Week 2 Graded Assignment Solution(September 2022)
Mathematics for Data Science - 1
Max Marks: 10

1 Instructions:

- There are some questions that have functions with discrete-valued domains (such as day, month, year, etc). For simplicity, we treat them as continuous functions.
- For NAT type question, enter only one right answer even if you get multiple answers for that particular question.
- **Notations:**
  - $\mathbb{R}$= Set of real numbers
  - $\mathbb{Q}$= Set of rational numbers
  - $\mathbb{Z}$= Set of integers
  - $\mathbb{N}$= Set of natural numbers
- The set of natural numbers includes 0.

1. A incident ray is passing through the point $(2, 3)$ makes an angle α with horizontal. The ray gets reflected at point M and passes through the point $(5, 2)$ as shown in figure below.

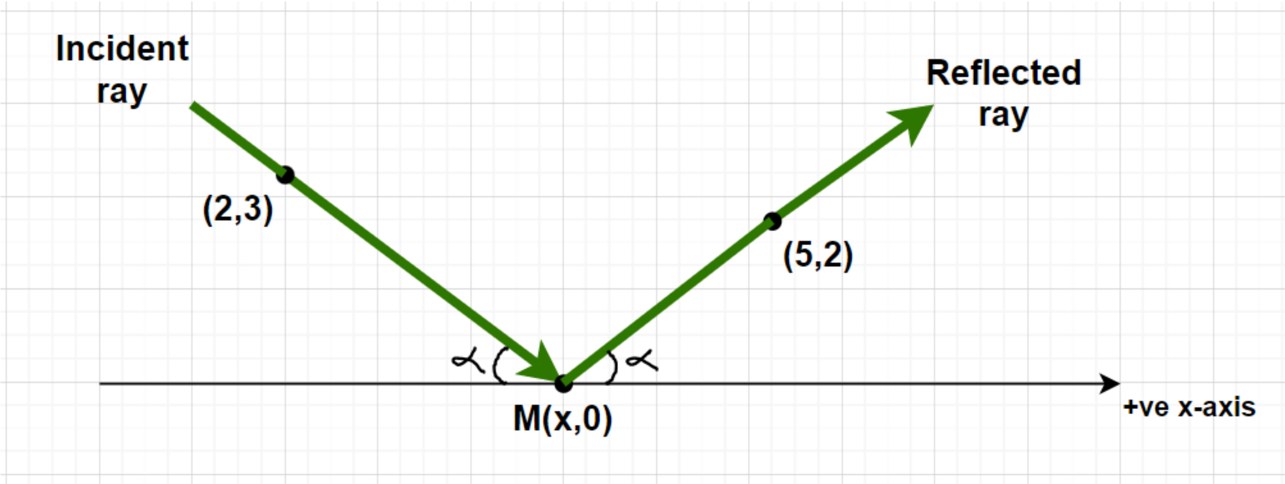


Fig:M1W2Q6

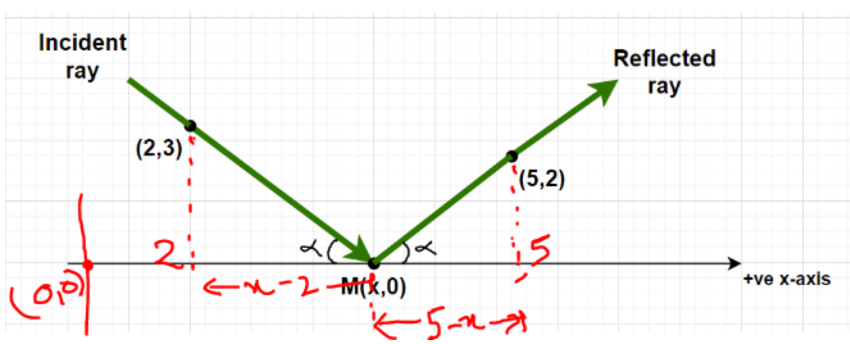
Choose the set of correct option(s).

Answer: Option a and c [MSQ:1 Marks]

- ◯ The equation of incident ray is $-5x - 3y + 19 = 0$
- ◯ The equation of incident ray is $3x + 2y - 12 = 0$
- ◯ The equation of reflected ray is $5x - 3y - 19 = 0$
- ◯ The equation of reflected ray is $2x + y - 12 = 0$

Solution:

Since, incident ray and reflected ray is making same angle α with horizontal, we can find out the value of x. From the fig shown below,



$$tan(\alpha) = \frac{3}{x-2} = \frac{2}{5-x}$$

$15 - 3x = 2x - 4$
$x = \dfrac{19}{5}$

The equation of incident ray is given by : $y - 3 = \dfrac{0-3}{\dfrac{19}{5} - 2}(x-2)$

$(y-3) = \dfrac{-5}{3}(x-2) \Rightarrow -5x - 3y + 19 = 0$

Thus, the equation of incident ray is $-5x - 3y + 19 = 0$.

Similarly, we can find the equation of reflected ray:
$(y-2) = \dfrac{0-2}{\dfrac{19}{5} - 5}(x-5)$

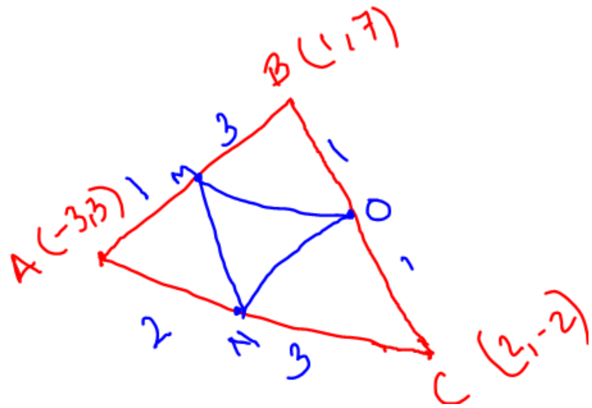
$(y-2) = \dfrac{5}{3}(x-5) \ \Rightarrow 5x - 3y - 19 = 0$

Thus, the equation of reflected ray is $5x - 3y - 19 = 0$.

2. Consider a triangle $\triangle ABC$, whose co-ordinates are $A(-3,3), B(1,7)$and $C(2,-2)$. Let the point M divides the line AB in 1:3, the point N divides the line AC in 2:3 and the point O is the mid-point of BC.
Find out the area of triangle $\triangle MNO$ (in sq. unit).
Answer: 4.5 [NAT: 1marks]

Solution

From the fig shown below, we can find out the co-ordinates of $M, N$and O.



Co-ordinates of M by using section formula:

$$x_1 = \frac{-3 \times 3 + 1 \times 1}{1+3} = -2$$

$$y_1 = \frac{3 \times 3 + 1 \times 7}{1+3} = 4$$

Co-ordinates of N by using section formula:

$$x_2 = \frac{-3 \times 3 + 2 \times 2}{2+3} = -1$$

$$y_2 = \frac{3 \times 3 + 2 \times (-2)}{2+3} = 1$$

Co-ordinates of O by using section formula:

$$x_3 = \frac{1 \times 1 + 1 \times 2}{1+1} = \frac{3}{2}$$

$$y_3 = \frac{1 \times (-2) + 1 \times 7}{1+1} = \frac{5}{2}$$

The area of triangle $\triangle\ MNO$ is given by :

$$= \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

$$=\frac{1}{2}|(-2)(1-\frac{5}{2})+(-1)(\frac{5}{2}-4)+\frac{3}{2}(4-1)|$$

$$=\frac{1}{2}|3+\frac{3}{2}+\frac{9}{2}|$$

$$= 4.5 \text{ sq. unit}$$

3. Ramesh works in the MNC ltd. company as a sales manager. He receives a monthly base salary and a ₹500 commission for each unit he sells. At the end of the month he figures out that he sold 100 units and received ₹80,000 at the end of the month. How much is Ramesh's monthly base salary?

   Answer: 30000 [NAT:1marks]

   Solution: Let x be the number of units and c be the base salary.

   The total salary Ramesh received at the end of month = Base salary + commission per unit × no. of units

   It is given that, he sold 100 units and received ₹80,000 at the end of the month.
   $80000 = c + 500 \times x$
   $80000 = c + 500 \times 100$
   $c = 80000 - 50000$
   $c = 30000$
   Thus, Ramesh's monthly base salary is 30000.

Consider the following scenario and answer the question 4 and 5

Consider the scenario where a fighter jet Mig-21 flying at a height of 2800m from origin $O(0,0)$. The air defense system S-400 is located at a distance of 400m from origin. The missile Brahmos is fired from Mig-21 towards the warship and it follows a straight line path. The missile makes an angle of $45°$ (measured in clockwise direction) with the ground (horizontal axis) if it hits the warship. The air defense system S-400 tracked the path of missile and fires a missile Triumpf towards it, missile Triumpf also follows a straight line path having slope 2. Let P be the point where Triumpf missile hits the Brahmos missile.

The below figure illustrates the given scenario.

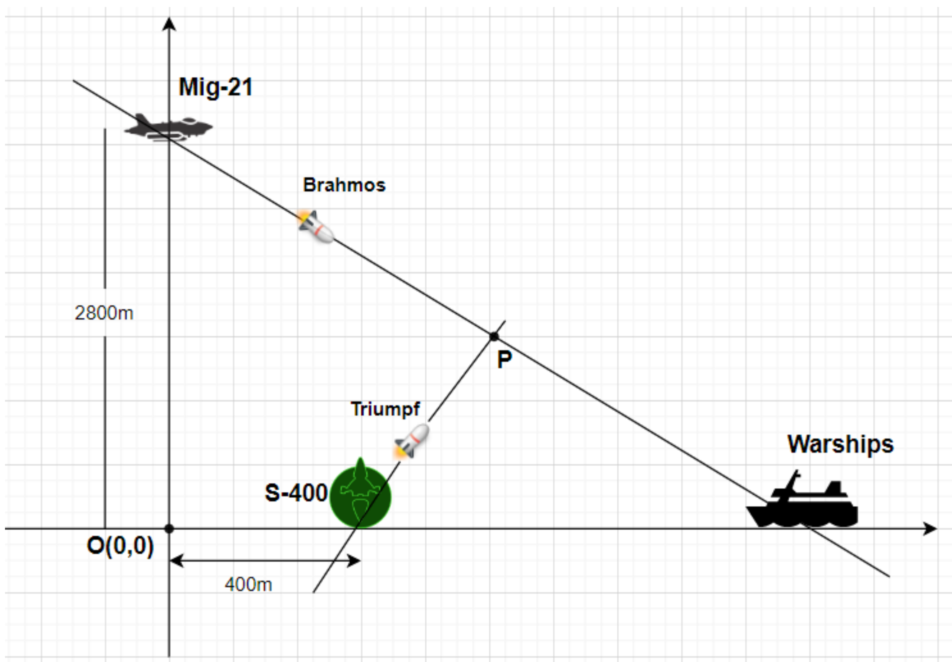
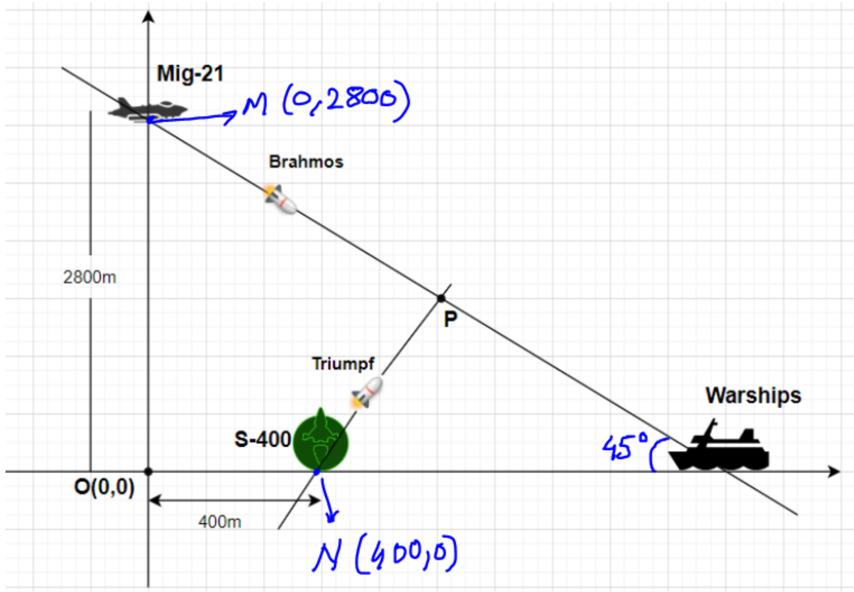


Fig:M1W2Q2

4. Find out the distance between point P and origin O in meters.

Answer: 2000 [NAT:1marks]

Solution:



Refer to the above figure:
The co-ordinates of M is $(0, 2800)$ and co-ordinates of N is $(400, 0)$. The path of brahmos missile makes an angle of $45°$ (measured in clockwise direction) with the ground (horizontal axis) and passing through M, and equation given by $y = mx + c$.
slope $m = tan(180 - 45) = -1$
So, equation becomes $y = -x + c$ and it passing through $M(0, 2800)$.
$2800 = 0 + c \Rightarrow c = 2800$
Thus, the equation of path followed by brahmos missile is $y = -x + 2800$......(1)

Similary, equation of path followed by Triumpf missile having slope 2 and passing through $N(400, 0)$ is $y = 2x - 800$.......(2).

In order to find out the co-ordinates of point P, solving equation 1 and 2 simultaneously, we get,
$-x + 2800 = 2x - 800 \Rightarrow x = 1200$ and $y = 1600$

The distance between point P and origin O is
$= \sqrt{(1200 - 0)^2 + (1600 - 0)^2}$
$= 2000$

5. If the speed of Brahmos missile is $96\sqrt{10}$ m/sec, then what should be the speed of Triumpf missile (in m/sec), so that it should hit the Brahmos missile at point P.
Answer: 320 [NAT:1marks]

Solution:
We know that $speed = \dfrac{Distance}{Time}$, the time required for both the missile are same.

$\dfrac{distance_{MP}}{speed_{Brahmos}} = \dfrac{distance_{SP}}{speed_{Triumpf}}$ ....(A)

$distance_{MP} = \sqrt{(1200-0)^2 + (1600-2800)^2} = 12\sqrt{2}\times 10^2$
$distance_{SP} = \sqrt{(1200-400)^2 + (1600-0)^2} = \sqrt{320}\times 10^2$

From (A), $\dfrac{12\sqrt{2}\times 10^2}{96\sqrt{10}} = \dfrac{\sqrt{320}\times 10^2}{speed_{Triumpf}}$

$speed_{Triumpf} = 320$

6. A bird is flying along the straight line $2y - 6x = 20$. In the same plane, an aeroplane starts to fly in a straight line and passes through the point $(4, 12)$. Consider the point where aeroplane starts to fly as origin. If the bird and plane collides then enter the answer as 1 and if not then 0.
Note: Bird and aeroplane can be considered to be of negligible size.
(NAT)(Answer: 0) [Marks: 1]
Solution:
As per the given statements, aeroplane passes through the point$(4, 12)$ and origin $O(0, 0)$.
Therefore, the slope of line will be $\dfrac{12-0}{4-0} = 3$ and the equation of line will be $y = 3x$

A bird is flying along the straight line $2y - 6x = 20$, the slope of line is 3. Since, the slope of both the path are same and y-intercept are different. So, they are parallel and hence, they will never collides.
The correct answer is 0.

7. Find out the perimeter of the triangle formed by the intersections of following 3 lines.
$L_1 : 2x + 3y - 6 = 0$
$L_2 : 3x + 2y + 6 = 0$
$L_3 : 3x - 3y + 6 = 0$
Answer: 17.25 [NAT:1 marks] Range: 17 to 18

Solution: Let $M(x_1, y_1)$ be the point of intersection of line L_1 and L_2.
By, solving the equations simultaneously,
$3L_1 - 2L_2 \Rightarrow y = 6$ and $x = -6$. Thus, the co-ordinates of $M(-6, 6)$.

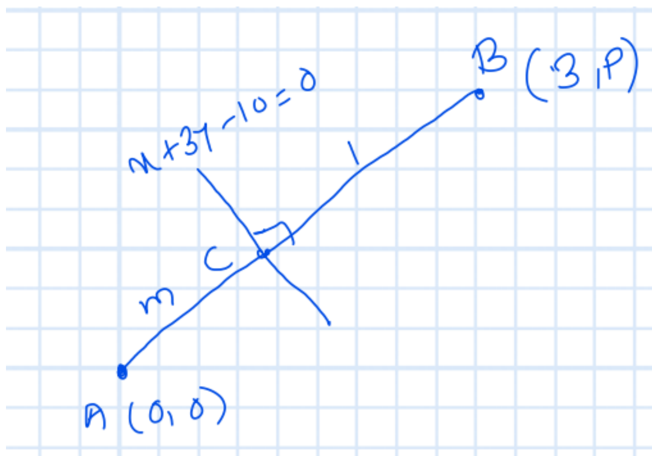
Let $N(x_2, y_2)$ be the point of intersection of line L_1 and L_3.
By, solving the equations simultaneously,
$3L_1 - 2L_3 \Rightarrow y = 2$ and $x = 0$. Thus, the co-ordinates of $N(0, 2)$.

Let $O(x_3, y_3)$ be the point of intersection of line L_2 and L_3.
By, solving the equations simultaneously,
$L_2 - L_3 \Rightarrow y = 0$ and $x = -2$. Thus, the co-ordinates of $O(-2, 0)$.
The perimeter of the triangle $\triangle MNO$ formed by the intersections of 3 lines is given by: $MN + NO + MO$.
$MN = \sqrt{(0 - (-6))^2 + (2 - 6)^2} = \sqrt{52}$
$NO = \sqrt{(-2 - 0)^2 + (0 - 2)^2} = \sqrt{8}$
$MO = \sqrt{(-2 - (-6))^2 + (0 - 6)^2} = \sqrt{52}$

So, perimeter of $\triangle MNO$is $\sqrt{52} + \sqrt{8} + \sqrt{52} = 17.25$

8. Suppose the equation $x + 3y - 10 = 0$ represents a straight line which is perpendicular to the line segment joining the points $A(0, 0)$ and $B(3, p)$, divides it at C in the ratio of $m : 1$, where $p, m \in \mathbb{R}$. Find the value of $p + 4m$. (NAT:1 mark)
Solution:

As per the given condition, following rough plot can be made:



The slope of $x + 3y - 10 = 0$ is $\frac{-1}{3}$. Since, it is given that, $x + 3y - 10 = 0$ is perpendicular to the line segment joining the points $A(0, 0)$ and $B(3, p)$. Therefore, the slope of line AB is 3.
$\frac{p - 0}{3 - 0} = 3 \Rightarrow p = 9$
The equation of line AB will be $y = 3x + c$, but since is passing through $A(0, 0)$, so $c = 0$. Thus, equation of line is $y = 3x$.
The point of intersection of line $y = 3x$ and $x + 3y - 10 = 0$, will be at c.
Solving these equations we get,
$10x - 10 = 0 \Rightarrow x = 1$ and $y = 3$
Co-ordinates of c is $(1, 3)$.
Using section formula (for x- coordinate of c)
$1 = \frac{3m + 0}{m + 1} \Rightarrow$m + 1 = 3m
m = $\frac{1}{2}$.
Thus, $p + 4m = 9 + 4 \times \frac{1}{2} = 11$

9. The distance between two parallel lines $3x + 4y + c_1 = 0$ and $3x + 4y + c_2 = 0$ is 4, where $c_2 > c_1 > 0$.The minimum distance between the point (2,3) and the line $3x + 4y + c_1 = 0$ is 6. Find out the value of $c_1 + c_2$.

   Answer: 44 [NAT:1marks]

   Solution:

   The distance between two parallel lines $3x + 4y + c_1 = 0$ and $3x + 4y + c_2 = 0$ is given by :

   $d = \dfrac{|c_2 - c_1|}{\sqrt{3^2 + 4^2}}$

   $4 = \dfrac{c_2 - c_1}{5}$.............{given that $c_2 > c_1$}

   $c_2 - c_1 = 20$......................(1)

   The minimum distance (means perpendicular distance)between the point (2,3) and the line $3x + 4y + c_1 = 0$ is given by:

   $d = \dfrac{3 \times 2 + 4 \times 3 + c_1}{\sqrt{3^2 + 4^2}}$

   $6 = \dfrac{18 + c_1}{5} \Rightarrow c_1 = 12$. From (1), we get $c_2 = 20 + 12 = 32$.

   $c_1 + c_2 = 32 + 12 = 44$

10. A rock is thrown in a pond, and creates circular ripples whose radius increases at a rate of 0.2 meter per second. What will be the value of $\frac{A}{\pi}$, where A is the area (in square meter) of the circle after 10 seconds?

Hint: The area of a circle $= \pi r^2$, where r is the radius of the circle. (NAT:1 marks)

Solution:
The area of a circle (A)$= \pi r^2$
The radius of circular ripples after 10 sec = 0.2 x 10 = 2m

$$r^2 = \frac{A}{\pi}$$

$$\frac{A}{\pi} = 2^2 = 4$$

Week 2

Graded Assignment Solution.

Mathematics for Data Science - 1

1. A ray of light passing through the point $A(1, 2)$ is reflected at a point B on X-axis and then passes through the point $(5, 3)$. Then the equation of straight line AB is (1 marks)

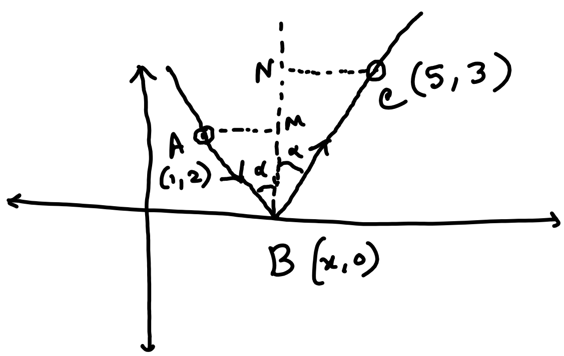
   ◯ **Option 1:** $5x + 4y = 13$

   ◯ Option 2: $5x - 4y = -3$

   ◯ Option 3: $4x + 5y = 14$

   ◯ Option 4: $4x - 5y = -6$

Soln.



$$\tan\alpha = \frac{AM}{BM} = \frac{CN}{BN}$$

$$\Rightarrow \frac{x-1}{2} = \frac{5-x}{3}$$

$$\Rightarrow 3x - 3 = 10 - 2x$$

$$\Rightarrow 5x = 13$$

$$\Rightarrow x = \frac{13}{5}$$

coordinate of B is $\left(\frac{13}{5}, 0\right)$

Hence equation of AB is,

$$\frac{y-0}{2-0} = \frac{x - \frac{13}{5}}{1 - \frac{13}{5}}$$

$$\Rightarrow \frac{y}{2} = \frac{5x-13}{-8} \Rightarrow y = \frac{5x-13}{-4}$$

$$\Rightarrow -4y = 5x - 13$$

$$\Rightarrow \underline{5x + 4y = 13}$$

2. If p is the length of perpendicular from the origin to the line whose intercept of the axes are a and b, then which of the following options always holds? (1 marks)

- ◯ Option 1: $\frac{1}{p^2} = \frac{1}{a^2} - \frac{1}{b^2}$
- ◯ Option 2: $\frac{1}{p^2} = \frac{1}{(a+b)^2} + \frac{1}{(a-b)^2}$
- ◯ **Option 3:** $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$
- ◯ Option 4: $\frac{1}{p^2} = (\frac{1}{a} + \frac{1}{b})^2$

Soln. Eqn of the line whose intercept of the axes are a and b is,

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\Rightarrow \frac{x}{a} + \frac{y}{b} - 1 = 0$$

Length of perp. from $(0,0)$ to the line $\frac{x}{a} + \frac{y}{b} - 1 = 0$ is,

$$\frac{\left|\frac{0}{a} + \frac{0}{b} - 1\right|}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}}$$

given that $p = \dfrac{|-1|}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}}$

$$\Rightarrow \sqrt{\frac{1}{a^2} + \frac{1}{b^2}} = \frac{|-1|}{p}$$

$$\Rightarrow \frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{p^2}.$$

3. If p and q are the lengths of the perpendiculars from the origin to the lines $x\ cos\theta - y\ sin\theta = k\ cos2\theta$ and $x\ sec\theta + y\ cosec\theta = k,\ k \neq 0$, repectively. Which of the following options always holds? [Hint: You may have to use the following formulas: $sin2\theta = 2sin\theta\ cos\theta$ and $sin^2\theta + cos^2\theta = 1$] (1 marks)

   - ◯ Option 1: $p^2 + q^2 = k^2$
   - ◯ **Option 2:** $p^2 + 4q^2 = k^2$
   - ◯ Option 3: $p^2 - q^2 = k^2$
   - ◯ Option 4: $4p^2 + q^2 = k^2$

Soln:

$$p = \frac{|0 \cdot \cos\theta - 0 \cdot \sin\theta - k\cos 2\theta|}{\sqrt{\cos^2\theta + \sin^2\theta}}.$$

$$\Rightarrow p^2 = k^2 \cos^2 2\theta.$$

$$q = \frac{|0 \cdot \sec\theta + 0 \cdot \operatorname{cosec}\theta - k|}{\sqrt{\sec^2\theta + \operatorname{cosec}^2\theta}}$$

$$\Rightarrow q^2 = \frac{k^2}{\sec^2\theta + \operatorname{cosec}^2\theta}$$

$$\Rightarrow q^2 = \frac{k^2 \cos^2\theta \sin^2\theta}{1}$$

$$\Rightarrow 4q^2 = k^2 \cdot (2\sin\theta\cos\theta)^2 = k^2 \sin^2 2\theta.$$

Hence, $p^2 + 4q^2 = k^2$.

4. A line l is such that its segment between the lines $x - y + 2 = 0$ and $x + y - 1 = 0$ is internally bisected at the point $(1, 1.5)$. What is the equation of the line l? (1 marks)

- ◯ Option 1: $x + 2y = 1$
- ◯ Option 2: $x - 2y = 3$
- ◯ Option 3: $y = 3x$
- ◯ **Option 4:** $x = 1$

Soln. Let l intersects $x - y + 2 = 0$ at $A(x_1, y_1)$ and $x + y - 1 = 0$ at $B(x_2, y_2)$

Hence, $\frac{x_1 + x_2}{2} = 1$ and $\frac{y_1 + y_2}{2} = 1.5$

$\Rightarrow x_1 + x_2 = 2$ and $y_1 + y_2 = 3$

Moreover, $x_1 - y_1 + 2 = 0$
and $x_2 + y_2 - 1 = 0$

$\Rightarrow (x_1 + x_2) - (y_1 - y_2) + 1 = 0$

$\Rightarrow 2 - (y_1 - y_2) + 1 = 0$

$\Rightarrow y_1 - y_2 = 3$

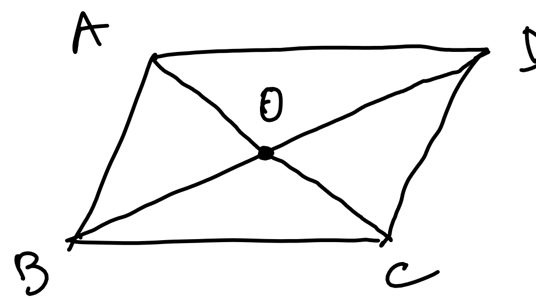
Hence, from $y_1 + y_2 = 3$ and $y_1 - y_2 = 3$, we get
$y_1 = 3$ and $y_2 = 0$.
Hence, $x_1 = 1$ and $x_2 = 1$.

Hence eqn. of l is $\underline{x = 1}$

5. Let $ABCD$ be a parallelogram with vertices $A(x_1, y_1)$, $B(x_2, y_2)$, and $C(x_3, y_3)$. Which of the following always denotes the coordinate of the fourth vertex D? (1 marks)

- ◯ Option 1: $(x_1 + x_2 + x_3, y_1 + y_2 + y_3)$
- ◯ **Option 2:** $(x_1 - x_2 + x_3, y_1 - y_2 + y_3)$
- ◯ Option 3: $(x_1 + x_2 - x_3, y_1 + y_2 - y_3)$
- ◯ Option 4: $(x_1 - x_2 - x_3, y_1 - y_2 - y_3)$

Soln:



For a parallelogram ABCD the diagonals AC and BD intersect each other at the midpoint.

Hence the coordinate of O is,

$$\left(\frac{x_1 + x_3}{2}, \frac{y_1 + y_3}{2}\right)$$

Let the coordinate of D is, (h, k)

then,

$$\frac{x_2 + h}{2} = \frac{x_1 + x_3}{2}$$

and $$\frac{y_2 + k}{2} = \frac{y_1 + y_3}{2}$$

Hence, $h = x_1 - x_2 + x_3$ and $k = y_1 - y_2 + y_3$.

6. A bird is flying along the straight line $2y - 6x = 20$. In the same plane, an aeroplane starts to fly in a straight line and passes through the point $(4, 12)$. Consider the point where aeroplane starts to fly as origin. If the bird and plane collides then enter the answer as 1 and if not then 0.
Note: Bird and aeroplane can be considered to be of negligible size.
(Answer: 0) [Marks: 1]

Soln. The eqn. of st. line along which the aeroplane flies is,

$$\frac{y-0}{12-0} = \frac{x-0}{4-0}$$

$$\Rightarrow \frac{y}{12} = \frac{x}{4} \Rightarrow y = 3x$$

The bird is flying along the st. line

$$2y - 6x = 20$$

$$\Rightarrow y = 3x + 10$$

Both the st. lines have the same slopes. Hence, they are parallel. So bird and plane donot collide.

7. To determine the gas constant R, two students A and B perform an experiment based on the ideal gas equation given as $Pv = RT$. Both use the same gaseous sample having $v = 16.6\ m^3/mol$ and reported the approximate value of R as $8.3\ J/(Kmol)$ using the minimisation of sum squared error. The data collected by both the students are reported below. Choose the correct options: (1 marks)

$T(K)$	274	276	278	282	290
$P(Pa)$	137	139	142	141	142

Data collected by student A.

$T(K)$	276	280	284	288	290
$P(Pa)$	137	141	142	148	145

Data collected by student B.

- ○ Option 1: A has better fit than B. ✗
- ○ **Option 2:** B has better fit than A. ✓
- ○ Option 3: A and B both have same fit. ✗
- ○ **Option 4:** SSE calculated by B is 18. ✗
- ○ Option 5: SSE calculated by A is 14. ✗
- ○ Option 6: SSE calculated by both A and B is 18. ✗

Soln:

$Pv = RT$

$\Rightarrow P(16.6) = (8.3)T \Rightarrow 2P = T$

For A

P	T collected	T original	Error	$(\text{Error})^2$
137	274	274	0	0
139	276	278	2	4
142	278	284	6	36
141	282	282	0	0
142	290	284	−6	36

Sum squared error

$= 76.$

For B

P	T (collected)	T original	Error	$(\text{Error})^2$
137	276	274	−2	4
141	280	282	2	4
142	284	284	0	0
148	288	296	8	64
145	290	290	0	0

Sum squared error

= 72

8. A carpenter has a call out fee (basic charges) of ₹100 and also charges ₹90 per hour. Which of the following are true? (1 marks)

   - ○ Option 1: Following the same notations of y, x, equation of the total cost is represented by $y = 100x + 90$. ✗
   - ○ **Option 2:** If y is the total cost in (₹) and x is the total number of working hours, then the equation of the total cost is represented by $y = 90x + 100$.
   - ○ **Option 3:** The total charges, if the carpenter has worked for 4 hours, would be ₹420. ✗
   - ○ Option 4: If the carpenter charged ₹350 for fixing a L-stand and changing door locks, then the number of working hours would be approximately one hour and 53 minutes. ✗

<u>Soln:</u>

Total cost (y)

$$y = 90x + 100.$$

9. A line perpendicular to the line segment joining the points $A\,(1,0)$ and $B\,(2,3)$, divides it at C in the ratio of $1:3$. Then the equation of the line is (1 marks)

- ◯ $2x+6y-9=0$
- ◯ $\mathbf{2x+6y-7=0}$
- ◯ $2x-6y-9=0$
- ◯ $2x-6y+7=0$

Soln:. Slope of the line segment AB is $= \dfrac{3-0}{2-1}$

$= 3$

Hence Slope of the line perp. to it is $= -1/3$

Let the eqn. of the line be

$$y = -\tfrac{1}{3}x + c.$$

The coordinate of C is $\left(\dfrac{1\times 2+3\times 1}{1+3}, \dfrac{1\times 3+3\times 0}{1+3}\right)$

$$= \left(\frac{5}{4}, \frac{3}{4}\right)$$

$$y = -\tfrac{1}{3}x + c$$

$$\Rightarrow \frac{3}{4} = -\frac{1}{3}\left(\frac{5}{4}\right) + c$$

$$\Rightarrow c = \frac{3}{4} + \frac{5}{12} = \frac{9+5}{12} = \frac{14}{12} = \frac{7}{6}$$

$$y = -\tfrac{1}{3}x + \frac{7}{6} \Rightarrow 6y = -2x+7$$

$$\Rightarrow 2x+6y-7=0.$$

10. A rock is thrown in a pond, and creates circular ripples whose radius increases at a rate of 0.2 meter per second. What will be the value of $\frac{A}{\pi}$, where A is the area (in square meter) of the circle after 5 seconds?

Hint: The area of a circle $= \pi r^2$, where r is the radius of the circle. (1 marks)

Soln:

$r = 0.2\,t$ After 5 seconds,

$r = 0.2 \times 5$

Hence after P_1 seconds,

$A = \pi r^2 = \pi(0.04 \times 5^2)$

$\Rightarrow \frac{A}{\pi} = 1$

Week 2 Graded assignment (May 2023)
Mathematics for Data Science - 1

1 Instructions:

- There are some questions that have functions with discrete-valued domains (such as day, month, year, etc). For simplicity, we treat them as continuous functions.
- For NAT type question, enter only one right answer even if you get multiple answers for that particular question.
- **Notations:**
  - $\mathbb{R}$= Set of real numbers
  - $\mathbb{Q}$= Set of rational numbers
  - $\mathbb{Z}$= Set of integers
  - $\mathbb{N}$= Set of natural numbers
- The set of natural numbers includes 0.

1. Find the y - coordinate of the point of intersection of straight lines represented by (1) and (2), given the following equations:

$$ax + by + c = E \tag{1}$$

$$bx + cy + d^2 = F \tag{2}$$

Given that
$E = F = 0$
Arithmetic mean of a and b is c
Geometric mean of a and b is d
Choose the set of correct option(s).
Note:
Arithmetic mean of m and n is $\frac{m+n}{2}$
Geometric mean of m and n is $\sqrt{mn}$
Answer: Option a [MCQ:1 Marks]

- ○ $\left(\frac{2a^2b-ab-b^2}{2b^2-a^2-ab}\right)$
- ○ $\left(\frac{a^2}{a-b}-1\right)$
- ○ $\left(\frac{2b^2b-ab-b^2}{2b^2-b^2-ab}\right)$
- ○ $\left(\frac{b^2}{a-b}-1\right)$

Sol^n

c is the arithmetic mean of a and b

so $c = \frac{a+b}{2}$

d is the geometric mean of a and b

so $d = \sqrt{ab}$

Once we will solve the equation (1) and (2) we get value of x and y

so, Let multiply by b both sides of equation (1) and multiply by a both sides of equation (2)

we get,

$$abx + b^2y + bc = 0 \quad \text{——} \quad (3)$$

$$abx + acy + ad^2 = 0 \quad \text{——} \quad (4)$$

Subtrating both sides of the equation (3) from equation both sides of the equation (4) we get,

$$(ac - b^2)y + ad^2 - bc = 0$$

$$\Rightarrow \quad y = \frac{bc - ad^2}{ac - b^2} = \frac{b\left(\frac{a+b}{2}\right) - a\left(\sqrt{ab}\right)^2}{a\left(\frac{a+b}{2}\right) - b^2}$$

$$= \frac{ab + b^2 - 2a^2b}{a^2 + ab - 2b^2}$$

$$y = \frac{2a^2b - ab - b^2}{2b^2 - a^2 - ab}.$$

Hence, y-co-ordinate of the intersection point is $\dfrac{2a^2b - ab - b^2}{2b^2 - a^2 - ab}.$

2. A mobile company wants to launch its new model in collaboration with a network provider named **Astron** to attract more customers. The following are two options the mobile company gives to buy the mobile.

Option 1 - Mobile and 1-year Astron Network costs 34000 rupees (Network offers unlimited calls for one year)
Option 2 - Only Mobile costs 22000 rupees
Now Lalith wants to buy this new model mobile. Lalith needs only 200 minutes per month. There are two network providers in the country.

Network Provider	Fixed Charge (Per month)	Per minute Charge
Astron	100	2
Proton	200	0.5

Answer the following questions:

(a) What is the best option for Lalith to buy the mobile?
Choose the set of correct option(s).
Answer: Option c [MCQ:1 Marks]

- ○ Buy only a mobile and take 1 year Astron.
- ○ Buy mobile along with 1-year Astron network offered by the mobile company
- ○ Buy only mobile and take 1 year Proton network
- ○ Buy only mobile and use Astron network for 6 months and Proton network for 6 months.

(b) How much will he save per year if he chooses the best option to buy the mobile compared to the collaborated offer given by the company?
Answer: 8400 [NAT:1 Marks]

(a) We have two cases and 3rd case we will see later.

Case ① If Lalith uses option ① for one year then he have to pay ₹ 34000

Case ② If Lalith uses option ② for one year then he have to pay, mobile costs + mobile recharge.

So lets form two equation according to the table as fixed charge and rate is given.

So for Astron → $y = 2x + 100$

and for proton $y = 0.5x + 200$

where x is the number of minutes using by a person and y is the cost per month.

So for 200 min in a month recharge cost when Lalith uses Astron network is $2 \times 200 + 100 = 500$

And so for one 1 year recharge cost $= 12 \times 500 = 6000$

Similarly for proton, recharge cost for 12 month for Lalith is $12 \times (0.5 \times 200 + 200) = 3600$

So case ② with proton network is $22000 + 3600 = 25600$

and case ② with Astron network is $22000 + 6000 = 28000$

Case ③ if Lalith use option ② with 6 month proton and 6 month Astron network then, for 12 month Lalith have to pay $22000 + 6(2 \times 200 + 100) + 6 \times (0.5 \times 200 + 200)$

$$= 22000 + 3000 + 1800$$
$$= 26800$$

Hence option ③ is true.

⑥ If Lalith uses best option to buy mobile then have to pay for one year (with mobile recharge) is 25600 ₹

And with collaborated offer is 34000 ₹

So save money $= 34000 - 25600$
$= 8400$ ₹

3. State Government wants to connect the state road to the national highway from a town. There are 3 possible locations in the town A,B and C to connect to the National Highway whose locations are given by coordinates $(3,8)$,$(5,7)$,$(6,9)$. The National Highway connects the 2 points $(2,1)$,$(10,7)$ and You, being the contractor, have the freedom to select any one of the 3 possible locations in the town.
Hint Always select the shortest path to construct the road.
Note: 1 unit = 100 meter
Answer the following questions:

(a) What point will you select to build the road?

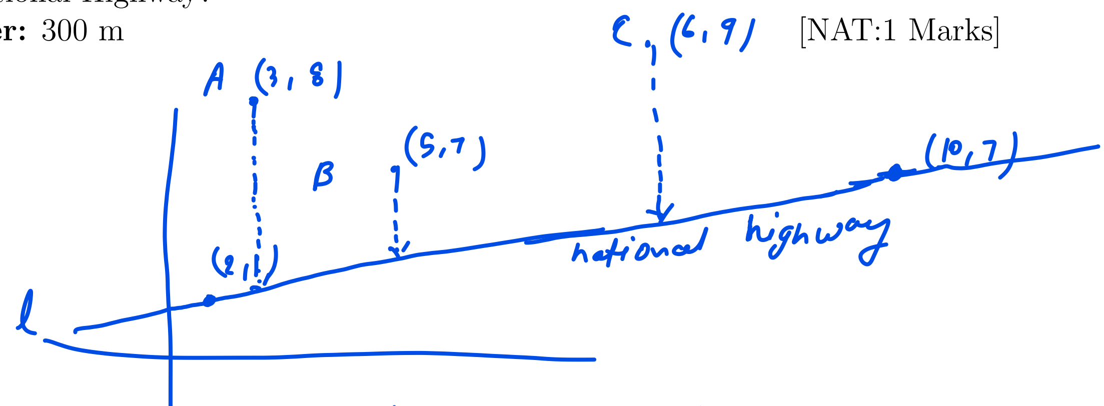
- ◯ A
- ◯ B
- ◯ C
- ◯ None

Answer: Point B [MCQ:1 Marks]

(b) What is the minimum length of road in meter required to construct to connect to the National Highway?

Answer: 300 m [NAT:1 Marks]

(a)



Let's find the equation of national highway.

$$y - 1 = \frac{7-1}{10-2}(x-2)$$

$$\Rightarrow y - 1 = \frac{6}{8}(x-2) \Rightarrow 8y - 8 = 6x - 12$$

$$\Rightarrow 6x - 8y - 4 = 0$$

Now lets find the distance of the points A, B, C from the lines l (national highway) for A, $d_1 = \dfrac{|6\times 3 - 8\times 8 - 4|}{\sqrt{6^2+8^2}} = \dfrac{|18-64-4|}{10} = 5$

for B, $d_2 = \dfrac{|6\times 5 - 8\times 7 - 4|}{\sqrt{6^2+8^2}} = \dfrac{|30-56-4|}{10} = 3$

for C, $d_3 = \dfrac{|6\times 6 - 8\times 9 - 4|}{\sqrt{6^2+8^2}} = \dfrac{|36-72-4|}{10} = 4$

Hence option B is correct.

(b) The minimum length = 300 meter.

4. The total expenses of mess consists of fixed cost and the variable cost, Variable cost is proportional to the number of inmates of the mess, The total expenses are 16000 rupees when 12 members in the mess,and 20000 rupees when 20 members in the mess, find the Fixed cost of the mess. **Answer:** 10000 rupees [NAT:1 Marks]

Solu

If 12 members are in the mess then total expenses = 16000

Similarly, if 20 member are in the mess, then total expenses = 20000

We can think it as point (12, 16000) and (20, 20000)

Let's find the equation of a line passes through these two points:

$$y - 16000 = \frac{4000}{8}(x - 12)$$

$$\Rightarrow y - 16000 = 500(x - 12)$$

$$\Rightarrow y - 16000 = 500x - 6000$$

$$\Rightarrow y = 500x + 10000$$

Hence fixed cost = 10000 ₹

5. A line perpendicular to the line segment joining A $(1, 0)$ and B $(2, 3)$, divides it at C in the ratio of 1:5 internally. Then the equation of line is

- ◯ $3x + 9y - 8 = 0$
- ◯ $3x + 9y + 8 = 0$
- ◯ $x + 3y - 8 = 0$
- ◯ $3x + 9y - 16 = 0$

Given ration $m_1 : m_2 = 1 : 5$

Answer: A [MCQ:1 Marks]

By using section formula :-

point C $(x, y) = \left(\frac{2+5}{6}, \frac{3+0}{6}\right)$

$$(x, y) = \left(\frac{7}{6}, \frac{1}{2}\right)$$

Let's substitute $\left(\frac{7}{6}, \frac{1}{2}\right)$ in equation $3x + 9y - 8 = 0$

$$\frac{7}{2} + \frac{9}{2} - 8 = 0$$

Hence respective line is $3x + 9y - 8 = 0$

This question can be solved by different method also (using slope point form).

6. A bird is flying along the straight line $2y - 6x = 20$. In the same plane, an aeroplane starts to fly in a straight line and passes through the point $(4, 12)$. Consider the point where aeroplane starts to fly as origin. If the bird and plane collides then enter the answer as 1 and if not then 0.
Note: Bird and aeroplane can be considered to be of negligible size.
(Answer: 0) [Marks: 1]

Solⁿ: Slope of the equation of the line

$2y - 6x = 20$ is 3. (we can get by changing equation is slope intercept form)

Slope of the line which passes through the origin the point $(4, 12)$ $= \frac{12}{4} = 3$

Since both lines have the same slope hence both line are parallel.

That means arrow will never hit the bird.

7. A surveyor needs to determine the area of a land show in Fig below. The coordinates of the four vertices of the land are as follows: A $(8, 13)$ B $(3, 10)$ C $(4, 4)$ D $(16, 5)$

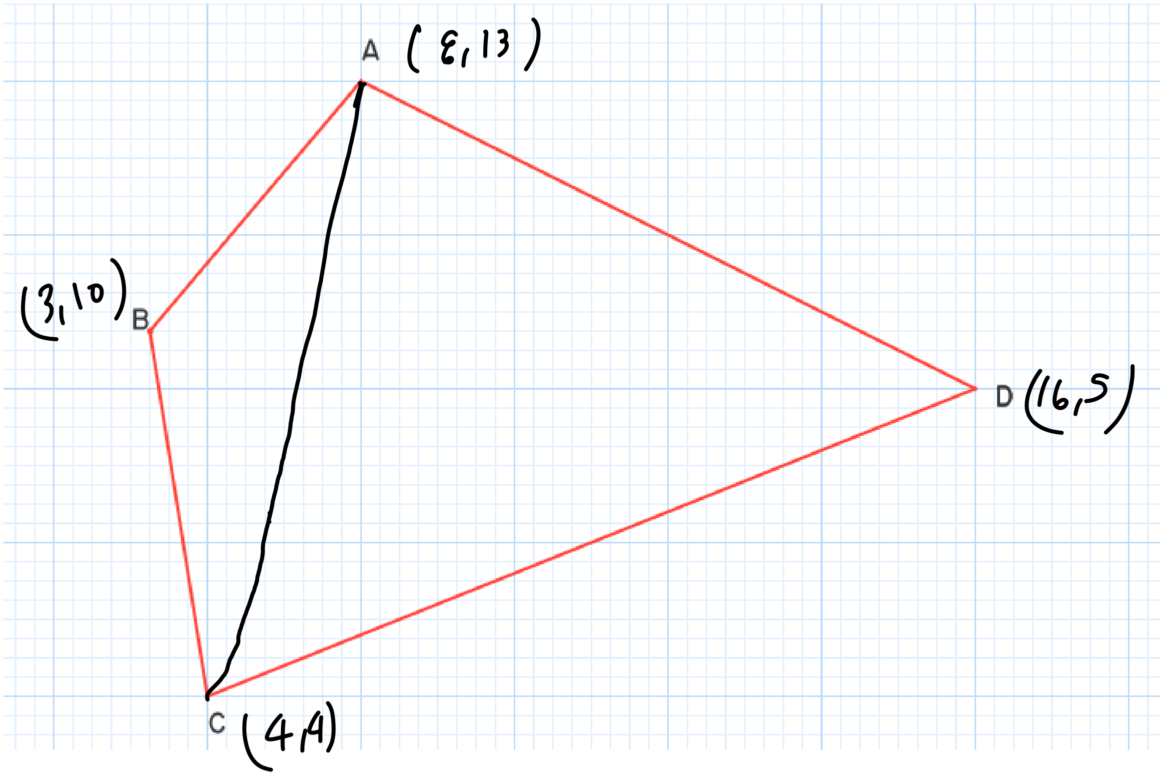


Fig:Survey Area

Answer: 68.5 Sq Units [NAT:1 Marks]

Let's find the find the area of the triangles ABC and triangle ADC

Area of $\Delta ABC = \frac{1}{2} \left| 8(10-4) + 3(4-13) + 4(13-10) \right|$

$= \frac{1}{2} \left| 48 - 27 + 12 \right| = \frac{33}{2} = 16.5$

Area of $\Delta ADC = \frac{1}{2} \left| 8(5-4) + 16(4-13) + 4(13-5) \right|$

$= \frac{1}{2} \left| 8 - 144 + 32 \right|$

$= 52$

Total area $= 52 + 16.5 = 68.5$ sq units.

8. A fitness trainer is analyzing the weight loss progress of his best client over a period of 6 months, to use it as for marketing. He recorded the weight of the client at the beginning and end of each month. Using straight line fitting, he came up with an equation $W = -8t + 98$, where W = Weight in Kg t= time in months. Now you want to check weather this equation is correct or not so you collected the data from the gym the data is given in table below.

Time (months)	Weight (Kgs)
0	98
1	90
2	82
3	74
4	66
5	57
6	49

Answer the following questions

(a) Equation that fitness trainer came up with $W = -8t + 98$ is well fitted to data. (Equation is said to be well fitted to data if the SSE is less than 5) True/False
Answer: True [MCQ:1 Marks]

(b) You were impressed by the performance of the fitness trainer, so you want to get trained under him, you assumed that the rate of weight loss (weight loss per month) will be same as the case of the best client mentioned in the question,Considering your assumption is true, How many days are required for you to loss weight from 100 kg to 72 kg.
Note: 1 month has 30 days
Answer: 105 days [NAT:1 Marks]

(a) We have $W = -8t + 98$

$$SSE = \sum (98-98)^2 + (90+8-98)^2 + (82+16-98)^2 + (74+24-98)^2 + (66+32-98)^2 + (57+40-98)^2 + (49+48-98)^2$$

$$= \sum 0 + 0 + 0 + 0 + 0 + 1 + 1$$

$$= 2 < 5$$

Hence equation is well fitted.

(b) We have

$W = -8t + 98$

So weight loss per month $= 8$ kg

Total weight loss from 100 kg to 72 kg

$= 100 - 72 = 28$ kg

So number of day to loss the weight 28 kg $= \frac{\cancel{28}^{\,7}}{\cancel{8}_{\,2}} \times \cancel{30}^{\,15} = 105$ day.

9. The equation used to measure the C_d (discharge coefficient) of the venturimeter value in the lab is:

$$Q = \frac{c_d}{\sqrt{1-\beta^4}}\sqrt{2g\Delta h} \tag{3}$$

c_d– discharge coefficient
β– $\frac{d}{D}$
A1 – Pipe section
A2 – Restriction area
Δh– head losses
Q – flow rate

Ramesh plotted the graph between Q on the y-axis and $\sqrt{\Delta h}$ on the x-axis and claimed that $\frac{c_d}{\sqrt{1-\beta^4}}\sqrt{2g}$ is the slope of the obtained straight line.

Suresh plotted the graph between $\sqrt{\Delta h}$ on the y-axis and Q on the x-axis and claimed that $\frac{\sqrt{1-\beta^4}}{c_d\sqrt{2g}}$ is the slope of the straight line.
Who is/are actually correct?

- ◯ Only Ramesh
- ◯ Only Suresh
- ◯ Both Ramesh and Suresh
- ◯ None of them are correct

Answer: C [MCQ:1 Marks]

Sol^n:

$$Q = \frac{c_d}{\sqrt{1-\beta^4}}\sqrt{2g\Delta h}$$

$$\Rightarrow Q = \frac{c_d}{\sqrt{1-\beta^4}}\sqrt{2h}\sqrt{\Delta h}$$

Comparing with slope intercept form $y = mx + c$

$$\Rightarrow \text{slope } m = \frac{c_d}{\sqrt{1-\beta^4}}\sqrt{2h}$$

again we have $Q = \frac{C_d}{\sqrt{1-\beta^4}} \sqrt{2g}\sqrt{\Delta h}$

$$\Rightarrow \quad \sqrt{\Delta h} = \frac{\sqrt{1-\beta^4}}{C_d\sqrt{2g}} Q$$

Same as above, slope $m = \dfrac{\sqrt{1-\beta^4}}{C_d\sqrt{2g}}$

Hence option c is true.

10. A rock is thrown in a pond, and creates circular ripples whose radius increases at a rate of 0.2 meter per second. What will be the value of $\frac{A}{\pi}$, where A is the area (in square meter) of the circle after 20 seconds?

<u>Hint:</u> The area of a circle $= \pi r^2$, where r is the radius of the circle. (1 marks)

radius = $0.2t$

So after 20 sec, radius $= 0.2 \times 20$
$= 4$ m.

So area $A = \pi 4^2$

$\Rightarrow \frac{A}{\pi} = 16$