MAX PAGE NO DATE

19/2/24

1. If Rohit Sharma scores the century and Y scores double century and Ravindra Jadeja takes 5 wickets then India wins the match. Find the primitives and represent using AND ($\wedge$) OR ($\vee$), NOT ($\neg$) and XOR.

X : Rohit Sharma Scores century

Y : Y scores double century. "

Z : 5 wickets

A : India wins match.

- $X \wedge Y \wedge Z \to A$.

2. A : Student scores less than 50 %.
   B : Fail grade can be given.
   C : Fail grade cannot be given when student scores more than 50 %.

- $A \wedge B \to \neg C$.
  $\neg A \wedge \neg B \to C$.

21/2/24

o + o + o = o

E + o + E = o

E + E + o :

o + E + E :

1, 3, 5   1 3 5   1 3 5 $= 3^3$

2, 4, 6 $= 27$.

$o + o + o \Rightarrow 3^3 \Rightarrow 27$.

$E + E + o \Rightarrow 27$

$E + o + E \Rightarrow 27$

$o + E + E \Rightarrow 27$

108

$\frac{108}{6^3} = \frac{108}{216} = \frac{1}{2}$ //

* $P \land Q$: True when both are true
* $P \lor Q$: True when either of them is true
* $\neg P$: True when P is false, false otherwise
* $P \veebar Q$: True when both have different values
* $P \to Q \Rightarrow \neg P \lor Q$
* $P \leftrightarrow Q \Rightarrow (P \to Q) \land (Q \to P)$

$P \land Q$

P	Q	$P \land Q$
1	1	1
1	0	0
0	1	0
0	0	0

$P \lor Q$

P	Q	$P \lor Q$
1	1	1
1	0	1
0	1	1
0	0	0

$P \land Q \to R$

P	Q	$P \land Q$
1	1	1
1	0	0
0	1	0
0	0	0

$P \to Q \Rightarrow \neg P \lor Q$

P	Q	$\neg P$	$\neg P \lor Q$
1	1	0	1
1	0	0	0
0	1	1	1
0	0	1	1

$P \lor Q \lor \neg R$

P	Q	R	$P \lor Q$	$\neg R$	$P \lor Q \lor \neg R$
1	1	1	1	0	1
1	1	0	1	1	1
1	0	1	1	0	1
1	0	0	1	1	1
0	1	1	1	0	1
0	1	0	1	1	1
0	0	1	0	0	0
0	0	0	0	1	1

Gray code   2's complement

8421 code

1's complement

$P \wedge Q \to R \Rightarrow \neg(P \wedge Q) \vee R$

P	Q	R	$P \wedge Q$	$\neg(P \wedge Q)$	$\neg(P \wedge Q) \vee R$
1	1	1	1	0	1
1	1	0	1	0	0
1	0	1	0	1	1
1	0	0	0	1	1
0	1	1	0	1	1
0	1	0	0	1	1
0	0	1	0	1	1
0	0	0	0	1	1

$(P \wedge Q) \to P$

P	Q	$P \wedge Q$	$\neg(P \wedge Q)$ ~~$(P \wedge Q) \to P$~~	$\neg(P \wedge Q) \vee P$ ~~$\neg(P \wedge Q)$~~
1	1	1	0	1
1	0	0	1	1
0	1	0	1	1
0	0	0	1	1

⇒ Tautology : If all the values are true.

⇒ Fallacy / contradiction : If all the values or false

* $\neg\neg P \Rightarrow P$ → Double negation
* $P \vee Q \Rightarrow Q \vee P$ } commutative
* $P \wedge Q \Rightarrow Q \wedge P$
* $(P \wedge Q) \wedge R \Rightarrow P \wedge (Q \wedge R)$ } Associative
* ~~$P \wedge (Q \wedge R) \Rightarrow (P \wedge Q)$~~ $\wedge R$
* $(P \vee Q) \vee R \Rightarrow P \vee (Q \vee R)$
* $(P \wedge Q) \vee R \Rightarrow (P \vee R) \wedge (Q \vee R)$ → Distributive
* $P \vee T \Rightarrow T$
* $P \wedge F \Rightarrow F$

PAGE NO.
DATE :

* $P \land P \Rightarrow P$

$P \lor P \Rightarrow P$

* $\left.\begin{array}{l} P \lor (P \land Q) \\ P \land (P \lor Q) \end{array}\right\} \Rightarrow P.$ } Absorption law

$P \leftrightarrow Q \Rightarrow (P \to Q) \land (Q \to P)$

P	Q	$\neg P$	$\neg Q$	$(\neg P \lor Q)$ $P \to Q$	$\neg Q \lor P$ $Q \to P$	$P \leftrightarrow Q$ $(P \to Q) \land (Q \to P)$
1	1	0	0	1	1	1
1	0	0	1	0	1	0
0	1	1	0	1	0	0
0	0	1	1	1	1	1

⇒ $P \to Q$

Converse ⇒ $Q \to P$

Inverse ⇒ $\neg P \to \neg Q$

Contrapositive ⇒ $\neg Q \to \neg P$.

P	Q	$\neg P$	$\neg Q$	~~$Q \to P$~~ $(\neg Q \lor P)$ $Q \to P$	$(P \lor \neg Q)$ $\neg P \to \neg Q$	$Q \lor \neg P$ $\neg Q \to \neg P$
1	1	0	0	1	1	1
1	0	0	1	1	1	0
0	1	1	0	0	0	1
0	0	1	1	1	1	1

$\neg P \lor Q$ $P \to Q$
1
0
1
1

→ P : ΔABC is equilateral

Q : ΔABC is isosceles

R : ΔABC is scalene

→ $[p \to (q \to r)] \to [(p \to q) \to (p \to r)]$ check whether it is a ~~tau~~ tautology, or fallacy or none.

~~$P \to (q \to r)$~~

						$\neg q \vee r$	$\neg P \vee (q \to r)$	$\neg P \vee r$
P	q	r	$\neg P$	$\neg q$	$\neg r$	$q \to r$	$P \to (q \to r)$	$P \to r$
1	1	1	0	0	0	1	1	1
1	1	0	0	0	1	0	0	0
1	0	1	0	1	0	1	1	1
1	0	0	0	1	1	1	1	0
0	1	1	1	0	0	1	1	1
0	1	0	1	0	1	0	1	1
0	0	1	1	1	0	1	1	1
0	0	0	1	1	1	1	1	1

$\neg P \vee q$		$\neg(P \to q) \vee (P \to r)$	
$P \to q$	$\neg(P \to q)$	$(P \to q) \to (P \to r)$	$[P \to (q \to r)] \to$
1	0	1	1
1	0	0	1
0	1	1	1
0	1	1	1
1	0	1	1
1	0	1	1
1	0	1	1
1	0	1	1

∴ It is a tautology.

$P \vee (P \wedge Q) \to P$ <u>Prove</u>

$LHS = P \vee (P \wedge Q)$

$= (P \vee P) \wedge (P \vee Q)$

$= P \wedge (P \vee Q)$

1. Prove that $P \vee (P \wedge Q) \Rightarrow P$.

~~P can be written as $(P \wedge T) \vee ($~~

P can be written as $(P \wedge T)$. (P and True)

$\therefore$ $LHS \Rightarrow (P \wedge T) \vee (P \wedge Q)$

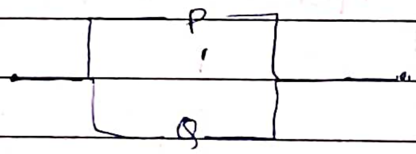
$\Rightarrow P \wedge (T \vee Q)$

$\Rightarrow P \wedge T$

$\Rightarrow P \to RHS$

<u>Series</u>:

$\Rightarrow$ ~~$P \vee Q$~~ $P \wedge Q$



$\Rightarrow$ ~~$P \wedge Q$~~ $P \vee Q$

2. Prove: $[P \to (q \vee r)] \Rightarrow [(P \wedge \neg q) \to r]$

$P \to (q \vee r)$

$\Rightarrow \neg P \vee q \vee r$

$\Rightarrow \neg((\neg\neg P) \wedge (\neg q)) \vee r$

$\Rightarrow \neg(P \wedge \neg q) \vee r$

$\Rightarrow (P \wedge \neg q) \to r$.

3. Prove : $P \to (q \wedge r) \Rightarrow (P \to q) \wedge (P \to r)$

$$P \to (q \wedge r) \Rightarrow \neg P \vee (q \wedge r)$$
$$\Rightarrow (\neg P \vee q) \wedge (\neg P \vee r)$$
$$\Rightarrow (P \to q) \wedge (P \to r)$$

//

4. ~~Prove~~ $(P \vee (q \wedge r)) \vee \neg (P \vee (q \wedge r))$

Verify if it is a tautology

$$\Rightarrow (P \vee q) \wedge (P \vee r) \vee \neg ((P \vee q) \wedge (P \vee r))$$
$$\Rightarrow (P \vee q) \wedge (P \vee r) \vee \neg (P \vee q) \vee \neg (P \vee r)$$

$$(A \wedge B) \vee \neg (A \wedge B)$$

$\Rightarrow$ Tautology

5. $P \to (q \vee r) \Rightarrow \neg r \to (P \to q)$

~~$\neg P \vee (q \vee r)$~~

RHS $r \vee (P \to q)$

$= r \vee (\neg P \vee q)$

$= \neg P \vee q \vee r$

$= P \to (q \vee r)$ //

$= \text{LHS}$

26/2/24 $P \to (Q \to R)$ is equivalent to $Q \to (P \to R)$ $\quad Q \to P \to R$

$P \to (Q \to R)$

P	Q	R	$Q \to R$ ($\neg Q \vee R$)	$P \to (Q \to R)$ ($\neg P \vee (Q \to R)$)	$P \to R$ ($\neg P \vee R$)	$Q \to (P \to R)$ ($\neg Q \vee (P \to R)$)
0	0	0	1	1	1	1
0	0	1	1	1	1	1
0	1	0	0	1	1	1
0	1	1	1	1	1	1
1	0	0	~~0~~ 1	1	0	1
1	0	1	1	1	1	1
1	1	0	0	0	0	0
1	1	1	1	1	1	1

= equivalent =

Proof :

$P \to (Q \to R)$

$= \neg P \vee (Q \to R)$

$= \neg P \vee \neg Q \vee R$

$= \neg Q \vee \neg P \vee R$ (Commutative law)

$= \neg Q \, (P \to R)$

$= Q \to (P \to R)$ //.

⇒ Verify whether $P \to [Q \to (P \wedge Q)]$

			$\neg Q \vee (P \wedge Q)$	
P	Q	$P \wedge Q$	$Q \to (P \wedge Q)$	$P \Rightarrow [Q \to (P \wedge Q)]$
0	0	0	1	1
0	1	0	0	1
1	0	0	1	1
1	1	1	1	1

Tautology //

⇒ $P \to [Q \to (P \wedge Q)]$

$\neg P \vee [Q \to (P \wedge Q)]$

$= \cancel{\neg P \vee [\neg}$ $\neg P \vee \neg Q \vee (P \wedge Q)$

$= \neg (P \wedge Q) \vee (P \wedge Q)$

$= T$ $(\because \neg A \vee A = \text{True})$

⇒ $(P \vee Q) \to (Q \to Q)$

$= \neg (P \vee Q) \vee (Q \to Q)$

$= \neg (P \vee Q) \vee (\neg Q \vee (Q \wedge Q))$

$= \neg (P \vee Q) \vee \neg Q \vee (Q \wedge Q)$

$= \neg (P \vee Q) \vee \neg Q \vee Q$

$= \neg (P \vee Q) \vee T$ ⇒ True.

Hence given expression is a tautology

$\Rightarrow (P \vee Q) \to [Q \to (P \wedge Q)]$

$\neg(P \vee Q) \vee [Q \to (P \wedge Q)]$

$\neg(P \vee Q) \vee \neg Q \vee (P \wedge Q)$

$(\neg P \wedge \neg Q) \vee \neg Q \vee (P \wedge Q)$

$\neg Q \vee (P \wedge Q)$ (Absorption law)

$(\neg Q \vee P) \wedge (\neg Q \vee Q)$

$(\neg Q \vee P) \wedge T$

$(\neg Q \vee P)$

$\therefore$ Given expression is not a tautology

$\Rightarrow$

$\neg(P \wedge Q) \Rightarrow (P \uparrow Q)$

$\neg(P \vee Q) \Rightarrow (P \downarrow Q)$

Prove :-

1. $\neg(P \downarrow Q) \Leftrightarrow \neg P \uparrow \neg Q$
2. $\neg(P \downarrow Q) \Leftrightarrow \neg P \downarrow \neg Q$

$\neg(P \downarrow Q) \Leftrightarrow \neg P \uparrow \neg Q \Rightarrow \neg(\neg(P \vee Q))$

$\Rightarrow \neg(\neg P \wedge \neg Q)$

$\Rightarrow \cancel{\neg(\neg P \downarrow}$

$\Rightarrow \neg P \uparrow \neg Q$

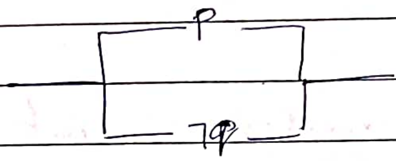
28/02/24

$(p \wedge (\neg r \vee q \vee \neg q)) \vee ((r \vee t \vee \neg r) \wedge \neg q)$

$= (p \wedge (\neg r \vee T))) \vee ((t \vee T) \wedge \neg q)$

$= (p \wedge T) \vee (T \wedge \neg q)$

$= p \vee \neg q$



* $P \rightarrow Q$ T

$Q \rightarrow R$ T

$\therefore P \rightarrow R$ T

* $P \rightarrow Q$

P

Q

Rules of Inference :

1. Modes Ponens :

$P \rightarrow Q$

P

$\therefore Q$

2. Modes ~~Pollens~~ Tollens :

$P \rightarrow Q$

$\neg Q$

$\therefore \neg P$

3. Hypothetical Syllogism :

$P \rightarrow Q$

$Q \rightarrow R$

$\therefore P \rightarrow R$

4. <u>Addition</u> :

$$\frac{P}{\therefore P \vee Q}$$

5. <u>Conjunction</u> :

P

Q

$P \wedge Q$

6. <u>Resolution</u> :

$P \vee Q$

$\neg P \vee R$

$\therefore Q \vee R$

7. <u>Disjunctive syllogism</u> :

$P \vee Q$

$\neg P$

$\therefore Q$

8. <u>Simplification</u> :

$P \wedge Q$	$P \wedge Q$
$\therefore P$	$\therefore Q$

* X works hard, if X works hard then X will get good at work, if X is good at work then X will get an appraisal

→ A : X works hard.
B : If X works hard then X gets good at work.
C : if X is good at work then X gets appraisal

PAGE NO
DATE

A

$A \rightarrow B$

$B \rightarrow C$ To prove

$\therefore H$

$A \rightarrow B$

$B \rightarrow C$

$A \rightarrow C$ (Hypothetical syllogism)

A

C (Model Ponens)

29/2/24

→ If it does not rain, or if it is not foggy then the sailing race will be held and the ~~race~~ ~~saft~~ life saving demonstration will go on. ~~If~~ the sailing race is held then the trophy will be awarded. The trophy was not awarded. Conclude that it rained.

→ $\underbrace{\neg R \vee \neg F}_{X} \rightarrow \underbrace{S \wedge D}_{Y}$

$S \rightarrow T$ To prove R

$\neg T$

$S \rightarrow T$

$\neg T$

$\therefore \neg S$ (MT)

$\neg S \vee \neg D$ (Addition)

$= \neg(S \wedge D)$

$\Rightarrow \neg Y$

$X \rightarrow Y$

$\neg Y$

$\neg X$ (MT)

~~R x 4~~

MAX PAGE NO.
DATE :

$\neg X$

$\rightarrow \neg(\neg R \vee \neg F)$

$: \quad \neg\neg(R \wedge F)$

$= \quad R \wedge F$

$\overline{\quad R \quad}$ (Simplification)

29/2/24

1. If I drive on the freeway I'll see the fire. I will drive on the freeway or take surface streets. I am not going to take the surface streets. Conclusion is I will see the fire.

- A → drive on the freeway ...

~~F~~ B → See the fire.

C → ~~drive on~~ ~~su~~ take on surface streets.

$A \rightarrow B$

$A \vee C$ conclusion B.

$\neg C$

$\therefore B$

$A \vee C$

$\neg C$

A (Disjunctive).

$A \rightarrow B$

A

$\therefore B.$ (MP

2. If I workout hard then I am tired. If I am tired I take vitamins. I did not take any vitamins. Conclusion is I did not work out hard.

→ $W \to T$

$T \to V$

$\neg V$

$\neg W$.

(or)

$W \to T$

$T \to V$

$W \to V$ (H.S)

$\neg V$

$\neg W$. (MT)

$T \to V$

$\neg V$

$\therefore \neg T$ (MT)

$W \to T$

$\neg T$

$\neg W$, (MT).

⇒ **Principle of duality :**

3. If a candidate is known to be corrupt then the candidate will not be ellected. If a candidate is kind then the candidate will be elected

A : If the person is known to be corrupt

~~B : If the person~~ then the person is kind

B : If the person is not known to be corrupt the the person is not kind

~~C~~ : If the person is kind, then the person is not known to be corrupt

D : If a person is not kind then the person is not known to be corrupt

$\Rightarrow$ $(P \wedge Q) \vee R$ $P \vee R$

$R \rightarrow S$ $P \vee S$ ✓

$P \vee Q$

$Q \vee R$

$=$ ~~$(P \vee R) \wedge (Q \vee R)$~~

$\underline{P \wedge Q}$

P (simplification)

$R \rightarrow S = \neg R \vee S$.

$P \vee R$.

$R \vee P$

$\neg R \vee S$

$\overline{P \vee S}$ //

1/3/24

1. If the band would not play a rock music or the refreshments were not delivered on time then the new year's party would have been cancelled and A would have been angry. If the party were cancelled then refunds would have had to be made. No refunds were made. Therefore the band could play the rock music.

→

$\neg M \rightarrow \neg P$

$\neg P \rightarrow R \qquad \therefore M.$

$\neg R$

$$\begin{array}{l} \neg M \rightarrow \neg P \\ \neg P \rightarrow R \\ \hline \neg M \rightarrow R \end{array} \qquad (HS).$$

$$\begin{array}{l} \neg M \rightarrow R \\ \neg R \\ \hline \neg(\neg M) = M \end{array} \qquad (MT).$$

→ $\neg R \vee \neg S \rightarrow \neg P \wedge A$

$$\begin{array}{l} \neg P \rightarrow Ref \\ \neg Ref \\ \hline \therefore R \end{array}$$

$$\begin{array}{l} \neg P \rightarrow Ref \\ \neg Ref \\ \hline \neg(\neg P) \Rightarrow P \end{array} \qquad (MT).$$

$$P \Rightarrow P \vee \neg A \quad \text{(Addition)}$$
$$= \neg\neg(P \vee \neg A)$$
$$P = \neg(\neg P \wedge A)$$

$$\neg R \vee \neg S \rightarrow \neg P \wedge A$$
$$\neg(\neg P \wedge A)$$
$$\overline{\quad}$$
$$\therefore \neg(\neg R \vee \neg S) \quad \text{(MT)}$$

$$\therefore R \wedge S$$
$$\Rightarrow R \quad \text{(Simplification)}$$

★ **Quantifiers:**

1. Everyone practices hard or plays badly. Someone does not practice hard. Therefore someone plays badly.

→ $P(x)$: x practices hard
$B(x)$: x plays badly.

~~$P(x) \vee B(x)$~~

$\forall x\, (P(x) \vee B(x))$
$\exists x\, \neg P(x)$
$\exists x\, B(x)$

2. All of the paintings by ~~A~~ are beautiful. The museum has a painting by ~~A~~. Therefore the museum has a beautiful painting.

→ $A(x)$: x is a painting by A
$B(x)$: Painting x is beautiful
$M(x)$: x is a painting in ~~meus~~ museum.

$\forall x\ (A(x) \rightarrow B(x))$

$\exists x\ (M(x) \wedge A(x))$

$\exists x\ (M(x) \wedge B(x))$

4/3/24.

1. * All lions are fierce
* Some lions don't drink coffee
* Some fierce creatures don't drink coffee.

$P(x)$: x is a lion.

$F(x)$: x is fierce.

$C(x)$: drinks coffee.

* $\forall(x)\ (P(x) \longrightarrow F(x))$.
* $\exists(x)\ (P(x) \wedge \neg C(x))$
* $\exists(x)\ (F(x) \wedge \neg C(x))$

2. * All humming birds are richly colored.
* No large birds live on honey
* Birds that do not live on honey are dull in color.
* humming birds are small

x : All birds.

$P(x)$: x is a humming bird.

$R(x)$: x is richly colored.

$H(x)$: x lives on honey

$L(x)$: x is large

~~S(x) : x is small.~~

* $\cancel{\forall(x)}$ $\forall x \, (P(x) \land R(x))$

* $\forall x \, (\neg L(x) \rightarrow \neg H(x))$

* $\forall x \, (\neg H(x) \rightarrow \neg G(x))$

* $\forall x \, (H(x) \rightarrow R(x))$

⇒ Equivalence :

$$\forall x \, (P(x) \land Q(x)) \equiv \forall x \, P(x) \land \forall x \, Q(x)$$

$$\exists x \, (P(x) \lor Q(x)) \equiv \exists x \, P(x) \lor \exists x \, Q(x)$$

6/3/24

→ $P(n)$: n is a prime no

$Q(n)$: n is even

$R(n)$: $n > 2$

$\forall n \in \mathbb{Z} \, (P(n) \rightarrow (Q(n) \land R(n)))$

$\exists n \in \mathbb{Z} \, (Q(n) \land P(n))$

$\forall n \in \mathbb{Z} \, ((GP(n) \land \neg R(n)) \rightarrow Q(n)))$

* ~~Every prime number~~

* For

PAGE NO.
DATE :

→ For any given prime number, the next number is a composite number.

$P(x)$: x is prime
$C(x)$: x is composite
~~$N(x)$~~

$\forall x\,(P(x) \rightarrow C(x+1))$.

$\neg(\forall x\,(P(x) \rightarrow C(x+1)))$
$= \neg(\forall x\,(\neg P(x) \vee C(x)))$
$: \exists x\,\neg(\neg P(x) \vee C(x))$.
$\exists x\,(P(x) \wedge \neg C(x))$

⇒ All students in this class studies physics and all students in this class codes in C.

→ $S(x)$: x is a student.
$P(x)$: x studies physics
$C(x)$: x codes in C

~~$\forall x\,(P(x) \wedge C(x))$~~
⇒ ~~$\neg(\forall x\,(P(x) \wedge C(x)))$~~
~~$\exists x\,\neg(P(x) \wedge C(x))$~~
~~$\exists x\,(\neg P(x) \vee \neg C(x))$~~ correct

~~$\forall x\,(P$~~ $\forall x\,P(x) \wedge \forall x\,C(x)$