B20AI013_A3_TimeSeries

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Part 1: ARIMA Model

ARIMA stands for Autoregressive Integrated Moving Average. It is a combination of the differenced autoregressive model with the moving average model. The AR part represents the dependency of the current value on its past values uptil P time steps, the MA part represents the dependency of the current value on the past errors uptil Q time steps, and the I part represents the differencing of the series to make it stationary.

The formula for the ARIMA model is given as:

$$Y_t' = L + \sum_{i=1}^{P} \alpha_i Y_{t-i}' + \sum_{j=1}^{Q} \theta_j Y_{t-j}'$$

where Y_t' is the D order difference of the Y_i terms, and a level L is learnt in order to offset the predictions properly.

Let us try to obtain the solution for the AR part of the model first.

The solutions to these equations can be found by solving the overdetermined system of equations with a lag p , also called the $AR(p)$:

$$x_{i+1} = \phi_1 x_i + \phi_2 x_{i-1} + \cdots + \phi_p x_{i-p+1} + \xi_{i+1}.$$

This can be solved by a minimization of least squares formulation. Eventually we arrive at the following formulation:

$$\begin{array}{ccccccccccccc}
r_1 & = & \phi_1 r_o & + & \phi_2 r_1 & + & \phi_3 r_2 & + & \cdots & + & \phi_{p-1} r_{p-2} & + & \phi_p r_{p-1} \\
r_2 & = & \phi_1 r_1 & + & \phi_2 r_o & + & \phi_3 r_1 & + & \cdots & + & \phi_{p-1} r_{p-3} & + & \phi_p r_{p-2} \\
 & & & & & & \vdots & & & & & & \\
r_{p-1} & = & \phi_1 r_{p-2} & + & \phi_2 r_{p-3} & + & \phi_3 r_{p-4} & + & \cdots & + & \phi_{p-1} r_o & + & \phi_p r_1 \\
r_p & = & \phi_1 r_{p-1} & + & \phi_2 r_{p-2} & + & \phi_3 r_{p-3} & + & \cdots & + & \phi_{p-1} r_1 & + & \phi_p r_o
\end{array}$$

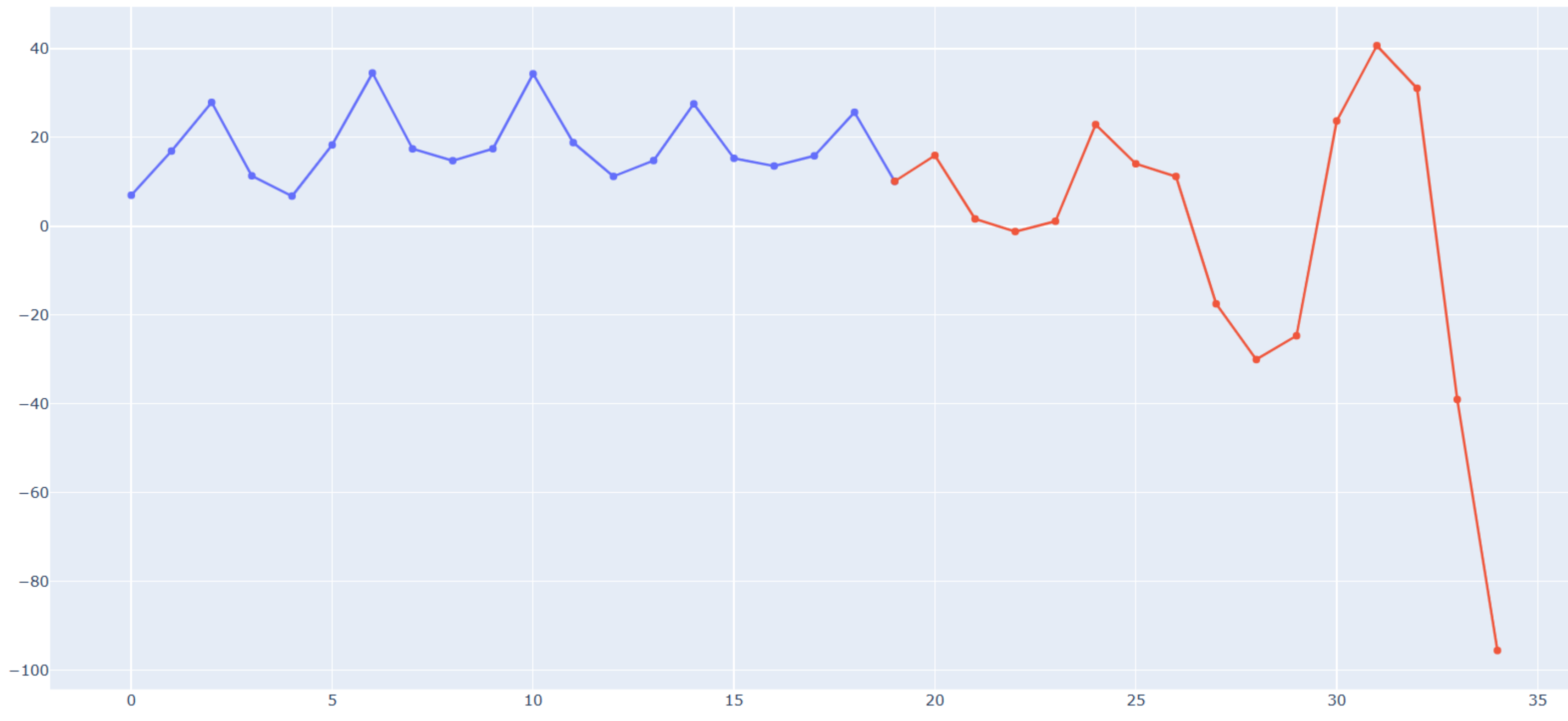
where the solutions to this equations are obtained by:

$$\hat{\mathbf{\Phi}} = \mathbf{R}^{-1}\mathbf{r}.$$

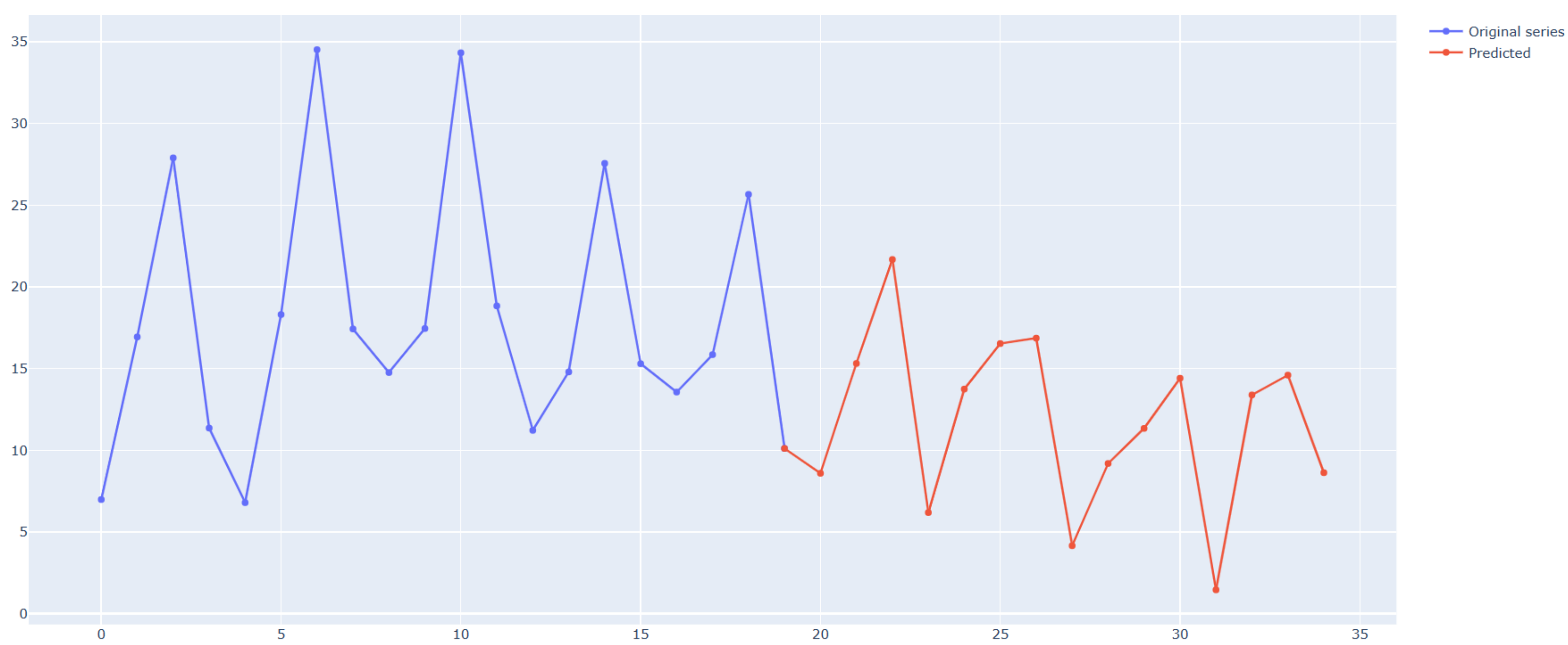
The MA part of the model can be calculated by using the residuals from the predictions of the $AR(p)$ model and using least squares regression upon the time series thus obtained. The MA model offers further scope of correction to the AR model being used.

One drawback of the ARIMA model is that since it is autocorrelated, the model can end up predicting values that blow up or oscillate around a certain level, or converge to a particular value, or have a tendency to tilt either up or down. This is because fitting the ARIMA model with least squares regression and the Yule-Walker equations only offers an approximate solution to the problem and does not guarantee freedom from biases in case the input time series is not stationary before feeding it into the model.

The predictions on the given sample in `tests.py` using ARIMA is shown below. Evidently, the ARIMA model is good at capturing the trend and level of the series however it is unable to capture the seasonality of the data very well and the variations explode. This is an often encountered artifact of using the ARIMA model and requires a lot of tuning and setting to thwart.

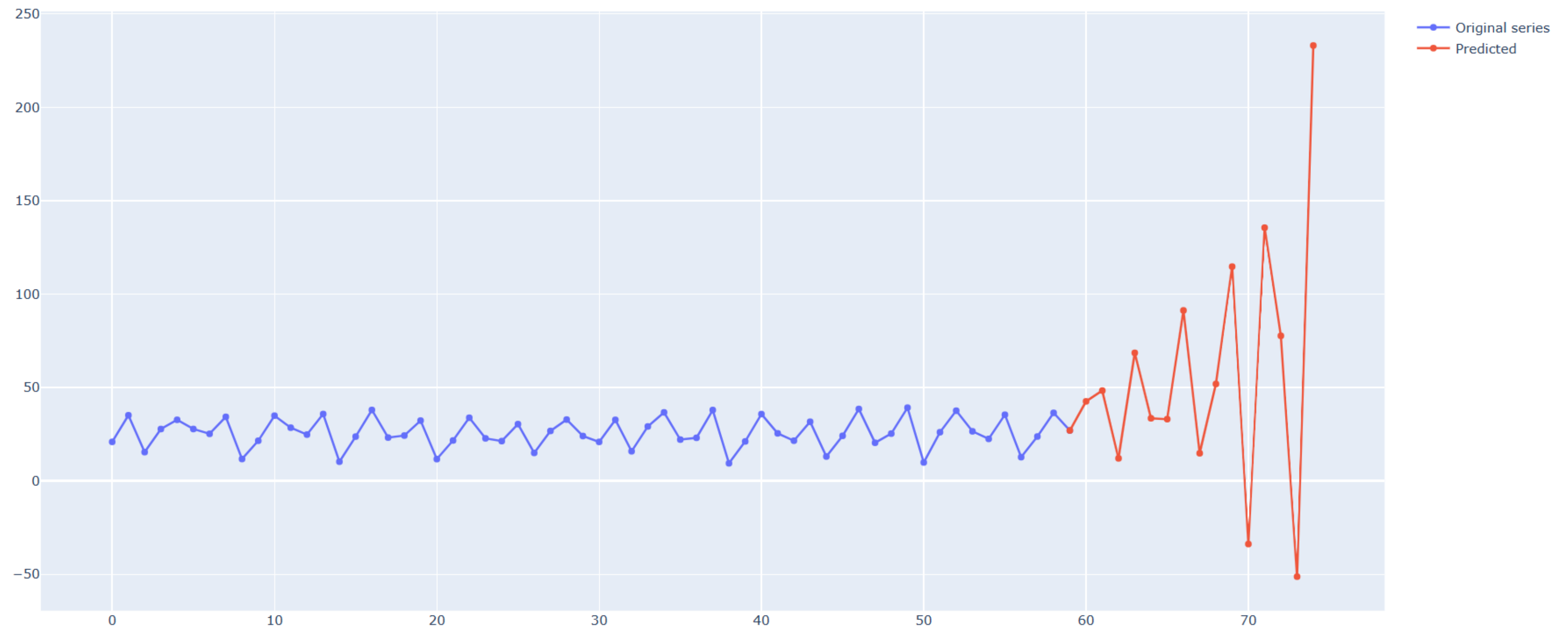


It is possible that the size of the past window ($P = 5$) is too small as the time period of the time series can visibly be seen to equal 4. It is generally appropriate to take a window of size around double of this. Let us try by changing the value of P to $P = 7$.

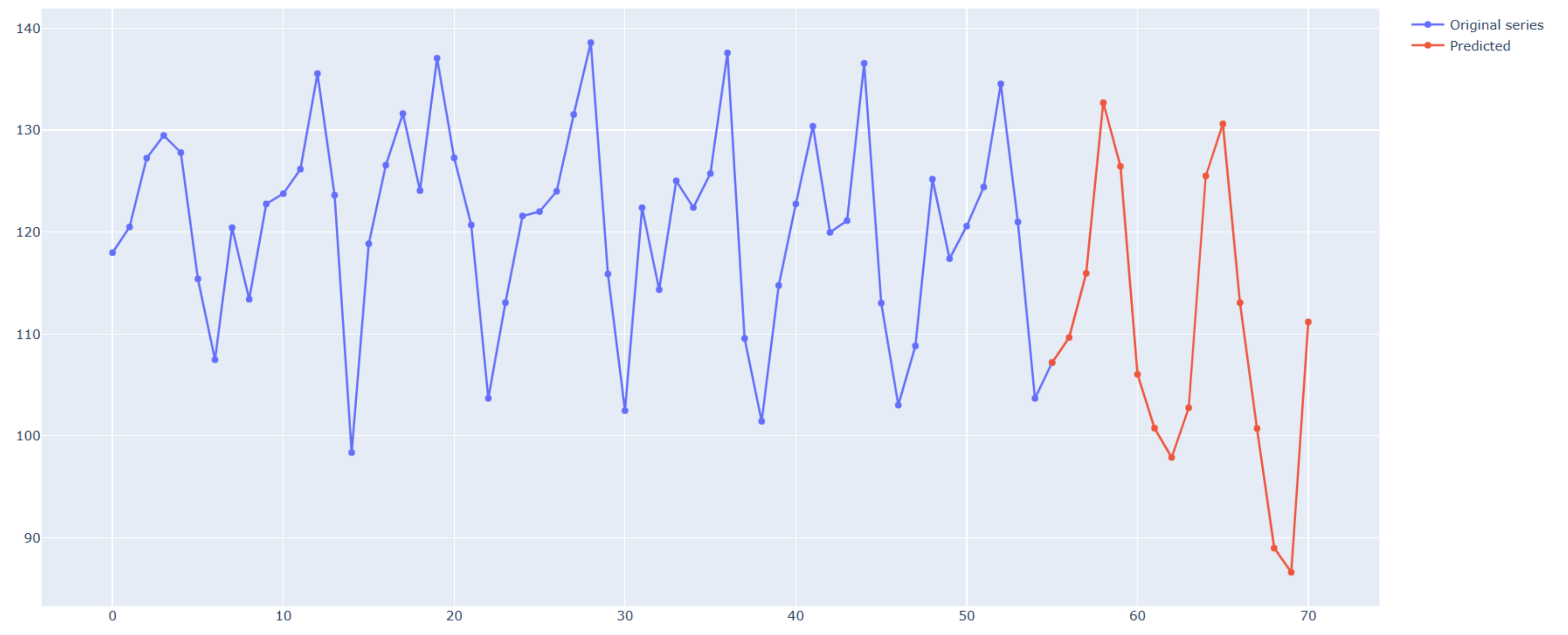


The resulting predictions are much better as a result and fit the shape of the time series much better, as hypothesized.

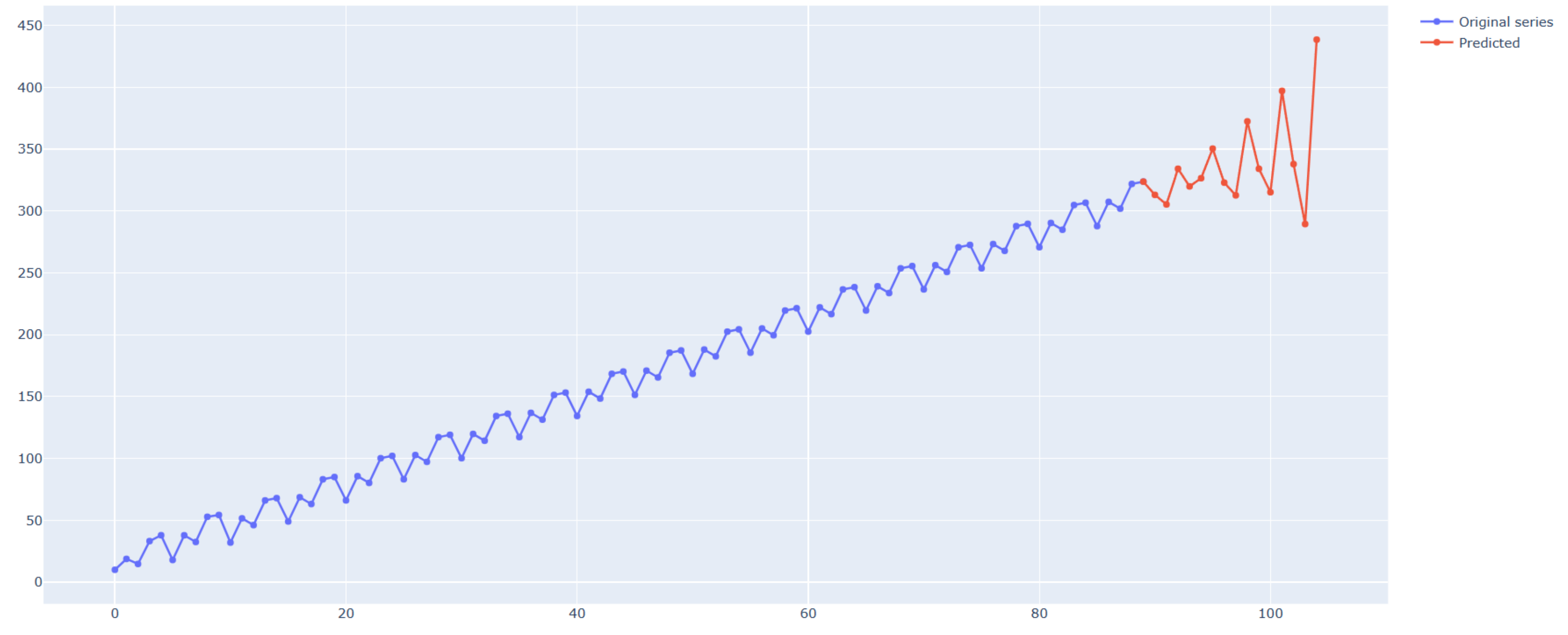
On trying to make the plots for the other time series, these are the plots I got:



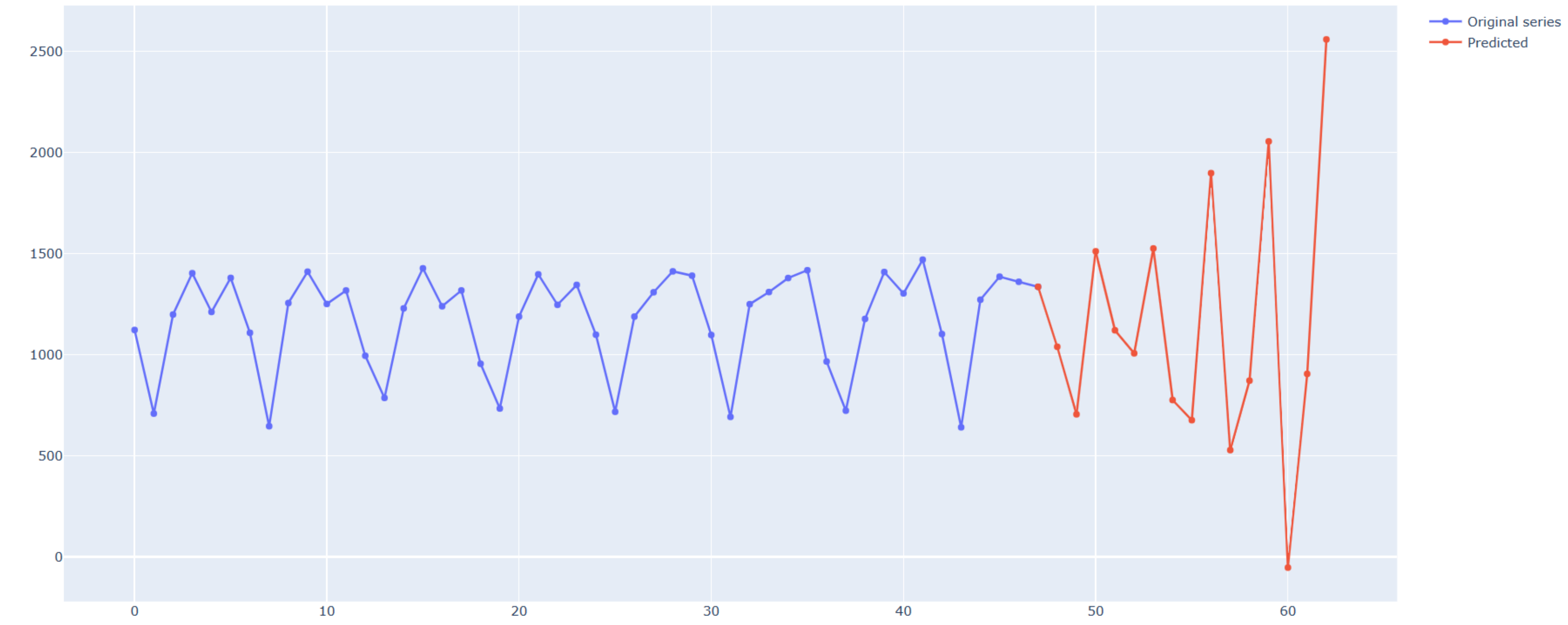
Original series
Predicted



Original series
Predicted



Original series
Predicted



Overall, it can be seen that the model generally does well at predicting the first few time steps autoregressively and then progressively gets worse as it has to make a forecast without taking into account new correct observations.

Part 2: Holt Winter's Forecast

Holt Winter's Forecast is an extension of linear exponential smoothing method, which is used for forecasting time series data with trend. It incorporates the use of an additional seasonality term S_t, on top of the terms already in the previous model, b_t (trend) and L_t (level).

Triple exponential smoothing is used to create forecasts using past values of a given time series. Three separate smoothing equations are fitted - one for the level, one for the trend, and one for the seasonal component - to the historical values of the time series. The forecast for a time step is computed as the summation of the trend, level and seasonal values for that time step. Given below is the formula for a Holt Winter forecast model where the variations in level and trend scale with the seasonality, which means that a multiplicative model is appropriate here.

$$\text{(Level) } L_t = \alpha * (Y_t \ / \ S_{t-s}) + (1 - \alpha) * (L_{t-1} + b_{t-1})$$

$$\text{(Trend) } b_t = \beta * (L_t - L_{t-1}) + (1 - \beta) * b_{t-1}$$

$$\text{(Seasonal) } S_t = \gamma * (Y_t \ / \ L_t) + (1 - \gamma) * S_{t-s}$$

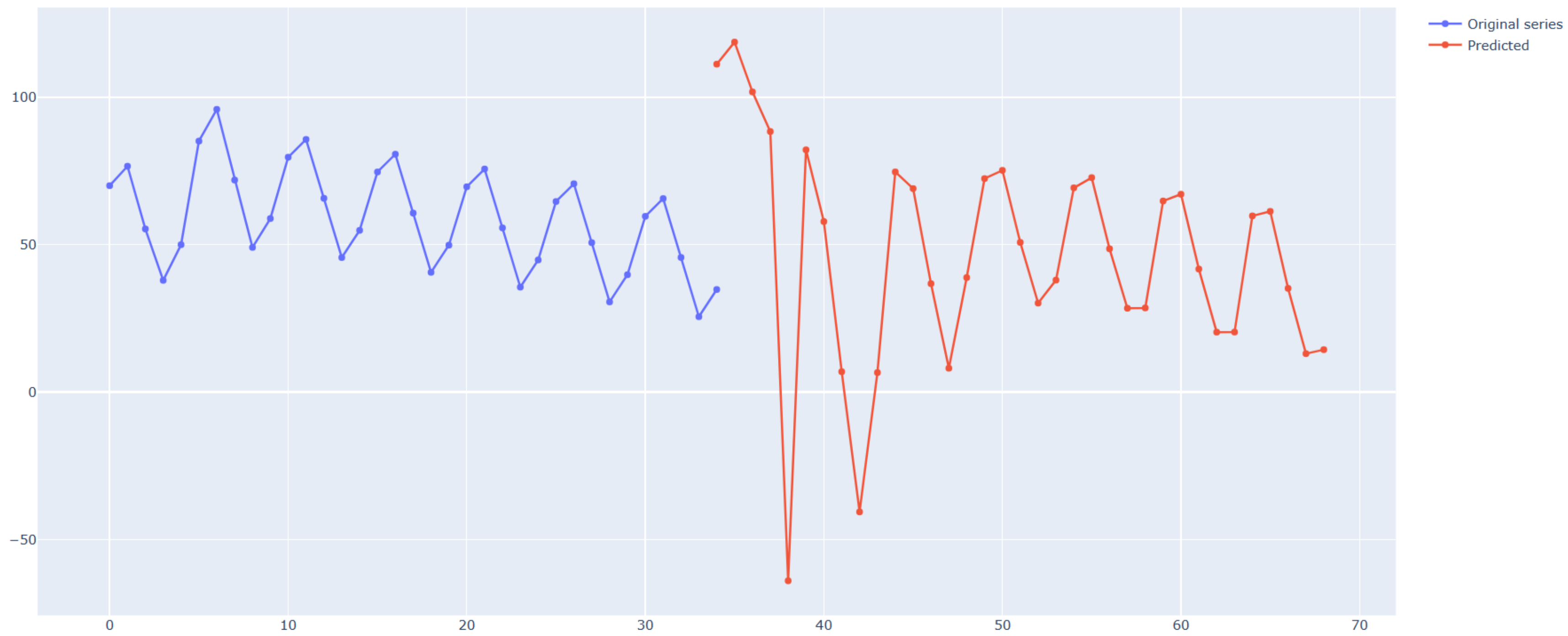
$$\text{(Forecast for period } m\text{) } F_{t+m} = (L_t + m{*}b_t) * S_{t+m-s}$$

The level smoothing equation is used to capture the long-term average of the series, the trend smoothing equation is used to capture the change in the series over time, and the seasonal smoothing equation is used to capture the seasonal fluctuations in the series.

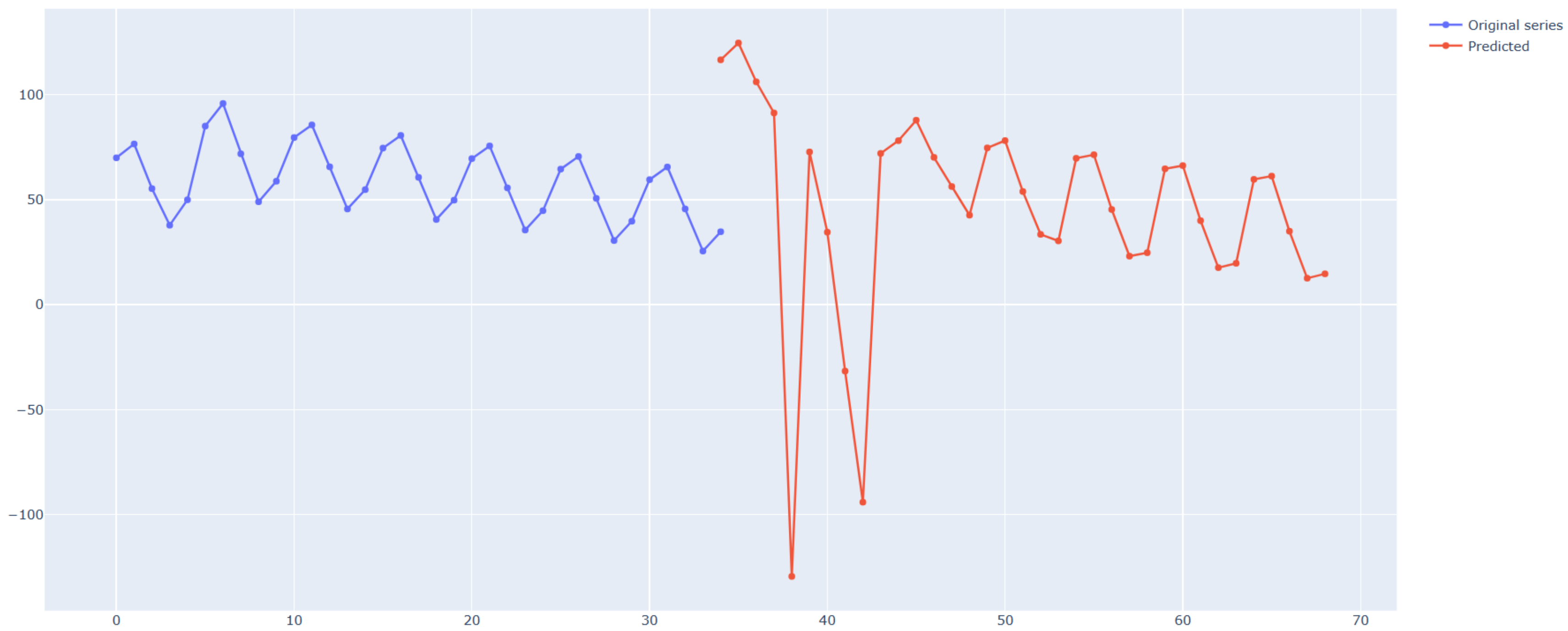
Example: This would be an appropriate modelling for a task such as, say, predicting the water level of a lake or a reservoir. Assuming that the expansion and contraction of the water due to heat is significant, it might result in higher variations exhibited in the other components. It would be appropriate to say in that case, that the Level and Trend term could model something similar to, say, the mass of water in the reservoir, and the seasonality term could represent the expansion and contraction of water with the daily variation in termperature.

Since Holt Winter's Forecast is a fairly complex method, it requires a large amount of historical data in order to obtain accurate forecasts and the dependence on the initial values taken is very high.

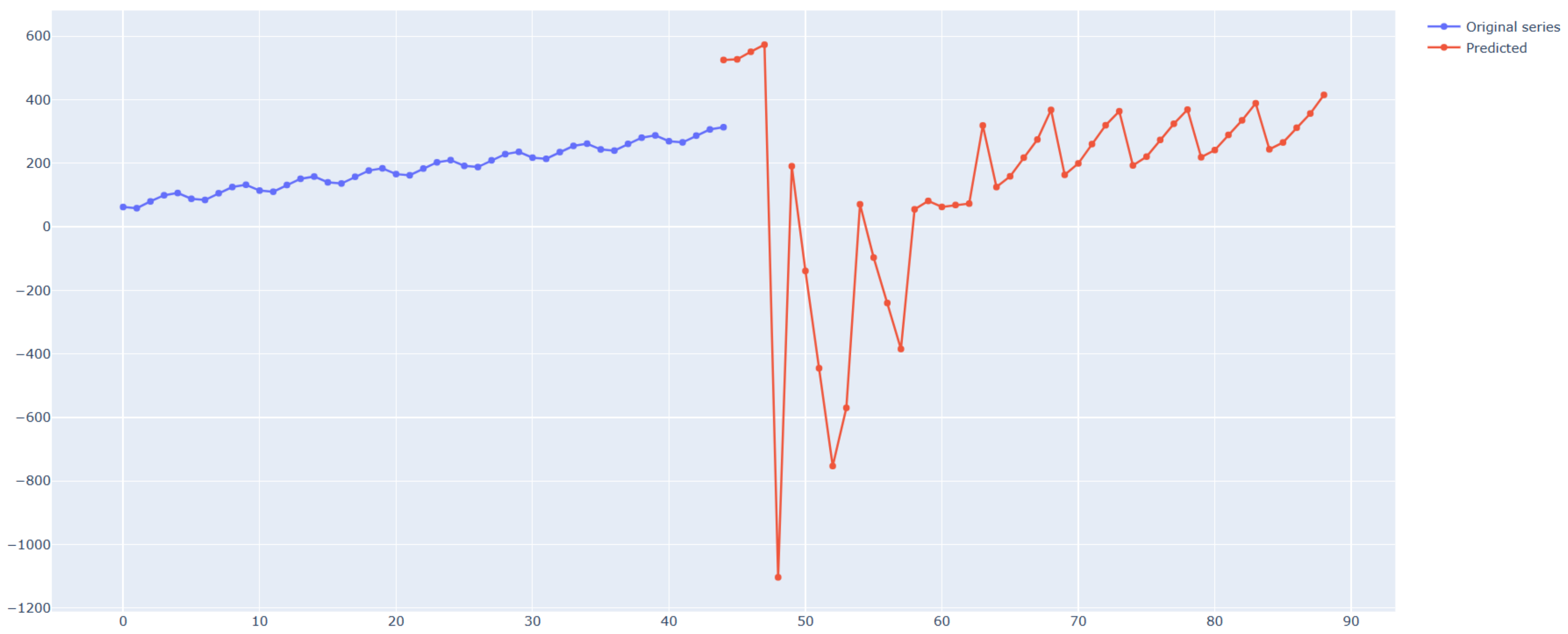
The predictions on the given sample in `tests.py` using Holt Winter's forecast method is shown below. Evidently, the model is very very good at capturing the trend and level of the series. It also takes some time to form the seasonality estimates however it ends up regressing to the a series shape that looks somewhat correct, albeit a bit skewed in the pattern because of the high dependence of the exponentially moving average on the more recent values of the time series.

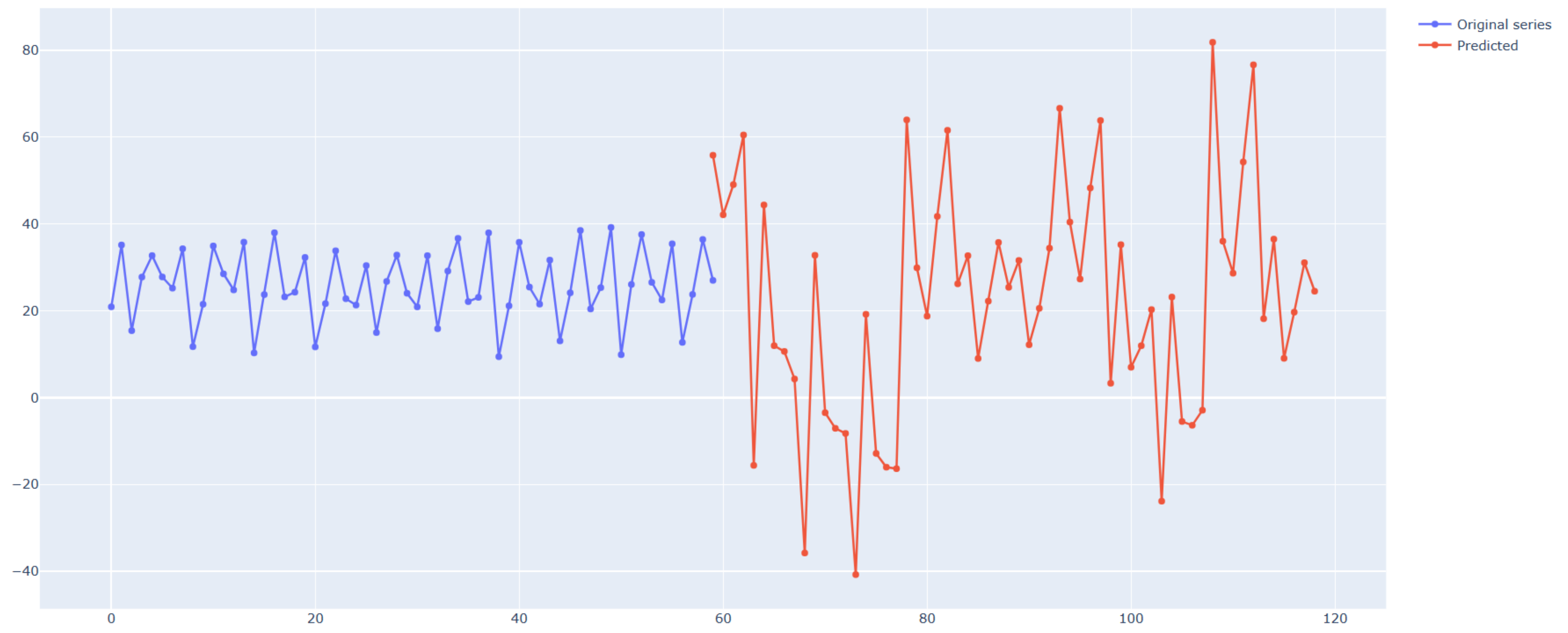


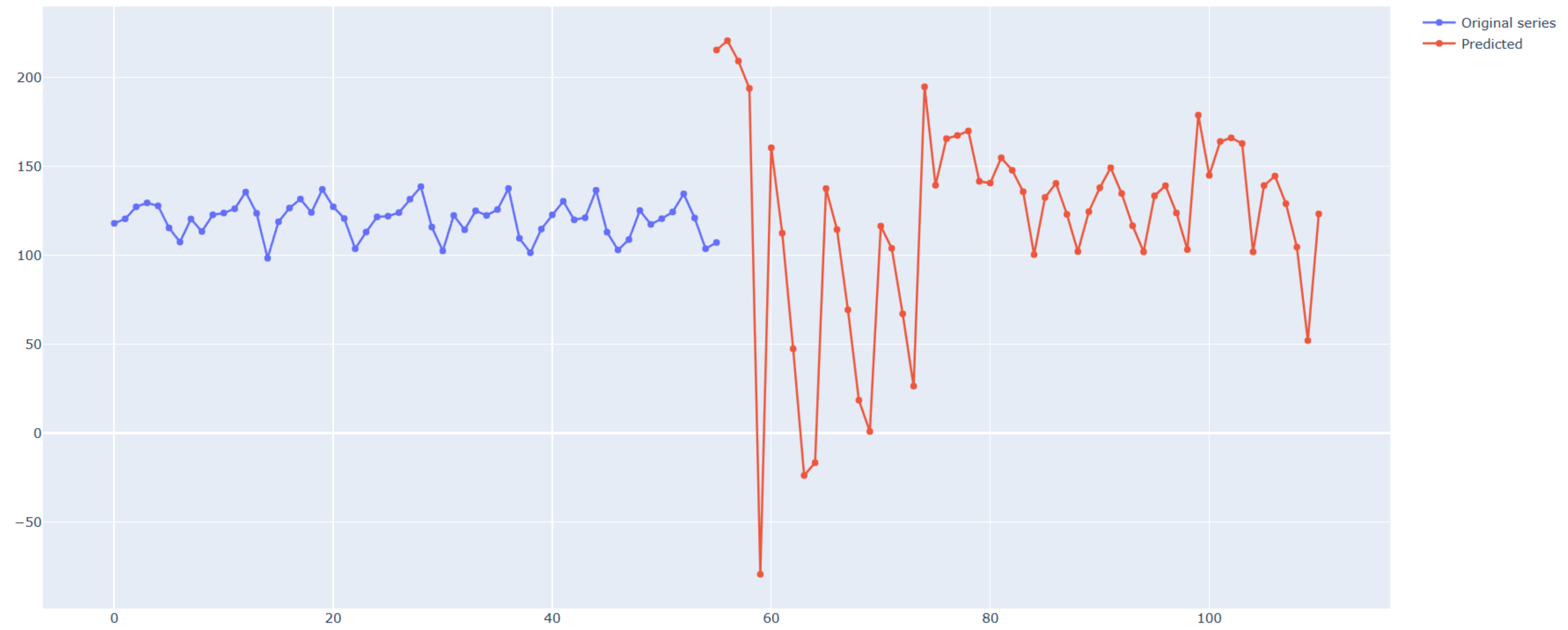
Considering the problem I stated above and my hypothesis about it, I tried increasing the values of the parameters (α, β, γ). The result is given below. It can be visibly seen that the prediction gets better and fits the trend better as well.

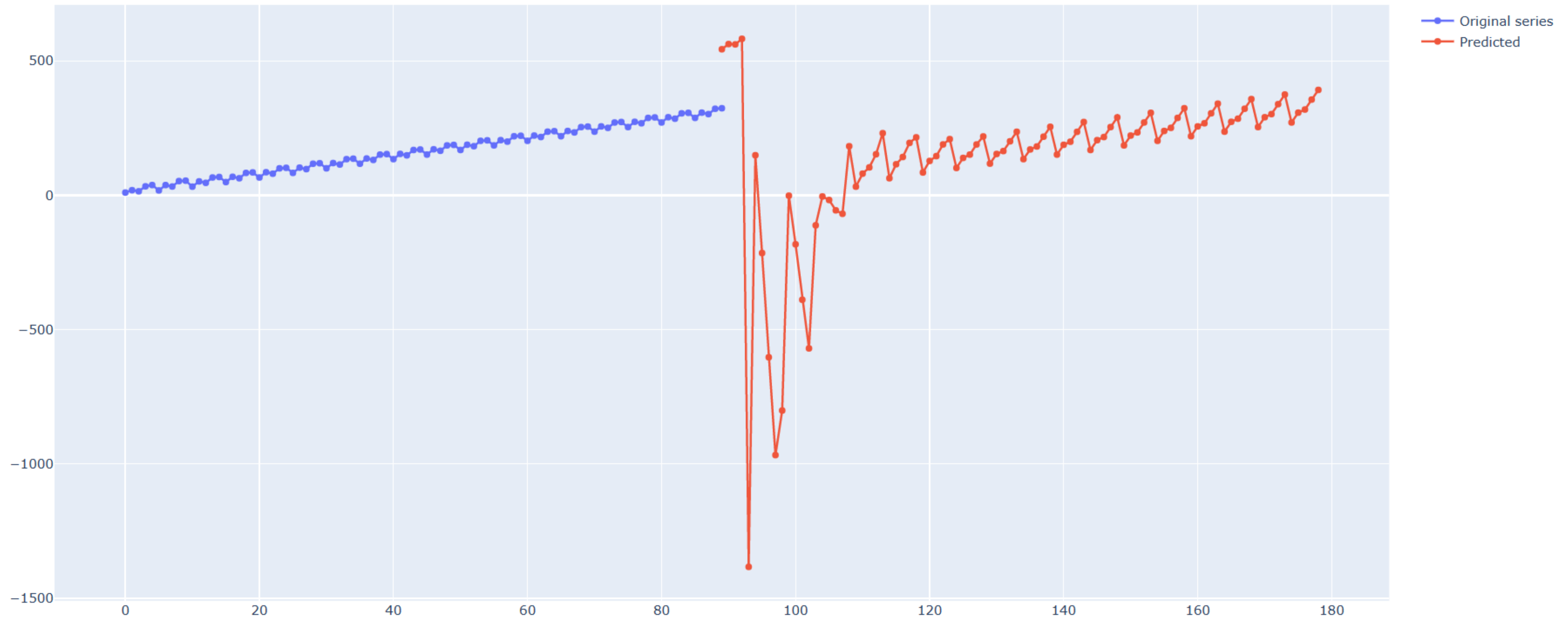


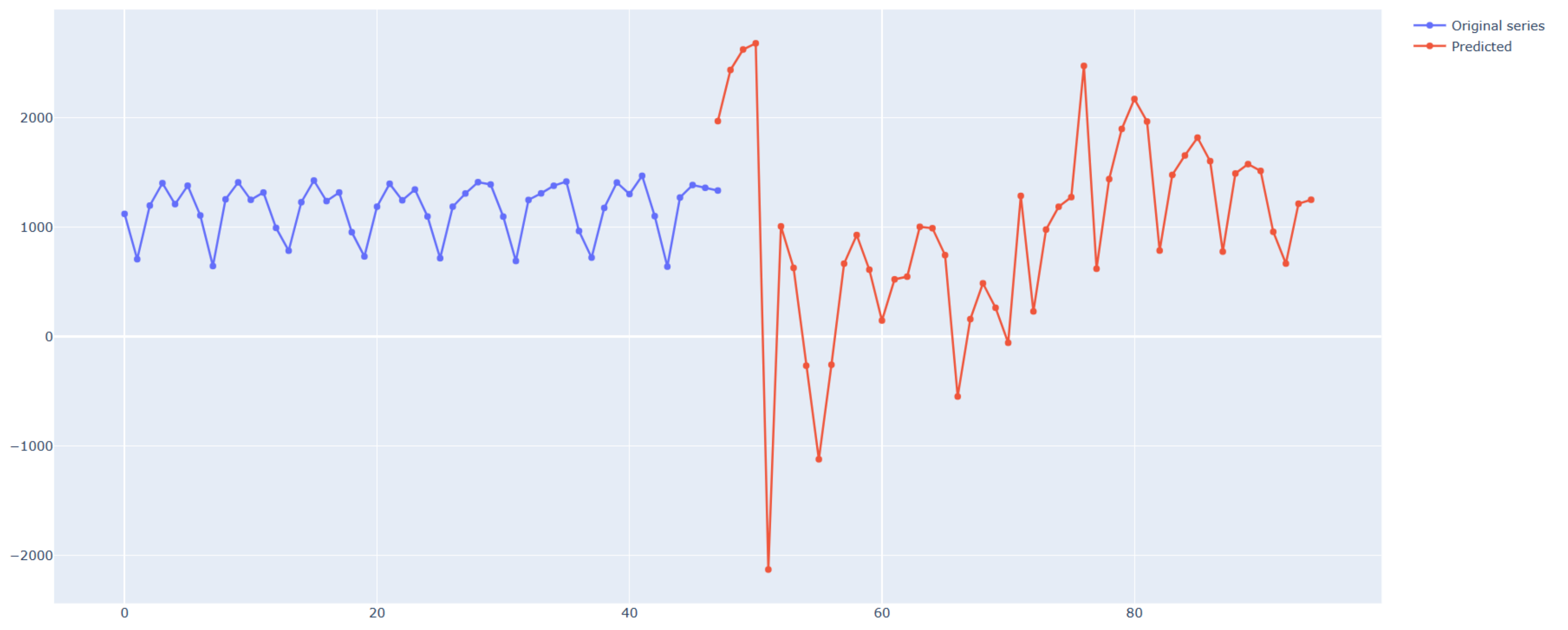
Next, I will show some plots for the remaining time series.




Original series
Predicted


Original series
Predicted


Original series
Predicted



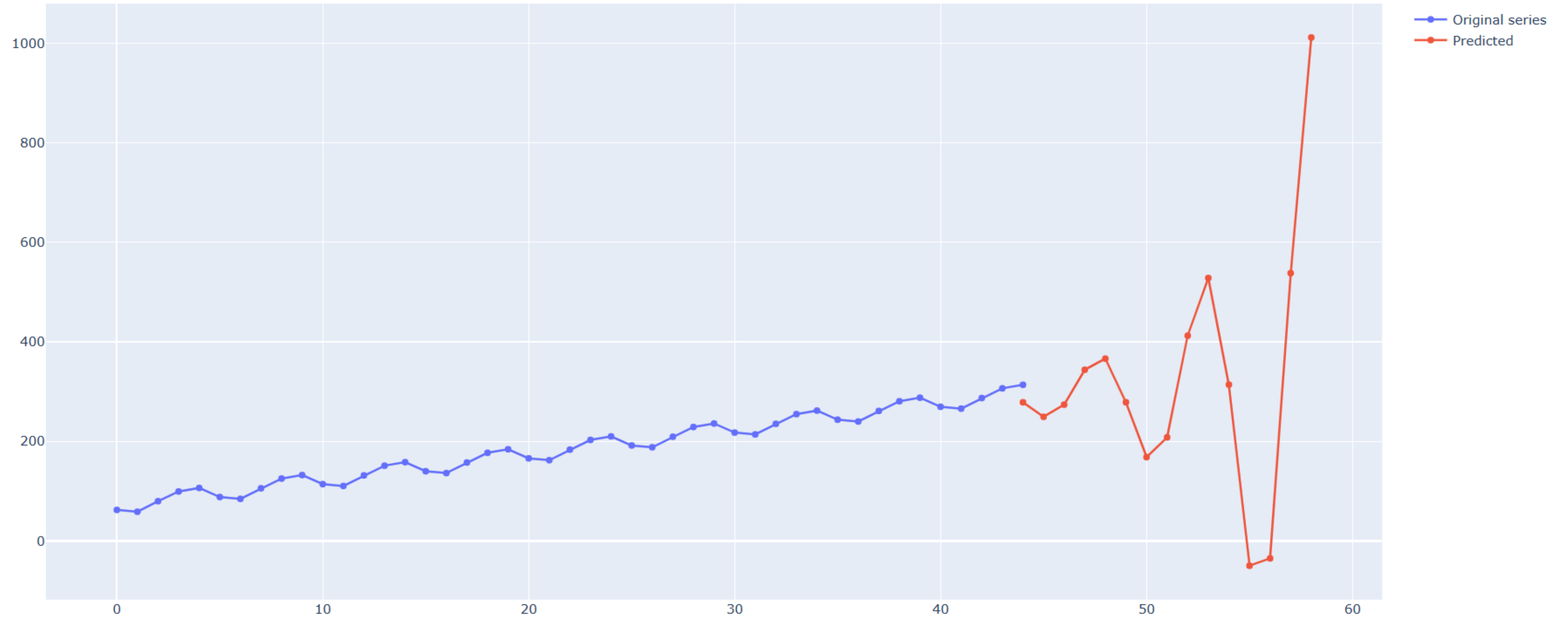
Overall, it can be seen very easily that the model has a difficult time autoregressing to a time series of relatively short length. It is generally noticed that the longer a time series is, the better its predictions are with the holt-winter model as it has more time to forget its initial estimate of the parameters and to learn the appropriate ones.

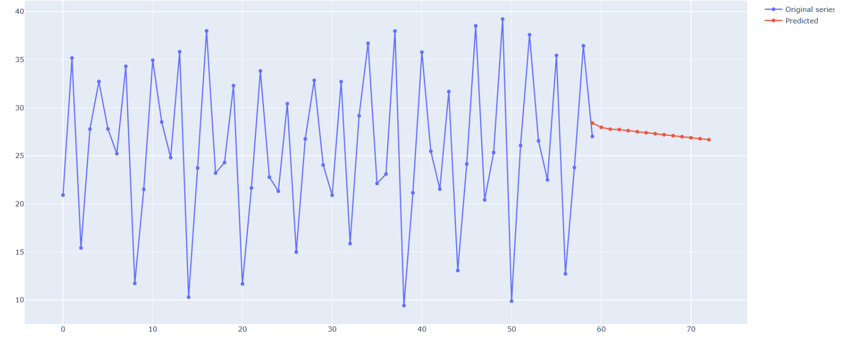
Part 3.

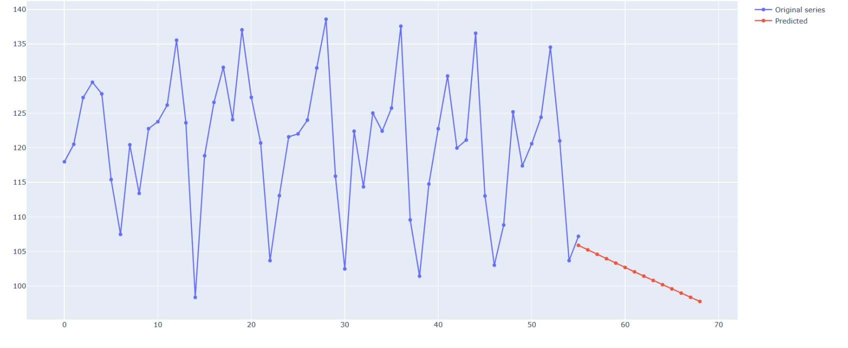
I have modified the function call to the following for this part: `ARIMA_Forecast=forecasting.ARIMA_Forecast(ARIMA_Sample,*forecasting.ARIMA_Paramters(ARIMA_Sample),15)` and the modification to the HoltWinter algorithm function call is similar to this. The resulting plots are modelled on models that are fitted using the values of the hyperparameters predicted by the functions instead of fixed values.

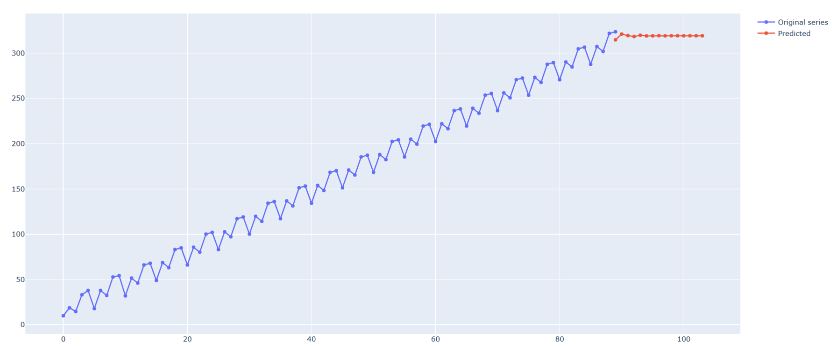
ARIMA

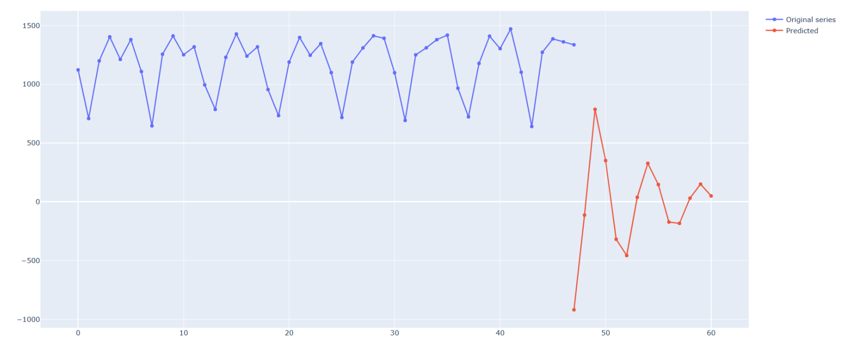
Overall, from the results below for the ARIMA model using the parameters predicted by `ARIMA_Paramters`, It is visible that the model does indeed learn the seasonality based variations in the data. However it is generallly veyr bad at capturing the trend and level. This may be because the appropriate depth is not being searched in the model.











Holt Winter

The Holt Winter model fares a little better. The `HoltWinter_Parameters` function involves a grid-serach over some predefined values to be searched, for the values of the hyperparameters $\alpha, \beta,$ and γ. The model subsequently takes a lot of time to be fitted, and the results are slightly better than that of the ARIMA model.
The model is able to, atleast for the initial few predictions, capture the variations in the trend and the seasonality of the data and although it is a little off-centre, it is able to predict the level decently too. However, the performance degrades quickly for unkown reasons. All in all, the Holt Winter model is able to capture valuable correlations and patterns in the time series and display them.