

1.

Given

$$g_1 = f_1 + h_2 * f_2 \quad \text{--- ①}$$

$$g_2 = f_2 + h_1 * f_1 \quad \text{--- ②}$$

h_1, h_2 are known blur kernels

g_1, g_2 are also known.

Need to determine f_1, f_2 , which are scene outside and scene inside respectively.

From ① and ②, applying Fourier Transform,

$$F(g_1) = F(f_1) + F(h_2 * f_2)$$

$$G_1 = F_1 + H_2 F_2 \quad \text{--- ③}$$

$$\text{let } F(g_i) = G_i$$

$$F(f_i) = F_i$$

$$F(h_i) = H_i \quad \text{for } i = 1, 2$$

$$\therefore F(h_2 * f_2) = F(h_2) \cdot F(f_2)$$

$$G_2 = F_2 + H_1 F_1 \quad \text{--- ④}$$

~~Also,~~

$$\hat{F}_1, \hat{F}_2, \hat{G}_1, \hat{G}_2$$

$$\text{③, ④} \Rightarrow G_1 = F_1 + H_2 (G_2 - H_1 F_1)$$

$$\hat{F}_1 = \frac{G_1 - H_2 G_2}{1 - H_1 H_2}$$

$$\hat{F}_2 = \frac{G_2 - H_1 G_1}{1 - H_1 H_2}$$

$$f_1 = F^{-1} \left(\frac{G_1 - H_2 G_2}{1 - H_1 H_2} \right)$$

$$f_2 = F^{-1} \left(\frac{G_2 - H_1 G_1}{1 - H_1 H_2} \right)$$

Problem with above formula for f_1, f_2 :

h_1 and h_2 are blur kernels.

So, H_1, H_2 are low-pass filters and

$H_1 \rightarrow 1$ and $H_2 \rightarrow 1$ for low frequencies.

This leads to $(1 - H_1 H_2) \rightarrow 0$ for low frequencies.

The value $\frac{G_1 - H_2 G_2}{1 - H_1 H_2}$, denominator becomes very small and whole value blows up. This will amplify the noise even if a small amount is present. Natural images have high contribution from low frequency component ^{with}. This method, it is difficult to extract them.