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Assignment 0

Problem 1

1. The derivative $f'(x) = \frac{-12x}{(6x^2-1)^2}$.
2. $f(0.408) = \infty$ (denominator became 0) using 3-digit arithmetic and $f(0.408) = 833.3$ using 4-digit arithmetic with rounding.
3. Horner's representation of the given polynomial is $y = -0.35 + x(8 + x(-7 + x))$. The value of y at $x = 1.37$ is 0.048 using 3-digit arithmetic rounding.
4. The value of y at $x = 1.37$ is 0.02 using direct substitution and 3-digit arithmetic rounding.
5. The value computed using Horner's rule is more accurate since it is closer the actual value without rounding which is 0.0431.

Problem 2

1. The RAM size necessary to store the given array is 49439 MB.
2. No. 32 GB is not sufficient to store the entire array in RAM alone. But swap space could be used to spill excess array to disk and still use the RAM.
3. No. It is not possible to use the workstation as this increase in resolution requires 111^2 more space than the previous array which amounts to roughly 600 TB.

Problem 3

1. The optimality condition is $f'(x) = (A + A^T)X + b = 0$. This optimum is for minimum since $f''(x) = A + A^T$ and $||f''(x)|| > 0$ since $||A + A^T|| > 0$ for the given A.
2. The algorithm for gaussian elimination is as follows,

   Algorithm 1: Gaussian Elimination (LU Decomposition)

   ```
   U = A + AT; L = I
   for k = 1 to m-1
       for j = k+1 to m
           l[j][k] = u[j][k] / u[k][k]
           u[j][k:m] = u[j][k:m] - l[j][k] * u[k][k:m]
   Y = L^-1 * -b // Back Substitution
   X = U^-1 * Y // Back Substitution
   ```

3. The above program is implemented in sgauss.m file.

4. There is no trouble for the current case. However, if the pivot element is 0 in any iteration, the algorithm breaks. Hence, Gaussian elimination with partial pivoting (interchange of rows) is required to overcome this problem.

5. The modified program with partial pivoting is implemented in sgausspivot.m file.

Problem 4

1. The functions have been implemented in scgs.m and smgs.m files.

2. Matrix generation function has been implemented in matgen.m file.

3. The script to run for different matrix sizes is written in exp.m file.

4. The plot for error is available below,

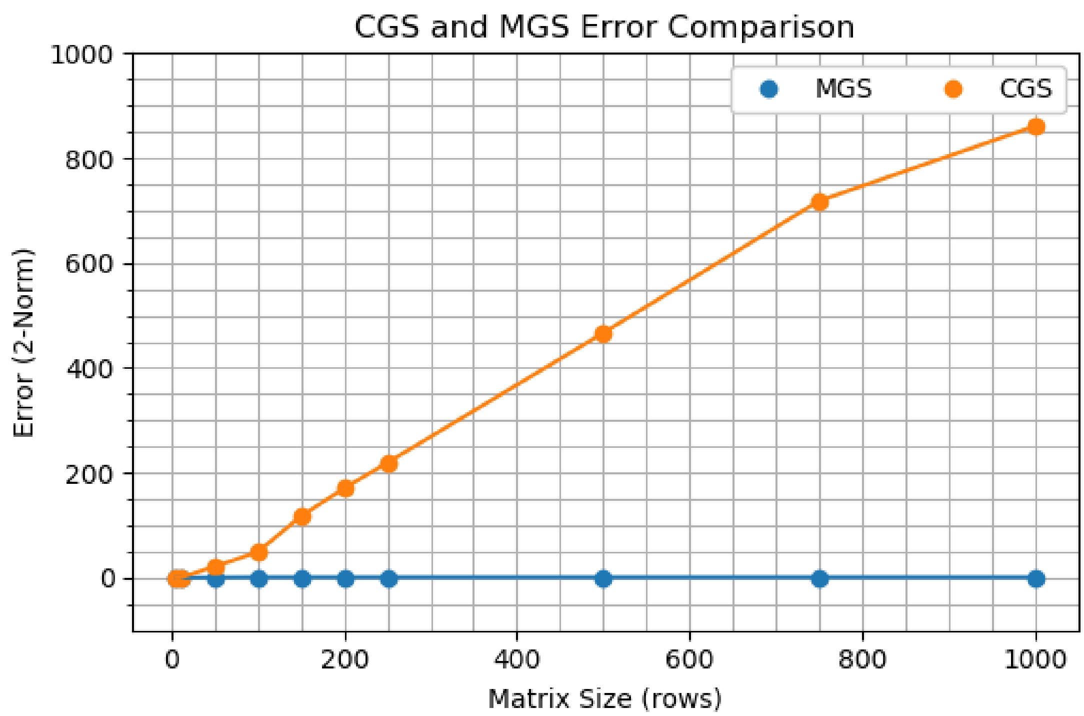

Figure 1: CGS and MGS Error Comparison

5. The modified Gram-Schmidt technique is more numerically stable than the classical version. Error remains low and constant for modified technique and increases linearly with respect to matrix size for classical version.

Problem 5

The following contour plot was taken gridded dataset downloaded from [1]

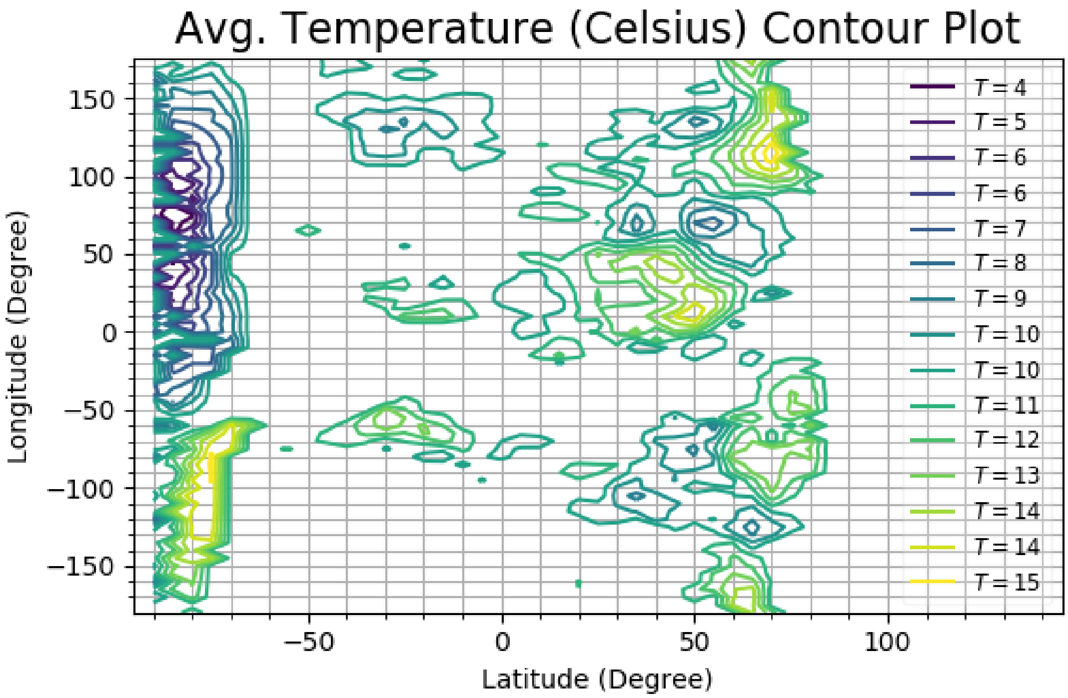

Figure 2: Average Temperature Contour Plot

References

1. http://berkeleyearth.lbl.gov/auto/Global/Gridded/Complete_TAVG_EqualArea.nc
2. Trefethen, Lloyd N., and David Bau III. Numerical linear algebra. Vol. 50. Siam, 1997.
3. DS 294 Course Lecture Notes.