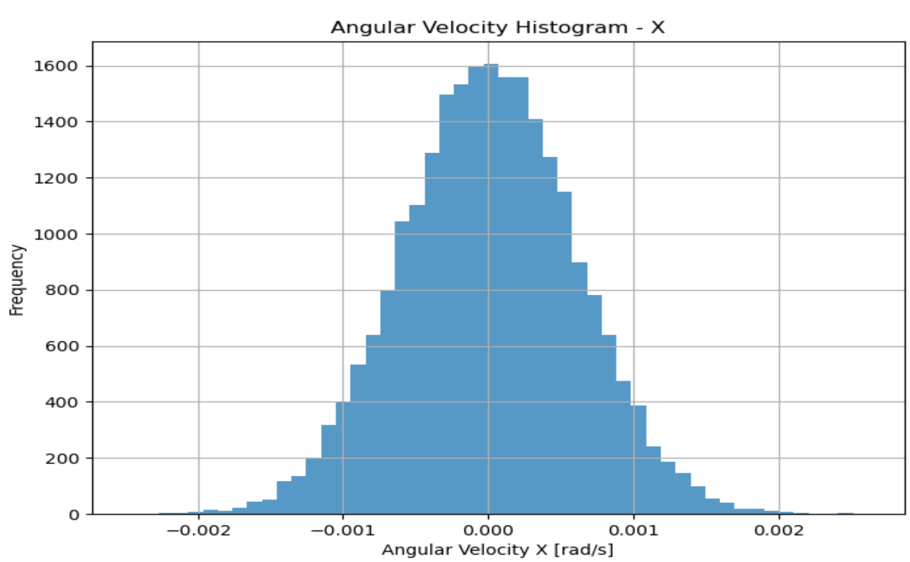

Angular Velocity - X
Angular Velocity X [rad/s]
Time [s]
0.002
0.001
0.000
−0.001
−0.002
200
300
400
500
600
700
+1.7287930000e9



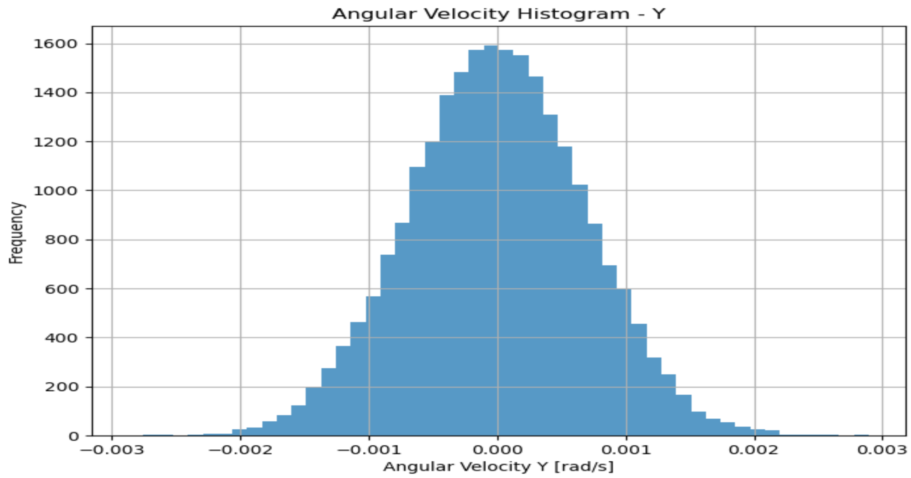
Angular Velocity - Y
Angular Velocity Y [rad/s]
Time [s]
+1.7287930000e9

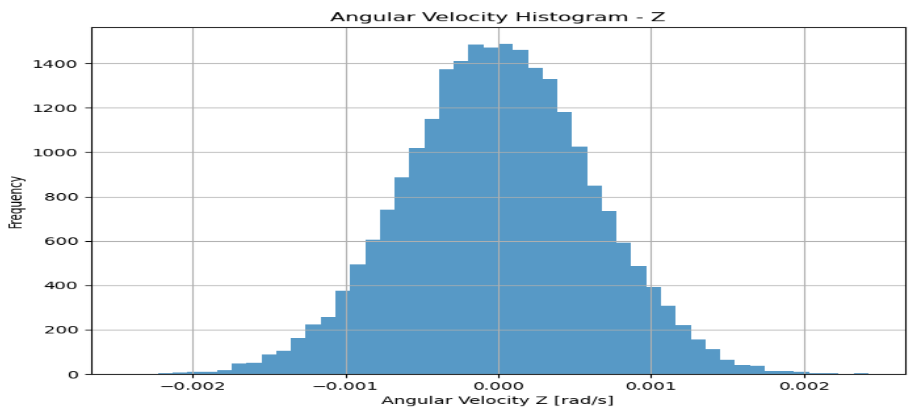


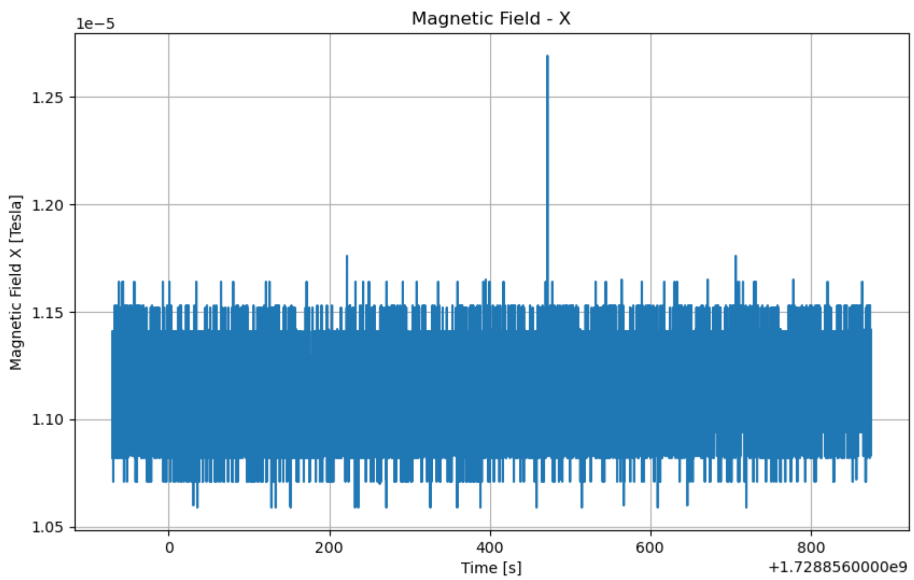
Angular Velocity - Z
Angular Velocity Z [rad/s]
Time [s]
+1.7287930000e9

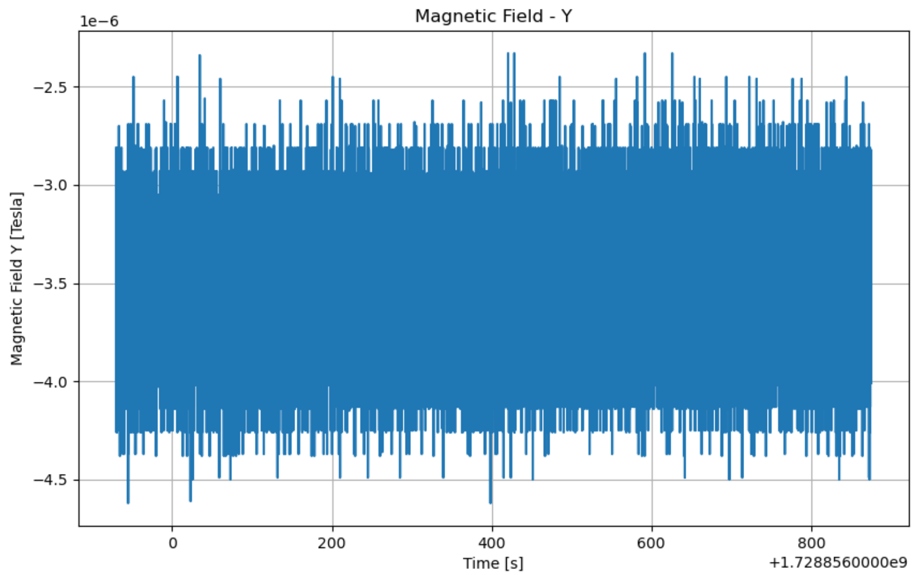


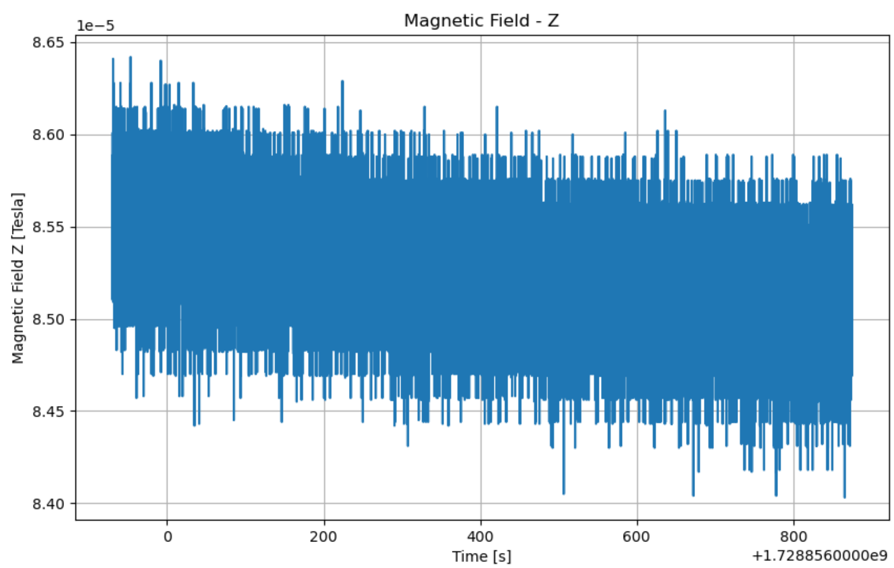
Angular Velocity Histogram - X
Frequency
Angular Velocity X [rad/s]

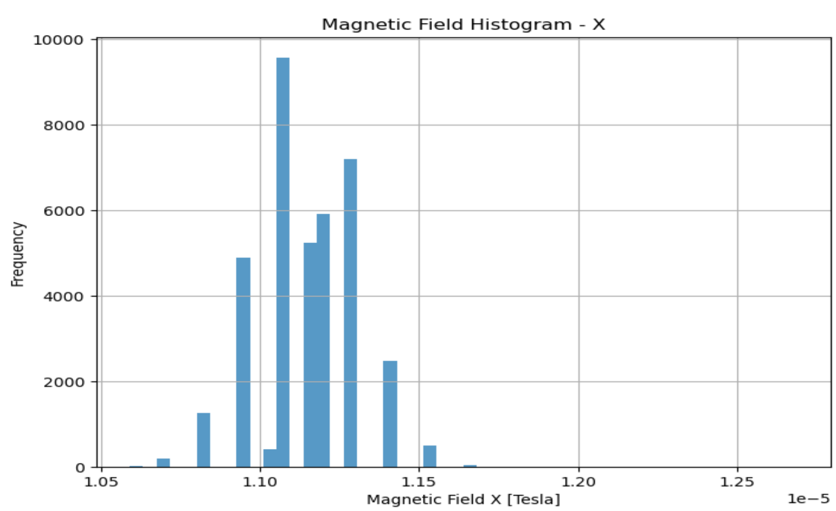

Angular Velocity Histogram - Y
Frequency
Angular Velocity Y [rad/s]
0
200
400
600
800
1000
1200
1400
1600
−0.003
−0.002
−0.001
0.000
0.001
0.002
0.003

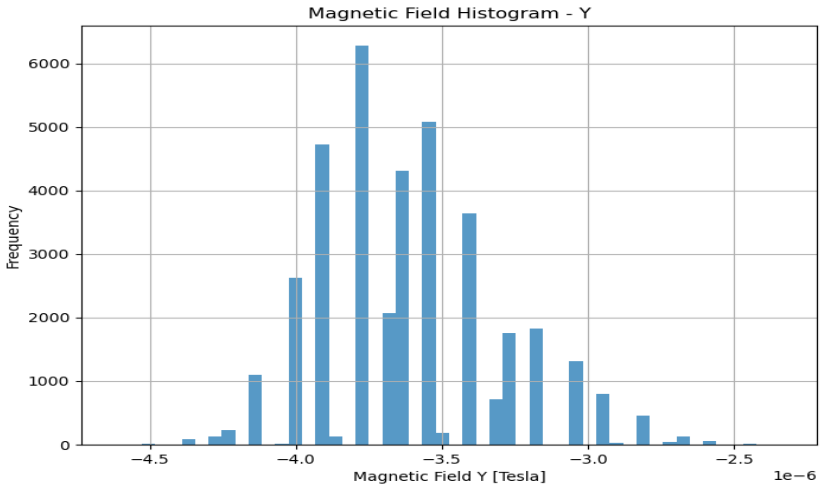

Angular Velocity Histogram - Z
Frequency
Angular Velocity Z [rad/s]
0
200
400
600
800
1000
1200
1400
−0.002
−0.001
0.000
0.001
0.002

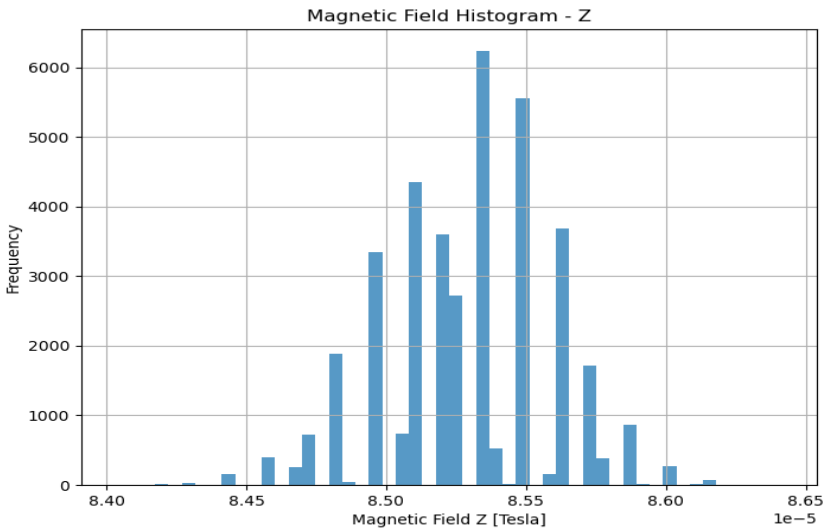

Magnetic Field - X
1e−5
Magnetic Field X [Tesla]
1.05
1.10
1.15
1.20
1.25
0
200
400
600
800
Time [s]
+1.7288560000e9

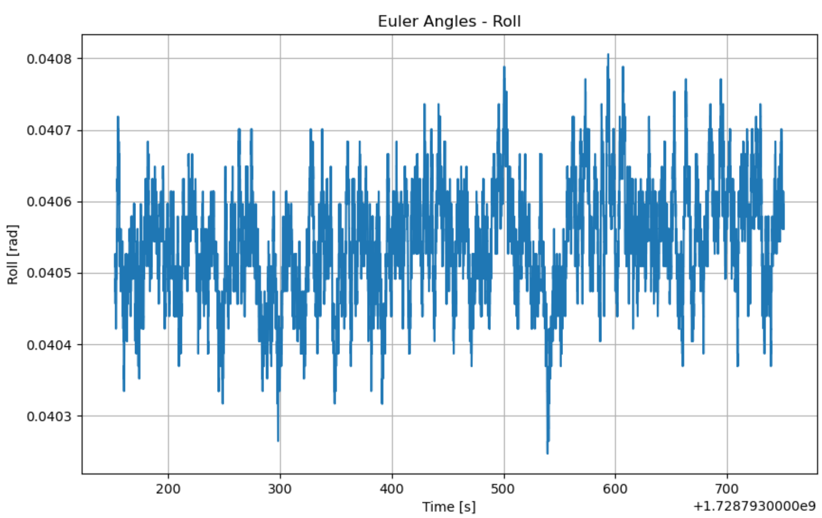

Magnetic Field - Y
1e−6
−2.5
−3.0
−3.5
−4.0
−4.5
Magnetic Field Y [Tesla]
0
200
400
600
800
Time [s]
+1.7288560000e9

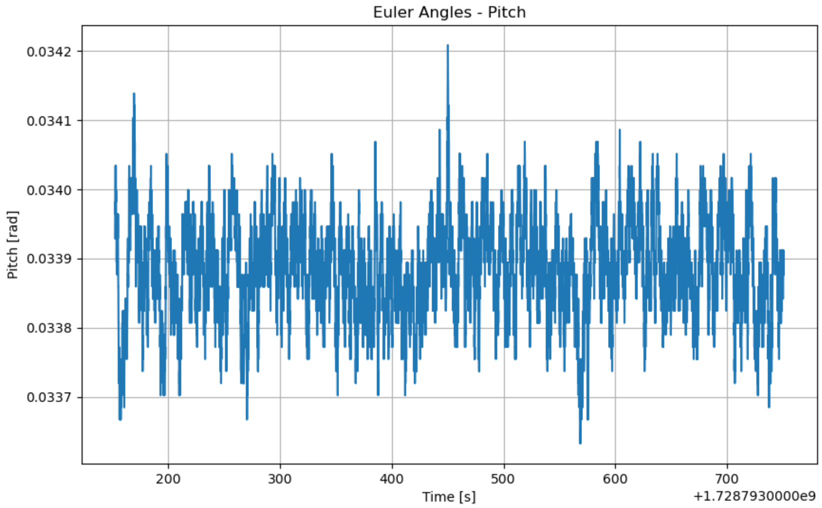

Magnetic Field - Z
1e−5
8.65
8.60
8.55
8.50
8.45
8.40
Magnetic Field Z [Tesla]
0
200
400
600
800
Time [s]
+1.7288560000e9

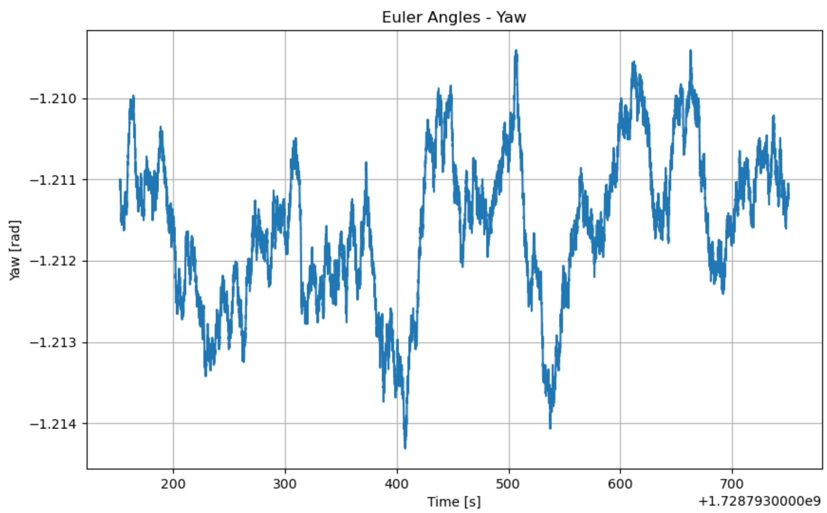

Magnetic Field Histogram - X
10000
8000
6000
4000
2000
0
Frequency
1.05
1.10
1.15
1.20
1.25
Magnetic Field X [Tesla]
1e−5

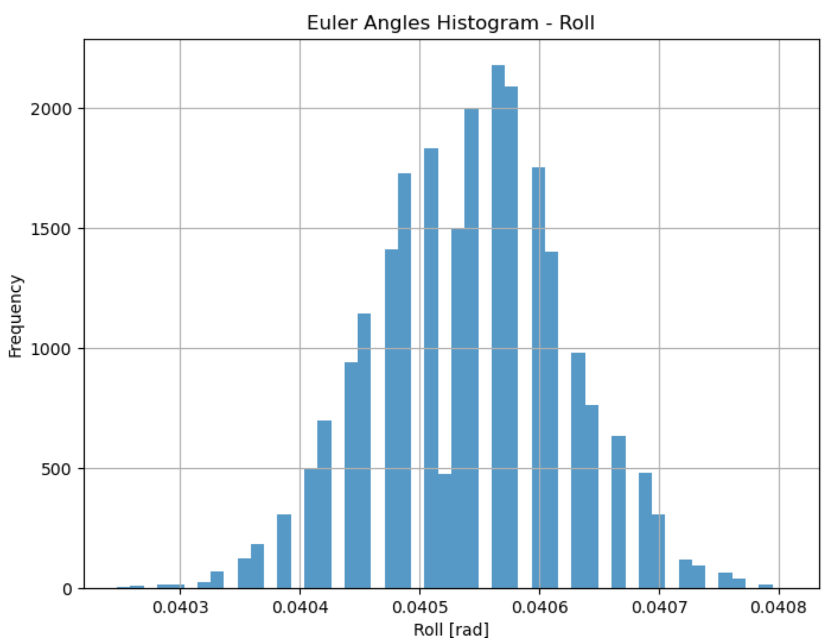

Magnetic Field Histogram - Y
Frequency
Magnetic Field Y [Tesla]
1e−6
−4.5
−4.0
−3.5
−3.0
−2.5
0
1000
2000
3000
4000
5000
6000

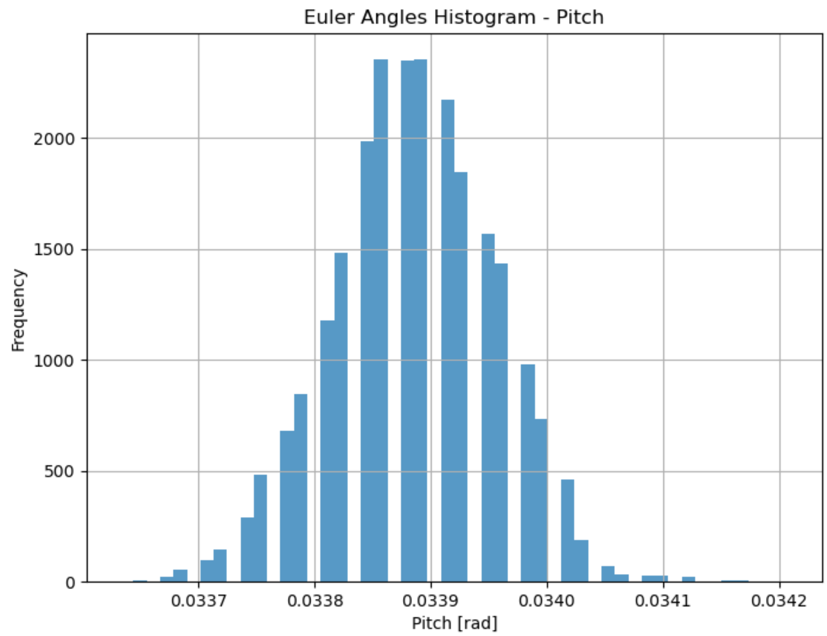

Magnetic Field Histogram - Z
Frequency
Magnetic Field Z [Tesla]
1e−5
8.40
8.45
8.50
8.55
8.60
8.65
0
1000
2000
3000
4000
5000
6000

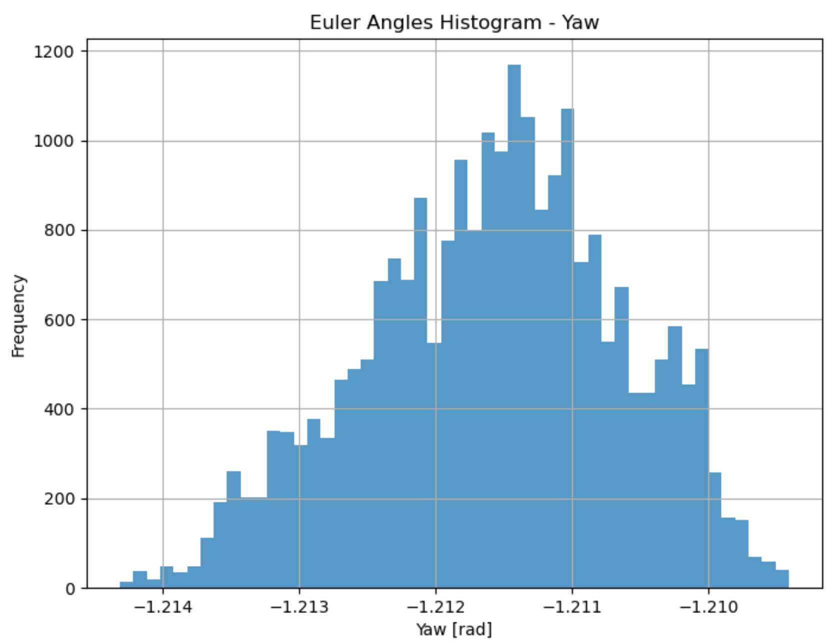

Euler Angles - Roll
Roll [rad]
Time [s]
+1.7287930000e9
200
300
400
500
600
700
0.0403
0.0404
0.0405
0.0406
0.0407
0.0408


Euler Angles - Pitch
0.3342
0.0341
0.0340
0.0339
0.0338
0.0337
Pitch [rad]
200
300
400
500
600
700
Time [s]
+1.7287930000e9


Euler Angles - Yaw
−1.210
−1.211
−1.212
−1.213
−1.214
Yaw [rad]
200
300
400
500
600
700
Time [s]
+1.7287930000e9


Euler Angles Histogram - Roll
2000
1500
1000
500
0
Frequency
0.0403
0.0404
0.0405
0.0406
0.0407
0.0408
Roll [rad]


Euler Angles Histogram - Pitch
Frequency
Pitch [rad]
0
500
1000
1500
2000
0.0337
0.0338
0.0339
0.0340
0.0341
0.0342


Euler Angles Histogram - Yaw
Frequency
Yaw [rad]
0
200
400
600
800
1000
1200
−1.214
−1.213
−1.212
−1.211
−1.210