Advanced Database Systems (ADBS), University of Edinburgh, 2023/24

Conjunctive Queries: Fast Evaluation

(Chapter 18 of DBT)

[DBT] Database Theory, https://github.com/pdm-book/community

Complexity of Query Evaluation

Theorem: CQ-Evaluation is NP-complete, and in PTIME in data complexity

Proof:

(NP-membership) Guess-and-check:

- Consider a database D, a CQ $Q(x_1,\dots,x_k)$:- body, and a tuple $(a_1,\dots,a_k)$ of values
- Guess a substitution h : terms(body) → terms(D)
- Verify that h is a match of Q in D, i.e., h(body) ⊆ D and $(h(x_1),\dots,h(x_k)) = (a_1,\dots,a_k)$

(NP-hardness) Reduction from 3-colorability

(in PTIME) For every substitution h : terms(body) → terms(D), check if h(body) ⊆ D and $(h(x_1),\dots,h(x_k)) = (a_1,\dots,a_k)$

Complexity of Query Evaluation

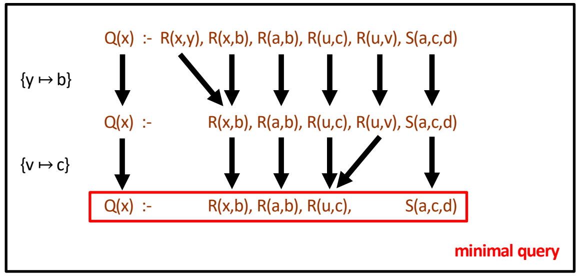
Theorem: CQ-Evaluation is NP-complete, and in PTIME in data complexity

Evaluating a CQ Q over a database D takes time $||D||^{O(||Q||)}$

Minimizing Conjunctive Queries

Database theory has developed principled methods for optimizing CQs:

- Find an equivalent CQ with minimal number of atoms (the core)
- Provides a notion of "true" optimality



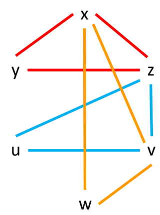
Minimizing Conjunctive Queries

- But, a minimal equivalent CQ might not be easier to evaluate - remains NP-hard
- “Good” classes of CQs for which query evaluation is tractable (*in combined complexity*):
  - Graph-based
  - Hypergraph-based

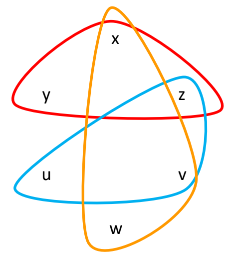
(Hyper)graph of Conjunctive Queries

Q :- R(x,y,z), R(z,u,v), R(v,w,x)

graph of Q - G(Q)



hypergraph of Q - H(Q)



“Good” Classes of Conjunctive Queries

- Graph-based
  - CQs of bounded treewidth - their graph has bounded treewidth

measures how close a graph is to a tree

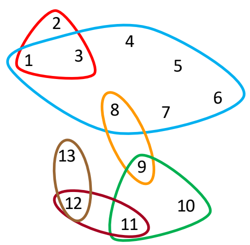
- Hypergraph-based:
  - CQs of bounded hypertree width - their hypergraph has bounded hypertree width
  - **Acyclic CQs - their hypergraph has hypertree width 1**

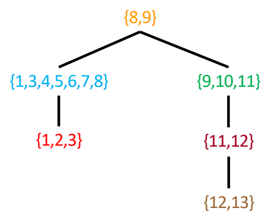
measures how close a hypergraph is to an acyclic one

Acyclic Hypergraphs

A join tree of a hypergraph **H** = (V,E) is a labeled tree **T** = (N,F,L), where L : N → E such that:

1. For each hyperedge e ∈ E of **H**, there exists n ∈ N such that e = L(n)
2. For each node u ∈ V of **H**, the set {n ∈ N | u ∈ L(n)} induces a connected subtree of **T**





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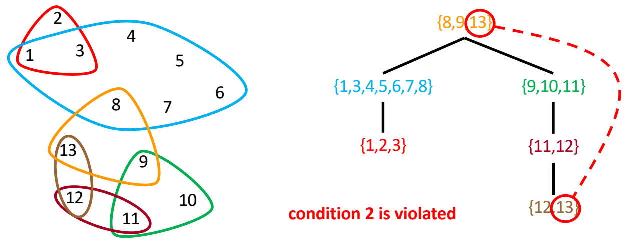

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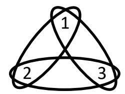
Definition: A hypergraph is acyclic if it has a join tree



prime example of a cyclic hypergraph

Acyclic Hypergraphs

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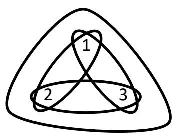


but this is acyclic

Relevant Algorithmic Tasks

ACYCLICITY
Input: a conjunctive query Q
Question: is Q acyclic? or is H(Q) acyclic?

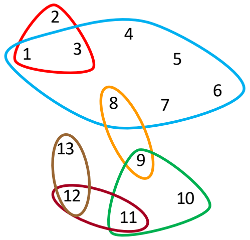
{Q ∈ **CQ** | H(Q) is acyclic}

ACQ-Evaluation
Input: a database D, an acyclic conjunctive query Q, and a tuple $(a_1,\ldots,a_k)$ of values
Question: $(a_1,\ldots,a_k) \in Q(D)$?

Checking Acyclicity

Via the GYO-reduction (Graham, Yu and Ozsoyoglu)

1. Eliminate nodes occurring in at most one hyperedge
2. Eliminate hyperedges that are empty or contained in other hyperedges



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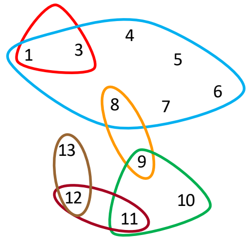

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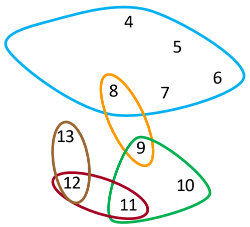

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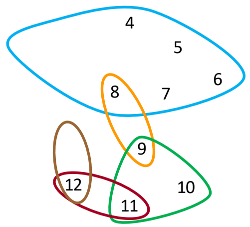

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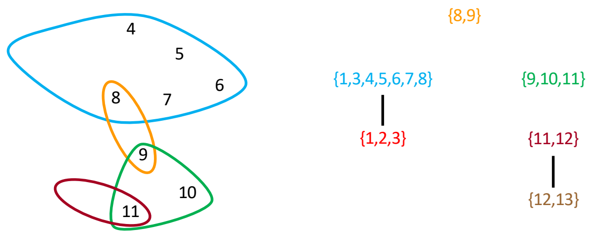

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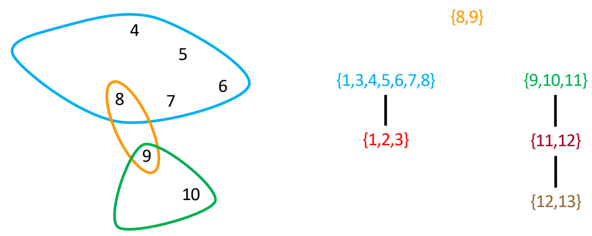

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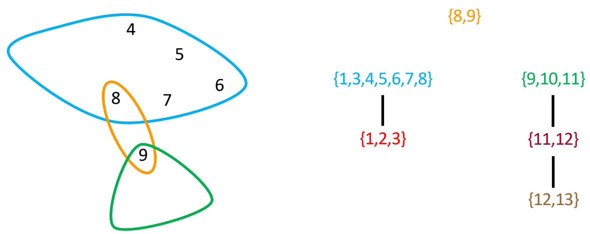

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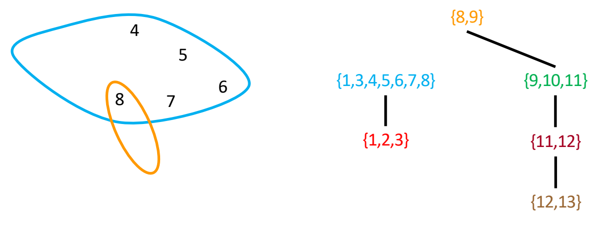

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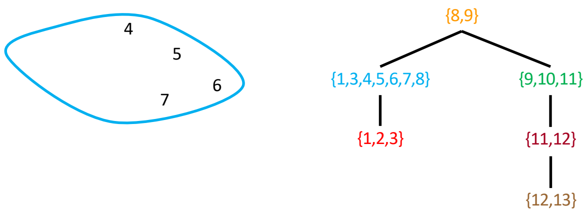

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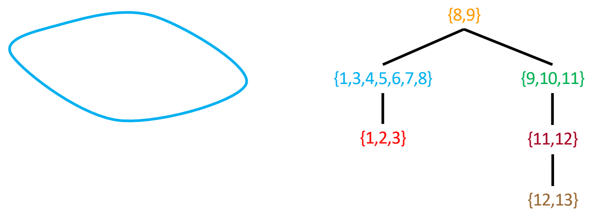

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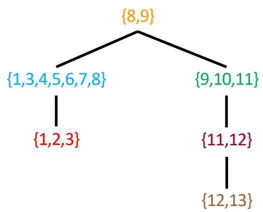
empty hypergraph



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Theorem: A hypergraph **H** is acyclic iff GYO(**H**) = Ø

$\Downarrow$

checking whether **H** is acyclic is feasible in polynomial time, and if it is the case, a join tree can be found in polynomial time

$\Downarrow$

Theorem: ACYCLICITY is in PTIME

Checking Acyclicity

Theorem: ACYCLICITY is in PTIME

NOTE: actually, we can check whether a CQ is acyclic in time O(||Q||)

linear time in the size Q

Evaluating Acyclic CQs

Theorem: ACQ-Evaluation is in PTIME

NOTE: actually, if H(Q) is acyclic, then Q can be evaluated in time O(||D|| · ||Q||)

linear time in the size of D and Q

Yannakaki’s Algorithm

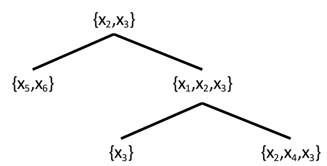
Dynamic programming algorithm over the join tree

Given a database D, and an acyclic Boolean CQ Q

1. Compute the join tree **T** of H(Q)
2. Assign to each node of **T** the corresponding relation of D
3. Compute semi-joins in a bottom up traversal of **T**
4. Return YES if the resulting relation at the root of **T** is non-empty; otherwise, return NO

Yannakaki’s Algorithm: Step 1

Q :- $R_1(x_1,x_2,x_3)$, $R_2(x_2,x_3)$, $R_2(x_5,x_6)$, $R_3(x_3)$, $R_4(x_2,x_4,x_3)$



Yannakaki’s Algorithm: Step 2

x2	x3
c1	b2
c1	b1
c4	b6

x5	x6
c1	b2
c1	b1
c4	b6

x1	x2	x3
s1	c1	b1
s1	c1	b2
s3	c3	b1
s3	c1	b4
s2	c2	b3

x3
b1
b2

x2	x4	x3
c1	a1	b1
c1	a1	b2
c1	a2	b2

Yannakaki’s Algorithm: Step 3

x2	x3
c1	b2
c1	b1
c4	b6

x5	x6
c1	b2
c1	b1
c4	b6

x1	x2	x3
s1	c1	b1
s1	c1	b2
s3	c3	b1
~~s3~~	~~c1~~	~~b4~~
s2	c2	b3

x3
b1
b2

x2	x4	x3
c1	a1	b1
c1	a1	b2
c1	a2	b2

Yannakaki’s Algorithm: Step 3

x2	x3
c1	b2
c1	b1
c4	b6

x5	x6
c1	b2
c1	b1
c4	b6

x1	x2	x3
s1	c1	b1
s1	c1	b2
s3	c3	b1
~~s3~~	~~c1~~	~~b4~~
~~s2~~	~~c2~~	~~b3~~

x3
b1
b2

x2	x4	x3
c1	a1	b1
c1	a1	b2
c1	a2	b2

Yannakaki’s Algorithm: Step 3

x2	x3
c1	b2
c1	b1
c4	b6

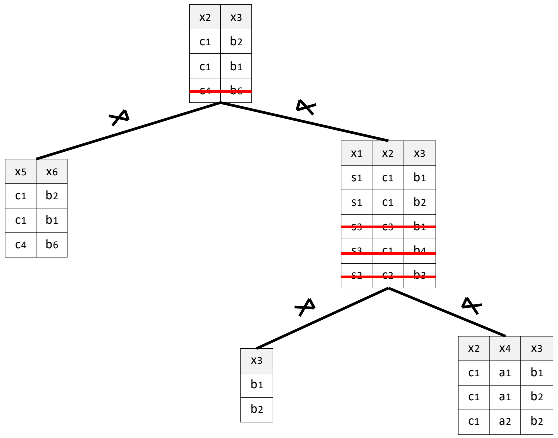
x5	x6
c1	b2
c1	b1
c4	b6

x1	x2	x3
s1	c1	b1
s1	c1	b2
~~s3~~	~~c3~~	~~b1~~
~~s3~~	~~c1~~	~~b4~~
~~s2~~	~~c2~~	~~b3~~

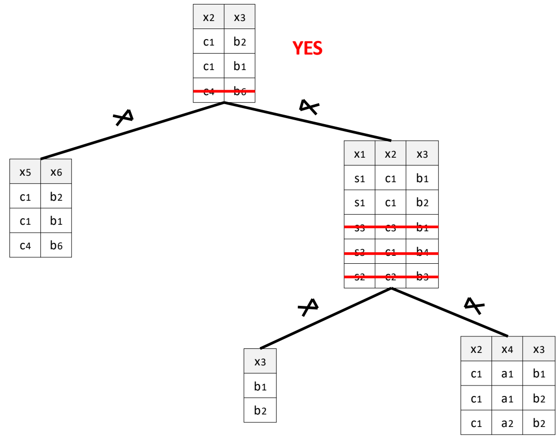
x3
b1
b2

x2	x4	x3
c1	a1	b1
c1	a1	b2
c1	a2	b2

Yannakaki's Algorithm: Step 3



Yannakaki's Algorithm: Step 4



Recap

- "Good" classes of CQs for which query evaluation is tractable - conditions based on the graph or hypergraph of the CQ
- Acyclic CQs - their hypergraph is acyclic, can be checked in linear time
- Evaluating acyclic CQs is feasible in linear time (Yannakaki's algorithm)