Statistics for Data Science-1

Week 7 Graded assignment

1. m boys and 2 girls are to be placed next to each other in the school ground for morning assembly. What is the probability that there are exactly 4 boys between the 2 girls?

   a. $\dfrac{2m-5}{^{m+2}P_2}$

   b. $\dfrac{2m-6}{^{m+2}P_2}$

   c. $\dfrac{2m-6}{^{m+3}P_2}$

   d. $\dfrac{2m-4}{^{m+2}P_2}$

   Answer: b

   Solution:
   There are a total of $(m+2)$ places to arrange the 2 girls.
   Therefore, the number of ways in which 2 girls can be arranged $= {}^{m+2}P_2$
   Positioning of the 2 girls such that there are exactly 4 boys between them can be done in the following ways:

   Case 1: First girl at 1^{st} place and second girl at 6^{th} place and vice-versa, i.e. 2 ways.
   Case 2: First girl at 2^{nd} place and second girl at 7^{th} place and vice-versa, i.e. 2 ways.
   Similarly,
   Case (m-3): First girl at $(m-3)^{th}$ place and second girl at $(m+2)^{th}$ place, and vice-versa, i.e. 2 ways.

   Hence, Number of possible ways such that there are exactly 4 boys between the 2 girls $= 2 \times (m-3) = 2m-6$ ways.
   Therefore, P[There are exactly 4 boys between the 2 girls] $= \dfrac{2m-6}{^{m+2}P_2}$

   Hence, option (b) is correct.

   Example: $m = 8$
   There are a total of 10 places to arrange the 2 girls.
   Therefore, the number of ways in which 2 girls can be arranged $= {}^{10}P_2$
   Positioning of the 2 girls such that there are exactly 4 boys between them can be done in the following ways:

   Case 1: First girl at 1^{st} place and second girl at 6^{th} place and vice-versa, i.e. 2 ways.

Case 2: First girl at 2^{nd} place and second girl at 7^{th} place and vice-versa, i.e. 2 ways.
Case 3: First girl at 3^{rd} place and second girl at 8^{th} place and vice-versa, i.e. 2 ways.
Case 4: First girl at 4^{th} place and second girl at 9^{th} place and vice-versa, i.e. 2 ways.
Case 5: First girl at 5^{th} place and second girl at 10^{th} place and vice-versa, i.e. 2 ways.

Hence, Number of possible ways such that there are exactly 4 boys between the 2 girls $= 2 \times 5 = 10$ ways.
Therefore, P[There are exactly 4 boys between the 2 girls] $= \dfrac{10}{{}^{10}P_2} = \dfrac{10}{90} = \dfrac{1}{9}$
Hence, option (b) is correct.

2. In a Multiple Select Question, there are m options, of which one or more can be correct. Let us define an event E that the option 'A' is correct. What is the cardinality of E?
Solution:
Case 1: Only option A is correct.
Number of possible elements = 1 i.e. $\{(A)\}$
Case 2: Two options are correct and A is one of them.
Number of possible elements $= {}^{m-1}C_1 = m-1$
Case 3: Three options are correct and A is one of them.
Number of possible elements $= {}^{m-1}C_2$
Similarly,
Case m: All options are correct.
Number of possible elements 1
Hence, Cardinality of $E = 1 + {}^{m-1}C_1 + {}^{m-1}C_2 + ... + 1 = 2^{m-1}$

Example: m=4
Case 1: Only option A is correct.
Number of possible elements = 1 i.e. $\{(A)\}$
Case 2: Two options are correct and A is one of them.
Number of possible elements $= {}^{3}C_1 = 3$ i.e. $\{(A,B),(A,C),(A,D)\}$
Case 3: Three options are correct and A is one of them.
Number of possible elements $= {}^{3}C_2 = 3$ i.e. $\{(A,B,C),(A,B,D),(A,C,D)\}$
Case 4: All options are correct.
Number of possible elements 1 i.e. $\{(A,B,C,D)\}$
Hence, Cardinality of $E = 1 + 3 + 3 + 1 = 2^3 = 8$

3. A person predicts daily whether the price of stocks of wrist watch companies will go up or down. If his prediction on stock price of Titan is correct a times out of b, for Rolex it is correct p times out of q and for Fossil it is correct x times out of y, then what is the probability that at least two of his predictions are correct on a given day?

a. $\left[\dfrac{a}{b} \times \dfrac{p}{q} \times \left(1 - \dfrac{x}{y}\right)\right] + \left[\dfrac{a}{b} \times \left(1 - \dfrac{p}{q}\right) \times \dfrac{x}{y}\right] + \left[\left(1 - \dfrac{a}{b}\right) \times \dfrac{p}{q} \times \dfrac{x}{y}\right] + \left[\dfrac{a}{b} \times \dfrac{p}{q} \times \dfrac{x}{y}\right]$

b. $\left[\frac{a}{b}\times\frac{p}{q}\times\left(1-\frac{x}{y}\right)\right]+\left[\frac{a}{b}\times\left(1-\frac{p}{q}\right)\times\frac{x}{y}\right]+\left[\left(1-\frac{a}{b}\right)\times\frac{p}{q}\times\frac{x}{y}\right]$

c. $\left[\frac{a}{b}\times\frac{p}{q}\times\frac{x}{y}\right]+\left[\frac{a}{b}\times\frac{p}{q}\times\frac{x}{y}\right]+\left[\frac{a}{b}\times\frac{p}{q}\times\frac{x}{y}\right]+\left[\frac{a}{b}\times\frac{p}{q}\times\frac{x}{y}\right]$

d. $\left[\frac{a}{b}\times\frac{p}{q}\times\frac{x}{y}\right]+\left[\frac{a}{b}\times\frac{p}{q}\times\frac{x}{y}\right]+\left[\frac{a}{b}\times\frac{p}{q}\times\frac{x}{y}\right]$

Answer: a

Solution:

Let us define the following events:
A : Prediction for Titan is correct.
B : Prediction for Rolex is correct.
C : Prediction for Fossil is correct.
We are given that :
$P(A)=\frac{a}{b}$, $P(B)=\frac{p}{q}$ and $P(C)=\frac{x}{y}$

Case 1: Prediction for only Titan and Rolex is correct
$P(A\cap B\cap C^c)=\frac{a}{b}\times\frac{p}{q}\times\left(1-\frac{x}{y}\right)$

Case 2: Prediction for only Titan and Fossil is correct
$P(A\cap B^c\cap C)=\frac{a}{b}\times\left(1-\frac{p}{q}\right)\times\frac{x}{y}$

Case 3: Prediction for only Rolex and Fossil is correct
$P(A^c\cap B\cap C)=\left(1-\frac{a}{b}\right)\times\frac{p}{q}\times\frac{x}{y}$

Case 4: All predictions are correct.
$P(A\cap B\cap C)=\frac{a}{b}\times\frac{p}{q}\times\frac{x}{y}$

Hence, $P(\text{At least two predictions are correct})$
$=\left[\frac{a}{b}\times\frac{p}{q}\times\left(1-\frac{x}{y}\right)\right]+\left[\frac{a}{b}\times\left(1-\frac{p}{q}\right)\times\frac{x}{y}\right]+\left[\left(1-\frac{a}{b}\right)\times\frac{p}{q}\times\frac{x}{y}\right]+\left[\frac{a}{b}\times\frac{p}{q}\times\frac{x}{y}\right]$
Hence, option (a) is correct.

Example: $a=4,b=5,p=,q=6,x=3,y=4$
Let us define the following events:
A : Prediction for Titan is correct.
B : Prediction for Rolex is correct.
C : Prediction for Fossil is correct.
We are given that :
$P(A)=\frac{4}{5}$, $P(B)=\frac{5}{6}$ and $P(C)=\frac{3}{4}$

Case 1: Prediction for only Titan and Rolex is correct

$P(A \cap B \cap C^c) = \frac{4}{5} \times \frac{5}{6} \times \frac{1}{4} = \frac{20}{120}$

Case 2: Prediction for only Titan and Fossil is correct

$P(A \cap B^c \cap C) = \frac{4}{5} \times \frac{1}{6} \times \frac{3}{4} = \frac{12}{120}$

Case 3: Prediction for only Rolex and Fossil is correct

$P(A^c \cap B \cap C) = \frac{1}{5} \times \frac{5}{6} \times \frac{3}{4} = \frac{15}{120}$

Case 4: All predictions are correct.

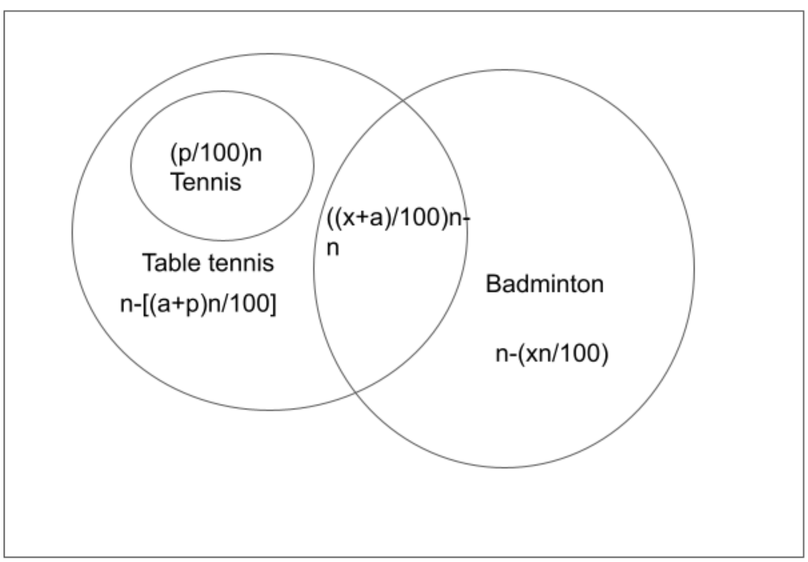
$P(A \cap B \cap C) = \frac{4}{5} \times \frac{5}{6} \times \frac{3}{4} = \frac{60}{120}$

Hence, P(At least two predictions are correct)$= \frac{20}{120} + \frac{15}{120} + \frac{12}{120} + \frac{60}{120} = \frac{107}{120}$

Hence, option (a) is correct.

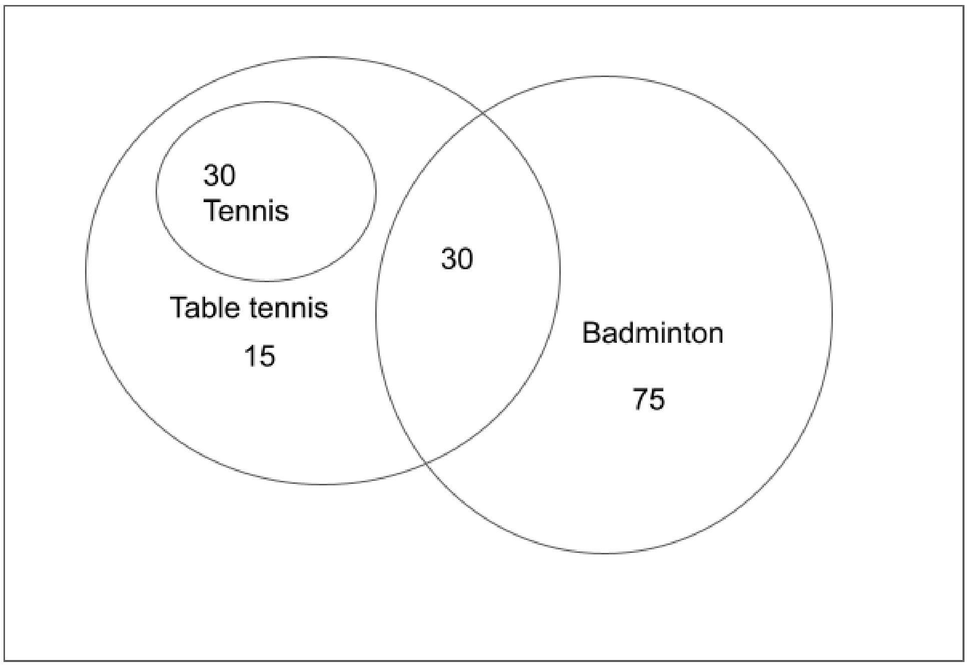
4. There are a total of n students who are part of badminton, table tennis and tennis team of the college. Of which $x\%$ of students play table tennis, $p\%$ play tennis and $a\%$ play badminton. It is also noticed that all students who play tennis also play table tennis, but not badminton. Now a student is selected at random, what is the probability that he/she is the part of table tennis team only? (Enter the answer correct to 1 decimal place.)

   Solution:



Therefore, probability that he/she is the part of table tennis team only $= \left(1 - \dfrac{a+p}{100}\right)$

Example: $n = 150, x = 50, p = 20, a = 70$



Therefore, probability that he/she is the part of table tennis team only $= \dfrac{15}{150} = \dfrac{1}{10}$

5. The chance that a student will clear the quiz 1 paper is a and the chance that he will clear both quiz 1 and quiz 2 papers is b. The chance that he will clear at least one quiz paper is c. What is the chance that he will clear quiz 2 paper? (Enter the answer correct to 2 decimal accuracy)
   Solution:
   Let us define the following events:
   A: Student will clear the quiz 1 paper ; B: Student will clear the quiz 2 paper.
   We are given that:
   $P(A) = a$, $P(A \cap B) = b$, $P(A \cup B) = c$ and we want, $P(B)$
   Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B) \implies c = a + P(B) - b$
   $P(B) = d = c - a + b$

For example: $a = 0.4$, $b = 0.3$, $c = 0.5$
Let us define the following events:
A: Student will clear the quiz 1 paper ; B: Student will clear the quiz 2 paper.
We are given that:
$P(A) = 0.4$, $P(A \cap B) = 0.3$, $P(A \cup B) = 0.5$ and we want, $P(B)$
Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B) \implies 0.5 = 0.4 + P(B) - 0.3$
$P(B) = 0.5 - 0.4 + 0.3 = 0.4$
Therefore, the chance that the student will clear the quiz 2 paper is 0.4

6. If $P(A) = x$ and $P(B) = y$ and probability of the complement of $(A \cup B)$ is z, then calculate $P(A \cup B)$? (Enter the answer correct to 2 decimal point accuracy)
Solution:
$P(A \cup B) = 1 - P(A \cup B)^c = 1 - z$
For example: $x = 0.2$, $y = 0.5$, and $z = 0.4$
$P(A \cup B) = 1 - P(A \cup B)^c = 1 - 0.4 = 0.6$

7. a cards are drawn at random (without replacement) from a pack of 52 cards. Find the probability that b are black and c are red. (Enter the answer correct to two decimal places)
Solution:
Since there are 26 black cards (of spades and clubs) and 26 red cards (of diamonds and hearts) in a pack of cards, the required probability=$\dfrac{^{26}C_b \times ^{26}C_c}{^{52}C_a}$
Example: $a = 4, b = 2$ and $c = 2$
Since there are 26 black cards (of spades and clubs) and 26 red cards (of diamonds and hearts) in a pack of cards, the required probability=$\dfrac{^{26}C_2 \times ^{26}C_2}{^{52}C_4} = 0.39$

Pramod goes to a shop to buy some clothes. Shopkeeper shows him x shirts, y pants and z t-shirts. If he selects three clothes at random, then based on the information, answer the questions 8, 9 and 10.

8. Find the probability that the randomly chosen clothes are of different type. (Enter the answer correct to three decimal places)
Solution:
The total number of cases are $^{x+y+z}C_3$.
Since the number of favourable cases of getting one cloth of each type is
$= {^xC_1} \times {^yC_1} \times {^zC_1}$
Therefore, the required probability=$\dfrac{^xC_1 \times ^yC_1 \times ^zC_1}{^{x+y+z}C_3}$

For example: $x = 5$, $y = 4$, $z = 10$
The total number of cases are $^{19}C_3$.
Since the number of favourable cases of getting one cloth of each type is

$= {}^5C_1 \times {}^4C_1 \times {}^{10}C_1$

Therefore, the required probability$=\dfrac{{}^5C_1 \times {}^4C_1 \times {}^{10}C_1}{{}^{19}C_3} = \dfrac{200}{969} = 0.2064$

9. Find the probability that the randomly chosen clothes does not contain pant. (Enter the answer correct to two decimal places)

   Solution:

   The total number of cases are ${}^{x+y+z}C_3$.

   If randomly chosen clothes does not contain pant, then all the three clothes must be from shirts and t-shirts, i.e., from $x+z$ clothes. Hence, the number of favourable cases for this event is ${}^{x+z}C_3$.

   Therefore, the required probability$=\dfrac{{}^{x+z}C_3}{{}^{x+y+z}C_3}$

   For example: $x = 5$**,** $y = 4$**,** $z = 10$

   The total number of cases are ${}^{19}C_3$.

   If randomly chosen clothes does not contain pant, then all the three clothes must be from shirts and t-shirts, i.e., from $5+10 = 15$ clothes. Hence, the number of favourable cases for this event is ${}^{15}C_3$.

   Therefore, the required probability$= \dfrac{{}^{15}C_3}{{}^{19}C_3} = \dfrac{2730}{5814} = 0.4695$

10. Find the probability that at least one of the clothes is a shirt. (Enter the answer correct to two decimal places)

    Solution:

    The total number of cases are ${}^{x+y+z}C_3$.

    P(at least one of the clothes is shirt) = 1-P(none of the three clothes is shirt)

    In order that none of the 3 clothes is shirt, all the 3 clothes must be from pants and t-shirts,i.e, from $y + z$ clothes and the number of favourable cases for this event is ${}^{y+z}C_3$.

    P(none of the three clothes is shirt)$=\dfrac{{}^{y+z}C_3}{{}^{x+y+z}C_3}$

    Hence, P(at least one of the clothes is shirt)$=1 - \dfrac{{}^{y+z}C_3}{{}^{x+y+z}C_3}$

    For example: $x = 5$**,** $y = 4$**,** $z = 10$

    The total number of cases are ${}^{19}C_3$.

    P(at least one of the clothes is shirt) = 1-P(none of the three clothes is shirt)

    In order that none of the 3 clothes is shirt, all the 3 clothes must be from pants and t-shirts, i.e, from $4 + 10 = 14$ clothes and the number of favourable cases for this event is ${}^{14}C_3$.

    P(none of the three clothes is shirt)$=\dfrac{{}^{14}C_3}{{}^{19}C_3} = \dfrac{2184}{5814}$

    Hence, P(at least one of the clothes is shirt)$=1 - \dfrac{2184}{5814} = 0.624$

11. An urn contains 3 balls numbered $1, 2$ and 3. The co-efficients of the equation

$px^2+qx+c=0$ is determined by drawing the numbered balls with replacement. What is the probability that the equation will have imaginary roots?

Nature of roots:

Consider a quadratic equation: $ax^2+bx+c=0$

Compute $D=b^2-4ac<0$

$$\text{Roots} = \begin{cases} D<0 & \text{imaginary roots} \\ D\geq 0 & \text{real roots} \end{cases}$$

a. $\dfrac{4}{27}$

b. $\dfrac{23}{27}$

c. $\dfrac{16}{27}$

d. None of the above

Answer: b

Solution:

Since each coefficient in equation $px^2+qx+c=0$ is determined by drawing a numbered ball from the urn, each of the coefficients p, q and c can take values from 1 to 3.

Therefore, total number of possible outcomes $=3\times 3\times 3=27$

P[Imaginary roots] $=1-P$[Real roots]

For a Quadratic equation to have real roots, the equation $q^2-4pc\geq 0$ should be satisfied.

The number of favourable cases for real roots are:

pc	p	c	$4pc$	q (such that $q^2-4pc\geq 0$)	No. of cases
1	1	1	4	2, 3	2
2	1	2	8	3	1
	2	1	8	3	1

Hence, Total number of favourable cases for real roots $=2+1+1=4$

Therefore, P[Real roots]$=\dfrac{4}{23}$

P[Imaginary roots]$=1-\dfrac{4}{23}=\dfrac{23}{27}$

Hence, option (b) is correct.

12. If A and B are mutually exclusive or disjoint events, then which of the following is/are always true:

a. $P(A)=P(B)$

b. $P(A) < P(B)$

c. $P(A) \leq P(B^c)$

d. $P(A) \geq P(B^c)$

Answer: c

Solution:

Since, A and B are mutually exclusive or disjoint events.
Therefore, $A \cap B = \phi$
$A = (A \cap B) \cup (A \cap B^c) = \phi \cup (A \cap B^c) = A \cap B^c$
Therefore, $A \subseteq B^c \Rightarrow P(A) \leq P(B^c)$