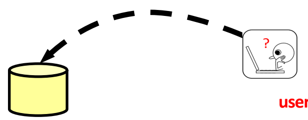
understand how a DBMS efficiently evaluates a query over a database

(the main focus of this course)



understand the mathematical foundations of query evaluation

(next two weeks)

A Quick Recap

Flight	***origin***	***destination***	***airline***

Airport	***code***	***city***

A Quick Recap

List all the airlines

Flight	***origin***	***destination***	***airline***
	VIE	LHR	BA
	LHR	EDI	BA
	LGW	GLA	U2
	LCA	VIE	OS

Airport	***code***	***city***
	VIE	Vienna
	LHR	London
	LGW	London
	LCA	Larnaca
	GLA	Glasgow
	EDI	Edinburgh

A Quick Recap

List all the airlines

Flight	***origin***	***destination***	***airline***
	VIE	LHR	$\boxed{\text{BA}}$
	LHR	EDI	BA
	LGW	GLA	U2
	LCA	VIE	OS

$\Downarrow$

{BA, U2, OS}

Airport	***code***	***city***
	VIE	Vienna
	LHR	London
	LGW	London
	LCA	Larnaca
	GLA	Glasgow
	EDI	Edinburgh

π_{airline} Flight

A Quick Recap

List the codes of the airports in London

Flight	origin	destination	airline
	VIE	LHR	BA
	LHR	EDI	BA
	LGW	GLA	U2
	LCA	VIE	OS

Airport	code	city
	VIE	Vienna
	LHR	London
	LGW	London
	LCA	Larnaca
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A Quick Recap

List the codes of the airports in London

Flight	origin	destination	airline
	VIE	LHR	BA
	LHR	EDI	BA
	LGW	GLA	U2
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Airport	code	city
	VIE	Vienna
	LHR	London
	LGW	London
	LCA	Larnaca
	GLA	Glasgow
	EDI	Edinburgh

$\Downarrow$

{LHR, LGW}

$\pi_{code}\ (\sigma_{city='London'}\ \text{Airport})$

A Quick Recap

List the airlines that fly directly from London to Glasgow

Flight	origin	destination	airline
	VIE	LHR	BA
	LHR	EDI	BA
	LGW	GLA	U2
	LCA	VIE	OS

Airport	code	city
	VIE	Vienna
	LHR	London
	LGW	London
	LCA	Larnaca
	GLA	Glasgow
	EDI	Edinburgh

$\pi_{airline}\ ((\text{Flight}\ \bowtie_{origin=code}\ (\sigma_{city='London'}\ \text{Airport}))\ \bowtie_{destination=code}\ (\sigma_{city='Glasgow'}\ \text{Airport}))$

A Quick Recap

$\pi_{airline}\ ((\text{Flight}\ \bowtie_{origin=code}\ \underbrace{(\sigma_{city='London'}\ \text{Airport})})\ \bowtie_{destination=code}\ \underbrace{(\sigma_{city='Glasgow'}\ \text{Airport})})$

code	city
LHR	London
LGW	London

code	city
GLA	Glasgow

A Quick Recap

$\pi_{airline}\ ((Flight\ \bowtie_{origin=code}\ (\sigma_{city='London'}\ Airport))\ \bowtie_{destination=code}\ (\sigma_{city='Glasgow'}\ Airport))$

code	city
LHR	London
LGW	London

code	city
GLA	Glasgow

origin	destination	airline	code	city
LHR	EDI	BA	LHR	London
LGW	GLA	U2	LGW	London

A Quick Recap

$\pi_{airline}\ ((Flight\ \bowtie_{origin=code}\ (\sigma_{city='London'}\ Airport))\ \bowtie_{destination=code}\ (\sigma_{city='Glasgow'}\ Airport))$

code	city
LHR	London
LGW	London

code	city
GLA	Glasgow

origin	destination	airline	code	city
LHR	EDI	BA	LHR	London
LGW	GLA	U2	LGW	London

origin	destination	airline	code	city	code	city
LGW	GLA	U2	LGW	London	GLA	Glasgow

A Quick Recap

$\pi_{airline}\ ((Flight\ \bowtie_{origin=code}\ (\sigma_{city='London'}\ Airport))\ \bowtie_{destination=code}\ (\sigma_{city='Glasgow'}\ Airport))$

code	city
LHR	London
LGW	London

code	city
GLA	Glasgow

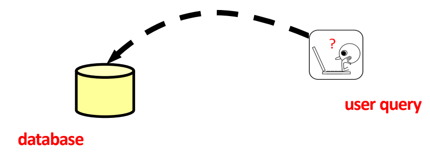
origin	destination	airline	code	city
LHR	EDI	BA	LHR	London
LGW	GLA	U2	LGW	London

origin	destination	airline	code	city	code	city
LGW	GLA	U2	LGW	London	GLA	Glasgow

airline
U2

understand how a DBMS efficiently evaluates a query over a database

(the main focus of this course)



understand the mathematical foundations of query evaluation

(next two weeks)

Advanced Database Systems (ADBS), University of Edinburgh, 2023/24

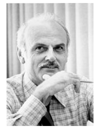
Conjunctive Queries: Syntax and Semantics

(Chapters 12 and 13 of DBT)

[DBT] Database Theory, https://github.com/pdm-book/community

Codd's Theorem

Relational Algebra = Relational Calculus



Edgar F. Codd
(1923 - 2003)
Turing Award 1981

- Queries are written using a declarative language
- DBMS converts declarative queries into procedural queries that are optimized and executed

Relational Calculus

List all the airlines

Flight	*origin*	*destination*	*airline*
	VIE	LHR	BA
	LHR	EDI	BA
	LGW	GLA	U2
	LCA	VIE	OS

Airport	*code*	*city*
	VIE	Vienna
	LHR	London
	LGW	London
	LCA	Larnaca
	GLA	Glasgow
	EDI	Edinburgh

⇩

{BA, U2, OS}

$\{z \mid \exists x \exists y\, (\text{Flight}(x,y,z))\}$

Relational Calculus

List the codes of the airports in London

Flight	*origin*	*destination*	*airline*
	VIE	LHR	BA
	LHR	EDI	BA
	LGW	GLA	U2
	LCA	VIE	OS

Airport	*code*	*city*
	VIE	Vienna
	LHR	London
	LGW	London
	LCA	Larnaca
	GLA	Glasgow
	EDI	Edinburgh

⇩

{LHR, LGW}

$\{x \mid \text{Airport}(x,\text{London})\}$

Relational Calculus

List the airlines that fly directly from London to Glasgow

Flight	*origin*	*destination*	*airline*
	VIE	LHR	BA
	LHR	EDI	BA
	LGW	GLA	U2
	LCA	VIE	OS

⇩

{U2}

Airport	*code*	*city*
	VIE	Vienna
	LHR	London
	LGW	London
	LCA	Larnaca
	GLA	Glasgow
	EDI	Edinburgh

$\{z \mid \exists x \exists y\, (\text{Airport}(x,\text{London}) \land \text{Airport}(y,\text{Glasgow}) \land \text{Flight}(x,y,z))\}$

A Core Relational Query Language

Conjunctive Queries (CQ)

= $\{\sigma,\pi,\bowtie\}$-fragment of relational algebra

= relational calculus without $\neg, \forall, \lor, =$

= simple SELECT-FROM-WHERE SQL queries (only AND and equality in the WHERE clause)

Syntax of Conjunctive Queries

$$Q(\mathbf{x}) := \exists \mathbf{y}\, (R_1(\mathbf{v_1}) \land \cdots \land R_m(\mathbf{v_m}))$$

- $R_1,\ldots,R_m$ are relation names
- $\mathbf{x}, \mathbf{y}, \mathbf{v_1},\ldots,\mathbf{v_m}$ are tuples of variables
- each variable mentioned in $\mathbf{v_i}$ appears either in $\mathbf{x}$ or $\mathbf{y}$
- the variables in $\mathbf{x}$ are free called distinguished or output variables

It is very convenient to see conjunctive queries as rule-based queries of the form

$$Q(\mathbf{x}) \;\text{:-}\; \underbrace{R_1(\mathbf{v_1}),\ldots,R_m(\mathbf{v_m})}$$

this is called the body of Q that can be seen as a set of atoms

Conjunctive Queries: Example 1

List all the airlines

Flight	*origin*	*destination*	*airline*
	VIE	LHR	BA
	LHR	EDI	BA
	LGW	GLA	U2
	LCA	VIE	OS

⇩

{BA, U2, OS}

Airport	*code*	*city*
	VIE	Vienna
	LHR	London
	LGW	London
	LCA	Larnaca
	GLA	Glasgow
	EDI	Edinburgh

π_{airline} Flight

$\{z \mid \exists x \exists y\, \text{Flight}(x,y,z)\}$

Q(z) :- Flight(x,y,z)

Conjunctive Queries: Example 2

List the codes of the airports in London

Flight	*origin*	*destination*	*airline*
	VIE	LHR	BA
	LHR	EDI	BA
	LGW	GLA	U2
	LCA	VIE	OS

Airport	*code*	*city*
	VIE	Vienna
	LHR	London
	LGW	London
	LCA	Larnaca
	GLA	Glasgow
	EDI	Edinburgh

⇩

{LHR, LGW}

$\pi_{code}\ (\sigma_{city='London'}\ \text{Airport})$

{x | Airport(x,London)}

Q(x) :- Airport(x,London)

Conjunctive Queries: Example 3

List the airlines that fly directly from London to Glasgow

Flight	*origin*	*destination*	*airline*
	VIE	LHR	BA
	LHR	EDI	BA
	LGW	GLA	U2
	LCA	VIE	OS

⇩

{U2}

Airport	*code*	*city*
	VIE	Vienna
	LHR	London
	LGW	London
	LCA	Larnaca
	GLA	Glasgow
	EDI	Edinburgh

$\pi_{airline}\ ((\text{Flight} \bowtie_{origin=code} (\sigma_{city='London'}\ \text{Airport})) \bowtie_{destination=code} (\sigma_{city='Glasgow'}\ \text{Airport}))$

{z | ∃x∃y (Airport(x,London) ∧ Airport(y,Glasgow) ∧ Flight(x,y,z))}

Conjunctive Queries: Example 3

List the airlines that fly directly from London to Glasgow

Flight	*origin*	*destination*	*airline*
	VIE	LHR	BA
	LHR	EDI	BA
	LGW	GLA	U2
	LCA	VIE	OS

⇩

{U2}

Airport	*code*	*city*
	VIE	Vienna
	LHR	London
	LGW	London
	LCA	Larnaca
	GLA	Glasgow
	EDI	Edinburgh

Q(z) :- Airport(x,London), Airport(y,Glasgow), Flight(x,y,z)

Pattern Matching Problem

List the airlines that fly directly from London to Glasgow

{
Flight(VIE,LHR,BA),
Flight(LHR,EDI,BA),
Flight(LGW,GLA,U2),
Flight(LCA,VIE,OS),
Airport(VIE,Vienna),
Airport(LHR,London),
Airport(LGW,London),
Airport(LCA,Larnaca),
Airport(GLA,Glasgow),
Airport(EDI,Edinburgh)
}

Q(z) :- Airport(x,London), Airport(y,Glasgow), Flight(x,y,z)

Pattern Matching Problem

List the airlines that fly directly from London to Glasgow

{
Flight(VIE,LHR,BA),
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Airport(VIE,Vienna),
Airport(LHR,London),
Airport(LGW,London),
Airport(LCA,Larnaca),
Airport(GLA,Glasgow),
Airport(EDI,Edinburgh)
}

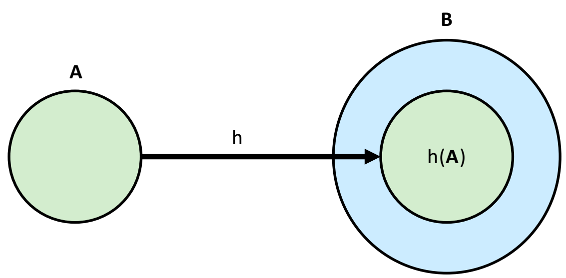
Q(z) :- Airport(x,London), Airport(y,Glasgow), Flight(x,y,z)

Homomorphism

- Pattern matching - properly formalized via the key notion of homomorphism
- A substitution from a set of terms **S** to a set of terms **T** is a function h : **S** → **T**, i.e., h is a set of mappings of the form s ↦ t, where s ∈ **S** and t ∈ **T**
- A homomorphism from a set of atoms **A** to a set of atoms **B** is a substitution h : terms(**A**) → terms(**B**) such that:
    1. t is a constant value ⇒ h(t) = t
    2. $R(t_1,\ldots,t_k) \in \mathbf{A} \Rightarrow h(R(t_1,\ldots,t_k)) = R(h(t_1),\ldots,h(t_k)) \in \mathbf{B}$

(terms(**A**) = {t | t is a variable or a constant value that occurs in **A**})

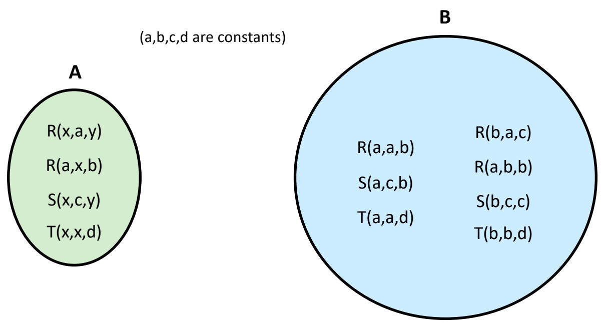
Homomorphism



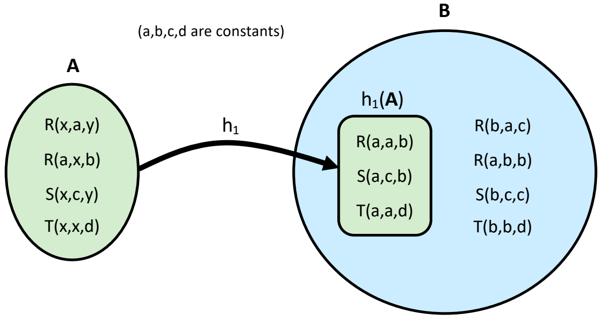
h : terms(**A**) → terms(**B**) that is the identity on constants

Homomorphism

(a,b,c,d are constants)

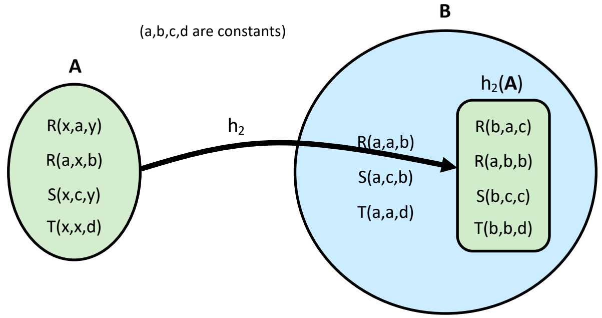


Homomorphism



$h_1 = \{a \mapsto a, b \mapsto b, c \mapsto c, d \mapsto d, x \mapsto a, y \mapsto b\}$

Homomorphism



$h_2 = \{a \mapsto a, b \mapsto b, c \mapsto c, d \mapsto d, x \mapsto b, y \mapsto c\}$

Find the Homomorphisms

$\mathbf{S_1} = \{P(x_1,y_1), P(y_1,z_1), P(z_1,w_1)\}$

$\mathbf{S_2} = \{P(x_2,y_2), P(y_2,z_2), P(z_2,x_2)\}$

$\mathbf{S_3} = \{P(x_3,y_3), P(y_3,x_3)\}$

$\mathbf{S_4} = \{P(x_4,y_4), P(y_4,x_4), P(y_4,y_4)\}$

$\mathbf{S_5} = \{P(x_5,x_5)\}$

Find the Homomorphisms

$\mathbf{S_1} = \{P(x_1,y_1), P(y_1,z_1), P(z_1,w_1)\}$

$\{x1 \mapsto x2, y1 \mapsto y2, z1 \mapsto z2, w1 \mapsto x2\}$

$\{x1 \mapsto x3, y1 \mapsto y3, z1 \mapsto x3, w1 \mapsto y3\}$

$\mathbf{S_2} = \{P(x_2,y_2), P(y_2,z_2), P(z_2,x_2)\}$

$\mathbf{S_3} = \{P(x_3,y_3), P(y_3,x_3)\}$

$\mathbf{S_4} = \{P(x_4,y_4), P(y_4,x_4), P(y_4,y_4)\}$

$\mathbf{S_5} = \{P(x_5,x_5)\}$

Find the Homomorphisms

$$S_1 = \{P(x_1,y_1), P(y_1,z_1), P(z_1,w_1)\}$$
$$S_2 = \{P(x_2,y_2), P(y_2,z_2), P(z_2,x_2)\} \qquad S_3 = \{P(x_3,y_3), P(y_3,x_3)\}$$
$$S_4 = \{P(x_4,y_4), P(y_4,x_4), P(y_4,y_4)\}$$
$$S_5 = \{P(x_5,x_5)\}$$

$$\begin{tikzcd}
 & S_1 \arrow[dl, "\{x1 \mapsto x2,\ y1 \mapsto y2,\ z1 \mapsto z2,\ w1 \mapsto x2\}"'] \arrow[dr, "\{x1 \mapsto x3,\ y1 \mapsto y3,\ z1 \mapsto x3,\ w1 \mapsto y3\}"] & \\
S_2 \arrow[dr, "\{x2 \mapsto y4,\ y2 \mapsto x4,\ z2 \mapsto y4\}"'] & & S_3 \arrow[dl, "\{x3 \mapsto x4,\ y3 \mapsto y4\}"] \\
 & S_4 & \\
 & S_5 &
\end{tikzcd}$$

Find the Homomorphisms

$$S_1 = \{P(x_1,y_1), P(y_1,z_1), P(z_1,w_1)\}$$
$$S_2 = \{P(x_2,y_2), P(y_2,z_2), P(z_2,x_2)\} \qquad S_3 = \{P(x_3,y_3), P(y_3,x_3)\}$$
$$S_4 = \{P(x_4,y_4), P(y_4,x_4), P(y_4,y_4)\}$$
$$S_5 = \{P(x_5,x_5)\}$$

$$\begin{tikzcd}
 & S_1 \arrow[dl, "\{x1 \mapsto x2,\ y1 \mapsto y2,\ z1 \mapsto z2,\ w1 \mapsto x2\}"'] \arrow[dr, "\{x1 \mapsto x3,\ y1 \mapsto y3,\ z1 \mapsto x3,\ w1 \mapsto y3\}"] & \\
S_2 \arrow[dr, "\{x2 \mapsto y4,\ y2 \mapsto x4,\ z2 \mapsto y4\}"'] & & S_3 \arrow[dl, "\{x3 \mapsto x4,\ y3 \mapsto y4\}"] \\
 & S_4 \arrow[d, "\{x4 \mapsto x5,\ y4 \mapsto x5\}"'] & \\
 & S_5 &
\end{tikzcd}$$

Find the Homomorphisms

$$S_1 = \{P(x_1,y_1), P(y_1,z_1), P(z_1,w_1)\}$$
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$$S_5 = \{P(x_5,x_5)\}$$

$$\begin{tikzcd}
 & S_1 \arrow[dl, "\{x1 \mapsto x2,\ y1 \mapsto y2,\ z1 \mapsto z2,\ w1 \mapsto x2\}"'] \arrow[dr, "\{x1 \mapsto x3,\ y1 \mapsto y3,\ z1 \mapsto x3,\ w1 \mapsto y3\}"] & \\
S_2 \arrow[dr, "\{x2 \mapsto y4,\ y2 \mapsto x4,\ z2 \mapsto y4\}"'] & & S_3 \arrow[dl, "\{x3 \mapsto x4,\ y3 \mapsto y4\}"] \\
 & S_4 \arrow[d, shift right, "\{x4 \mapsto x5,\ y4 \mapsto x5\}"'] & \\
 & S_5 \arrow[u, shift right, "\{x5 \mapsto y4\}"'] &
\end{tikzcd}$$

Homomorphisms Compose

$$S_1 = \{P(x_1,y_1), P(y_1,z_1), P(z_1,w_1)\}$$
$$S_2 = \{P(x_2,y_2), P(y_2,z_2), P(z_2,x_2)\}$$
$$S_4 = \{P(x_4,y_4), P(y_4,x_4), P(y_4,y_4)\}$$

$$\begin{tikzcd}
 & S_1 \arrow[dl, "\{x1 \mapsto x2,\ y1 \mapsto y2,\ z1 \mapsto z2,\ w1 \mapsto x2\}"'] \arrow[dd, "\{x1 \mapsto y4,\ y1 \mapsto x4,\ z1 \mapsto y4,\ w1 \mapsto y4\}"] \\
S_2 \arrow[dr, "\{x2 \mapsto y4,\ y2 \mapsto x4,\ z2 \mapsto y4\}"'] & \\
 & S_4
\end{tikzcd}$$

Homomorphisms Compose

$$\begin{tikzcd}
\mathbf{S_1} = \{P(x_1,y_1), P(y_1,z_1), P(z_1,w_1)\} \arrow[d, "\{x1 \mapsto x4,\ y1 \mapsto y4,\ z1 \mapsto x4,\ w1 \mapsto y4\}"'] \arrow[dr, "\{x1 \mapsto x3,\ y1 \mapsto y3,\ z1 \mapsto x3,\ w1 \mapsto y3\}"] & \\
 & \mathbf{S_3} = \{P(x_3,y_3), P(y_3,x_3)\} \arrow[dl, "\{x3 \mapsto x4,\ y3 \mapsto y4\}"] \\
\mathbf{S_4} = \{P(x_4,y_4), P(y_4,x_4), P(y_4,y_4)\} &
\end{tikzcd}$$

Semantics of Conjunctive Queries

- A match of a conjunctive query $Q(x_1,\ldots,x_k)$:- body in a database D is a homomorphism h from the set of atoms body to the set of atoms D
- The answer to $Q(x_1,\ldots,x_k)$:- body over D is the set of k-tuples
  $$Q(D) := \{(h(x_1),\ldots,h(x_k)) \mid h \text{ is a match of } Q \text{ in } D\}$$
- The answer consists of the witnesses for the distinguished variables of Q

Pattern Matching Problem

List the airlines that fly directly from London to Glasgow

	Airport(VIE,Vienna),
Flight(VIE,LHR,BA),	Airport(LHR,London),
Flight(LHR,EDI,BA),	Airport(LGW,London),
Flight(LGW,GLA,U2),	Airport(LCA,Larnaca),
Flight(LCA,VIE,OS),	Airport(GLA,Glasgow),
	Airport(EDI,Edinburgh)

Q(z) :- Airport(x,London), Airport(y,Glasgow), Flight(x,y,z)

Pattern Matching Problem

List the airlines that fly directly from London to Glasgow

	Airport(VIE,Vienna),
Flight(VIE,LHR,BA),	Airport(LHR,London),
Flight(LHR,EDI,BA),	**Airport(LGW,London)**,
Flight(LGW,GLA,U2),	Airport(LCA,Larnaca),
Flight(LCA,VIE,OS),	**Airport(GLA,Glasgow)**,
	Airport(EDI,Edinburgh)

{x ↦ LGW, y ↦ GLA, z ↦ U2,
London ↦ London, Glasgow ↦ Glasgow}

Q(z) :- Airport(x,London), Airport(y,Glasgow), Flight(x,y,z)