

2.

1D case:

Given  $g, h$  and  $g = h * f$ , find  $f$ .

Applying Fourier Transform to  $g = h * f$  - ①

$$G = H F$$

$$F = \frac{G}{H} \quad f = F^{-1}\left(\frac{G}{H}\right)$$

$$\text{let } F(g) = G$$

$$F(h) = H$$

$$F(f) = F$$

$$F(h * f) = F(h)F(f)$$

$h$  is gradient kernel. Hence,

$H$  is high pass filter.

$H(u) \approx 0$  for small  $u$ .

The value  $\frac{G}{H}$  blows up for small  $u$ , and it is  
But not possible to retrieve  $F(u)$  ~~when~~ <sup>for</sup>  $u \rightarrow 0$  small  $u$

But natural images have large contribution from small frequencies in  $F(u)$ . So, it is difficult to retrieve the image. Also, ~~small~~ even small noise get amplified at small  $u$  and retrieval of  $F(u)$  for small  $u$  is made difficult by even presence of small noise.

For 2D case :

$$g_x = h_x * f \quad \text{--- (1)} \quad g_y = h_y * f \quad \text{--- (2)}$$

$h_x, h_y$  are derivative kernel in  $x$  and  $y$  directions respectively.

Given  $g_x, g_y, h_x, h_y$ , need to find  $f$ .

Applying Fourier transform to (1) and (2)

$$G_x = H_x F \quad G_y = H_y F$$

$$\hat{F} = \frac{G_x}{H_x} \quad \text{so, } f = F^{-1}\left(\frac{G_x}{H_x}\right) \quad \text{--- (3)}$$

$$\hat{F} = \frac{G_y}{H_y} \quad f = F^{-1}\left(\frac{G_y}{H_y}\right) \quad \text{--- (4)}$$

$f$  can be obtained using either (3) or (4)

Problem is  $h_x$  and  $h_y$  are derivative kernels.

$$\therefore H_x(u, v) \rightarrow 0 \quad \text{for small } u$$

$$H_y(u, v) \rightarrow 0 \quad \text{for small } v$$

So, when both  $u$  and  $v$  are small, both  $H_x(u, v) \rightarrow 0$  and  $H_y(u, v) \rightarrow 0$ .

$\frac{G_x}{H_x}$  and  $\frac{G_y}{H_y}$  blow up, making retrieval of  $F(u, v)$  for small  $u, v$  difficult.

Even small noise, if present in  $G_x, G_y$ , blow up significantly.

Natural images have large components of low  $(u, v)$  in  $F(u, v)$  and ~~losing~~ this makes retrieval difficult.

Intuitively, if  $f$  is constant,  ~~$G_x, G_y = 0$~~   $G_x, G_y = 0$ . There is no way to retrieve  $f$ . This corresponds to retrieving  $F(0, 0)$  since for constant image only  $F(0, 0) > 0$  and  $F(u, v) = 0 \quad \forall (u, v) \neq (0, 0)$ .

Q2 continued:

Also, if a constant value is added to each pixel in the  $f$ ,  $g_x$  and  $g_y$  do not change. So, multiple images can correspond to same  $g_x$  and  $g_y$  for fixed  $h_x$  and  $h_y$ . Here, the constant value added corresponds to  $F(0, 0)$ . That constant cannot be retrieved as  $F(0, 0)$  cannot be retrieved.