6. Here, we have $J(x, y) = I(x - x_0, y - y_0)$

where $x_0 = -30$, $y_0 = 70$

x represents the row number, y represents the column number

cross-power spectrum,

$$\frac{F_I(\xi, \eta)\, F_J^*(\xi, \eta)}{|F_I(\xi, \eta)\, F_J(\xi, \eta)|} = e^{j2\pi(\xi x_0 + \eta y_0)}$$

Taking IDFT of $e^{j2\pi(\xi x_0 + \eta y_0)}$,

$$F(u, v) = \sum_{x=0}^{W_1 - 1} \sum_{y=0}^{W_2 - 1} f(x, y)\, e^{-j2\pi v y / W_2}\, e^{-j2\pi u x / W_1}$$

$f(-x_0, W_2 - y_0) = 1$ & $f(x, y) = 0$ $\forall$ $x \neq -x_0$, $y \neq W_2 - y_0$ works.

gives $F(\xi, \eta) = e^{j2\pi(\xi x_0 + \eta y_0)}$

So, ~~IDFT~~ $\mathcal{F}^{-1}\left(e^{j2\pi(\xi x_0 + \eta y_0)}\right) = f$

where $f(-x_0, W_2 - y_0) = 1$ and

$f(x, y) = 0$ $\forall$ $x \neq -x_0$, $y \neq W_2 - y_0$

$W_2 - y_0$ was chosen since $y_0 > 0$

So, peak will occur at $(30, 230)$. $W_1 = W_2 = 300$

~~This is~~

This is equal to the one obtained in experiment

In experiment, $(31, 231)$ was obtained. But since coordinates start at ~~(0,0)~~ $(1,1)$ instead of $(0,0)$, this is actually equal to that derived above, which is $(30, 230)$.

Image, DFT, IDFT are periodic.

From ~~value~~ position of peak in IDFT - $(30, 230)$, it can be concluded that shift, $x_0 = -30$, $y_0 = -230$. But since image is periodic, $y_0 = 70$ is equivalent to $y_0 = -230$.

So, shift $= (-30, 70)$.

Complexity of this methods for finding shift:

$N \times N$: DFT $\times 2$: $N^2 \log(N^2)$

~~N×N IDFT~~

Pointwise multiplication of $N \times N$: $O(N^2)$

$N \times N$: IDFT : $N^2 \log(N^2)$

~~Esta~~ Then find position maximum element in $N \times N$ matrix : $O(N^2)$.

Total time : $O(N^2 \log N)$

Naive method of pixel wise comparison for finding shift:

~~For~~ each
To check if image is shifted by (i,j), need to compare all pixels, so For every (i,j), N^2 comparisons.

Total possible values of (i,j) : N^2.

Total worst case complexity : $O(N^2 \cdot N^2)$
: $O(N^4)$

It can be seen that using DFT is much faster than naive pixel wise comparison