2. 1D case:

Given g, h and $g = h * f$, find f.

Applying Fourier Tranform to $g = h * f$ — ①

Let $\mathcal{F}(g) = G$

$\mathcal{F}(h) = H$

$\mathcal{F}(f) = F$

$\mathcal{F}(h * f) = \mathcal{F}(h)\mathcal{F}(f)$

$$G = HF$$

$$F = \frac{G}{H} \qquad f = \mathcal{F}^{-1}\left(\frac{G}{H}\right)$$

h is gradient kernel. Hence,

H is high pass filter.

$H(u) \approx 0$ for small u.

The value $\frac{G}{H}$ blows up for small u and it is

~~But~~ not possible to retrieve $F(u)$ ~~when~~ for ~~$u \to 0$~~ small u

But natural images have large contribution from small frequencies in $F(u)$. So, it is difficult to retrieve the image. Also, ~~small~~ even small noise get amplified at small u and retrieval of $F(u)$ for small u is made difficult by even presence of small noise.

For 2D case :

$$g_x = h_x * f \quad -① \qquad g_y = h_y * f \quad -②$$

h_x, h_y are derivative kernel in x and y directions respectively.

~~Ap~~ Given g_x, g_y, h_x, h_y, need to find ~~E~~ f.

Applying Fourier transform to ① and ②

$$G_x = H_x F \qquad G_y = H_y F$$

$$\hat{F} = \frac{G_x}{H_x} \qquad \text{so,} \quad f = \mathcal{F}^{-1}\left(\frac{G_x}{H_x}\right) \quad -③$$

$$\hat{F} = \frac{G_y}{H_y} \qquad f = \mathcal{F}^{-1}\left(\frac{G_y}{H_y}\right) \quad -④$$

f can be obtained using either ③ or ④

Problem is h_x and h_y are derivative kernels.

$\therefore$ $H_x(u,v) \to 0$ for small u

$H_y(u,v) \to 0$ for small v

~~\$~~ So, when both u and v are small, both $H_x(u,v) \to 0$, $H_y(u,v) \to 0$.

$\frac{G_x}{H_x}$ and $\frac{G_y}{H_y}$ blow up, making retrieval of $F(u,v)$ for small u,v difficult.

Even small noise, if present in G_x, G_y, will blow up significantly.

Natural images have large components of low (u,v) in $F(u,v)$ and ~~losing~~ ~~this~~ this makes retrieval difficult.

Intuitively, if f is constant, ~~G_x, G_y~~ $g_x, g_y = 0$. There is no way to retrieve f. This corresponds to retrieving $F(0,0)$ since for constant image only $F(0,0) > 0$ and $F(u,v) = 0$ $\forall (u,v) \neq (0,0)$