Statistics for Data Science-1

Week 6 Graded Assignment

1. How many 5-digit numbers can be formed from the numbers $0, 2, 4, 5, 7$ and 9 (without repetition), such that it is divisible by 4?

   a. 120

   b. 144

   c. 132

   d. 104

   Answer: b

   Solution:

   By the divisibility rule of 4, for a number to be divisible by 4, the last two digits of the number should be divisible by 4.

   Using the divisibility rule of 4, any 5-digit number formed from the numbers $0, 2, 4, 5, 7$ and 9 will be divisible by 4 if its last two digits are: 04, 20, 24, 40, 52, 72 or 92.

   Case 1- Number ends with 04:

   The last two digits of a five digit number is fixed (i.e. 04). We have to fill the first three digits using the remaining four numbers (as repetition is not allowed) i.e. 2,5,7 and 9.

   Now, the three places can be filled with these 4 numbers in ${}^4P_3$ ways.

   Similarly, in the cases where the five digit number ends with 20 and 40, the number of ways to fill in the last three digits is ${}^4P_3$ in each case.

   Case 2- Number ends with 24:

   The last two digits of a five digit number is fixed (i.e. 24). We have to fill the first three digits using the remaining four numbers (as repetition is not allowed) i.e. 0,5,7 and 9.

   0 cannot be the first digit as it will make the number a four digit number. Therefore, the number of ways to fill in the first place is 3.

   Now, 0 can be used along with the numbers 5,7 and 9 in the remaining places. Hence, the remaining two places can be filled with these 4 digits in ${}^3P_2$ ways.

   Therefore, total number of ways to complete a five-digit number ending with 24 = $3 \times {}^3P_2$

   Similarly, in the cases where the five digit number ends with 52, 72 and 92, the number of ways to fill in the last three digits is $3 \times {}^3P_2$ in each case.

Hence, the total number of 5-digit numbers that can be formed from the digits $0, 2, 4, 5, 7$ and 9 (without repetition), such that it is divisible by 4 are:
$={}^4 P_3 +{}^4 P_3 +{}^4 P_3 + 3 \times{}^3 P_2 + 3 \times{}^3 P_2 + 3 \times{}^3 P_2 + 3 \times{}^3 P_2$
$= 3 \times{}^4 P_3 + 4 \times 3 \times{}^3 P_2$
$= 3 \times 24 + 4 \times 3 \times 6$
$= 144$

2. There are n train stops between Chennai and Assam. How many train tickets are to be printed, so that a person can travel between any of the two stations (irrespective of direction of travel)?
Solution:
There are a total of $(n + 2)$ stations including Assam and Chennai along with the n stations between them.
Now, one can travel from any one station to another (irrespective of direction of travel).
Hence, total number of tickets to be printed $={}^{(n+2)} P_2 = (n + 2)(n + 1)$ tickets.
Example: n=7
There are a total of 9 stations including Assam and Chennai along with the 7 stations between them.
Now, one can travel from any one station to another (irrespective of direction of travel).
Hence, total number of tickets to be printed $={}^9 P_2 = 72$ tickets.

3. A man desires to throw a party for some of his friends. In how many ways can he select m friends from a group of n friends, if the two of his friends (say 'A' and 'B') will not attend the party together?
Solution:
Total number of ways in which he can invite his friends to the party (without any condition) $={}^n C_m$ ways.
Now, number of ways in which he can invite his friends so that 2 of his friends will attend the party $={}^{n-2} C_{m-2}$
Hence, Number of ways in which two of his friends will not attend the party together $={}^n C_m -{}^{n-2} C_{m-2}$
Example: n=10 and m=7
Total number of ways in which he can invite his friends to the party (without any condition) $={}^{10} C_7$ ways.
Now, number of ways in which he can invite his friends so that 2 of his friends will attend the party $={}^8 C_5$
Hence, Number of ways in which two of his friends will not attend the party together $={}^{10} C_7 -{}^8 C_5$
$= 120 - 56$
$= 64$ ways.

4. Suman has m clothes of different types, say, $C_1, C_2, ..., C_m$ and she wants to wear all these clothes at different days, say, $D_1, D_2, ..., D_m$. Due to some reason, C_1 must be used either at D_{m-2} or at D_{m-1} and C_2 can be used either at D_{m-1} or at D_{m-2} and

at D_m. Every cloth is to be used at only one day, in how many ways can clothes be used?

Solution:

C_1 must be used at either D_{m-2} or D_{m-1}, thus there are 2 ways in which C_1 can be used.

C_2 can be used in only 2 ways ,i.e, at D_{m-2} or D_{m-1} and D_m. (As C_1 is already used at anyone of D_{m-2} or D_{m-1})

Now , for C_3, $(m-2)$ days are available, i.e., there are $(m-2)$ ways for using C_3, $(m-3)$ ways for using C_4, $(m-4)$ ways for using C_5,..., 1 way for using C_m

Therefore, total number of ways clothes can be used will be $= 2 \times 2 \times (m-2) \times (m-3) \times (m-4), ..., \times 3 \times 2 \times 1$

For example:

If $m = 6$, then we will have 6 clothes and 6 days.

Corresponding number of ways will be;

C_1 must be used at D_4 or $D_5 = 2$ ways

C_2 can be used in 2 ways i.e., 1 way will be using at D_4 or D_6 (if C_1 is used at D_5) and 1 way will be at D_5 or D_6 (if C_1 is used at D_4) = total 2 ways for using C_2.

for C_3 four days are available= 4 ways.

for C_4 three days are available= 3 ways.

for C_5 two days are available= 2 ways.

for C_6 only one day is left = 1 way.

Therefore, total number of ways $= 2 \times 2 \times 4 \times 3 \times 2 \times 1 = 96$ ways.

5. How many n-digit numbers can be formed such that they read the same way from either of the side (i.e. the number should be a palindrome)?

   a. $9 \times 10^{\left(\frac{n-1}{2}\right)}$
   b. $9 \times 10^{\left(\frac{n+1}{2}\right)}$
   c. $9 \times 10^{n-1}$
   d. 10^n

   Answer: a

   Solution:

   For a n-digit number to read the same way from either side, the $1^{st}, 2^{nd}, ...$ and $\left(\frac{n-1}{2}\right)^{th}$ digits needs to be the same as $n^{th}, (n-1)^{th}, ...$ and $\left(\frac{n+3}{2}\right)^{th}$ digits respectively.

   So, number of ways to fill first place = 9, as the number will become a $(n-1)$ digit number if 0 is in the first place.

   And, $2^{nd}, 3^{rd}..., \left(\frac{n-1}{2}\right)^{th}$ and $\left(\frac{n+1}{2}\right)^{th}$ can be filled in $10^{\left(\frac{n-1}{2}\right)}$ ways as repeti-

tion is allowed.
$n^{th}, (n-1)^{th}, ...$ and $\left(\frac{n+3}{2}\right)^{th}$ digit should be same as $1^{st}, 2^{nd}, ...$ and $\left(\frac{n-1}{2}\right)^{th}$ digit respectively, so number of ways to fill that place is 1
Hence, the total number of n-digit numbers that can be formed such that they read the same way from either side $= 9 \times 10^{\left(\frac{n-1}{2}\right)}$ ways.
Hence, option a is correct.

Example: $n = 5$
For a 5-digit number to read the same way from either side, the first and second digit needs to be same as the fifth and fourth digit respectively.
So, number of ways to fill first place $= 9$, as the number will become a 4-digit number if 0 is in the first place.
And, second and third place can be filled in 10×10 ways as repetition is allowed.
Fourth and fifth digit should be same as first and second digit respectively, so number of ways to fill that place is 1×1
Hence, the total number of 5-digit number that can be formed such that they read the same way from either side $= 9 \times 100 = 9 \times 10^2$
Hence, option a is correct.

6. Find the total number of ways to form a m digit number (without repetition) from the digits $0, 1, 2, ..., n$.

   (a) $n \times {}^{n}P_{m-1}$
   (b) ${}^{n+1}P_{m}$
   (c) $(n-1) \times {}^{n-1}P_{m-1}$
   (d) $n + {}^{n}P_{m-1}$

   Answer: a
   Solution:
   We have to form m digit number with the digits $0, 1, 2, ..., n$ and we have $n+1$ digits in total.
   For the first digit we have n ways (as 0 can't be considered as first digit). Now, we have n digits remaining (because one digit is already used for first digit) and $m-1$ places. Therefore, number of ways to arrange n digits at $m-1$ places is ${}^{n}P_{m-1}$ ways.
   Hence, total number of ways to form a m digit number from the digits $0, 1, 2, ..., n$ (without repetition) is $n \times {}^{n}P_{m-1}$.
   For example: $m = 6$ **and** $n = 7$
   We have to form 6 digit numbers from digits $0, 1, 2, 3, 4, 5, 6, 7$ and we have 8 digits in total.
   For the first digit we have 7 ways(as 0 can't be considered as first digit). Now, we have 7 digits remaining (because one digit is already used for first digit) and 5 places.

Therefore, number of ways to arrange 7 digits at 5 places is ${}^7P_5$ ways.
Hence, total number of ways to form a 6 digit number from the digits $0, 1, 2, ..., 7$ (without repetition) is $7 \times {}^7P_5$.

7. In a restaurant, x men and y women are seated on $(x + y)$ chairs at a round table. Find the total number of possible ways such that x men are always sitting next to each other.

   (a) $x! \times y!$
   (b) $(x - 1)! \times (y - 1)!$
   (c) $x! \times (y + 1)!$
   (d) $(x + y - 1)!$

   Answer: a
   Solution:
   Considering the x men $M_1, M_2, ..., M_x$ who sit together as one, we get $(y + 1)$ persons in all, who can be seated at a round table in $y!$ ways. Further, since $M_1, M_2, ..., M_x$ can interchange their positions in $x!$ ways, the total number of possible ways of getting $M_1, M_2, ..., M_x$ together is $y! * x!$
   For example: $y = 2, x = 3$
   Considering the 3 men M_1, M_2 and M_3 who sit together as one, we get $2 + 1 = 3$ persons in all, who can be seated at a round table in 2! ways. Further, since M_1, M_2 and M_3 can interchange their positions in 3! ways, the total number of possible ways of getting M_1, M_2 and M_3 together is $2! * 3! = 12$.

8. In how many ways can a group of $n - m$ players be formed from n state level players and m district level players such that the group contains exactly 1 district level player?
   Answer: $\dfrac{m \times n!}{(m+1)!(n-m-1)!}$

   Solution:
   The group must have $(n - m)$ players and must contain exactly 1 district level player. Hence, we will select $(n - m - 1)$ persons from n state level players and 1 from m district level players.
   The total number of ways to form a group of $(n - m)$ players is:
   $= {}^nC_{n-m-1} \times {}^mC_1$

   $$= \frac{m \times n!}{(n-m-1)! \times (m+1)!}.$$

   For example: $n = 10, m = 6$
   The group must have 4 players. But the group must contain exactly 1 district level

player. Hence, we will select 3 players from 10 state level players and 1 from 6 district level players.

Total number of ways to form a group of 4 players = ${}^{10}C_3 \times^6 C_1 = \dfrac{6 \times 10!}{3! \times 7!} = 720$.

9. Find the value of r such that the ratio of ${}^3P_r$ and ${}^4P_{r-1}$ will be $\dfrac{1}{2}$?

   Answer: 3

   Solution:

   Given,

$$\frac{3P_r}{{}^4P_{r-1}} = \frac{1}{2}$$

$$\frac{3!/(3-r)!}{4!/(4-(r-1))!} = \frac{1}{2}$$

$$\frac{3!/(3-r)!}{4!/(5-r)!} = \frac{1}{2}$$

$$\frac{3!/(3-r)!}{4 \times 3!/(5-r)(4-r)(3-r)!} = \frac{1}{2}$$

$$\frac{(5-r)(4-r)}{4} = \frac{1}{2}$$

$$(5-r)(4-r) = 2$$

   By solving above equation,
   $r = 6$ and $r = 3$.
   $r = 3$ is the answer since $r = 6$ is greater than $n = 3$.

10. Choose the incorrect option/s for $n > 2$:

    a. ${}^nC_r +^n C_{r-1} =^{n+1} C_r$

    b. ${}^nC_r = 1$ for $r = 0$ and $r = n$

    c. ${}^nC_r =^{n-1} C_{r-1} +^{n-1} C_r$

    d. None of the above

    Answer: d

    Solution:

    Going through the options and solving for given conditions, options a, b, c are the true relations and hence, are incorrect. Therefore, the correct option is (d).