5 Probability and Statistics

5.1 Probability Fundamentals

5.1.1 Definitions

Sample Space and Event: Consider an experiment whose outcome is not predictable with certainty. Such an experiment is called a **random experiment**. However, although the outcome of the experiment will not be known in advance, let us suppose that the set of all possible outcomes is known. This set of all possible outcomes of an experiment is known as the **sample space** of experiment and is denoted by S. Some examples follow.

1. If the outcome of an experiment consist in the determination of the sex of a newborn child, then $S = \{g, b\}$ where the outcome g means that the child is a girl and b is the boy.
2. If the outcome of an experiment consist of what comes up on a single dice, then $S = \{1, 2, 3, 4, 5, 6\}$.
3. If the outcome of an experiment is the order of finish in a race among the 7 horses having post positions 1, 2, 3, 4, 5, 6, 7; then S = {all 7! permutations of the (1, 2, 3, 4, 5, 6, 7)}.

The outcome (2, 3, 1, 6, 5, 4, 7) means, for instances, that the number 2 horse comes in first, then the number 3 horse, then the number 1 horse, and so on.

Any subset E of the sample space is known as **Event**. That is, an event is a set consisting of some or all of the possible outcomes of the experiment. For example, in the throw of a single dice $S = \{1, 2, 3, 4, 5, 6\}$ and some possible events are

$$E_1 = \{1, 2, 3\}$$
$$E_2 = \{3, 4\}$$
$$E_3 = \{1, 4, 6\} \text{ etc.}$$

If the outcome of the experiment is contained in E, then we say that E has occurred. Always $E \subseteq S$.

Since E & S are sets, theorems of set theory may be effectively used to represent and solve probability problems which are more complicated.

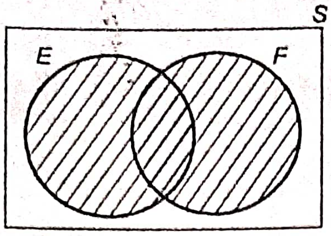
Example: If by throwing a dice, the outcome is 3, then events E_1 and E_2 are said to hare occurred.

In the child example – (i) If $E_1 = \{g\}$, then E_1 is the event that the child is a girl.

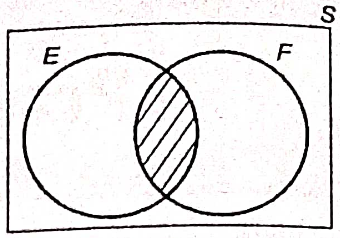
Similarly, if $E_2 = \{b\}$, then E_2 is the event that the child is a boy. These are examples of **Simple** events. **Compound** events may consist of more than one outcome. Such as $E = \{1, 3, 5\}$ for an experiment of throwing a dice. We say event E has happened if the dice comes up 1 or 3 or 5.

For any two events E and F of a sample space S, we define the new event $E \cup F$ to consists of all outcomes that are either in E or in F or in both E and F. That is, the event $E \cup F$ will occur if either E or F or both occurs. For instances, in the dice example (i) if event $E = \{1, 2\}$ and $F = \{3, 4\}$, then $E \cup F = \{1, 2, 3, 4\}$.

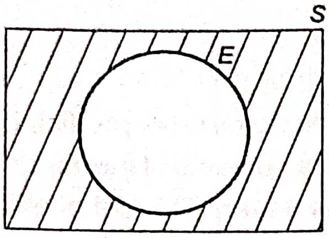
That is $E \cup F$ would be another event consisting of 1 or 2 or 3 or 4. The event $E \cup F$ is called **union** of event E and the event F. Similarly, for any two events E and F we may also define the new event $E \cap F$, called **intersection** of E and F, to consists of all outcomes that are common to both E and F.



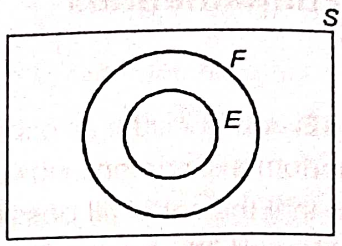
(a) Shaded region : $E \cup F$



(b) Shaded region : $E \cap F$



(c) Shaded region : E^C



(d) $E \subset F$

5.1.2 Types of Events

5.1.2.1 Complementary Event

The event E^C is called complementary event for the event E. It consists of all outcomes not in E, but in S. F
example, in a dice throw, if E = {Even nos} = {2, 4, 6} then E^C = {Odd nos} = {1, 3, 5}.

5.1.2.2 Equally Likely Events

Two events E and F are equally likely iff

$$p(E) = p(F)$$

For example,

$$E = \{1, 2, 3\}$$
$$F = \{4, 5, 6\}$$

are equally likely, since $p(E) = p(F) = 1/2$.

5.1.2.3 Mutually Exclusive Events

Two events E and F are mutually exclusive, if $E \cap F = \phi$ i.e. $p(E \cap F) = 0$. In other words, if E occurs, F cann
occur and if F occurs, then E cannot occur (i.e. both cannot occur together).

5.1.2.4 Collectively Exhaustive Events

Two events E and F are collectively exhaustive, if $E \cup F = S$.i.e. together E and F include all possib
outcomes, $p(E \cup F) = p(S) = 1$.

5.1.2.5 Independent Events

Two events E and F are independent iff

$$p(E \cap F) = p(E) * p(F)$$

Also

$$p(E \mid F) = p(E) \text{ and } p(F \mid E) = p(F).$$

Whenever E and F are independent. i.e. when two events E and F are independent, the conditional probabil
becomes same as marginal probability. i.e. probability E is not affected by whether F has happened or not, a
vice-versa i.e., when E is independent of F, then F is also independent of E.

5.1.3 DeMorgan's Law

1. $\left(\bigcup_{i=1}^{n} E_i\right)^C = \bigcap_{i=1}^{n} E_i^C$

2. $\left(\bigcap_{i=1}^{n} E_i\right)^C = \bigcup_{i=1}^{n} E_i^C$

Example:

$$(E_1 \cup E_2)^C = E_1^C \cap E_2^C$$
$$(E_1 \cap E_2)^C = E_1^C \cup E_2^C$$

Note that $E_1^C \cap E_2^C$ is the event **neither** E_1 **nor** E_2.

$E_1 \cup E_2$ is the event **either** E_1 **or** E_2 **(or both).**

Demorgan's law is often used to find the probability of neither E_1 nor E_2.

i.e. $p(E_1^C \cap E_2^C) = p[(E_1 \cup E_2)^C] = 1 - p(E_1 \cup E_2)$.

5.1.4 Approaches to Probability

There are 2 approaches to quantifying probability of an Event E.

1. **Classical Approach:**

$$P(E) = \frac{n(E)}{n(S)} = \frac{|E|}{|S|}$$

i.e. the ratio of number of ways an event can happen to the number of ways sample space can happen, is the probability of the event. Classical approach assumes that all outcomes are equally likely.

Example 1.

If out all possible jumbles of the word "BIRD", a random word is picked, what is the probability, that this word will start with a "B".

Solution:

$$p(E) = \frac{n(E)}{n(S)}$$

In this problem

$n(S)$ = all possible jumbles of BIRD = 4!

$n(E)$ = those jumbles starting with "B" = 3!

So,

$$p(E) = \frac{n(E)}{n(S)} = \frac{3!}{4!} = \frac{1}{4}.$$

Example 2.

From the following table find the probability of obtaining "A" grade in this exam.

Grade	A	B	C	D
No. of Students	10	20	30	40

Solution:

$$N = \text{total no of students} = 100$$

By frequency approach,

$$p(A\text{ grade}) = \frac{n(A\text{ grade})}{N} = \frac{10}{100} = 0.1$$

5.1.5 Axioms of Probability

Consider an experiment whose sample space is S. For each event E of the sample space S we assume that a number $P(E)$ is defined and satisfies the following three axioms.

Axiom-1: $0 \le P(E) \le 1$

Axiom-2: $P(S) = 1$

Axiom-3: For any sequence of mutually exclusive events E_1, E_2, ...(that is, events for which $E_i \cap E_j = \phi$ when $i \neq j$

$$P\left(\bigcup_{i=1}^{\infty} E_i\right) = \sum_{i=1}^{\infty} P(E_i)$$

Example: $P(E_1 \cup E_2) = P(E_1) + P(E_2)$ where (E_1, E_2 are mutually exclusive).

5.1.6 Rules of Probability

There are six rules of probability using which probability of any compound event involving arbitrary events A and B, can be computed.

Rule 1:

$$p(A \cup B) = p(A) + p(B) - p(A \cap B)$$

This rule is also called the inclusion-exclusion principle of probability.

This formula reduces to

$$p(A \cup B) = p(A) + p(B)$$

if A and B are mutually exclusive, since $p(A \cap B) = 0$ in such a case.

Rule 2:

$$p(A \cap B) = p(A) * p(B/A) = p(B) * p(A/B)$$

where $p(A/B)$ represents the **conditional probability of A given B** and $p(B/A)$ represents the conditional probability of B given A.

(a) $p(A)$ and $p(B)$ are called the **marginal probabilities** of A and B respectively. This rule is also called as the multiplication rule of probability.

(b) $p(A \cap B)$ is called the **joint probability** of A and B.

(c) If A and B are **independent** events, this formula reduces to

$$p(A \cap B) = p(A) * p(B).$$

since when A and B are independent

$$p(A/B) = p(A)$$

and $p(B/A) = p(B)$

i.e. the conditional probabilities become same as the marginal (unconditional) probabilities.

(d) If A and B are independent, then so are A and B^C; A^C and B and A^C and B^C.

(e) Condition for three events to independent:

Events A, B and C are independent iff

$$p(ABC) = p(A)\,p(B)\,p(C)$$

and $p(AB) = p(A)\,p(B)$

and $p(AC) = p(A)\,p(C)$ — A, B, C are pairwise independent

and $p(BC) = p(B)\,p(C)$

Note: *If A, B, C are independent, then A will be independent of any event formed from B and C.* For instance, A is independent of $B \cup C$.

Rule 3: Complementary Probability

$$p(A) = 1 - p(A^C)$$

$p(A^C)$ is called the complementary probability of A and $p(A^C)$ represents the probability that the event A will not happen.

$$\therefore \quad p(A) = 1 - p(A^C)$$

$p(A^C)$ is also written as $p(A')$

Notice that $$p(A) + p(A') = 1$$

i.e. A and A' are mutually exclusion as well as collectively exhaustive.

Also notice that by Demorgan's law since $A^C \cap B^C = (A \cup B)^C$

$$p(A^C \cap B^C) = p(A \cup B)^C = 1 - p(A \cup B)$$

i.e. $$\text{p(neither } A \text{ nor } B) = 1 - p(\text{either } A \text{ or } B)$$

Rule 4: Conditional Probability Rule

Starting from the multiplication rule

$$p(A \cap B) = p(B) * p(A/B)$$

by cross multiplying we get the conditional probability formula

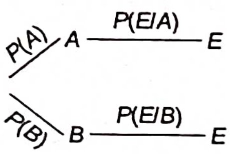
$$p(A/B) = \frac{p(A \cap B)}{p(B)}$$

By interchanging A and B in this formula we get

$$p(B/A) = \frac{p(A \cap B)}{p(A)}$$

Rule 5: Rule of Total Probability

Consider an event E which occurs via two different events A and B. Further more, let A and B be mutually exclusive and collectively exhaustive events. This situation may be represented by following tree diagram



Now, the probability of E is given by value of total probability as

$$P(E) = P(A \cap E) + P(B \cap E) = P(A) * P(E/A) + P(B) * (E/B)$$

This is called rule of total probability.

Sometimes however, we may wish to know that, given that the event E has already occurred, what is the probability that it occurred with A? In this case we can use Bayes Theorem given below.

Example:

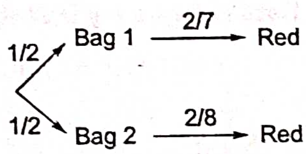
Suppose we have 2 bags. Bag 1 contains 2 red & 5 green marbles. Bag 2 contains 2 red and 6 green marbles. A person tosses a coin & if it is heads goes to bag 1 and draws a marble. If it is tails, he goes to bag 2 and draws a marble. In this situation.

(a) What is the probability that the marble drawn this is Red?

(b) Given that the marble draw is red, what is probability that it came from bag 1.

Solution:

The tree diagram for above problem, is shown below:



(a) $\therefore$ $$P(\text{Red}) = 1/2 \times 2/7 + 1/2 \times 2/8$$

(b) $$P(\text{bag1} \mid \text{Red}) = \frac{P(\text{bag1} \cap \text{Red})}{P(\text{Red})} = \frac{1/2 \times 2/7}{1/2 \times 2/7 + 1/2 \times 2/8} = \frac{1/7}{15/56} = 8/15$$

5.2 Statistics

5.2.1 Introduction

Statistics is a branch of mathematics which gives us the tools to deal with large quantities of data and derive meaningful conclusions about the data. To do this, statistics uses some numbers or measures which describe the general features contained in the data. In other words, using statistics, we can summarise large quantities of data, by a few descriptive measures.

Two descriptive measures are often used to summarise data sets. These are

1. Measure of central tendency
2. Measure of dispersion

The central tendency measure indicates the average value of data, where "average" is a generic term used to indicate a representative value that describes the general centre of the data.

The dispersion measure characterises the extent to which data items differ from the central tendency value. In other words dispersion measures and quantifies the variation in data. The larger this number, the more the variation amongst the data items.

Mean, Median and Mode are some examples of central tendency measures.

Standard deviation, variance and coefficient of variation are examples of dispersion measures.

Now we will study each of these six statistical measures in greater detail.

5.2.2 Arithmetic Mean

5.2.2.1 Arithmetic Mean for Raw Data

The formula for calculating the arithmetic mean for raw data is: $\bar{x} = \frac{\Sigma x}{n}$

$\bar{x}$ - arithmetic mean

x - refers to the value of an observation

n - number of observations.

Example:

The number of visits made by ten mothers to a clinic were; 8 6 5 5 7 4 5 9 7 4
Calculate the average number of visits.

Solution:

Σx = Total of all these numbers of visits, that is the total number of visits made by all mothers.

$8 + 6 + 5 + 5 + 7 + 4 + 5 + 9 + 7 + 4 = 60$

Number of mothers $n = 10$

$$\bar{x} = \frac{\Sigma x}{n} = \frac{60}{10} = 6$$

5.2.2.2 The Arithmetic Mean for Grouped Data (Frequency Distribution)

The formula for the arithmetic mean calculated from a frequency distribution has to be amended to include the frequency. It becomes

$$\bar{x} = \frac{\Sigma(fx)}{\Sigma f}$$

Example:

To show how we can calculate the arithmetic mean of a grouped frequency distribution, there is a example of weights of 75 pigs. The classes and frequencies are given in following table:

Weight (kg)	Midpoint of class x	Number of pigs f (frequency)	fx
0 & under 20	15	1	15
20 & under 30	25	7	175
30 & under 40	35	8	280
40 & under 40	45	11	495
50 & under 60	55	19	1045
60 & under 70	65	10	650
70 & under 80	75	7	525
80 & under 90	85	5	425
90 & under 100	95	4	380
100 & under 110	105	3	215
Total		75	4305

Solution:

With such a frequency distribution we have a range of values of the variable comprising each group. As our values for x in the formula for the arithmetic mean we use the midpoints of the classes.

In this case $$\bar{x} = \frac{\Sigma(fx)}{\Sigma f} = \frac{4305}{75} = 54.4 \text{ kg}$$

5.2.3 Median

Arithmetic mean is the central value of the distribution in the sense that positive and negative deviations from the arithmetic mean balance each other. It is a quantitative average.

On the other hand, **median** is the central value of the distribution in the sense that the number of values less than the median is equal to the number of values greater than the median. So, median is a positional average. Median is the central value in a sense different from the arithmetic mean. In case of the arithmetic mean it is the **"numerical magnitude"** of the deviations that balances. But, for the median it is the 'number of values greater than the median which balances against the number of values of less than the median.

5.2.3.1 Median for Raw Data

In general, if we have n values of x, they can be arranged in ascending order as:

$$x_1 < x_2 < \dots < x_n$$

Suppose n is odd, then Median = the $\dfrac{(n+1)}{2}$-th value

However, if n is even, we have two middle points

$$\text{Median} = \frac{\left(\frac{n}{2}\right)^{\text{th}} \text{value} + \left(\frac{n}{2}+1\right)^{\text{th}} \text{value}}{2}$$

Example:

The heights (in cm) of six students in class are 160, 157, 156, 161, 159, 162. What is median height?

Solution:

Arranging the heights in ascending order 156, 157, 159, 160, 161, 162

Two middle most values are the 3rd and 4th.

$$\text{Median} = \frac{1}{2}(159 + 160) = 159.5$$

5.2.3.2 Median for Grouped Data

1. Identify the median class which contains the middle observation $\left(\frac{N+1}{2}\right)^{th}$ observation) This can be done by observing the first class in which the cumulation frequency is equal to or more than $\frac{N+1}{2}$.

   Here, $N = \Sigma f$ = total number of observations.

2. Calculate Median as follows:

$$\text{Median} = L + \left[\frac{\left(\frac{N+1}{2}\right) - (F+1)}{f_m}\right] \times h$$

Where,
L = Lower limit of median class
N = Total number of data items = ΣF
F = Cumulative frequency of the class immediately preceding the median class
f_m = Frequency of median class
h = width of median class

Example:

Consider the following table giving the marks obtained by students in an exam

Mark Range	f No. of Students	Cumulative Frequency
0 – 20	2	2
20 – 40	3	5
40 – 60	10	15
60 – 80	15	30
80 – 100	20	50

Solution:

Here, $\frac{N+1}{2} = 25.5$

The class 60-80 is the median class since cumulative frequency is 30 >25.5

$$\text{Median} = \frac{60 + [25.5 - (15+1)]}{15} \times 20 = 69.66 \simeq 69.7$$

$\therefore$ Median marks of the class is approximately 69.7.

i.e. (at least) half the students got less than 69.7 and (almost) half got more than 69.7 marks.

5.2.4 Mode

Mode is defined as the value of the variable which occurs most frequently.

5.2.4.1 Mode for Raw Data

In raw data, the most frequently occurring observation is the mode. That is data with highest frequency is mode. If there is more than one data with highest frequency, then each of them is a mode. Thus we have Unimodal (single mode), Bimodal (two modes) and Trimodal (three modes) data sets.

Example:

Find the mode of the data set: 50, 50, 70, 50, 50, 70, 60.

Solution:

1. Arrange in ascending order: 50, 50, 50, 50, 60, 70, 70
2. Make a discrete data frequency table:

Data	Frequency
50	4
60	1
70	2

Since, 50 is the data with maximum frequency, mode is 50. This is a unimodal data set.

5.2.4.2 Mode for Grouped Data

Mode is that value of x for which the frequency is maximum. If the values of x are grouped into the classes (such that they are uniformly distributed within any class) and we have a frequency distribution then:

1. Identify the class which has the largest frequency (modal class)
2. Calculate the mode as

$$\text{Mode} = L + \frac{f_0 - f_1}{2f_0 - f_1 - f_2} \times h$$

Where,

L = Lower limit of the modal class

f_0 = Largest frequency (frequency of Modal Class)

f_1 = Frequency in the class preceding the modal class

f_2 = Frequency in the class next to the modal class

h = Width of the modal class

Example:

Data relating to the height of 352 school students are given in the following frequency distribution. Calculate the modal height.

Height (in feet)	Number of students
3.0 – 3.5	12
3.5 – 4.0	37
4.0 – 4.5	79
4.5 – 5.0	152
5.0 – 5.5	65
5.5 – 6.0	7
Total	352

Solution:

Since, 152 is the largest frequency, the modal class is (4.5 – 5.0).

Thus, $L = 4.5$, $f_0 = 152$, $f_1 = 79$, $f_2 = 65$, $h = 0.5$.

$$\text{Mode} = 4.5 \times \frac{152-79}{2(152)-79-65} \times 0.5 = 4.73 \text{ (approx.)}$$

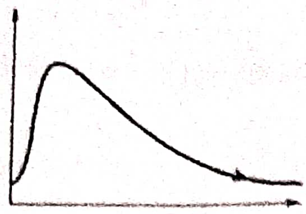
5.2.5 Properties Relating Mean, Median and Mode

1. Empirical mode = 3 median – 2 mean

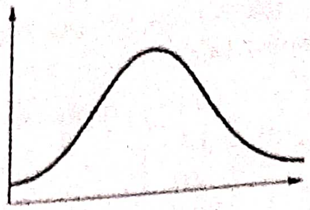
   when an approximate value of mode is required above empirical formula for mode may be used.

2. There are three types of frequency distributions.

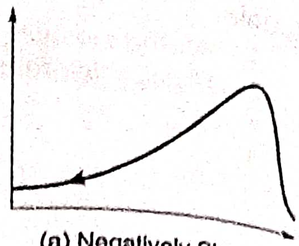
   Positively skewed, symmetric and negatively skewed distribution.



(a) Positively Skewed



(a) Symmetric



(a) Negatively Skewed

(a) In positively skewed distribution:
$$\text{Mode} \le \text{Median} \le \text{Mean}$$

(b) In symmetric distribution:
$$\text{Mean} = \text{Median} = \text{Mode}$$

(c) In negatively skewed distribution:
$$\text{Mean} \le \text{Median} \le \text{Mode}$$

5.2.6 Standard Deviation

Standard Deviation is a measure of dispersion or variation amongst data.

Instead of taking absolute deviation from the arithmetic mean, we may square each deviation and obtain the arithmetic mean of squared deviations. This gives us the 'variance' of the values.

The positive square root of the variance is called the **'Standard Deviation'** of the given values.

5.2.6.1 Standard Deviation for Raw Data

Suppose $x_1, x_2, \ldots x_n$ are n values of the x, their arithmetic mean is:

$\bar{x} = \frac{1}{N}\Sigma x_i$ and $x_1 - \bar{x}, x_2 - \bar{x}, \ldots x_n - \bar{x}$ are the deviations of the values of x from $\bar{x}$. Then

$\sigma^2 = \frac{1}{n}\Sigma(x_i - \bar{x})^2$ is the variance of x. It can be shown that

$$\sigma^2 = \frac{\Sigma(x_i - \bar{x})^2}{n} = \frac{1}{n}\Sigma x_i^2 - \bar{x}^2 = \frac{n\sum x_i^2 - \left(\sum x_i\right)^2}{n^2}$$

It is conventional to represents the variance by the symbol σ^2. Infact, σ is small sigma and Σ is capital sigma.

Square root of the variance is the standard deviation

$$\sigma = +\sqrt{\frac{1}{n}\Sigma(x_i - \bar{x})^2} = \sqrt{\frac{1}{n}\Sigma x_i^2 - \bar{x}^2} = \sqrt{\frac{n\sum x_i^2 - \left(\sum x_i\right)^2}{n^2}}$$

Example:

Consider three students in a class, and their marks in exam was 50, 60 and 70. What is the standard deviation of this data set?

Solution:

Student	x_i Marks	x_i^2
A	50	2500
B	60	3600
C	70	4900
	180	11000

Here, $n = 3$

$$\text{Standard Deviation } (\sigma) = \sqrt{\frac{n\sum x_i^2 - (\sum x_i)^2}{n^2}} = \sqrt{\frac{3 \times 11000 - (180)^2}{3^2}}$$

$$= 8.165$$

$$\text{Variance} = \sigma^2 = 66.67$$

5.2.6.2 Standard Deviation for Grouped Data

Calculation for standard deviation for grouped data can be shown by this example:

Example:

The frequency distribution for heights of 150 young ladies in a beauty contest is given below for which we have to calculate standard deviation.

Solution:

Height (in inches)	Mid values x	Frequency f	$f \times x$	$f \times x^2$
62.0 – 63.5	62.75	12	753.00	47250.75
63.5 – 65.0	64.25	20	1285.00	82561.25
65.0 – 66.5	65.75	28	1841.00	121045.75
66.5 – 68.0	67.25	18	1210.50	81406.125
68.0 – 69.5	68.75	19	1306.25	89806.125
69.5 – 71.0	70.25	20	1405.00	89804.6875
71.0 – 72.5	71.75	30	2152.50	98701.25
72.5 – 74.0	73.25	3	219.75	154441.875
				16096.6875
Total		150	10173.00	691308.375

Thus, $\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{10173}{150} = 67.82$

where, $N = \sum f_i = 150$

Therefore, the standard deviation of x is

$$= \sigma_x = \sqrt{\frac{\sum f_i x_i^2}{N} - \bar{x}^2}$$

$$= \sqrt{\frac{N\sum f_i x_i^2 - (\sum f_i x_i)^2}{N^2}} = \sqrt{\frac{150 \times 691308.375 - (10173)^2}{(150)^2}}$$

$$= 3.03$$

$$\text{Variance} = \sigma_x^2 = (3.03)^2 = 9.170$$

5.2.7 Variance

The square of standard deviation (σ) is called as the variance (σ^2).

So if $\sigma = 10$, then variance $= \sigma^2 = 100$.

Alternatively if variance $= \sigma^2 = 100$ then standard deviation $= \sqrt{\text{Variance}} = \sqrt{100} = 10$

The larger the standard deviation, larger will be the variance.

5.2.8 Coefficient of Variation

The standard deviation is an absolute measure of dispersion and hence can not be used for comparing variability of 2 data sets with different means.

Therefore, such comparisons are done by using a relative measure of dispersion called coefficient of variation (CV).

$$CV = \frac{\sigma}{\mu}$$

where σ is the standard deviation and μ is the mean of the data set.

CV is often represented as a percentage,

$$CV\ \% = \frac{\sigma}{\mu} \times 100$$

When comparing data sets, the data set with larger value of CV% is more variable (less consistent) as compared to a data set with lesser value of CV%.

For example:

	μ	σ	CV%
Data set 1	5	1	20%
Data set 2	20	2	10%

Although $\sigma = 2$ for data set 2 is more than $\sigma = 1$ for data set 1, data set 2 is actually less variable compared to data set 1, as can be seen by the fact that data set 2 has a CV % of 10%, while data set 1 has a CV % of 20%.

So comparison of variability between 2 or more data sets (with different means) should be done by comparing CV % and not by comparing standard deviations.

5.3 Probability Distributions

5.3.1 Random Variables

It is frequently the case when an experiment is performed that we are mainly interested in some function of the outcome as opposed to the actual outcome itself.

For instances, in tossing dice we are often interested in the sum of two dice and are not really concerned about the separate value of each die. That is, we may be interested in knowing that the sum is 7 and not be concerned over whether the actual outcome was (1, 6) or (2, 5) or (3, 4) or (4, 3) or (5, 2) or (6, 1).

Also, in coin flipping we may be interested in the total number of heads that occur and not care at all about the actual head tail sequence that results. These quantities of interest, or more formally, these real valued functions defined on the sample space, are known as random variables.

Because the value of a random variable is determined by the outcome of the experiment, we may assign probabilities to the possible values of the random variable.

Types of Random Variable: Random variable may be discrete or continuous.

Discrete Random Variable: A variable that can take one value from a discrete set of values.

Example: Let x denotes sum of 2 dice, Now x is a discrete random variable as it can take one value from the set {2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}, since the sum of 2 dice can only be one of these values.

Continuous Random Variable: A variable that can take one value form a continuous range of values.

Example: x denotes the volume of Pepsi in a 500 ml cup. Now x may be a number from 0 to 500, any of which value, x may take.

5.3.1.1 Probability Density Function (PDF)

Let x be continuous random variable then its PDF $F(x)$ is defined such that

1. $F(x) \geq 0$ 2. $\int_{-\infty}^{\infty} F(x)dx = 1$ 3. $P(a < x < b) = \int_a^b F(x)dx$

5.3.1.2 Probability Mass Function (PMF)

Let x be discrete random variable then its PMF $p(x)$ is defined such that

1. $p(x) = P[X = x]$ 2. $p(x) \geq 0$ 3. $\Sigma p(x) = 1$

5.3.2 Distributions

Based on this we can divide distributions also into **discrete distribution** (based on a discrete random variable) or **continuous distribution** (based on a continuous random variable).

Examples of discrete distribution are binomial, Poisson and hypergeometric distributions.

Examples of continuous distribution are uniform, normal and exponential distributions.

5.3.2.1 Properties of Discrete Distribution

$$\Sigma P(x) = 1$$

$$E(x) = \Sigma x\, P(x)$$

$$V(x) = E(x^2) - (E(x))^2 = \Sigma x^2 P(x) - [\Sigma x\, P(x)]^2$$

$E(x)$ denotes expected value or average value of the random variable x, while $V(x)$ denotes the variance of the random variable x.

5.3.2.2 Properties of Continuous Distribution

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

$$F(x) = \int_{-\infty}^{x} f(x)dx \text{ (cumulative distribution function)}$$

$$E(x) = \int_{-\infty}^{\infty} xf(x)dx$$

$$V(x) = E(x^2) - [E(x)]^2 = \int_{-\infty}^{\infty} x^2 f(x)dx - \left[\int_{-\infty}^{\infty} xf(x)dx\right]^2$$

$$p(a < x < b) = p(a \leq x < b) = P(a < x \leq b) = P(a \leq x \leq b) = \int_a^b f(x)dx$$

5.3.3 Types of Distributions

Discrete Distributions:

1. General Discrete Distribution
2. Binomial Distribution
3. Hypergeometric Distribution
4. Geometric Distribution
5. Poisson Distribution

5.3.3.1 General Discrete Distribution

Let X be a discrete random variable.

A table of possible values of x versus corresponding probability values $p(x)$ is called as its probability distribution table.

Example:

Let X be the number which comes on a single throw of a dice.

Then probability distribution table of x is given by

x	1	2	3	4	5	6
$p(x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

In this case $p(x)$ is same for all values of x, but this is not necessary, as following example shows.

For example, let x be the sum of the numbers coming on a pair of dice thrown.

Now the probability distribution table can be constructed as follows

x	2	3	4	.....	10	11	12
$p(x)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$		$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

Notice, that here $p(x)$ is not same for all values of x.

In any probability distribution table

$$\Sigma p(x) = 1 \text{ is always true}$$

Take the case of simple dice

x	1	2	3	4	5	6
$p(x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

Notice that $$\Sigma p(x) = \frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6} = 1$$

From above table, we can compute the following:

$$p(x = 3) = \frac{1}{6}$$

$$p(x \geq 3) = \frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}=\frac{4}{6} = \frac{2}{3}$$

$$p(x \leq 3) = \frac{1}{6}+\frac{1}{6}+\frac{1}{6}=\frac{3}{6} = \frac{1}{2}$$

$$p(x < 4) = \frac{1}{6}+\frac{1}{6}+\frac{1}{6}=\frac{3}{6} = \frac{1}{2}$$

Also from above table, we can compute the expected value and variance of x.

$$E(x) = \Sigma\, x\, p(x)$$

$$V(x) = E(x^2) - [E(x)]^2 = \Sigma x^2 p(x) - [\Sigma\, x\, p(x)]^2$$

$E(x)$ is the expected value of x and is similar to an average value of x after infinite number of trials. So, $E(x)$ is sometimes also written as μ_x.

$V(x)$ represents the variability of X. So it is sometimes written as σ_x^2.

So, $\sigma_x = \sqrt{V(x)}$, which is the standard deviation of X.

Also expected value of any function $g(x)$ of x can be computed as follows:

$$E(g(x)) = \Sigma g(x)p(x)$$

For example,

$$E(x^3) = \Sigma x^3\, p(x) \text{ and } E(x^2+1) = \Sigma(x^2+1)\, p(x)$$

For the single dice probability distribution table,

$$P_x = E(x) = \Sigma x\, p(x) = 1\times\frac{1}{6}+2\times\frac{1}{6}+\ldots+6\times\frac{1}{6} = 3.5$$

and

$$\sigma_x^2 = V(x) = \Sigma x^2 p(x) - [\Sigma x\, p(x)]^2$$

$$= \left[1^2\times\frac{1}{6}+2^2\times\frac{1}{6}+3^2\times\frac{1}{6}\ldots+6^2\times\frac{1}{6}\right]-(3.5)^2 = 2.917$$

$\therefore$

$$\sigma_x = \sqrt{2.917} = 1.7078$$

Properties of Expectation and Variance:

If x_1 and x_2 are two random variance and a and b are constants,

$$E(ax_1+b) = a\,E(x_1)+b \quad \text{... (i)}$$

$$V(ax_1+b) = a^2\,V(x_1) \quad \text{... (ii)}$$

$$E(ax_1+bx_2) = a\,E(x_1)+b\,E(x_2) \quad \text{... (iii)}$$

$$V(ax_1+bx_2) = a^2\,V(x_1)+b^2V(x_2)+2ab\,\text{cov}(x_1,x_2) \quad \text{... (iv)}$$

where $\text{cov}(x_1, x_2)$ represents the covariance between x_1 and x_2

If x_1 and x_2 are independent, then $\text{cov}(x_1, x_2) = 0$ and the above formula reduces to

$$V(ax_1+bx_2) = a^2V(x_1)+b^2V(x_2) \quad \text{... (v)}$$

For example, from above formula we can say

$$E(x_1+x_2) = E(x_1)+E(x_2)$$

$$E(x_1-x_2) = E(x_1)-E(x_2)$$

$$V(x_1+x_2) = V(x_1-x_2) = V(x_1)+V(x_2)$$

Formula for calculating covariance between X and Y

$$\text{Cov}(X, Y) = E(XY)-E(X)\,E(Y)$$

$\therefore$ If X, Y are independent $\quad E(XY) = E(X)\,E(Y)$

and hence $\quad \text{Cov}(X, Y) = 0$

5.3.3.2 Binomial Distribution

Suppose that a trial or an experiment, whose outcome can be classified as either a success or a failure is performed.

Suppose now that n independent trials, each of which results in a success with probability p and in a failure with probability $1-p$, are to be performed.

If X represents the number of successes that occur in the n trials, then X is said to be binomial random variable with parameters (n, p).

The Binomial distribution occurs when experiment performed satisfies the three assumptions of Bernoulli trials, which are:

1. Only 2 outcomes are possible, success and failure
2. Probability of success (p) and failure ($1-p$) remains same from trial to trial.
3. The trials are statistically independent. i.e The outcome of one trial does not influence subsequent trials. i.e. No memory.

These assumptions are satisfied in following types of problems:

(a) dice problems.

(b) coin toss problems.

(c) sampling with replacement from a finite population.

(d) sampling with or without replacement from an infinite (large) population.

The probability of obtaining x successes from n trials is given by the binomial distribution formula,

$$P(X=x) = nC_x\, p^x (1-p)^{n-x}.$$

Where p is the probability of success in any trial and $(1-p) = q$ is the probability of failure.

Example 1.

10 dice are thrown. What is the probability of getting exactly 2 sixes.

Solution:

$$P(X=2) = 10C_2\left(\tfrac{1}{6}\right)^2\left(\tfrac{5}{6}\right)^8 = 0.2907$$

Example 2.

It is known that screws produced by a certain company will be defective with probability 0.01 independently of each other. The company sells the screws in packages of 10 and offers a replacement guarantee that at most 1 of the 10 screws is defective. What proportion of packages sold must the company replace?

Solution:

If X is the number of defective screws in a packages, then X is a binomial variable with parameters (10, 0.01). Hence, the probability that a package will have to be replaced is:

$$P(X \geq 2) = 1 - [P(X \leq 1)] = 1 - [P\{X=0\} + P\{X=1\}]$$

$$= 1 + \left[\binom{10}{0}(0.01)^0(0.99)^{10} + \binom{10}{1}(0.01)^1(0.99)^9\right]$$

$$\approx 0.004$$

Hence only 0.4% of packages will have to be replaced.

For Binomial Distribution:

$$\text{Mean} = E[X] = np$$

$$\text{Variance} = V[X] = np(1-p)$$

Recurrence Relation

For binomial distribution, (n, p, q)

$$P(r) = {}^nC_r p^r q^{n-r} \quad \text{...(i)}$$

$$P(r+1) = {}^nC_{r+1} p^{r+1} q^{n-r-1} \quad \text{...(ii)}$$

By dividing (ii) by (i), we get

$$\frac{P(r+1)}{P(r)} = \frac{{}^nC_{r+1} p^{r+1} q^{n-r-1}}{{}^nC_r p^r q^{n-r}}$$

$$\frac{P(r+1)}{P(r)} = \frac{(n-r)p}{(r+1)q}$$

$\Rightarrow$
$$P(r+1) = \frac{(n-r)p}{(r+1)q}P(r)$$

Example 3.
100 dice are thrown. How many are expected to fall 6. What is the variance in the number of 6's?

Solution:

$$E(x) = np = 100 \times 1/6 = 16.7 \approx 17$$

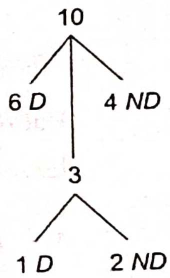
So, 17 out of 100 are expected to fall 6.

$$V(x) = np\,(1-p) = 100 \times 1/6 \times (1 - 1/6) = 13.9$$

So, variance is number of 6's = 13.9.

5.3.3.3 Hypergeometric Distribution

If the probability changes from trial to trial, one of the assumptions of binomial distribution gets violated and hence binomial distribution cannot be used. In such lases hypergeometric distribution is used. This is particularly used is cases of sampling without replacement from a finite population.



Example:
There are 10 markers on a table, of which 6 are defective and 4 are not deflective. If 3 are randomly taken from above lot, what is the probability that exactly 1 of markers is defective?

Solution:
The above problem is tackled by hypergeometric distribution as follows.
D is defective and ND is non defective.

$$p(X=1) = \frac{{}^6C_1 \times {}^4C_2}{10C_3} = 0.3$$

The above problem can be generalised into a distribution if we make X as the number of defective markers.
X can now take the values 0, 1, 2 or 3.

$$p(X=x) = \frac{{}^6C_x \times {}^4C_{3-x}}{{}^{10}C_3}$$

This is the hypergeometric distribution for above problem.
from above formula, we can calculate the following:

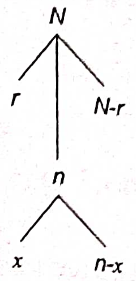
$$p(x=1) = \frac{{}^6C_1 \times {}^4C_2}{{}^{10}C_3}$$

$$p(x \geq 1) = p(x=0) + p(x=1) = \frac{{}^6C_0 \times {}^4C_3}{{}^{10}C_3} + \frac{{}^6C_1 \times {}^4C_2}{{}^{10}C_3}$$

$$p(x \geq 1) = 1 - p(x=0) = 1 - \left[\frac{{}^6C_0 \times {}^4C_3}{{}^{10}C_3}\right]$$

The hypergeometric distribution can be written is a more general way as follows.
Consider N objects of which r are of type 1 and N-r are of type 2.
from this n objects are drawn without replacement. What is the probability that x objects drawn are of type 1?

The diagram for above problem is



$$p(X=x) = \frac{{}^{r}C_x \times {}^{N-r}C_{n-x}}{{}^{N}C_n}$$

This is the general formula for hypergeometric distribution.
The expected value of this distribution is given by,

$$E(x) = n^*\left(\frac{r}{N}\right)$$

5.3.3.4 Geometric Distribution

Consider repeated trials of a Bernoulli experiment $\in$ with probability P of success and $q = 1 - P$ of failure. Let x denote the number of times $\in$ must be repeated until finally obtaining a success. The distribution of random variable x is given as follows:

k	1	2	3	4	5	....
$P(k)$	P	qP	q^2P	q^3P	q^4P	

The experiment $\in$ will be repeated k times only in the case that there is a sequence of $k-1$ failures followed by a success.

$$P(k) = P(x - k) = q^{k-1}P$$

The geometric distribution is characterized by a single parameter P.

Points to Remember:

Let x be a geometric random variable with distribution GEO(P). Then

1. $E(x) = \frac{1}{P}$
2. $Var(x) = \frac{q}{P^2}$
3. Cumulative distribution $F(k) = -1 - q^k$
4. $P(x > r) = q^r$

Geometric distribution possesses "no-memory" or "lack of memory" property which can be stated as

$$P(x > a + r \mid x > a) = P(x > r)$$

1. Suppose the probability that team A wins each game in a tournament is 60 percent. A plays until it loses.
   (a) Find the expected number E of games that A plays
   (b) Find the probability P that A plays in at least 4 games
   (c) Find the probability P that A wins the tournament if the tournament has 64 teams. (Thus, a team winning 6 times wins the tournament).

Sol. 1

This is a geometric distribution with $P = 0.4$ and $q = 0.6$ (A plays until A loses)

(a) Since $E(x) = \frac{1}{P} = \frac{1}{0.4} = 2.5$

(b) The only way A plays at least 4 games is if A wins the first 3 games. Thus,
$$P = P(x > 3) = q^3 = (0.6)^3 = 0.216 = 21.6\%$$

(c) Here A must win all 6 games;
So $$P = (0.6)^6 = 0.0467 = 4.67\%$$

3.3.5 Poisson Distribution

A random variable X, taking on one of the values 0, 1, 2 .... is said to be a Poisson random variable with parameter λ if for some $\lambda > 0$,

$$P(x = x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$$

For Poisson distribution:

$$\text{Mean} = E(x) = \lambda$$

$$\text{Variance} = V(x) = \lambda$$

Therefore, expected value and variance of a Poisson random variable are both equal to its parameter λ.

Here λ is average number of occurrences of event in an observation period Δt. So, $\lambda = \alpha\, \Delta t$ where α is number of occurrences of event per unit time.

Recurrence Relation (x, λ)

$$P(x) = \frac{e^{-\lambda}\lambda^{x}}{x!} \qquad \text{...(i)}$$

$$P(x+1) = \frac{e^{-\lambda}\lambda^{x+1}}{(x+1)!} \qquad \text{...(ii)}$$

By dividing (ii) by (i)

$$\frac{P(x+1)}{P(x)} = \frac{e^{-\lambda}\lambda^{x+1}}{(x+1)!} \times \frac{x!}{e^{-\lambda}\lambda^{x}} = \frac{\lambda}{x+1}$$

$$P(x+1) = \frac{\lambda}{x+1}P(x)$$

Example 1.

A certain airport receives on an average of 4 air-crafts per hour. What is the probability that no aircraft lands in a particular 2 hr period?

Solution:

Given equation, α = rate of occurrence of event per unit time = 4/hr

λ = average no. of occurrences of event in specified observation period $= \alpha\Delta t$

In this case α = 4/hr and $\Delta t = 2h$

$\therefore$ So, $\lambda = 4 \times 2 = 8$

Now we wish that no aircraft should land for 2 hrs. i.e. $x = 0$

$$P(x = 0) = \frac{e^{-\lambda}\lambda^{0}}{0!} = \frac{e^{-8}8^{0}}{0!} = e^{-8}$$

Frequently, Poisson distribution is used to approximate binomial distribution when n is very large and p is very small. Notice that direct computation of $nC_x p^x (1-p)^{n-x}$ may be erroneous or impossible when n is very large and p is very small. Hence, we resort to a Poisson approximation with $\lambda = np$.

Example 2.

A certain company sells tractors which fail at a rate of 1 out of 1000. If 500 tractors are purchased from this company what is the probability of 2 of them failing within first year?

Solution:

$$\lambda = np = 500 \times \frac{1}{1000} = \frac{1}{2}$$

$$P(x = 2) = \frac{e^{-1/2}(1/2)^2}{2!} = 0.07582$$

Continuous Distributions:

1. General Continuous Distribution
2. Uniform Distribution
3. Exponential Distribution
4. Normal Distribution
5. Standard Normal Distribution

5.3.3.6 General Continuous Distribution

Let X be a continuous random variable. A continuous distribution of X can be defined by a probability density function $f(x)$ which is such a function such that

$$p(-\infty \le X \le \infty) = \int_{-\infty}^{\infty} f(x)dx = 1$$

The expected value of x is given by

$$\mu_x = E(x) = \int_{-\infty}^{\infty} xf(x)dx$$

i.e.

$$V(x) = E(x^2) - [E(x)]^2 = \int_{-\infty}^{\infty} x^2 f(x)dx - \left[\int_{-\infty}^{\infty} xf(x)dx\right]^2$$

$$\sigma_x^2 = \sqrt{V(x)}$$

The cumulative probability function (sometimes also called as probability distribution function), is given by $F(x)$, where

$$F(x) = p(X \le x) = \int_{-\infty}^{x} f(x)dx$$

Note: From distribution function we can get probability density franction by formula below:

$$f(x) = \frac{dF}{dx}$$

5.3.3.7 Uniform Distribution

In general we say that X is a uniform random variable on the interval (a, b) if its probability density function is given by:

$$f(x) = \begin{cases} \dfrac{1}{\beta - \alpha} & \text{if } \alpha < x < \beta \\ 0 & \text{otherwise} \end{cases}$$

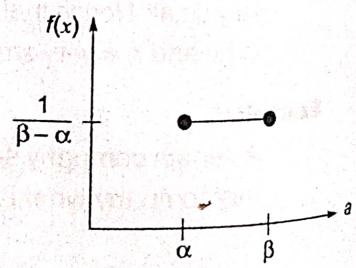
Since $f(x)$ is a constant, all values of x between α and β are equally likely (uniform).

Graphical Representation:

For Discrete Uniform Distribution:

$$\text{Mean} = E[X] = \frac{\beta + \alpha}{2}$$

$$\text{Variance} = V(X) = \frac{(\beta - \alpha)^2}{12}$$



Example:

If X is uniformly distributed over (0, 10), calculate the probability that

(a) $X < 3$

(b) $X > 6$

(c) $3 < X < 8$.

Solution:

$$f(x) = \frac{1}{10-0} = \frac{1}{10}$$

$$P\{X < 3\} = \int_0^3 \frac{1}{10}dx = \frac{3}{10}$$

$$P\{X > 6\} = \int_6^{10} \frac{1}{10}dx = \frac{4}{10}$$

$$P\{3 < X < 8\} = \int_3^8 \frac{1}{10}dx = \frac{1}{2}$$

5.3.3.7 Exponential Distribution

A continuous random variable whose probability density function is given for some $\lambda > 0$ by

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

is said to be exponential random variable with parameter λ. The cumulative distributive function $F(a)$ of an exponential random variable is given by:

$$F(a) = P(X \leq a) = \int_0^a \lambda e^{-\lambda x} dx = \left(-e^{-\lambda x}\right)_0^a = 1 - e^{-\lambda a},\ a \geq 0$$

For Exponential Distribution:

$$\text{Mean} = E[X] = 1/\lambda$$
$$\text{Variance} = V(x) = 1/\lambda^2$$

Example:

Suppose that the length of a phone call in minutes is an exponential random variable with parameter $\lambda = \frac{1}{10}$. If someone arrives immediately ahead of you at a public telephone booth, find the probability that you will have to wait,

(a) More than 10 minutes

(b) Between 10 and 20 minutes.

Solution:

Letting X denote the length of the call made by the person in the booth, we have that the desired probabilities are:

(a)
$$\begin{aligned} P\{X > 10\} &= 1 - P(x < 10) \\ &= 1 - F(10) = 1 - (1 - e^{-\lambda \times 10}) \\ &= e^{-10}\lambda = e^{-1} = 0.368 \end{aligned}$$

(b)
$$\begin{aligned} P\{10 < X < 20\} &= F(20) - F(10) \\ &= (1 - e^{-20\lambda}) - (1 - e^{-10\lambda}) = e^{-1} - e^{-2} = 0.233 \end{aligned}$$

5.3.3.8 Normal Distribution

We say that X is a normal random variable, or simply that X is normally distributed, with parameters μ and σ^2, if the probability density function is given by:

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \quad -\infty < x < \infty$$

The density function is a bell-shaped curve that is symmetric about μ.

For Normal Distribution:

$$\text{Mean} = E(X) = \mu$$

$$\text{Variance} = V(X) = \sigma^2$$

5.3.3.8.1 Standard Normal Distribution

Since the for $N(\mu, \sigma^2)$ varies with μ and σ^2 and the integral can only be evaluated numerically, it is more reasonable to reduce this distribution to another distribution called Standard normal distribution $N(0, 1)$ for which, the shape and hence the integral values remain constant.

Since all $N(\mu, \sigma^2)$ problems can be reduced to $N(0, 1)$ problems, we need only to consult a standard table giving calculations of area under $N(0, 1)$ from 0 to any value of z.

The conversion from $N(\mu, \sigma^2)$ to $N(0, 1)$ is effected by the following transformation,

$$Z = \frac{X-\mu}{\sigma}$$

Where Z is called standard normal variate.

For Standard Normal distribution:

$$\text{Mean} = E(X) = 0$$

$$\text{Variance} = V(X) = 1$$

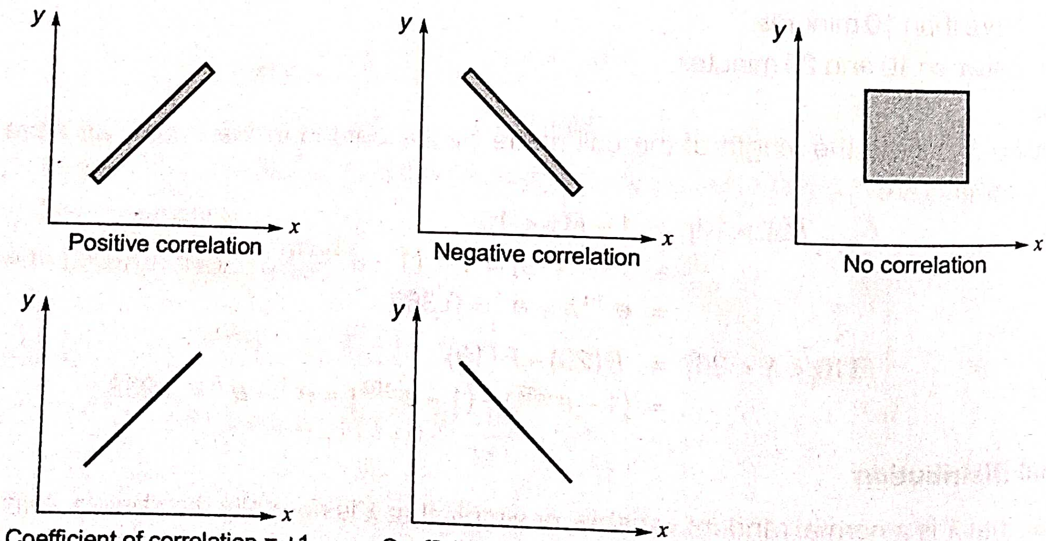
Hence the standard normal distribution is also referred to as the $N(0, 1)$ distribution.

Correlation: Whenever two variables x and y are so related that an increase in the one is accompanied by an increase or decrease in the other, then the variables are said to be correlated.

For example, the yield of crop varies with the amount of rainfall.

If two variables vary in such a way that their ratio is always constant, then the correlation is said to be perfect.

Scatter or Dot diagram: When we plot the corresponding value of two variables, taking one on x-axis and another on y-axis, it shows a collection of dots.



Positive correlation Negative correlation No correlation

Coefficient of correlation = +1 Coefficient of correlation = −1

Positive correlation means if value of one variable is increased then value of other variable also increases.

Negative correlation means if value of one variable is increased then value of other variable decreases.

If variation of x doesn't have any effect on y, then there is no correlation.

Coefficient of correlation

$$r = \frac{P}{\sigma_x \cdot \sigma_y} = \frac{\text{Covariance }(x,y)}{\sqrt{\text{Variance } x} \times \sqrt{\text{Variance } y}}$$

$$= \frac{\Sigma XY}{\sqrt{(\Sigma X^2)(\Sigma Y^2)}}$$

The correlation coefficient satisfies $-1 \le r \le 1$.

(i) If $r = 0$, then there is no correlation.

(ii) If $r = 1.$, then there is perfect positive correlation.

(iii) If $r = -1$, then there is perfect negative correlation.

(iv) If $0 < r < 1$, then there is positive correlation.

(v) If $-1 < r < 0$, then there is negative correlation.

Regression: If the scatter diagram indicate some relationship between two variable x and y, then the dots of the scatter diagram will be concentrated round a curve. This curve is called the curve of regression.

Regression analysis is the method used for estimating the unknown values of one variable corresponding to the known value of another variable.

Line of Regression: When the curve is a straight line, it is called a line of regression. A line of regression is the straight line which gives the best fit in the least square sense to the given frequency.

In regression analysis, one of the two variable, can be regarded as an ordinary variable because we can measure it without substantial error.

In correlation analysis both quantities are random variables and we evaluated a relation between them. x is independent variable or controlled variable. y is random variable in regression analysis, dependency of y on x is checked.

Let the line of regression is given by

$$y = a + bx \quad \text{...(i)}$$

Since it is the line of best fit the value of a and b are given by normal equation

$$\Sigma y = na + b\Sigma x$$

where n is the number of pairs of values of x and y.

$$\frac{\Sigma y}{n} = a + b\frac{\Sigma x}{n}$$

or

$$\bar{y} = a + b\bar{x} \quad \text{...(ii)}$$

where $\bar{x}$, $\bar{y}$ are means of variable x and y respectively.

The line of regression passes through $(\bar{x}, \bar{y})$.

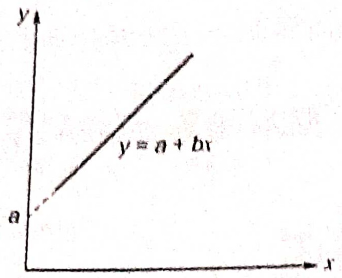
By substracting equation (ii) from equation (i), we get

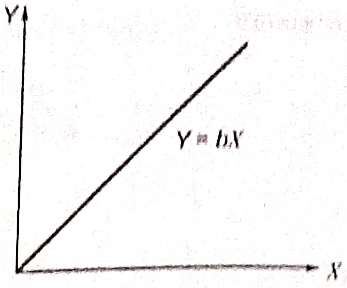
$$y - \bar{y} = b(x - \bar{x})$$

$$Y = bX$$

Now shift the origin at $(\bar{x}, \bar{y})$, the y will reduce to $Y = bX$, where $X = x - \bar{x}$ and $Y = y - \bar{y}$.

As the line of regression is $Y = bX$ is passing through new origin.





■■■■

Previous GATE and ESE Questions

Q.1 Let $P(E)$ denote the probability of the event E. Given $P(A) = 1$, $P(B) = 1/2$, the values of $P(A/B)$ and $P(B/A)$ respectively are

(a) 1/4, 1/2 (b) 1/2, 1/4
(c) 1/2, 1 (d) 1, 1/2

[CS, GATE-2003, 1 mark]

Q.2 A box contains 10 screws, 3 of which are defective. Two screws are drawn at random with replacement. The probability that none of the two screws is defective will be

(a) 100% (b) 50%
(c) 49% (d) None of these

[CE, GATE-2003, 1 mark]

Q.3 A box contains 5 black and 5 red balls. Two balls are randomly picked one after another from the box, without replacement. The probability for both balls being red is

(a) $\frac{1}{90}$ (b) $\frac{1}{2}$
(c) $\frac{19}{90}$ (d) $\frac{2}{9}$

[ME, GATE-2003, 2 marks]

Q.4 A program consists of two modules executed sequentially. Let $f_1(t)$ and $f_2(t)$ respectively denote the probability density functions of time taken to execute the two modules. The probability density function of the overall time taken to execute the program is given by

(a) $f_1(t) + f_2(t)$ (b) $\int_0^t f_1(x) f_2(x) dx$
(c) $\int_0^t f_1(x) f_2(t-x) dx$ (d) $\max\{f_1(t), f_2(t)\}$

[CS, GATE-2003, 2 marks]

Q.5 A hydraulic structure has four gates which operate independently. The probability of failure of each gate is 0.2. Given that gate 1 has failed, the probability that both gates 2 and 3 will fail is

(a) 0.240 (b) 0.200
(c) 0.040 (d) 0.008

[CE, GATE-2004, 2 marks]

Q.6 An examination paper has 150 multiple-choice questions of one mark each, with each question having four choices. Each incorrect answer fetches – 0.25 mark. Suppose 1000 students choose all their answers randomly with uniform probablity. The sum total of the expected marks obtained by all these students is

(a) 0 (b) 2550
(c) 7525 (d) 9375

[CS, GATE-2004, 2 marks]

Q.7 If a fair coin is tossed four times. What is the probability that two heads and two tails will result?

(a) 3/8 (b) 1/2
(c) 5/8 (d) 3/4

[CS, GATE-2004, 1 mark]

Two n bit binary strings, S_1 and S_2 are chosen randomly with uniform probability. The probability that the Hamming distance between these strings (the number of bit positions where the two strings differ) is equal to d is

(a) $^nC_d/2^n$
(b) $^nC_d/2^d$
(c) $d/2^n$
(d) $1/2^d$

[CS, GATE-2004, 2 marks]

From a pack of regular playing cards, two cards are drawn at random. What is the probability that both cards will be Kings, if first card in NOT replaced?

(a) $\frac{1}{26}$
(b) $\frac{1}{52}$
(c) $\frac{1}{169}$
(d) $\frac{1}{221}$

[ME, GATE-2004, 2 marks]

A point is randomly selected with uniform probability in the X-Y plane within the rectangle with corners at (0, 0), (1, 0), (1, 2) and (0, 2). If p is the length of the position vector of the point, the expected value of p^2 is

(a) 2/3
(b) 1
(c) 4/3
(d) 5/3

[CS, GATE-2004, 2 marks]

Q.11 If P and Q are two random events, then the following is TRUE

(a) Independence of P and Q implies that probability $(P \cap Q) = 0$
(b) Probability $(P \cup Q) \geq$ Probability (P) + Probability (Q)
(c) If P and Q are mutually exclusive, then they must be independent
(d) Probability $(P \cap Q) \leq$ Probability (P)

[EE, GATE-2005, 1 mark]

Q.12 A fair dice is rolled twice. The probability that an odd number will follow an even number is

(a) $\frac{1}{2}$
(b) $\frac{1}{6}$
(c) $\frac{1}{3}$
(d) $\frac{1}{4}$

[EC, GATE-2005, 1 mark]

Q.13 A single die is thrown twice. What is the probability that the sum is neither 8 nor 9?

(a) 1/9
(b) 5/36
(c) 1/4
(d) 3/4

[ME, GATE-2005, 2 marks]

Q.14 A fair coin is tossed three times in succession. If the first toss produces a head, then the probability of getting exactly two heads in three tosses is

(a) 1/8
(b) 1/2
(c) 3/8
(d) 3/4

[EE, GATE-2005, 2 marks]

Q.15 Which one of the following statements is NOT true?

(a) The measure of skewness is dependent upon the amount of dispersion
(b) In a symmetric distribution, the values of mean, mode and median are the same
(c) In a positively skewed distribution: mean > median > mode
(d) In a negatively skewed distribution: mode > mean > median

[CE, GATE-2005, 1 mark]

Q.16 A lot has 10% defective items. Ten items are chosen randomly from this lot. The probability that exactly 2 of the chosen items are defective is

(a) 0.0036
(b) 0.1937
(c) 0.2234
(d) 0.3874

[ME, GATE-2005, 1 mark]

Q.17 Let $f(x)$ be the continuous probability density function of a random variable X. The probability that $a < X \leq b$, is

(a) $f(b-a)$
(b) $f(b) - f(a)$
(c) $\int_a^b f(x)dx$
(d) $\int_a^b xf(x)dx$

[CS, GATE-2005, 1 mark]

Q.18 If P and Q are two random events, then the following is TRUE

(a) Independence of P and Q implies that probability $(P \cap Q) = 0$
(b) Probability $(P \cup Q) \geq$ Probability (P) + Probability (Q)
(c) If P and Q are mutually exclusive, then they must be independent
(d) Probability $(P \cap Q) \leq$ Probability (P)

[EE, GATE-2005, 1 mark]

Q.19 The value of the integral $I = \frac{1}{\sqrt{2\pi}}\int_0^{\infty} \exp\left(-\frac{x^2}{8}\right)dx$ is

(a) 1
(b) π
(c) 2
(d) 2π

[EC, GATE-2005, 2 marks]

Q.20 Two fair dice are rolled and the sum r of the numbers turned up is considered

(a) $Pr(r > 6) = (1/6)$
(b) $Pr(r/3 \text{ is an integer}) = (5/6)$
(c) $Pr(r = 8 \mid r/4 \text{ is an integer}) = (5/9)$
(d) $Pr(r = 6 \mid r/5 \text{ is an integer}) = (1/18)$

[EE, GATE-2006, 2 marks]

Q.21 Three companies *X*, *Y* and *Z* supply computers to a university. The percentage of computers supplied by them and the probability of those being defective are tabulated below:

Company	% of computer	Probability of being supplied defective
X	60%	0.01
Y	30%	0.02
Z	10%	0.03

Given that a computer is defective, the probability that it was supplied by *Y* is

(a) 0.1
(b) 0.2
(c) 0.3
(d) 0.4

[EC, GATE-2006, 2 marks]

Q.22 For each element in a set of size 2n, an unbiased coin is tossed. All the 2n coin tossed are independent. An element is chosen if the corresponding coin toss were head. The probability that exactly n elements are chosen is

(a) $\binom{2n}{n}\Big/4^n$
(b) $\binom{2n}{n}\Big/2^n$
(c) $1\Big/\binom{2n}{n}$
(d) $\frac{1}{2}$

[CS, GATE-2006, 2 marks]

Q.23 There are 25 calculators in a box. Two of them are defective. Suppose 5 calculators are randomly picked for inspection (i.e., each has the same chance of being selected), what is the probability that only one of the defective calculators will be included in the inspection?

(a) $\frac{1}{2}$
(b) $\frac{1}{3}$
(c) $\frac{1}{4}$
(d) $\frac{1}{5}$

[CE, GATE-2006, 2 marks]

Q.24 A box contains 20 defective items and 80 non-defective items. If two items are selected at random without replacement, what will be the probability that both items are defective?

(a) $\frac{1}{5}$
(b) $\frac{1}{25}$
(c) $\frac{20}{99}$
(d) $\frac{19}{495}$

[ME, GATE-2006, 1 mark]

Q.25 Consider the continuous random variable with probability density function

$$f(t) = 1 + t \text{ for } -1 \le t \le 0$$
$$= 1 - t \text{ for } 0 \le t \le 1$$

The standard deviation of the random variable is

(a) $\frac{1}{\sqrt{3}}$
(b) $\frac{1}{\sqrt{6}}$
(c) $\frac{1}{3}$
(d) $\frac{1}{6}$

[ME, GATE-2006, 2 marks]

Q.26 A probability density function is of the form $p(x) = Ke^{-\alpha|x|}, x \in (-\infty, \infty)$.
The value of *K* is

(a) 0.5
(b) 1
(c) 0.5 α
(d) α

[EC, GATE-2006, 1 mark]

Q.27 A class of first year *B*. Tech. students is composed of four batches *A*, *B*, *C* and *D*, each consisting of 30 students. It is found that the sessional marks of students in Engineering Drawing in batch *C* have a mean of 6.6 and standard deviation of 2.3. The mean and standard deviation of the marks for the entire class are 5.5 and 4.2, respectively. It is decided by the course instructor to normalize the marks of the students of all batches to have the same mean and standard deviation as that of the entire class. Due to this, the marks of a student in batch *C* are changed from 8.5 to

(a) 6.0
(b) 7.0
(c) 8.0
(d) 9.0

[CE, GATE-2006, 2 marks]

Q.28 Suppose we uniformly and randomly select a permutation from the 20! permutations of 1, 2, 3 ...., 20. What is the probability that 2 appears at an earlier position that any other even number in the selected permutation?

(a) $\frac{1}{2}$ (b) $\frac{1}{10}$

(c) $\frac{9!}{20!}$ (d) None of these

[CS, GATE-2007, 2 marks]

Q.29 A loaded dice has following probability distribution of occurrences

DiceValue	1	2	3	4	5	6
Probability	1/4	1/8	1/8	1/8	1/8	1/4

If three identical dice as the above are thrown, the probability of occurrence of values 1, 5 and 6 on the three dice is

(a) same as that of occurrence of 3, 4, 5
(b) same as that of occurrence of 1, 2, 5
(c) 1/128
(d) 5/8

[EE, GATE-2007, 2 marks]

Q.30 An examination consists of two papers, Paper 1 and Paper 2. The probability of failing in Paper 1 is 0.3 and that in Paper 2 is 0.2. Given that a student has failed in Paper 2, the probability of failing in Paper 1 is 0.6. The probability of a student failing in both the papers is

(a) 0.5 (b) 0.18
(c) 0.12 (d) 0.06

[EC, GATE-2007, 2 marks]

Q.31 If the standard deviation of the spot speed of vehicles in a highway is 8.8 kmph and the mean speed of the vehicles is 33 kmph, the coefficient of variation in speed is

(a) 0.1517 (b) 0.1867
(c) 0.2666 (d) 0.3646

[CE, GATE-2007, 2 marks]

Q.32 Let X and Y be two independent random variables. Which one of the relations between expectation (E), variance (Var) and covariance (Cov) given below is FALSE?

(a) $E(XY) = E(X)\,E(Y)$
(b) $\text{Cov}(X, Y) = 0$
(c) $\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y)$
(d) $E(X^2 Y^2) = (E(X))^2 (E(Y))^2$

[ME, GATE-2007, 2 marks]

Q.33 A person on a trip has a choice between private car and public transport. The probability of using a private car is 0.45. While using the public transport, further choices available are bus and metro, out of which the probability of commuting by a bus is 0.55. In such a situation, the probability (rounded up to two decimals) of using a car, bus and metro, respectively would be

(a) 0.45, 0.30 and 0.25
(b) 0.45, 0.25 and 0.30
(c) 0.45, 0.55 and 0.00
(d) 0.45, 0.35 and 0.20

[CE, GATE-2008, 2 marks]

Q.34 Aishwarya studies either computer science or mathematics everyday. if she studies computer science on a day, then the probability that she studies mathematics the next day is 0.6. If she studies mathematics on a day, then the probability that she studies computer science the next day is 0.4. Given that Aishwarya studies computer science on Monday, what is the probablity that she studies computer science on Wednesday?

(a) 0.24 (b) 0.36
(c) 0.4 (d) 0.6

[CS, GATE-2008, 2 marks]

Q.35 A coin is tossed 4 times. What is the probability of getting heads exactly 3 times?

(a) 1/4 (b) 3/8
(c) 1/2 (d) 3/4

[ME, GATE-2008, 1 mark]

Q.36 If probability density function of a random variable x is

$$f(x) = x^2 \text{ for } -1 \le x \le 1, \text{ and}$$
$$= 0 \text{ for any other value of } x$$

then, the percentage probability $P\left(-\frac{1}{3} \le x \le \frac{1}{3}\right)$ is

(a) 0.247 (b) 2.47
(c) 24.7 (d) 247

[CE, GATE-2008, 2 marks]

Q.37 Let X be a randon variable following normal distribution with mean +1 and variance 4. Let Y be another normal variable with mean −1 and variance unknown,. If $P(X \le -1) = P(Y \ge 2)$ the standard deviation of Y is

(a) 3
(b) 2
(c) $\sqrt{2}$
(d) 1

[CS, GATE-2008, 2 marks]

Q.38 A fair coin is tossed 10 times. What is the probability that ONLY the first two tosses will yield heads?

(a) $\left(\frac{1}{2}\right)^2$
(b) ${}^{10}C_2\left(\frac{1}{2}\right)^2$
(c) $\left(\frac{1}{2}\right)^{10}$
(d) ${}^{10}C_2\left(\frac{1}{2}\right)^{10}$

[EC, GATE-2009, 1 mark]

Q.39 An unbalanced dice (with 6 faces, numbered from 1 to 6) is thrown. The probability that the face value is odd is 90% of the probability that the face value is even. The probability of getting any even numbered face is the same.
If the probability that the face is even given that it is greater than 3 is 0.75, which one of the following options is closest to the probability that the face value exceeds 3?

(a) 0.453
(b) 0.468
(c) 0.485
(d) 0.492

[CS, GATE-2009, 2 marks]

Q.40 If two fair coins are flipped and at least one of the outcomes is known to be a head, what is the probability that both outcomes are heads?

(a) 1/3
(b) 1/4
(c) 1/2
(d) 2/3

[CS, GATE-2009, 1 mark]

Q.41 If three coins are tossed simultaneously, the probability of getting at least one head is

(a) 1/8
(b) 3/8
(c) 1/2
(d) 7/8

[ME, GATE-2009, 1 mark]

Q.42 The standard deviation of a uniformly distributed random variable between 0 and 1 is

(a) $\frac{1}{\sqrt{12}}$
(b) $\frac{1}{\sqrt{3}}$
(c) $\frac{5}{\sqrt{12}}$
(d) $\frac{7}{\sqrt{12}}$

[ME, GATE-2009, 2 marks]

Q.43 The standard normal cumulative probability function (probability from $-\infty$ to x_n) can be approximated as

$$F(x_n) = \frac{1}{1+\exp(-1.7255\, x_n\, |x_n|^{0.12})}$$

where x_n = standard normal deviate. If mean and standard deviation of annual precipitation are 102 cm and 27 cm respectively, the probability that the annual precipitation will be between 90 cm and 102 cm is

(a) 66.7%
(b) 50.0%
(c) 33.3%
(d) 16.7%

[CE, GATE-2009, 2 marks]

Q.44 A box contains 2 washers, 3 nuts and 4 bolts. Items are drawn from the box at random one at a time without replacement. The probability of drawing 2 washers first followed by 3 nuts and subsequently the 4 bolts is

(a) 2/315
(b) 1/630
(c) 1/1260
(d) 1/2520

[ME, GATE-2010, 2 marks]

Q.45 A box contains 4 white balls and 3 red balls. In succession, two balls are randomly selected and removed from the box. Give that the first removed ball is white, the probability that the second removed ball is red is

(a) 1/3
(b) 3/7
(c) 1/2
(d) 4/7

[EE, GATE-2010, 2 marks]

Q.46 Consider a company that assembles computers. The probability of a faulty assembly of any computer is p. The company therefore subjects each computer to a testing process. This testing process gives the correct result for any computer with a probability of q. What is the probability of a computer being declared faulty?

(a) $pq + (1 - p)(1 - q)$
(b) $(1 - q)p$
(c) $(1 - p)q$
(d) pq

[CS, GATE-2010, 2 marks]

Q.47 A fair coin is tossed independently four times. The probability of the event "the number of times heads show up is more than the number of times tails show up" is

(a) $\frac{1}{16}$ (b) $\frac{1}{8}$
(c) $\frac{1}{4}$ (d) $\frac{5}{16}$

[EC, GATE-2010, 2 marks]

Q.48 A deck of 5 cards (each carrying a distinct number from 1 to 5) is shuffled thoroughly. Two cards are then removed one at a time from the deck. What is the probability that the two cards are selected with the number on the first card being one higher than the number on the second card?
(a) 1/5 (b) 4/25
(c) 1.4 (d) 2/5

[CS, GATE-2011, 2 marks]

Q.49 There are two containers, with one containing 4 red and 3 green balls and the other containing 3 blue and 4 green balls. One ball is drawn at random from each container. The probability that one of the balls is red and the other is blue will be
(a) 1/7 (b) 9/49
(c) 12/49 (d) 3/7

[CE, GATE-2011, 1 mark]

Q.50 A fair dice is tossed two times. The probability that the second toss results in a value that is higher than the first toss is
(a) 2/36 (b) 2/6
(c) 5/12 (d) 1/2

X

[EC, GATE-2011, 1 mark]

Q.51 Consider the finite sequence of random values $X = [x_1, x_2, ..., x_n]$. Let μ_x be the mean and σ_x be the standard deviation of X. Let another finite sequence Y of equal length be derived from this as $y_i = a * x_i + b$, where a and b are positive constant. Let μ_y be the mean and σ_y be the standard deviation of this sequence. Which one of the following statements INCORRECT?
(a) Index position of mode of X in X is the same as the index position of mode of Y in Y.
(b) Index position of median of X in X is the same as the index position of median of Y in Y
(c) $\mu_y = a\mu_x + b$
(d) $\sigma_y = a\sigma_x + b$

[CS, GATE-2011, 2 marks]

Q.52 If the difference between the expectation of the square of a random variable ($E[x^2]$) and the square of the expectation of the random variable $(E[x])^2$ is denoted by R, then
(a) $R = 0$ (b) $R < 0$
(c) $R ^3 0$ (d) $R > 0$

[CS, GATE-2011, 1 mark]

Q.53 An unbiased coin is tossed five times. The outcome of each toss is either a head or a tail. The probability of getting at least one head is
(a) $\frac{1}{32}$ (b) $\frac{13}{32}$
(c) $\frac{16}{32}$ (d) $\frac{31}{32}$

[ME, GATE-2011, 2 marks]

Q.54 A fair coin is tossed till a head appears for the first time. The probability that the number of required tosses is odd, is
(a) 1/3 (b) 1/2
(c) 2/3 (d) 3/4

[EC, EE, IN, GATE-2012, 2 marks]

Q.55 Suppose a fair six-sided die is rolled once. If the value on the die is 1, 2 or 3 the die is rolled a second time. What is the probability that the sum of total values that turn up is at least 6?
(a) 10/21 (b) 5/12
(c) 2/3 (d) 1/6

[CS, GATE-2012, 2 marks]

Q.56 In an experiment, positive and negative values are equally likely to occur. The probability of obtaining at most one negative value in five trials is
(a) $\frac{1}{32}$ (b) $\frac{2}{32}$
(c) $\frac{3}{32}$ (d) $\frac{6}{32}$

[CE, GATE-2012, 2 marks]

Q.57 Consider a random variable X that takes values +1 and –1 with probability 0.5 each. The values of the cumulative distribution function $F(x)$ at $x = -1$ and $+1$ are
(a) 0 and 0.5 (b) 0 and 1
(c) 0.5 and 1 (d) 0.25 and 0.75

[CS, GATE-2012, 1 mark]

Q.58 A box contains 4 red balls and 6 black balls. Three balls are selected randomly from the box one after another, without replacement. The probability that the selected set contains one red ball and two black balls is
(a) 1/20 (b) 1/12
(c) 3/10 (d) 1/2
[ME, GATE-2012, 2 marks]

Q.59 Two independent random variables X and Y are uniformly distributed in the interval [–1, 1]. The probability that max [X, Y] is less than 1/2 is
(a) 3/4 (b) 9/16
(c) 1/4 (d) 2/3
[EE, GATE-2012, 1 mark]

Q.60 The annual precipitation data of a city is normally distributed with mean and standard deviation as 1000 mm and 200 mm, respectively. The probability that the annual precipitation will be more than 1200 mm is
(a) < 50% (b) 50%
(c) 75% (d) 100%
[CE, GATE-2012, 1 mark]

X **Q.61** Suppose p is the number of cars per minute passing through a certain road junction between 5 PM, and p has Poisson distribution with mean 3. What is the probability of observing fewer than 3 cars during any given minute in this interval?
(a) $8/(2e^3)$ (b) $9/(2e^3)$
(c) $17/(2e^3)$ (d) $26/(2e^3)$
[CS, GATE-2013, 1 Mark]

Q.62 Find the value of λ such that function $f(x)$ is valid probability density function

$$f(x) = \begin{cases} \lambda(x-1)(2-x) & 1 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

[CE, GATE-2013, 2 Mark]

Q.63 A continuous random variable X has a probability density function $f(x) = e^{-x}$, $0 < x < \infty$. Then $P\{X > 1\}$ is
(a) 0.368 (b) 0.5
(c) 0.632 (d) 1.0
[EE, GATE-2013, 1 Mark]

Q.64 A continuous random variable X has a probability density $f(x) = e^{-x}$, $0 < x < \infty$. Then $P\{X > 1\}$ is
(a) 0.368 (b) 0.5
(c) 0.632 (d) 1.0
[IN, GATE-2013 : 1 mark]

X **Q.65** Let X be a normal random variable with mean 1 and variance 4. The probability $P\{X < 0\}$ is
(a) 0.5
(b) greater than zero and less than 0.5
(c) greater than 0.5 and less than 1.0
(d) 1.0
[ME, GATE-2013, 1 Mark]

Q.66 A nationalized bank has found that the daily balance available in its savings accounts follows a normal distribution with a mean of Rs. 500 and a standard deviation of Rs. 50. The percentage of savings account holders, who maintain an average daily balance more than Rs. 500 is ____.
[ME, GATE-2014 : 1 Mark]

Q.67 A fair (unbiased) coin was tossed four times in succession and resulted in the following outcomes: (i) Head, (ii) Head, (iii) Head, (iv) Head. The probability of obtaining a 'Tail' when the coin is tossed again is
(a) 0 (b) $\frac{1}{2}$
(c) $\frac{4}{5}$ (d) $\frac{1}{5}$
[CE, GATE-2014 : 1 Mark]

Q.68 A box contains 25 parts of which 10 are defective. Two parts are being drawn simultaneously in a random manner from the box. The probability of both the parts being good is
(a) $\frac{7}{20}$ (b) $\frac{45}{125}$
(c) $\frac{25}{29}$ (d) $\frac{5}{9}$
[ME, GATE-2014 : 1 Mark]

Q.69 A group consists of equal number of men and women. Of this group 20% of the men and 50% of the women are unemployed. If a person is selected at random from this group, the probability of the selected person being employed is ____.
[ME, GATE-2014 : 1 Mark]

Q.70 A fair coin is tossed n times. The probability that the difference between the number of heads and tails is $(n-3)$ is
(a) 2^{-n} (b) 0
(c) ${}^nC_{n-3}2^{-n}$ (d) 2^{-n+3}
[EE, GATE-2014 : 2 Marks]

Q.71 Consider a dice with the property that the probability of a face with n dots showing up is proportional to n. The probability of the face with three dots showing up is ____ .

[EE, GATE-2014 : 1 Mark]

Q.72 In a housing society, half of the families have a single child per family, while the remaining half have two children per family. The probability that a child picked at random, has a sibling is

[EC, GATE-2014 : 1 Mark]

Q.73 An unbiased coin is tossed an infinite number of times. The probability that the fourth head appears at the tenth toss is

(a) 0.067 (b) 0.073
(c) 0.082 (d) 0.091

[EC, GATE-2014 : 1 Mark]

Q.74 Parcels from sender S to receiver R pass sequentially through two post-offices. Each post-office has a probability 1/5 of losing an incoming parcel, independently of all other parcels. Given that a parcel is lost, the probability that it was lost by the second post-office is ________.

[EC, GATE-2014 : 2 Marks]

Q.75 Four fair six-sided dice are rolled. The probability that the sum of the results being 22 is X/1296. The value of X is ______.

[CS, GATE-2014 : 2 Marks]

Q.76 The probability that a given positive integer lying between 1 and 100 (both inclusive) is NOT divisible by 2, 3 or 5 is _______.

[CS, GATE-2014 : 2 Marks- Set-2]

Q.77 Let S be a sample space and two mutually exclusive events A and B be such that $A \cup B = S$. If $P(\cdot)$ denotes the probability of the event, the maximum value of $P(A)\ P(B)$ is _______.

[CS, 2014 : 2 Marks-Set-3]

Q.78 In the following table, x is a discrete random variable and $p(x)$ is the probability density. The standard deviation of x is

x	1	2	3
$p(x)$	0.3	0.6	0.1

(a) 0.18 (b) 0.36
(c) 0.54 (d) 0.6

[ME, 2014 : 2 Marks]

Q.79 A machine produces 0, 1 or 2 defective pieces in a day with associated probability of 1/6, 2/3 and 1/6, respectively. The mean value and the variance of the number of defective pieces produced by the machine in a day, respectively, are

(a) 1 and 1/3 (b) 1/3 and 1
(c) 1 and 4/3 (d) 1/3 and 4/3

[ME, 2014 : 2 Marks]

Q.80 Consider an unbiased cubic dice with opposite faces coloured identically and each face coloured red, blue or green such that each colour appears only two times on the dice. If the dice is thrown thrice, the probability of obtaining red colour on top face of the dice at least twice is _____.

[ME, GATE-2014 : 2 Marks]

Q.81 The security system at an IT office is composed of 10 computers of which exactly four are working. To check whether the system is functional, the officials inspect four of the computers picked at random (without replacement). The system is deemed functional if at least three of the four computers inspected are working. Let the probability that the system is deemed functional be denoted by p. Then 100p = ________.

[CS, GATE-2014 : 1 Mark]

Q.82 A traffic office imposes on an average 5 number of penalties daily on traffic violators. Assume that the number of penalties on different days is independent and follows a Poisson distribution. The probability that there will be less than 4 penalties in a day is __________.

[CE, GATE-2014 : 2 Marks]

Q.83 The number of accidents occurring in a plant in a month follows Poisson distribution with mean as 5.2. The probability of occurrence of less than 2 accidents in the plant during a randomly selected month is

(a) 0.029 (b) 0.034
(c) 0.039 (d) 0.044

[ME, GATE-2014 : 2 Marks]

Q.84 The probability density function of evaporation E on any day during a year in a watershed is given by

$$f(E) = \begin{cases} \frac{1}{5} & 0 \le E \le 5 \text{ mm/day} \\ 0 & \text{otherwise} \end{cases}$$

The probability that E lies in between 2 and 4 mm/day in a day in the watershed is (in decimal) ______.

[CE, GATE-2014 : 1 Mark]

Q.85 Let X be a random variable with probability density function

$$f(x) = \begin{cases} 0.2, & \text{for } |x| \le 1 \\ 0.1, & \text{for } 1 < |x| \le 4 \\ 0, & \text{otherwise} \end{cases}$$

The probability $P(0.5 < X < 5)$ is ______.

[EE, GATE-2014 : 2 Marks]

Q.86 If $\{x\}$ is a continuous, real valued random variable defined over the interval $(-\infty, +\infty)$ and its occurrence is defined by the density function given as:

$f(x) = \frac{1}{\sqrt{2\pi} * b} e^{-\frac{1}{2}\left(\frac{x-a}{b}\right)^2}$ where 'a' and 'b' are the statistical attributes of the random variable $\{x\}$.

The value of the integral $\int_{-\infty}^{a} \frac{1}{\sqrt{2\pi} * b} e^{-\frac{1}{2}\left(\frac{x-a}{b}\right)^2} dx$

(a) 1
(b) 0.5
(c) π
(d) $\pi/2$

[CE, GATE-2014 : 1 Mark]

Q.87 Suppose A and B are two independent events with probabilities $P(A) \neq 0$ and $P(B) \neq 0$. Let $\bar{A}$ and $\bar{B}$ be their complements. Which one of the following statements is FALSE?

(a) $P(A \cap B) = P(A)\,P(B)$
(b) $P(A/B) = P(A)$
(c) $P(A \cup B) = P(A) + P(B)$
(d) $P(\bar{A} \cap \bar{B}) = P(\bar{A})P(\bar{B})$

[EC, GATE-2015 : 1 Mark]

Q.88 Suppose X_i for $i = 1, 2, 3$ are independent and identically distributed random variables whose probability mass functions are

$Pr[X_i = 0] = Pr[X_i = 1] = 1/2$ for $i = 1, 2, 3$

Define another random variable $Y = X_1 X_2 \oplus X_3$, where $\oplus$ denotes XOR. Then $Pr[Y = 0 \mid X_3 = 0]$ = ______.

[CS, 2015 : 2 Marks]

Q.89 Three vendors were asked to supply a very high precision component. The respective probabilities of their meeting the strict design specifications are 0.8, 0.7 and 0.5. Each vendor supplies one component. The probability that out of total three components supplied by the vendors, at least one will meet the design specification is ______.

[ME, GATE-2015 : 1 Mark]

Q.90 The chance of a student passing an exam is 20%. The chance of a student passing the exam and getting above 90% marks in it is 5%. Given that a student passes the examination, the probability that the student gets above 90% marks is

(a) $\frac{1}{18}$
(b) $\frac{1}{4}$
(c) $\frac{2}{9}$
(d) $\frac{5}{18}$

[ME, GATE-2015 : 2 Marks]

Q.91 If $P(X) = \frac{1}{4}$, $P(Y) = \frac{1}{3}$, and $P(X \cap Y) = \frac{1}{12}$, the value of $P(Y/X)$ is

(a) $\frac{1}{4}$
(b) $\frac{4}{25}$
(c) $\frac{1}{3}$
(d) $\frac{29}{50}$

[ME, GATE-2015 : 1 Mark]

Q.92 Two players, A and B, alternately keep rolling a fair dice. The person to get a six first wins the game. Given that player A starts the game, the probability that A wins the game is

(a) $\frac{5}{11}$
(b) $\frac{1}{2}$
(c) $\frac{7}{13}$
(d) $\frac{6}{11}$

[EE, GATE-2015 : 2 Marks]

Q.93 Consider the following probability mass function (p.m.f.) of a random variable X.

$$p(X,q) = \begin{cases} q & \text{If } X = 0 \\ 1-q & \text{If } X = 1 \\ 0 & \text{otherwise} \end{cases}$$

If $q = 0.4$, the variance of X is ______.

[CE, GATE-2015 : 1 Mark]

Q.94 Two coins R ans S are tossed. The 4 joint events H_RH_S, T_RT_S, H_RT_S, T_RH_S have probabilities 0.28, 0.18, 0.30, 0.24, respectively, where H represents head and T represents tail. Which one of the following is TRUE?
(a) The coin tosses are independent
(b) R is fair, S is not
(c) S is fair, R is not
(d) The coin tosses are dependent

[EE, GATE-2015 : 2 Marks]

Q.95 The probability of obtaining at least two "SIX" in throwing a fair dice 4 times is

(a) $\frac{425}{432}$ (b) $\frac{19}{144}$

(c) $\frac{13}{144}$ (d) $\frac{125}{432}$

[ME, GATE-2015 : 2 Marks]

Q.96 The probability that a thermistor randomly picked up from a production unit is defective is 0.1. The probability that out of 10 thermistors randomly picked up, 3 are defective is
(a) 0.001 (b) 0.057
(c) 0.107 (d) 0.3

[IN, GATE-2015 : 2 Marks]

Q.97 The probability density function of a random variable, x is

$$f(x) = \frac{x}{4}(4-x^2) \text{ for } 0 \le x \le 2 = 0$$

The mean, μ_x of the random variable is ________.

[CE, GATE-2015 : 2 Marks]

Q.98 A random variable X has probability density function $f(x)$ as given below:

$$f(x) = \begin{cases} a+bx & \text{for } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

If the expected value $E[X] = 2/3$, then $Pr[X < 0.5]$ is ________.

[EE, GATE-2015 : 1 Mark]

Q.99 An urn contains 5 red and 7 green balls. A ball is drawn at random and its colour is noted. The ball is placed back into the urn along with another ball of the same colour. The probability of getting a red ball in the next draw is

(a) $\frac{65}{156}$ (b) $\frac{67}{156}$

(c) $\frac{79}{156}$ (d) $\frac{89}{156}$

[IN, 2016 : 2 Marks]

Q.100 The probability of getting a "head" in a single toss of a biased coin is 0.3. The coin is tossed repeatedly till a "head" is obtained. If the tosses are independent, then the probability of getting "head" for the first time in the fifth toss is ________.

[EC, 2016 : 1 Mark]

Q.101 X and Y are two random independent events. It is known that $P(X) = 0.40$ and $P(X \cup Y^C) = 0.7$. Which one of the following is the value of $P(X \cup Y)$?
(a) 0.7 (b) 0.5
(c) 0.4 (d) 0.3

[CE, 2016 : 1 Mark]

Q.102 Consider the following experiment.
Step 1. Flip a fair coin twice.
Step 2. If the outcomes are (TAILS, HEADS) then output Y and stop.
Step 3. If the outcomes are either (HEADS, HEADS) or (HEADS, TAILS), then output N and stop.
Step 4. If the outcomes are (TAILS, TAILS), then go to Step 1.

The probability that the output of the experiment is Y is _____ (up to two decimal places).

[CS, 2016 : 2 Marks]

Q.103 Suppose that a shop has an equal number of LED bulbs of two different types. The probability of an LED bulb lasting more than 100 hours given that it is of Type 1 is 0.7, and given that it is of Type 2 is 0.4. The probability that an LED bulb chosen uniformly at random lasts more than 100 hours is______.

[CS, 2016 : 1 Mark]

Q.104 Three cards were drawn from a pack of 52 cards. The probability that they are a king, a queen, and a jack is

(a) $\frac{16}{5525}$ (b) $\frac{64}{2197}$

(c) $\frac{3}{13}$ (d) $\frac{8}{16575}$

[ME, 2016 : 2 Marks]

Q.105 Type II error in hypothesis testing is

(a) acceptance of the null hypothesis when it is false and should be rejected
(b) rejection of the null hypothesis when it is true and should be accepted
(c) rejection of the null hypothesis when it is false and should be rejected
(d) acceptance of the null hypothesis when it is true and should be accepted

[CE, 2016 : 1 Mark]

Q.106 The spot speeds (expressed in km/hr) observed at a road section are 66, 62, 45, 79, 32, 51, 56, 60, 53, and 49. The median speed (expressed in km/hr) is ______.
(**Note:** answer with one decimal accuracy)

[CE, 2016 : 1 Mark]

Q.107 If $f(x)$ and $g(x)$ are two probability density functions,

$$f(x) = \begin{cases} \frac{x}{a}+1 & : -a \le x < 0 \\ -\frac{x}{a}+1 & : 0 \le x \le a \\ 0 & : \text{otherwise} \end{cases};$$

$$g(x) = \begin{cases} -\frac{x}{a} & : -a \le x < 0 \\ \frac{x}{a} & : 0 \le x \le a \\ 0 & : \text{otherwise} \end{cases}$$

Which one of the following statements is true?

(a) Mean of $f(x)$ and $g(x)$ are same; Variance of $f(x)$ and $g(x)$ are same
(b) Mean of $f(x)$ and $g(x)$ are same; Variance of $f(x)$ and $g(x)$ are different
(c) Mean of $f(x)$ and $g(x)$ are different; Variance of $f(x)$ and $g(x)$ are same
(d) Mean of $f(x)$ and $g(x)$ are different; Variance of $f(x)$ and $g(x)$ are different

[CE, 2016 : 2 Marks]

Q.108 The probability that a screw manufactured by a company is defective is 0.1. The company sells screws in packets containing 5 screws and gives a guarantee of replacement if one or more screws in the packet are found to be defective. The probability that a packet would have to be replaced is______.

[ME, 2016 : 2 Marks]

Q.109 The second moment of a Poisson-distributed random variable is 2. The mean of the random variable is ______.

[EC, 2016 : 1 Mark]

Q.110 Consider a Poisson distribution for the tossing of a biased coin. The mean for this distribution is μ. The standard deviation for this distribution is given by

(a) $\sqrt{\mu}$
(b) μ^2
(c) μ
(d) $\frac{1}{\mu}$

[ME, 2016 : 1 Mark]

Q.111 A probability density function on the interval $[a, 1]$ is given by $1/x^2$ and outside this interval the value of the function is zero. The value of a is___.

[CS, 2016 : 1 Mark]

Q.112 Two random variables X and Y are distributed according to

$$f_{X,Y}(x,y) = \begin{cases} (x+y) & 0 \le x \le 1, \quad 0 \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$$

The probability $P(X + Y \le 1)$ is ______.

[EC, 2016 : 2 Marks]

Q.113 Probability density function of a random variable X is given below

$$f(x) = \begin{bmatrix} 0.25 & \text{if } 1 \le x \ge 5 \\ 0 & \text{otherwise} \end{bmatrix} \quad P(X \le 4)$$

(a) $\frac{3}{4}$
(b) $\frac{1}{2}$
(c) $\frac{1}{4}$
(d) $\frac{1}{8}$

[CE, 2016 : 2 Marks]

Q.114 Let the probability density function of a random variable, X, be given as:

$$f_x(x) = \frac{3}{2}e^{-3x}u(x) + ae^{4x}u(-x)$$

where $u(x)$ is the unit step function. Then the value of 'a' and Prob $\{X \le 0\}$, respectively, are

(a) $2, \frac{1}{2}$
(b) $4, \frac{1}{2}$
(c) $2, \frac{1}{4}$
(d) $4, \frac{1}{4}$

[EE, 2016 : 2 Marks]

Q.115 The area (in percentage) under standard normal distribution curve of random variable Z within limits from -3 to $+3$ is______

[ME, 2016 : 1 Mark]

Q.116 Two coins are tossed simultaneously. The probability (upto two decimal points accuracy) of getting at least one head is ______.

[ME, GATE-2017 : 1 Mark]

Q.117 A sample of 15 data is as follows: 17, 18, 17, 17, 13, 18, 5, 5, 6, 7, 8, 9, 20, 17, 3. The mode of the data is

(a) 4 (b) 13
(c) 17 (d) 20

[ME, GATE-2017 : 1 Mark]

Q.118 A six-face fair dice is rolled a large number of times. The mean value of the outcomes is ______.

[ME, GATE-2017 : 1 Mark]

Q.119 Assume that in a traffic junction, the cycle of the traffic signal lights is 2 minutes of green (vehicle does not stop) and 3 minutes of red (vehicle stops). Consider that the arrival time of vehicles at the junction is uniformly distributed over 5 minute cycle. The expected waiting time (in minutes) for the vehicle at the junction is______.

[EE, GATE-2017 : 1 Mark]

Q.120 An urn contains 5 red balls and 5 black balls. In the first draw, one ball is picked at random and discarded without noticing its colour. The probability to get a red ball in the second draw is

(a) $\frac{1}{2}$ (b) $\frac{4}{9}$
(c) $\frac{5}{9}$ (d) $\frac{6}{9}$

[EE, GATE-2017 : 1 Mark]

Q.121 If a random variable X has a Poisson distribution with mean 5, then the expectation $E[(X + 2)^2]$ equals ______.

[CS, GATE-2017 : 2 Marks]

Q.122 A two-faced coin has its faces designated as head (H) and tail (T). This coin is tossed three times in succession to record the following outcomes; H, H, H. If the coin is tossed one more time, the probability (up to one decimal place) of obtaining H again, given the previous realizations of H, H and H, would be ______.

[CE, GATE-2017 : 1 Mark]

Q.123 The number of parameters in the univariate exponential and Gaussian distributions, respectively, are

(a) 2 and 2 (b) 1 and 2
(c) 2 and 1 (d) 1 and 1

[CE, GATE-2017 : 1 Mark]

Q.124 For the function $f(x) = a + bx$, $0 \le x \le 1$, to be a valid probability density function, which one of the following statements is correct?

(a) $a = 1, b = 4$ (b) $a = 0.5, b = 1$
(c) $a = 0, b = 1$ (d) $a = 1, b = -1$

[CE, GATE-2017 : 2 Marks]

■■■■

Answers Probability and Statistics

1.	(d)	2.	(c)	3.	(d)	4.	(c)	5.	(c)	6.	(d)	7.	(a)	8.	(a)	9.	(d)
10.	(d)	11.	(d)	12.	(d)	13.	(d)	14.	(b)	15.	(d)	16.	(b)	17.	(c)	18.	(d)
19.	(a)	20.	(c)	21.	(d)	22.	(a)	23.	(b)	24.	(d)	25.	(b)	26.	(c)	27.	(d)
28.	(d)	29.	(c)	30.	(c)	31.	(c)	32.	(d)	33.	(a)	34.	(c)	35.	(a)	36.	(b)
37.	(a)	38.	(c)	39.	(b)	40.	(a)	41.	(d)	42.	(a)	43.	(b)	44.	(c)	45.	(c)
46.	(a)	47.	(d)	48.	(a)	49.	(c)	50.	(c)	51.	(d)	52.	(c)	53.	(d)	54.	(c)
55.	(b)	56.	(d)	57.	(c)	58.	(d)	59.	(b)	60.	(a)	61.	(c)	63.	(a)	64.	(a)
65.	(b)	67.	(b)	68.	(a)	70.	(b)	73.	(c)	78.	(d)	79.	(a)	83.	(b)	86.	(b)
87.	(c)	90.	(b)	91.	(c)	92.	(d)	94.	(d)	95.	(b)	96.	(b)	99.	(a)	101.	(a)
104.	(a)	105.	(a)	107.	(b)	110.	(a)	113.	(a)	114.	(a)	117.	(c)	120.	(a)	123.	(b)
124.	(b)																

Explanations Probability and Statistics

1. (d)

Given, $P(A) = 1$

$P(B) = 1/2$

Both events are independent

So, $P(A \cap B) = 1/2$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{1/2}{1/2} = 1$$

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)} = \frac{1/2}{1} = 1/2$$

2. (c)

This problem is to be solved by binomial distribution, since although population is finite, sampling is done with replacement and so probability does not change from trial to trial.

Here, $n = 2$

$x = 0$ (no defective)

$$p = \text{p(defective)} = \frac{3}{10}$$

So, $$p(x=0) = 2C_0\left(\frac{3}{10}\right)^0\left(1-\frac{3}{10}\right)^2$$

$$= 0.49 = 49\%$$

3. (d)

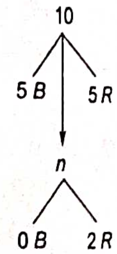
Probability of drawing two red balls

= p(first is red) × p(second is red given that first is red)

$$= \frac{5}{10} \times \frac{4}{9} = \frac{2}{9}$$

Alternatively this problem can be done as hypergeometric distribution, since it is sampling without replacement from finite population.

From above diagram,



$$p(X=2) = \frac{{}^5C_2}{{}^{10}C_2}$$

$$= \frac{5 \times 4}{10 \times 9} = \frac{2}{9}$$

4. (c)

Let the time taken for first and second modules be represented by x and y and total time = t.

$\therefore$ $t = x + y$ is a random variable.

Now the joint density function,

$$g(t) = \int_0^t f(x, y)dx = \int_0^t f(x, t-x)dx$$

$$= \int_0^t f_1(x)\, f_2(t-x)dx$$

which is also called as convolution of f_1 and f_2, abbreviated as $f_1 * f_2$.

Correct answer is therefore, choice (c).

5. **(c)**

Since all three gates are independent

p(gate 2 and gate 3 fail | gate 1 failed)

$= p$(gate 2 and gate 3 fail)

$= p$(gate 2) $\times$ p(gate 3)

[gate 2 and 3 fail independently]

$= 0.2 \times 0.2 = 0.04$

6. **(d)**

Let the marks obtained per question be a random variable X.

Its probability distribution table is given below:

X	1	–0.25
p(X)	1/4	3/4

Expected marks per question

$= E(x) = \Sigma X\, p(X)$

$= 1 \times 1/4 + (-0.25) \times 3/4$

$= 1/4 - 3/16 = 1/16$ marks

Total marks expected for 150 questions

$= \frac{1}{16} \times 150 = \frac{75}{8}$ marks per student

Total expected marks of 1000 students

$= \frac{75}{8} \times 1000 = 9375$ marks

So, correct answer is (d).

7. **(a)**

The condition getting 2 heads and 2 tails is same as getting exactly 2 heads out of 4 tosses.

Given, $p = P(H) = 1/2$

Applying the formula for binomial distribution, we get,

$$P(X=2) = {}^4C_2(1/2)^2\left(1-\frac{1}{2}\right)^{4-2}$$

$$= {}^4C_2\left(\frac{1}{2}\right)^2(1/2)^2 = \frac{{}^4C_2}{2^4} = \frac{6}{16} = \frac{3}{8}$$

8. **(a)**

If hamming distance between two n bit strings is d, we are asking that d out of n trials to be success (success here means that the bits are different). So this is a binomial distribution with n trials and d successes and probability of success

$p = 2/4 = 1/2$

(Since out of the 4 possibilities {(0, 0), (0, 1), (1, 0), (1, 1)} only two of then (0, 1) and (1, 0) are success)

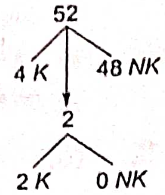
So, $p(X=d) = {}^nC_d\,(1/2)^d\,(1/2)^{n-d} = \frac{{}^nC_d}{2^n}$

Correct choice is therefore (a).

9. **(d)**

Problems can be solved by hypergeometric distribution as follows:

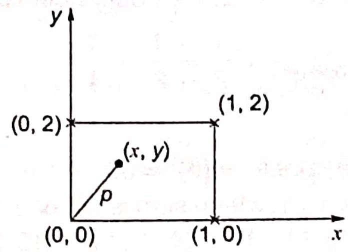
$$p(X=2) = \frac{{}^4C_2 \times {}^{48}C_0}{{}^{52}C_2} = \frac{1}{221}$$



10. **(d)**

Length of position vector of point

$= p = \sqrt{x^2 + y^2}$



$p^2 = x^2 + y^2$

$E(p^2) = E(x^2 + y^2) = E(x^2) + E(y^2)$

Now x and y are uniformly distributed $0 \le x \le 1$ and $0 \le y \le 2$

Probability density function of $x = \frac{1}{1-0} = 1$

Probability density function of $y = \frac{1}{2-0} = 1/2$

$$E(x^2) = \int_0^1 x^2 p(x)\,dx = \int_0^1 x^2 \cdot 1 \cdot dx$$

$$= \left[\frac{x^3}{3}\right]_0^1 = \frac{1}{3}$$

$$E(y^2) = \int_0^2 y^2\, p(y)\, dy = \int_0^2 y^2 \cdot 1/2 \cdot dy$$

$$= \left[\frac{y^3}{6}\right]_0^2 = \frac{8}{6} = \frac{4}{3}$$

$\therefore \quad E(p^2) = E(x^2) + E(y^2) = \frac{1}{3} + \frac{4}{3} = \frac{5}{3}$

11. **(d)**

(a) is false since of P and Q are independent

$pr(P \cap Q) = pr(P) * pr(Q)$

which need not be zero.

(b) is false since

$pr(P \cup Q) = pr(P) + pr(Q) - pr(P \cap Q)$

$\therefore\ pr(P \cup Q) \le pr(P) + pr(Q)$

(c) is false since independence and mutually exclusion are unrelated properties.

(d) is true

since $P \cap Q \subseteq P$

$\Rightarrow n(P \cap Q) \leq n(P)$

$\Rightarrow pr(P \cap Q) \leq pr(P)$

12. (d)

$$P_o = \frac{3}{6} = \frac{1}{2}$$

$$P_e = \frac{3}{6} = \frac{1}{2}$$

Since both events are independent of each other.

$$P_{(\text{odd/even})} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

13. (d)

Sample space $= (6)^2 = 36$

Total ways in which sum is either 8 or 9 is

(2, 6), (3, 5), (3, 6), (4, 4), (4, 5), (5, 3), (5, 4), (6, 2), (6, 3) = 9 ways

$\therefore$ Probability of coming sum 8 or 9 $= \frac{9}{36} = \frac{1}{4}$

So probability of not coming sum 8 or 9

$= 1 - \frac{1}{4} = \frac{3}{4}$

14. (b)

Sample space = {HHH, HTH, HHT, HTT}

Favourable (2 heads in 3 tosses) = {HTH, HHT}

Required probability $= \frac{2}{4} = \frac{1}{2}$

15. (d)

a, b, c are true but (d) is not true since in a negatively skewed distribution, mode > median > mean.

16. (b)

This problem can be done using binomial distribution since population is infinite.

Probability of defective item,

$$p = 0.1$$

Probability of non-defective item,

$$q = 1 - p = 1 - 0.1 = 0.9$$

Probability that exactly 2 of the chosen items are defective

$$= {}^{10}C_2(p)^2(q)^8$$

$$= {}^{10}C_2(0.1)^2(0.9)^8 = 0.1937$$

17. (c)

If $f(x)$ is the continuous probability density function of a random variable X then,

$$p(a < x \leq b) = p(a \leq x \leq b) = \int_a^b f(x)dx$$

18. (d)

(a) is false since if P & Q are independent

$pr(P \cap Q) = pr(P) * pr(Q)$

which need not be zero.

(b) is false since

$pr(P \cup Q) = pr(P) + pr(Q) - pr(P \cap Q)$

$\therefore pr(P \cup Q) \leq pr(P) + pr(Q)$

(c) is false since independence and mutually exclusive are unrelated properties.

(d) is true

since $P \cap Q \subseteq P$

$\Rightarrow n(P \cap Q) \leq n(P)$

$\div$ both sides by $n(S)$ we get,

$$\frac{n(P \cap Q)}{n(S)} \leq \frac{n(P)}{n(S)}$$

$\Rightarrow pr(P \cap Q) \leq pr(P)$

19. (a)

$$I = \frac{1}{\sqrt{2\pi}} \int_0^\infty e^{(-x^2/8)} dx$$

Comparing with

$$\frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{\infty} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx = 1$$

We can put μ and σ as any thing:

Here, putting $\mu = 0$

$$2\int_0^\infty \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}} = 1$$

Putting, $\quad -\frac{x^2}{8} = -\frac{x^2}{2\sigma^2}$

$\Rightarrow \quad \sigma = 2,$

Now putting $\sigma = 2$, in above equation, we get,

$$\therefore \quad \int_0^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{8}} = 1$$

20. (c)

If two fair dices are rolles the probability distribution of r where r is the sum of the numbers on each die is given by

Y	2	3	4	5	6	7	8	9	10	11	12
P(Y)	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

The above table has been obtained by taking all different ways of obtaining a particular sum. For example, a sum of 5 can be obtained by (1, 4), (2, 3), (3, 2) and (4, 1).

$\therefore \quad p(x=5) = 4/36$

Now let us consider choice (a)

$$Pr(r>6) = pr(r \geq 7)$$

$$= \frac{6}{36}+\frac{5}{36}+\frac{4}{36}+\frac{3}{36}+\frac{2}{36}+\frac{1}{36} = \frac{21}{36} = \frac{7}{12}$$

$\therefore$ choice (a) pr (r > 6) = 1/6 is wrong.

Consider choice (b)

$Pr(r/3 \text{ is an integer}) = pr(r=3) + pr(r=6) + pr(r=9) + pr(r=12)$

$$= \frac{2}{36}+\frac{5}{36}+\frac{4}{36}+\frac{1}{36} = \frac{12}{36} = \frac{1}{3}$$

$\therefore$ choice (b) $pr\ r/3$ is an integer = 5/6 is wrong

Consider choice (c)

$$pr(r=8 \mid r/4 \text{ isan integer}) = \frac{1}{36}$$

Now, $pr(r/4 \text{ is an integer}) = pr(r=4) + pr(r=8) + pr(r=12)$

$$= \frac{3}{36}+\frac{5}{36}+\frac{1}{36} = \frac{9}{36} = \frac{1}{4}$$

$$\text{pr(r = 8 and r/4 is an integer)} = pr(r=8) = \frac{5}{36}$$

$$\therefore pr(r=8 \mid r/4 \text{ is aninteger}) = \frac{5/36}{1/4} = \frac{20}{36} = \frac{5}{9}$$

$\therefore$ Choice (c) is correct.

21. (d)

$S \rightarrow$ supply by y, $d \rightarrow$ defective

Probability that the computer was supplied by y, if the product is defective

$$P(s/d) = \frac{P(s \cap d)}{P(d)}$$

$$P(s \cap d) = 0.3 \times 0.02 = 0.006$$

$$P(d) = 0.6 \times 0.1 + 0.3 \times 0.02 + 0.1 \times 0.03 = 0.015$$

$$P(s/d) = \frac{0.006}{0.015} = 0.4$$

22. (a)

The probability that exactly n elements are chosen = The probability of getting n heads out of $2n$ tosses

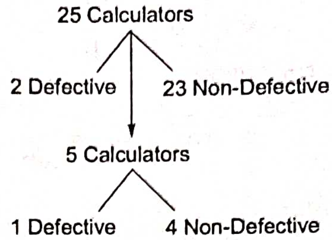
$$= 2nC_n (1/2)^n (1/2)^{2n-n} \text{ (Binomial formula)}$$

$$= 2nC_n (1/2)^n (1/2)^n$$

$$= \frac{^{2n}C_n}{2^{2n}} = \frac{^{2n}C_n}{(2^2)^n} = \frac{^{2n}C_n}{4^n}$$

23. (b)

Since population is finite, hypergeometric distribution is applicable

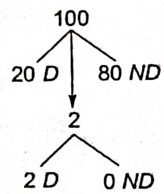


$$p(\text{1 defective in 5 calculators}) = \frac{2C_1 \times 23C_4}{25C_5} = \frac{1}{3}$$

24. (d)

Problem can be solved by hypergeometric distribution

$$p(X=2) = \frac{20C_2 \times 80C_0}{100C_2} = \frac{19}{495}$$



25. (b)

$$\text{Mean} \quad \mu_t = E(t) = \int_{-\infty}^{\infty} t \cdot f(t) \cdot dt = \int_{-1}^{1} t \cdot f(t) \cdot dt$$

$$= \int_{-1}^{0} t(1+t)dt + \int_{0}^{1} t(1-t)dt$$

$$= \left[\frac{t^2}{2}+\frac{t^3}{3}\right]_{-1}^{0} + \left[\frac{t^2}{2}-\frac{t^3}{3}\right]_{0}^{1}$$

$$= -\left[\frac{1}{2}-\frac{1}{3}\right] + \left[\frac{1}{2}-\frac{1}{3}\right]$$

$$= -\left[\frac{1}{6}+\frac{1}{6}\right] = 0$$

$$\text{Variance} = E(t^2) - [E(t)]^2$$

$$= \int_{-\infty}^{\infty} t^2 f(t)dt - [E(t)]^2$$

$$= \int_{-\infty}^{\infty} t^2 f(t)dt - (0)^2 = \int_{-\infty}^{\infty} t^2 f(t)dt$$

$$= \int_{-1}^{0} t^2(1+t)dt + \int_{0}^{1} t^2(1-t)dt$$

$$= \int_{-1}^{0} (t^2+t^3)\cdot dt + \int_{0}^{1} t^2(1-t)dt$$

$$= \left[\frac{t^3}{3}+\frac{t^4}{4}\right]_{-1}^{0} + \left[\frac{t^3}{3}-\frac{t^4}{4}\right]_{0}^{1}$$

$$= \frac{1}{12}+\frac{1}{12} = \frac{1}{6}$$

Standard deviation

$$= \sqrt{\text{variance}} = \frac{1}{\sqrt{6}}$$

26. (c)

$$\int_{-\infty}^{\infty} p(x)\cdot dx = 1$$

$$\int_{-\infty}^{\infty} K\cdot e^{-\alpha|x|}\cdot dx = 1$$

$$\int_{-\infty}^{0} K\cdot e^{\alpha x}\cdot dx + \int_{0}^{\infty} K\cdot e^{-\alpha x} = 1$$

$$\Rightarrow \quad \frac{K}{\alpha}\left(e^{\alpha x}\right)_{-\infty}^{0} + \frac{K}{-\alpha}\left(e^{-\alpha x}\right)_{0}^{\infty} = 1$$

$$\Rightarrow \quad \frac{K}{\alpha}+\frac{K}{\alpha} = 1$$

$$2K = \alpha$$

$$\Rightarrow \quad K = 0.5\,\alpha$$

27. (d)

Let the mean and standard deviation of the students of batch C be μ_c and σ_c respectively, and the mean and standard deviation of entire class of first year students be μ and σ respectively.

Now given, $\mu_c = 6.6$

$\sigma_c = 2.3$

and $\mu = 5.5$

$\sigma = 4.2$

In order to normalise batch C to entire class, the normalised score (z scores) must be equated.

since $$Z = \frac{x-\mu}{\sigma} = \frac{x-5.5}{4.2}$$

$$Z_c = \frac{x_c-\mu_c}{\sigma_c} = \frac{8.5-6.6}{2.3}$$

Equating these two and solving, we get

$$\frac{8.5-6.6}{2.3} = \frac{x-5.5}{4.2}$$

$$\Rightarrow \quad x = 8.969 \simeq 9.0$$

28. (d)

Number of permutations with '2' in the first position = 19!

Number of permutations with '2' in the second position = $10 \times 18!$

(fill the first space with any of the 10 odd numbers and the 18 spaces after the 2 with 18 of the remaining numbers in 18! ways)

Number of permutations with '2' in 3rd position = $10 \times 9 \times 17!$

(fill the first 2 places with 2 of the 10 odd numbers and then the remaining 17 places with remaining 17 numbers)

and so on until '2' is in 11th place. After that it is not possible to satisfy the given condition, since there are only 10 odd numbers available to fill before the '2'. So the desired number of permutations which satisfies the given condition is

$19! + 10 \times 18! + 10 \times 9 \times 17! + 10 \times 9 \times 8 \times 16! + \ldots + 10! \times 9!$

Now the probability of this happening is given by

$$\frac{19! + 10\times 18! + 10\times 9\times 17! \ldots + 10!\times 9!}{20!}$$

Which is clearly not choices (a), (b) or (c). $\therefore$ Answer is (d) none of these.

29. (c)

Dice value	1	2	3	4	5	6
Probability	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{4}$

Since the dice are independent,

$$p(1, 5, 6) = \frac{1}{4}\times\frac{1}{8}\times\frac{1}{4} = \frac{1}{128}$$

30. (c)

(A denote the event of failing in paper 1)

(B denote the event of failing in paper 2)

Given, $P(A) = 0.3$, $P(B) = 0.2$

$P(A/B) = 0.6$

Probability of failing in both

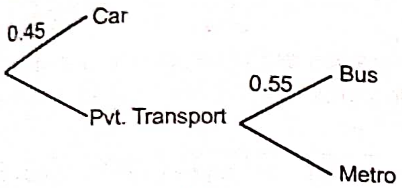
$$P(A\cap B) = P(B) * p(A \mid B)$$
$$= 0.2 * 0.6 = 0.12$$

31. (c)

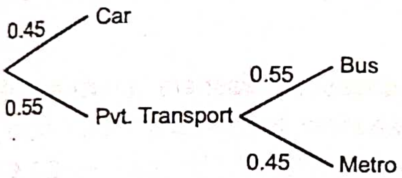
$$CV = \frac{\sigma}{\mu} = \frac{8.8}{33} = 0.2666$$

32. (d)
(a) is true, (b) is true, (c) is true.
(d) is false.
since,
$$E(X^2\,Y^2) = E(X^2)\,E(Y^2)$$
But since X is not independent of X,
$$E(X^2) \neq [E(X)]^2$$
$$\therefore\quad E(X^2\,Y^2) = E(X^2)\,E(Y^2) \neq [E(X)]^2\,[E(Y)]^2$$

33. (a)
The information given in the problem can be represented by the tree diagram given below:

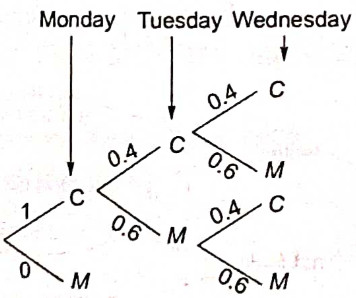


Now completing the blanks in the above diagram we have the final diagram as shown below:



From above diagram
$$\therefore\quad p(\text{Car}) = 0.45$$
$$p(\text{Bus}) = 0.55 \times 0.55 = 0.30$$
and $p(\text{Metro}) = 0.55 \times 0.45 = 0.25$

34. (c)
Let C denote computes science study and M denotes maths study. The tree diagram for the problem can be represented as shown below:



Now by rule of total probability we total up the desired branches and get the answer as shown below:
$p(C$ on monday and C on wednesday$)$
$= p(C$ on monday, C on tuesday and C on wednesday$) + p(C$ on monday, M on tuesday and C on wednesday$)$
$$= 1 \times 0.6 \times 0.4 + 1 \times 0.4 \times 0.4 = 0.24 + 0.16 = 0.40$$

35. (a)
Binomial distribution is used, since this problem involves coins.
$$p = p(H) = 0.5$$
Probability of getting head exactly 3 times is
$$p(X = 3) = {}^4C_3 (0.5)^3 (0.5)^1 = 1/4$$

36. (b)
Given, $f(x) = x^2 \qquad -1 \le x \le 1$
$\qquad\quad = 0 \qquad$ elsewhere
$$p\left(-\frac{1}{3} \le x \le \frac{1}{3}\right) = \int_{-\frac{1}{3}}^{\frac{1}{3}} f(x)\,dx$$
$$= \int_{-\frac{1}{3}}^{\frac{1}{3}} x^2\,dx = \left[\frac{x^3}{3}\right]_{-\frac{1}{3}}^{\frac{1}{3}} = \frac{2}{81}$$
The probability expressed in percentage,
$$p = \frac{2}{81} \times 100 = 2.469\% = 2.47\%$$

37. (a)
Given, $\mu_X = 1,\ \sigma_x^2 = 4 \Rightarrow \sigma_x = 2$
Also given, $\mu_Y = -1$ and σ_Y is unknown
given,
$$p(X \le -1) = p(Y \ge 2)$$
Converting into standard normal variates,
$$p\left(z \le \frac{-1-\mu_x}{\sigma_x}\right) = p\left(z \ge \frac{2-\mu_y}{\sigma_y}\right)$$
$$p\left(z \le \frac{-1-1}{2}\right) = p\left(z \ge \frac{2-(-1)}{\sigma_y}\right)$$
$$p(z \le -1) = p\left(z \ge \frac{3}{\sigma_y}\right) \qquad \text{... (i)}$$
Now since us know that in standard normal distribution,
$$p(z \le -1) = p(z \ge 1) \qquad \text{... (ii)}$$
Comparing (i) and (ii) we can say that
$$\frac{3}{\sigma_y} = 1 \Rightarrow \sigma_y = 3$$

38. (c)
p(only first two tosses are heads) $= p(H, H, T, T, T, \ldots T)$
Now, each toss is independent.
So required probability
$$= p(H) \times p(H) \times [p(T)]^8 \ldots$$
$$= \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^8 = \left(\frac{1}{2}\right)^{10}$$

39. (b)

It is given that

$$p(\text{odd}) = 0.9\,p(\text{even})$$

Now since, $\Sigma p(x) = 1$

$\therefore$ $p(\text{odd}) + p(\text{even}) = 1$

$\Rightarrow 0.9\,p(\text{even}) + p(\text{even}) = 1$

$$\Rightarrow \quad p(\text{even}) = \frac{1}{1.9} = 0.5263$$

Now, it is given that p (any even face) is same

i.e. $p(2) = p(4) = p(6)$

Now since, $p(\text{even}) = p(2)$ or $p(4)$ or $p(6)$

$$= p(2) + p(4) + p(6)$$

$$\therefore \quad p(2) = p(4) = p(6) = \frac{1}{3}$$

$$p(\text{even}) = \frac{1}{3}(0.5263) = 0.1754$$

It is given that

$$p(\text{even} \mid \text{face} > 3) = 0.75$$

$$\Rightarrow \frac{p(\text{even} \cap \text{face} > 3)}{p(\text{face} > 3)} = 0.75$$

$$\Rightarrow \frac{p(\text{face} = 4,6)}{p(\text{face} > 3)} = 0.75$$

$$\Rightarrow \quad p\,(\text{face} > 3) = \frac{p(\text{face} = 4,6)}{0.75}$$

$$= \frac{p(4) + p(6)}{0.75}$$

$$= \frac{0.1754 + 0.1754}{0.75}$$

$$= 0.4677 \simeq 0.468$$

40. (a)

Sample space = {HT, TH, HH}

Both outcomes head = {HH}

$$\text{Required probability} = \frac{1}{3}$$

41. (d)

Binomial distribution is used since this problem involves coins.

Here, $n = 3$

$p = p(H) = 1/2$

$x \geq 1$

Now, $p(x \geq 1) = 1 - p(x = 0)$

$$= 1 - 3C_0\left(\frac{1}{2}\right)^0\left(1 - \frac{1}{2}\right)^3$$

$$= 1 - \frac{1}{8} = \frac{7}{8}$$

42. (a)

$$\sigma = \sqrt{\frac{(\beta - \alpha)^2}{12}} = \sqrt{\frac{(1-0)^2}{12}} = \frac{1}{\sqrt{12}}$$

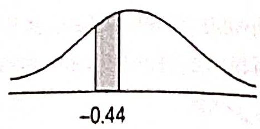
43. (b)

Here $\mu = 102$ cm; $\sigma = 27$ cm

$$p(90 \leq x \leq 102) = p\left(\frac{90 - 102}{27} \leq x_n \leq \frac{102 - 102}{27}\right)$$

$= p(-0.44 \leq x_n \leq 0)$

This area is shown below:



The shades area in above figure is given by $F(0) - F(-0.44)$

$$= \frac{1}{1 + \exp(0)} - \frac{1}{1 + \exp(-1.7255(-0.44)(0.44)0.12)}$$

$= 0.5 - 0.3345 = 0.1655 \simeq 16.55\%$

closest answer is 16.7%.

44. (c)

Box contains 2 washers, 3 nuts and 4 bolts

p(2 washers, then 3 nuts, then 4 bolts)

$$= \left(\frac{2}{9} \times \frac{1}{8}\right) \times \left(\frac{3}{7} \times \frac{2}{6} \times \frac{1}{5}\right) \times \left(\frac{4}{4} \times \frac{3}{3} \times \frac{2}{2} \times \frac{1}{1}\right) = \frac{1}{1260}$$

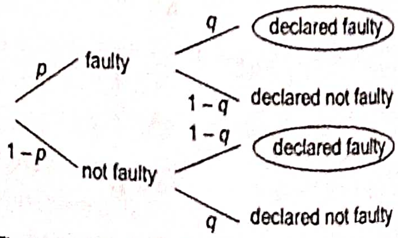
45. (c)

$p(\text{II is red} \mid \text{I is white})$

$$= \frac{p(\text{II is red and I is white})}{p(\text{I is white})}$$

$$= \frac{p(\text{I is white and II is red})}{p(\text{I is white})} = \frac{\frac{4}{7} \times \frac{3}{6}}{\frac{4}{7}} = \frac{3}{6} = \frac{1}{2}$$

46. (a)



The tree diagram of probabilities is shown above.

From above tree, by rule of total probability,

p(declared faulty) = $pq + (1 - p)(1 - q)$

47. (d)

Coin is tossed 4 times.

p(number of heads > number of tails)

$= p(4H \,\&\, OT \text{ or } 3H \,\&\, IT)$
$= p(\text{Exactly 4 Heads}) + p(\text{Exactly 3 Heads})$

$$= 4C_4\left(\frac{1}{2}\right)^4\left(1-\frac{1}{2}\right)^0 + 4C_3\left(\frac{1}{2}\right)^3\left(1-\frac{1}{2}\right)^1$$

$$= \frac{1}{16} + \frac{4}{16} = \frac{5}{16}$$

48. (a)
The five cards are {1, 2, 3, 4, 5}
Sample space = 5 × 4 ordered pairs.
[Since there is a I[st] card and II[nd] card we have to take ordered pairs]
p(I[st] card = II[nd] card + 1)

$$= P\{(2, 1), (3, 2), (4, 3), (5, 4)\} = \frac{4}{5\times 4} = \frac{1}{5}$$

49. (c)
p(one ball is Red & another is blue)
$= p$(first is Red and second is Blue)

$$= \frac{4}{7}\times\frac{3}{7} = \frac{12}{49}$$

50. (c)
Total cases = 36 [(1, 1) (1, 2) (1, 3) and so on]
Favourable case = $(x_1 > x_2) = 15$

$$P[x_1 > x_2] = \frac{15}{36} = \frac{5}{12}$$

51. (d)
Standard deviation is affected by scale but not by shift of origin.
So $\quad y_i = ax_i + b$
$\Rightarrow \quad \sigma_y = a\sigma_x$
(if a could be negative then $s_y = |a|s_x$ is more correct since standard deviation cannot be negative)
Clearly, $\sigma_y = a\sigma_x + b$ is false
So (d) is incorrect.

52. (c)
$V(x) = E(x^2) - [E(x)]^2 = R$
where $V(x)$ is the variance of x,
Since variance is σ_x^2 and hence never negative, $R \geq 0$.

53. (d)

$$p(x \geq 1) = 1 - p(x = 0)$$

$$= 1 - {}^5C_0\left(\frac{1}{2}\right)^0\left(1-\frac{1}{2}\right)^5 = 1 - \frac{1}{32} = \frac{31}{32}$$

54. (c)

$$P(H) = \frac{1}{2} \quad ; \quad P(T) = \frac{1}{2}$$

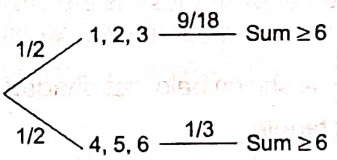
Favourable situation: H or TTH or TTTT H and so on
Probability of odd number of tosses

$$= \frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} + \ldots$$

$$= \frac{1}{2}\left[1 + \frac{1}{4} + \frac{1}{4^2} + \ldots\right]$$

$$= \frac{1}{2}\left[\frac{1}{1-1/4}\right] = \frac{2}{3}$$ (sum of infinite geometric series with a = 1 and r = 1/4)

55. (b)
If first throw is 1, 2 or 3 then sample space is only 18 possible ordered pairs. Out of this only (1, 5), (1, 6), (2, 4), (2, 5), (2, 6), (3, 3), (3, 4), (3, 5) and (3, 6) i.e. 9 out of 18 ordered pairs gives a Sum ≥ 6.
If first throw is 4, 5 or 6 then second throw is not made and therefore the only way Sum ≥ 6 is if the throw was 6. Which is one out of 3 possible.
So the tree diagram becomes as follows:



From above diagram

$$P(\text{sum} \geq 6) = \frac{1}{2}\times\frac{9}{18} + \frac{1}{2}\times\frac{1}{3} = \frac{15}{36} = \frac{5}{12}$$

56. (d)
Since negative and positive are equally likely, the distribution of number of negative values is binomial with $n = 5$ and $p = \frac{1}{2}$

Let X represent number of negative values in 5 trials.
p(at most 1 negative value)

$$= p(x \leq 1)$$

$$= p(x = 0) + p(x = 1)$$

$$= 5C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^5 + 5C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^4$$

$$= \frac{6}{32}$$

57. (c)

The p.d.f. of the random variable is

x	-1	$+1$
$P(x)$	0.5	0.5

The cumulative distribution function F(x) is the probability upto x as given below:

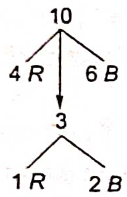
x	-1	$+1$
$F(x)$	0.5	1.0

So correct option is (c).

58. (d)

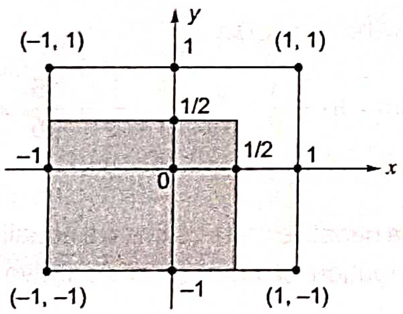
The problem can be represented by the following diagram.

$$p(\text{1R and 2B}) = \frac{{}^4C_1 \times {}^6C_2}{{}^{10}C_3} = \frac{60}{120} = \frac{1}{2}$$



59. (b)

$-1 \le x \le 1$ and $-1 \le y \le 1$ is the entire rectangle. The region in which maximum of $\{x, y\}$ is less than $\frac{1}{2}$ is shown below as shaded region inside this rectangle.



$$p\left(\max\{x, y\} < \frac{1}{2}\right) = \frac{\text{Area of shaded region}}{\text{Area of entire rectangle}}$$

$$= \frac{\frac{3}{2} \times \frac{3}{2}}{2 \times 2} = \frac{9}{16}$$

60. (a)

The annual precipitation is normally distributed with $\mu = 1000$ mm and $\sigma = 200$ mm

$$p(x > 1200) = p\left(z > \frac{1200 - 1000}{200}\right) = p(z > 1)$$

Where z is the standard normal variate.

In normal distribution

Now, since $p(-1 < z < 1) \approx 0.68$

($\simeq$ 68% of data is within one standard deviation of mean)

$$p(0 < z < 1) = \frac{0.68}{2} = 0.34$$

So, $p(z > 1) = 0.5 - 0.34 = 0.16 \simeq 16\%$

Which is <50%

So choice (a) is correct.

61. (c)

Poisson formula for $(P = x)$ given as

$$\frac{e^{-\lambda}\lambda^x}{x!}$$

λ : mean of Poisson distribution = 3 (given)

Probability of observing fewer than 3 cars.

$(P = 0) + (P = 1) + (P = 2)$

$$\frac{e^{-3}\lambda^0}{0!} + \frac{e^{-3}\lambda^1}{1!} + \frac{e^{-3}\lambda^2}{2!} = \frac{17}{2e^3}$$

(c) is correct option.

62. Sol.

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

$$f(x) = \begin{cases} \lambda(-x^2 + 3x - 2) & 1 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

$$\therefore \quad \int_1^2 \lambda(-x^2 + 3x - 2)dx = 1$$

$$\Rightarrow \quad \lambda\left[-\frac{x^3}{3} + 3\frac{x^2}{2} - 2x\right]_1^2 = 1$$

$$\Rightarrow \lambda\left[-\left(\frac{8}{3} - \frac{1}{3}\right) + \frac{3}{2}(4-1) - 2(2-1)\right] = 1$$

$$\Rightarrow \quad \lambda\left[-\frac{7}{3} + \frac{9}{2} - 2\right] = 1$$

$$\Rightarrow \quad \lambda\left[\frac{-14 + 27 - 12}{6}\right] = 1$$

$$\Rightarrow \quad \lambda = \frac{6}{1} = 6$$

$$\lambda = 6$$

63. (a)

$$P = \int_1^{\infty} f(x)dx = \int_1^{\infty} e^{-x}dx = -e^{-x}\Big|_1^{\infty}$$

$$= -(e^{-\infty} - e^{-1}) = e^{-1} = 0.368$$

64. (a)

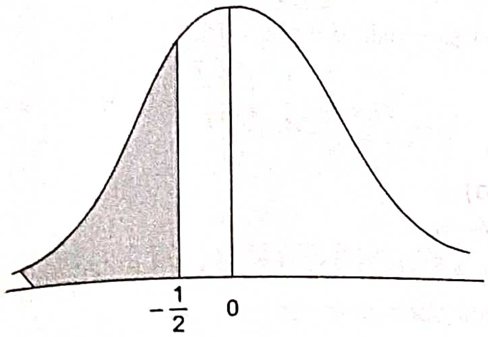
$$P = \int_1^{\infty} f(x)\,dx = \int_1^{\infty} e^{-x}dx$$

$$= e^{-x}\Big|_{\infty}^{1} = e^{-1} = 0.368$$

65. (b)

Here, $\sigma^2 = 4 \quad \Rightarrow \sigma = 2$

$$P(x<0) = p\left(z < \frac{0-\mu}{\sigma}\right) = p\left(z < \frac{0-1}{2}\right) = p\left(z < -\frac{1}{2}\right)$$



Which is the shaded area in the picture and its value is clearly between 0. and 0.5

66. Sol.

Given X is normally distributed,

Given, $\mu = 500$, $\sigma = 50$ $\quad p(x > 500) =$

$$p\left(z > \frac{500-\mu}{\sigma}\right) = p\left(z > \frac{500-500}{50}\right) = p(z>0) = 0.5$$

which is equal to 50%.

67. (b)

$$P(E) = \frac{n(E)}{n(S)}$$

$$n(s) = [\{H\}, \{T\}] = 2$$

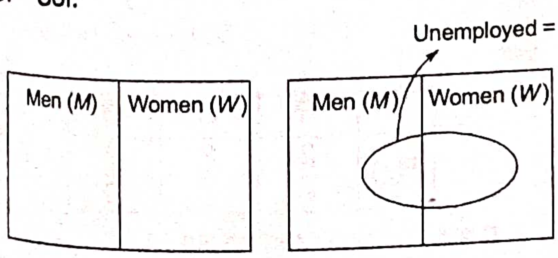
$$n(E) = \{(T)\} = 1$$

$$\therefore \quad P(E) = \frac{1}{2}$$

68. (a)

$$\text{required prob} = \frac{{}^{15}C_2}{{}^{25}C_2} = \frac{14 \times 15}{25 \times 24} = \frac{7}{20}$$

69. Sol.



$$P(M) = \frac{1}{2} \quad P(U \mid M) = 0.2$$

$$P(W) = \frac{1}{2} \quad P(U \mid W) = 0.5$$

Let $\quad E$ = Employed person

$$P(E \mid M) = 1 - 0.2 = 0.8\,;$$

$$P(E \mid W) = 1 - 0.5 = 0.5$$

By total probability

Probability of selecting employed person,

$$P(E) = P(M)\cdot P(E \mid M) + P(W)\cdot P(E \mid W)$$

$$= \frac{1}{2} \times 0.8 + \frac{1}{2} \times 0.5 = 0.65$$

70. (b)

Let number of heads = x. So number of tails will be $n - x$. We want the difference between the number of heads and number of tails to be $n - 3$ i.e. $x - (n - x) = n - 3$.

$$\Rightarrow x = \frac{2n-3}{2} = n - \frac{3}{2} \text{ which is not an integer}$$

$\therefore$ which is an impossible event so, the required probability is zero.

71. Sol.

Let probability of occurence of one dot is P.

So, writing total probability

$$P + 2P + 3P + 4P + 5P + 6P = 1$$

$$P = \frac{1}{21}$$

hence problem of occurence of 3 dot is

$$= 3P = \frac{3}{21} = \frac{1}{7} = 0.142$$

72. Sol.

Let there n families. Now $\frac{n}{2}$ families have single child and $\frac{n}{2}$ families have two children. So total number of children is

$$= \frac{n}{2} \times 1 + \frac{n}{2} \times 2 = \frac{3n}{2}$$

Now, favourable case is the child picked at random has sibling = n.

So probability (a child picked at random, has a sibling) $= \dfrac{n}{\frac{3n}{2}} = \dfrac{2}{3} = 0.666$

73. (c)

It means 3-head appears in 1st 9 trials.

Probability of getting exactly 3 head in 1st 9 trials

$$= {}^9C_3 \times \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^6 = {}^9C_3 \times \left(\frac{1}{2}\right)^9$$

and in 10th trial head must appears.

So required probability

$$= {}^9C_3 \left(\frac{1}{2}\right)^9 \times \frac{1}{2} = \frac{84}{1024} = 0.082$$

74. Sol.

Probability to lost at post-office 1 = $\frac{1}{5}$

Probability to lost at post-office 2 = $\frac{4}{5} \times \frac{1}{5}$

Total probability to lost = $\frac{1}{5} + \frac{4}{25} = \frac{9}{25}$

Required probability = $\frac{4/25}{9/25} = \frac{4}{9} = 0.444$

75. Sol.

6, 6, 6, 4 $\Rightarrow$ $\frac{4!}{3!} = 4$ ways

6, 6, 5, 5 $\Rightarrow$ $\frac{4!}{2!\,2!} = 6$ ways

Probability of sum to be 22

$$= \frac{6+4}{6^4} = \frac{6+4}{1296} = \frac{x}{1296}$$

$\Rightarrow$ $x = 10$

76. Sol.

$1 \le x \le 100$

$P(x$ is not divisible by 2, 3 or 5) = 1 − $P(x$ is divisible by 2, 3 or 5)

$$= 1 - \left[\left\lfloor\frac{100}{2}\right\rfloor + \left\lfloor\frac{100}{3}\right\rfloor + \left\lfloor\frac{100}{5}\right\rfloor - \left\lfloor\frac{100}{6}\right\rfloor - \left\lfloor\frac{100}{15}\right\rfloor - \left\lfloor\frac{100}{10}\right\rfloor + \left\lfloor\frac{100}{30}\right\rfloor\right]$$

$$1 - \frac{74}{100} = 0.26$$

77. Sol.

It is given that A and B are mutually exclusive also it is given that $A \cup B = S$ which means that A and B are collectively exhaustive.

Now if two events A and B are both mutually exclusive and collectively exhaustive, then $P(A) + P(B) = 1 \Rightarrow P(B) = 1 - P(A)$

Now we wish to maximize $P(A)\,P(B)$

$$= P(A)\,(1 - P(A))$$

Let $P(A) = x$

Now, $P(A)\,(1 - P(A)) = x(1-x) = x - x^2$

Say $y = x - x^2$

$$\frac{dy}{dx} = 1 - 2x = 0 \Rightarrow x = \frac{1}{2}$$

$$= \frac{d^2y}{dx^2} = -2 < 0; \left(\frac{d^2y}{dx^2}\right)_{x=\frac{1}{2}}$$

$$= -2 < 0$$

y has maximum at $x = 1/2$,

$$y_{max} = \frac{1}{2} - \left(\frac{1}{2}\right)^2 = 0.25$$

78. (d)

Mean,

$$\bar{x} = \Sigma\, xp(x) = 1\times 0.3 + 2\times 0.8 + 3\times 0.1 = 1.8$$

Standard deviation,

$$\sigma = \left(\Sigma x^2 p(x) - (\Sigma xp(x))^2\right)^{1/2}$$

$$\therefore \sigma = \left(0.3\times 1^2 + 0.6\times 2^2 + 0.1\times 3^2 - 1.8^2\right)^{1/2}$$

$$= (3.6 - 1.8^2)^{1/2} = (0.36)^{1/2} = 0.6$$

79. (a)

x	0	1	2
$p(x)$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{1}{6}$

$$\text{mean} = \Sigma x \,.\, p(x) = 0\left(\frac{1}{6}\right) + 1\left(\frac{2}{3}\right) + 2\left(\frac{1}{6}\right)$$

$$= \frac{2}{3} + \frac{2}{6} = \frac{2}{3} + \frac{1}{3} = \frac{3}{3} = 1$$

$$E(x^2) = 0\left(\frac{1}{6}\right) + 1\left(\frac{2}{3}\right) + 4\left(\frac{1}{6}\right)$$

$$= \frac{2}{3} + \frac{4}{6} = \frac{2}{3} + \frac{2}{3} = \frac{4}{3}$$

$$\text{Variance} = E(x^2) - (E(x)^2) = \frac{4}{3} - 1 = \frac{1}{3}$$

80. Sol.

x	R	B	G
$P(x)$	$\frac{2}{6}$	$\frac{2}{6}$	$\frac{2}{6}$

$n = 3$ $\quad$ x : red colour

$$p = p(\text{Red}) = \frac{2}{6}$$

$$q = 1 - p = 1 - \frac{2}{6} = \frac{4}{6}$$

Prob. of getting red colour on top face atleast twice is

$$= p(x = 2) + p(x = 3)$$
$$= {}^nC_2 p^2 q^{n-2} + {}^nC_3 p^3 q^{n-3}$$
$$= {}^3C_2 \left(\frac{2}{6}\right)^2 \left(\frac{4}{6}\right)^1 + {}^3C_3 \left(\frac{2}{6}\right)^3 \left(\frac{4}{6}\right)^0$$
$$= 3 \cdot \frac{4}{36} \cdot \frac{4}{6} + 1 \cdot \frac{8}{216}$$
$$= \frac{48+8}{216} = \frac{56}{216} = 0.259$$

81. Sol.

The tree diagram for the problem is shown below:

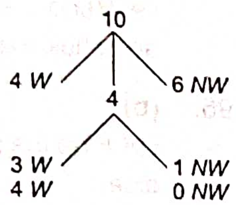
Required probability

$$= \frac{{}^4C_3 \cdot {}^6C_1}{{}^{10}C_4} + \frac{{}^4C_4 \cdot {}^6C_0}{{}^{10}C_4}$$

$$= \frac{24}{210} + \frac{1}{210} = \frac{25}{210}$$

$$p = 0.1190$$

$$\Rightarrow \quad 100\,p = 11.90$$



82. Sol.

Mean $\lambda = 5$

$$P(x < 4) = p(x = 0) + p(x = 1) + p(x = 2) + p(x = 3)$$

$$= \frac{e^{-5}5^0}{0!} + \frac{e^{-5}5^1}{1!} + \frac{e^{-5}5^2}{1!} + \frac{e^{-5}5^3}{3!}$$

$$= e^{-5}\left[1 + 5 + \frac{25}{2} + \frac{125}{6}\right]$$

$$= e^{-5}\left(\frac{118}{3}\right) = 0.265$$

83. (b)

$$P(x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

As λ(mean) = 5.2

$$P(x < 2) = P(0) + P(1) = e^{-5.2}\left[\frac{5.2^0}{0!} + \frac{5.2^1}{1!}\right]$$

$$\therefore \quad P(x < 2) = \frac{6.2}{e^{5.2}} = 0.0342$$

84. Sol.

$$f(E) = \begin{cases} \frac{1}{5} & 0 \le E \le \text{mm/day} \\ 0 & \text{Otherwise} \end{cases}$$

$$P(2 < E < 4) = \int_2^4 f(E)dE = \int_2^4 \frac{1}{5}dE = \frac{1}{5}[E]_2^4$$

$$= \frac{1}{5}(4-2) = \frac{2}{5} = 0.4$$

85. Sol.

Probability ($0.5 < n < 5$)

$$= \int_{0.5}^{5} f(x)dx$$

$$= \int_{0.5}^{1} 0.2dx + \int_1^4 0.1\,dx + \int_4^5 0dx$$

$$= 0.2[1-0.5] + 0.1[4-1] + 0[5-4]$$
$$= 0.2 \times 0.5 + 0.1 \times 3 = 0.1 + 0.3 = 0.4$$

86. (b)

In normal distribution, the area under the normal curve from $-\infty$ to the mean = 0.5

Here, 'a' is the mean. So, The value of the integral

$$\int_{-\infty}^{a} \frac{1}{\sqrt{2\pi} * b} e^{\frac{-1}{2}\left(\frac{x-a}{b}\right)^2} dx$$ = the area under the normal curve from $-\infty$ to the mean = 0.5

87. (c)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Since

$$P(A \cap B) = p(A)\,p(B)$$

(not necessarily equal to zero).

So, $P(A \cup B) = P(A) + P(B)$ is false.

88. Sol.

X_1	X_2	X_3	X_1X_2	$Y = X_1X_2 \oplus X_3$
0	0	0	0	0
0	1	0	0	0
1	0	0	0	0
1	1	0	1	1

$Y = 0$ and $X_3 = 0$ in 3 cases

$X_3 = 0$ in 4 cases

$$P(Y = 0 / X_3 = 0) = \frac{P(Y = 0 \cap X_3 = 0)}{P(X_3 = 0)} = \frac{3}{4} = 0.75$$

89. Sol.

Probability of atleast one meet the specification

$$= 1-\left(\bar{A}\times\bar{B}\times\bar{C}\right)$$

$$= 1-(0.2\times0.3\times0.5) = 0.97$$

90. (b)

Given, p(passing the exam) = 0.2

p(passing the exam $\cap$ > 90%) = 0.05

The desired probability

= p(> 90% | passing the exam)

$$= \frac{p(\text{passing the exam} \cap > 90\%)}{p(\text{passing the exam})} = \frac{0.05}{0.2} = \frac{1}{4}$$

91. (c)

$$P\left(\frac{Y}{X}\right) = \frac{P(Y\cap X)}{P(X)} = \frac{1/12}{1/4} = \frac{1}{3}$$

92. (d)

P(A wins) = p(6 in first throw by A) + p(A not 6, B not 6, A 6) + ...

$$= \frac{1}{6}+\frac{5}{6}\cdot\frac{5}{6}\cdot\frac{1}{6}+\cdots$$

$$= \frac{1}{6}\left(1+\left(\frac{5}{6}\right)^2+\left(\frac{5}{6}\right)^4+\cdots\right) = \frac{1}{6}\cdot\frac{1}{1-\left(\frac{5}{6}\right)^2} = \frac{6}{11}$$

93. Sol.

Given, $q = 0.4$

X	0	1
$p(X)$	0.4	0.6

Required value = $V(X) = E(X^2) - [E(X)]^2$

$$E(X) = \sum_i X_i p_i = 0\times0.4+1\times0.6 = 0.6$$

$$E(X^2) = \sum_i X_i^2 p_i = 0^2\times0.4+1^2\times0.6 = 0.6$$

$$\therefore\ V(X) = E(X^2)-[E(X)]^2 = 0.6-0.36 = 0.24$$

94. (d)

From the given information, we can create a joint probability table as follows:

S \ R	H_R	T_R	
H_S	0.28	0.24	0.52
T_S	0.30	0.18	0.48
	0.58	0.42	1

From the table, we can get

$P(H_R) = 0.58$, $P(T_R) = 0.42$, $P(H_S) = 0.52$

$P(T_S) = 0.48$

So, Coins R and S are biased (not fair). So choices (b) and (c) are both false.

The coin tosses are not independent since their probability of heads and tails is not 0.5.

R and S are dependent.

If R and S were independent then all the joint probabilites will be equal to the product of the marginal probabilities.

For example

$$P(H_R\cap H_S) = 0.28$$

$$P(H_R)\cdot P(H_S) = 0.58\times0.52 = 0.3016$$

Clearly $P(H_R\cap H_S) \neq P(H_R)\cdot P(H_S)$

So R and S are not independent.

i.e. R and S are dependent. So, choice (a) is false and choice (d) is true.

95. (b)

Let P be the probability that six occurs on a fair dice,

$$\therefore\quad P = \frac{1}{6}$$

$$\therefore\quad q = \frac{5}{6}$$

Let X, be the number of times 'six' occurs,

Probability of obtaining at least two 'six' in throwing a fair dice 4 times is

$$= 1-\{P(X=0)+P(X=1)\}$$

$$= 1-\{{}^4C_0\,p^0q^4+{}^4C_1\,p^1q^3\}$$

$$= 1-\left\{\left(\frac{5}{6}\right)^4+\left[4\times\frac{1}{6}\times\left(\frac{5}{6}\right)^3\right]\right\}$$

$$= 1-\left\{\frac{125}{144}\right\} = \frac{19}{144}$$

96. (b)

$$\text{Probability} = {}^{10}C_3(0.1)^3(0.9)^7 = 0.057$$

97. Sol.

$$f(x) = \begin{cases}\frac{x}{4}\left(4-x^2\right), & 0\le x\le 2\\ 0 & \text{, Otherwise}\end{cases}$$

$$\mu_x = \int_0^2 xf(x)\,dx$$

$\therefore$ Mean $(\mu_x) = \int_0^2 x \cdot \frac{x}{4}(4-x^2)\,dx$

$$= \int_0^2 \left(x^2 - \frac{x^4}{4}\right)dx = \left[\frac{x^3}{3} - \frac{x^5}{20}\right]_0^2$$

$$= \frac{8}{3} - \frac{32}{20} = \frac{16}{15} = 1.066$$

98. **Sol.**

$$f(x) = \begin{cases} a+bx & \text{for } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

Now given $E(X) = 2/3$

$$\Rightarrow \int_0^1 x f(x)\,dx = \frac{2}{3}$$

$$\Rightarrow = \int_0^1 x(a+bx)\,dx = \frac{2}{3}$$

$$\Rightarrow a\left(\frac{x^2}{2}\right)_0^1 + b\left(\frac{x^3}{3}\right)_0^1 = \frac{2}{3}$$

$$a\left(\frac{1}{2}\right) + b\left(\frac{1}{3}\right) = \frac{2}{3}$$

$$\Rightarrow 3a + 2b = 4 \quad \text{...(i)}$$

Now, $\int_0^1 f(x)\,dx = 1$

(Total probability is always equal to 1)

$$\Rightarrow = \int_0^1 (a+bx)\,dx$$

$$= \left(ax + \frac{bx^2}{2}\right)_0^1 = 1$$

$$\Rightarrow a + \frac{b}{2} = 1$$

$$\Rightarrow 2a + b = 2 \quad \text{...(ii)}$$

Now solving (i) and (ii), we get

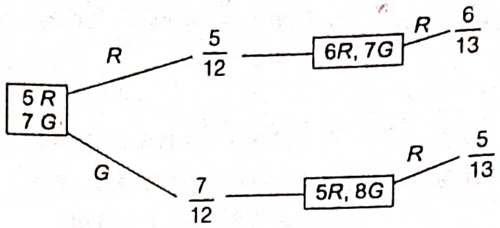
$$a = 0,\ b = 2$$

So

$$f(x) = \begin{cases} 2x & \text{for } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

Now we need $\int_0^{1/2} 2x\,dx = \frac{1}{4}$

99. **(a)**



$$P(\text{Red}) = \frac{5}{12}\times\frac{6}{13} + \frac{7}{12}\times\frac{5}{13} = \frac{65}{156}$$

100. **Sol.**

$$P(H) = 0.3$$
$$P(T) = 0.7$$

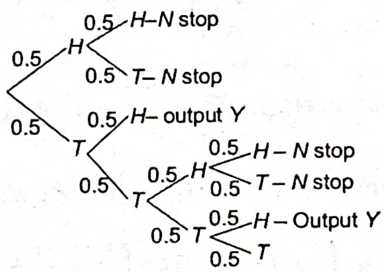
since all tosses are independent
so, probability of getting head for the first time in 5^{th} toss is

$$= P(T)\,P(T)\,P(T)\,P(T)\,P(H)$$
$$= 0.7 \times 0.7 \times 0.7 \times 0.7 \times 0.3$$
$$= 0.072$$

101. **(a)**

$$P(X \cup Y^c) = 07$$
$$\Rightarrow P(X) + P(Y^c) - P(X)\,P(Y^c) = 0.7$$
(Since X, Y are independent events)
$$\Rightarrow P(X) + 1 - P(Y) - P(X)\,\{1 - P(Y)\} = 0$$
$$\Rightarrow P(Y) - P(X \cap Y) = 0.3 \quad \text{...(i)}$$
$$P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$$
$$= 0.4 + 0.3 = 0.7$$

102. **Sol.**



The tree diagram for the problem is given above.
The desired output is Y.
Now by rule of total probability
$p(\text{output} = Y) = 0.5 \times 0.5 + 0.5 \times 0.5 \times 0.5 \times 0.5 + \ldots$
Infinite geometric series with

$$a = 0.5 \times 0.5$$

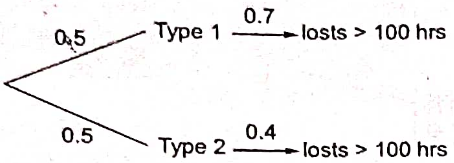
and

$$r = 0.5 \times 0.5$$

so $p(\text{output} = Y) = \frac{0.5\times0.5}{1-0.5\times0.5} = \frac{0.25}{0.75}$

$$\frac{1}{3} = 0.33 \text{ (upto 2 decimal places)}$$

103. Sol.



$\therefore$ P(losts > 100 hr) $= 0.5 \times 0.7 + 0.5 \times 0.4$

$= 0.35 + 0.2 = 0.55$

104. (a)

$$\frac{{}^4C_1 \cdot {}^4C_1 \cdot {}^4C_1}{{}^{52}C_3} = \frac{64}{\frac{52\times 51\times 50}{3\times 2\times 1}} = \frac{64}{22100} = \frac{16}{5525}$$

106. Sol.

Median speed is the speed at the middle value in series of spot speeds that are arranged in ascending order. 50% of speed values will be greater than the median 50% will be less than the median.

Ascending order of spot speed studies are 32, 39, 45, 51, 53, 56, 60, 62, 66, 79

Median speed $= \frac{53+56}{2} = 54.5$ km/hr

107. (b)

Mean of $f(x)$ is $E(x)$

$$= \int_{-a}^{0} x\left(\frac{x}{a}+1\right)dx + \int_{0}^{a} x\left(\frac{-x}{a}+1\right)dx$$

$$= \left(\frac{x^3}{3a}+\frac{x^2}{2}\right)_{-a}^{0} + \left(\frac{-x^3}{3a}+\frac{x^2}{2}\right)_{0}^{a} = 0$$

Variance of f(x) is $E(x)^2 - \{E(x^2)\}$ where

$$E(x)^2 = \int_{-a}^{0} x^2\left(\frac{x}{a}+1\right)dx + \int_{0}^{a} x^2\left(\frac{-x}{a}+1\right)dx$$

$$= \left(\frac{x^4}{4a}+\frac{x^3}{3}\right)_{-a}^{0} + \left(\frac{-x^4}{4a}+\frac{x^3}{3}\right)_{0}^{a} = \frac{a^3}{6}$$

$\Rightarrow$ Variance is $\frac{a^3}{6}$

Next, mean of $g(x)$ is $E(x)$

$$= \int_{a}^{0} x\left(\frac{-x}{a}\right)dx + \int_{0}^{a} x\left(\frac{x}{a}\right)dx = 0$$

Variance of $g(x)$ is $E(x^2) - \{E(x)\}^2$, where

$$E(x^2) = \int_{-a}^{0} x^2\left(\frac{-x}{a}\right)dx + \int_{0}^{a} x^2\left(\frac{x}{a}\right)dx = \frac{a^3}{2}$$

$\Rightarrow$ Variance is $\frac{a^3}{2}$

$\therefore$ Mean of $f(x)$ and $g(x)$ are same but variance of $f(x)$ and $g(x)$ are different.

108. Sol.

$n = 5$, $P = 0.1$, $q = 0.9$

X: no of defectives

$P(X \geq 1) = 1 - P(X=0) = 1 - {}^5C_0\,(0.1)^0(0.9)^5$

$= 1 - (0.9)^5 = 0.4095$

109. Sol.

In Poisson distribution,

Mean = First moment $= \lambda$

Second moment $= \lambda^2 + \lambda$

Given that second moment is 2

$\therefore \quad \lambda^2 + \lambda = 2$

$\lambda^2 + \lambda - 2 = 0$

$(\lambda + 2)(\lambda - 1) = 0$

$\lambda = 1$

110. (a)

In poisson distribution mean = Variance

Given that mean = Variance = m

Standard deviation = $\sqrt{\text{Variance}} = \sqrt{\mu}$

111. Sol.

Given, $f(x) = \frac{1}{x^2} \quad a \leq x \leq 1$

$= 0$ elsewhere

So $\int_a^1 f(x) = 1$

$\Rightarrow \int_a^1 \frac{1}{x^2} = 1$

$\Rightarrow \left[\frac{-1}{x}\right]_a^1 = 1$

$-\left[\frac{1}{1} - \frac{1}{a}\right] = 1$

$\Rightarrow \frac{1}{a} = 2$

$\Rightarrow a = \frac{1}{2} = 0.5$

112. Sol.

$$P(X+Y\le 1) = \int_{x=0}^{1}\int_{y=0}^{(1-x)} f_{xy}(x,y)\,dx\,dy$$

$$= \int_{x=0}^{1}\int_{y=0}^{1-x} (x+y)\,dx\,dy$$

$$= \int_{x=0}^{1}\left(xy+\frac{y^2}{2}\right)_0^{1-x}$$

$$= \int_{x=0}^{1}\left(x(1-x)+\frac{(1-x)^2}{2}\right)dx$$

$$= \int_{x=0}^{1}\left(\frac{1}{2}-\frac{x^2}{2}\right)dx = \left(\frac{x}{2}-\frac{x^3}{6}\right)_0^1$$

$$= \frac{1}{2}-\frac{1}{6} = \frac{1}{3} = 0.33$$

113. (a)

$$P(x\le 4) = \int_{-\infty}^{4} f(x)\,dx$$

$$= \int_{-\infty}^{1}(0)\,dx + \int_{1}^{4}(0.25)\,dx + \int_{4}^{\infty}(0)\,dx$$

$$= \frac{1}{4}(x)_1^4 = \frac{1}{4}(4-1) = \frac{3}{4}$$

114. (a)

$$f_x(x) = \begin{cases} ae^{4x} & x<0 \\ \frac{3}{2}e^{-3x} & x\ge 0 \end{cases}$$

$$\int_{-\infty}^{\infty} f_x(x) = 1$$

$$\int_{-\infty}^{0} ae^{4x}dx + \int_{0}^{\infty}\frac{3}{2}e^{-3x}dx = 1$$

$$\left[\frac{ae^{4x}}{4}\right]_{-\infty}^{0} + \left[\frac{\frac{3}{2}e^{-3x}}{-3}\right]_{0}^{\infty} = 1$$

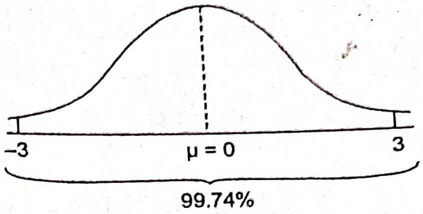
$$\frac{a}{4}+\frac{3}{6} = 1$$

$$a = 2$$

$$P(x\le 0) = \int_{-\infty}^{0} 2e^{4x}dx$$

$$= \left[\frac{e^{4x}}{2}\right]_{-\infty}^{0} = \frac{1}{2}$$

115. Sol.



116. Sol.

Sample space [HH, HT, TH, TT]

Probability of getting at least one head $= \frac{3}{4}$.

117. (c)

Mode means highest number of observations or occurrence of data most of the time as data 17, occurs four times, i.e., highest time. So mode is 17.

118. Sol.

Face	1	2	3	4	5	6
Prob.	1/6	1/6	1/6	1/6	1/6	1/6

mean $= E(x) = \Sigma x \cdot P(x)$

$= 1(1/6) + 2(1/6) + 3(1/6) + 4(1/6) + 5(1/6) + 6(1/6)$

$=$

$$\frac{1}{6}(1+2+3+4+5+6) = \frac{21}{6} = 3.5$$

119. Sol.

t be arrival time of vehicles of the junction is uniformaly distributed in [0, 5].

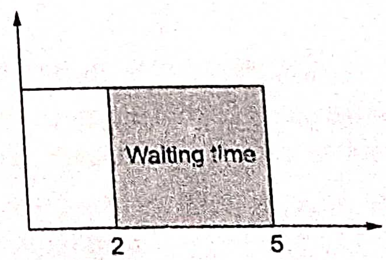
Let y be the waiting time of the junction.

Then, $$y = \begin{bmatrix} 0 & t<2 \\ 5-t & 2\le t<5 \end{bmatrix}$$

$$y \to [0, 5]$$

$$f(y) = \frac{1}{5-0} = \frac{1}{5}$$

$$E(y) = \int_{-\infty}^{0} y(y)\,dy = \int_{0}^{5} y f(y)\,dy$$



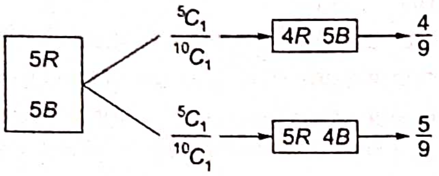
$$= \int_2^5 y\left(\frac{1}{5}\right)dy = \frac{1}{5}\int_2^5 (5-t)\,dt$$

$$= \frac{1}{5}\left(5t - \frac{t^2}{2}\right)\Bigg|_2^5$$

$$= \frac{1}{5}\left\{\left(25 - \frac{25}{2}\right) - \left(10 - \frac{4}{2}\right)\right\}$$

$$= \frac{1}{5}\left(\frac{25}{2} - 8\right) = \frac{1}{5}\left(\frac{9}{2}\right) = 0.9$$

120. (a)



$$P(\text{red}) = \frac{5}{10}\cdot\frac{4}{9} + \frac{5}{10}\cdot\frac{5}{9} = \frac{45}{90} = 0.5$$

121. Sol.

Given, Poisson distribution $\lambda = 5$

We know that in Poisson distribution

$$E(X) = V(X) = \lambda$$

so here $E(X) = V(X) = 5$

now, we need $E[(X+2)^2]$

$$= E(X^2 + 4X + 4)$$

$$= E(X^2) + 4E(X) + 4$$

To find $E(X^2)$ we write, $V(X) = E(X^2) - (E(X))^2$

$$5 = E(X^2) - 5^2$$

So, $E(X^2) = 5^2 + 5 = 30$

required value $= 30 + 4 \times 5 + 4 = 54$

122. Sol.

Since the coin is fair, outcome of next experiment will be independent of previous outcome.

$$\Rightarrow \quad P(H) = \frac{1}{2}$$

123. (b)

In exponential,

$$f(x) = \lambda e^{-\lambda x}; \quad x = 0$$

The parameter is λ.

In Gaussian,

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}; \quad -\infty < x < \infty$$

The parameters are μ and σ.

Therefore, answer is (b).

124. (b)

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_0^1 (a + bx)\,dx = 1$$

$$\left(ax + \frac{bx^2}{2}\right)\Bigg|_0^1 = 1$$

$$a + \frac{b}{2} = 1$$

Option (b) is satisfying the above equation.

■■■■